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Soliton Management for Ultra-high Speed Telecommunications

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Manejo de solitones para telecomunicaciones de alta velocidad

Resumen. *La metodología desarrollada provee un método sistemático de encontrar un número infinito de las novedosas islas solitónicas ("soliton islands") estables, brillantes y obscuras en un mar de olas solitarias, para la ecuación no lineal de Schrödinger con dispersión y no linealidad variables y con ganancia o absorción. Se muestra que los solitones existen sólo bajo ciertas condiciones y las funciones paramétricas que describen la dispersión, la no linealidad, la ganacia o absorción no homógenea, no pueden ser electas independientemente. Se han descubierto los regímenes de manejo fundamental solitónico para comunicaciones a velocidades ultra-rápidas a través de fibras ópticas.*

Palabras clave: *telecomunicaciones en fibras ópticas, solitones.*

Abstract. *The methodology developed provides for a systematic way to find an infinite number of the novel stable bright and dark "soliton islands" in a "sea of solitary waves" of the non linear Schrödinger equation model with varying dispersion, non linearity and gain or absorption. It is shown that solitons exist only under certain conditions and the parameter functions describing dispersion, non linearity and gain or absorption inhomogeneities cannot be chosen independently. Fundamental soliton management regimes for ultra-high speed fiber optics telecommunications are discovered.*

Keywords: *fiber optics telecommunications, solitons.*

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Introduction

The soliton –a solitary wave with the properties of a moving particle– is a fundamental object of nature. Solitons of the non linear Schrödinger equation model (NLSE solitons) appear in many branches of modern science including physics and applied mathematics, non linear quantum field theory, condensed matter and plasma physics, nonlinear optics and quantum electronics, fluid mechanics, theory of turbulence and phase transitions, biophysics, and star formation.

The current state-of-the-art in this very active field is reviewed, for instance [1, 2]. Characteristic properties of NLSE solitons include a localized wave form that is retained upon interaction with other solitons, giving them a "particle-like" quality. The theory of NLSE solitons was developed for the first time in 1971 by Zakharov and Shabad [3]. Over the years, there have been many significant contributions to the development of the NLSE soliton theory (see, for example, [1-9] and references therein). After predictions of the possibility of the existence [10] and experimental discovery by Mollenauer, Stolen and Gordon [11], today, NLSE optical solitons are regarded as the natural data bits and as an important

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alternative for the next generation of ultra-high speed optical telecommunication systems [12-17].
Ultra-high speed optical communication is attracting word-wide interests as the 21st century infra-structure for computer based information age. Most agrees that optical soliton will play major role as the means of transmission, however, it has not yet demonstrated decisive merit over linear transmission scheme because of it's intrinsic problems. The quasi-soliton concept is developed to overcome these difficulties and to demonstrate decisively its merit as the mean of information carrier of expected terabit/second ultra-high speed network.

The problem of soliton management in the non linear systems described by the NLSE model with varying coefficients is a new and important one (see, for example, the review of optical soliton dispersion management principles and research as it currently stands in [18-23] and references therein). Soliton interaction in optical telecommunication lines has attracted considerable attention in view of its effect on achievable bit rates.

Dispersion managed soliton technique now is the most powerful technique in the fiber optics communication systems. The 40 Gbit/s - 1000 km soliton transmission test was realized recently [23] in which Nakazawa group from NTT Network Innovation Laboratories, Japan, used the part of the Tokyo metropolitan optical loop network.

In this Report we show that methodology based on the quasi-soliton concept, provides for a systematic way to discover a novel stable soliton management regimes for the non linear Schrödinger equation (NLSE) model with varying dispersion, non linearity and gain or absorption. Quasi-soliton solutions for this model must be of rather general character than canonical solitons of standard NLSE, because the generalized model takes into account arbitrary variations of group velocity dispersion, non linearity and gain or absorption.

I. Novel soliton solutions for non linear Schrödinger equation model

Our starting point is the NLSE model with varying coefficients:

$$i\frac{\partial q^{\pm}}{\partial Z} \pm \frac{1}{2} D_2(Z)\frac{\partial^2 q^{\pm}}{\partial T^2} + N_2(Z)\left|q^{\pm}\right|^2 q = i\Gamma(Z)q^{\pm} \qquad (1)$$

NLSE (1) is written here in standard soliton units, as they are commonly known. There it is assumed that the perturbations to the dispersion parameter $D_2(Z)$, non linearity $N_2(Z)$ and to the amplification or absorption coefficient $\Gamma(Z)$ are not limited to the regime where they are smooth and small. Due to the well known spatial-temporal analogy [3] both temporal and spatial solitons are described by (1). In the case of temporal solitons T is the non dimentional time in the retarded frame associated with the group velocity of wave packets at a particular optical carrier frequency. In the case of two-dimensional spatial solitons T=X represents a transverse coordinate.

Theorem 1

Consider the NLSE model Eq. (1) with varying dispersion, nonlinearity and gain or absorption. Suppose that the Wronskian $W\{N_2, D_2\}$ of the functions $N_2(Z)$ and $D_2(Z)$ is nonvanishing, the two functions $N_2(Z)$ and $D_2(Z)$ are thus linearly independent. There are then an infinite number of a solitary wave solutions for the Eq. (1) written in the following form:

$$q^{\pm}(Z,T) = \sqrt{\frac{D_2(Z)}{N_2(Z)}} P(Z)Q^{\pm}[P(Z)T]\exp\left[\pm i\frac{P(Z)}{2}T^2 + i\int_0^Z K^{\pm}(Z')dZ'\right] \qquad (2)$$

where the real function $Q^{+}(S)$ describes canonical functional form of bright (sign= +1, $Q^{+}(S)= \eta\,\mathrm{sech}(\eta P(Z)T)$) or dark (sign= -1, $Q^{-}(S)= \eta\tanh(\eta P(Z)T)$) NLSE solitons [1,2,3], and the real functions $D_2(Z)$, $N_2(Z)$, $\Gamma(Z)$ and $P(Z)$ satisfy the equation system:

$$\frac{\partial P(Z)}{\partial Z} + P^2(Z)D_2(Z) = 0;\ \frac{W\{N_2(Z), D_2(Z)\}}{D_2(Z), N_2(Z)} - D_2(Z)P(Z) = 2\Gamma(Z) \qquad (3)$$

Theorem 2

Consider the NLSE model (1) with varying dispersion, nonlinearity and gain or absorption. Suppose that the Wronskian $W\{N_2, D_2\}$ is vanishing, the two functions $N_2(Z)$ and $D_2(Z)$ are thus linearly dependent. There are then an infinite number conserving the pulse area solitary wave solutions for the Eq. (1):

$$q^{\pm}(Z,T) = CP(Z)Q^{\pm}[P(Z)T]\exp\left[\pm i\frac{P(Z)}{2}T^2 + i\int_0^Z K^{\pm}(Z')dZ'\right] \qquad (4)$$

where the real functions $Q\pm(S)$ describe a canonical form of bright ($Q^{+}(S)$) or dark ($Q^{-}(S)$) NLSE solitons, and the real functions $P(Z)$, $D_2(Z)$, $N_2(Z)$ and $\Gamma(Z)$ satisfy the equation system:

$$2\Gamma(Z) = \frac{1}{P}\frac{\partial P(Z)}{\partial Z};\ C^2 N_2(Z) = D_2(Z) = -\frac{1}{P^2(Z)}\frac{\partial P(Z)}{\partial Z} \qquad (5)$$

The explicit solutions for the travelling solitary waves can easily be constructed by applying the Galilei transformation and by using the equation for the "soliton center" $T_{sol}(Z)$ given by

$$\frac{\partial T_{sol}(Z)}{\partial Z} = -VD_2(Z) \tag{6}$$

where V is a soliton group velocity (in the case of spatial soliton V=tanθ, and θ is the angle of propagation in the X-Z plane).

The phase function $K^+(Z)$ for the bright soliton solution of Eq. (1) is given by

$$K^+(Z) = \frac{1}{2}\eta^2 D_2(Z)P^2(Z)$$

and the correspondent phase function $K^-(Z)$ for the dark soliton solution is represented by

$$K^-(Z) = \eta^2 D_2(Z)P^2(Z)$$

By applying Theorems 1 and 2 we develope a systematic analytical approach to find the fundamental set of the different NLSE solitons management regimes.

Case 1. Soliton dispersion management. In this case the dispersion management function $D_2(Z)$ is assumed to be given: $D_2(Z)=\Phi(Z)$ (wecall it control function here). The function $\Phi(Z)$ is required only to be a once-differentiable and once integrable, but otherwise arbitrary function, there are no restrictions. There are then an infinite number of solutions for the Eq. (1) of the form of bright and dark dispersion managed solitons represented by the Eq. (2), where the main functions P(Z) and $\Gamma(Z)$ are given by:

$$P(Z) = -\frac{1}{\left[C - \int \Phi(Z)dZ\right]};\ \Gamma(Z) = \frac{1}{2}\frac{\partial}{\partial Z}\ln\left(\left|\frac{P(Z)\Phi(Z)}{N_2(Z)}\right|\right) \tag{7}$$

In the limit of $N(Z)$=const Eq. (7) reduces to:

$$\Gamma(Z) = \frac{1}{2}\frac{\Phi(Z)}{\left[C - \int \Phi(Z)dZ\right]} + \frac{1}{2}\frac{1}{\Phi(Z)}\frac{\partial \Phi(Z)}{\partial Z} \tag{8}$$

where C is the constant of integration.

Case 2. Soliton energy control. In this case the soliton energy control function $E(Z)=2D_2(Z)P(Z)/N_2(Z)$ is assumed to be given. The function $E(Z)$ is required only to be a once-differentiable and once integrable, but otherwise arbitrary function, there are no restrictions. There are then an infinite number of solutions for the Eq. (1) of the form of bright and dark solitons represented by the Eq. (2), where the main functions $D_2(Z)$, $P(Z)$ and $\Gamma(Z)$ are given by:

$$D_2(Z) = \frac{E(Z)N_2(Z)}{2P(Z)};\ 2\Gamma(Z) = \frac{\partial}{\partial Z}\ln(E(Z)/2) \tag{9}$$

$$P(Z) = \pm\exp\left[-\frac{1}{2}\int E(Z)N_2(Z)dZ + C\right] \tag{10}$$

Case 3. Soliton intensity management. In this case the soliton pulse intensity (peak power) is assumed to be controlled by the function $\Theta(Z)=D_2(Z)P^2(Z)/N_2(Z)$, where the control function $\Theta(Z)$ is required only to be a once-differentiable and once integrable. There are then an infinite number of solutions for the Eq. (1) of the form of bright and dark solitons represented by the Eq. (2), where the main functions, $D_2(Z)$, $P(Z)$ and $\Gamma(Z)$ are given by quadratures:

$$D_2(Z) = \frac{\Theta(Z)}{\left[C - \int\Theta(Z)dZ\right]^2};\ \mathrm{P}(Z) = -\int\Theta(Z)dZ + C$$
$$2\Gamma(Z) = \frac{\Theta(Z)}{\left[C - \int\Theta(Z)dZ\right]} + \frac{1}{\Theta(Z)}\frac{\partial\Theta(Z)}{\partial Z} \tag{11}$$

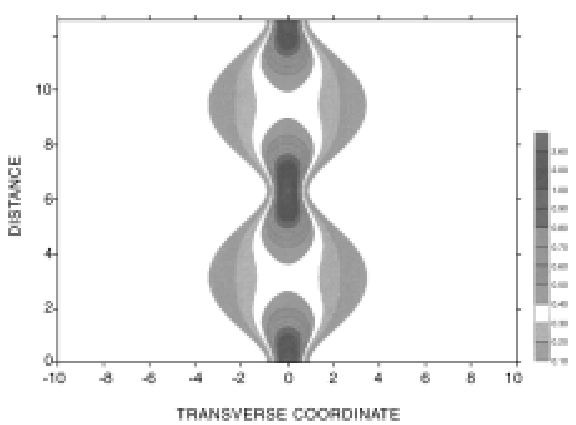
and the non linearity is assumed to be a constant ($N_2(Z)$° 1).

Case 4. Soliton pulse width management and the problem of optimal soliton compression. In this case the soliton pulse width control function is assumed to be given: $\Upsilon=P^{-1}(Z)$ The real function $\Upsilon(Z)$ is required only to be a twice-differentiable, but otherwise arbitrary function, there are no restrictions. There are then an infinite number of solutions for the Eq. (1) of the form of bright and dark solitons represented by the Eq. (2), where the main coefficients of the NLSE model $D_2(Z)$ and $\Gamma(Z)$ are given by:

$$D_2 = \frac{\partial\Upsilon}{\partial Z};\ 2\Gamma = -\frac{1}{\Upsilon}\frac{\partial}{\partial Z} + \left(\frac{\partial\Upsilon}{\partial Z}\right)^{-1}\frac{\partial^2\Upsilon}{\partial Z^2} \tag{12}$$

Case 5. Soliton amplification management and the problem of optimal soliton amplification. In this case the gain (or loss) function $\Gamma(Z)$ is assumed to be given: $\Gamma(Z)=\Lambda(Z)$. The gain control function $\Lambda(Z)$ is required only to be once

FIGURE 1. CONTOURS PLOT FOR THE EVOLUTION OF THE DISPERSION MANAGED BRIGHT SOLITARY WAVES (EQS. 2, AND 18-19) AS A FUNCTION OF THE PROPAGATION DISTANCE. INITIAL CONDITIONS: $m=1;\ k=8;\ \partial=0.5$ AND $C=10^4$.



integrable. There are then an infinite number of solutions for the Eq. (1) of the form of bright and dark solitons represented by the Eq. (2), where the main functions $D_2(Z)$ and $P(Z)$ are given by quadratures:

$$|P(Z)||D_2(Z)| = \exp\left[\int 2\Lambda(Z)\,dZ + C_1\right] \tag{13}$$

$$|D_2| = \exp\left\{\int\left[2\Lambda(Z) \pm |P(Z)||D_2(Z)|\right]dZ + C_2\right\} \tag{14}$$

where the integration constants $C_{1,2}$ are determined by initial conditions.

Case 6. Combined non linear and dispersion soliton management regimes. In this case the Wronskian $W\{N_2, D_2\}$ is assuming to be vanishing, that means the non linearity and dispersion are linearly dependent functions. The main feature of soliton solutions given by Theorem 2 consists in the fact that the soliton pulse area is conserved during propagation. Suppose that the dispersion management function $D_2(Z)$ is determined by the known control function $D_2(Z) = \Xi(Z)$, where the function $\Xi(Z)$ is required only to be a once integrable. There are then an infinite number of solutions for the Eq. (1) of the form of bright and dark conserving pulse area dispersion managed solitons represented by the Eq. (4), where the main functions $D_2(Z)$, $P(Z)$, $N_2(Z)$ and $\Gamma(Z)$ are given by quadratures:

$$P(Z) = -\frac{1}{\left[C - \int \Xi(Z)\,dZ\right]};\quad N_2(Z) = D_2(Z)/C \tag{15}$$

$$2\Gamma(Z) = \frac{\Xi(Z)}{\left[C - \int \Xi(Z)\,dZ\right]} \tag{16}$$

The interested reader can take different control functions $\Phi(Z)$ (Eqs. 7-8); E(Z) (Eqs. 9-10); $\Theta(Z)$ (Eqs. 11); $\Upsilon(Z)$ (Eqs. 12); $\Lambda(Z)$ (Eqs.13-14) and $\Xi(Z)$ (Eqs. 15-16) to find the novel "soliton islands" in a "sea of solitary waves" for the NLSE model (1) by using algorithms developed in this work.

Notice, that soliton solutions exist only under certain conditions and the parameter functions $D(Z)$, $R(Z)$ and $\Gamma(Z)$ cannot be chosen independently (see Eqs. 3,4 and Eqs. 5.6).

Examples. Let us consider some examples. The fundamental set of dispersion managed (DM) solutions can be expressed in trigonometric and hyperbolic functions. Assume that dispersion coefficient of the NLSE model (1) is a periodically varying control function:

$$D(Z) = \Phi(Z) = 1 + \delta \sin^m kZ \tag{17}$$

Then an infinite number of the DM-soliton solutions are given by the Eqs.(2) and (8). Integration in (8) is elementary for any value of the parameter m, and in the simplest case (m = 1) is given by:

$$P(Z)^{-1} = -\left[C - Z + \delta\cos(kZ)/k\right] \tag{18}$$

$$2\Gamma(Z) = -\Phi P + \Phi^{-1}\,\delta k \cos kZ \tag{19}$$

Let us consider periodical soliton dispersion management regimes in the case of Theorem 2. The main feature of soliton solutions given by Theorem 2 consists in the fact that the soliton pulse area is conserved. Assume the dispersion inhomogeneity $D(Z)$ to be a potential barrier, for instance, of the cos or sin functional form. Then the combined nonlinear and dispersion management regime is given by:

$$D = \cos Z;\ P = -(C - \sin Z)^{-1};\ 2\Gamma = -PD \tag{20}$$

where arbitrary constant $|C| > 1$.

Let us consider the soliton pulse width management regimes. One of the simplest periodical soliton solution is given by:

$$P(Z) = \Theta(Z) = -(1 + \delta \sin^2 Z);\quad 2\Gamma = -PD \tag{21}$$

$$D(Z) = \partial(1 + \partial \sin^2 Z)^{-2} \sin 2Z \tag{22}$$

The main features of analytical solutions predicted (Theorem 1 and 2) have been investigated by using direct computer simulations. Their soliton-like features have been proved in our computer simulations with the accuracy as high as 10^{-9}. The time-space evolution of DM-solitons for the case represented by the Theorem 1 (Eqs. 2 and 18-19) is shown in figure 1. The time-space dynamics of the propagation and interaction of DM solitons for the case of the Theorem 2 (Eqs. 4 and 21) is shown in figures 2 and 3. An important feature of the solitary waves solutions given by Eqs. 2-5 consists of the elastical character of their interaction. We also have investigated the nonlinear dynamics of high-order solitons generation in the frame of the *NLSE* model (1). Computer experiments show periodical time-space evolution of the bound state of two -solitons and represents the decay of this bound state produced by self-induced Raman soliton scattering effect which have been considered within the framework of the oscillator model [24]. This remarkable fact also emphasizes the full soliton features of solutions discussed. They not only interact elastically but they can form bound states and these bound states split under perturbations.

FIGURE 2. Contours plot for the evolution of the DM bright solitary waves (Eqs.4, and 21) as a function of the propagation distance (DM-soliton "snake" effect).

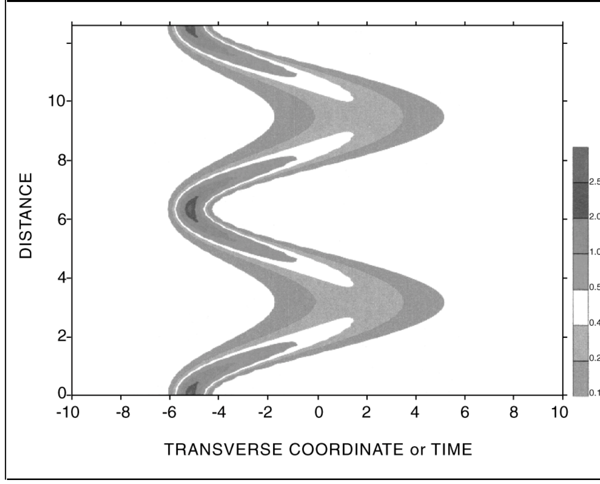
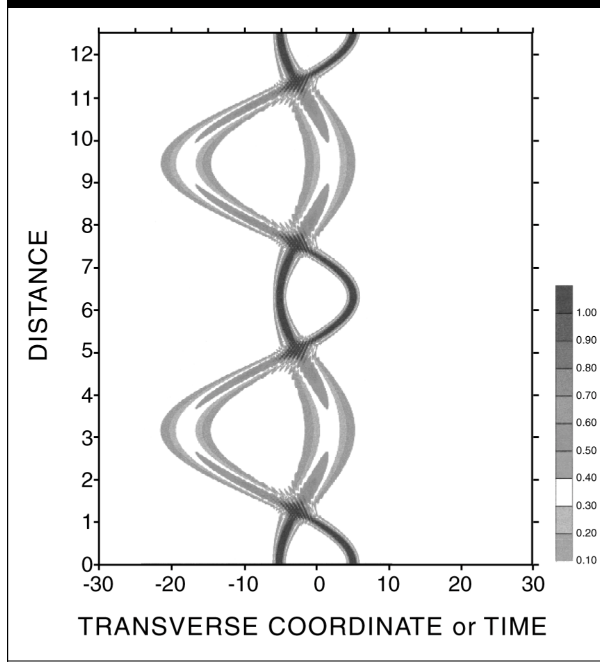


FIGURE 3. Contours plot for the nonlinear trapping of two DM-soliton pulses in the periodically dispersion managed structure (21). Initial conditions: $\Gamma(Z=0)=0.5; \partial=0.9$ and $V_{1,2}(Z=0)=\pm 10.0$.

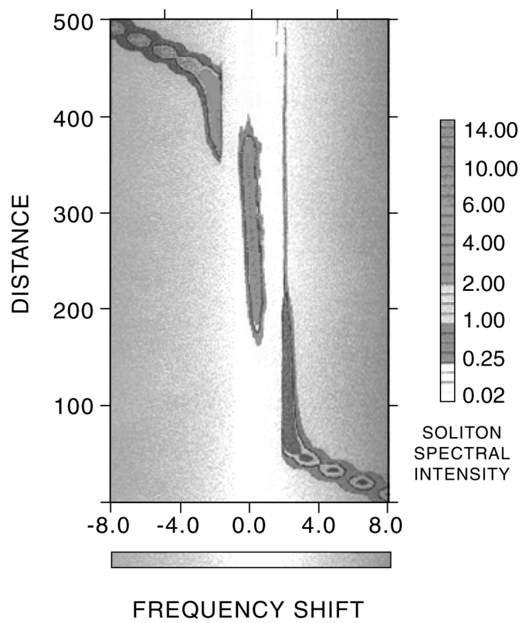


Conclusions

In sumary, the methodology developed (theorems 1 and 2) provides for systematic way to discover and investigate an infinite number of the novel solitary waves for the NLSE model with varying dispersion, non linearity and gain or absorption. The surprising aspect is that analytical solutions are obtained here in quadratures. Their pure soliton-like features are confirmed by the accurate direct computer simulations.

Finally, let us consider an example of the dynamic dispersion soliton management technique. It is the well known fact that due to the Raman self-scattering effect [25](called soliton self-frequency shift [26] the central femtosecond soliton frequency shifts to the red spectral region and the so-called colored solitons are generated [27]. This effect decreases significantly the efficiency of the resonant soliton amplification in the femtosecond time duration range. The mathematical model is based on the modified *NLSE* including the effects of molecular vibrations and soliton amplification processes (see details in [24]). As numerical experiments showed the dispersion inhomogeneity in the spectral domain allows one to capture a soliton in a sort of spectral trap. As soliton approaches the dispersion well, it has got into the well and is trapped as shown in figure 4. There exists a long time of soliton trapping in internal region of the spectral dispersion well (see figure 4). This effect opens a controlled possibility to increase the energy of a soliton. As follows from our computer

FIGURE 4. OPTICAL SOLITON MANAGEMENT EFFECT IN THE SPECTRAL DOMAIN: CAPTURE OF THE SOLITON SPECTRA BY A DISPERSIVE TRAP FORMED IN AN AMPLIFYING FIBER.



simulations the capture of a moving in the frequency space femtosecond colored soliton by a dispersive trap formed in an amplifying optical fiber makes it possible to accumulate an additional energy and to reduce significantly the soliton pulse duration. This effect can be considered as the spectral soliton management regime.

The results obtained in this Letter are of general physics interest and should be readily experimentally verified. The finding of a new mathematical algorithm to discover solitary wave solutions in non linear dispersive systems with spatial parameters variations is important to the field, and might have significant impact on future research.

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