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Mathematical knowledge in early school grades: advanced mathematics from an elementary point of view?

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ABSTRACT. This text aims to discuss the mathematics of early school years from a historical perspective. Supported on recent research results derived from projects implemented through national and international cooperation, the work to be published analyzes the difficulties of conducting historical studies about the presence of mathematics in the initial training of students. Said difficulties include the way to treat the types of mathematical teaching in the form of subjects rather than discipline-like contents. This implies a special attention to the articulation that occurs between pedagogies and pieces of knowledge to be taught. Additionally, this case requires the analysis of the cultural purposes in each school time, and of the role of mathematical knowledge. Among the findings of the study, there is an evident watershed with the emergence of modern pedagogies in the last decades of the 19th century and first decades of the 20th century, and the redefinition of mathematical basics.

Keywords: elementary knowledge, primary school mathematics, history of mathematical education.

Saberes escolares matemáticos nos primeiros anos escolares: a matemática superior de um ponto de vista elementar?

RESUMO. O texto tem por objetivo problematizar a matemática dos primeiros anos escolares em perspectiva histórica. Apoiado em recentes resultados de pesquisa vindos de projetos realizados em cooperação nacional e internacional, o trabalho a ser divulgado analisa as dificuldades de realização de estudos históricos sobre a presença da matemática na formação inicial dos alunos. Dentre essas dificuldades, destaca-se o modo de tratar os ensinos matemáticos sobre a forma de matérias e não conteúdos disciplinarizados. Isso implica especial atenção para a articulação que se dá entre pedagogias e saberes a serem ensinados. E, neste caso, impõe-se a análise das finalidades culturais, em cada tempo escolar, do papel do conhecimento matemático. Dentre os resultados do estudo, evidencia-se um divisor de águas com a emergência das pedagogias modernas nas últimas décadas do século XIX e primeiras do século XX, e a redefinição do elementar em matemática.

Palavras-chave: saber elementar, ensino primário de matemática, história da educação matemática.

Saberes escolares matemáticos en los primeros años escolares: ¿la matemática superior de un ponto de vista elementar?

RESUMEN. El texto tiene por objetivo problematizar la matemática de los primeros años escolares en una perspectiva histórica. Apoyado en recientes resultados de investigación oriundos de proyectos realizados en cooperación nacional e internacional, este trabajo analiza las dificultades de realización de estudios históricos sobre la presencia de la matemática en la formación inicial de los alumnos. Entre estas dificultades, se destaca el modo de tratar las enseñanzas matemáticas sobre la forma de asignaturas y no contenidos disciplinarizados. Esto implica especial atención para la articulación que ocurre entre pedagogías y saberes a ser enseñados. Y, en este caso, se impone el análisis de las finalidades culturales, en cada tiempo escolar, del papel del conocimiento matemático. Entre los resultados del estudio, se evidencia un punto de inflexión con la urgencia de las pedagogías modernas en las últimas décadas del siglo XIX y primeras del siglo XX, y la redefinición de lo elementar en matemática.

Palabras clave: saber elementar, enseñanza primario de matemática, historia de la educación matemática.

Introduction

The aim of this article is to problematize the characterization of what we know as 'elementary knowledge'; more specifically, it analyzes, what meaning is given to elementary mathematics over time.

The text is built as to defend and support this thesis: the different pedagogies that are established in

different historical periods change the notion of what is elementary for mathematics teaching. To defend said thesis, this work will analyze three distinct eras that comprehend the intuitive, Progressive Education and New Maths Mouvement, evidencing the diverse manner in which each one of them characterizes what we could call 'mathematical basics'. On the other hand, the analysis of said 'basics' in each pedagogical time will evidence the transformations of the mathematics to be taught in early school grades.

The development path of this study will go through broader issues that will allow answering, finally, the following guiding question of this investigation: what transformations have occurred in the mathematics of primary school that was subject to pedagogical waves of late 19th and 20th centuries?

To begin with, in this process of formulation of broad questions that, in the course of the text, with the expansion of data and characterization of contexts, will become more and more precise, a first interrogation proves worthy: what transformations has the mathematics of first grades suffered?

The historical analysis needs a temporal delimitation. Thus, the last decades of the 19th century will be considered herein – age when the intuitive teaching pedagogical wave was introduced, with studies by Rui Barbosa (1882); the analysis moves further to the moment of expansion of mandatory schooling, in early 1970s, with the 1971 Guidelines and Bases for National Education Law, LDB No. 5692. In this way, the question is brought again, now a bit more precisely, though broadly, but in accordance with this Introduction: what changes has the mathematics to be taught in the primary school suffered between 1882 and 1971?

The emergence of modern pedagogies and the types of mathematics teaching

The analyses of the transformations that took place regarding mathematics in the first school grades seem to present a watershed with the emergence of modern pedagogies from the late 19th century. Such pedagogies refer to the intuitive teaching/lessons of things and, later, to the movement known as Progressive School; for mathematics, it seems, in a somewhat different way from that of other school rubrics, there is also a real revolution occurred by the end of the 1950s, with what was called New Maths Movement.

In the first case – that of the intuitive pedagogy/object lessons – Rui Barbosa is the protagonist. He will be the one who, by manifesting opinions in 1882, will reference demands for changes in the education of the first school grades, with the circulation of proposals for the intuitive method, objectified in the object lessons[1].

The installation of the Republic seeks to establish a new way to treat education in Brazil. According to the education historian Maria Cecília Cortez de Souza,

> The monumental report and the set of opinions by Rui Barbosa, proposing a reform in primary education, prepared in 1882, served as a guide, a source and a diagnosis for a good portion of the republicans that were, in a way or another, concerned with the public instruction (SOUZA, 1998, p. 83).

The main point of Rui Barbosa's writings touches the teaching method. They declare a true war against the mechanical processes of repetition through memorization. The work exalts the need for fighting this tradition. Rui Barbosa is actually characterized as one of the builders of the representation that we call up to present days 'traditional school': a methodical school, centered on the teacher, with an evaluation apparatus that imposes intense memorization.

Having built this representation of teaching that must be overcome, Rui Barbosa seeks to bring the modern intuitive pedagogy to combat this traditional past[2]. This overtaking means the diffusion of the lessons of things.

In the late 1920s, the Progressive Education stands up, in its different nuances and proposals, including the 'scientific pedagogy' – a pedagogy that borrows from experimental psychology its processes and methods, resorting to statistics for reference to the standardization of pedagogical contents and evaluation. Since Alfred Binet[3], at least, such pedagogy is disseminated among mathematicians and mathematics teachers[4]:

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[1]The 'objetct lessons way through which the intuitive teaching method has been vulgarized, are actually the first form of intuition – the sensitive intuition. The term was popularized by Mme. Pape-Carpentier and officially employed during her conferences addressed to teachers present at the International Exposition of 1867. Pestalozzi is also pointed as reference of this method for having identified the main points of the pedagogical renewal that the lessons preconized "[...] things before words; education for things and not education for words". [...] Its diffusion in late 19th century generated the production of a large number of school manuals for teaching lessons of things, including: *Primary Object Lessons*, by Norman allison Calkins, originally published in the United States, in 1861, and translated by Rui Barbosa in 1886.

[2]A deeper study on the period, its proposals, didactical material, etc, for mathematics teaching in the first school grades can be read in the text by Valente (2011).

[3]Alfred Binet was born in 1857, in Nice, France. His formation comprises several studies. Around 1880, he started to dedicate himself to psychological studies. In 1886, he publishes 'La psychologie du raisonnemnet'. He heads Sorbonne's psychophysiology research laboratory. He develops, along with Théodore Simon, scales to measure intelligence, elaborating the concept of mental age. In 1905, he introduces the 'Metric Intelligence Scale'. According to Almeida (2010, p. 30), "[...] the golden period of Binet's reception in Brazil comprehends between 1906 and 1919; therefore, between the creation of the first Pedagogical Psychology Laboratory, idealized by himself, and the translation by Lourenço Filho of the 'Tests for Measuring the Intelligence Development in Children'".

[4]The creation of the CIEM 'Commission Internationale de l´Enseignement Mathématique / IMUK – Internationale Mathematische Unterrichtskommission' is the beginning of concerns about the teaching of mathematics, bringing it into

> The terms 'scientific pedagogy, experimental pedagogy' and 'pedology' are employed as synonyms nowadays to designate an absolutely new movement of researches that have been produced for the past few years in the pedagogical world. [...] The new pedagogy differs above all from the former one for the important place it reserves to observation and to experience; it seeks to substitute 'a priori' affirmations with accurate results and numbers. This revolution, if successful, will be nothing but a logical consequence of what has been happening to psychology, and what is about to happen to all sciences judged moral, in which we see the nonstop-talking period being replaced by the observations period (BINET, 1899, p. 29-30, our translation).

Among us, one of the most prominent representatives of this pedagogy was Lourenço Filho. This teacher had important teaching heading positions, wrote books and formulated didactical-pedagogical proposals, including for mathematics teaching[5].

Finally, in this very brief retrospective that evokes the mathematics taught in the first school grades in different pedagogical times, it is worth mentioning the creation of study groups aimed at the transformation of the mathematics curriculum since primary education from the 1960s, with highlight to the pioneer *Grupo de Estudos do Ensino de Matemática* [Study Groups for Mathematics Teaching] – G.E.E.M, under the coordination of teacher Osvaldo Sangiorgi[6].

We have mentioned the watershed… Indeed, the emergence of these new pedagogies, at the counterpoint to what they would call 'traditional pedagogy' themselves puts the study of the transformation of school knowledge and, in particular, of mathematics in primary education on a different level of analysis.

These pedagogical waves, taking into account studies by the researcher André Chervel, shall be analyzed in a way different from that is normally attributed to pedagogies: a restrict role of 'lubricant' (words of the author himself) in the teaching process. Thus, a true epistemological revolution in the form of analyzing school contents is brought by Chervel. The theme arises when the author approaches relations between science, pedagogy and school subjects. The common conception that exists about school knowledge hangs on a classic manner of perceiving pedagogy: a facilitating ingredient that acts on the contents produced by the scientific community, in order to vulgarize science for children and adolescents. It would be a methodology, a didactics, ways to teach contents in order to enable their acquisition by students. According to this common view, there is on one side the scientific knowledge and, on the other side, the methods. In short, sciences detached from pedagogy.

However, André Chervel's work breaks with this perspective as it makes the following observations:

> To exclude pedagogy from the study of contents is to condemn oneself to comprehend nothing about the actual operation of teaching. Pedagogy, far from being a lubricant spread over the mechanism, is nothing but an element of said mechanism; that which transforms teaching into knowledge (CHERVEL, 1990, p. 182).

Based on the above considerations, it is possible to elaborate questions relating to changes in the mathematics present in the first school years, having in mind that pedagogical waves play a primary role in the transformations of school knowledge. Once again, the question that guides this study can be resumed, now written as follows: what transformations has the mathematics taught in the first school grades suffered, subject to pedagogical waves, during the period from 1882 to 1971?

Pedagogies, cultural purposes and types of mathematics teaching

Studying the pieces of knowledge present in primary education – those that are part of the formation of students in the first school years – from a historical perspective is not an easy task, as it necessarily means a need for learning the ambitions of an era, in terms of the culture that one intends to explore, and the actions of the school, in this context, in the conflicts of these pretensions of a society taken in a broader manner, with the school culture. In the technical words of the very same André Chervel, it will be necessary to analyze the 'goal purposes' and 'real purposes' dyad. Goal purposes express the wishes of a school time, of a proposal socially accepted in a certain age, with a culture to be transmitted. Real purposes refer to those that were actually part of the school routine, of a moment in the history of teaching, of the elements that give a meaning to the practices of the school, the school culture, etc (CHERVEL, 1990).

Perhaps, this first level of teaching – the initial school grades, the primary education, a school modality existing until late 1960s; the first years of

debate in the heart of the international community of mathematicians, from 1908. However, it is necessary to be attentive to the existence of the 'L'Enseignement Mathématique' maganize, which came before the creation of that Commission. Moreover, it is this journal, right in the first issue, of 1899, that brings the dialogue between Alfred Binet and people interested in the teaching of mathematics.

[5]Recent studies by Bassinello (2014) and Soares (2014) analyze the production by Lourenço Filho regarding the teaching of mathematics in the first years of school.

[6]Several works have already been developed on the New Maths Movement theme. An important reference on the matter is the collective work written by approximately 45 researchers, 'O Movimento da Matemática Moderna – história de uma revolução curricular' [The New Maths Movement – History of a Curricular Revolution], organized by Oliveira et al. (2011). In said work, a special chapter is dedicated to new maths for children.

elementary school, from late 1970s – is the place where the connection of pedagogies with the subjects to be taught is more evident. Unlike more advanced grades, in which, progressively, teaching contents are endowed with a relative autonomy over pedagogies in the first years, reaffirming Chervel, pedagogy is an element intrinsic to the contents themselves.

There is then a huge difficulty: how to study, in particular, a piece of knowledge present in the schools, as mathematics, without separating it from pedagogy? This difficulty may be expressed, in a more systematized manner, in the condition of thinking about the mathematics present in early school years as a 'subject', rather than a 'school discipline'. In the analysis of different teaching subjects, thus, since basic education, there is the triad: 'reading, writing and counting'. In a miscellaneous of pedagogies and specific pieces knowledge, subjects are historically produced. Thus, the triad has been chosen a long time ago as representative of the fundamental subjects to be taught in the initial school grades. 'Counting', for instance, should be thought as a miscellaneous resulting from the encounter between the real purposes (how the school sees mathematics) and the goal purposes (how society wants mathematics in the school), in a time when certain pedagogical ideas prevail. In this text, 'counting' presents itself as the first expression of what herein is being called 'mathematics in early school grades'.

Thus, let us go back to the question left earlier, seeking to better make it explicit before these last considerations: What path has mathematics taken in the first school grades midst the intuitive, Progressive Education and structuralist pedagogies as a New Maths Mouvement?

Modern pedagogies, teaching subjects and elementary mathematics

The teaching subjects, in each time, when it comes to initial school years, are always considered as the first steps, the elements from a '~~superior~~ advanced culture' that should be taught and needs to be acquired by the individual. Said acquisition, in a given historical context, will be 'based on' the school, or, progressively, will take place *in* the school itself. Let us explain: in times when mandatory school comprehended four years – basic education – teaching should provide those who participated in it with tools for life and, with them, they could better acquire other pieces of knowledge in the real world. Thus, for that time, the progression to a broader culture, for most of the population, only happens outside the school[7]. Later, with the expansion of mandatory schooling, such perspective changes, and the teachings of the first years were then considered, to a good extent, as bases to the following years. The goal purposes of teaching change and, certainly, the real ones as well, with the creation of the eight-year elementary school[8]. Thus, the 'counting' that was thought as useful for the practical purposes of life begins to be important for the acquisition of a more systematized knowledge of numbers, that is, the 'arithmetic' that progressively and in an increasingly deeper manner will be part of post-primary course years.

In any way, teaching subjects are always regarded, as it has been said, as elementary pieces of knowledge, the foundations of knowledge, in this case, the first elements of mathematical knowledge.

Resuming the idea previously exposed, modern pedagogies will constitute a watershed, as already said, because they will deal differently with the conception of elementary knowledge. In other words, when the pedagogies change, so does the concept of elementary. This is the most important meaning in analyzing the transformation of school knowledge – elementary knowledge – in face of new pedagogies. Thus, the guiding question of this work takes on a new configuration: How do the intuitive, the Progressive Education and the structuralist pedagogies, as a New Maths Movement characterize the elements of mathematics teaching – the 'mathematical basics'? The analysis of these *basics* in each pedagogical time will provide an answer to the transformations of the mathematics of the first grades.

Mathematical basics in different pedagogies

Let us begin from the beginning… The historical origin of the work 'element' is found in the Latin term 'elementum', which derives from the Greek term 'stoikheîon'. The Greek word means: 'that which is aligne', 'in a line', 'in a sequence'. Employed in its plural form, *stoikheîa* designates the characters of writing, precisely the letters of the alphabet, arranged in a sequence. From this initial sense, the term acquires the meaning of 'principles' or "[…] fundamental elements […]" (TROUVÉ,

[7]The text of the 1946 Organic Law for Primary Education is emblematic when mentioning, among the purposes of this teaching level, in its Article 1: "[…] to promote a cultural initiation that leads all to the knowledge of the national life and to the exercise of the moral and civic virtues that sustain and exalt it, within an elevated spirit of human naturalness" (LEI ..., 1946).

[8]An important study that analyzes this transition to mathematics teaching in early school years is the thesis by Denise Medina (2012) titled *Do primário ao primeiro grau: as transformações da Matemática nas orientações das Secretarias de Educação de São Paulo (1961- 1979) e o conceito de número* [From Basic Education to Elementary School: The Transformations of Mathematics in the Orientations of São Paulo's Secretariats of Education (1961-1979) and the Concept of Number].

2008, p. 21). This analysis will be able to lead us, also, to the 'Euclides Elements': a chain of premises, theorems… And allows us, additionally, to think how much the idea of linearity in teaching is present in mathematics: the so-called pre-requisites. A given theme depends on the previous one in order to be understood and so on… However, let us stop by here for now.

Returning to the question that guides this text, we seek to analyze the transformation of elementary knowledge, in particular, of the mathematical basics, from late 19th century to early 20th century. In this way, the investigation is centered on changes in the basics. If that is so, each pedagogy is responsible for making explicit the subjects that should be taught in a certain school time, the elementary pieces of knowledge. As said before, there is the question about how each pedagogical wave characterizes the basics. It seems that the role of each trend in this characterization is heir of two big philosophical chains: rationalism and empiricism.

In the first case, it is possible to cite as an important representative the illuminist, philosopher, mathematician and political man engaged with the French Revolution, Condorcet. Such character elaborated a complete system of public instruction, integrating elementary levels of teaching into higher levels. In this system, the basics take a strategic and fundamental place: they encompass the beginning of a progression towards more advanced and superior knowledge. Condorcet, with an encyclopedial and systematized conception of knowledge, defines hierarchies and levels to be achieved in every schooling stage (TROUVÉ, 2008)

At this point, it is interesting to mention how Maria Montessori, in one of her first books[9], addresses this rationalist inheritance of thinking pedagogy – without directly citing it – mentioning how the proposal for the teaching of arithmetic and geometry was established in the first school grades:

> […] the teacher will begin teaching by the lines, by the angles or with the numbers, and the first problem will be, before anything else, learning what the simplest thing is; from then, teaching shall start. I remember discussions of wannabe teachers in a mathematics congress wondering what was simpler: to count the numbers in a natural succession (cardinal numbers) or to consider them according to the order and the position they appear in a reciprocal manner (ordinal numbers). Being the problem relating to the order of successive knowledge resolved, teaching is carried out; making the simplest thing to be comprehended first, and then chaining, consequently, the preceding with the following, by order of difficulty (complexity), moving from the unknown to the known (MONTESSORI, 1934 [2007], p. 14).

From this rationalist tradition, it is possible to say, concerning mathematics and the mathematical basics, that the latter refer to the first steps towards superior mathematics. Having found the simplest thing, one can, in a progression, move on to advanced contents. In times of a school aimed at reading, writing and counting, the subject 'counting', for instance, refers to the advanced mathematics from an elementary point of view[10]. Thus, by counting the student will have access to the superior knowledge of mathematics. Counting constitutes the basics of mathematics.

The new pedagogical theories arising from late 20th century, such as the intuitive pedagogy and the Progressive Education trend, have another inheritance: empiricism. And it is here where the watershed mentioned earlier can be found.

If, to Condorcet, the knowledge principle, the basics, is essentially epistemological and chained in a whole that must be gradually taught, to Pestalozzi, on the contrary, the basics are psychological and empirical (they refer to an existential and sensitive subject). To Pestalozzi, it is about leaving the existence of things to have access to the words, according to mechanisms deemed natural. Pestalozzi's basics reside in simple intuitions (TROUVÉ, 2008).

Pestalozzi's thinking, just as that of most modern pedagogues of his time, is inserted into the empiricist perspective. It considers that simple sensitive intuitions do not only represent an immediate data of consciousness, but are also a knowledge principle (TROUVÉ, 2008). From this empiricist tradition, of which Pestalozzi is a major representative, it could be said that the access to superior mathematics depends on the empirical basics, on the first sensitive forms. Here, the superior mathematics does not govern the basics, setting access levels to them.

A notable example is geometry. The 'shapes', for instance, are the first geometrical approaches in the intuitive pedagogy, constituting the basics. The latter, however, have another nature, different from that coming from the hierarchy of the mathematical knowledge, referring to a maximum simplification

[9]The Italian educator and physician Maria Montessori has written many books, including 'Scientific Pedagogy', 'Education for a New World', 'The Stages of Education'. 'Psychogeometry – The Study of Geometry Founded on the Child's Psychology' was published in 1934, in Barcelona, Spain, being one of her first written productions.

[10]Certainly, on purpose, we parodied the title of the book by the mathematician Félix Klein who, in the beginning of the 20th century, publishes a work titled 'Elementary Mathematics from an Advanced Standpoint', intended to teachers, working with elementary concepts in a mathematically sophisticated manner.

of the advanced mathematics. Such basics have their origin in the sensitive intuition of the shape (Pestalozzi)[11].

Likewise, the Progressive Education trend has an empiricist inheritance. The basics, in this case, also refer to the psychological individual, but not to his or her sensitive reception; it relates to his or her ways of acting about the things. From them, he or she derives the first steps towards knowledge. In this sense, the title itself of the work by Montessori, previously mentioned, is elucidative: 'Psychogeometry', a work whose objective "[…] is not to teach geometry, but to help the development of the child's mathematical spirit" (MONTESSORI, 1934 [2007], p. 6). In the same way, the subtitle of the book is elucidative as well: 'The study of the geometry founded on the child's psychology'.

The empiricist inheritance that marks the Progressive Education chains, especially that aimed at scientific pedagogy, reveals in a way other than that of the intuitive pedagogy. Its basics are grounded on the statistic-based experimental psychology. The first elements will come from the standardization of mental and pedagogical tests, in the formulation of minimum teaching programs (BASSINELLO, 2014).

The New Maths Movement, in the terms of the previous analysis, seems to resume the rationalist inheritance. From the perspective of structuralist mathematics, the tall mathematical building shall be constructed from basics taught in the first school years. The goal is to seek maximum simplification for the Set Theory so it can take the lower levels of teaching and serve as a basis to the definition of numbers, of the properties of operations, etc. 'Counting', in this case, seems not to have importance as basics anymore[12].

Final considerations

The historical investigation on mathematical education in the first school grades poses huge challenges to researchers. After all, how can one characterize mathematics historically in early school years? Calculus, Arithmetic, Geometry, Design, Shapes, Cartography, Manual Works, etc, a set of teaching subjects derived from pedagogies and pieces of knowledge that reveal the basics of mathematical knowledge. Carrying out the historical analysis, in this case, means a work with the set of those subjects, in addition to their articulation with historical processes of reading and writing… That is, a very hard task,

In any way, simplification does not fit pedagogy, just as the autonomy of pieces of knowledge in face of it. The pedagogical theories and their movements along with the pedagogical waves are ingredients that compose school subjects. In a large-scale historical analysis, it is possible to notice a constant redefinition of basics.

Finally, let us go back to the guiding question of this study: what transformations have occurred in the mathematics of the first school grades that was subject to the pedagogical waves of late 19th century and early 20th century?

The answer to the question identifies the transformations in the mathematics of the first years in terms of changes in that which is regarded as elementary mathematics, mathematical basics. In short, there is a pedagogical waves movement in the end of the 19th century that opposes to the rationalist tradition, characterizing the basics in a deductive manner – from the advanced mathematics, one deduces what its basics are. This contraposition generates a watershed in terms of how, traditionally, the elementary knowledge is thought. The context of modern pedagogies points to inductive processes: the experiences of individuals, organized and interpreted, will enable the constitution of the basics, taken as foundations to other learning types. This empiricist trend changes in early 20th century, taking a deductive character again: new maths points to what the new basics shall be in face of the acquisition of a knowledge taken by structuralism.

[11]Pestalozzi's sensitive intuitions will be systematized in the basics relative to 'number', 'language' and 'shape'. They will be considered as elements of all types of knowledge. These elements will first change the order of the school aimed at reading, writing and counting. It will be seen as the place for "[…] speaking, feeling and counting" (TROUVÉ, 2008, p. 240).

[12]See, as an illustration, the study that dealt with the concept of number in the different pedagogies in Valente (2012).

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