

A New (Non-andersonian) Attitude to Reducing the Deontic Modalities to a Combination of the Alethic Ones with the Constant “A Sanction”

(An Unknown Evaluation-Functional Alternative for the Modal Logic of Norms)

Vladimir O. Lobovikov*

Recepción: enero 7 de 2003

Aceptación: mayo 7 de 2003

* Universidad Estatal de los Montes Urales, Ekaterinburg, Rusia.

Director de *International Journal of Ural's Philosophy*.

Una nueva actitud (no andersoniana) para reducir las modalidades deonticas a una combinación de las aleticas con la constante “sanción”.

Resumen. El artículo expresa una nueva actitud hacia el problema de reducción de las modalidades deonticas a las aleticas. Se analizan las proposiciones originales de esta reducción. El algebra de acciones propuesta por el autor usa la imaginación lógica de modo indirecto (por analogía). La acción algebraica es un sistema evaluativo-funcional de caracter moral y legal. De este modo el texto representa un enfoque evaluativo-funcional hacia modalidades y sus interconexiones.

Palabras clave: aletic, modalidad equivalencia, obligación, sanción.

Abstract. This paper presents a new attitude to the problem of reducing the deontic modalities to the alethic ones. G. H. von Wright's and A. R. Anderson's original submissions of such reducing are analyzed. An algebra of actions submitted by the author uses the logic maginary indirectly (by analogy). The action algebra is a system of moral-legal evaluation-functions. Thus it represents an unknown evaluation-functional approach to the modalities and to their interconnection.

Key words: alethic, modality, equivalence, obligation, sanction.

According to A. R. Anderson (1967), the deontic modality ‘obligatory’ is adequately defined as follows: $Op \leftrightarrow \bullet (\sim p \rightarrow S)$. Here the symbol O stands for the deontic modality *obligatory*, \bullet stands for the alethic modality *necessary*, the letter p means a propositional variable, the signs \leftrightarrow , \sim , \rightarrow stand for the classical symbolic logic operations *equivalence*, *negation* and *implication*, respectively. The symbol S means a constant, which is named *a sanction*. The constant is interpreted as some negative consequence for the person who has violated the norm.

Anderson considers that if one adds the above mentioned definition of the concept *obligatory* to an appropriate system of alethic modal logic, then the result of this addition is an

adequate system of deontic logic. Thus the deontic modalities are reduced to the alethic ones combined with the constant called *a sanction*. Is this reducing the deontic modalities to the alethic ones realistic? From the one hand, yes, it is, because the inevitability (alethic necessity) of an adequate punishment is essential for any legal order.

However, on the other hand, no, it is not, because, in fact, there are such violations of norms, which do not result in punishing the violator. Hence, in fact, the correlation between an obligation and the sanction for violating this obligation is not the alethically necessary relationship. Therefore, in my opinion, Anderson's system of normative logic describes not the factual, but the desirable state of

affairs. As in real world it is possible that Op is true and $\bullet (\sim p \rightarrow S)$ is false, there is no basis for accepting the above given Andersonian definition of the notion ' p is obligatory'. Is it possible to save the rational grains of truth existing in the Andersonian approach under review? I think, yes. But how could we save them? For an adequate answering this question one should go into the very grounds of the logic paradigm within which the approach under consideration was developed.

In my opinion, the above reviewed theory of Anderson and the theory of normative logic elaborated by G. H. von Wright belong to one and the same logic paradigm in spite of the fact that in some aspects their theories are different. For better understanding the logic paradigm, which is common for the two, let us consider the normative logic theory of von Wright in comparison with the one of Anderson. In several papers and books von Wright investigated G. W. Leibniz's intuition (presented in 1672 in *Elementa juris naturalis*) concerning the essential structural-functional (formal) interconnection of the deontic (juridic) and the alethic (Aristotelian) modalities. In 1951 von Wright suggested an original system of non-classical modal logic, which is called the deontic one. From that moment the evident progress of the modal logic of norms is achieved in both the logical syntax and the logical semantics of artificial languages constructed for representing ideal norm systems and processes of normative reasoning. Step by step scientists came to the conviction that the deontic modal logic is the only true direction of investigating the above-mentioned intuition of Leibniz.

Therefore, they decided that the problem of adequate formalizing this intuition absolutely coincides with the problem of true success in constructing an adequate modal logic of norms. It is implied that the true success means a complete and precise representation and mathematical simulation of the intuition of Leibniz by means of some artificial language. Also it is implied that the success could (and should) be reached only within the framework of the subject matter of logic as such. This paradigm is tolerant to the expectation that the successful system would be a very non-

The creator of the modern deontic logic himself indicated some evident contradictions between the logic of norms and the classical (alethic) modal logic.

classical one, but it strongly insists that it should be a logic system. However, from the abstract methodology viewpoint, the following questions arise. Is this concrete direction of structural-functional explication of the idea of Leibniz the only possible one? Could one find or create a substantially new attitude to formalizing and interpreting Leibniz's statement about the existence of a fundamental formal identity (equivalence)

of the alethic and the deontic modalities?

If we accept the direction, suggested by von Wright, then we arrive to the following difficult problem. Leibniz wrote about a fundamental formal unity (complete identity of forms) of the corresponding deontic and alethic modalities. In contrast to him, von Wright wrote about an analogy (similarity), *i.e.* only partial correspondence (not complete identity of forms) of the two types of modalities (Wright, 1983).

The creator of the modern deontic logic himself indicated some evident contradictions between the logic of norms and the classical (alethic) modal logic. He convincingly demonstrated that there is no serious factual basis for affirming a formal equivalence (complete identity of forms) of corresponding alethic and deontic notions in modal logic (*ibid.*). Therefore he insisted that the formal correlation under investigation is nothing but a similarity (analogy). It is well known that the analogy (similarity) relation is not a transitive one and, consequently, it is not a relation of equivalence. Hence, it is necessary to recognize that there is some important contradiction between the intuition of Leibniz and the paradigm of von Wright. As we have got a contradiction, there is an

alternative: either we are to reject the intuition of Leibniz, or we are to reject the paradigm of von Wright. Nowadays one of these possibilities – accepting and elaborating the idea of von Wright – is investigated comparatively well, but the other part of the alternative is absolutely ignored, or, perhaps, scientists even do not see it at all. Let us investigate just this still not studied possibility. I believe that, generally speaking, von Wright's approach to explication of Leibniz's intuition is not the only possible one. Moreover, taking into



Table 1. Part 1.

a	Na	Wa	Ha	Oa	Fa	La	Za	Ua	Ja	Ta
g	b	b	b	g	b	g	b	b	g	g
b	g	g	g	b	g	b	g	g	b	b

Table 1. Part 2.

a	Sa	Da	Ba	Ma	Pa	Ia	Ya	Va	Xa	Ra
g	b	g	g	g	g	b	b	g	g	g
b	g	b	b	b	b	b	b	b	b	b

an account the above fixed contradiction, it is natural to suppose that, probably, von Wright's attitude is a rational one not in all relations. Perhaps, there is a specific relation, in which some other paradigm of formal analysis of the modalities is more adequate and fruitful. Moreover, perhaps, in this hypothetical specific relation Leibniz's affirming the fundamental unity under consideration could be represented as affirming not only a similarity (analogy) but an equivalence (complete identity).

The above hypothetical reasoning about the existence of such an unknown approach to formalizing the intuition of Leibniz, which is more adequate in comparison with the approach of von Wright, is based upon the fact that the contrary implies a logical contradiction. Consequently, this reasoning is not a *constructive* one. For *constructive* demonstration of its truth it is necessary to *construct* the above mentioned more adequate abstract theory using an artificial language of mathematics for representing forms of the modalities. In this paper an algebra of actions is defined and exposed in capacity of the hypothetical theory in question.

Let us introduce symbols of the action-algebra language used in this paper. The letters *a* and *c* mean (any) actions, *i.e.* (by definition) *free* operations possessing one of the two *moral-legal values*: either *g* (good), or *b* (bad). Symbols *Na*, *Wa*, *Ha*, *Ia*, *Fa*, *La*, *Za*, *Ua*, *Sa*, *Da*, *Ba*, *Ma*, *Pa*, *Ia*, *Ya*, *Va*, *Xa*, *Ra* (respectively) stand for unary moral-legal operations (evaluation functions): 'abstaining from *a*'; 'a resistance (handicap) to *a*'; 'non-being (absence) of *a*'; 'making *a* obligatory (commanding to perform *a*)'; 'making *a* forbidden (prohibiting *a*)'; 'a freedom for *a*'; 'a freedom from *a*'; 'a cognition of *a*'; 'making a secret of *a*'; 'doing *a* suddenly'; 'making *a* an alethically impossible operation'; 'making *a* an alethically necessary operation'; 'realizing *a*, *i.e.* making *a* an

alethically actual (really existing) operation'; 'possibility of *a*, *i.e.* making *a* an alethically possible operation'; 'a permission of *a*'; 'normative indifference to *a*'; 'making *a* an alethically accidental operation'; 'depriving (or loss) of *a*'; giving (or receiving) *a*'; 'rationalizing *a*, *i.e.* making *a* a rational operation'. In the algebra of actions the evaluation functional sense of the above mentioned moral legal operations is defined by the following evaluation table 1 consisting of two parts.

Let the symbol ' $a=+=c$ ' stand for the relation: 'an action possessing the moral-legal form *a* is *formally-ethically equivalent* to an action possessing the moral legal form *c*'. In the algebra under review, actions are called formally ethically equivalent if and only if their moral legal forms are formally ethically equivalent. By definition, a moral legal form *a* is called formally ethically equivalent to a moral legal form *c* if and only if these moral legal forms (*a* and *c*) acquire identical moral legal values (*g* or *b*) under any possible combination of moral legal values of the variables occurring in these forms. The role of moral legal variables is played by elementary moral legal forms, *i.e.* by moral legal forms of elementary actions deprived of their contents. The variables take their values from the set {*g b*}. Values of the moral-legal evaluation-functions belong to this set as well.¹

By means of the above said it is easy to demonstrate the following formal-ethical equivalences:

$$Oa = + = Da, \tag{1}$$

$$Fa = + = Sa, \tag{2}$$

$$Pa = + = Ma, \tag{3}$$

$$Ia = + = Ya. \tag{4}$$

The conjunction of the formal-axiological equations 1-4 may be called 'the formal-ethical principle of the *equivalence of corresponding deontic and alethic modalities*'. I consider that this principle is more relevant (fits much better) to the above discussed intuition of Leibniz, than the existing systems of deontic logic created by von Wright and his (numerous followers).

In addition to the equations 1-4 it is easy to prove the following formal-ethical equivalences:

$$Oa = + = Pa, \tag{5}$$

$$Oa = + = Ba, \tag{6}$$

$$Ba = + = Da, \tag{7}$$

$$Ma = + = Ba, \tag{8}$$

$$Ma = + = Da, \tag{9}$$

$$Ba = + = Ra. \tag{10}$$

1. The author of this paper constructed the algebra of actions and started investigating it in 70th of xx century. The book (Lobovikov, 1999) written by the author in English is a representative although not complete introduction to the system of basic notions of the algebra of actions.

The last equation may be called ‘the principle of Spinoza-Leibniz-Hegel’. Now let us indicate some formal-ethical equivalences concerning the two freedoms (for *a* and from *a*). Taking into an account the above said, it is easy to prove the following equations:

- $Sa = + = VLa;$ (11)
- $Da = + = VZa;$ (12)
- $La = + = HWa;$ (13)
- $Za = + = MNa;$ (14)
- $Za = + = HDa;$ (15)
- $Fa = + = VLa;$ (16)
- $Oa = + = VZa;$ (17)
- $Pa = + = La;$ (18)
- $Za = + = PNa;$ (19)
- $Sa = + = LaNa;$ (20)
- $Za = + = UDa.$ (21)

The last equation may be called ‘the principle of Spinoza-Hegel’.

Thinking about the above given equivalences one naturally gets an impression that they are paradoxical. However this impression is nothing but an intellectual illusion made up by the ambiguity of the language we use. In the natural language the words ‘is’, ‘means’, ‘implies’, ‘entails’ could stand for the relation ‘ $=+=$ ’.

Moreover, in my opinion, they were actually used just in this nonstandard meaning by B. Spinoza, G. W. Leibniz, G. W. F. Hegel and many others. (Although it is obvious, that these philosophers were not able precisely to formulate such an unusual meaning of the mentioned words). As in the natural language the same words ‘is’, ‘means’, ‘implies’, etc. also could stand for the logical operations ‘equivalence (\leftrightarrow)’ and ‘implication (\rightarrow)’, there is a real possibility of a confusion-absolute identifying and, hence, substituting for each other the substantially different notions ‘ $=+=$ ’ and ‘ \leftrightarrow ’ (or ‘ $=+=$ ’ and ‘ \rightarrow ’). Such mixing and substituting is strictly forbidden in the algebra of actions (Lobovikov, 1999). Ignoring this ban indispensably leads to paradoxical results. Let Ea stand for an act of informing (true or false affirming) that the action *a* takes place in reality. The above said may be formulated as the following rule (A-D). (A) From the truth of $a=+=c$ it does not follow logically that $Ea\leftrightarrow Ec$ is true. (C) From the truth of $Ea\leftrightarrow Ec$ it does not follow logically that $a=+=c$ is true. (C) From the truth

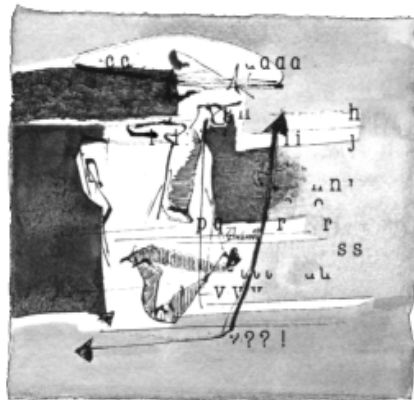


Table 2.

A	c	Kac	K ^o ac	A ^o ac	Aac	Gac	Cac	C ^f ac	C ^N ac	C ^E ac
G	g	g	b	b	g	g	g	g	g	g
G	b	b	b	g	g	b	g	b	b	b
B	g	b	g	g	g	b	b	g	b	b
B	b	b	b	b	b	g	g	b	g	b

of $a=+=c$ it does not follow logically that either $Ea\rightarrow Ec$ or $Ec\rightarrow Ea$ is true. (D) From the fact that either $Ea\rightarrow Ec$ or $Ec\rightarrow Ea$ is true, it does not follow logically that $a=+=c$ is true.

Now let us return to the discussion of Anderson’s approach to reducing the deontic modalities to the alethic ones. In my opinion, the theories of Anderson and von Wright belong to one and the same paradigm, namely, the one of modal logic. As we have shown that in some respect the algebra of actions is more powerful in explaining the juridical intuition of Leibniz, it is natural to expect that the action-algebra is also more powerful in explaining the juridical principle of the inevitability of punishment for a crime. At the very beginning of this paper it was shown that starting with Anderson’s formalization of this juridical principle, one arrives to a contradiction with empirical data.

Let us try to use the algebra of actions for making such an explication of the juridical principle of the inevitability of punishment for a crime, which does not lead to contradictions with facts. To do this we have to introduce some binary operations of the action-algebra, namely, Kac - ‘uniting *a* and *c* in a behavior’, K^oac - “switching from *a* to *c* (leaving *a* for *d*)”, A^oac - “exclusive choosing and performing the best (the most good or the least bad) action between *a* and *c*”, Aac - “non-exclusive choosing and performing the best (the most good or the least bad) action from the couple *a* and *c*”, Gac - “an exchange of actions *a* and *c*”, Cac - “performing *c* in classical response (reaction) to performing *a*”, C^fac - “performing *c* in L. Tolstoi’s response (reaction) to performing *a*”, C^Nac - “executing *c* in F. Nietzsche’s response (talion-reaction) to executing *a*”, C^Eac - “executing *c* in strange

response (unjust-equalizing-reaction) to executing *a*. In the algebra of actions the moral-legal evaluation-functional sense the above mentioned binary operations is defined by the following table (table 2).

I consider that by means of the above said (about the algebra of actions) it is possible now to suggest a substantially new (non-Andersonian) approach to reducing the deontic modalities to the alethic ones by virtue of using the

constant 'a sanction'. Instead of the above-discussed Anderson's definition of the term 'obligation' I suggest the following equi-valence: $Oa=+=DCHab$ (or $Oa=+=DCNab$). Here the letter a means a moral-legal variable of the action-algebra, but it is relevant to emphasize that in this paper the letter b stands for the moral-legal constant 'bad'. This *negative moral-legal constant* plays the role of 'a sanction' (a retribution) for performing Ha (or Na). By comparing the definition of Anderson with the suggestion of mine it is easy to see that the new approach developed by me is fundamentally analogous to the one of Anderson. However they are not absolutely identical.

From $Oa=+=DCHab$ it does not logically follow that $Op \leftrightarrow \bullet (\sim p \rightarrow S)$. Therefore the above-mentioned criticizing the definition $Op \leftrightarrow \bullet (\sim p \rightarrow S)$ as logically contradicting with empirical data of crimino-logy, has nothing to do with the equation $Oa=+=DCHab$.

The formal-ethical equivalence $Oa=+=DCHab$ deals not with the *real existence or non-existence* of operations, but with moral-

legal *values* of ones. It asserts nothing about facts of performing or non-performing relevant actions. Hence, it is impossible to refute $Oa=+=DCHab$ by showing that in real life very often the truth-values of propositions EOa and $EDCHab$ are different and, hence, generally speaking, the logical equivalence ($EOa \leftrightarrow EDCHab$) is false. It is impossible to falsify $Oa=+=DCHab$ by falsifying ($EOa \leftrightarrow EDCHab$) because the last does not logically follow from the first and, therefore, using the 'modus tollens' in this case is not relevant. ■

Bibliography

- Anderson, A. R. (1967). *The Formal Analysis of Normative Systems. The Logic of Decision and Action*. The University Press, Pittsburgh.
- Lobovikov, V. O. (1999). *Mathematical Jurisprudence and Mathematical Ethics*. Ekaterinburg.
- Wright, G. H. von (1983). *Practical Reason*. Basil Blackwell, Oxford.