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PROBLEM-POSING AS A DIDACTIC RESOURCE IN FORMAL MATHEMATICS COURSES TO TRAIN FUTURE SECONDARY SCHOOL MATHEMATICS TEACHERS

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Abstract

Beginning university training programs must focus on different competencies for mathematics teachers, i.e., not only on solving problems, but also on posing them and analyzing the mathematical activity. This paper reports the results of an exploratory study conducted with future secondary school mathematics teachers on the introduction of problem-posing tasks in formal mathematics courses, specifically in abstract algebra and real analysis courses. Evidence was found that training which includes problem-posing tasks has a positive impact on the students' understanding of definitions, theorems and exercises within formal mathematics, as well as on their competency in reflecting on the mathematical activity.

Keywords – Posing problems, Initial training, Mathematics education, Groups, Continuity.

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1 INTRODUCTION

In the last decade, there has been an increase in the number of research studies on the knowledge and competencies that future mathematics teachers need to acquire in order to succeed in their profession (Rubio, 2012; Ball, Thames & Phelps, 2008; Hill et al., 2008; Silverman & Thompson, 2008; Font, 2011). For example, Rubio (2012) points out that a mathematics teacher must not only attain competency in mathematics, but also competency in the analysis of the mathematical activity. This work intends to show how training including problem-posing tasks is a powerful tool to facilitate not only the comprehension of mathematical concepts, but also the reflection on the mathematical activity. Specifically, the objective of this research was:

Objective: To study the effect that introducing problem-posing tasks in abstract algebra and real analysis courses has on the comprehension of mathematical concepts and the academic achievement of prospective secondary school mathematics teachers.

The structure of this paper is as follows: after this introduction, which also explains the objective of the research, a review of the literature on task design and problem-posing is carried out to provide a theoretical framework. Next, the methodology used in this research is explained and the experiment itself is described. The article ends with some concluding remarks.

2 THEORETICAL FRAMEWORK

2.1 Problem-Posing

Several researchers in the field of mathematics education have focused their attention not only on problem-solving, but also on problem-posing. For example, Malaspina (2013) states that problem-posing is closely related to problem-solving and contributes to the development of mathematical thinking, by providing opportunities for students and teachers to discuss generalizations and take initial steps towards mathematical research.

Beginning university training programs must develop the problem-posing —or at least the problem-reformulating— skills of future mathematics teachers in order to achieve their educational objective. Singer and Voica (2013) report that despite teachers being naturally predisposed to posing problems, they need to be adequately trained in this skill as part of their university studies in order to acquire an effective technique. In this research with future mathematics teachers, this strategy seeks, on the one hand, to facilitate the assimilation of proper mathematical concepts and, on the other hand, to promote reflection on mathematics and future professional work.

Malaspina (2013) states that this strategy stimulates the ability to pose and solve problems, leads to reflections on teaching and mathematics, may give problems a greater potential than that they were originally conceived to have, and shows the importance of properly drafting the statement, as varying the requirement and the mathematical environment of a problem creates opportunities for generalizations and the extension of the mathematical horizon. Furthermore, according to Espinoza, Lupiáñez and Segovia (2014), posing problems is a way to develop the students' creativity and encourage them to take responsibility for their own learning. On the other hand, it appears to be a window into mathematical understanding, as it can be used to assess the students' acquisition of mathematical skills. It also improves the students' disposition and attitude towards mathematics. In this document, problem-posing is not only a means of achieving formal understanding of mathematical concepts; it is also a means through which future teachers can reflect on the mathematical activity.

2.2 Task design

When designing the tasks, we considered the four aspects of problem-posing described by Malaspina (2013): information, requirement, mathematical context and mathematical environment. In this case, the information consisted of definitions, theorems and problems from textbooks; the requirement was to achieve a basic level in the areas of abstract algebra and real analysis; the mathematical context was intra-mathematical, specifically the group-concept and the continuous functions topics; and the mathematical environment was the demonstration process in work groups.

Should any particular process be followed in order to pose a problem? Problem-posing is closely related to problem-solving. Singer and Voica (2013) state that "when the process of solving is a successful one, a solver successively changes his/her cognitive stances related to the problem via transformations that allow different levels of description of the initial wording." They claim that problem-solving involves four operational categories: decoding, representing, processing and implementing. This framework can be helpful when analyzing the original problem, modifying it or posing a new problem.

For this study, different types of tasks were designed to train students in problem-posing: comprehension of definitions, modification of problems from a textbook, modification of modified problems, creation of counter examples and generalization.

2.3 Methodology

This research was based on the MAB500 Introduction to Analysis and MA0371 Abstract Algebra courses in the mathematics-teaching programs at the Universidad Nacional de Costa Rica and the Universidad de Costa Rica, respectively. Eight students attended the analysis course and 20 students attended the algebra course. Both courses can be described as traditional formal mathematics courses and are generally characterized by lacking any relation to secondary-school mathematics and not requiring any sort of didactic reflection on the mathematical activity. The context used for the algebra course was the concept of the group as an algebraic structure, whereas for the analysis course, the context was the composition of continuous functions.

For data-collection purposes, a diary was used in which all classroom events were recorded. The problems posed by the working groups were collected as evidence. A week later, a specific individual written test was administered in order to assess the understanding of mathematical concepts. Finally, the students answered a questionnaire to evaluate the activity. Only some of the students' problems, comments and results are reported here, due to space restrictions.

3 THE EXPERIMENT IN THE ABSTRACT ALGEBRA COURSE

During the lessons based on traditional methodology, the teacher merely gave the definition of a group followed by some examples and immediately moved on to the inherent properties of this concept. However, in this experiment, the teacher gave the formal definition of a group and proposed a task that required the students to reflect on the details of each part of the definition before proceeding to superior levels of understanding, i.e., creating their own groups from a given set.

3.1 Task 1

A group (G, *) is a non-empty set (G) with an operation (*) that fulfills these characteristics:

1) The operation (*) satisfies the closure and association properties.

2) An identity element $e \in G$ exists, so that $g*e = e*g = g$ in all cases.

3) For each element $g \in G$, there is an inverse element $g' \in G$, so that $g*g' = g'*g = e$.

Express in your own words the meaning of each point (1 to 3) of the definition of a group. Then, if possible, formulate an operation for each of the following sets in order to obtain a group:

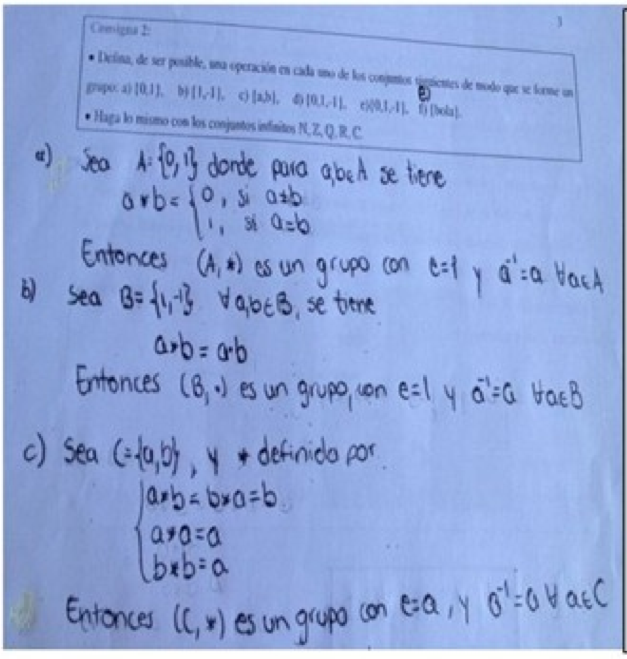
U={ball}, A={0,1}, B={1,-1}, C={a,b}, D={0,1,-1}, E={a,b,c}.

Apparently, the students had no problems with understanding each part of the definition of a group. However, when trying to create groups, they realized that some things were not as clear as they thought. When trying to formulate an operation (*) for the set U={ball} that would result in a group (U ,*), they encountered difficulties. Some of them mistakenly formulated the operation *ball + ball = 2ball*, but the element *2ball* was not in the set, which did not satisfy the closure property. After a few discussions, they correctly formulated that

Figure 1. Operation in a unitary group, ball + ball = ball

They reflected on mathematical definitions and the fact that nothing that is not explicitly stated can be assumed. We should not adhere to a preconceived idea regarding the symbolism. + in this case is only a symbol and does not refer to the usual addition of natural numbers. This fact, which they discovered, would have gone unnoticed if a traditional teaching methodology had been applied, as happened in other semesters.

For the set {0,1}, some students formulated an operation (*) as follows: 0 if they are equal, and 1 if they are unequal. For {1,-1}, another group of students formulated the operation * as the usual product of integers, and for the set {a, b}, they formulated an operation (*) as follows: a if they are equal, and b if they are unequal, repeating the same argument used before for {0,1}. After that, they were asked to draw the operation tables for each group, which made them realize that the three groups were actually the same. This was an introduction to the isomorphism of groups. Finally, they concluded that groups of order two could be structurally characterized by $G = \{e, a\}$, where e plays the identity role and $a = a^{-1}$. This type of strategy generates additional questions by the students. For example, some asked themselves how they could characterize a group of order three or four, although this was not part of the original task. Using the closure and association properties of groups and the cancellation law several times, they found the answer. Without much difficulty, they simply reached the conclusion that there was one and only one group of order three, $G_2 = \{e, a, a^{-1}\}$, where $a^2 = a^{-1}$. They also concluded by themselves that there were only two types of groups of order four. One is the group $G_1 = \{e, a, b, c\}$, in which each element is its own inverse. In the other group, $G_2 = \{e, a, b, b^{-1}\}$, one of the elements, in this case a, is its own inverse, b has a different inverse, and $b^2 = a$. The operation tables for these groups are shown in Figure 3.



If $A = \{0,1\}$ where

$$a * b = \begin{cases} 0, & a \neq b \\ 1, & a = b \end{cases}$$

, then $(A,*)$ is a group where e=1 and $a^{-1} = a\ \forall a \in A$.

If $B = \{1,-1\}\ \forall a,b \in B\ a * b = a \cdot b$, then $(B,\cdot)$ is a group where e=1 and $a^{-1} = a\ \forall a \in B$.

If $C = \{a,b\}$ and $*$ is defined by

$$\begin{cases} a * b = b * a = b \\ a * a = a, b * b = a \end{cases},$$

then $(B,*)$ is a group where e=a and $a^{-1} = a\ \forall a \in C$.

Figure 2. Creation of groups of order two

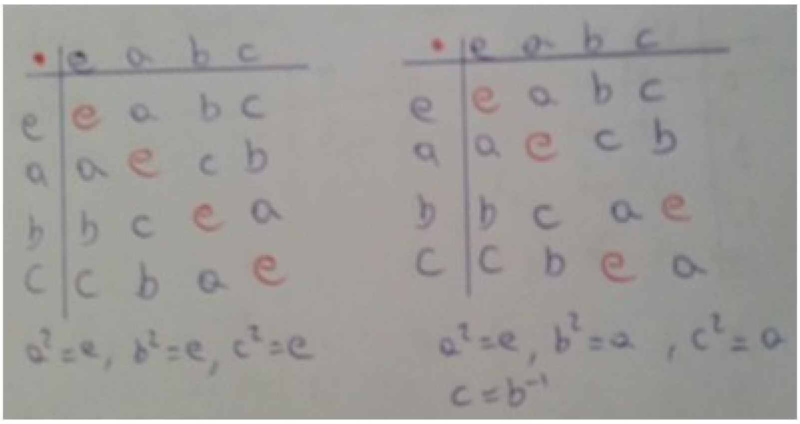


Figure 3. Operation tables for the groups of order four created by the students

3.2 Task 2

Imagine you have two different coins with a value of ₡100 and ₡50 on a table. Create a group where the operation (◦) is the composition of the movements of the coins and G is the set of those movements (Pinter, 2010).

Figure 4. Costa Rican coins

After several attempts, they described some movements, such as changing the position of the coins on the table, turning them upside down and tossing them in the air (they realized that the coins could come up either heads or tails, and since the result was not the same, they discarded this movement). Another point of discussion was whether it was the coin itself or its position that mattered. One group, taking into account the position of the coins, defined the following movements:

V_1: flipping over the coin with a value of ₡100, V_2: flipping over the coin with a value of ₡50, V: flipping over both coins, C: switching the coins, V_1C: switching the coins and then flipping over the coin with a value of ₡100, V_2C: switching the coins and then flipping over the coin with a value of ₡50, CV: flipping over both coins and then switching them and I: not changing anything.

Using $G = \{V_1, V_2, V, C, V_1C, V_2C\ CV, I\}$ and the operation $\circ$, which consists of performing any two movements in succession, they obtained the group that is shown in Figure 5.

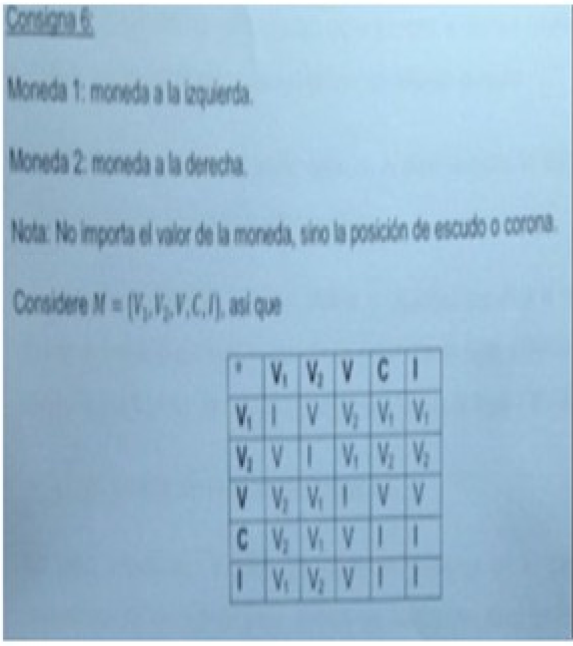
	V_1	V_2	V	C	V_1C	V_2C	CV	I
V_1	I	V	V_2	V_1C	C	CV	V_2C	V_1
V_2	V	I	V_1	V_2C	CV	C	V_1C	V_2
V	V_2	V_1	I	CV	V_2C	V_1C	C	V
C	V_1C	V_2C	CV	I	V_1	V_2	V	C
V_1C	C	CV	V_2C	V_1	I	V	V_2	V_1C
V_2C	CV	C	V_1C	V_2	V	I	V_1	V_2C
CV	V_2C	V_1C	C	V	V_2	V_1	I	CV
I	V_1	V_2	V	C	V_1C	V_2C	CV	I

Figure 5. Group created with eight coin movements

Another group of students opted not to take into account the position of the coins, focusing on whether they came up heads or tails. Although they had originally stated that the value of the coin did not matter, they actually took it into account in their table, as they differentiated the coin with a value of ₡100 from the coin with a value of ₡50. When they created the table, they realized that this was not a group. For example, in the table they obtained $V_2 \circ C = V_2 \circ I \Rightarrow C = I$, which contradicts the principle that all elements in a group are different. This is why they decided to eliminate movement C, in order to obtain a group of order four.

These two interpretations proved to be a source of interesting discussions and reflections on both mathematics and the interpretation of problems. One student said: "Professor, it is important to be clear in the statements; I never imagined that someone could interpret it in this other way." The original problem was not clear enough, so it allowed for different interpretations.

Students were told that this is actually a subgroup of the group of order eight created by the first group of students, which is a good example of isomorphic groups. What matters is the structure of the group, not the characterization of the elements, the operation used, or the different interpretations.



Consigna 6:

Moneda 1: moneda a la izquierda.

Moneda 2: moneda a la derecha.

Nota: No importa el valor de la moneda, sino la posición de escudo o corona.

Considere $M = \{V_1, V_2, V, C, I\}$, así que

*	V_1	V_2	V	C	I
V_1	I	V	V_2	V_1	V_1
V_2	V	I	V_1	V_2	V_2
V	V_2	V_1	I	V	V
C	V_2	V_1	V	I	I
I	V_1	V_2	V	I	I

Task 6:

Coin 1: to the left

Coin 2: to the right

The value of the coin does not matter, only whether it comes up heads or tails.

Consider $V = \{V_1, V_2, V, C, I\}$.

Figure 6. Operations table with five coin movements that are not a group

4 THE EXPERIMENT IN THE ANALYSIS COURSE

The experiment was also carried out in a real analysis course, on the topic of continuous functions. The teacher explained the composition theorem for continuous functions (CTCF), the proof of which was presented in the traditional way. After that, a new methodology was implemented.

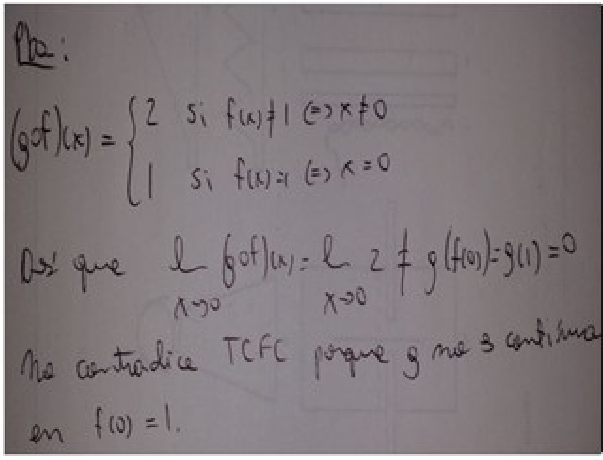
4.1 Strategy used: What if not?

Posing problems is not an easy and immediate task for students, but they have to be trained to do it. According to Sang-Hun, Jae-Hoon, Eun-Ju and Hyang-Hoon (2007), “there are some strategies necessary to help students to pose new problems: posing of new auxiliary problems, changing of conditions, or combination and disassembly. Among these strategies, the so-called ‘What if not?’ strategy suggested by Brown & Walter (1990) is one of the most widely used strategies.” In this experiment, the students were provided with an original problem, taken from Bartle and Sherbert (2010, pp. 160), and asked questions of the type “What if not?”

Original problem

f: $\mathbb{R} \to \mathbb{R}$ and g: $\mathbb{R} \to \mathbb{R}$ are two functions defined by $g(1) = 0$ and $g(x) := 2$ if $x \neq 1$, and $f(x) := X + 1\ \forall x \in IR$. You have to demonstrate that $\lim_{x\to 0} (g \circ f)(x) \neq (g \circ f)(0)$. Why doesn't this fact contradict the composition theorem for continuous functions (CTCF)?

The students solved the problem without any difficulty. One of the solutions is shown in the next figure.



Proof:

$$(gof)(x) = \begin{cases} 2, & x \neq 1 \\ 0, & x = 1 \end{cases}$$

$$\lim_{x\to 0}(gof)(x) = \lim_{x\to 0} 2 = 2 \neq g(f(0)) = g(1) = 0$$

Therefore,

this does not contradict the CTCF because g is not continuous in $f(0) = 1$.

Figure 7. Solution of the original problem

After this, they were asked to change the conditions of the above problem in order to pose a new problem. Since this was the first time that they were formulating problems, a guide was provided. They were then asked to complete the following sentence:

$$\text{What if} \begin{bmatrix} \text{the function } f \\ \text{the function } g \end{bmatrix} \begin{matrix} \text{...is not...} \\ \text{...does not have...} \end{matrix} \begin{bmatrix} \text{continuous?} \\ \text{an avoidable discontinuity at } x=1\text{?} \end{bmatrix}$$

4.2 Some problems posed by the students

Some problems posed by the students are shown below. Note that they began with variations of the original problem and then moved on to variations of their own problems.

Problem 1: (application of the CTCF at x=3)

If $g(x) = x - 2$ and $f(x) = x^2$, justify why$\lim_{x\to 3} (g \circ f)(x) = g(f(3))$.

Problem 2: (generalization of Problem 1)

If $g(x) = x - 2$ and $f(x) = x^2$, justify why $\lim_{x\to c} g(f(x)) = g(f(c))$.

Problem 3: (application of the CTCF)

If f: $\mathbb{R} \to \mathbb{R}^+$ and g: $\mathbb{R}^+ \to \mathbb{R}$, where $f(x) = x^2 - x + 1$ and $g(x) = 1nx$, demonstrate that $\lim_{x\to c} (g \circ f)(x) = g(f(c))$ $\forall c \in \mathbb{R}$.

Problem 4: (modification of Problem 3)

If $g(x) = 1nx$ and $f(x) = x^2 - x$, can you guarantee that $\lim_{x\to c} (g \circ f)(x) = g(f(c))$?

Problem 5: (modification of Problem 4)

If $g(x) = \sqrt{x}$ and $f(x) = x^2 - x$, determine the values of $x = c$ for which $\lim_{x\to c} (g \circ f)(x) = g(f(c))$.

Problem 6: (modification of Problem 5)

If *f, g* are two functions defined by $f(x) = \sqrt{x-1}$ and $g(x) = x + 1$, determine the values of *c* for which $\lim_{x\to c} (g \circ f)(x) = g(f(c))$. Do the same for $f \circ f$, $g \circ g$, $g \circ f \circ g$.

Problem 7: (modification of Problem 4, with a mistake)

If g(x) = 1n(x) and $f(x) = x + 1$, why $\lim_{x\to -1} (g \circ f)(x) \neq g(f(-1))$?

In Problem 1, the CTCF is applied at a specific point, x = 3, while in Problem 2, it is applied at any point x = c. Problem 3 is a higher-level variation: it is concerned with this composition being well-defined, so it verifies that it only contains positive images. Problem 4 is a variation of the above problem, in which f includes some negative images, so the composition might not be well-defined. Problem 5 is a variation of Problem 4 which, instead of asking whether the equality is correct, asks for the values of x at which the equality holds. Problem 6 generalizes other types of compositions.

Problem 7 asks for proof that two things are different, losing sight of the existence of the mathematical objects involved. It must be proven that $\lim_{x\to -1} (g \circ f)(x) \neq g(f(-1))$, regardless of whether these mathematical objects exist. In this case, the limit of the composition does not exist because $\lim_{x\to -1^+} (g \circ f)(x) = \lim_{x\to -1^+} 1n(x + 1) = -\infty$. On the other hand, $\lim_{x\to -1^+} (g \circ f)(x)$ does not exist, and neither does $g(f(-1)) = g(0) = 1n(0)$.

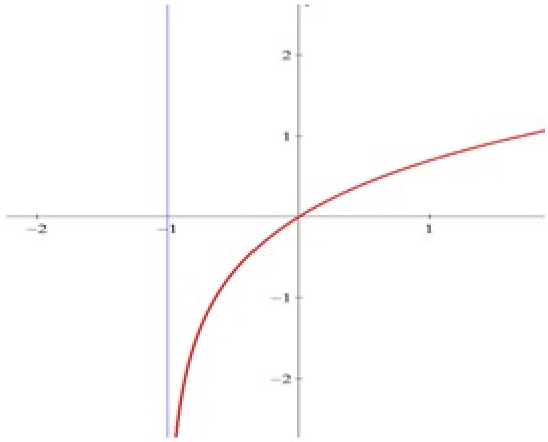


Figure 8. Graphical representation of g(f(x)) =1n(x + 1), mentioned in Problem 7

After these reflections, the students reformulated the problem with the following corrections:

Correction of Problem 5:

If $g(x) = 1nx$ and $f(x) = x + 1$ determine whether $\lim_{x\to -1^+} (g \circ f)(x)$ and $g(f(-1))$ exist, and justify your answer. Can you say that $\lim_{x\to -1} (g \circ f)(x) \neq g(f(-1))$?

Problem 8: (another incorrect problem)

If g(x) = sin(x) and f(x) = 1/*x*, justify the fact that $\lim_{x\to 0} (g \circ f)(x) \neq g(f(0))$.

Problem 9: (problem with a non-removable discontinuity)

If $f(x) = [|x|]$ and $g(x) = \frac{1}{x^2+1}$, is it true that $\lim_{x\to 1} (g \circ f)(x) = g(f(1))$?

Problem 10: (modification of Problem 9)

If $f(x) = [|x|]$ and $g(x) = \dfrac{1}{x^2+1}$, determine whether $\lim\limits_{x\to 1} g(f(x)) = g(\lim\limits_{x\to 1} f(x))$.

Problem 11: (modification of Problem 10)

If $f(x) = [|x|]$ and $g(x) = \dfrac{1}{x^2+1}$, determine whether $\lim\limits_{x\to 1} f(g(x)) = f(\lim\limits_{x\to 1} g(x))$.

Problem 12: (two discontinuous functions)

If $f(x) = x + 1$, where $x \neq 0$ and $f(0) = -5$, and $g(x) = 2x$, where $x \neq -5$ and $g(-5)$ 2, is it true that $\lim_{x\to 0} (g \circ f)(x) = g(f(0))$?

In Problem 8, the left-hand limit does not exist because it ranges from -1 to 1. Moreover, f(x) = 1/x is not defined at X = 0, so f(0) is not defined. The students corrected this problem. Sometimes mistakes can be a very good motivation for learning. It is important to note that this problem involves oscillatory discontinuities, not simply removable discontinuities, as the original problem did. Problems 9, 10 and 11 also illustrate other non-removable discontinuities, such as the floor function.

In Problem 9, the equality $\lim_{x\to 1} (g \circ f)(x) = g(f(1))$ is not satisfied because the limit as x approaches 1 does not exist. Problem 10 shows one of errors most commonly made by calculus students, namely, when it is possible to "insert" the limit within the composition, i.e., when it is true to say that

$$\lim_{x\to 1} g(f(x)) = g(\lim_{x\to 1} f(x))$$

In this case (Problem 10), the equality is not valid since $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} [|x|]$ does not exist. Problem 11 changes $g \circ f$ to $f \circ g$, so that the equality $\lim\limits_{x\to 1} f(g(x)) = f(\lim\limits_{x\to 1} g(x))$ is now true because it is a direct application of the CTCF. The function g is continuous at $x=1$ and the function f is continuous at $g(1) = 1/2$. Moreover, since $1 \leq 1 + x^2$, $[|1/(1 + x^{2)}|] = 0$, then the composition $f \circ g$ is zero. Finally, Problem 12 shows two discontinuous functions whose composition turns out to be continuous.

4.3 Generalizing a problem

In the last task, students were asked to generalize the original problem, so that it becomes only a specific case. This task clearly required that they have a higher level of understanding, because generalization implies "doing math." They were given a guide of questions to help them in their task. After a great deal of discussion within the groups and then at class level, using a collaborative method, they detailed the characteristics of each of the functions until they achieved the following generalization, to the great satisfaction of everyone.

Generalization of the original problem

If f, g and h are three functions defined by g(c) = a and g(x) = h(x), where $x \neq c$, and f, h are continuous and f is also invertible, demonstrate that if $d = f^{-1}(c)$, then $\lim_{x\to d} (g \circ f)(x) \neq (g \circ f)(d)$. Why doesn't this fact contradict the CTCF?

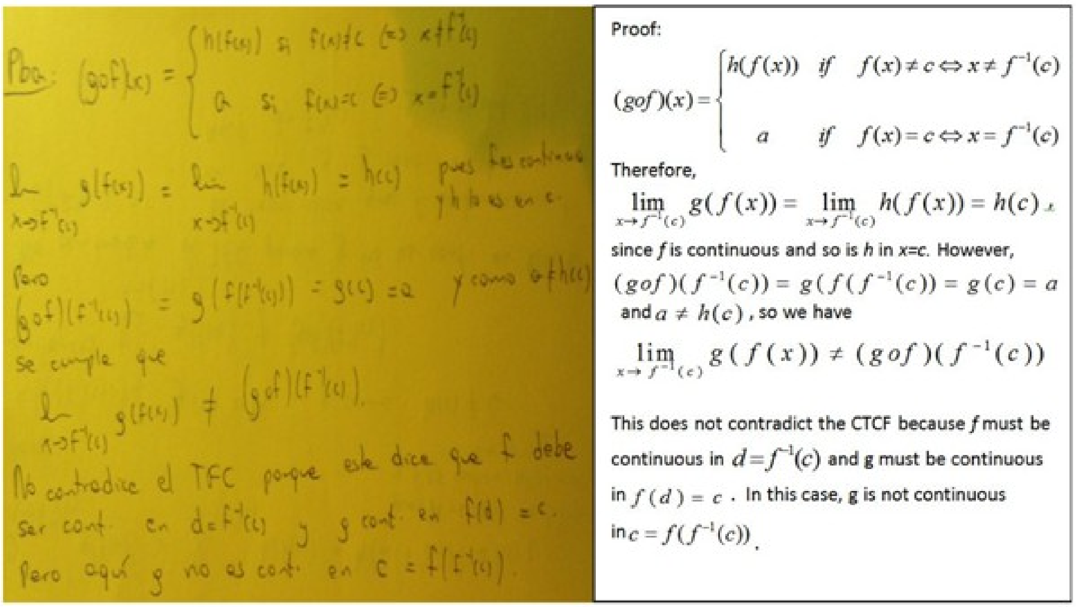


Figure 9. Generalization of the original problem by a group of students

5 CONCLUSIONS

The introduction of problem-posing and problem-variation tasks increased the students' understanding and academic performance in both courses (algebra and analysis). Compared to other semesters, the scores on the written test were very much improved. These tasks increased the students' performance in different ways, thanks to their active involvement in them. Tasks related to understanding a problem, definition or theorem and the consequences of modifying some of its assumptions led to a better (and faster) understanding of formal demonstrations. This was evident from the results of a written test that we administered. Moreover, a change in the students' attitude towards problems posed in textbooks was observed: now they do not simply attempt to solve a problem, but also analyze its statement, thereby achieving a better understanding of the theory that they have to apply to solve it. Another positive outcome was an increase in the students' motivation, particularly among students with lower academic performance, who did not participate much in class. Although these tasks were more time-consuming than traditional lecture sessions, they facilitated the understanding of the subsequent topics, which could then be presented more quickly. Finally, it is important to note that these tasks can ease the transition from traditional lessons to innovative lessons, in which students not only do math, but also reflect on mathematics.

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