A Simulation of a Virtual Qubits on a Classical Computer has Been Developed Recently

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Simulación de una computadora cuántica virtual sobre una computadora clásica operando con un número infinito de bits

Resumen. Una simulación en términos de un número infinito de señales analógicas de los qubits virtuales generados aleatoriamente es hecha. Las compuertas cuánticas asociadas con la computadora cuántica aleatoria son simuladas con amplificadores operacionales. La simulación satisface los cinco criterios de Di Vincenzo impuestos a una computadora cuántica operativa. La medición cuántica y entrelazamiento son explicados perfectamente. Dentro de nuestro enfoque, decoherencia no es un problema ya que el tiempo de ejecución de una compuerta es menor que el tiempo de calibración del amplificador operacional. En suma, la presente simulación de una computadora cuántica en señales y sistemas de Electrónica, es completa y proporciona información importante acerca de los detalles de decoherencia.

Palabras clave: qubits, compuertas, analógicas, señales, sistemas, entrelaamiento, SO(2), decoherencia.

Abstract. A simulation of virtual and randomly generated qubits in terms of an infinite number of analogical signals is performed. The quantum gates associated with the random quantum computer are simulated through operational amplifiers. The present simulation satisfies the five Di Vincenzo requirements imposed to an operative quantum computer. Quantum measurement and entanglement are perfectly well accounted for. Within our approach, decoherence is not a problem since the time of execution of a gate is less than the calibration time of the operational amplifier. In short, the present simulation of a quantum computer in electronics signals and systems is complete and gives important information about the details on decoherence.

Key words: qubits, gates, analogical signals, systems, entanglement, SO(2), decoherence.

Introduction

A Simulation of a virtual qubits on a classical computer has been developed recently (Kamalov, 2006; Kamalov, 2009). From this, a classical computer model of quantum entangled states is proposed. The approach relies in a controlled correlation or anti-correlation of the EPR-Bohm type where the Bell inequalities are not violated. The authors of (Kamalov, 2006; Kamalov, 2009) argue that the construction of virtual quantum states is possible due to the hypothesis on the nature of quantum states (Kamalov, 2001). Thus, they construct a correlated quantum objects from the stochastic geometrical background (*i. e.* gravitational fields and waves background) whose solutions are two particles oscillating with a suggestive random phase. This phase is the same in all of the geometric area of coherence (Kamalov, 2006).

By proposing an algorithm for the coherent phase, in (Kamalov, 2006; Kamalov, 2009) a virtual qubit is modeled on a classical computer. At this point we want to stress, that Feynman's original idea of quantum calculation, relies on the impossibility of calculating through a classical computer, the action of a transformation on a superposition of entangled states (Feynman, 1982; Feynman, 1986). The above is a serious obstacle for the approach practiced in (Kamalov, 2006; Kamalov, 2009) although certainly this last is not completely wrong. That is, the approach is overall semi-classical since the discretization process for obtaining the Boolean values 0 and 1 is achieved without making use of the azimuthally of the single valued coherent phase. In this way, they simulate a virtual quantum computer with just one classical bit. Consequently, the agreement of (Kamalov, 2006; Kamalov, 2009) with Feynman's quantum computation is not direct unless we work within a very limited semiclassical scheme.

On the other hand, it is worth to note that in the original papers (Feynman, 1982; Feynman, 1986) nothing was mentioned about the limit case of a classical computer operating with an infinite number of classical bits. By this reason, in the present work the simulation of the virtual quantum computer of Kamalov et al (Kamalov, 2006) is done in terms of a computer operating with an infinite number of classical bits. The utility of the present approach is the suggestion of a new approach for generating quantum computer technologies. The initialization state of the respective non-finite classical machine is implemented by subjecting the resulting signal to a rectifier whose time of operation is very long. Furthermore, in the image of the classical computer operating with an infinite number of bits, an unitary operation (quantum gate) is thought of as phase shifting in discrete steps.

The paper is organized as follows: In section II a brief review on the construction of virtual quantum states is given, in Section III the equivalence between a virtual quantum computer and a computer of an infinite number of bits is proved. In section IV the initialization is discussed and the CNOT gate is also simulated in the image of a computer of an infinite number of bits. Furthermore, quantum entanglement is also simulated in terms of signals and systems. Finally in Conclusions a discussion about the results is given.

1. Virtual quantum states

The theoretical basis for the construction of virtual quantum states is accounted in Ref. (Kamalov, 2009). In such an approach, the generator of various non-interacting objects is the generating correlation (coherence) of a stochastic geometrical background. The region of the background localization is called the area of coherence zone. The explanation about how this occurs is quite simple; let us consider a situation where the background of stochastic gravitational fields and waves represent the effect of the stochastic geometrical background. This makes that the mathematical metric fluctuates. To use the representation of extended particles as localized self-gravitating structures within the Einstein-De Broglie approach, we obtain the self-gravitating solution with the non-resonance quantization mechanism as a result (Kamalov, 2009). The accounting for the gravitational field in the vacuum within the Einstein theory (with invariants of Ricci tensor R and metric tensor g) is the action function (Saharov, 1967; Saharov, 1975).

$$S(R) = -\frac{1}{16\pi G}\int dx\,\sqrt{-gR}. \tag{1}$$

The resultant action of all these gravitational fields with number j is the functional

$$S_0(\psi) = \Sigma_{j=1}^{\infty} S_j, \tag{2}$$

being $\psi(x)$ the external field given by the metric tensor g_{ik} of the gravitational field (Kamalov, 2009). The metric in the linear approach is:

$$g_{ik} = \eta_{ik} + h_{ik}, \tag{3}$$

where η_{ik} is the Minkowski metric (unity diagonal matrix) and $h_{\mu\nu}$ satisfies the gravitational field equations

$$\Box h_{mn} = 16\pi G S_{mn}, \tag{4}$$

G being the gravitational constant, S_{mn} the energy-momentum tensor of gravitational field sources and $\Box$ the d'Alembertian operator. The solution of Eq. (4) is:

$$h_{\mu\nu} = e_{\mu\nu}\exp(ik_\nu x^\nu) + e^*_{\mu\nu}\exp(-ik_\nu x^\nu), \tag{5}$$

the tensor $h_{\mu\nu}$ is called the metric perturbation, $e_{\mu\nu}$ is the polarization, and k^ν the 4-dimensional wave vector. In the approach of Kamalov et al it is assumed that the metric perturbation is distributed in space with an unknown distribution function $\rho = \rho(h_{\mu\nu})$.

The relative displacement δ of two particles is described in General Theory of Relativity by deviation equations (Kamalov, 2009):

$$\frac{D^2}{D\tau^2}\delta^i(j) = R^i_{kmn}(j)\delta^m \frac{dx^k}{d\tau}\frac{dx^n}{d\tau}, \quad (6)$$

where $R^i_{kmn}(j)$ is the Riemann's tensor with gravitational field number j of the stochastic gravitational fields. Eq. (3) reduces to an equation of two particles oscillations whose solution is:

$$d^1(j) = d_0 exp(k_a x^a + i\omega(j)t) \quad (7)$$

with $a = 1, 2, 3$. To sum over all of the fields the phase transforms as $\Phi(j) = \omega(j)t \rightarrow \Phi(t) = \omega(t)t$, where t is the time coordinate. According with present approach, which is close of Ref. (Kamalov, 2009) this random phase is the same for various quantum microobjects in the area of this coherent background localization (Kamalov, 2004). This area is such that the correlation factor for these two particles is nonzero *i. e.* presence of quantum entanglement.

2. Equivalence of a virtual quantum computer and a computer of an infinite number of bits

The generation of several dichotomic random signals with controlled mutual correlation factor out of a single continuous stochastic process has been implemented (Kamalov, 2006). In fact, in the Eqs. (37) and (38) of Ref. (Kamalov, 2006) a random signal was generated on its basis with the help of the algorithm:

$$b(\alpha,t) = sign\{\cos(\Phi(t) + \alpha)\}, \quad (8)$$

where α is an arbitrary parameter and $\Phi(t)$ a random quantity, which satisfy $\langle\Phi(t)\rangle = 0$. Paraphrasing the first line after Eq. (37) of (Kamalov, 2006) "The random phase (37) can be used for simulating quantum computing via generating the following K random dichotomic functions". However, we want to stress that in the Kamalov's random dichotomic functions given by Eq. (8) it was not mentioned explicitly that there are an infinite number of degrees of freedom although certainly such a functions contains them. In fact he claims that "are arbitrary fixed phases". It is worth mentioning that the infinite number of Boolean signals of Eq. (8) is quantized due that,

$$B_n = sign\{\cos(\Phi(t) + \alpha + n\pi/2)\}, \quad (9)$$

is also a Boolean signal for each $n = 0, \pm1, \pm2, \pm3,...$. The set of bits B_n of the above equation is non finite. The correlation of signals $b(\alpha)$ and $b(\alpha + \Delta\alpha)$ is (Kamalov, 2006):

$$M(\Delta\alpha) = \langle x(\alpha)x(\alpha+\Delta\alpha)\rangle = 1 - \frac{2|\Delta\alpha|}{\pi} \quad (10)$$

We observe that for a couple of phases $\alpha_1 = \alpha + (n+1)\pi/2$ and $\alpha_2 = \alpha + n\pi/2$ a classical computer is achieved because in such a case the entanglement (*i. e.* correlation) is (Kamalov, 2006):

$$M(\Delta\alpha) = 1 - \frac{2|\Delta\alpha|}{\pi} = 0.$$

2.1. Experimental simulation of the probabilistic quantum states.

The central idea about quantum computers proposed by Feynman is that they cannot be simulated by a finite number of classical bits. We point out that if the computer works with an infinite number of bits then there is not contradiction with Feyman's idea. Furthermore, the simulation is possible.

Let us define the orthogonal O-bits in terms of typical electrical engineering analogical signals as follows

$$|0) = \frac{a_0}{2} + \Sigma_{n=1}^{\infty} a_n \cos(n\pi x), \quad (11)$$

$$|1) = \Sigma_{n=1}^{\infty} b_n \sin(n\pi x), \quad (12)$$

where the coefficients a_n y b_n, which are obtained as usual by using the ortho normality of the functions sin and cos, satisfy

$$(0|0) = |a_0/2|^2 + \Sigma_{n=1}^{\infty} |a_n|^2 = 1, \quad (13)$$

$$(1|1) = \Sigma_{n=1}^{\infty} |b_n|^2 = 1, \quad (14)$$

$$(0|1) = (1|0) = 0. \quad (15)$$

If q: $[0, 1] \rightarrow \mathbb{R}$ is a continuous periodic function then a general O-bit satisfy

$$|q) = A|0) + B|1), \quad (16)$$

where $|A|^2 + |B|^2 = 1$.

The way in which the quantum bits (dichotomic random signals) of Ref. (Kamalov, 2009) can be experimentally simulated by the signals (16) is illustrated in figure 1 where micro controller 16F877A system has as input a random generation of a function obtained from a C^{++} program, which is sent to a deriver LM741 (integrer) transformer from where a signal called basic O-bit emerges with a definite parity.

It is worth to observe that the simulations (11) and (12) of the quantum bits involve an infinite number of degrees of freedom or component modes. In other words, the simulation is done on a Hilbert space of order 1.

Figure 1. Experimental setup for the generation of an O-bit from a random signal.

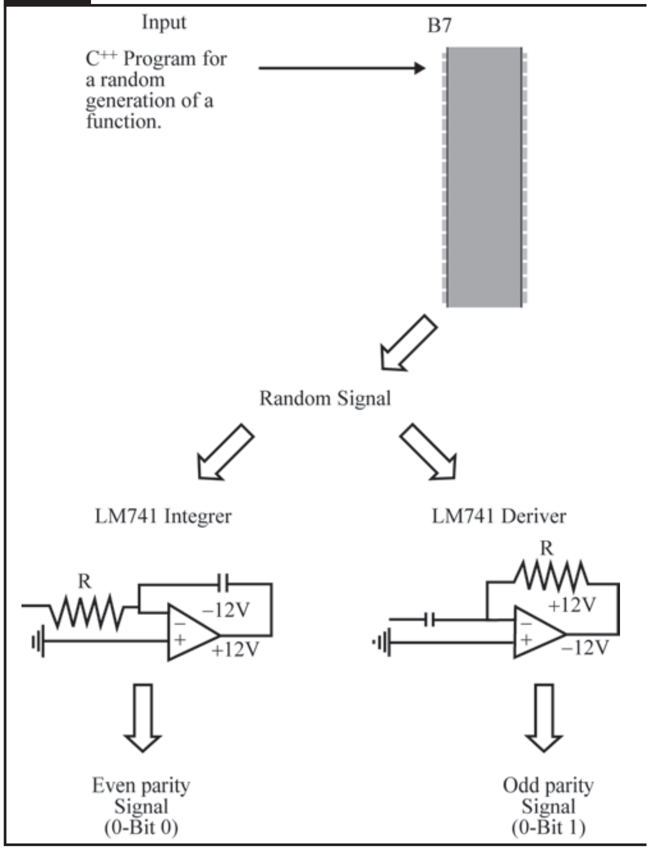
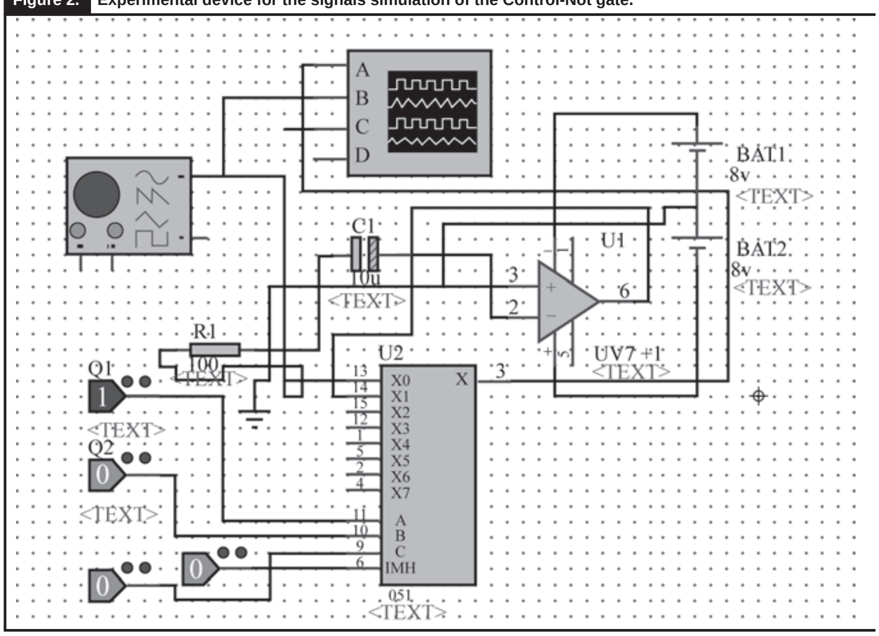


Figure 2. Experimental device for the signals simulation of the Control-Not gate.



3. Di Vincenzo criteria imposed to an operative quantum computer

In order to complete the simulation of random quantum computers by classical computers of an infinite number of bits, we need first to simulate the five requirements imposed by DiVincenzo (Nakahara, 2004) on operative quantum computers. These five requirements are the following:

a) *The quantum degrees of freedom*: the qubits required to hold data and perform computation should be available as dimensions of the Hilbert space of a quantum system. This requirement is trivially simulated by the Eqs (11) and (12).

b) *Initialization*: this requirement indicates that it must be possible to place the quantum system in a fiducial starting quantum state. The idea is then to set the system in an "all spin down" configuration for a chain of nuclear spins. This can be prepared by cooling the system to its ground state. The respective simulation of this initial state is achieved if we start the signal with an $|1\rangle$ obtained from the procedure of figure 1.

c) *Precision Required* (*decoherence*): this requirement indicates that the quantum computer must be to a high degree isolated from coupling with the environment. That is, the entanglement (Schroedinger, 1983; Schroedinger, 1935) between the quantum computer system and the environment system must be very small. Otherwise the system would collapse in a phase-breaking or decoherent state. This might cause lost of precision due that the time which has the quantum computer for performing logic primitive operations is negligible. In such a case an error correction theory would become necessary and this is precisely a paradigm of quantum information theory. In figure 2 is sketched the way the O-bit is processed through a logic gate. In the signal image the noiseless requirement is achieved by an optimal calibration of the device with which unwelcome fluctuations are avoided. The common source of noise in the signal image come from the external currents to the apparatus. In this way, the above gauge must be complimented with an ideal isolation of the system for repelling external currents which could deform the signal under study.

d) *One qubit logic gate:* in Ref. (Oshima, 2003) it has been shown that in presence of an appropriate magnetic field, which act on a single qubit there is a non adiabatic unitary operation on its wave function. This consists of a phase shifting $\exp(i\Phi_g)$ which define an universal quantum gate. This gate can be accomplished in the probabilistic quantum computer as a simple modulation in the amplitude of the signal. This simulation is faithful since there is a correspondence one to one and onto between an unitary operation acting on the qubit and an appropriate modulation in the amplitude of the signal. From the experimental point of view it is a trivial task to implement this through the use of a capacitor for eliminating the loops and a standard diode (Oppenheim, 1999).

e) *Quantum Measurement*: the framework of quantum mechanics requires a careful definition of measurement. The issue of measurement lies at the heart of the problem of the interpretation of quantum mechanics, for which there is currently no consensus (Feynman, 1982; Wheeler, 1983).

Measurement is viewed in different ways in the many interpretations of quantum mechanics; however, despite the considerable philosophical differences, they almost universally agree on the practical question of what results from a routine quantum physics laboratory measurement (Wheeler, 1983). To describe this, a simple framework to use is the Copenhagen interpretation, the utility of this approach has been verified countless times. All other interpretations are necessarily constructed so as to give the same quantitative predictions as this in almost every case.

The expected result of the measurement is in general described by a probability distribution that specifies the likelihoods that the various possible results will be obtained (This distribution can be either discrete or continuous, depending on what is being measured e.g. parity, energy, momentum, etc). What is universally agreed, however, is that if the measurement is repeated, without re-preparing the state, one finds the same result as the first measurement. As a result, after measuring some aspects of the quantum state, we normally update the quantum state to reflect the result of the measurement. This updating ensures that if an immediate re-measurement is repeated without re-preparing the state, one finds the same result as the first measurement. The updating of the quantum state model is called wave-function collapse (Feynman, 1982; Wheeler, 1983), which can be formulated as follows.

Let the one qubit system be prepared in a state (in the signals and systems image this means to prepare the initial signal as a linear combination of sinusoidal and cosenoid functions)

$$|\psi\rangle = C_0|0\rangle + C_1|1\rangle, \tag{17}$$

where $|C_0|^2 + |C_1|^2 = 1$. To measure the observable $\hat{H}$ it will be obtained the result h with probability given by:

$$|p = |\langle \psi|\hat{H}|\psi\rangle|^2 = h_0|C_0|^2 + h_1|C_1|^2, \tag{18}$$

where $\hat{H}|n\rangle = h_n|n\rangle$ with $n = 0, 1$.

In Quantum Mechanics for the measurement process we need to take into consideration the measuring apparatus. By following Von Neumann (Neumann, 1955) we visualize the measurement as beginning with the surroundings S interacting with our machine M in the ready state $|\psi\rangle$. During this "pre-measurement" phase, the interaction Hamiltonian entangles S with M. Assuming that the apparatus can be characterized by a single degree of freedom (Srikanth, 2003), represented by the $\{|\zeta_i\rangle\}$ that span the pointer basis, one obtains the state

$$|\psi\rangle \xrightarrow{\hat{H}} |\Psi\rangle = h_0|C_0|^2|\zeta_0\rangle + h_1|C_1|^2|\zeta_1\rangle. \tag{19}$$

Certainly it is not so simple to determine the general form of the interaction which leads to a specific measurement. However, in Quantum Computation and Quantum Information very often it is suggested that the more general form of the interaction of the measurement is (Nielsen, 2000):

$$\hat{H} = e^{-i}(\alpha_1\sigma_x + \alpha_2\sigma_y + \alpha_3\sigma_z), \tag{20}$$

where $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$. By the way, the above interaction leads to bit flipping through σ_x, a phase with σ_z or both within σ_y (Calderbank, 1997).

Within the signal and system approach, the simulation of the measurement process, which is represented by the Eq. (20), is then achieved by simulating in terms of signals the Pauli matrices σ_x, σ_y and σ_z. This means to perform appropriate changes in the phases of the O-bits signals to turn $|0)$ into $i|1)$, $|0)$ or $|1)$ into $i|0)$, $-|1)$.

f) *Simulation of the Control-Not gate*: in order to proceed further for a more complete signals and systems simulation of the random quantum computer, we must be able of simulating the Control-Not gate. For this purpose the electronic circuit shown in the figure 2 is proposed.

The circuit of figure 2 is fed through two channels corresponding to the two input signals, which are simulating to the two initial qubits. This is composed of the following components: oscilloscope, LM741 amplifier, capacitor, a 100 k Ohms, a 4045 gate and two push buttons representing the two initial qubits. To push the buttons, the incoming current activates the gate, which decides the parity of the output signal through the pin 3.

g) *Entanglement*: quantum entanglement (Nielsen, 2000) is a basic ingredient for Quantum Computation.

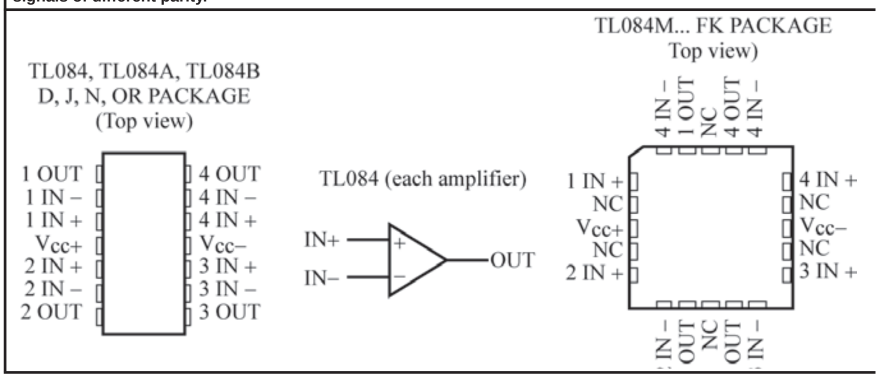
The definition of an entangled state is that it is not entirely independent of other states. This intuitive point of view led to the formulation of the EPR paradox (Einsten, 1935) where the validity of the Quantum Mechanics was drastically questioned. In his paper of 1964 John Bell (Bell, 1964) proved that quantum mechanics and Einstein's assumptions lead to different results. In recent Bell test experiments (Aspect, 1999) it has been shown that pure Quantum Mechanics is acceptable and that Einstein's "local realism" does not. The so called Bell states are an important tool for the discussion of bi-partite entanglement which play a fundamental role for Quantum Computation and Quantum Information sciences (Nielsen, 2000).

A complete simulation of the random quantum computer in terms of signal and systems demands an accounting of quantum entanglement. In this way, we stress that this is possible if we employ analogical signals since the digital ones are manipulated in terms of classical logic gates, which are unsuitable for Quantum Computation. Thus, for accomplishing simple bi-partite entanglement in terms of signals, two random signals are produced as it is sketched in figure 1. Then the relative parity of these two signals which are different, are made dependent each other to subject them to an TL084 operational amplifier as it is illustrated in figure 3.

In figure 3 the amplifier works in such a way that if the parity of one analogical signal changes then the parity of the other changes too in such a way that the products of the parities is invariant under rotations. In this way, the simulation of simple entanglement in terms of analogical signals is done.

Let us proceed now to simulate the so called Bell states:

Figure 3. The TL084 operational amplifier which makes dependent each other the parities of two random signals of different parity.



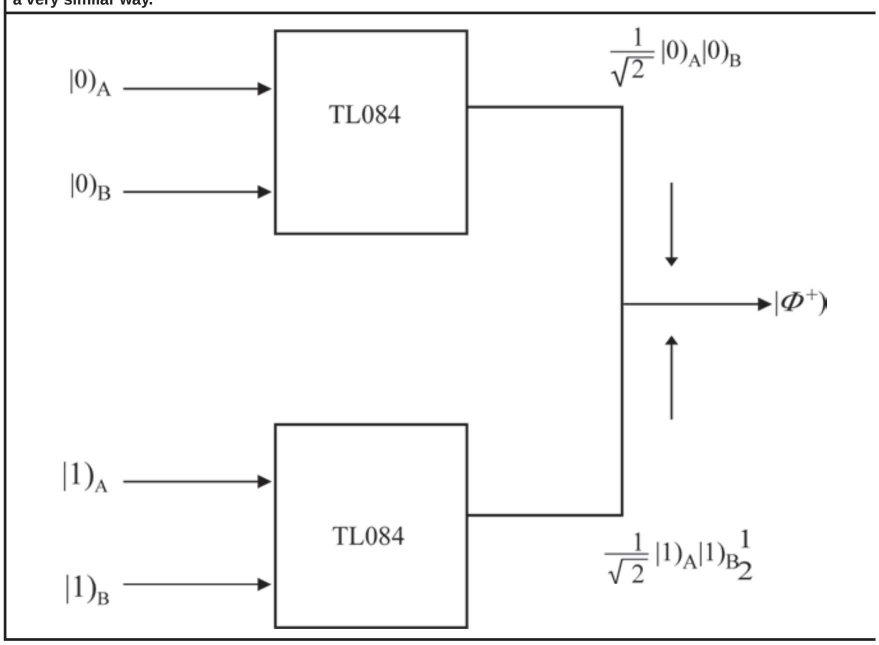
$$|\Phi^{+}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B), \quad (21)$$

$$|\Phi^{-}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B), \quad (22)$$

$$|\Psi^{+}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B), \quad (23)$$

$$|\Psi^{-}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B). \quad (24)$$

Figure 4. Generation of the maximally entangled Bell signal $|\Phi^{+}\rangle$. The respective Bell signal $|\Psi^{+}\rangle$ is obtained in a very similar way.



The idea is to employ a couple of amplifiers of the type TL084. For instance, to create the signal state $|\Phi^{+}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$, a couple of analogical states $|0\rangle_A$ and $|0\rangle_B$ are first randomly created in an independent way in the form illustrated in figure 1. Then they are introduced in a TL084 operational amplifier with which the entangled signal $\frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B$ is created, providing an amplitude modulation $1 \rightarrow \frac{1}{\sqrt{2}}$ is performed through the process. In exactly the same way it is created the entangled signal $\frac{1}{\sqrt{2}}(|1\rangle_A \otimes |1\rangle_B$ Therefore, to superpose the two above signals in the usual way, it is obtained the Bell signal $|\Phi^{+}\rangle$, which is maximally entangled since this cannot be decomposed by a product of signals. This follows from the simple rules of the product of sine and cosine functions. All this is sketched in the figure 4. From such figure it is easily seen that the state $|\Phi^{+})$ is even under exchange of the couples of entangled signals A and B. On the other hand, the respective Bell signal $|\Psi^{+})$ is obtained in a very similar way.

In order to create the Bell signal $|\Phi^{-})$ we follow a procedure analog to the employed for the creation of the signal $|\Phi^{+})$. The only difference is that the input signal $|1)_A$ is firstly subjected to a phase shifting $|1)_A \rightarrow -|1)_A$. In this way the Bell signal $|\Phi^{-}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$, is obtained. This process is sketched in figure 5.

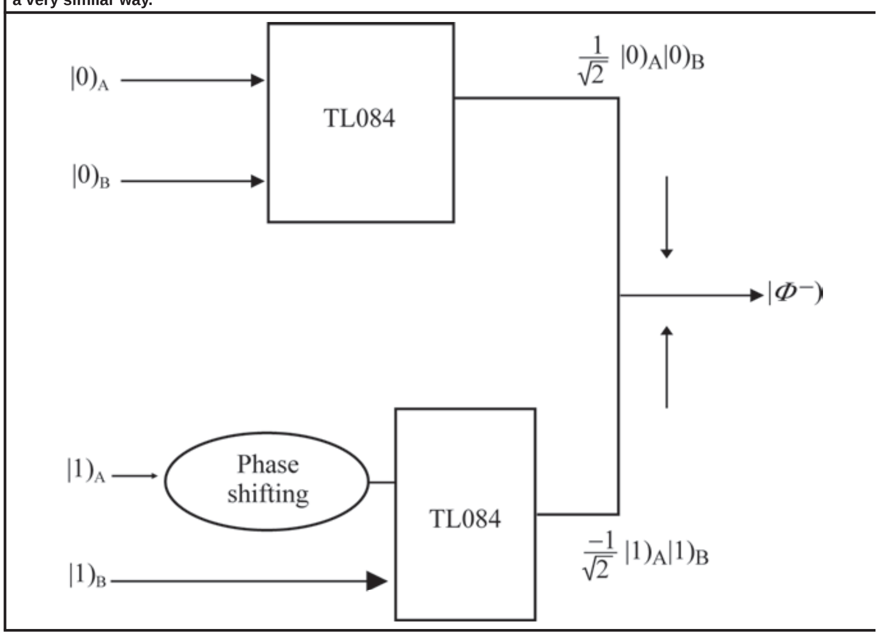
Conclusions

The construction of virtual quantum states from a stochastic geometrical background has been investigated. A simple method of generating a dichotomic signal has also been accomplished. In fact, this can be thought of a probabilistic quantum computer. We argue here that a quantum computer can be simulated on a classical computer with an infinite degrees of freedom. In fact, we have found a simulation of the virtual quantum computer in terms of a non finite number of analogical signals defining the orthogonal qubits (O-bits). We have also verified that such simulation satisfies the five requirements imposed by Di Vincenzo imposed to an operative quantum computer. The decoherence time of the random quantum computer is thought of in the signals and systems image as the amplifiers calibration time which goes from 1 μs to a few minutes. Decoherence is not a problem since the signals operation time is constrained to be less than the amplifiers calibration time. In particular, the Control-Not gate which has been successfully simulated in the present work has a time of execution less than the calibration time, of the amplifier LM741. Quantum entanglement is a basic tool of communication and processing of the information. By this reason, in the present work entanglement has been simulated through the operational amplifier TL084. Furthermore, by manipulation of the signals in a two amplifiers of the type TL084, the simulation of the four maximally entangled Bell states $|\Phi^{+}\rangle$, $|\Phi^{-}\rangle$, $|\Psi^{+}\rangle$ and $|\Psi^{-}\rangle$ has been done in terms of four randomly generated analogical signals $|\Phi^{+})$, $|\Phi^{-})$, $|\Psi^{+})$, and $|\Psi^{-})$ respectively. By using the definitions of odd and even functions we observe from figure 3 that the product of the parities is invariant in the amplifier TL084 (which simulates bi-partite entanglement) under rotations of the signals. This suggests that the system TL084 is a gate simulating an interaction that has a SO(2) symmetry.

Retrospective

The development of an operative quantum computer is a novel paradigm of the Computer Sciences Technologies. The subject becomes rather abstract for many scientist and students. The importance of the present paper lies in the four following points. *a*) The operational amplifiers employed in the present work are fed by signals (called O-bits) represented by $|0) = (a_0, a_1, a_2, a_3,...)$ and $|1) = (b_0, b_1, b_2, b_3,...)$ in Eqs. (11) and (12). The O-bits can be thought of as a non-deterministic Turing machines with an infinite number of degrees of freedom. *b*) This helps to make more tangible and concrete the concept of a quantum computer to non specialists. *c*) This prompts electronics science to develop more efficient chips for accounting for quantum entanglement and control not gates. *d*) The present approach is a novel technology for developing quantum computers, which may compete with others previously developed.

Figure 5. Generation of the maximally entangled Bell signal $|\Phi^{-})$. The respective Bell signal $|\Psi^{-})$ is obtained in a very similar way.



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