# Equiprobable Growth Model and Growth Cycles: An Approximation to Persistence and Volatility

Zeus Salvador Hernández Veleros*

To Estrella, Héctor, Alfonso and Brisa.

**Recepción: 17 de agosto de 2007**
**Aceptación: 7 de enero de 2008**

***Universidad Autónoma del Estado de Hidalgo, Ciudad Universitaria, Instituto de Ciencias Económico Administrativas.**
**Carretera Pachuca-Tulancingo Km. 4.5, col. Carboneras, Mineral de la Reforma, Hidalgo, México, C. P. 42184.**
**Correo electrónico: zshveleros@yahoo.com.**
**I am grateful to José Antonio García-Durán de Lara, Pablo Mejía Reyes, Juan Francisco Islas Aguirre and two anonymous referees, for their helpful comments. To the Universidad Autónoma del Estado de Hidalgo and PROMEP-SEP who financed my scholarship at the Universidad de Barcelona. Any errors are the sole responsibility of the author.**

**Modelo de crecimiento equiprobable y ciclos de crecimiento: una aproximación a la persistencia y volatilidad**

**Resumen.** Este artículo presenta una aproximación novedosa al estudio de los ciclos de crecimiento, para lo cual se emplea la distribucion de probabilidad multinomial y el denominado modelo de crecimiento equiprobable. El estudio considera información de 142 economías durante el periodo 1950-2000, las cuales son agrupadas en siete clusters. El análisis planteado define tres regímenes de crecimiento y sus características (duración, volatilidad y correlación entre tales regímenes). A partir del ciclo de crecimiento estimado para cada cluster se obtiene una tendencia de largo plazo mediante el uso del valor esperado. Las hipótesis del artículo son examinadas mediante bootstrap.

**Palabras clave:** ciclos de crecimiento, tendencia de largo plazo, distribución multinomial, análisis de cluster, bootstrap.

**Abstract:** This paper proposes an innovative approach, based on the multinomial distribution and on the equiprobable growth model, to define and characterize growth cycles during the period 1950-2000, for 142 countries by using three regimes of growth for seven cluster of economies, and from these growth cycles, we obtained the long term trend. Through bootstrap we contrast the main hypothesis of the paper by means of the expected value of the chosen regimes.

**Key words:** growth cycles, trend, multinomial distribution, clustering, bootstrap.

## Introduction

Lucas (1988) establishes, without proving it, that in the most advanced economies, growth rates tend to be very stable over long periods of time, since the averages are long enough to eliminate business cycles effects (or to correct for short term fluctuations in some other way), while in poorer countries there are sudden, large changes in growth rates, both up and down. Thus, the poorest countries tend to have the lowest growth; the richest countries experience a bigger growth, and the middle income countries show the highest.

Therefore, in different groups of economies, classified by their income, there are different durations and volatilities in their growth rates, and long term growth trends are affected by such situations; so, in order to examine these situations we present the following hypotheses: *a)* The multinomial distribution with three growth rates regimes: increase, decrease and stagnation of the real *per capita* Gross Domestic Product (PCGDP), allows us to examine the relationship between persistence (duration) and volatility (variability) for different economic groups: While poor economies are characterized by a regime of positive growth with low persistence and large volatility in comparison with rich economies; in the regimes of no significance or stagnation, poor economies display low persistence and large volatility and, rich economies show low persistence and low volatility; whereas in the regime of negative growth, poor economies display large persistence and low volatility when compared with rich economies.

*b)* From the growth cycle defined by the multinomial distribution for a cluster of economies, it is possible to derive a long term growth trend.

This paper consists of six sections. The first presents the theorical works on the relationship between the business cycles and the long-term growth trends, the studies of the classic business cycles and the real business cycles, the different studies on co-movements and the asymmetries of series, as well as international synchronization and decline in the volatility in some economies. In the second section, we show some theorical models that explain the low persistence of growth. Next, we expose the multinomial distribution and the theoretical aspects about simulation tests based on bootstrap. In the fourth, we define the equiprobable growth model, which is supported on the multinomial distribution. With that model, we examine three regimes of growth rates (increase, decrease and stagnation) and estimate the PCGDP according to different options for the duration of stagnation. We choose the best option for stagnation by minimizing the sum of errors squared between the estimate and actual PCGDP. Then, in the fifth section, we examine the persistence and volatility for each regime and the covariances between regimes with bootstrap. After that, we obtain a long term growth trend for each group of economies during the period 1950-2000. Finally, we present some of our conclusions.

## 1. Long term growth and business cycle: debates

For Higgins (1955) there are three perspectives about the relationship between cycle and trend: some argue that cycle and trend are indistinguishable; for others trend can be introduced into, or derived from, a model of trade cycle or business cycle, so that a trade cycle theory can be used to explain the trend; while still others maintain that information about the trend is essential to have a satisfactory explanation of the economic fluctuations.

For Kaldor (1954) the same forces that produce violent booms and slumps will tend to produce a high trend rate of progress, whereas Higgins (1955) contends that the causes for fluctuations and growth must be analyzed separately, but, that at the same time, both elements affect each other. Zarnowitz (1991) states that business cycles are most probably caused, in part, by uncontrollable outside disturbances and, in part, by errors of public policy and private decision makers that may be avoidable; but they are also, just as plausibly, to a large extent self-sustained and self-evolving. Zarnowitz (1997) establishes that trends and cycles are interrelated and have common causes: time periods and economies with high growth have low instability, whereas great fluctuations reduce growth. Argandoña *et al.* (1997) say that keynesians, monetarists and neoclassic of the balance cycle models with incomplete information presume that the variables of the supply side determine the trend, whereas the demand disturbances generate cyclical oscillations, such variables being independent from each other; while theoreticians of the real business cycle consider the same real factors in low frequency (growth), as well as in high frequency (cycle).

Zarnowits (1991) indicates that the modern macroeconomic analysts narrowed, in diverse forms, the approach to the study of the business cycle, doing it more aggregative and less dynamic; whereas the pioneers in this field stressed the importance of the endogenous cyclical processes and their connection with long term economic growth and development. Argandoña *et al.* (1997) indicate that the cycle theory should not be artificially separated from the theory of growth.

Lucas (1987) establishes that the elimination of the cycle brings about gains equivalent to a permanent increase on consumption of 0.1 percent, whereas an increase in a percentage point in the rate of growth implies a permanent increase of 20 percent in consumption. Ramey and Ramey (1994) point out that the studies about the relationship between growth and business cycle volatility are scarce because there are authors, like Lucas (1987), who establishes that there is no correlation between these two aspects of the economy; whereas for other investigators growth and volatility are linked positively –Black (1987) and Mirman (1971)–. On the other hand, Ramey and Ramey (1994) find an inverse link between volatility and growth, using a panel of 92 countries in the period 1960-1985 and a sample of 24 countries of the Organization for Economic Co-operation and Development (OECD) for the period 1950-1988. Their results remain strong against any conceivable controls that vary with time or country.[1]

Easterly *et al.* (1993) state that growth rates are highly unstable over time (with a correlation across decades of

1. Otrok (1999) defends Lucas´s (1987) view: the models that gain from the elimination of aggregate fluctuations are trivial because they are not rigorous about preference parameters; so, Otrok (1999) uses a model of consistent preferences with the observed fluctuations in a model of business cycle.

0.1 to 0.3), in comparison with country characteristics, which are considered decisive for growth, and are very stable or persistent (with cross-decade correlation of 0.6 to 0.9). These authors also explain that the low persistence of growth rates reconciles the enormous variation in growth rates across countries with the remarkable stability of relative incomes across countries. In addition, they show that much of the variance in growth rates can be explained by shocks (these indirectly influence growth by changing the variables in economic policy), thus, one should be careful when attributing the high growth rates to good economic policies, they might simply be good luck.

It is necessary to make some clarifications about two terms: Easterly *et al*. (1993) use the adjectives "stable" and "persistent" as synonymous of independent, whereas Lucas (1988) uses the term "stable" as synonymous of variability. We use the words "persistence" as duration and "volatility" as variability.

Another aspect to debate is expressed by Lucas (1977) and Zarnowitz (1991), for the first:

> […] a unified explanation (of the cycles) based on the general laws that rule market economies must be sought, instead of one based on the political or institutional characteristics distinctive for each country or period of time". On the contrary, the second author states that "The nature of the business cycle depends and changes with the characteristics of economy, society and politics.

But Lucas (1988) accepts the capacity or the incapacity to eliminate the business cycles effects between advanced and poor economies, and that the poorest countries register the lower growth rates, the richest have higher rates, but those with the highest rates are the countries with an average income. He also declares that little variability is registered between the richest countries, while in the poor countries and those with an average income, an enormous variability exists. That is to say, the author goes against his own affirmations made in his articles from 1977 and 1987.

Not everybody, Hall (2005) for example, considers that the economy moves along a smooth growth trend with temporary cyclical departures, as Lucas (1977) and Kydland and Prescott (1990) do. Hall (2005) declares that important ideas contributed by Friedman, Lucas and the developers of the macro of sticky prices models generate this type of added behavior, for which they use a representative variable, without considering the parts of the cycle and the interconnections between the same. In a similar way, Hall (2005) indicates that the real business cycle model shows that the neoclassic model implies anything but smooth growth.[2]

For Zarnowitz (1991) growth cycles (fluctuations around the upward trend in a nation's economic activity expressed in real terms) cannot be considered true cycles if changes are not generated in the signs of activity: "a slow expansion is still an expansion; the problems that a contraction causes are totally different." Argandoña *et al.* (1997) mention that other authors do not agree with this viewpoint.

Two approaches were developed to analyze the dynamics of the business cycles during the XX century: the approach of the classic business cycles (which emphasizes the importance of the analysis of the characteristics of the business cycle regimes), has its beginnings in the works of Mitchell (1927) and Burns and Mitchell (1946), who provided the basis for the identification of the business cycles turning points; these cycles can be divided in four successive phases: prosperity, crisis, depression and recovery. This approach was criticized by Koopmans (1947); such critics are known as "measurement without an economic theory» and «measurement without a statistical theory". The first one referring to the inclusion *ad hoc* of specific variables in the conducted studies and the second one because the probability distribution of the variables were not specified. In opposition to this position, Kydland and Prescott (1990) consider it a necessity to identify empirical regularities to develop theoretical models that explain them.

The second approach, linked to the real business cycles, emphasizes the analysis of the classic macroeconomic variables co-movements, once its trend component has been eliminated. The work of Lucas (1977) is fundamental in this approach; such work is influenced by Frish (1933), Slutsky (1937) and Adelman and Adelman (1959). Frish studies the problems of impulse (in the form of random shocks) and the spreading of the business cycle. Slutsky (1937) shows that the combination of random causes could be enough to generate regular cycles. Adelman and Adelman (1959) add random shocks to the model of Klein-Goldberger and reproduce added time series for the economy of the United States (US) similar to those of the postwar.

Crum (1923), Mitchell (1927), Keynes (1936) and Neftxi (1984) agree with the asymmetry between expansions and contractions. For DeLong and Summers (1986) the affirmations of these economists are mistaken because they do not take into consideration growth trends; if there was

2. Higgins (1955) mentions that there are different theories about trend (Tinbergen, Kalecki, Hicks). Furthermore, he defines his particular viewpoint about it. And Canova (1998a) mentions that Beveridge and Nelson (1981), Watson (1986), Hamilton (1989) and Quah (1992) have proposed alternative definitions of the trend.

asymmetry between expansions and contractions, the distribution of growth rates should have significantly less than half of observations below the mean; and the average deviation from the mean of the observations below average, should be significantly more than the average deviation of the observations above the mean; thus the median output growth rate should exceed the mean by a significant amount. DeLong and Summers (1986) do not find asymmetry in gross national product or industrial production, but they find it in unemployment for six developed countries. McKay and Reis (2006) establish that the expansions and contractions in the output are equally brief and violent, whereas the contractions in the employment are briefer and more violent than the expansions, using data from US and some European countries.

For us, DeLong and Summers (1986) commit a conceptual error in their application of the measurement of asymmetry when they consider that the recessions are inferior growth rates than the average growth rate (with or without a trend): what should be consider for recessions are the negative growth rates if we did not take into account the trend, and such negative rates less the trend in case we have considered it.

DeLong and Summers (1986) and Sichel (1989) use nonparametric methods to measure the characteristics of different sub periods; whereas Artis, Kontolemis and Osborn (1997) use nonparametric methods to measure the characteristics of the regimes of the business cycle, and to study the international synchronization of the business cycles. Hamilton (1989), Teräsvirta and Anderson (1992), and Potter (1995) have modeled non-linear dynamic and asymmetries using models of Markovian change in regime, models of smooth transition regression and autoregressive threshold models, respectively. Cermeño (2002) built a two-regime model for the *per cápita* growth rates in different groups of countries, where the low growth regime shows high volatility but not much persistency, while the high growth regime is less volatile and more persistent, for which he considers a mixture of distributions that rule the different regimes.

Others studies about the business cycle analyze the decline in the volatility of the same; for instance, the investigations of Cecchetti *et al.* (2006) on the relationship between volatility and growth use quarterly data for the period 1970-2003, for 25 countries; the study of the OECD (2002) uses data for 13 member countries of that organism; the research of Gordon (2005) about the decrease in the volatility of the U. S. business cycle since the mid-eighties and the causes for the same; or the article of Arias *et al.* (2006) who shows, with a real business cycle model, that productivity shocks can explain the decline in the U. S. business cycle volatility gross domestic product beginning in the first quarter of 1984; as well as the studies of Christina Romer (1986a, 1986b, 1989, 1994, 1999) about the main macroeconomic time series of U. S. and the Watson's (1994) paper.

For Kose *et al.* (2003) diverse papers of international business cycles do not refer to the world itself (Backus, *et al.*, (1995), Baxter (1995), Mendoza (1995), Kose (2002), Gregory *et al.* (1997), etc.) because of data limitations and econometric intractability, these researches do not respond to the question of whether fluctuations are associated with worldwide, regional, or country-specific shocks.

Kyndland and Prescott (1990) consider that the decomposition of a series in its cyclical and trend components must be a statistical process guided by the following criteria:

*a)* The trend component must be, approximately, the curve that a student of the cycle would draw to summarize the cloud of points of this series.

*b)* This trend must be a linear transformation of the series, and the same transformation should be used for all the series.

*c)* The extension of the sample period should not significantly alter the values of the data deviations, except perhaps near the final end of the series.

*d)* The process must well be defined, free of judgments and easily reproducible.[3]

Canova (1998a) establishes that there is no consensus about what constitutes business fluctuations nor a trend. Furthermore, there is a division between those who disregard trends based on statistical approaches, and those who use economic approaches. Thus, different statistical representations of a trend involve different economic concepts of business cycle fluctuations and, choosing one detrending method over another implies selecting one particular economic object over another.[4] If we use a detrending method based on a statistical approach we have a problem of "measurement without theory"; as long as the theoretical relationship between trend and cycle is unknown, the choice among diverse methods of decomposition based on economic approach will be arbitrary.

3. Argandoña *et al.* (1997) establishe that we must not formulate simple assumptions about the trend's deterministic or stochastic character, constant or variable, linear or not. This character and the cycle are part of an integrated process; according to the dominant component of the series, it should be the economic policy, and we do not have to artificially separate the cycle theory and the growth theory.

4. Burnside (1998) writes, in reference to Canova's document (1998a), that there is nothing misleading in the fact that different filtering techniques lead to different facts about macroeconomic time series.

Different methodologies have been used to eliminate economic series linear deterministic trends, as well as, stochastic trends. The indicator of the cycle would be the series left, once the trend of the original series has been subtracted. For example, the Hodrick-Prescott (HP) filter is used to obtain a smooth non-linear representation of a time series, one that is more sensitive to long-term than to short-term fluctuations. This filter is widely used by macroeconomists to obtain a smooth nonlinear estimate of the trend component at long term of a series.

But the problem, as Canova (1998a) shows, is that the different methods to eliminate trends generate different stylized facts from business cycle, since the different filters for trend elimination extract different types of information from data. Likewise, King and Rebelo (1993, 1999) have found that the application of filters can remove important time series components; components which economists have considered elements of the cycle. Furthermore, the use of filter in simulated time series or in historical time series can reveal than, in both cases, appear similar cyclical characteristics, which were not necessarily present in the original series. Osborn (1995) reaches similar conclusions.

Lucas (1977) defines economic cycle regularities as "the deviation co-movements around a trend of different temporary added series", and the economic cycle as "the GNP movements around a trend".

Other important definitions are: A regime is an episode through which the conduct of the series is evidently different from another episode (Mejia-Reyes, 2003). The intensity, volatility or amplitude, which is the difference in the value of a variable of reference between its maximum and its consecutive minimum, or between the average value and its maximum value, or the deviation with respect to its trend; also, can be the variance of the series with relation to the variance of the reference series. While periodicity or length is the measurement of time between a cycle and the next one (Argandoña *et al.*, 1997). The depth is the relation between the maximum positive growth rate with respect to the maximum negative growth rate (in absolute value).

This article uses the growth cycles approach in which the periods of expansion and contraction are represented as cyclical movements around a trend, and not the approach of classic business cycles, in which the periods of expansion and contraction are represented by the level of activity.

We established as definition of cycle, the statistical relation between persistence (duration) and volatility (variability) for different economic performance regimes (increase, decrease and stagnation) of the *per capita* Gross Domestic Product (PCGDP). From this relationship it is possible to define a long term growth trend and its respective volatility.

This approach, used here for the first time, takes into consideration the observations pointed out by Lucas (1988) about the differences between the business cycles from rich countries and poor countries, as well as those indicated by Easterly *et al.* (1993) referring to the low persistence among growth rates; it also relies on the comments of Kydland and Prescott (1990) regarding identification of empirical regularities as a necessary condition for the development of theoretical models that try to explain them.

The multinomial distribution and the equiprobable growth model allow us to define and analyze the statistical relationship between persistence and volatility for three economic performance regimes; and to define a long term trend and a relationship (positive, negative or no significant) between growth and volatility, according to the recommendations of Kyndland and Prescott's (1990).

## 2. Growth models with low persistence of growth rates

Easterly *et al.* (1993) expose that growth rates are very unstable (correlation across decades of 0.1 to 0.3) over time, in comparison with the characteristics of countries (correlation across decades of 0.6 to 0.9), considered as determinants of growth which are stable or more persistent. Under this situation (low persistence of growth rates), what explains long-run growth? Easterly *et al.* (1993) examine two types of growth models of low persistence. In the first type, long-run growth depends on the countries characteristics: on tax rates in the Rebelo (1991) model, on the country's patent system and its market size in the Romer (1990) model and in the Aghion and Howitt (1992) model, respectively. The persistence coefficient can be interpreted as a reflection of the magnitude of variance in underlying growth rates across countries, in respect to the variance of random shocks: low persistence of growth rates implies that random shocks (luck) are important in determining the long-run path of growth. Thus, this model leaves much of growth unexplained; or deterministic spurts of growth are important in order to explain the low persistence of growth rates.

In the second type of model, growth depends on a world-wide process (for example, technological progress), and country characteristics determine the relative level of income. Low persistence is consistent with shocks of any size. These define only fluctuations around a long-run path of output, despite being an important determinant of variance in decade-long growth rates. This type of models includes the neoclassical model of Solow (1956), and some models of technological diffusion which incorporate the advantages of backwardness. Persistence depends on the distribution of the relative income of countries in regard to their steady state income, which is determined by politics: if the country is near to a steady state, then random shocks increase their importance.

The different viewpoints about economic fluctuations and trends we have presented (in sections I and II), are summarized in Table A1 of the annex.

## 3. Multinomial distribution and bootstrap

### *3.1. The multinomial distribution*

The probability mass function of the multinomial distribution is:

$$f(x_1,\ldots,x_n; m, p_1,\ldots,p_n) = p[X_1 = k_1,\ldots,X_n = k_n] = \begin{cases} \dfrac{m!}{k_1!\ldots k_n!} p_1^{k_1}\cdots p_n^{k_n} \\ 0 \end{cases}, \quad (1)$$

when $\sum_{i=1}^{n} x_i = m$ otherwise

for non-negative integers $x_1 \ldots, x_n$.

The expected value is:

$$E(X_i) = mp_i \quad (2)$$

The covariance matrix is as follows. Each diagonal entry is the variance of a binomially distributed random variable, and it is therefore

$$Var(X_i) = mp_i\,(1 - p_i), \quad (3)$$

The off-diagonal entries are the co-variances:

$$Cov(X_j, X_l) = m(m-1)\, p_j\, p_l - m\, p_j\, m\, p_l = -mp_j\, p_l \quad (4)$$

for $i$, $j$ distinct. All co-variances are negative because for fixed $N$, an increase in a component of a multinomial vector requires a decrease in another component.

This is a $k \times k$ nonnegative-definite matrix of rank $k - 1$.

The off-diagonal entries of the corresponding correlation matrix are:

$$\rho(X_j, X_i) = -\sqrt{\frac{p_j\, p_i}{(1-p_j)(1-p_i)}}. \quad (5)$$

Note that the sample size drops out of this expression.

Each of the $k$ components separately has a binomial distribution with parameters $n$ and $p_i$, for the appropriate value of the subscript $i$.

The mode (modes) is (are) located near of $E(X_i) = mp_i$, in the next interval

$$mp_i < M\,[X_i] \leq (m + n - 1)\, p_i\,. \quad (6)$$

The support of the multinomial distribution is the set:

$$\{(k_1, ..., k_n) \in \mathrm{N}^n \mid k_1 + \cdots + k_n = m\}. \quad (7)$$

### *3.2. Bootstrap*

Bootstrap is a data-based simulation method for statistical inference, which can be used to define measures of accuracy to statistical estimates. Bootstrapping is a statistical method to get the distribution of an estimator by sampling with replacement from the original data, most often with the purpose of deriving robust estimates of standard errors and confidence intervals of a population parameter like a mean, median, proportion, odds ratio, correlation coefficient or regression coefficient. It may also be used for constructing hypothesis tests.

It is often used as a robust alternative to inference based on parametric assumptions when those assumptions are in doubt, or where parametric inference is impossible or requires very complicated formulas for the calculation of standard errors.

Efron and Tibshirani (1993) explain that the bootstrap algorithm for estimating standard errors selects $B$ independent bootstrap sample $x^{*1}$, $x^{*2}$, …, $x^{*B}$, each consisting of $n$ data values drawn with replacement from $x$. The number $B$ will be in the range 25 to 200.

Then it evaluates the bootstrap replication $s(x^{*b}) = \hat{\theta}(b)$ corresponding to each bootstrap sample of our statistic of interest ($\hat{\theta}$)(mean, median, etc.). So it calculates the standard error $\hat{\theta}$.

$$\hat{se}_{boot} = \left\{ \sum \left[ s(x^{*b}) - s(\cdot) \right]^2 / (B-1) \right\}^{1/2}, \quad (8)$$

where $s(\cdot) = \sum_{b=1}^{B} s(x^{*b}) / B$.

## 4. Equiprobable Growth Model

The source for PCGDP of 142 economies is Maddison (2003). In the model, the periods considered are decades and the frequencies are obtained using averages in each cluster of significant positive rates, significant negative rates and no significant rates (positive and negative).

We use clustering algorithms from the 1950 PCGDP to determine the groups or clusters. To begin with, from the information of 142 economies we define three clusters: the first with 115 economies, the second with 17, and a third cluster with 10 economies. Because the first cluster included too many economies (115), and because these economies were heterogeneous in the variable used for their definition, we define five more clusters within the first cluster. That is to say, we first use a no hierarchical clustering and then, a hierarchical clustering. So, we obtained seven clusters using the 1950 PCGDP. In Cluster 1 we find the poorest economies and, in cluster 7, we have the richest economies (table A2).

Three regimes were used: increase ($k_1$), defined by significant positive rates; decrease ($k_2$), defined by negative significant rates, and stagnation ($k_3$), defined by positive and negative rates closer to zero that could be consider no significant.

We chose nine options to evaluate for each cluster with respect to the stagnation: *a)* In the first one, zero percent of the rates represented stagnation; *b)* In the second option, five percent of the rates represented stagnation: 2.5 percent from negative rates and 2.5 percent from positive rates nearest to zero rate, which could be consider as no significant. We applied this increase associated with no significant rates, up to the ninth option with 40 percent of no significant rates, equivalent to 20 years of stagnation. We chose among these nine options for stagnation, using the minimization of the sum of errors squared principle, between the expected data and the observed data.

Due to the previous considerations: three regimes ($n$) and five decades ($m$), we obtained 21 different estimations of the PCGDP in each cluster,[5] from a total of 243 (= $3^5$) possible estimations, and, also, from the corresponding frequencies of the number of economies that should be expected for each one of the previous estimations.

For the equiprobable model we use the geometric average of the 1950 PCGDP by cluster; the geometric average of the annual growth rates for each one of the three regimes and its corresponding frequencies, and the geometric average of the annual growth rates by regime was the same for all the economies in that cluster and for all the period (51 years).[6]

In order to construct our equiprobable growth model, let us define the following elements:

$m$: decades = 5;

$X_i$: event or regime $i$, with $i$ = 1, 2, 3, where $X_1$ is the regime of increase, $X_2$ is the regime of decrease and $X_3$ the regime of stagnation;

$n$: total events or total regimes = 3;

$k_t$: = number of times in which event $X_i$ or regime $i$ appears with $k_1 + k_2 + k_3 = m$, here we have two random variables;

$PR_m^{k_1,k_2,k_3}$: multinomial coefficient = $\frac{m!}{\Pi_1^3 X_i!}$;

$p_{ij}$: probability of or frequency of regime $i$ at cluster $j$, with $p_{1j} + p_{2j} + p_{3j} = 1$; for $j$ = 1, 2..., 7;

$AAAGR(-)_j$: geometric average of annual decrease growth rate of cluster $j$, with $j$ = 1, 2..., 7;

$AAAGR(+)_j$: geometric average of annual increase growth rate of cluster $j$, with $j$ = 1, 2..., 7;

$AAAGR(0)_j$: geometric average of annual stagnation growth rate of cluster $j$, with $j$ = 1, 2..., 7;

$C_j$: total number of economies of cluster $j$, with $j$ = 1, 2..., 7;

$PCGDP_{j,1950}$: geometric average of 1950 PCGDP of cluster $j$; with $j$ = 1, 2..., 7.

Thus, the fitted PCGDP for the year 2000 of cluster $j$ results from the times $k_1$, $k_2$ and $k_3$ that the respective events happened $X_1$, $X_2$ and $X_3$, is given by:[7]

$$PCGDP_{j,2000}^{k_1,k_1,k_3} = PCGDP_{j,1950}\left[(1+AAGR(-)_j)^{10}\right]^{k_1} \left[(1+AAGR(0)_j)^{10}\right]^{k_2}\left[(1+AAGR(+)_j)^{10}\right]^{k_3}. \quad (9)$$

The number of economies corresponding to each $PCGDP_{j,2000}^{k_1,k_2,k_3}$ is given by

$$NC_{j,2000}^{k_1,k_2,k_3} = C_j \cdot p_{1j}^{k_1} \cdot p_{2j}^{k_2} \cdot p_{3j}^{k_3} \cdot PR_m^{k_1,k_2,k_3}, \quad (10)$$

rounded to the closest whole number.[8]

5. This number is simply obtained using this combinatory: $\binom{m+n-1}{n-1}$, with $m$ = 5 and $n$ = 3.

6. McGrattan and Schmitz (1999) use the geometric average of the *per capita* GDP for 137 countries. The geometric average is employed with rates, ratios, means, geometrics progressions, it is less affected by extreme values that the arithmetic average.

7. We can change the base $(1 + c_{ij})^{tk_i}$ by an exponential base ($e^{r_{ij}tk_i}$), with $r_{ij} = \ln(1 + c_{ij})$, for $c_{ij}$ = AAGR$(-)_j$, AAGR$(0)_j$ or AAGR$(+)_j$.

8. We got these by using the first moment or expected value.

## 5. Economic growth and growth cycle

Can we define a growth cycle for a group of economies and distinguish it from other groups' cycles? These are the questions that we tried to answer using data from Maddison (2003) for 142 economies during the period 1950-2000.

For depth, the maximum positive growth rates by cluster are greater than the maximum negative growth rates (in absolute value), except for cluster three. The greatest positive rate belongs to cluster one and the greatest negative rate (in absolute value) belongs to cluster three. Clusters one and two show positive skewness (the right tail is longer), because some unusually high values appear, the mean is greater than the median. The other five clusters display negative skewness or to the left, because of the presence of unusually low values, the mean is lower than the median. The distributions of the growth rates for all clusters show high kurtosis (For DeLong and Summers (1986) a significant kurtosis is the test of the existence of outliers) (table 1).

In respect to the stationarity of series, we applied tests for panel data and rejected the hypothesis that the series are integrated of order one when we use test for unit root in first difference with individual intercept; while Hadri's test rejected the null hypothesis that claims that series are stationary for all clusters; if we use individual trend and intercept, all the results are the same, except for Hadri's test for clusters six and seven, here we can't refuse the null hypotheses (table 2).

The number of economies that displayed high growth rates diminished during the two last decades of the last century. While during the 1951-1960 decade there were 95 economies that displayed annual growth rates superior in average to 0.02, in the 1991-2000 decade there were just 56; for the complete period only 63 economies had greater registries than the mentioned or 44.36 percent of the 142 economies. Between the first and the fifth decades all clusters diminished in the number of economies with growth rates greater than 0.02, but the most important fall correspond to clusters 1 and 2. By cluster, the only ones with growth rates greater to 0.02 are the fifth and the sixth.

We estimated the fitted 2000 PCGDP and the respective number of economies for each stagnation option using the equiprobable growth model for all clusters, then we minimized the sum squared errors between fitted and actual PCGDP, and we chose the best stagnation option. So, for cluster one, it is 35 percent, not significant data; for cluster two, 25 percent; for cluster three, 10 percent; for cluster four, 15 percent; for cluster five, 20 percent; for cluster six, 25 percent and for cluster seven, the 10 percent of data is not significant (table 3).

**Table 1.** Data of seven clusters of economies (1990 international dollars from Geary-Khamis and growth rates).

| | Cluster | | | | | | |
|---|---|---|---|---|---|---|---|
| Variable | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1950 *per capita* GDP (*PCGDP* 50)* | 447.000 | 740.000 | 1,139.000 | 1,710.000 | 2,239.000 | 4,014.000 | 7,586.000 |
| Number of economies | 24.000 | 28.000 | 23.000 | 19.000 | 21.000 | 17.000 | 10.000 |
| Minimum annual growth rate (*AGR*) | –0.420 | –0.292 | –0.614 | –0.288 | –0.300 | –0.294 | –0.108 |
| Maximum *AGR* | 0.848 | 0.768 | 0.367 | 0.316 | 0.362 | 0.313 | 0.112 |
| Positive *AGR* | 829.000 | 970.000 | 785.000 | 682.000 | 814.000 | 711.000 | 404.000 |
| Negative *AGR* | 371.000 | 430.000 | 365.000 | 268.000 | 236.000 | 139.000 | 96.000 |
| Average *AGR* (*AAGR*) | 0.018 | 0.018 | 0.011 | 0.020 | 0.023 | 0.025 | 0.019 |
| *AGR* upper that *AAGR* | 588.000 | 680.000 | 640.000 | 500.000 | 550.000 | 459.000 | 276.000 |
| *AGR* lower that *AAGR* | 612.000 | 720.000 | 510.000 | 450.000 | 500.000 | 391.000 | 224.000 |
| Standard Deviation of *AGR* | 0.069 | 0.064 | 0.057 | 0.050 | 0.047 | 0.041 | 0.026 |
| Skewness of *AGR* | 2.310 | 1.887 | –1.440 | –0.411 | –0.309 | –0.823 | –0.783 |
| Average deviation above of *AAGR* | 0.035 | 0.031 | 0.024 | 0.022 | 0.022 | 0.016 | 0.009 |
| Average deviation below of *AAGR* | 0.031 | 0.030 | 0.032 | 0.026 | 0.025 | 0.022 | 0.015 |
| Kurtosis of *AGR* | 31.899 | 24.663 | 20.598 | 7.350 | 9.619 | 15.762 | 5.501 |
| Median of *AGR* | 0.017 | 0.017 | 0.016 | 0.022 | 0.025 | 0.028 | 0.022 |

* The cluster center corresponds to the arithmetic average defined at the procedure of clustering from the *PCGDP*.

**Table 2.** Panel unit root and stationarity tests of *PCGDP*, 1950-2000 (first difference with intercept).

| Test | Cluster *i* | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Levin, Lin and Chu | I(0) | I(0) | I(0) | I(0) | I(0) | I(0) | I(0) |
| Breitung | I(0) | I(0) | I(0) | I(0) | I(0) | I(0) | I(0) |
| Im, Pesaran and Shin | I(0) | I(0) | I(0) | I(0) | I(0) | I(0) | I(0) |
| Fisher-ADF | I(0) | I(0) | I(0) | I(0) | I(0) | I(0) | I(0) |
| Fisher-PP | I(0) | I(0) | I(0) | I(0) | I(0) | I(0) | I(0) |
| Hadri | No stat | No stat | No stat | No stat | No stat | No stat | No stat |

**Table 3.** Relative frequency and positive significant, negative significant and no significant growth rates by cluster, 1950-2000.

| Cluster | Relative frequency / *AAGR* | | | | | | *AAGR** % | S. D. of *AAGR* % | C. V. of *AAGR* % |
|---|---|---|---|---|---|---|---|---|---|
| | Negative | | Non significant | | Positive | | | | |
| 1 | 13.42 | –0.07 | 34.92 | 0.00 | 51.67 | 0.04 | 1.3527 | 1.5085 | 1.1151 |
| 2 | 18.21 | –0.05 | 25.00 | 0.00 | 56.79 | 0.04 | 1.3341 | 1.5019 | 1.1257 |
| 3 | 26.70 | –0.04 | 10.00 | 0.00 | 63.30 | 0.03 | 0.9344 | 1.3735 | 1.4699 |
| 4 | 10.74 | –0.06 | 34.95 | 0.00 | 54.32 | 0.05 | 1.8452 | 1.5856 | 0.8593 |
| 5 | 12.29 | –0.05 | 20.29 | 0.00 | 67.43 | 0.04 | 2.0143 | 1.6081 | 0.7983 |
| 6 | 3.76 | –0.09 | 25.06 | 0.00 | 71.18 | 0.04 | 2.3033 | 1.6364 | 0.7105 |
| 7 | 14.20 | –0.02 | 10.00 | 0.00 | 75.80 | 0.03 | 1.7169 | 1.4209 | 0.8276 |

* We get these rates using non significance rates, geometric averages of positive and negative rates, and the respective frequencies. The original rates by cluster are: 0.016, 0.016, 0.010, 0.020, 0.023, 0.025 and 0.019, respectively.
** C. V. = Coefficient of variation.

**Table 4.** Expected values, variances, co-variances, coefficients of correlation, eigenvalues and eigenvectors from multinomial model by cluster.

| | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cluster | | | 1 | | | 2 | | | 3 | | | 4 | | | 5 | | | 6 | | | 7 | |
| $m$ (decades) | | | 5 | | | 5 | | | 5 | | | 5 | | | 5 | | | 5 | | | 5 | |
| $n$ (regimens) | | | 3 | | | 3 | | | 3 | | | 3 | | | 3 | | | 3 | | | 3 | |
| $p_1$ (frec (+) rates) | AAGR (+) | 0.52 | | 0.04 | 0.57 | | 0.04 | 0.63 | | 0.03 | 0.54 | | 0.05 | 0.67 | | 0.04 | 0.71 | | 0.04 | 0.76 | | 0.03 |
| $p_2$ (frec (-) rates) | AAGR (-) | 0.13 | | -0.07 | 0.18 | | -0.05 | 0.27 | | -0.04 | 0.11 | | -0.06 | 0.12 | | -0.05 | 0.04 | | -0.09 | 0.14 | | -0.02 |
| $p_3$ (frec non sign rates) | AAGR (0) | 0.35 | | 0.00 | 0.25 | | 0.00 | 0.10 | | 0.00 | 0.35 | | 0.00 | 0.20 | | 0.00 | 0.25 | | 0.00 | 0.10 | | 0.00 |
| $E(X_1)$ | Years | 2.58 | | 25.83 | 2.84 | | 28.39 | 3.17 | | 31.65 | 2.72 | | 27.16 | 3.37 | | 33.71 | 3.56 | | 35.59 | 3.79 | | 37.90 |
| $E(X_2)$ | Years | 0.67 | | 6.71 | 0.91 | | 9.11 | 1.33 | | 13.35 | 0.54 | | 5.37 | 0.61 | | 6.14 | 0.19 | | 1.88 | 0.71 | | 7.10 |
| $E(X_3)$ | Years | 1.75 | | 17.46 | 1.25 | | 12.50 | 0.50 | | 5.00 | 1.75 | | 17.47 | 1.01 | | 10.14 | 1.25 | | 12.53 | 0.50 | | 5.00 |
| $(m+n-1)p_1$ | | 3.62 | | | 3.98 | | | 4.43 | | | 3.80 | | | 4.72 | | | 4.98 | | | 5.31 | | |
| $(m+n-1)p_2$ | | 0.94 | | | 1.28 | | | 1.87 | | | 0.75 | | | 0.86 | | | 0.26 | | | 0.99 | | |
| $(m+n-1)p_3$ | | 2.44 | | | 1.75 | | | 0.70 | | | 2.45 | | | 1.42 | | | 1.75 | | | 0.70 | | |
| $Var(X_1)$ $Cov(X_1,X_2)$ $Cov(X_1,X_3)$ | | 1.25 | -0.35 | -0.90 | 1.23 | -0.52 | -0.71 | 1.16 | -0.84 | -0.32 | 1.24 | -0.29 | -0.95 | 1.10 | -0.41 | -0.68 | 1.03 | -0.13 | -0.89 | 0.92 | -0.54 | -0.38 |
| $Cov(X_2,X_1)$ $Var(X_2)$ $Cov(X_2,X_3)$ | | -0.35 | 0.58 | -0.23 | -0.52 | 0.74 | -0.23 | -0.84 | 0.98 | -0.13 | -0.29 | 0.48 | -0.19 | -0.41 | 0.54 | -0.12 | -0.13 | 0.18 | -0.05 | -0.54 | 0.61 | -0.07 |
| $Cov(X_3,X_1)$ $Cov(X_3,X_2)$ $Var(X_3)$ | | -0.90 | -0.23 | 1.14 | -0.71 | -0.23 | 0.94 | -0.32 | -0.13 | 0.45 | -0.95 | -0.19 | 1.14 | -0.68 | -0.12 | 0.81 | -0.89 | -0.05 | 0.94 | -0.38 | -0.07 | 0.45 |
| $\rho(X_1,X_2)$ | | -0.41 | | | -0.54 | | | -0.79 | | | -0.38 | | | -0.54 | | | -0.31 | | | -0.72 | | |
| $\rho(X_1,X_3)$ | | -0.29 | | | -0.27 | | | -0.20 | | | -0.25 | | | -0.19 | | | -0.11 | | | -0.14 | | |
| $\rho(X_2,X_3)$ | | -0.76 | | | -0.66 | | | -0.44 | | | -0.80 | | | -0.73 | | | -0.91 | | | -0.59 | | |
| Eigenvalues | | | | | | | | | | | | | | | | | | | | | | |
| $\lambda_1$ | | 0.00 | | | 0.00 | | | 0.00 | | | 0.00 | | | 0.00 | | | 0.00 | | | 0.00 | | |
| $\lambda_2$ | | 0.86 | | | 1.03 | | | 0.66 | | | 0.71 | | | 0.74 | | | 0.27 | | | 0.58 | | |
| $\lambda_3$ | | 2.10 | | | 1.88 | | | 1.93 | | | 2.14 | | | 1.71 | | | 1.88 | | | 1.40 | | |
| $\Sigma \lambda_t$ | | 2.97 | | | 2.91 | | | 2.59 | | | 2.86 | | | 2.45 | | | 2.15 | | | 1.98 | | |
| $\theta_{est}$ | | 0.71 | | | 0.64 | | | 0.75 | | | 0.75 | | | 0.70 | | | 0.88 | | | 0.71 | | |
| Eigenvectors | | | | | | | | | | | | | | | | | | | | | | |
| $v_1$ | | -0.58 | 0.81 | -0.06 | -0.58 | 0.78 | 0.26 | -0.58 | 0.49 | -0.65 | -0.58 | 0.81 | -0.05 | -0.58 | 0.79 | -0.22 | -0.58 | 0.82 | -0.04 | -0.58 | 0.63 | -0.52 |
| $v_2$ | | -0.58 | -0.46 | -0.67 | -0.58 | -0.61 | 0.54 | -0.58 | -0.81 | -0.10 | -0.58 | -0.45 | -0.68 | -0.58 | -0.58 | -0.57 | -0.58 | -0.44 | -0.69 | -0.58 | -0.77 | -0.28 |
| $v_3$ | | -0.58 | -0.35 | 0.74 | -0.58 | -0.17 | -0.80 | -0.58 | 0.32 | 0.75 | -0.58 | -0.36 | 0.73 | -0.58 | -0.20 | 0.79 | -0.58 | -0.37 | 0.73 | -0.58 | 0.14 | 0.80 |

As we can see in table 4, the greatest persistence or duration of the increase regime (measured by the expected value), expressed in years, is for cluster seven with 37.90 years and the lowest persistence for cluster one with 25.83 years. When we applied the respective bootstrap analysis in order to compare the persistence at this regime, we found the following order: $c_1 \leq c_4 = c_2 < c_3 \leq c_5 = c_6 < c_7$.

Cluster three has an expected value of decrease of 13.35 years, which is the maximum persistence among the seven clusters, whereas cluster six has a persistence of 1.88 years at this regime. For the decrease regime we tested the persistence in the decrease regime with bootstrap and we found this next situation: $c_6 < c_4 = c_5 = c_1 = c_7 < c_2 < c_3$ (table 4).

Clusters seven and three showed an expected value of stagnation of five years (minimum persistence), whereas clusters one and four exhibited a value of 17.47 years (maximum persistence). The bootstrap analysis indicates the following order in the persistence of the stagnation regime by cluster: $c_7 = c_3 < c_5 < c_6 = c_2 < c_4 = c_1$ (table 4).

The greatest volatility (measured by the variance) of the increase regime corresponds to cluster one and the lowest volatility to cluster seven. So the bootstrap analysis indicates the following ascending order of volatility at this regime by cluster: $c_7 < c_6 < c_5 \leq c_3 < c_2 \leq c_4 = c_1$ (table 4).

The greatest volatility of the decrease regime belongs to cluster one and the lowest volatility at this regime is for cluster six. Using bootstrap, the ordering is: $c_6 < c_4 < c_5 < c_1 < c_7 < c_2 < c_3$ (table 4).

The greatest variance of the stagnation regime corresponds to clusters one and four, and the lowest one to clusters three and seven, for all the clusters, we have the next ordering: $c_7 = c_3 < c_5 < c_6 = c_2 < c_4 = c_1$ (table 4).

Because we are comparing different means, a right measurement for the dispersion analysis is the coefficient of variation, since the standard deviation is only significant in relation to the mean to which it is calculated. In this case we have the following ascending ordering for the positive growth regime: $c_4 < c_7 < c_6 < c_5 < c_3 < c_2 < c_1$ (table 4).

In the decrease regime, the ascending order of the coefficient of variation is this: $c_4 < c_3 < c_2 < c_7 < c_1 < c_5 < c_6$ (table 4).

Finally, the ascending order of the variation coefficient of the stagnation regime is the following one: $c_4 < c_1 < c_6 = c_2 < c_5 < c_3 < c_7$ (table 4).

These results show that for the period 1950-2000 the best growth rates,[9] which define the long term trend, were those from cluster six and five, with a 2.30 percent and 2.04 annual percent, respectively; with the worst results for cluster three with a growth rate of 0.93 annual percent (table 4). The ascending order of the coefficients of variation of the long term growth rates is: $c_6 < c_5 \leq c_7 < c_4 < c_1 \leq c_2 < c_3$ (table 4).

9. We got these by using the first moment or expected value.

As far as regression lines for the seven clusters that represent the regression equations of $Xi$ in $Xj$ $(\mu_{X_i|X_j})$ among two events of the three possible ones: increase ($X1 = X_C$), decrease ($X_2 = X_D$) and stagnation ($X_3 = X_D$), we found the best situations in clusters six and seven, and the worse ones in cluster three (table 4).

## Conclusions

The equiprobable growth model is an alternative window to look at the low dependence among growth rates across decades.

The approach used for this work establishes that long-term growth trends can be derived from a multinomial model of three regimes for seven clusters of economies during the second half of the century XX; thus, our procedure can be placed in the second vision raised by Higgins (1955): the trend can be derived from a model of growth cycle.

Our concept of cycle gives alternative information to analyze persistence and volatility, and so we define a relation between growth and volatility as suggested by Zarnowitz (1997): periods and countries with high growth had low instability, whereas great fluctuations reduce growth.

This document shows that similar economies have similar economic fluctuations: persistence and volatility by growth regime; but, we can't use a unified explanation for all economies of the world, nor idiosyncratic explanations.

Our results can't refute Lucas´ (1988) affirmations: Within advanced countries, growth rates tend to be very stable over long periods of time, but for poor countries there are many examples of sudden, large changes in growth rates; the richest countries register little variability, whereas in poor countries and in the average income countries an enormous variability exists. So the poorest countries register the lowest growth, the wealthiest countries obtain the next rates, but middle income countries have the highest of them.

We employ the growth cycles approach (in which the periods of expansion and contraction are represented as cyclical movements around a trend, but not every negative growth rate or positive growth rate is significant), and we follow Kyndland and Prescott's (1990) criteria in order to decompose a series in its cyclical and trend components.

In this approach, the probabilities or frequencies of occurrence of the various events or regimes are regarded as constant from trial to trial (decade to decade), but it's possible to consider a case where the probabilities are constant within a set of trials, but vary randomly from set to set, at this situation we can use the multivariate *b*-distribution or Dirichlet distribution and the compound multinomial distribution or Dirichlet multinomial distribution.

These probabilities are comprised of non-negative components and which's sum is one, so that we can use compositional time series to estimate trends, covariates, interventions and forecasting. ergo

## References


Adelman, I. and F. L. Adelman (1959). "The Dynamic Properties of the Klein-Goldberger Model," *Econometrica*. Vol. 27, Num. 4.

Aghion, P. and P. Howitt (1992). "A Model of Growth Through Creative Destruction," *Econometrica*. Vol. 60.

Argandoña, A.; C. Gámez and F. Mochón (1997). *Macroeconomía Avanzada II: Fluctuaciones Cíclicas y Crecimiento Económico*. McGraw-Hill, Sapain.

Arias, A.; G. D. Hansen and L. E. Ohanian (2006). "Why Have Business Cycle Fluctuations Become Less Volatile?" *Working Paper* No. 12079, National Bureau of Economic Research.

Artis, M.; Z. G. Kontolemis and D. R. Osborn (1997). "Business Cycles for G7 and European Countries," *Journal of Business*. Vol. 70, Num. 2.

Avella, M. y L. Ferguson (2003). "El ciclo económico. Enfoques e ilustraciones: Los ciclos económicos de Estados Unidos y Colombia". Banco de la República. <http://www.banrep.gov.co/docum/ftp/borra284.pdf>.

Backus, D. K.; P. J. Kehoe and F. E. Kydland (1995). "International Business Cycles: Theory and Evidence", in Cooley, T. F. (ed.) *Frontiers of Business Cycle Research*. Princeton University Press.

Baxter, M. (1995). "International Trade and Business Cycles", in Grossman, G. M. and K. Rogoff (eds.). *Handbook of International Economics*. Vol. 3, Amsterdam: North Holland.

Black, F. (1987). *Business Cycles and Equilibrium*. Cambridge, Mass: Basil Blackwell Inc.

Bernanke, B. S. (1983). "Irreversibility, Uncertainty, and Cyclical Investment", *Quarterly Journal of Economics*. Vol. 106, Num. 2.

Burns, A. F. and W. C. Mitchell (1946). "Measuring Business Cycles", *Studies in Business Cycles*. Num. 2, New York: National Bureau of Economic Research.

Burnside, C. (1998). "Detrending and Business Cycles Facts: A comment", *Journal of Monetary Economics*. Vol. 41, Num. 3.

Canova, F.

______ (1998a). "Detrending and Business Cycles Facts", *Journal of Monetary Economics*. Vol. 41, Num. 3.

______ (1998b). "Detrending and Business Cycles Facts: A user's guide", *Journal of Monetary Economics*. Vol. 41, Num. 3.

Cecchetti, S. G.; A. Flores-Lagunes and S. Krause (2006). "Assessing the Sources of Changes in the Volatility of Real Growth", *Working Paper No. 11946*. National Bureau of Economic Research.

Cermeño, R. (2002). "Growth Convergence Clubs: Evidence from Markov Switching Models Using Panel Data", División de Economía, CIDE. <http://econpapers.repec.org/cpd/2002/115_Cermeno.pdf>.

Córdova, J. C. and G. Verdier (2007). "Lucas vs. Lucas: On Inequality and Growth", *Working Paper No. 17*. International Monetary Fund. Vol. 7.

Crum, W. L. (1923). "Cycles of Rates of Commercial Papers", *Review of Economics and Statistics*. Vol. 5.

DeLong, J. B. and L. H. Summers (1986). "Are business cycles symmetrical?", *The American Business Cycle*. Chicago: Chicago University Press.

Easterly, W.; M. Kremer; L. Pritchett and L. H. Summers (1993). "Good Policy or Good Luck? Country Growth Performance and Temporary Shocks", *Journal of Monetary Economics*. Vol. 32.

Brandley, E. and R. J. Tibshirani (1993). *An Introduction to the Bootstrap*. U. S.: Chapman and Hall/CRC.

Frish, R. (1933). "Propagation Problems and Impulse Problems in Dynamic Economics", *Economic Essays in Honor of Gustave Cassel*. London: Allen and Unwin.

Gallego, F. and N. Loayza (2001). "The Golden Period for Growth in Chile Explanations and Forecasts", *Working Paper No. 146*. Central Bank of Chile.

Gancia, G. A. (2003). "Globalization, Divergence and Stagnation", *Seminar Paper*, No. 270. Institute for International Economic Studies.

Gordon, R. J. (2005). "What Caused the Decline in U. S. Business Cycle Volatility?" *Working Paper* No. 11777. National Bureau of Economic Research.

Granger, C. W. J. (2001). "Macroeconometrics - Past and Future", *Journal of Econometrics*. Vol. 100.

Gregory, A. W.; A. C. Head and J. Raynauld (1997). "Measuring World Business Cycles", *International Economic Review*. Vol. 38, Num. 3.

Hall, R. E. (2005). "Separating the Business Cycle from other Economic Fluctuations", *Working Paper No. 11651*, National Bureau of Economic Research.

Hamilton, J. D. (1989). "A New Approach to the Economic Analysis of Nonstationary Time Series and Business Cycle", *Econometrica*. Vol. 57, Num. 2.

Hernández-Veleros, Z. S. (2006). *Equiprobable Growth Model and World Income Distribution*. Research for DEA, Universidad de Barcelona.

Hicks, J. R. (1950). *A Contribution to the Theory of the Trade Cycle*. Oxford.

Higgins, B. (1955). "Interactions of Cycles and Trends", *The Economic Journal*. Vol. 65, Num. 260.

Hodrick, R. J. and E. C. Prescott (1997). "Postwar U. S. Business Cycles: An Empirical Investigation", *Journal of Money, Credit and Banking*. Vol. 29, Num. 1.

Kaldor, N. (1954). "The Relation of Economic Growth and Cyclical Fluctuations", *The Economic Journal*. Vol. 64, Num. 253.

Kalecki, M. (1954). *The Theory of Economic Dynamics*. London.

Keynes, J. M. (1936). *The General Theory of Employment, Interest, and Money*. London: Macmillan.

Kim, C. J.; C. Nelson and J. Piger (2004). "The Less-Volatile U. S. Economy: A Bayesian Investigation of Timing, Breadth, and Potential Explanations", *Journal of Business and Economic Statistics*. Vol. 22, Num. 1.

King, R. G. and S. T. Rebelo (1993). "Low Frequency Filtering and Real Business Cycles", *Journal of Economic Dynamics and Control*. Vol. 17.

King, R. G. and S. T. Rebelo (1999). "Resuscitating Real Business Cycles", in Taylor, J. B. and M. Woodford (eds.) *Handbook of Macroeconomics*. Vol. 1. Amsterdam: North Holland, pp. 927-1007.

Kose, M. A.

______ (2002). "Explaining Business Cycles in Small Open Economies: How Much Do World Prices Matter", *Journal of International Economics*. Vol. 56, Num. 2.

______; C. Otrok and C. H. Whiteman (2003) "International Business Cycles: World, Region, and Country-Specific Factors", *The American Economic Review*. Vol. 93, Num. 4.

Koopmans, T. C. (1947). "Measurement without Theory". *Review of Economic Statistics*. Vol. 29.

Kydland, F. E. and E. C. Prescott (1990). "Business Cycles: Real Facts and Monetary Myth", *Federal Reserve Bank of Minneapolis Quarterly Review*. Vol. 14, Num. 2.

Lucas, R. E. Jr.

_______ (1977). "Understanding business cycles", *Carnegie-Rochester Conference Series in Public Policy*. Vol. 5.

_______ (1987). *Models of Business Cycles*. United Kingdom: Basil Blackwell.

_______ (1988). "On the Mechanics of Economic Development", *Journal of Monetary Economics*. Vol. 22.

_______ (2000). "Some Macroeconomics for the 21st Century", *Journal of Economics Perspectives*. Vol. 14, Num. 1.

Maddison, A. (2003). *The World Economy: Historical*

*Statistics*. Paris: Organisation for Economic Co-operation and Development.

McGrattan, E. R. and J. A. Schmitz Jr. (1999). "Explaining Cross-Country Income Differences", in J. B. Taylor and M. Woodford (eds.). *Handbook of Macroeconomics*. Vol. 1. Amsterdam: North Holland.

McKay, A. and R. Reis (2006). "The Brevity and Violence of Contractions and Expansions", *Working Paper No. 12400*. National Bureau of Economic Research.

Mendoza, E. G. (1995). "The Terms of Trade, the Real Exchange Rate, and Economic Fluctuations", *Economic Journal*. Vol. 113, Num. 484.

Mejía-Reyes, P.

______ (1999). "Classical Businesss Cycles in Latin America: Turning Points, Asymmetries and International Synchro-nisation", *Estudios Económicos*. Vol. 14, Num. 2.

______ (2003). *No-linealidades y Ciclos Económicos*, México: El Colegio Mexiquense, A. C. and Universidad Autónoma del Estado de México, 237 pp.

Mills, T. C.

______ (2002a). *Long Term Trends and Business Cycles I*. The International Library of Critical Writings in Economics.

______ (2002b). *Long Term Trends and Business Cycles II*. The International Library of Critical Writings in Economics.

Mirmall, L. (1971). "Uncertainty and Optimal Consumption Decisions", *Econometrica*. Vol 39, Num. 1.

Mitchell, W. C. (1927). *Business Cycles: The Problems and its Setting*. New York: National Bureau of Economic Research.

Morgenstern, O. (1959). "International Financial Transactions and Business Cycles", *Studies in Business Cycles*. Num. 8. New York: National Bureau Economic Research.

NeftxI, S. N. (1984). "Are Economic Time Series Asymmetric over the Business Cycle?", *Journal of Political Economy*. Vol. 92.

OECD (Organization for Economic Co-operation and Development) (2002). *Economic Outlook*. Num. 71.

Osborn, D. R. (1995). "Moving Average Detrending and the Analysis of Business Cycles", *Oxford Bulletin of Economics and Statistics*. Vol. 57.

Otrok, C. (1999). "On Measuring the Welfare Cost of Business Cycles", *Virginia Economics Online Papers*. Num. 318. <http://ideas.repec.org/s/vir/virpap.html>

Parente, S. L. and E. C. Prescott (2004). "A Unified Theory of the Evolution of International Income Levels", *Staff Report No. 333*. Federal Reserve Bank of Minneapolis, Research Department.

Potter, S. M. (1995). "A Nonlinear Approach to US GDP", *Journal of Applied Econometrics*. Vol. 10.

Pindyck, R. S. (1991). "Irreversibility, Uncertainty, and Investment", *Journal of Economic Literature*. Vol. 29, Num. 3.

Ramey, G. and V. A. Ramey

______ (1991). "Technology Commitment and the Cost of Economic Fluctuations", *Working Paper No. 3755*. National Bureau of Economic Research.

______ (1994). "Cross-Country Evidence on the Link Between Volatility and Growth", *Working Paper No. 4959*. National Bureau of Economic Research.

Rebelo, S. (1991). "Long Run Policy Analysis and Long Run Growth", *Journal of Political Economy*. Vol. 99, pp. 500-521.

Romer, C. D.

______ (1986a). "Spurious Volatility in Historical Unemployment Data", *Journal of Political Economy*. Vol. 94.

______ (1986b). "Is the Stabilization of the Postwar Economy a Figment of the Data", *American Economic Review*. Vol. 76.

______ (1989). "The Prewar Business Cycle Reconsidered: New Estimates of Gross National Product, 1869-1908", *Journal of Political Economy*. Vol. 97.

______ (1994). "Remeasuring Business Cycles", *Journal of Economic Perspectives*. Vol. 7.

______ (1999). "Changes in Business Cycles: Evidence and Explanations", *Journal of Economic Perspectives*. Vol. 73.

Romer, D. (2002). *Macroeconomía Avanzada*. Ed. McGraw Hill.

Romer, P. (1990). "Endogenous Technological Change", *Journal of Political Economy*. Vol. 98.

Schumpeter, J. (1939). *Business Cycles: A Theorical, Historical and Statistical Analysis of the Capitalism Process*. New York: Harvard Published.

Sichel, D. E. (1989). "Are Business Cycles Asymmetries?", *Journal of Political Economy*. Vol. 97.

Simkins, S. (1999). "Measurement and Theory in Macrecocomics", *Working papers in Economics*. <www. ncat. edu/cecondept/wp/mitchell paper.pdf>

Slutsky, E. E. (1937). "The Summation of Random Causes as the Source of the Cyclic Processes", *Econometrica*. Vol. 5.

Solow, R. (1956). "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics*. Vol. 70.

Stock, J. H. (2001) "Macro-econometrics", *Journal of Econometrics*. Vol. 100.

Teräsvirta, T. and H. M. Anderson (1992). "Characterising Nonlinearities in Business Cycles Using Smooth Transition Regression Models", *Journal of Applied Econometrics*. Vol. 7.

Tinbergen, J. and J. J. Polak (1950). *The Dynamics of Business Cycle*. Chicago. *Economica*. Vol. 18, No. 71.

Watson, Mark W. (1994). "Business Cycle Durations and Potswar Stabilization of the U. S. Economy", *American Economic Review*. Vol. 84.

Zarnowitz, V.

______ (1991). "What is a Business Cycle?" *Working Paper No. 3863*. National Bureau of Economic Research.

______ (1997). "Business Cycles Observed and Assessed: Why and How They Matter", *Working Paper No. 6230*. National Bureau of Economic Research.

## Annex

**Table A1.** **Debates about economic fluctuations.**

| Topic | Views and authors |
|---|---|
| Interaction cycles and trends | 1. Cycle and trend are indistinguishable (Schumpeter, 1939); 2. The trend can be derived from a model of trade cycle or business cycle (Tinbergen and Polak, 1950; Kalecki, 1954); 3. The information about the trend is essential to have a satisfactory explanation of the economic fluctuations (Hicks, 1950). |
| Causes of cycles and trends | 1. The same forces which produce violent booms and slumps will also tend to produce a high trend rate of progress (Kaldor, 1954). 2. Economic fluctuations have different causes than growth, but both are affected mutually by each other (Higgins, 1955); 3. Business cycles are most probably caused in part by uncontrollable outside disturbances and in part by errors of public policy and private decision makers that may be avoidable; but they are also, just as plausibly, to a large extent self-sustained and self-evolving (Zarnowitz, 1991). 4. Trend and cycles are interrelated and have common causes. Periods and countries with high growth had low instability (Zarnowitz, 1997); 5. Keynesian, monetarist and neoclassic of the models of the balance cycle with incomplete information consider that the variables of the supply side determine the trend, whereas the demand disturbances generate cyclical oscillations, being such independent variables (Argandoña *et al.*, 1997); 6. The theoreticians of the real business cycle consider real factors as causes of low frequency (growth) and high frequency (cycle) –Argandoña *et al.* (1997). |
| Simultaneous analysis of cycle and growth | 1. The modern macroeconomic analysts narrowed the approach to the study of the business cycle (Zarnowits, 1991); 2. Pioneers in this field stressed the importance of the endogenous cyclical processes and their connection with long term economic growth and development (Zarnowits, 1991). |
| Effects from elimination of the cycle on growth | 1. Growth and business cycle volatility are unrelated (Lucas, 1987); 2. Growth and volatility are linked positively (Black, 1987 and Mirman, 1971); 3. There is an inverse link between volatility and growth (Bernanke, 1983; Pindyck, 1991; Ramey and Ramey, 1991 and 1994). |
| When we have low persistence of growth rates, what does long-run depend growth on? | 1. It depends on country characteristics, and low persistence of growth rates implies that random shocks (luck) are important relative to policies in determining the long-run growth or deterministic spurts of growth are important in determining the long-run path of output (Rebelo, 1991; Romer, 1990 and Aghion and Howitt, 1992); 2. Growth is a worldwide process and country characteristics, like policies, determine steady state relative levels of income, shocks may play only a minor role in determining the long-run path of output, but except for those countries large and advanced enough to generate a significant share of world technology, long-run growth is exogenously determined. If there is a wide dispersion of distances between countries´ initial income and their steady states the countries furthest below their steady state will grow the fastest, growth rates will initially be highly persistent, here worldwide technological process and their policies define long-run growth; but if a country were close to its steady state income (determined by its policies) its growth rate will fall, and a large percentage of the time series variance in its growth rate would be explained by random shocks, at this situation we will have low persistence (Solow, 1956 and Easterly e*t al.*, 1993). |
| Is it correct a unified explanation about economic fluctuations? or, Do we need idiosyncratic explanations? | 1. We require an unified explanation them based on the general laws that prevail to the market economies (Lucas, 1977); 2. The nature of business cycles depends on, and changes with, the major characteristics of the economy, society and the polity (Zarnowitz, 1991). |
| Can a country eliminate the business cycles effects? | 1. Advances economies can do it (Lucas, 1988); 2. Poor economies can not eliminate the business cycles effects (Lucas, 1988). These paragraphs go against the affirmations of the same author in his articles of 1977 and 1987. |
| Controlled by income, does it change the growth-volatility relationship? | 1. The poorest countries register the lower growth rates, richest have higher rates, but than those of highest rates are the countries of average income; and also, Lucas (1988) affirms that between the richest countries little variability is registered, whereas in the poor countries and of average income an enormous variability exists. 2. These paragraphs go against the affirmations of the same author in his articles of 1977 and 1987. |
| Does economic moves along a smooth growth trend with temporary cyclical departures? | 1. Yes, it does (Lucas, 1977 and Kydland and Prescott, 1990);2. No, it doesn't - Hall (2005). |
| How do we eliminate trends? | 1. Using statistical approaches (we have a problem of "measurement without theory") (Canova, 1998a); 2. Using economic approaches (the choice among diverse methods of decomposition based on economic approach is arbitrary) (Canova, 1998a). |
| Are symmetric expansions and contractions? | 1. Yes, they are (Crum, 1923; Mitchell, 1927; Keynes, 1936 and Neftxi, 1984);2. No, they aren't (DeLong and Summers, 1986). |

**Table A2.** Economies by cluster grouped according to their 1950 *PCGDP*.

| Cluster | Economies |
|---|---|
| 1 | Equatorial Guinea, Botswana, China, Cape Verde, Lesotho, Burma, Cambodia, Mongolia, Mauritania, Nepal, Bangladesh, Burkina Faso, Mali, Rwanda, Guinea Bissau, Malawi, Eritrea and Ethiopia, Comoro Islands, Togo, Burundi, Guinea, Tanzania, Chad, Zaire. |
| 2 | Taiwan, South Korea, Oman, Thailand, Indonesia, Egypt, Swaziland, Yemen, Libya, Pakistan, India, Vietnam, Mayotte, S. Helena, West Sahara, Zimbabwe, São Tomé and Principe, Laos, North Korea, Nigeria, Cameroon, Kenya, Sudan, Gambia, Uganda, Zambia, Central African Republic, Afghanistan, Niger y Sierra Leone. |
| 3 | Palestine and Gaza, Tunisia, Dominican Republic, Sri Lanka, Jamaica, Romania, Algeria, 20 small Asian Countries, Albania, Philippines, Congo, Honduras, Senegal, Mozambique, Côte d'Ivoire, Benin, Ghana, Iraq, Somalia, Liberia, Haiti, Angola y Madagascar. |
| 4 | Japan, Greece, Malaysia, Turkey, Seychelles, Costa Rica, Panama, Brazil, Bulgaria, Iran, Yugoslavia, Jordan, Ecuador, Paraguay, El Salvador, Morocco, Bolivia, Nicaragua y Djibouti. |
| 5 | Singapore, Hong Kong, Spain, Puerto Rico, Portugal, Mauritius, South Arabia, Syria, Mexico, Poland, Hungary, 24 small Caribbean countries, Colombia, Bahrain, Reunion, South Africa, Namibia, Peru, Lebanon, Guatemala y Cuba. |
| 6 | Norway, Ireland, France, Belgium, Finland, Austria, 13 small Western Europe, Italy, Germany, Israel, Trinidad and Tobago, Chile, Czechoslovakia, Argentina, Uruguay, USSR y Gabon. |
| 7 | US, Denmark, Canada, Switzerland, Netherlands, Australia, Sweden, UK, New Zealand y Venezuela. |