Revista de Economía del Rosario. Vol. 14. No. 1. Enero - Junio 2011. 57 - 60

Adding one risk to another: generalizing the unavoidable (background) risk

Received: January, 2011 – Accepted: March, 2011

Moawia Alghalith*
Economics Department.
University of West Indies.

Resumen

Este artículo ofrece una generalización del modelo de riezgo subyacente. En primer lugar, se relajan los supuestos de independencia. En segundo lugar, se adopta una forma funcional general. Tercero, se adopta un tipo de riesgo general. Adicionalmente se presenta una nueva forma general de riesgo subyacente.

Clasificación JEL: D8, D2.
Palabras clave: Riesgo subyacente, incertidumbre.

Abstract

This paper provides a complete generalization of the background risk models. In so doing, first, it relaxes the independence assumption. Second, it adopts a general functional form. Third, it adopts a general type of risk. Furthermore, it introduces a new general form of background risk.

JEL Classification: D8, D2.
Keywords: Background risk, uncertainty.

*UWI. St. Augustine, Trinidad. Tel: (868) 7245804. E-mail: malghalith@gmail.com

ISSN 0123-5362 - ISSNE 2145-454x

1 Introduction

A large number of studies examined background risk due to the belief that background risk impacts the wealth and hence the welfare of the agent (see, for example Menoncin (2002), among others). However, the impact of background risk on the agent's decision variable(s) was not examined by the literature. Since the agent has control over the decision variable(s) only, it is much more useful to investigate the impact of background risk on the agent's decision variable(s).

The literature focused on the impact of the additive form of background risk. Examples include Gollier and Pratt (1996), Quiggin (2003) and Machina (1982). On the other hand, the multiplicative background risk was largely neglected. Exceptions include Franke et al. (2003) and Pratt (1988). Studies dealing with more general forms of background risk are virtually non-existent. Even the ones that examined the additive or multiplicative form provided restrictive models and results. They placed restrictions on the functional form, probability distributions, and/or the characteristics of the risk such as undesirable risk. For example, Gollier and Pratt (1996) and Franke et al. (2003) adopted undesirable risk and restricted the functional form of utility.

Another important restriction which is imposed by the previous models is the statistical independence assumption. That is, the background risk is independent of the other risk(s). Moreover, background risk is normally incorporated in choice models (Quiggin, 2003). Therefore, the impact of background risk on production decisions is rarely investigated. In sum, all the previous models are restrictive in multiple aspects.

This paper overcomes all the restrictions and limitations of the previous models. First, it relaxes the independence assumption. Second, it adopts a general functional form. Third, it adopts a general type of risk (as opposed to undesirable risk or mean-zero risk). Fourth, it introduces a new general form of background risk. Finally, it incorporates background risk into theory of the firm, as opposed to choice models. In sum, this paper serves as a general theory of modeling background risk and the impact of adding one risk to another.

2 The model

Profit is given by $\tilde{\pi} = \tilde{\eta}(\tilde{\varpi} + py = c(y)) \equiv \tilde{\eta}(\tilde{\varpi} + \Pi)$, where y is output, p is price, c is the cost function, $\tilde{\varpi}$ is a random variable representing wealth (Franke et al., 2003) or additive background risk, $\tilde{\eta} > 0$ is a multiplicative background risk.[1] The risk averse firm maximizes the expected utility of the profit

$$\underset{y}{Max}\ Eu(\tilde{\pi})$$

where u is a von Newmann-Morgestern utility function.

[1]Franke et al. (2003) used the restriction $E\tilde{\eta} \leq 1$, while Gollier and Pratt (1996) used the restriction $E\tilde{\varpi} \leq 0$

Lemma 1. *In the presence of a multiplicative background risk only (i.e., $\tilde{\varpi} = 0$) output is equal to its certainty-equivalent level.*[2]

Proof. The first order condition is

$$(p - c')(y^*)Eu'(\tilde{\pi}^*)\tilde{\eta} = 0 = (p - c'(y*)) \tag{1}$$

since $Eu'(\tilde{\pi}*)\tilde{\eta} > 0$, clearly the certainty equivalent output $\bar{y}$ satisfies the condition $p = c'(\bar{y})$ and thus $\bar{y} = y^*$. □

Theorem 2. *A. Starting with wealth risk, adding a multiplicative background risk does not affect the optimal output.*

B. Starting with a multiplicative background risk, adding a wealth risk does not affect the optimal output

Proof. A. The first-order condition in the presence of both risks

$$(p - c')(y^{**})Eu'(\tilde{\pi}^{**})\tilde{\eta} = 0 = (p - c'(y^{**})) \tag{2}$$

therefore, using the lemma, $\bar{y} = y^* = y^{**}$.

The proof of B is similar and thus omitted. □

3 General background risk

A general form of gackground risk is specified as $\tilde{\pi} = g(\tilde{\varpi} + \Pi, \tilde{\eta})$. The first order condition is

$$(p - c')(y^{**})Eu'(\tilde{\pi})g'(\cdot) = 0 = (p - c'(y)) \tag{3}$$

since $Eu'(\tilde{\pi})g'(\cdot) \neq 0$.

Thus the results of the theorem hold under a general background risk. That is, a general background risk does not impact the optimal output. Clearly, under the multiplicative form $g'(\cdot) = \tilde{\eta}$ and $g'(\cdot) = 1$ under the additive form. An example of another form of background risk is $\tilde{\pi} = ()^{\tilde{\eta}}$.

An important implication of this result is that even though, the addition of the background risk may affect the welfare of the agent, it does not influence his decision if the price of the asset is certain.

References

Franke, G., H. Schlesinger, and R. C. Stapleton (2003, May). Multiplicative background risk. (52), 146–153.

Gollier, C. and J. W. Pratt (1996). Risk vulnerability and the tempering effect of background risk. *Econometrica 64*(5), 1109–1123.

Machina, M. J. (1982). Expected utility analysis without the independence axiom. *Econometrica 50*(2), 277–323.

[2]

Menoncin, F. (2002). Optimal portfolio and background risk an exact and an approximated solution. *Insurance: Mathematics and Economics 31*(2), 249–265.

Pratt, J. (1988). Aversion to one risk in the presence of others. *Journal of Risk and Uncertainty 1*(4), 395–413.

Quiggin, J. (2003). Background risk in generalized expected utility theory. *Economic Theory 22*(3), 607–611.