REDIMAT
Revista de Investigación en Didáctica de las Matemáticas
Hipatia Press
www.hipatiapress.com

Instructions for authors, subscriptions and further details:

http://redimat.hipatiapress.com

Elementary Students' Spontaneous Metacognitive Functions in Different Types of Mathematical Problems

Evagelos Mokos[1], Sonia Kafoussi[1]

1) University of the Aegean.

Date of publication: June 24th, 2013

To cite this article: Mokos, E., Kafoussi, S. (2013). Elementary Students' Spontaneous Metacognitive Functions in Different Types of Mathematical Problems. *Journal of Research in Mathematics Education,* 2 (2), 242-267. doi: 10.4471/redimat.2013.29

To link this article: http://dx.doi.org/10.4471/redimat.2013.29

REDIMAT - Journal of Research in Mathematics Education Vol. 2 No. 2 June 2013 pp. 242-267.

Elementary Students' Spontaneous Metacognitive Functions in Different Types of Mathematical Problems

Evagelos Mokos & Sonia Kafoussi
University of the Aegean

Abstract

Metacognition is the mind's ability to monitor and control itself or, in other words, the ability to know about our knowing (Dunlosky & Bjork, 2008). In mathematics education, the importance of the investigation of students' metacognition during their mathematical activity has been focused on the area of mathematics problem solving. This study investigates the spontaneous emergence of the metacognitive functions of control and monitoring, during the solving of different types of mathematical problems with fifth grade students. We used the "think aloud" method on a group of ten year old students and the results showed that metacognitive strategies were used by the students so as the metacognitive functions of control and monitoring to be achieved.

Keywords: metacognition, control, monitoring, problem solving, elementary education.

2013 Hipatia Press
ISSN 2014-3621
DOI: 10.4471/redimat.2013.29

REDIMAT - Journal of Research in Mathematics Education Vol. 2 No. 2 June 2013 pp. 242-267.

Funciones Metacognitivas Espontáneas de los Estudiantes de Primaria en Diferentes Tipos de Problemas Matemáticos

Evagelos Mokos & Sonia Kafoussi
University of the Aegean

Abstract

Metacognición es la habilidad de la mente de monitorear y autorregular los procesos propios o, en otras palabras, la capacidad de conocer nuestro propio razonamiento (Dunlosky & Bjork, 2008). En la enseñanza de la matemática, la importancia de la investigación de la metacognición en escolares durante su actividad matemática está enfocada en el área de la resolución de problemas matemáticos. Este trabajo investiga el afloramiento espontáneo de las funciones metacognitivas de control y monitoreo, durante la resolución de diversos tipos de problemas matemáticos en escolares de quinto grado. Utilizamos el método "pensar en voz alta" en un grupo de escolares de diez años y los resultados indicaron que las estrategias metacognitivas fueron utilizadas por los escolares para lograr así las funciones megacognitivas de control y de monitoreo.

Keywords: Metacognición, control, monitoreo, resolución de problemas, educación primaria.

2013 Hipatia Press
ISSN 2014-3621
DOI: 10.4471/redimat.2013.29

The concept of metacognition has gained a lot of interest in mathematical education research and practice (cf. Ku & Ho, 2010; Mevarech & Kramarski, 1997; NCTM, 2000; Schoenfeld, 1985). In general the usefulness of the research of student's metacognitive strategies during mathematical activities is connected to the efforts made by students to acquire consciousness on their actions while they are learning mathematics. The researches on metacognition emphasize on the importance of the conscious control of the thought upon cognition during problem solving and support the impact of metacognitive strategies on the construction of new knowledge, so that metacognition can facilitate the development of students' learning.

Most of the researches have been focused on the relationship between mathematical problem solving and the use of metacognitive strategies. A review of the literature indicates that metacognition can reinforce the ability of students to become better problem solvers, because metacognitive strategies support the efforts during problem solving (Fortunato et al., 1991; Kapa, 2001; Mevarech & Kramarski, 1997; Mohini & Nai, 2005; Schoenfeld, 2007). The more the students control and monitor the strategies they use, they acquire better abilities to solve problems (Kapa, 2001; Mevarech & Fridkin, 2006; Schoenfeld, 1992). In other words, metacognition supports the cognitive level, through the activation of the monitoring and control functions during mathematical problem solving.

The purpose of our study is to investigate the spontaneous emergence of the metacognitive functions of control and monitoring, of elementary students at the age of 10 years old (fifth grade), during the solving of different types of mathematical problems. The contribution of this study to the literature will be to present proposals related to education planning for instructional intervention on the problem solving according to different types of mathematical problems. In primary education the need of the emergence of metacognitive functions is a vital aspect of mathematical problem solving, as it makes students better problem solvers (Shoenfeld, 1992).

Theoretical Background

Theoretical Definitions

John Flavell in 1976 defined metacognition as follows:

> "In any kind of cognitive transaction with the human or non-human environment, a variety of information processing activities may go on. Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in service of some concrete goal or objective." (p.232).

In his article "Metacognition and cognitive monitoring" (1979), he had proposed a model of metacognition, which included four stages of phenomena and their relation among them: a) metacognitive knowledge (one's knowledge or beliefs about the factors that affect cognitive activities), b) metacognitive experiences (the subjective internal responses of an individual to his/her own metacognitive knowledge, goals or strategies), c) tasks and goals (the desired outcomes or objectives of a cognitive venture) and d) strategies (ordered processes used to control one's own cognitive activities and to ensure that a cognitive goal). Flavell mentioned the usefulness of metacognition in a wide range of appliances, that included reading, oral speech, writing, the acquisition of speaking, memory, attention, social interactions, self-teaching, the evolution of personality and education (Flavell, 1979). According to Flavell, metacognitive procedures can be used consciously or unconsciously.

Later, Ann Leslie Brown (1987) separated metacognition into two categories. The first one is related to the knowledge of cognition which in turn includes reflection on cognitive skills and activities. The second one is related to self – monitoring mechanisms that are activated during the procedure of learning or problem solving. This procedure is according to Brown the regulation of cognition. These two categories, knowledge of cognition and regulation of cognition are very close related to each other (Brown, 1987). Brown's knowledge of cognition corresponds to Flavell's metacognitive knowledge and metacognitive experiences, while regulation of cognition corresponds to tasks, goals and strategies (Gama, 2004).

In the 1990s Nelson and Narens (1990) managed to organize and compose almost the whole existing research on metacognition (Schraw & Moshman, 1995). This model focuses on the interaction between two metacognitive functions: monitoring and control. Nelson and Narens proposed a theoretical mechanism, which is necessary so as to have a metacognitive system, and is composed of two structures: the meta-level and the object-level, and also the flow of information relationship between the two levels. In this model, information flows with the meta-level acquiring information from the object-level (monitoring) and the meta-level sending information to and thereby changing the object-level (control) (Dunlosky & Bjork, 2008).

Nelson and Narens (1994) argued that the meta-level includes the following components: a) a dynamic model of the existing situation of the object-level which is based on information from the monitoring procedure, b) a representation of a target or a situation, c) a list of possible control actions with which the meta-level can change/control the object-level, and it also includes details related to the time needed for a control action to be used, as well as the consequences of this action, d) a list of restrictions on potential control actions (e.g. restrictions on time, beliefs, expectations), e) a judgment or a procedure of decision making which assess the meta-model and leads to a decision according to which course of action might be implemented or which answer might be given for a target to be achieved (Van Overschelde, 2008).

According to the Nelson's and Narens model, metacognitive control includes the conscious or unconscious decisions that we make and are based on the outcome of the monitoring procedures that are made by ourselves. The control actions are revealed by the behaviors that one adopts as a result of the function of monitoring. So if someone feels that an item has not been adequately coded in his mind, then he may continue studying it. For example, if someone feels that he has not comprehended adequately a passage, then he may restudy it.

On the other hand, metacognitive monitoring includes the procedures that allow to the person to observe, to reflect or to have experiences on his own cognitive procedures. So, someone knows that he may have acquired a mathematical procedure or that he has understood the meaning of a passage that has already studied. Monitoring informs

persons for the statement of their knowing in accordance with the ongoing target (Schwartz & Perfect, 2001).
The following diagram is a representation of Nelson and Narens' metacognitive model with a meta-level and an object-level.

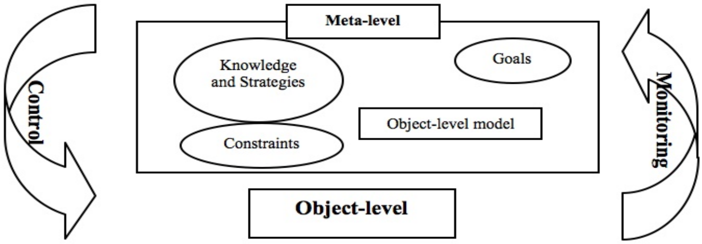

Figure 1. Nelson and Naren's metacognitive model (Van Overschelde, 2008, p. 48)

Instructional Approaches

In mathematics education, firstly Schoenfeld (1985) presented a theory of the interaction between the cognitive and metacognitive procedures that take place while students solve mathematical problems and denoted four aspects of knowledge and behavior: sources (mathematical knowledge), heuristics (ways of solving a mathematical problem), control (metacognition) and beliefs (attitudes). While teaching tends to focus on the two first aspects, the failure of the students to solve problems seems to appear due to the malfunction of the two latter ones. This means that, students have the required mathematical knowledge, but they fail to use it, because they cannot control and monitor it (Schoenfeld, 1992). According to Schoenfeld (1992), we could mention that metacognition helps students to become more effective problem solvers, because they are capable of defining their targets, monitoring their thoughts and assessing whether their actions lead to the target.

Montague (1992) defined three metacognitive strategies which support the functions of monitoring and control: a) self – instruction which helps students to discriminate the components of a mathematical problem, before the total solution of the problem, b) self – question, which is guided by the self – dialogue, that means a methodological analysis of the problem, based on the discovery of the relationships among the components of the problem, c) self – monitoring which encourage the student to control the whole procedure. These metacognitive strategies control and monitor the cognitive ongoing procedures.

More recently, Kapa (2001) suggested a model where separate metacognitive functions appear for each of the phases of a problem-solving process. The metacognitive knowledge (meta-level) may affect cognitive tasks (object-level) in each problem-solving phase as described in Table 1.

Table 1
Metacognitive functions classified according to the problem-solving process phases (Kapa, 2001, p. 318)

a) Problem identification	Collecting data, coding and remembering
b) Problem representation	Analogy, inference, imaginativeness, selective comparison and combination
c) Planning how to solve	Integration, conceptualization, heuristic choosing and formulating
d) Planning performance	Controlling and monitoring performance components of algorithmic mathematical knowledge and appropriate rules
e) Evaluation	Adjusting and contradicting a few possible solutions or suggesting alternative solution methods

Many researchers have studied the effect of metacognitive strategies on mathematical problem solving situations, aiming at the investigation of the existence of something that can be taught, which in turn would help students to better succeed on the process of the solving procedure (Biryukov, 2004; Kapa, 2001; Kramarski et al., 2002; Mevarech & Fridkin, 2006; Mohini &Nai, 2005).

Some researchers showed that mathematical problem solving in a cooperative environment (Goos et al., 2002; Kramarski et al., 2002) can be better succeeded if there is simultaneously a metacognitive kind of teaching or some kind of metacognitive questions. Kramarski, Mevarech and Arami (2002) investigated the different results of collaborative learning with or without metacognitive tutoring, on high or low mathematical performance students. A critical aspect of metacognitive teaching is the training of the students to work in small groups, on mathematical reasoning, by answering specific metacognitive questions. The aim of that study was to compare the results of cooperative – metacognitive and cooperative instruction, while solving authentic mathematical tasks, as well as to replicate the findings of previous researches on cooperative – metacognitive learning by teachers who use metacognitive teaching in their classrooms. They concluded that students can obtain better scores on mathematical problem solving when they perform in small groups, creating social interactive environments which are based on a series of metacognitive kind of questions. Moreover, Mevarech and Fridkin (2006) studied the effect of a metacognitive teaching method, called IMPROVE, on problem solving and mathematical reasoning. The results showed that the students who were taught by the IMPROVE method cultivated a higher level of metacognition than those of the control team.

Goos, Galbraith and Renshaw (2002) conducted a research which was focused on problem solving in a collaborative sociocultural context with metacognitive cooperation. The results showed that teacher has to play a crucial role during the collaboration among students.

Mohini and Nai (2005) analysed the comprehension and the decision of which metacognitive performance is associated with the successful problem solving. The “thinking aloud” method was used and the results showed that students with high level of self-reflection can ask themselves continually about the process of the problem solving.

Generally in the researches that have been held for the scrutiny of the relationship between mathematics and meacognition, authentic and open – ended mathematical problems have been used as this kind of problems seem to be more suitable for the trace of metacognitive behavior (Biryukov, 2004; Kapa, 2001; Kramarski et al., 2002; Mevarech & Fridkin, 2006). Additionally the problems that were given were of one specific kind in each research (i.e. open-ended or authentic problems). Moreover the researches conducted for the investigation of metacognitive functions in the learning of mathematics have mainly focused on problem solving, after the students have been given a metacognitive instruction. Finally, the majority of the researches were held in secondary education schools.

According to the above annotation the present study orientates its research question in the following way: Which metacognitive behaviors do students spontaneously emerge when they solve different kinds of mathematical problems, without a previous metacognitive instruction?

In our study we chose three different kinds of mathematical problems: open-ended, authentic and complex problems, as these problems are mostly presented in elementary school textbooks in our country. An open-ended problem is a problem which has more than one possible solutions. An authentic problem is the one that is encountered in a student's everyday life. A complex problem is the problem for which more than one mathematical operation are needed so as to reach its solution and it is the most often appeared in the Greek mathematical school textbooks. Although this distinction can't be considered as an absolute one, it was considered useful for the purpose of our study. The choice of these different kinds of the problems was done as we would like to investigate if the different types of problems influence the metacognitive behaviour of the students through the use of different metacognitive control and monitoring functions.

Methodology

This study is a qualitative research of a case study, as the research concerned all the students of one class. The study was conducted in a fifth grade of a typical[1] public elementary school in Athens, in Greece. The sample was 20 students (10 boys and 10 girls). The study lasted one month (April 2010). The fifth grade was chosen because the students at this grade seem to be capable of understanding and producing metacognitive type questions (Focant et al., 2006).

Three kinds of problems were given to the students: an open-ended problem, an authentic problem and a complex problem. The problems were the following:

a) When we play a game and we create pairs, one kid is left alone. When we create triples, one kid is left alone. Again when we create quadruples, one kid is left alone. How many kids are we?

b) You want to buy refreshments for your party in various packings:
- 500 ml which cost 1 €
- 1 litre which cost 1,20 €
- 1,5 litre which cost 1,50 €

You know from theory that 1 litre equals 1.000 ml. In your party you will need 5 litres of refreshments. Which packing is the cheapest for you to buy for your party?

c) Someone bought a 90 square meter house for 2.300 € a square meter. He paid half of the price in advance and the rest of the sum in 25 monthly installments. Which was the price of each installment?

Three meetings were realized by the researcher with each student, one meeting for each type of problem correspondingly. Each meeting lasted about 10-20 minutes.

The trace of the metacognitive functions of control and monitoring was made by the "thinking aloud" method, during which the students solved the mathematical problems. "Thinking aloud" is a verbal method which can be used either by the teacher or by two students working together or by one student working individually (Goos & Galbraith, 1996; Hartman, 2001). According to Ericsson and Simon (1980) during the talk/think aloud method, the subjects declare every thought they make. They denote loudly their thoughts during an activity without the researcher's intervention. In a case of silence the researcher just says "please continue thinking aloud" or "please keep on talking". So the subject has to explain loudly why he/she took into consideration some data or how he/she solved the problem.

Each student was videotaped and individual metacognitive behaviors were traced to each student during problem solving process. Then the verbal reports were transferred as "thinking aloud" protocols. The sessions were held in the school, out of the classroom, in the computer laboratory. The subjects insured permission from their parents so as to participate in the research.

For the analysis of the data we used an analysis protocol for the "thinking aloud" method, which was based on the Metacognitive Awareness Index (MAI) from Schraw & Dennison (1994). The subject's verbal reports were attributed to the suitable metacognitive area that is controlled by MAI, that is they attributed to the level to which every metacognitive strategy from the Metacognitive Awareness Index referred to. For the purpose of the research, in order to adjust the theory of Nelson and Narens to our study we used the following analysis protocol:

Table 2
The adjusted MAI[2]

Object Level	
Control	Monitoring
Information management stragegies	Comprehension monitoring
1. He slows down when he encounters important information.	11. He periodically reviews to help him understand important relationships.
2. He consciously focuses his attention on important information.	12. He finds himself analyzing the usefulness of strategies while he studies.
3. He tries to break studying down into smaller steps.	13. He finds himself pausing regularly to check his comprehension.
4. He focuses on overall meaning rather than specifics.	Evaluation
Debugging strategies	14. He knows how well he did once he finishes a test.
5. He re-evaluates his assumptions when he gets confused.	15. He summarizes what he has learned after he finishes.
6. He stops and rereads when he gets confused.	16. He asks himself if he has considered all options after he solves a problem.
Planning	
7. He sets specific goals before he begins a task.	
8. He asks himself questions about the material before he begins.	

Object Level	
Control	Monitoring
9. He thinks of several ways to solve a problem and chooses the best one. 10. He reads instructions carefully before he begins a task.	
Meta-level	
Procedural knowledge	Declarative knowledge
17. He has a specific purpose for each strategy he uses. 18. He finds himself using helpful learning strategies automatically.	19. He is a good judge of how well he understands something.

The verbal reports (statements) referring to procedural and declarative knowledge agree with the meta-level of Nelson and Naren's model of metacognition, while Information Management Strategies, Debugging strategies, Planning, Comprehension Monitoring and Evaluation agree with the object-level of Nelson and Naren's model of metacognition (Dunlosky et al., 2008). Moreover, the verbal reports (statements) referring to Information Management Strategies, Debugging strategies, Planning and Procedural knowledge are control functions, while Comprehension Monitoring, Evaluation and Declarative knowledge are monitoring functions (Dunlosky et al., 2008).

Five per cent of the pupils' verbal reports were analyzed by an external researcher who was aware of the topic and there was a 99% correlation (cf. appendix for an illustrative example about the analysis of a pupils' verbal report that was chosen incidentally). The following table gives some examples of the verbal reports which indicate a metacognitive strategy used by the students when they were solving different types of mathematical problems.

Table 3
Verbal reports that indicate metacognitive strategies

	Verbal report	Metacognitive strategy
Evaluation	Do I have to do multiplication? I must think to do something.	He asks himself if he has considered all the possible solutions after he has solved a problem.
	So lets consider the problem once again …	The student revises what he has learned after he has finished.
Information Management Strategies	I am trying to think what mathematical operation should I use…	He is consciously focused on valuable information.
Declarative knowledge	I cannot solve it.	He is good at judging how well he has understood something.

Results

Firstly, the presentation of the results has been separately done for each type of the problems the students solved. Then there is a comparison of the results among the different kinds of the problems.

Open-Ended Problem

The next table (table 4) shows the metacognitive functions of the students for the open-ended problem. This table reveals that in the open-ended problem there was a strong metacognitive control action, in the Meta-level concerning the Procedural Knowledge (83,9% verbal reports). According to Nelson and Narens (1994) the students who solved the open-ended mathematical problem tried through the Procedural Knowledge to modify the Object-level, by taking the correct control action. Moreover, in the Object-level the students' verbal reports concerning Debugging strategies and Information Management Strategies were dominant, and control function is showed to be very high performed during the solution procedure of the open-ended problem. According to the adjusted MAI questionnaire for the

Information Management Strategies, the sub strategy which was dominant was the one that denotes that the students "consciously focus their attention on important information", while for the Debugging Strategies the sub strategy that was dominant was the one that denotes that the student "stops and rereads when gets confused". We should also mention that the verbal reports, which appeared during the solving process, were not in accordance with the number of the students that expressed these verbal reports.

Table 4
The open-ended problem

		Verbal reports	Number of students
Object-level	Information Management Strategies (Control)	10 (22,2%)	5
	Debugging Strategies (Control)	16 (35,7%)	9
	Planning (Control)	6 (13,3%)	6
	Comprehension Monitoring (Monitoring)	7 (15,5%)	6
	Evaluation (Monitoring)	6 (13,3%)	5
	Total	45	
Meta-level	Procedural Knowledge (Control)	47 (83,9%)	18
	Declarative Knowledge (Monitoring)	9 (16,10%)	8
	Total	56	

Authentic Problem

The following table (table 5) shows the metacognitive functions of the students for the authentic problem.

In the authentic task we could mention that there was a strong metacognitive control action in the Object-level concerning Information Management Strategies (48,1% verbal reports). According to the adjusted MAI questionnaire for the Information Management Strategies, the sub strategy which was dominant was the one that the student "tries to break studying down into smaller steps". Moreover, many verbal reports were also made referring to the Meta-level concerning the Procedural Knowledge strategies. So, according to Nelson and Narens' model, when the students solved the authentic problem tried to control their object-level through Information Management Strategies and they also tried to monitor it by evaluating the cognitive process as we can see from the high performance of the evaluation metacognitive strategy (18,1%).

Table 5
The authentic problem

		Verbal reports	Number of students
Object-level	Information Management Strategies (Control)	37 (48,1%)	15
	Debugging Strategies (Control)	9 (11,7%)	5
	Planning (Control)	6 (7,8%)	4
	Comprehension Monitoring (Monitoring)	11 (14,3%)	6
	Evaluation (Monitoring)	14 (18,1%)	11
	Total	77	
Meta-level	Procedural Knowledge (Control)	37 (75,5%)	17
	Declarative Knowledge (Monitoring)	12 (24,5%)	10
	Total	49	

Complex Problem

The table 6 shows the results of the metacognitive functions of the students for the complex problem.

In the complex problem there is a strong metacognitive control action in the Meta-level concerning Procedural Knowledge (87,3% verbal reports). This means that when the students solved the complex problem, they tried to control their Meta-level through strategies that refer to Procedural knowledge and monitor the same level through strategies that refer to Declarative knowledge. So the students revealed the same metacognitive behaviour to the one that showed at the solution of the open-ended problem. In the Object-level we cannot mention great differences in the appearance of the metacognitive functions the students revealed when they solved the complex problem. Their metacognitive functions were at a very low performance.

Table 6
The complex problem

		Verbal reports	Number of students
Object-level	Information Management Strategies (Control)	7 (26%)	7
	Debugging Strategies (Control)	6 (22,2%)	5
	Planning (Control)	4 (14,8%)	4
	Comprehension Monitoring (Monitoring)	6 (22,2%)	5
	Evaluation (Monitoring)	4 (14,8%)	4
	Total	27	
Meta-level	Procedural Knowledge (Control)	62 (87,3%)	20
	Declarative Knowledge (Monitoring)	9 (12,7%)	9
	Total	71	

The following table (table 7) shows the results according to each metacognitive function of control and monitoring throughout the three kinds of problems.

Table 7
Comparing the three kinds of problems

	Open-ended	Authentic	Complex	Total
Information Management Strategies (Control)	10 (18,5%)	37 (68,5%)	7 (13%)	54
Debugging Strategies (Control)	16 (51,6%)	9 (29%)	6 (19,4%)	31
Planning (Control)	6 (37,5%)	6 (37,5%)	4 (25%)	16
Comprehension Monitoring (Monitoring)	7 (29,2%)	11 (45,8%)	6 (25%)	24
Evaluation (Monitoring)	6 (25%)	14 58,3%)	4 (16,7%)	24
Procedural Knowledge (Control)	47 (32,2%)	37 (25,3%)	62 (42,5%)	146
Declarative Knowledge (Monitoring)	9 (30%)	12 (40%)	9 (30%)	30

Analysing the results according to the spontaneous appearance of each metacognitive function of control and monitoring throughout the three kinds of problems, this table shows that, in general, fewer strategies appeared in the Object-level than in the Meta-level. This reveals that when the students solved the mathematical problems, they used many metacognitive functions which activated the Meta-level, that is they tried to orient metacognitive goals, to overcome constraints, to adjust the incomplete model of the Object-level which exists in their Meta-level in order to coincide it with the real Object level, and finally to take the right control action and change the Object-level.

Moreover, concerning the different kinds of metacognitive strategies in the Object and the Meta-level, the metacognitive function of control was dominant in each type of the mathematical problems. More specifically, 54 verbal reports emerged that implied Information Management Strategies, 31 verbal reports that implied Debugging Strategies, 16 verbal reports that implied Planning strategies and 146 verbal reports that implied Procedural Knowledge strategies appeared throughout the three different types of mathematical problems. Concerning the appearance of monitoring actions, the students developed strategies in order to monitor their thought, so as to reach suitable control actions and consequently solve the problem. These findings mean that the students tried to assure themselves that they had understood something, scanning their declarative knowledge to find the suitable information or knowledge stored in the memory, so as to be used for the solution (Lenat, 1983).

Furthermore, the students spontaneously emerged the most meta-cognitive control and monitoring actions during the solution of the authentic problem. We can observe that:

- The Information Management, Comprehension Monitoring and Evaluation strategies were stronger in the authentic problem.
- The Debugging strategies were stronger in the open-ended problem.
- The Planning strategies were almost equal for the three types of mathematical problems.
- The strategies used by the Procedural Knowledge were stronger in the complex problem.

- The strategies used by the Declarative Knowledge were almost equal for the three types of mathematical problems.

These findings can maybe be interpreted if we mainly think about the nature of each problem and the students' experiences with them in the mathematics classroom. The open-ended problems are such by their nature that they need more Debugging strategies when students try to solve them, because as these problems have many solutions and not only one, the students try a solution and change it or transform it when they comprehend the particularities. The authentic problems need Information Management, Comprehension Monitoring and Evaluation

Conclusions

This study set out to examine spontaneous metacognitive functions which students emerge when they are engaged in different types of mathematical problems without previous metacognitive instruction. Our framework followed Nelson and Narens' (1990) model for metacognition. This framework provides a solid structure that accommodates the ideas presented in this study. The model that is constructed by two levels, the Object-level and the Meta-level and the interrelation between them by the Control and the Monitoring functions, worked on the dynamic aspects of personal learning. From the methodological perspective, the techniques developed to analyse the spontaneous appearance of metacognitive functions were based on the Metacognitive Awareness Index (Schraw & Dennison, 1994). This method helped us to underline the verbal reports which included metacognitive behaviour and to set off the interrelation between the Object and the Meta-level by the metacognitive functions of control and monitoring. Of course the small sample of the students that participated in this study cannot lead us to generalise the results, but we can mention some issues based on our findings.

A first conclusion of our study is the appearance of control and monitoring actions in both levels of our cognitive system, in each type of mathematical problems. We observed that in each type of problem, metacognitive strategies were spontaneously emerged, as these strategies were traced by the verbal reports of the students. Moreover, we could say that the metacognitive function of control was dominant in each type of mathematical problem. Furthermore, based on the data collected in this study, whatever type of problem the students solved, the Meta-level had a great role in the mathematical problem solving procedure. The Meta-level, with its components that consist it, such as constraints, goals, metacognitive knowledge and strategies and especially with its incomplete object-level model, seems to work on the problem solving procedure, tries to complete this model so as to provide through the control actions, the necessary changes to the real Object-level during the solution of the mathematical problem. So, as the students do have strong metacognitive behaviors in the Meta-level, a reinforcement from teachers in the Object-level and in specific metacognitive behaviors, through a metacognitive didactical intervention or a metacognitive program, could help.

A second issue of our study is that we can denote in each type of problem how each metacognitive function appeared. Although the metacognitive actions appeared to have a "normal" dispersion in Planning strategies, through the three types of problems, the Information Management, Comprehension Monitoring and Evaluation strategies were dominant in authentic problems. The Debugging strategies were stronger in the open-ended problem. This finding means that the spontaneous emergence of the metacognitive strategies isn't the same through the three types of problems. The acknowledgement of the students' possibilities could help the teacher to emphasize concrete aspects of their metacognitive strategies and to design a planned intervention in order to improve metacognitive functioning. Maybe there is a special care to be taken into consideration when complex and open-ended problems are taught in mathematics classrooms, as students may need more efforts by their side so as to reach a solving procedure.

In the Meta-level and for the Procedural knowledge the control actions showed a regularity and only in complex problems we can notice that the control actions were much more than in the other kind of problems. We can also notice that the monitoring strategies were very close and in about the some range in each of the three types of mathematical problems. That is, we didn't observe great differences considering the monitoring function in the meta-level. This means that the students used their monitoring actions when they solved mathematical problems and they tried to comprehend and evaluate the solving process, through their declarative knowledge, in a stable and consistent manner.

According to our findings, fourteen students solved correctly the open ended problem and less students solved correctly the authentic and the complex problem (six and eight students correspondingly). So the unfolding of the metacognitive functions of control and monitoring simply means that the students only monitored the solving procedure and took the right control actions so as to promote this procedure. However, this finding has to be searched in more depth as the verbal reports that were made by the students when they were solving different types of mathematical problems were not in accordance with the number of the students that participated in the research. This means that some students have made more than one verbal report of the same kind. So these students seemed to be more metacognitive compared to the others who had not made a verbal report of any kind. Hence, if a spontaneous appearance of a metacognitive function helps a student to solve a type of mathematical problem and if a student who uses more than once a metacognitive function in a specific type of mathematical problem, is helped to succeed during a solving procedure, then a guided metacognitive instruction is required so as to achieve better results in the mathematical classroom.

Finally, further research is needed on the interpretation of the spontaneous metacognitive functions of control and monitoring in connection with the metacognitive feelings and metacognitive judgments students have while solving different types of mathematical problems. Moreover, the research could focus on the teacher's teaching practices which would be more effective for the use of the metacognitive functions of control and monitoring during problem solving.

Notes

[1] A public primary school of education where the population of the pupils does not have considerable individual differences (i.e. the students are all Greek citizens). There also exist private primary schools and pilot primary schools under the supervision of the Universities.

[2] The adjusted MAI was implemented in a pilot research and the results revealed reliability of this instrument for the expanded research. The whole MAI includes 52 questions in a likert climax and it registers the knowledge of cognition and the regulation of cognition.

References

Biryukov, P. (2004). Metacognitive Aspects of Solving Combinatorics Problems. *International Journal for Mathematics Teaching and Learning*, 1 – 19.

Brown, A. (1987). Metacognition, executive control, self-regulation and other more mysterious mechanisms. In Frann Weinert & Rainer Kluwe (Eds.), *Metacognition, Motivation and Understanding* (65-115). London: LEA.

Dunlosky, J. & Bjork, R. (2008). The Integrated Nature of Metamemory and Memory. In Dunlosky, J., Bjork, R (Eds.), *Handbook of Memory and Metamemory* (pp. 11-28). Psychology Press.

Ericsson, K. & Simon, H. A. (1980). Verbal reports as data. *Psychological Review, 87*, 215–251.

Ericsson, K. & Simon, H. A. (1993). *Protocol analysis: Verbal reports as data*. Cambridge: MIT.

Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), *The nature of intelligence* (pp. 231–235). Hillsdale, NJ: Erlbaum.

Flavell, J. (1979). Metacognition and cognitive monitoring. *American Psychologist, 34*, 906-911.

Focant, J., Grégoire, J. & Desoete, A. (2006). Goal setting, planning and control strategies and arithmetical problem solving at grade 5.In A. Desoete and M. Veenman (Eds.), *Metacognition and Mathematics Education* (pp. 51-71). Nova Science Publishers, Inc, New York.

Fortunato, I., Hecht, D., Tittle, C.K. & Alvarez, L. (1991). Metacognition and problem solving. *Arithmetic Teacher 39*(4), 38–40.

Gama C., (2004). *Integrating Metacognition Instruction in Interactive Learning Environmets*. Doctoral Thesis, University of Sussex.

Goos, M., Galbraith P., (1996). Do it this way! Metacognitive strategies in collaborative mathematical problem solving. *Educational Studies in Mathematics, 30*(3), 229 – 260. Doi: 10.1007 /BF00304567.

Goos, M., Galbraith P., Renshaw, P., (2002). Socially Mediated Metacognition: Creating Collaborative Zones of Proximal Development in Small Group Problem Solving, *Educational*

Studies in Mathematics, 49(2), 193-223. Doi: 10.1023 /A:1016209010120.

Hartman, H. J., (2001). Developing students' metacognitive knowledge and strategies. In H. J. Hartman (ed.), *Metacognition in Learning and Instruction: Theory, Research, and Practice* (pp. 33–68). Kluwer Academic Publishers, Dordrecht, The Netherlands.

Kapa, E., (2001). A metacognitive support during the process of problem solving in a computerized environment. *Educational Studies in Mathematics, 47*(3), 317–336. Doi: 10.1023 /A:1015124013119.

Kramarski, B., Mevarech, Z., Arami, M., (2002). The Effects of Metacognitive Instruction on Solving Mathematical Authentic Tasks. *Educational Studies in Mathematics, 49*(2), 225 – 250. Doi:10.1023/A:1016282811724.

Ku, K., Ho, I., (2010). Metacognitive strategies that enhance critical thinking. *Metacognition and Learning*, 5(3), 251-267. Doi: 10.1007/s11409-010-9054-4.

Lenat D. B., (1983). Theory Formation by Heuristic Search. The nature of heuristics II: background and examples. *Artificial Intelligence 21,* 31-60.

Mevarech, Z., Kramarski, B., (1997). IMPROVE: A multidimentional method for teaching mathematics in heterogeneous classrooms. *American Educational Research Journal, 34*(2), 365-394. Doi: 10.3102/00028312034002365.

Mevarech Z., Fridkin, S., (2006). The effects of IMPROVE on mathematical knowledge, mathematical reasoning and metacognition. *Metacognition Learning, 1*(1), 85-97. Doi: 10.1007/s11409-006-6584-x.

Mohini, M., Nai, Tan Ten, (2005). *The use of metacognitive process in learning mathematics, Reform, Revolution and Paradigm Shifts in Mathematics Education* (pp. 159-162). Johor Bahru, Malaysia, Nov 25th – Dec 1st.

Montague, M., (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. *Journal of Learning Disabilities 25*(4), 230–248. Doi: 10.1177 /002221949202500404.

National Council of Teachers of Mathematics, (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Nelson, T. O., & Narens, L., (1990). Metamemory: A theoretical framework and new findings. *The Psychology of Learning and Motivation, 26,* 125–141.

Nelson, T. O., & Narens, L., (1994). Why investigate metacognition? In J. Metcalfe & A. P. Shimamura (Eds.), *Metacognition: Knowing about knowing* (pp. 1–25). Cambridge, MA: MIT Press.

Schoenfeld, A. H., (1985). *Mathematical Problem Solving*. Orlando, Florida, Academic Press.

Schoenfeld, A. H., (2007). A theory of teaching and its applications. *The Montana Mathematics Enthusiast. Monograph 3*, 33-38.

Schoenfeld, A. H., (1992). Learning to think mathematically: Problem solving, metacognition and sense-making in mathematics. In D. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 334-370). Macmillan, New York.

Schraw, G. & Dennison, R.S., (1994). Assessing metacognitive awareness. *Contemporary Educational Psychology, 19*, 460-475.

Schraw, G., Moshman, D., (1995). Metacognitive Theories. *Educational Psychology Review, 7*(4), 351–371.

Schwartz, B., Perfect, T., (2001). Introduction: Toward an applied metacognition. In T. Perfect and B. Schwartz (Eds.). *Applied Metacognition*. Cambridge University Press.

Van Overschelde, J. P., (2008). Metacognition: Knowing About Knowing. In J. Dunlosky and R. Bjork (Eds.), Handbook of Metamemory and Memory (pp. 47-71). Hillsdale, NJ: Psychology Press.

Appendix

This appendix contains an example protocol of the study concerning the open-ended problem, which was used to trace the verbal reports that revealed the metacognitive strategies.

Open-Ended Problem

[1] Student: (He reads the problem. He rereads it). Well, I am thinking of multiplying 2 times 3 times 4 and I believe that this is the way of solving this problem. Now because I have no more information, I believe that this is the correct way. Well … 2 times 3 equal 6. Six times 4 equal 24.

[2] How many kids are we? I think we are 24 kids. It's better to reread the problem as I don't have many data, in case I am wrong. (He reads the problem once again.)

[3] I wonder if I have to find the Least Common Multiple. I want to think something else. This is the solution (he shows the previous solution he gave), but … I don't know … I want to try something else.

[4] Well, I make pairs. The first one, the second, the third (he draws shapes in the paper). No! … wait a minute to make it better.

Ok! Pairs of 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, up to 40.

Now let's make triads: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, up to there.

Now sets of four: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40.

[5] Now that I am rethinking the solution the previous one was wrong. I will erase it.

[6] This problem must be an open-ended one. I will find the cases that suit.

2, 3, 4, doesn't suit

4, 6, 8, the same

6, 9, 12, the same

Oh! No! … Number 2 doesn't't suit, number four the same, number 8 the same … number 12! …

One solution is for the kids to be 12

Let's see another solution. Number 14 is wrong, number 16, 18, 20, 22 the same, but number 24 may be another solution of the problem. Also number 26, 28, 30, 32, 34 are not what we seek, but number 36 suits.

[7] Now I am sure about the solution of the problem, but there may be more solution as this problem is an open-ended one.
[8] Now I am going to write down the correct solution. I'll write down the multiples of 2, 3, 4 and I will find the common multiples.
Well
2, 4, 6… 40
3, 6, 9… 39
4, 8, 12… 40
[9] I check again the procedure in case we make a mistake. Now the common multiples are 12, 24, 36 and many more. So as it says that one kid is left alone, I have to add one to the common multiple. So the correct answer is that the kids may be 13, or 25, or 37 etc. I've finished.

The following table mentions the number of the verbal report of the child who solved the open-ended problem and the Metacognitive strategy which was implied by the certain verbal report.

Table 8

Verbal reports	Metacognitive strategy
[1] Well, I am thinking of multiplying 2 times 3 times 4 and I believe that this is the way of solving this problem.	In verbal report [1], the student has a specific purpose for each strategy he uses, and he justifies his thought by saying that he has not many data (Procedural knowledge).
[2] It's better to reread the problem as I don't have many data, in case I am wrong.	In verbal report [2], he consciously focuses his attention on important information (Information Management Strategies).
[3] I wonder if I have to find the Least Common Multiple.	In verbal report [3], he asks himself if he has considered all options after he solves a problem. He wonders if he must find the Least Common Multiple (Evaluation).

Verbal reports	Metacognitive strategy
[4] Well, I make pairs. The first one, the second, the third (he draws shapes in the paper). No! … wait a minute to make it better.	In verbal report [4], he tries to break studying down into smaller steps, by making pairs, triples, quadruples (Information Management Strategies).
[5] Now that I am rethinking the solution the previous one was wrong. I will erase it.	In verbal report [5], he re-evaluates his assumptions when he gets confused (Debugging strategies).
[6] This problem must be an open-ended one. I will find the cases that suit.	In verbal report [6], he sets specific goals before he begins a task saying that the problem is an open-ended one and that he will find the solutions that suit (Procedural knowledge).
[7] Now I am sure about the solution of the problem …	In verbal report [7], he is a good judge of how well he understands something (Declarative knowledge).
[8] Now I am going to write down the correct solution. I'll write down the multiples of 2, 3, 4 and I will find the common multiples.	In verbal report [8], he finds himself using helpful learning strategies automatically by making a diagram (Procedural knowledge).
[9] I check again the procedure in case we make a mistake. Now the common multiples are 12, 24, 36 and many more. So as it says that one kid is left alone, I have to add one to the common multiple. So the correct answer is that the kids may be 13, or 25, or 37 etc.	In verbal report [9], he summarizes what he has learned after he finishes (Evaluation).

Evagelos Mokos is director of the 50th Elementery School of Athens, Greece.

Sonia Kafoussi is associate professor in Mathematics Education in the Department of Sciences of Preschool Education and Educational Design, at the University of the Aegean, Greece.

Contact Address: Direct correspondence concerning this article should be addressed to the author at: University of the Aegean, Rue Demokratias, 1, 85100, Rhodes, Greece. E-mail address: Kafoussi@aegean.gr.