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12TH CENTURY ALGEBRA IN AN ARABIC POEM: IBN AL-YĀSAMĪN'S *URJŪZA FI'L-JABR WA'L-MUQĀBALA*

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SUMMARY

Ibn Al-Yāsamīn (d. 1204) received his higher education in Western Maghrib *and, for a period of time, in Sevilla. He is known to have taught in this andalusian town around 1190 using his poem on algebra intitled* al-Urjūza fi'l-jabr wa'l-muqābala *as a basis.*

In this paper we propose a translation into English of al-Urjūza, *with a descriptive analysis of the terminology and concepts used in its mathematical section.*

RÉSUMÉ

Ibn Al-Yāsamīn (mort en 1204) a poursuivi une formation supérieure au Maghreb Extrême et aussi pendant quelque temps à Séville. On rapporte que dans cette cité andalouse il aurait utilisé vers 1190 le poème al-Urjūza fi'l-jabr wa'l-muqābala *comme base de son enseignement de l'algèbre.*

*Dans cet article, nous présentons la traduction en anglais de l'*Urjūza *accompagnée d'une analyse descriptive de la terminologie et des concepts utilisés dans sa partie mathématique.*

Key words: Arab medieval algebra; mathematical poems; Ibn al-Yāsamīn.

I. Who is Ibn Al-Yāsamīn?

Ibn Al-Yāsamīn,[1] Abū M. ᶜAbd Allah b. M. b. Hajjāj Al-'Adrīnī, who was descended from the Berber tribe of Banū Hajjāj of Qalaᶜat Fandalāwa, was born in the Twelfth century and was black like his mother. He received his higher education for a period of time in Sevilla where, around 1190, he taught his poem in algebra *al-Urjūza fi'l-jabr wa'l-muqābala*.[2] He lived during a period of upheavals, corresponding to the advent of the Almohād Dynasty whose rulers extended their dominion over the whole of North Africa and part of Spain from 1155 to 1269.

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The city of Marrakech was then the capital of the Almohāds, but some other cities also grew into important cultural and scientific centers: such was the case with Sevilla and Grenada in the south of *Al-Andalus*, and with Ceuta, Fès, Bejaïa and Tunis in North Africa.[3] The chronicler Ibn al-Abbār (d. 1260) emphasizes the fame of Ibn al-Yāsamīn, as a great figure well-versed in the fields of literature and languages, able to produce texts in prose or verse, such as *muwashshaḥāt* set to music and sung in his time. Ibn al-`Ābār further suggests that he was on familiar terms with the Almohād Caliph al-Mansūr (1184-1199), as well as with his son, Caliph an-Nāṣīr (1199-1213).

However, Ibn al-Yāsamīn was murdered in 1204 ; his mangled body was found lying in front of his house in Marrakech.

II. Ibn Al-Yāsamīn's mathematical production

Ibn Ibn al-Yāsamīn fame today rests chiefly on his mathematical work and its impact on the teaching of mathematics in North Africa and the Middle-East. Among his works,[4] two of his mathematical poems are still extant: the first on roots (*al-Urjūza fi'l-judhūr*) and the most famous one on algebra (*al-Urjūza fi'l-jabr wa'l-muqābala*). A third poem dealing with proportions (*al-Urjūza fi'l-kaffāt*) is attributed by Shawqi (1988) and Zemouli (1999) to ibn al-Yāsamīn. However his most elaborate, though not so well known mathematical work, is his treatise on algebra (*Talqīh al-'afkār fi-l ᶜamali bi rushūm al-ghubār*).[5]

The Poem on algebra (*al-Urjūza fi'l-jabr wa'l-muqābala*) consists of 54 lines, introducing different terms used in algebra and standard resolutions of all the six canonical equations. It includes:

— Thanks addressed to the Creator and His Prophet (lines 1-2);
— A tribute to the author's mathematics teacher: Muhammad ibn Qāsim to whom he dedicates *al-Urjūza* (the poem) and justifications for composing it (lines 3-10);
— The terminology of algebra (lines 11-13);
— Presentation of the six types of canonical equations of degrees 1 and 2 (lines 14-15);
— Solutions of the three types of canonical equations of degree 1 (lines 16-21);
— Solutions of the three types of canonical equations of degree 2 (lines 22-34);
— Solutions of quadratic equations where the coefficient of x^2 is not equal to 1 (lines 35-38);

— Restoration and confrontation (lines 39-40);
— Operations on monomial expressions (lines 41-50);
— The rule of signs (lines 51-52);
— A prayer (line 53).

Presenting this poem, Ibn Al-Hāim (d. 1412) says: «*Ibn Yāsamīn's expression is so delightful that many memorized the poem, and the propositions that it contains are so elaborate that many had to explain them.*» (Abdeljaouad, [2003, 57])

From the Thirteenth century on, Ibn Al-Yāsamīn's poem on algebra became indeed a prerequisite for students wishing to learn algebra. Commentaries on the poem flourished in the *Maghrib* and in Egypt. There are 20 commentaries known extant,[6] including those of Ibn Qunfudh (d. 1404), Ibn Al-Hā'īm (d. 1412), Al-ᶜIrāqi (d. 1423), Al-Qalaṣādī (d. 1486), Al-Māradīnī (d. 1506) and Al-Anṣāri (d. 1661).

III. *Al-Urjūzā Al-Yāsaminīya fi'l-jabr wa'l-muqābala* (Ibn Al-Yāsamīn's mathematical poem on algebra)

1. Praise be to God for blessing us with
His teaching and enlightenment.
2. God's prayers be for ever
on his chosen Prophet Muhammad.
3. Thanks are due to the bright learned scholar,
our master Muhammad ibn Qāsim,
4. Who elucidated what was obscure,
and made easier and understandable baffling intricacies.
5. May God bless him for it and bestow
His reward on him in the hereafter.
6. Those in need of assistance have made a request,
which I cannot do otherwise than fulfill.
7. They have asked me to introduce algebra
in few metrically ordered lines,
8. Arranged according to the *Rajz* meter,
highly meaningful and concisely put.
9. Though I made every effort to be excused,
it was all in vain;
10. I wrote it with due apologies,
so let the reader forgive any lapses therein.
11. Algebra rests upon three:
amwāl , numbers, then roots.

12. The *māl* is every square number,
    and its root one of its sides.
13. The absolute number relates
    neither to the *amwāl* nor to the roots. Do understand!
14. Thing and root mean the same,
    just as the terms father and progenitor.
15. They may be equal to an isolated number,
    or added to other species.
16. These are six well-ordered equations,
    half of which compound, and half simple.
17. The first one, under the current terminology,
    consists in equating *amwāl* with roots.
18. When they are equated with numbers,
    you obtain the next equation. Mind what I say!
19. And when you equate roots with a number,
    you get the third equation accordingly.
20. Divide by the *amwāl* if you find them,
    and by the roots if you don't.
21. These are the simple equations,
    the solution of which is a root, except for the intermediate one.
22. For this, the solution is one *māl,*
    if the problem leads to it.
23. Do be aware, may God guide you, that the number
    is isolated in the first compound equation.
24. And in the second, roots are also isolated,
    and the *amwāl* in the next.
25. Take the square of half the things,
    and carefully add it to the numbers;
26. Extract the root of the result,
    then subtract the half, and you will unveil the secret.
27. What remains corresponds to the root of the *māl,*
    and that is your fourth type of equation.
28. In the next, from the squaring subtract the number,
    and the root of what remains will serve your purpose,
29. And you may subtract it from or add it to
    your halving of the roots, as you choose.
30. One is the root of the *māl* by subtraction,
    and the other the root by addition.
31. If the squaring is equal to the number,
    then its root is the halving without diminution.

32. And if it is smaller than the number,
    you realize that there is no solution.
33. Now that we have done with the fifth equation,
    let us explain the solution of the sixth.
34. Append the squaring to your numbers,
    and extract the root of this sum,
35. Add to the halving what you got,
    and you obtain the root sought.
36. Reduce the *amwāl* if many,
    and restore the fraction if incomplete,
37. So that it yields a full *māl*,
    and take either into account for the remaining terms.
38. Or you may multiply the *māl* by the numbers
    and apply on it what precedes,
39. Then divide the intermediary root by
    the number of *amwāl*. There is your final solution.
40. If there is a subtractive term in an equation,
    add its additive form to the equal term.
41. Once restoration is done, do the confrontation
    by taking off corresponding equal terms.
42. Then I shall deal with ranks,
    in few words, but comprehensively.
43. The root comes first, followed by the *māl*,
    then by the cube which is autonomous.
44. On these they are all based,
    whatever their number, indefinitely.
45. In multiplications, take the rank of both factors,
    and you will thus know the rank of the product.
46. For each cube repeated count three,
    and two for each *māl*.
47. For each root, count invariably one,
    but for numbers, no rank is known.
48. If you multiply a number by a species,
    the result is beyond doubt this same species.
49. In both species, the result of any division
    is undoubtedly a number.
50. The rank of the result of any division of the higher species
    is the excess of the ranks.
51. I mean its positioning.
    The result of the reverse is the original statement.

52. The multiplication of any additive or any subtractive term
 by a term of the same kind is additive for the seeker,
53. Its multiplication by an opposite is subtractive.
 Do understand and may God guide you !
54. Then God's peace and prayers
 be on the Prophet, as long as Light prevails.

IV. Descriptive analysis of the mathematical sections of the poem: lines 11-53.

Lines 11-12-13-14: Terminology

al-jabr (algebra) is the name of the art of algebra. This is an abbreviation for *al-jabr wa'-muqābala.*

The three terms *māl*, *jidhr* and *ᶜadad* relates to species which are defined in the next lines.

Māl is clearly defined as any square number. We shall find it used 14 times in the poem, sometimes as *mālan* (exactly one *māl*), *al-māl* (the square number, the square of the unknown), *amwāl* (plural of *māl*). In medieval Europe *māl* was translated by the Latin word *census* and the Italian word *censo.* In modern notation it is represented by x^2.

Jidhr (root) is certainly the most recurrent term in this poem (20 times), sometimes as *jidhrun* (a root), a*l-jidhr* (the root), *ajdhār* (roots), *al-ajdhār* (the roots).

In line 14, the two words *shay* (thing) and *jidhr* are said to be equivalent. That means that *jidhr* is also used for the unknown (our modern x). In medieval Europe *shay* was translated by the Latin word *res* and the Italian word *cosa.*

Lines 15-16 : Presentation of the six canonical equations of degrees 1 and 2

The first type, called simple equation, are obtained by equating one species to another species. The second type, called compound equations, are obtained by equating one species to the sum of the two other species.

Lines 17-19: Definition of the three simple equations

Type I: *Amwāl* equal roots. (modern representation: $ax^2 = bx$ where a and b $\in \mathbf{Q}_+$.)

Type II: *Amwāl* equal a number. (modern representation: $ax^2 = c$ where a and c $\in \mathbf{Q}_+$.)

Type III: Roots equal a number. (modern representation: $bx = c$ where b and c $\in \mathbf{Q}_+$.)

This is the order for simple equations adopted by most successors of al-Khwārizmī (9th c.) and Abū Kāmil (10th c.), but al-Karājī (11th c.), as-Samaw'al (12th c.), al-Kāshī (15th c.), and al-ᶜĀmilī (17th c.) adopt the III-I-II sequence, while al-Miṣṣīṣī (10th c.), al-Birūnī (11th c.), al-Khayyām (12th c.), al-Qurāshī (13th c.) and Sharaf ad-Dīn at-Ṭūsī (13th c.) follow the III-II-I sequence [Djebbar, 1980, 9-10].

Lines 20-21-22: Solutions of simple equations

«Divide the amwāl» means divide terms on both side of the equation by the number of *amwāl*.

«if you find them» means whenever the coefficient of the x^2 is different from 0, as is the case in type I and II equations.

$$ax^2 = bx \longrightarrow \frac{a}{a}x^2 = \frac{b}{a}x \longrightarrow x^2 = \frac{b}{a}x \longrightarrow x = \frac{b}{a}.$$

$$ax^2 = c \longrightarrow \frac{a}{a}x^2 = \frac{c}{a} \longrightarrow x^2 = \frac{c}{a}.$$

«and by the roots if you don't» means divide terms on both side of the equation by the number of roots when $a = 0$, that is for the type III equations.

$$bx = c \longrightarrow \frac{b}{b}x^2 = \frac{c}{b} \longrightarrow x = \frac{c}{b}.$$

For type I and III equations the solution is a root, while for the second type it is one *māl.*

Lines 23-24 : Definition of the three compound equations

The order of each type is given according to which species is isolated from the two others.

Type IV: Isolated numbers.

Amwāl and roots equal a number. ($ax^2 + bx = c$ where a , b and c $\in \mathbf{Q}_+$)

Type V: Isolated roots.

Amwāl and a number equal roots. ($ax^2 + c = bx$ where a , b and c $\in \mathbf{Q}_+$)

Type VI: Isolated *amwāl.*

Roots and a number equal *amwāl.* ($bx + c = ax^2$ where a , b and c $\in \mathbf{Q}_+$)

This is the traditional order for compound equations found in most all Arabic treatises.

Lines 25-26-27: Solution of normalized equations of type IV

$x^2 + bx = c$ where b and $c \in \mathbf{Q}_+$)

Ibn Al-Yasāmīn provides the reader with traditional algorithmic recipe-like method for finding the solution of compound equations. These rules work only for a normalized equation, that is an equation where the coefficient a of the *amwāl* is equal to 1. This precaution is not explicitly enunciated by the author.

«half the thing»: this is the only time the term *shay* (the unknown number, the *thing*) is used. So, in spite of the logical equivalence between *jidhr* and *shay,* the latter is clearly associated with setting up the equation, while the former is the root of the equation, but the author prefers *jidhr.*

The algorithm as stated in these verses can be shown in diagrams as follows:

$$\frac{b}{2} \to \left(\frac{b}{2}\right)^2 \to \left(\frac{b}{2}\right)^2 + c \to \sqrt{,\left(\frac{b}{2}\right)^2 + c} \to x = \sqrt{,\left(\frac{b}{2}\right)^2 + c} - \frac{b}{2}.$$

Lines 28-29-30-31-32-33: Solution of normalized equations of type V

$x^2 + c = bx$ where b and $c \in \mathbf{Q}_+$)

In verse 28 we are asked to subtract the number from the squaring done in line 24.

$$\left(\frac{b}{2}\right)^2 \to \left(\frac{b}{2}\right)^2 - c \to \sqrt{\left(\frac{b}{2}\right)^2 - c}.$$

In line 29, we are left with two choices, either subtracting the radical from $\frac{b}{2}$ or adding it to $\frac{b}{2}$

First solution: $x = \frac{b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 - c}$. (*root of the māl by subtraction)*

Second solution: $x = \frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - c}$. (*root of the māl by addition)*

Line 31: If $\left(\frac{b}{2}\right)^2 = c$, then the solution is x = $x = \frac{b}{2}$.

Line 32: If $\left(\frac{b}{2}\right)^2 < c$, then the equation has no solution.

Lines 33-34: Solution of normalized equations of type VI

$bx + c = x^2$ where b and $c \in \mathbf{Q}_+$)

$$\left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{b}{2}\right)^2 + c \rightarrow \sqrt{\left(\frac{b}{2}\right)^2 + c} \rightarrow x = \frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + c}.$$

Lines 35-36-37-38: Normalizing equations.

Ibn Al-Yasāmīn proposes two ways for dealing with non-normalized equations.

The first method consists in normalizing them by either *ḥaṭṭ* or *jabr.*

— *ḥaṭṭ* (reducing to 1) is the operation consisting in replacing coefficient «a» of the *amwāl* with «1» whenever it is greater than 1. Al-Khwārizmī and other Arab authors use instead the term *radd* translated into Latin by *reducere* and sometimes by *convertere*.

— *jabr* (augmenting to 1) is the operation consisting in replacing by 1 the coefficient a of the *amwāl* with «1» whenever it is a fraction smaller than 1. Al-Khwārizmī and other Arab authors use instead the term *ikmāl*, translated into Latin by *complere*, *reintegrare* and sometimes *restaurare*.

The second method adopted by Ibn al-Yāsamīn himself uses an intermediate equation and an intermediate root.

— *you multiply the amwāl by the number.*
— *you apply on <the intermediate equation> the preceding.*
— *you divide the resulting intermediate root by the number of amwāl.*
— *It is the solution.*

Suppose that you need to resolve an equation of type IV: $ax^2 + bx = c$.

— multiply by the number «a» of *amwāl* : $a(ax^2 + bx) = ac$.
— you get an intermediate equation: $X^2 + bX = ca$, with $X = ax$.
— you look at solution X_0 of this intermediate equation. Ibn Al-Yasāmīn calls this solution *naḍir al-jidhr* [the counterpart of the root or the intermediate root].
— The final solution of the given equation is $x = \frac{X_0}{a}$.

As stated in the *Urjūza*, this rule is unclear, however a few commentators understood it. In his more elaborate treatise on algebra, *Talqīh al-afkār,* Ibn al-Yāsamīn gives many applications of this technique, and, in his commentary on Ibn al-Yāsamīn'*urjūza*, Ibn al-Hā'im explains this technique thoroughly and illustrates it with many examples.[7]

Line 40: The second meaning of the term *al-jabr* [restoration]

Al-jabr is the operation consisting in suppressing any subtractive terms from an equation by adding its additive form to both sides of the equation.

Line 41: Definition of the term *al-muqābala* [confrontation]

This is a very concise definition of the term *al-muqābala* which consists in the removal of equal corresponding quantities occurring on both sides of the equation.

Lines 42-43-44: *manzila, pl. manāzil,*

This word could be translated into «house» pl. «houses», but we prefer using instead the term «rank» which indicates that roots can be ranked at the first place, followed by *amwāl* , then cubes, and so on. All (positive) powers of the unknown will have therefore a rank, (also a degree or an exponent), but no word is said on monomials of the form x^m for $m < 0$.

Line 45-46-47: The additiveness of ranks is stated ($x^n x^m = x^{n+m}$) and illustrated . The rank of a cube is three, of a mal is 2, of a root is 1, and for numbers, *no rank is known for them* is the usual way of signifying that their rank is nil.

Line 48: If N is a number and X a monomial expression, then NX is a monomial expression.

Line 49: If a and b are positive numbers then $ax^n : bx^n$ is a:b. The result of the division of two monomials of the same degree is a number.

Lines 50: By *division of the higher species* the author means the division of a higher species by a lower species, as for example in $ax^n : bx^m$ where $n > m$. The excess meant here is the difference $n – m$ of the exponents.

Lines 51: *The result of the reverse is* the *same as the original* concerns the division of a lower species by a higher one. When dividing ax^m by bx^n where $m < n$, the result is a sentence enunciating «ax^m divided by bx^n.»

Lines 52-53: These are the standard rules of signs.

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NOTES

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1. Carl Brockelmann, [1938, GLS I p. 471 and p. 621. GLS Supplement I p. 858]. Driss Lamrabet, [1994, 66-71]. Rosenfeld B.A - Ihsanôglu E. [2003, n.° 521, p.192].
2. This is reported by Ibn al-Abbār (d. 1260) who do not give any details concerning the place and the date of composition of this poem.
3. Ahmed Djebbar [2003, 106].
4. Ibn al-Yāsamīn's poems were edited by Jalal Shawqi in 1988.
5. Ibn al-Yāsamīn's works were edited by Touhami Zemouli in 1999.
6. Jalal Shawqi [1988, pp. 59-112] gives the complete list of extant commentaries.
7. Mahdi Abdeljaouad [2003, 34-35].

ِجوزة ابن الياسمين في الجبر و المقابلة

1. الحمد لله على ما ألهمـــا   ومن من تعليمه وفهما .
2. وصلوات الله طـول الأبد   على النبي المصطفي محمد .
3. والشكر للحبر الذكي العالم   أستاذنا محمد بن قاسم .
4. فهو الذي بين ما قد أشكلا   وقرب القاصي حتى سهلا .
5. جزاه رب الناس عنـا خيرا   وأجزل الأجر له في الأخرى .
6. كلف من لا بد من إسعـافه   ولا أرى وجها إلى خلافه .
7. أن أوضح الجبرية المقدمة   في أحرف قليلة منظمه .
8. موزونة على عروض الرجز   كثيرة المعنى بلفظ موجز .
9. فلم أزل معتذرا عن هــذا   ولم أجد عن أمره ملاذا .
10. فقلتها قولا على اعتـــذاري   فليغفر الزلة فيها القاري .
11. على ثلاثة يدور الجبـــــر   المال والأعداد ثم الجذر.
12. فالمال كل عدد مربــــع   وجذره واحد تـــلك الأضلع .
13. والعدد المطلق ما لم ينسب   للمال أو للجذر فافهم تصب .
14. والشيء والجذر بمعنى واحد   كالقول في لفظ أب و والد .
15. فبعضها يعدل بعضا عددا   مركبا مع غيره أو مفردا.
16. فتلك ست نصفها مركـــبة   ونصفها بسيطة مرتـبة .
17. أولها في الاصطلاح الجاري   أن تعدل الأموال للأجذار .
18. وإن تكن عـادلت الأعـداد   فهي تليهـــا فـــافهم المـــرادا .
19. وإن تعـادل بالجذور عددا   فتلك تتلوها على ما حددا .
20. فاقسم على الأموال إن وجدتها   واقسم على الأجذار إن عدمتها .
21. فهذه المسائل البسيطـــة   خارجها الجذر سوى الوسيطة .
22. فإنما يخرج فيها المـــال   بحسب ما قد اقتضى السؤال .
23. واعلم هداك ربنا أن العدد   في أول المركبات انفرد .
24. ووحدوا أيضا جذور الثانية   وأفردوا أموالهم في التالية .

25. فربع النصف من الأشياء	واحمل على الأعداد باعتناء .
26. وخذ من الذي تناهي جذره	ثم انقص التنصيف تفهم سره .
27. فما بقي فذاك جذر المال	وهذه رابعة الأحوال .
28. واسقط من التربيع في الأخرى العدد	وجذر ما يبقى عليه يعتمد .
29. وانقصه من تنصيفك الأجذار	وإن تشأ جمعته اختيارا .
30. فذاك جذر المال بالنقصان	وذاك جذر المال بالحملان .
31. وإن غدا التربيع مثل العدد	فجذره التنصيف دون فند .
32. وإن يكن يربى عليه العدد	أيقنت أن ذلك لا ينعضد .
33. وإذ فرغنا من بيان الخامسة	فلنوضح الآن بيان السادسة .
34. فاجمع إلى أعدادك التربيعا	واستخرجن جذرهما جميعا .
35. واحمل على التنصيف ما أخذتا	فذلك الجذر الذي أردتا .
36. وحط الأموال إذا ما كثرت	واجبر كسورها إذا ما قصرت .
37. حتى يصير الكل مالا مفردا	وخذ بذاك الاسم مما عدا.
38. أو فاضرب الأموال في الأعداد	وكن على ما مر ذا اعتماد .
39. واقسم نظير الجذر من بعد على	عدد الأموال وخذ ما أصلا .
40. وكلما استثنيت في المسائل	صيره إيجابا مع المعادل .
41. وبعد ما تجبر فلتقابل	بطرح ما نظيره يماثل .
42. ثم أقول بعد في المنازل	مقال إيجاز بلفظ شامل .
43. الجذر في الأولى يليه المال	وبعده كعب له استقلال .
44. وهكذا ركب عليه أبدا	ما بلغت وما تناهت عددا .
45. وما ضربته فخذ منازله	تعرف بذاك الأخذ أسا لحاصله .
46. ثلاثة لكل كعب كررا	واثنان للمال متى ما ذكرا .
47. وواحد للجذر ولا ينحرف	وليس للأعداد أس يعرف .
48. وإن ضربت عددا في جنس	فالخارج الجنس بغير لبس .
49. وخارج القسمة في النوعين	مقامه عد بغير مين .

50. وقسمة الأعلى من الجنسين خـــارجها زيادة الأسين .

51. أعني بهذا ما له من منزلة وعكسه جوابه كالمسألة .

52. وضرب كل زائد ونـاقص في نوعه زيادة للفاحص .

53. وضربه في ضده نقصان فافهم هداك الملك الديان .

54. ثم صلاة الله والســـلام على النبي ما انجلى الظلام .