

The term structure, latent factors and macroeconomic data: A local linear level model

Jakas, Vicente

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Abstract


This paper applies a local linear level model to European yields using the state space methodology to structural equation models in order to obtain an unobserved state vector containing the level, slope and seasonal component of the yields. In addition, this has been performed by differentiating money markets from capital markets' yields. Also an affine term structure model has been calibrated using the estimated level, slope and seasonality from the local linear level model. It is shown that both, the local level model as well as the no-arbitrage approach, perform quite well in replicating the yields. The model also shows that there is strong evidence of macroeconomic effects influencing the level, the slope and the seasonal components common to a set of yields (the yield curve). However, this paper shows that there is weak evidence of yields influencing European macroeconomic variables. This could be interpreted as the central bank and markets responding to macroeconomic releases, which is observed in yield movements, but there is weak evidence of yield innovations influencing the macroeconomy.

Keywords:

European benchmark yields, Local linear model, Affine term structure model, Financial simulation, State space model, Latent factors.

JEL classification:

B22, C5, C58, E4, E27, G17, G1.

Jakas, V.  is with Deutsche Bank AG in the Finance Analytics for the area of Corporate Banking and Securities; this work is part of a research work he is currently performing in Saarland University, Germany. E-mail: vicente.jakas@db.com

Estructura temporal, factores latentes y datos macroeconómicos: Un modelo de tendencia lineal local

Jakas, Vicente

Resumen

En este artículo se aplica un modelo de tendencia lineal local a los rendimientos europeos aplicando la metodología de espacios de estado a modelos de ecuaciones estructurales, para así obtener un vector de estados no observable que incluya el nivel, la pendiente y la componente estacional de dichos rendimientos. Además, ello se ha llevado a cabo diferenciando los rendimientos de los mercados monetarios y de los mercados de capitales. También se han calibrado modelos afines de estructura temporal utilizando las estimaciones del nivel, pendiente y estacionalidad proporcionadas por el modelo de tendencia lineal local. Se demuestra que tanto el modelo de tendencia lineal local como el enfoque de no arbitraje replican ciertamente bien los tipos. El modelo también muestra que existe una fuerte evidencia de que los efectos macroeconómicos influyen en el nivel, la pendiente y la componente estacional común a un conjunto de tipos (la curva de tipos). Sin embargo, en lo que se refiere a la influencia de los rendimientos sobre las variables macroeconómicas europeas, la evidencia es débil. Una posible interpretación de este hecho puede formularse en términos de respuesta del Banco Central y los mercados a los datos macroeconómicos, lo que se observa en los movimientos de los tipos, pero la evidencia de innovaciones de tipos influenciando la macroeconomía es débil.

Palabras clave:

Tipos europeos de referencia, Modelo local lineal, Modelo Afín de estructura temporal, Simulación financiera, Modelo de espacio de estados, Factores latentes.

■ I. Introduction

This paper is an attempt to apply a local level model as seen in Commandeur *et al.* (2011) to yield curve dynamics in a similar fashion to the latent factor approach described in the paper by Diebold *et al.* (2006) and following the contributions from Diebold and Li (2006). The first stage of this analysis is to use a local level model –with other unobserved components– in order to identify latent factors such as the level, the slope and a seasonal factor. Subsequently, on a second stage, the model links macroeconomic data to these latent factors. Here, the intention is to model the latent factors using the same macroeconomic data as those in Jakas (2011 and 2012) and Jakas and Jakas (2013), and trying to understand which unobserved components are influenced most by the macroeconomy. This model differs from that of Diebold *et al.* (2006), as they use a state space model which nests a *VAR* in order to identify the latent factors such as level, slope and curvature. They then expand the model by incorporating three macroeconomic variables to the state vector. In contrast, in this essay the local level approach is used to identify the latent factors in a state space model and, in a second stage, these latent factors are modelled using macroeconomic data. In addition, since the local level model is used, we incorporate the seasonal component in lieu of the curvature. It could be said that the approach used here is closer to the works of Ang and Piazzesi (2003) and Hördahl *et al.* (2002) however, this research uses different European data and focused on European yields instead of US data to calibrate the models. In a first step this research departs from the no-arbitrage approach, as our intention is to estimate the latent factors *via* a state space model with an observation equation depicting the level, slope and seasonal components. In a second step an affine term structure model is calibrated with the unobserved states or latent factors in a similar set up as in Jakas (2012). In contrast to Jakas (2012) and Jakas and Jakas (2013) the affine model is calibrated with the latent factors instead of macroeconomic data. It should be said that some of the empirical literature in no-arbitrage such as in Backus *et al.* (1998), Duffie and Kan (1996) and Dai and Singleton (2000), do not link latent variables to macroeconomic data or when they do so, empirics have been mostly limited to the short rate. In addition, this paper segregates money markets from capital markets and by doing so performance improves significantly for longer term maturities. This is a reasonable approach, as it could be considered that there are two markets governing the yield curve, somehow contradicting the no-arbitrage approach. The reason for taking this approach is mainly because traders and fixed income strategists make a difference between the two, as liquidity risks and market conventions are different. Notwithstanding, an affine term structure model is calibrated with latent factors and despite results are encouraging for money market yields they are observed to be less impressive for long term maturities.

The paper is organised as follows: section 2 presents the local linear level model and the no-arbitrage approach; section 3 presents discussion of results; section 4 an affine term structure model is calibrated using the latent factors estimated in previous section; section 5 presents some discussion on policy implications; and 6 outlines main conclusions and final remarks.

■ 2. Yields' unobserved components

In this section we specify yields as a state space model with an unobserved state or transition equation which is linked to an observation or measurement equation. This state space model nests a local level model with a stochastic slope and a stochastic seasonal component. We define the state and observation equations following the notation from Commandeur *et al.* (2011),

$$u_t = u_{t-1} + v_{t-1} + \xi_t, \quad (1)$$

$$v_t = v_{t-1} + \zeta_t, \quad (2)$$

$$\gamma_{1,t+1} = -\gamma_{1,t} - \gamma_{2,t} - \gamma_{3,t} + \omega_t, \quad (3)$$

$$\gamma_{2,t} = \gamma_{1,t-1}, \quad (4)$$

$$\gamma_{3,t} = \gamma_{2,t-1}, \quad (5)$$

$$y_t = \beta_1 u_t + \beta_2 v_t + \beta_3 \gamma_{1,t} + \varepsilon_t, \quad (6)$$

where $\xi_t = NID(0, \sigma_\xi^2)$, $\zeta_t = NID(0, \sigma_\zeta^2)$, $\omega_t = NID(0, \sigma_\omega^2)$, and $\varepsilon_t = NID(0, \sigma_\varepsilon^2)$, $NID(x, \sigma^2)$ being a normal independent-distributed variable with mean x and variance $\sigma^2 (> 0)$. Equation (6) is the observation or measurement equation for y_t , namely, the yield of a zero coupon bond with a given maturity at time t , β_1 , β_2 and β_3 are parameters. As shown below for the multivariate case, yields will be assumed to be a function of these latent variables or unobserved states, which comprise: (i) the linear trend or level u_t , (ii) the stochastic slope v_t and (iii) the stochastic seasonal component $\gamma_{1,t}$.

We estimate the unobserved states and parameters by maximum likelihood using a Kalman (1960) filter, which can be specified as follows,

$$z_t = \alpha z_{t-1} + \theta \varepsilon_t, \quad (7)$$

$$y_t = \beta z_{t-1} + \varphi \eta_t, \quad (8)$$

where,

z_t : 5×1 vector of unobserved state variables;

ε_t : 5×1 vector of state-error terms;

y_t : $n \times 1$ vector of observed endogenous variables depicting the yields;

η_t : $n \times 1$ vector of observation-error terms and,

α , β , θ and φ : $n \times n$ parameter matrices.

Combining equations (1) to (8) in matrix form yields

$$\mathbf{z}_t = \begin{pmatrix} u_t \\ v_t \\ \gamma_{1,t} \\ \gamma_{2,t} \\ \gamma_{3,t} \end{pmatrix}, \quad \boldsymbol{\alpha} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad (9)$$

$$\mathbf{z}_{t-1} = \begin{pmatrix} u_{t-1} \\ v_{t-1} \\ \gamma_{1,t-1} \\ \gamma_{2,t-1} \\ \gamma_{3,t-1} \end{pmatrix}, \quad \boldsymbol{\theta} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{\varepsilon}_t = \begin{pmatrix} \xi_t \\ \zeta_t \\ \omega_t \\ 0 \\ 0 \end{pmatrix}, \quad (10)$$

$$\mathbf{y}_t = \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{n,t} \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ \beta_{1,1} & \beta_{1,2} & \beta_{1,3} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \beta_{n,1} & \beta_{n,2} & \beta_{n,3} & 0 & 0 \end{pmatrix}, \quad \boldsymbol{\varphi}\boldsymbol{\eta}_t = \begin{pmatrix} \varepsilon_{y_1} \\ \varepsilon_{y_2} \\ \vdots \\ \varepsilon_{y_n} \end{pmatrix}, \quad (11)$$

Notice that in the matrix $\boldsymbol{\alpha}$ we applied the usual constraints for a local level model with a stochastic slope and a stochastic seasonal component as seen in Commandeur *et al.* (2011). Despite that we use monthly data, we set the number of seasons to 4. This will not be an issue mainly because we let the seasonal component to remain flexible thanks to the random error term ω_t . In addition, $\boldsymbol{\theta}$ is diagonal as this ensures that random error terms remain uncorrelated, notice that the diagonal elements in $\boldsymbol{\theta}$ are set to one in order to allow the components of $\boldsymbol{\varepsilon}_t$ in Equation (10) to be free parameters. This is a standard assumption as seen in the no-arbitrage literature (see Piazzesi, 2010; Dai and Singleton, 2000; or Duffie and Kan, 1996; or Backus *et al.*, 1998). For simplicity, and without loss of generality, we can also assume that all components in vector $\boldsymbol{\varphi}\boldsymbol{\eta}_t$ are free parameters in the model. By doing so, we follow the local level model as in Commandeur *et al.* (2011) and depart from the local level model presented in Drukker and Gates (2011). Finally, we apply a constraint to the coefficients in the measurement equation $y_{1,t}$, however we let all other parameters in matrix $\boldsymbol{\beta}$ free, this does not necessary have to be the case, however it does not affect our analysis and simplifies the estimation. In addition, by letting $\boldsymbol{\beta}$ free for the rest of the maturities it is possible to observe or account for the existence of a term structure effect.

We compute maximum likelihood using the diffuse Kalman filter with the De Jong (1988, 1991) method for estimating the initial values, as our model is non-stationary.

For convenience, we have also applied the optimization algorithm Newton-Raphson technique instead of the Marquart and Berndt-Hall-Hall-Hausman, as seen in the works from Diebold *et al.* (2006).

In a second stage we estimate via OLS, the effects of macroeconomic data on the latent factors. The macroeconomic data used are the natural logarithms of Euro-Zone Unemployment, Euro-Zone Consumer Confidence Index, ECB *M3* levels and Euro-Zone Production Price Index, thus a possible specification could be:

$$\begin{pmatrix} u_t \\ v_t \\ \gamma_{1,t} \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_1 \end{pmatrix} + \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{11} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{11} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{11} \end{pmatrix} \begin{pmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \\ x_{4,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix}, \quad (12)$$

where $\varepsilon_{1,t} = NID(0, \sigma_u^2)$, $\varepsilon_{2,t} = NID(0, \sigma_v^2)$, and $\varepsilon_{3,t} = NID(0, \sigma_\gamma^2)$.

We will perform estimations (1-12) twice. Firstly, for Money Market yields: EONIA; Euribor 3M, Euribor 6M, 2 and 5 year German Government Benchmark and, secondly, for the Capital Markets yield curve comprising the 5, 10, 15, 20 and 30 year German Government Benchmarks. The yields and the macroeconomic data are on a monthly basis available in Bloomberg and as of end of month. The period considered is from December 1999 until January 2010, hence resulting in 122 observations.

Linking Latent Factors with a no-arbitrage term structure model

The expected price at t with maturity $N+1$ of a bond that redeems at $t+1$ is usually specified as follows,

$$P_t^{N+1} = E[m_{t+1} P_{t+1}^N], \quad (13)$$

where P_t^{N+1} is the price of a zero coupon bond of maturity $N+1$ at time t , m_{t+1} being the stochastic discount factor and P_{t+1}^N being the price of the same bond at $t+1$. By applying natural logarithms one has,

$$\ln[P_t^{(N+1)}] = \ln[m_{t+1}] + \ln[P_{t+1}^{(N)}]. \quad (14)$$

Whereby log prices are related to yields and this can be described as follows,

$$y_{t+1}^{(N)} = \frac{\ln[P_{t+1}^{(N)}]}{N}. \quad (15)$$

As seen in most recent no-arbitrage affine term structure literature log prices can be specified as a linear function of a state vector \mathbf{x}_{t+1} as follows:

$$-\ln[\mathcal{P}_{t+1}^{(N)}] = A(N) + \mathbf{B}(N)' \mathbf{x}_{t+1}, \quad (16)$$

where $A(N)$ is a scalar, $\mathbf{B}(N)'$ a $1 \times k$ vector of coefficients and \mathbf{x}_{t+1} a $k \times 1$ vector of state variables, which for this case $k=3$ for the level, slope and seasonal components. Note that the transpose of a vector or matrix is specified with a “ ‘ ”.

From (16) it is possible to find a closed solution and estimate the parameters $A(N)$ and $\mathbf{B}(N)'$. These parameters are obtained by linking observable yields to an observation equation describing the behaviour of a space state vector. This can be done by combining equations (15) and (16) at $t+1$ for any maturity, thus yielding,

$$y_{t+1}^{(N)} = \frac{A(N)}{N} + \frac{\mathbf{B}(N)'}{N} \mathbf{x}_{t+1}. \quad (17)$$

Intuitively, the short rate could be specified as follows,

$$y_{t+1}^{(1)} = A(1) + \mathbf{B}(1)' \mathbf{x}_{t+1}. \quad (18)$$

Empirically, equation (18) looks like,

$$y_{t+1}^{(1)} = a_0 + \mathbf{a}_1' \mathbf{x}_{t+1}. \quad (19)$$

However, from the restrictions in (11) it is possible to set $a_0=0$ and $\mathbf{a}_1'=(1 \ 1 \ 1)$. In addition, the state space vector \mathbf{x}_t is calibrated as follows,

$$\mathbf{x}_t = \begin{pmatrix} u_t \\ v_t \\ \gamma_{1,t} \end{pmatrix}. \quad (20)$$

The stochastic processes for \mathbf{x}_{t+1} and for the stochastic discount factor shown in (13) can be specified similarly to the pricing kernel à la Backus *et al.* (1998) which here is combined with the Vasicek (1977), for which a possible specification would be like,

$$\mathbf{x}_{t+1} = \mathbf{x}_t + \mathbf{\Phi}(\bar{\mathbf{x}} - \mathbf{x}_t) + \mathbf{\Sigma}_x \boldsymbol{\varepsilon}_{t+1}, \quad (21)$$

$$-\ln[m_{t+1}] = \delta + y_t^{(1)} + \boldsymbol{\lambda}' \boldsymbol{\varepsilon}_{t+1}. \quad (22)$$

Equation (21) describes the stochastic process of the independent state variables. Where \mathbf{x}_t and $\bar{\mathbf{x}}$ are both 3-dimensional vectors. $\mathbf{\Phi}$ is a 3×3 diagonal matrix, i.e. $\Phi_{i,i} = \Phi_i$, which represent the speed of adjustment at which each $x_{i,t}$ of elements reverse to their means. $\mathbf{\Sigma}_x$ is a diagonal 3×3 matrix comprising the volatility of the state variables. $\boldsymbol{\varepsilon}_{t+1}$ is a (3×1) -vector of shocks moving \mathbf{x}_t away from $\bar{\mathbf{x}}$ and with $\varepsilon_{i,t+1}$ elements being normally distributed with mean zero and variance unity.

Equation (22) is the stochastic discount factor as seen in Backus *et al.* (1998), however here with somehow a different setting, as (22) was originally the univariate Vasicek (1977) case and this paper calibrates using latent factors instead of the short rate. In this paper and similar to Jakas (2012), the multifactor case of a 3-dimension state variable is used. Furthermore, same as in Backus *et al.* (1998), δ is specified as follows,

$$\delta = \frac{1}{2} \sum_{i=1}^3 \lambda_i^2. \quad (23)$$

Clearly, specification (23) is fortuitous, the only aim is to normalise the stochastic discount factor so that it becomes the inverse of the short rate. Notice that with (23), now (22) has the following conditional mean and variance,

$$E \left[-\frac{1}{2} \sum_{i=1}^3 \lambda_i^2 - y_t^{(1)} - \boldsymbol{\lambda}' \boldsymbol{\epsilon}_{t+1} \right] = -\frac{1}{2} \sum_{i=1}^3 \lambda_i^2 - y_t^{(1)},$$

$$\text{Var} \left[-\frac{1}{2} \sum_{i=1}^3 \lambda_i^2 - y_t^{(1)} - \boldsymbol{\lambda}' \boldsymbol{\epsilon}_{t+1} \right] = \sum_{i=1}^3 \lambda_i^2,$$

where $\boldsymbol{\lambda}' = (\lambda_1 \lambda_2 \lambda_3)$. Therefore, assuming $E[\ln x] = \mu(x) + \frac{1}{2} \sigma^2(x)$ it yields, $E[\ln m_{t+1}] = -y_t^{(1)}$.

Here it is shown how to get to the solution. Starting first with equation (14) and substituting the right hand term for (22) and (16) one obtains,

$$\ln [P_t^{(N+1)}] = -\delta - y_t^{(1)} - \boldsymbol{\lambda}' \boldsymbol{\epsilon}_{t+1} - A(N) - \mathbf{B}(N)' \mathbf{x}_{t+1}. \quad (24)$$

In order to solve recursively δ is replaced by (23) and $y_t^{(1)}$ is replaced by (19). In addition, \mathbf{x}_{t+1} is also replaced for (21) to account for the Vasicek (1977) process. In sum one has,

$$\ln [P_t^{(N+1)}] = -\frac{1}{2} \sum_{i=1}^3 \lambda_i^2 - \gamma_1' - \boldsymbol{\lambda}' \boldsymbol{\epsilon}_{t+1} - A(N) - \mathbf{B}(N)' [\mathbf{x}_t + \boldsymbol{\Phi}(\bar{\mathbf{x}} - \mathbf{x}_t) + \boldsymbol{\sigma}_x \boldsymbol{\epsilon}_{t+1}]. \quad (25)$$

Notice that a_0 does not appear in equation (25) because $a_0 = 0$. The constant terms and the terms multiplying \mathbf{x}_t and $\boldsymbol{\epsilon}_{t+1}$ are grouped, thus yielding,

$$\ln [P_t^{(N+1)}] = -\left(\frac{1}{2} \sum_{i=1}^3 \lambda_i^2 + A(N) + \mathbf{B}(N)' \boldsymbol{\Phi} \bar{\mathbf{x}} \right) - [a_1' + \mathbf{B}(N)' (\mathbf{I} - \boldsymbol{\Phi})] \mathbf{x}_t - [\boldsymbol{\lambda}' + \mathbf{B}(N)' \boldsymbol{\sigma}_x] \boldsymbol{\epsilon}_{t+1}. \quad (26)$$

where \mathbf{I} denotes the (3×3) -identity matrix. The right hand side of equation (25), which has now developed into (26), has the following conditional moments,

$$E[\ln m_{t+1} + \ln P_{t+1}^{(N)}] = -\left(\frac{1}{2} \sum_{i=1}^3 \lambda_i^2 + A(N) + \mathbf{B}(N)' \boldsymbol{\Phi} \bar{\mathbf{x}} \right) - [a_1' + \mathbf{B}(N)' (\mathbf{I} - \boldsymbol{\Phi})] \mathbf{x}_t, \quad (27)$$

and,

$$\text{Var}[\ln m_{t+1} + \ln P_{t+1}^{(N)}] = (\boldsymbol{\lambda}' + \mathbf{B}(N)' \boldsymbol{\sigma}_x)^2. \quad (28)$$

Bearing in mind that the implied present-value of a fixed income security yields,

$$-E[\ln P_t^{(N+1)}] = -E[\ln m_{t+1} + \ln P_{t+1}^{(N)}] - \frac{1}{2} \text{Var}[\ln m_{t+1} + \ln P_{t+1}^{(N)}] . \quad (29)$$

By substituting [27] and [28] into [29] one obtains,

$$-E[\ln P_t^{(N+1)}] = \frac{1}{2} \sum_{i=1}^3 \lambda_i^2 + A(N) + \mathbf{B}(N)' \Phi \bar{\mathbf{x}} + [\mathbf{a}'_1 + \mathbf{B}(N)'(\mathbf{I} - \Phi)] \mathbf{x}_t - \frac{1}{2} (\boldsymbol{\lambda} + \mathbf{B}(N)' \boldsymbol{\sigma}_x)^2 . \quad (30)$$

Rearranging the constant terms and the terms multiplying \mathbf{x}_t and lining up with (16) yields,

$$A(N+1) = A(N) + \mathbf{B}(N)' \Phi \bar{\mathbf{x}} + \frac{1}{2} \left(\sum_{i=1}^3 \lambda_i^2 - [\boldsymbol{\lambda}' + \mathbf{B}(N)' \boldsymbol{\sigma}_x]^2 \right) , \quad (31)$$

$$\mathbf{B}(N+1)' = \mathbf{a}'_1 + \mathbf{B}(N)'(\mathbf{I} - \Phi) . \quad (32)$$

The solution is obtained by computing the present value recursively using (14) for some guess of coefficients from (17). Since $P_{t+1}^{(0)} = 1$, $A(0) = 0$ and $\mathbf{B}(0)' = \mathbf{0}$, which means this can be solved recursively, as for one period would imply $A(1) = 0$ and $\mathbf{B}(1)' = \mathbf{a}'_1$ which means that equals the short rate as described in (19). Now for any set of state variables the resulting yield curve can be computed. As this author is trying to compute the coefficients for maturity N , all is needed is to use (17) to compute the present value of an $N+1$ maturity bond. Subsequently, we replace (31) and (32) into (17) and solve numerically by fitting the curve to the observed yields by adjusting λ for a given choice of maturities, recalling that parameters a_0 and \mathbf{a}'_1 are restricted to discussion in (11) and (19).

■ 3. Discussion of results

Tables 1 and 2 show the coefficients and standard errors obtained from the state space model discussed in equations (1) to (11) for Money Market yields (comprising the maturities ranging from EONIA to 5 years) and for Capital Market yields (hence, the maturities ranging from 5 years to 30 years). Notice that it is reasonable to let an overlapping between 2 years and 5 years, as it is generally accepted that between these maturities often Capital and Money Market instruments act as substitutes. Both tables show that the coefficients are very significant with the exception of the seasonal component for the case of the Money Markets, as the seasonal component appears to be significant only for the case of the Capital Markets curve.

● **Table 1. Money Market Curve: EONIA, Euribor 3M, Euribor 6M, 2Yr and 5Yr German Govy Bond**

	Level u_t	Slope v_t	Season γ_{1t}
EONIA	1	1	1
Euribor 3M	1.042 (.0108)	1.7072 (.2309)	.1453 (.1585)
Euribor 6M	1.0636 (.0107)	1.4985 (.2346)	.0375 (.1644)
BRD 2 Years	1.018 (.0143)	-.9528 (.2650)	.0005 (.1608)
BRD 5 Years	1.1148 (.0233)	-2.9520 (.4203)	-.0945 (.2291)
$\sigma_{u,v,\gamma}^2$ (state)	.0333 (.0048)	-	-
σ_y^2 (observation)	.0655 (.0049)	-	-
$\hat{\mu}$	3.0781	-.0460	.0023
$\hat{\delta}$	1.1236	.1691	.1935

Note: With the exception of the seasonal component all other parameters are very significant with p-values below 0.05, thus $P > |z| = 0$.

Tables 1 and 2 also show that for both yield curves – hence the Money Market as well as the Capital Market yield curves – the coefficients for the Level u_t increases with the maturity. Interestingly, the coefficients for the Slope factor v_t in the Money Market curve exhibit different behaviour to increasing maturities with respect to those seen for the Capital Market curve. The coefficients for v_t in the Money Market curve start with a positive value and becomes negative for the 2 years onwards thus is decreasing. However, the coefficients for v_t in the capital market curve increase as maturities become longer.

● **Table 2. Capital Market Curve: 5, 10, 15, 20 and 30 year German Govy Bonds**

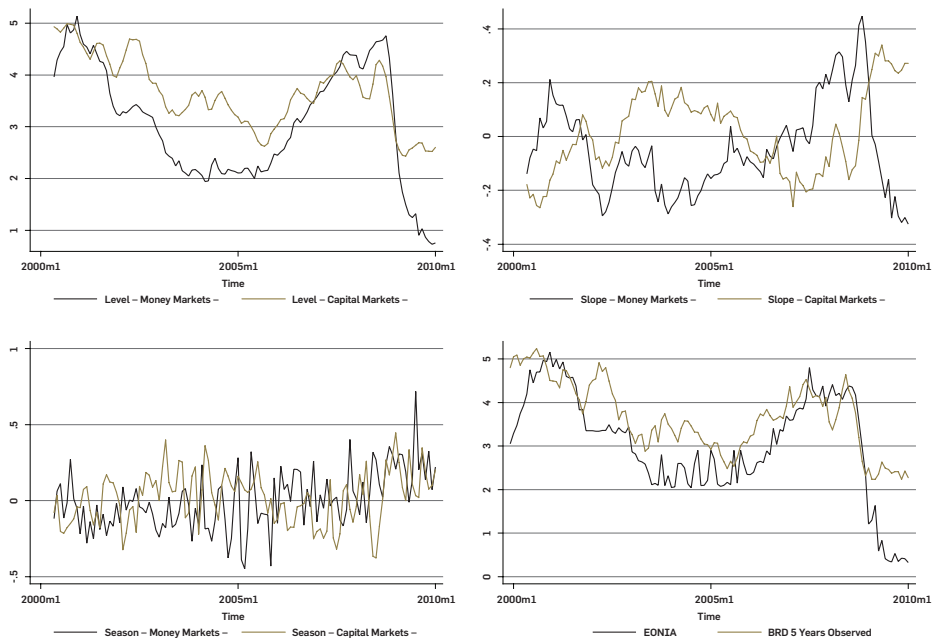
	Level u_t	Slope v_t	Season γ_{1t}
BRD 5 Years	1	1	1
BRD 10 Years	1.1242 (.0098)	3.3261 (.2345)	1.0536 (.1971)
BRD 15 Years	1.1921 (.0143)	4.4612 (.3414)	1.1864 (.2864)
BRD 20 Years	1.2360 (.0177)	5.3056 (.4228)	1.1558 (.3334)
BRD 30 Years	1.2541 (.0174)	5.257 (.4159)	1.0366 (.3343)
$\sigma_{u,v,\gamma}^2$ (state)	.0305 (.0045)	-	-
σ_y^2 (observation)	.0065 (.0006)	-	-
$\hat{\mu}$	3.6729	.0129	.0197
$\hat{\delta}$.6855	.1493	.1717

Note: All estimates very significant with p-values $P > |z| = 0$.

The seasonal component γ_{1t} for the Money Market curve is insignificantly close to zero, and for the capital markets curve is significantly close to one.

Figure 1 shows the latent factors (level, slope and seasonal components) as well as the EONIA and 5 year German Government bond. By comparing the top left hand chart which shows the levels for the money and capital markets with the bottom right hand chart which shows the EONIA and the 5 year German Government yield, it is possible to recognise that the level is possibly the most relevant parameter as it appears to follow almost the same stochastic path. The top right hand chart showing the time path for both slopes which vary mostly between -0.25 and $+0.25$ and breach these boundaries towards around pre and post Lehman's collapse. The capital markets slope appears to lag the money market slope at the beginning of the series and exhibits rather smoother turnarounds. The seasonality component for both time series seems stationary with no apparent trend.

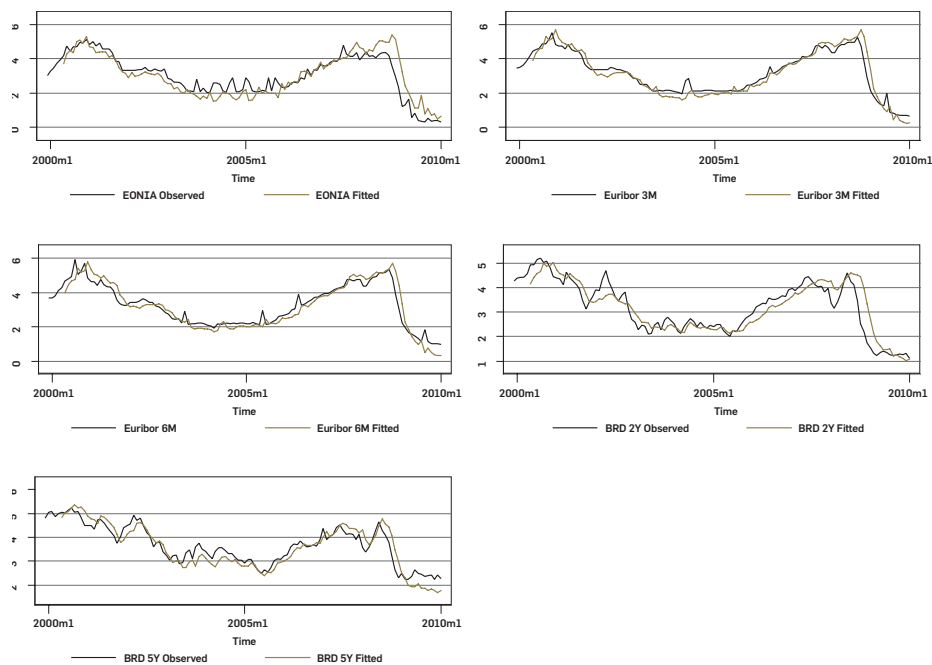
Figure 1. Latent factors level, slope, curvature and rates for money markets and capital markets rates



Figures 2 and 3 show fitted versus observed values obtained by running the state space model described in (1) to (11). The fitted values seem to follow quite close the observed yields. These results are encouraging, as they are very similar to those seen in Jakas (2011, 2012) and Jakas and Jakas (2013). Not surprisingly, this stems from the fact that the latent factors estimated do a good job in replicating the yields, as most of the effect comes from the Level, which shows a very similar be-

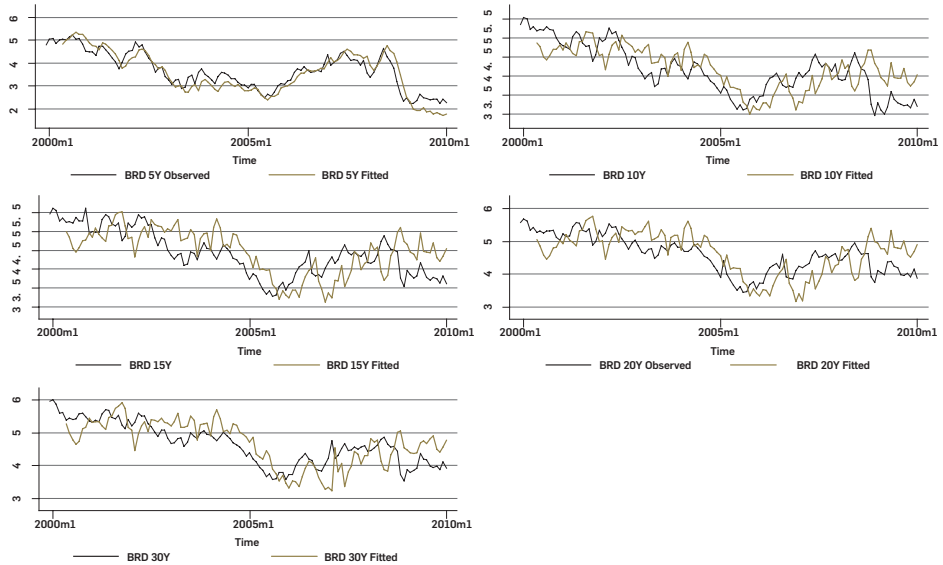
behaviour to the EONIA rate. In addition, it can be seen that as maturities become larger the model performs poorer, but still better than the results seen in Jakas (2012) and Jakas and Jakas (2013), where solely macro data were used for calibrating the model. As maturities become longer, the Capital Market Level is likely to be more influential than the Money Market Level, thus suggesting that there are long term components evidencing a different structure between the front and the long end of the curve. This improvement is mostly due to the fact that the yield curve has been segregated between money and capital markets and hence latent factors for longer maturities are different, as they carry information which is more relevant to yields on the long end, whereas latent factors influencing the short end have less predictive ability on long end yields.

Figure 2. Fitted versus observed money market yields



It should be mentioned that most of the research has been focused on yields up to 10 year maturities. Models fitting yields in the short end up to 10 years always perform better than for those trying to fit longer maturities such as 20 and 30 years. If the state space for the capital markets is run by dropping from the model the 5 years and leaving only the maturities comprising 10, 15, 20 and 30 years the fitted values become even closer to the observed long end yields. This effect is mostly attributed to the fact that the front end of the curve appears to have less information influencing yields on the longer maturities.

Figure 3. Fitted versus observed capital market yields



Tables 3 and 4 show the OLS (robust) results as discussed in (12). Recalling that the Level u_t is the most important factor governing the yields, the macroeconomic data influencing this factor is analysed in this section. Table 3 and Table 4 show that coefficients for the macroeconomic factors influencing the Level, Slope and Seasonal components are smaller for capital market latent factors compared to money market latent factors. In fact, the signs of the coefficients only seem to be in agreement for the case of the Level. For the Slope factor, only the consumer confidence coefficients are similar in size and exhibit the same sign. For the Seasonal component the coefficients for unemployment and consumer confidence exhibit same signs however, they differ in size significantly.

Table 3. OLS (robust) results of money market latent factors versus macroeconomic data

	Level u_t	Slope v_t	Season γ_{1t}	$\hat{\mu}$
$\text{Ln } U_t$	-9.655936 (.5953772)	-1.348114 (.1271292)	.4216055 (.301257)	2.1329
$\text{Ln } PPI_t$	18.67764 (2.140462)	3.700983 (.7038401)	1.658626 (1.0074)	4.6036
$\text{Ln } M3_t$	-8.055411 (.7723205)	-1.299466 (.2410876)	-.1551028 (.3770993)	8.8127
$\text{Ln } CC_t$	-1.070341 (.556391)	-.9127945 (.1554375)	-.1715393 (.1920829)	4.4767
Intercept	13.47607 (5.312529)	1.323285 (1.304486)	-6.404112 (2.249984)	
R-squared	0.9267	0.7702	0.2578	

$\text{Ln } U_t$ = Euro-Zone Unemployment rate; $\text{Ln } PPI_t$ = Euro-Zone Production Price Index; $\text{Ln } M3_t$ = ECB M3 Money Aggregate and, $\text{Ln } CC_t$ = Euro-Zone Consumer Confidence Index.

● **Table 4. OLS results of capital market latent factors versus macroeconomic data**

	Level u_t	Slope v_t	Season γ_{1t}	$\hat{\mu}$
$\text{Ln } U_t$	-4.60098 (.5729704)	.4937988 (.1106354)	.083843 (.2393815)	2.1329
$\text{Ln } PPI_t$	2.715887 (2.305689)	-2.10111 (.3090523)	-1.455954 (1.062349)	4.6036
$\text{Ln } M3_t$	-2.710198 (.7925468)	.7113546 (.1122904)	.3672107 (.378095)	8.8127
$\text{Ln } CC_t$.9895847 (.4375565)	-.8902952 (.084333)	-.9289367 (.2541476)	4.4767
Intercept	20.46274 (4.951114)	6.344468 (.8901571)	7.462491 (2.146008)	
R-squared	0.8047	0.8633	0.3356	

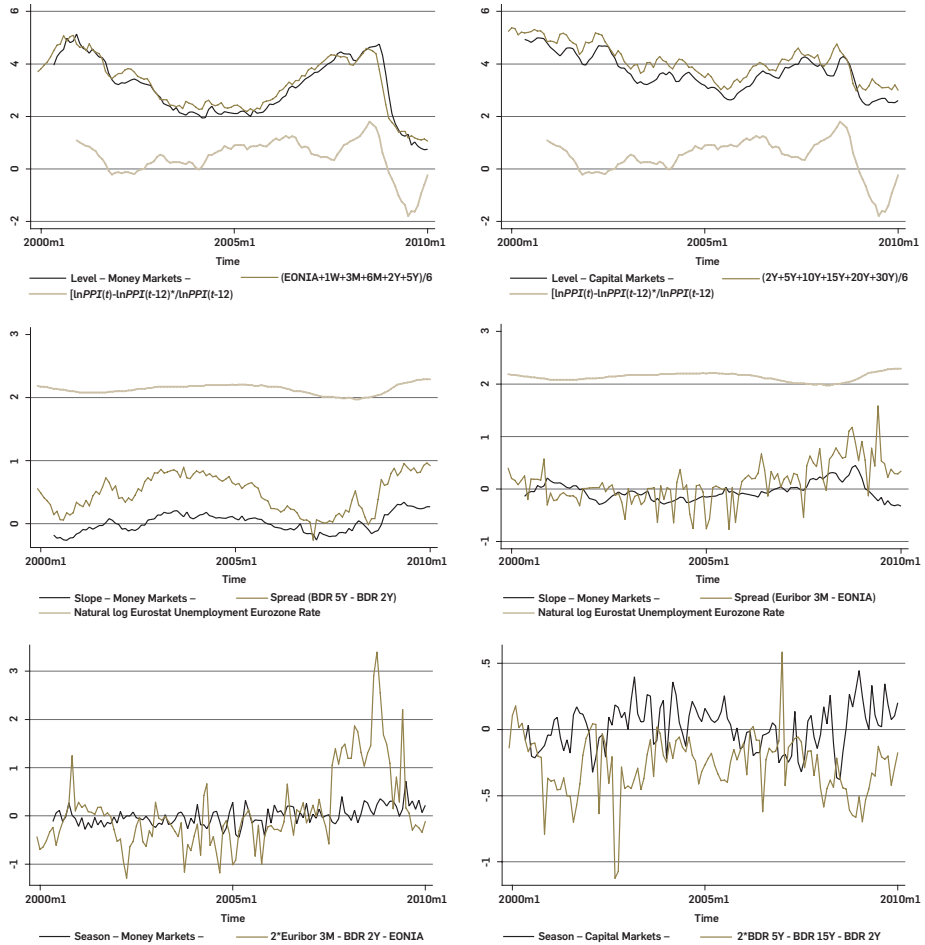
$\text{Ln } U_t$ = Euro-Zone Unemployment rate; $\text{Ln } PPI_t$ = Euro-Zone Production Price Index; $\text{Ln } M3_t$ = ECB M3 Money Aggregate and, $\text{Ln } CC_t$ = Euro-Zone Consumer Confidence Index.

The theoretical interpretation of the effects of the macroeconomic data on the Slope and Seasonal component factors are left for the reader to go through the exercise. However, the interpretation of the level is less challenging, as it appears to be pretty much in line with economic theory. For example Tables 3 and 4 show that increases in unemployment rate result in a fall in yields. This makes sense as the yield curve studied is the risk free curve and hence if unemployment increases, expected aggregate consumption growth is expected to be lower with the subsequent fall in risk-free asset yields. On the other hand, a fall in unemployment is expected to decrease inflationary pressures so that central banks have no reason for keeping policy rates high and hence are likely to introduce rate cuts. In same fashion, if the price level PPI increases this is expected to result in an increase of the short rate as a consequence of central bank policy, but also an increase in the price level is expected to rise the overall level of interest rates, mostly, in order to compensate investors for the loss in value on real money balances. An increase in money supply $M3$ is expected to result in a fall in interest rates, as seen in the classical IS-LM models. The sign of the consumer confidence index is not as expected by the theory. In addition, the coefficient does not appear to be very significant and its contribution to the overall variance is negligible.

Figure 4 shows the latent factors level, the slope and seasonal component and their empirical counterparts. Here, the empirical counterparts differ to those of Diebold *et al.* (2006) as in this research the yield curve is segregated into money markets and capital markets. Notice that by doing so it is possible to account for different behaviours of the level in the front end and the level in the long end of the curve. For both cases the level fits very well the observed empirical counterparts. Correlations between Money and Capital Market levels u_t with their empirical counterparts (EONIA+...+5Y)/6 and (2Y+...+30Y)/6 is of 0.93 and 0.88 respectively. The correlation between

the money and capital market levels with current inflation ($(\ln PPI_t - \ln PPI_{t-12}) / \ln PPI_{t-12}$) is of 0.34 and 0.06 respectively.

Figure 4. Level, slope, seasonal components and their empirical counterparts



For the case of the slope, it can be shown that in both cases it is possible to replicate very well the trend however Money Market slope is less volatile than its empirical counterpart. Interestingly, the slope for the Capital Market follows a similar pattern to that of its empirical counterpart. Correlations between Money and Capital Market slopes with their empirical counterparts (Euribor3M-EONIA) and (5Y-2Y) is of 0.14 and 0.78 respectively. The correlation between the money and capital market levels with current unemployment is of 0.57 and 0.53 respectively. So these results appear to be in support representing interest rates levels for money and capital markets and being the slope which seems to be more relevant for the Capital Market and less relevant for the Money market curve. The seasonal component is rather inconclusive, as for the Money Market

the empirical counterpart is much more volatile, particularly during the Lehman collapse. Somehow a better picture is observed for the Capital Markets seasonal component however still, they do not seem to match as nicely as it did for the level or the slope factors. So far, this paper has concentrated in applying a local level model with a stochastic slope and a stochastic seasonal component with no feedback. Thus, innovations in the latent variables do not feed back to the macroeconomy. This assumption can be tested via a basic *VAR* model, orthogonal impulse response functions as well as the forecast error variance decompositions and the classical Granger Causality test, all of these will be taken care of in this section.

Results for lag selection

For the lag order selection criteria for a series vector autoregressions of order 1, a prediction error (FPE), Akaike's information criterion (AIC), Schwarz's Bayesian information criterion (SBIC), and the Hannan and Quinn information criterion (HQIC) are used. According to our results, the FPE and AIC selected 2 lags, and SBIC and HQIC selected 1 lag for the Money Markets curve. So for simplicity's sake and following the theoretical advantages of using SBIC and HQIC over FPE and AIC, as discussed in Lütkepohl (2005, 148-152) 1 lag is selected for the *VAR*. For the Capital Markets curve the SBIC and HQIC selected 4 and 1 lags accordingly. Here, in order to keep consistency, the lags selected have also been of 1 order.

Tables 5 and 6 show *VAR* results for the coefficients and standard errors for the Money Market and Capital Market latent factors with the respective macroeconomic variables. The macroeconomic variables used exhibit a significant autocorrelation to their one-period lags and appear not to be a function of the other macroeconomic or latent factors. On the contrary, the latent factors do seem to be influenced by some of its own lags, as well as by lagged macroeconomic data. For the reader's convenience, coefficients which are significantly different from zero have been bold highlighted. According to the *VAR* results, there appears to be very little feedback from latent factors to the macroeconomy and a rather significant feedback from the macroeconomy to the latent factors. This does not mean that there is no feedback at all from latent factors to the macroeconomy, but rather that this feedback is weaker. This is a striking result, as it would appear that interest rate levels provide little feedback to the macroeconomy according to the period analysed.

Figures 5 and 6 show the orthogonalised impulse response functions and, they seem to confirm this view. These results are in line to those seen in Jakas (2011) and in line with the *VAR* results, which suggests that little or no feedback is observed between latent factors and the macroeconomy, however there is a clear statistical relationship from lagged macro variables to the latent factors particularly to the level.

● Table 5. VAR results of money market latent factors versus macroeconomic data

	$\text{Ln } U_{t-1}$	$\text{Ln } PPI_{t-1}$	$\text{Ln } M3_{t-1}$	$\text{Ln } CC_{t-1}$	Level u_{t-1}	Slope v_{t-1}	Season γ_{1t-1}	Intercept
$\text{Ln } U_t$	1.007 (.0274)	-.1177 (.0650)	.0385 (.0256)	-.1051 (.0143)	.0074 (.0028)	-.0092 (.0102)	.0038 (.0044)	.6337 (.1336)
$\text{Ln } PPI_t$	-.0270 (.0157)	.9863 (.0372)	.0049 (.0146)	.0462 (.0082)	-.0047 (.0016)	.0083 (.0058)	.0003 (.0025)	-.1130 (.0764)
$\text{Ln } M3_t$	-.0531 (.0187)	.1021 (.0444)	.9559 (.0174)	.0012 (.0097)	-.0050 (.0019)	.0094 (.0069)	-.0053 (.0030)	.0470 (.0911)
$\text{Ln } CC_t$	-.0738 (.0595)	.0794 (.1410)	-.0366 (.0553)	1.015 (.0311)	-.0131 (.0061)	-.0156 (.0222)	.0100 (.0096)	.0830 (.2895)
Level u_t	-1.2932 (.4520)	5.7258 (1.0709)	-1.828 (.4207)	.9297 (.2356)	.8646 (.0465)	-6.903 (.1688)	-.1798 (.0730)	-11.2878 (2.1988)
Slope v_t	.0765 (.1868)	1.292 (.4426)	-.3223 (.1737)	-.0614 (.0974)	.04016 (.0192)	.6247 (.0698)	-.0715 (.0301)	-3.140 (.9087)
Season γ_{1t}	1.8691 (.5679)	-.3229 (1.3452)	.8502 (.5284)	-.3829 (.2961)	.2130 (.0585)	-4.671 (.2120)	.0437 (.0917)	-8.9613 (2.7621)

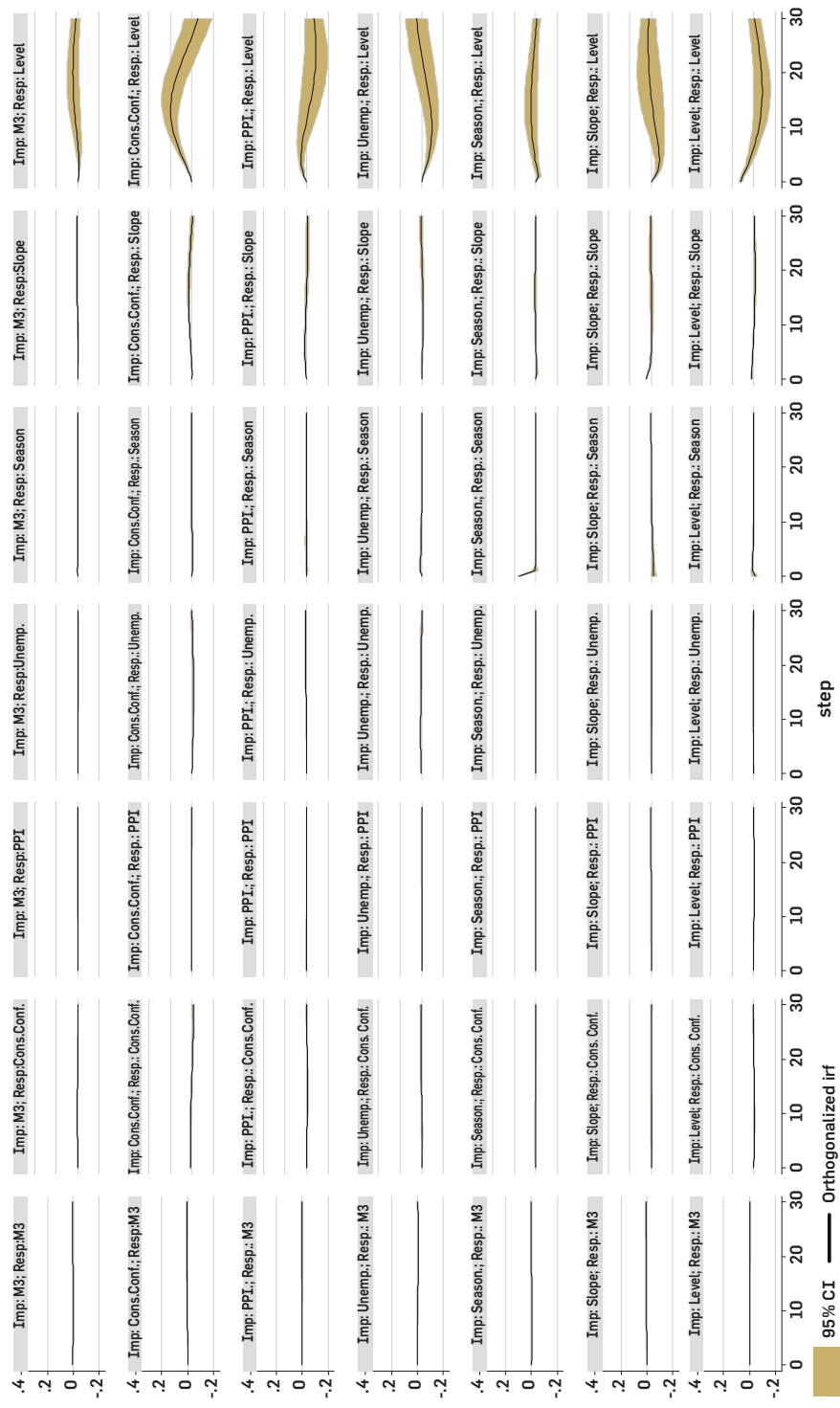
Note: Sample: 2000m6 - 2010m1; No. of obs = 116; all equations significant with $P > \chi^2$ at P -values of 0.0000; All equations with $R\text{-sq} > 0.91$ except for season at 0.36, and $\text{Ln } U_t$ = Euro-Zone Unemployment rate; $\text{Ln } PPI_t$ = Euro-Zone Production Price Index; $\text{Ln } M3_t$ = ECB M3 Money Aggregate and, $\text{Ln } CC_t$ = Euro-Zone Consumer Confidence Index.

● Table 6. VAR results of capital market latent factors versus macroeconomic data

	$\text{Ln } U_{t-1}$	$\text{Ln } PPI_{t-1}$	$\text{Ln } M3_{t-1}$	$\text{Ln } CC_{t-1}$	Level u_{t-1}	Slope v_{t-1}	Season γ_{1t-1}	Intercept
$\text{Ln } U_t$.9861 (.0174)	.0261 (.0531)	-.0018 (.0190)	-.0865 (.0161)	.0098 (.0024)	.0222 (.0139)	.0076 (.0052)	.2763 (.1667)
$\text{Ln } PPI_t$.1667 (.0310)	.9456 (.0111)	(.0102) (.0094)	.0477 (.0013)	-.0041 (.0081)	.0057 (.0030)	-.0051 (.0974)	-.0784
$\text{Ln } M3_t$	-.0371 (.0123)	.0257 (.0376)	.9793 (.0134)	-.0033 (.0114)	-.0052 (.0016)	-.0163 (.0099)	.0079 (.0036)	.1833 (.1181)
$\text{Ln } CC_t$.0395 (.0395)	-.2772 (.1202)	.0874 (.0430)	1.003 (.0365)	-.0115 (.0053)	-.0351 (.0316)	-.0178 (.0117)	.4482 (.3774)
Level u_t	-5.870 (2.156)	-.8975 (.6563)	.0902 (.2352)	.4419 (.1993)	.8358 (.0291)	.2504 (.1727)	-6.970 (.0640)	3.2088 (2.0599)
Slope v_t	.1872 (.0951)	-.2362 (.2896)	.1164 (.1038)	-3.608 (.0879)	.0319 (.0126)	.7204 (.0762)	.0794 (.0282)	1.1639 (.9091)
Season γ_{1t}	.3303 (.3049)	.6967 (.9279)	-.0833 (.3325)	-.1485 (.2818)	.1340 (.0412)	.7537 (.2443)	.2891 (.0905)	-3.0001 (2.9124)

Note: Sample: 2000m6 - 2010m1; No. of obs = 116; all equations significant with $P > \chi^2$ at P -values of 0.0000; All equations with $R\text{-sq} > 0.92$ except for season at 0.46, and $\text{Ln } U_t$ = Euro-Zone Unemployment rate; $\text{Ln } PPI_t$ = Euro-Zone Production Price Index; $\text{Ln } M3_t$ = ECB M3 Money Aggregate and, $\text{Ln } CC_t$ = Euro-Zone Consumer Confidence Index.

Figure 5. Impulse response functions for money market latent and macroeconomic variables



Graphs by irfname, impulse variable, and response variable

Note: "M3" for ECB M3 Money Aggregate; "Cons.Conf." as Euro-Zone Consumer Confidence Index; "PPI" for Euro-Zone Production Price Index and "Unemp." for Euro-Zone Unemployment Rate.

In light of the above results, a Granger Causality test is performed. Here it is possible to observe in Tables 7 and 8 below that feedback from latent factors to the macro variables exists. The test though does not tell us the size of these feedbacks. For example, in the case of Money Market yields (see Table 7) it is possible to see that Consumer Confidence and the Level Granger-cause unemployment rate and *PPI*, amongst others (see bold font in columns $\ln CC_t$ and Level_t). It can also be seen that monetary aggregate *M3* is Granger-caused by unemployment, *PPI* and the Level (with *p*-values: 0.005, 0.021 and 0.009 respectively). Consumer Confidence is only Granger-caused by the Level (with a *p*-value: 0.032). The Level is Granger-caused by all macro and latent factors. The Slope is Granger-caused by *PPI*, the Level and by the Seasonal component (with *p*-values: 0.004, 0.037 and 0.018 respectively). Interestingly, the Seasonal component is Granger-caused by the unemployment rate, the Level and the Slope (with *p*-values: 0.001, 0.000 and 0.028 respectively). The Granger Causality test using Capital Market latent factors have the following discrepancies with respect to the Money Market latent factors; 1) *PPI* does not Granger-cause *M3* (*p*-value: 0.495), 2) *PPI*, *M3* and the Slope do not Granger-cause the Level (*p*-values: 0.172, 0.701 and 0.147, respectively), 3) Unemployment and Consumer Confidence index Granger-cause the Slope, 4) *PPI* does not Granger-cause the slope and 4) Unemployment does not Granger-causes the seasonal component.

● **Table 7. Money markets Granger causality test for Prob > chi2, so that if below *p*-values ≤ 0.05 the *h*₀ “excluded variable does not Granger cause the equation of the endogenous variable” is rejected**

Excluded	$\ln U_{t-1}$	$\ln PPI_{t-1}$	$\ln M3_{t-1}$	$\ln CC_{t-1}$	Level_t u_{t-1}	Slope_t v_{t-1}	Season_t $\gamma_{1,t-1}$	All
Equation								
$\ln U_t$	-	0.070	0.132	0.000	0.008	0.365	0.388	0.000
$\ln PPI_t$	0.085	-	0.735	0.000	0.004	0.152	0.883	0.000
$\ln M3_t$	0.005	0.021	-	0.894	0.009	0.175	0.075	0.000
$\ln CC_t$	0.214	0.573	0.508	-	0.032	0.481	0.297	0.000
$\text{Level } u_t$	0.004	0.000	0.000	0.000	-	0.000	0.014	0.000
$\text{Slope } v_t$	0.682	0.004	0.064	0.528	0.037	-	0.018	0.000
$\text{Season } \gamma_{1,t}$	0.001	0.810	0.108	0.196	0.000	0.028	-	0.000

Note: $\ln U_t$ = Euro-Zone Unemployment rate; $\ln PPI_t$ = Euro-Zone Production Price Index; $\ln M3_t$ = ECB *M3* Money Aggregate and $\ln CC_t$ = Euro-Zone Consumer Confidence Index.

● **Table 8.** Capital markets Granger causality test for Prob > chi2, so that if below *p*-values ≤ 0.05 the ho “excluded variable does not Granger cause the equation of the endogenous variable” is rejected

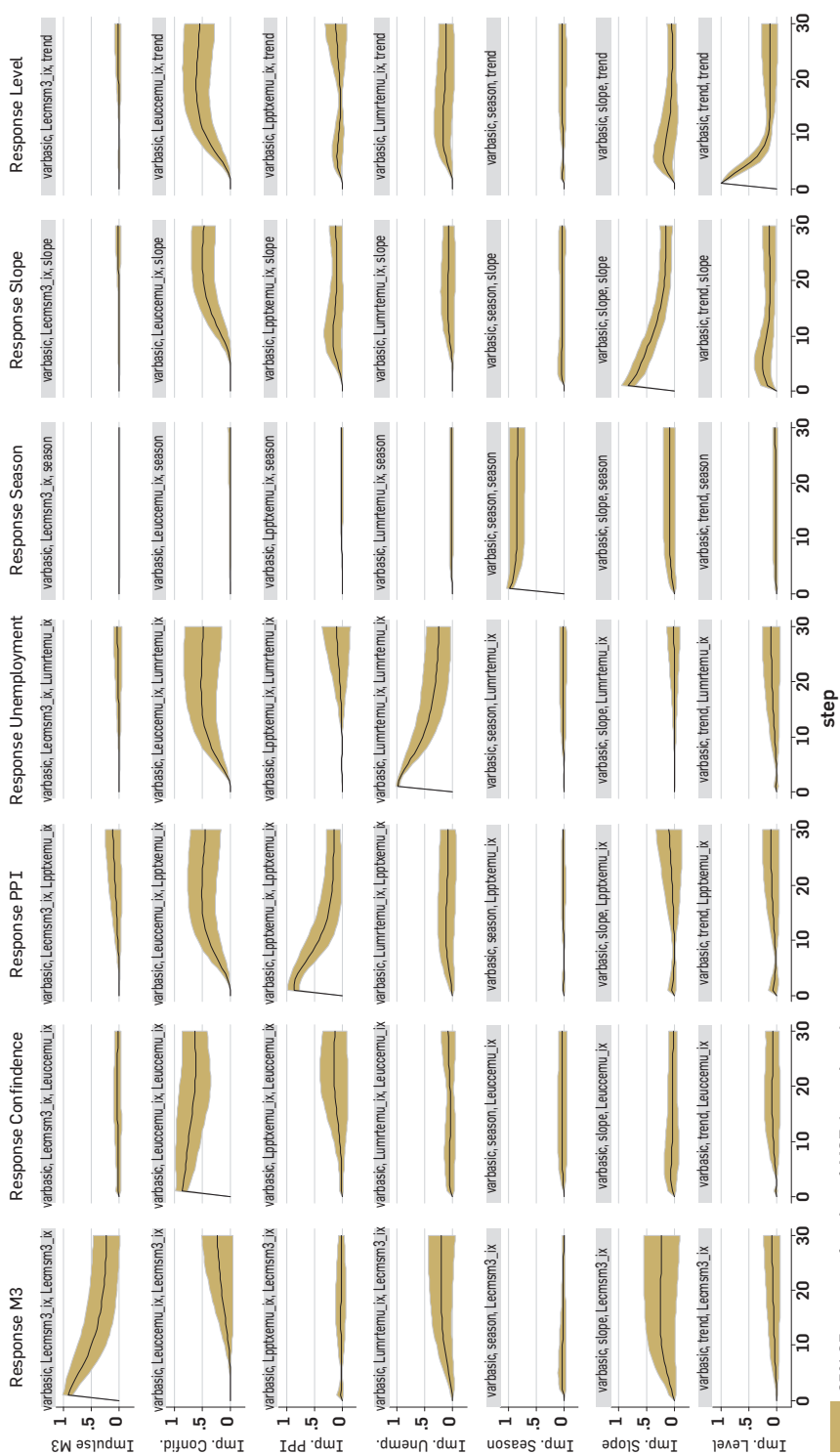
Excluded	$\ln U_{t-1}$	$\ln PPI_{t-1}$	$\ln M3_{t-1}$	$\ln CC_{t-1}$	Level u_{t-1}	Slope v_{t-1}	Season γ_{1t-1}	All
Equation								
$\ln U_t$	-	0.624	0.927	0.000	0.000	0.112	0.138	0.000
$\ln PPI_t$	0.155	-	0.097	0.000	0.003	0.481	0.090	0.000
$\ln M3_t$	0.003	0.495	-	0.769	0.002	0.100	0.030	0.000
$\ln CC_t$	0.318	0.021	0.042	-	0.030	0.268	0.129	0.000
Level u_t	0.006	0.172	0.701	0.027	-	0.147	0.000	0.000
Slope v_t	0.049	0.415	0.262	0.000	0.013	-	0.005	0.000
Season γ_{1t}	0.279	0.453	0.802	0.598	0.001	0.002	-	0.000

Note: $\ln U_t$ = Euro-Zone Unemployment rate; $\ln PPI_t$ = Euro-Zone Production Price Index; $\ln M3_t$ = ECB M3 Money Aggregate and, $\ln CC_t$ = Euro-Zone Consumer Confidence Index.

In summary, it could be said that there is a clear effect from the macroeconomy to the yield curve and from the yield curve to the macroeconomy however, from what we have learned from VAR Tables 5 and 6 and impulse response Figures 5 and 6, feedback from the yield curve to the macroeconomy seems to be weak.

Figures 7 and 8 show the Cholesky forecast error variance decomposition (FEVD) to orthogonal shocks. Here, it is also possible to observe that orthogonal shocks cause permanent effects on some of the latent and macro factors. For example, shocks on consumer confidence index (second row) has permanent effects on PPI, unemployment, the slope and the level, thus these shocks do not die away, but in turn persist in time. Interestingly, own variable orthogonal shocks, thus auto-shocks, in some factors appear to have permanent effects too or exhibit persistence, as they appear to last several periods before they die away (see for example the diagonal charts in figures 7 and 8 below, for instance, see the consumer confidence index and the seasonal component diagonal charts). Another interesting outcome is that the level, the slope and the monetary aggregate M3 have weak impact on the price level (PPI). This can be seen – presumably – as a result of the ECB being successful in anchoring long term inflation expectations at low levels. The price level seems to be mostly influenced by orthogonal shocks on consumer confidence index which in turn seems to be independent of all other factors.

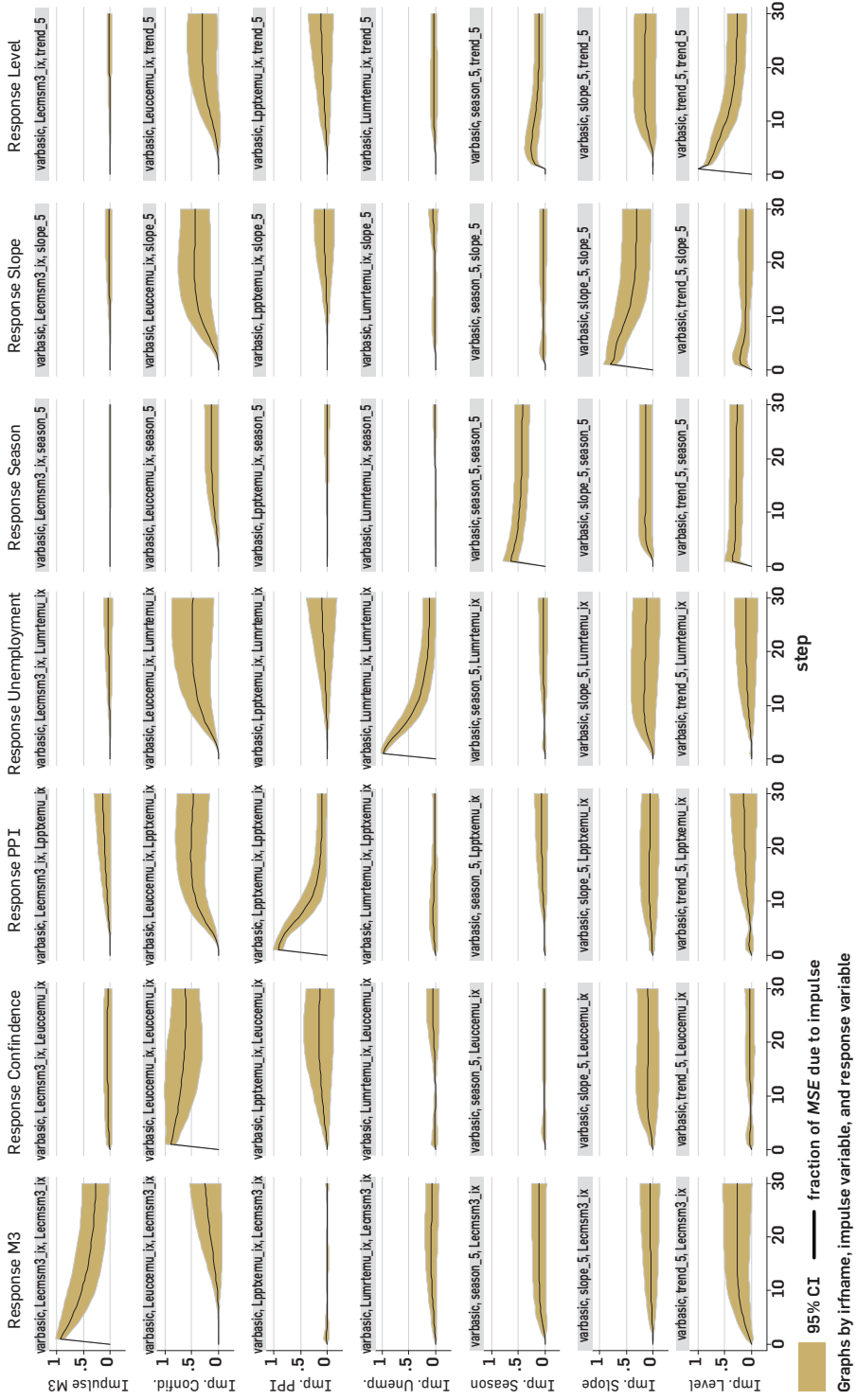
Figure 7. Forecast variance decomposition to cholesky orthogonal shocks for money market latent and macroeconomic variables



95% CI — fraction of MSE due to impulse
 Graphs by irfname, impulse variable, and response variable

Note: "M3" for ECB M3 Money Aggregate; "Confid." or "Confidence" as Euro-Zone Consumer Confidence Index; "???" for Euro-Zone Production Price Index and "Unemp." or "Unemployment" for Euro-Zone Unemployment Rate.

Figure 8. Forecast variance decomposition to cholesky orthogonal shocks for capital market latent and macroeconomic variables



Note: "M3" for ECB M3 Money Aggregate; "Confid." or "Confidence" as Euro-Zone Consumer Confidence Index; "Unemp." or "Unemployment" for Euro-Zone Unemployment Rate.

Interestingly enough, a Johansen test for cointegration has been also performed for both the Money Markets as well as the Capital Markets latent factors applying a vector error correction model of two lags on the Level, Slope, Season, and unemployment rate, $M3$, PPI and Consumer Confidence index. The results show that Money Markets exhibit at least four cointegrating equations and that Capital Markets exhibit three cointegrating equations. This suggests that the latent factors and macroeconomic variables used are highly cointegrated.

■ 4. Calibrating an affine term structure model with latent factors

In this section an affine term structure model is calibrated with the money market latent factors discussed in previous sections and entered into equations (13) to (32). Thus a no-arbitrage model is fitted by calibrating the state vector with the latent variables: obtained from the local level model. Figure 9 below shows that the affine approach to yield curve modelling seems to fit quite well the observed yields, even for the 10 year maturities. However, this deteriorates as the maturity gets longer as seen also in most of the empirical research. Clearly, these results suggest that short term components in the yield curve (money market level, slope and season) which have high predictive power on the front end of the curve, exhibits a diminishing predictive power as maturities become larger. It could also be interpreted that the front and the long end of the curve are governed by different factors which appear not to have much in common, thus casting some doubt on the use of a no-arbitrage model for these maturities spectrum. The results shown in Figure 9 seem to support the approach of breaking the yield curve in two types of markets: the money markets and the capital markets. For money markets being the yields governed by short term latent factors, hence comprising the maturities from overnight (EONIA) to 2-5 year German Govies, and for capital markets being the yields governed by long term latent factors, hence for maturities ranging from 2-5 years up to the 30 year German Govies. Figures 10 and 11 show the average yield curve fitted using the affine term structure model and the coefficients for equation (17) discussed in section 2. Here it can be seen that all coefficients are positive and decrease as maturities get closer to the 30 years (or 360 months), with the exception of the $A(N)/N$ which increases as maturity becomes larger.

Figure 9. Yield curve fitted with an affine term structure model using latent factors as state variables

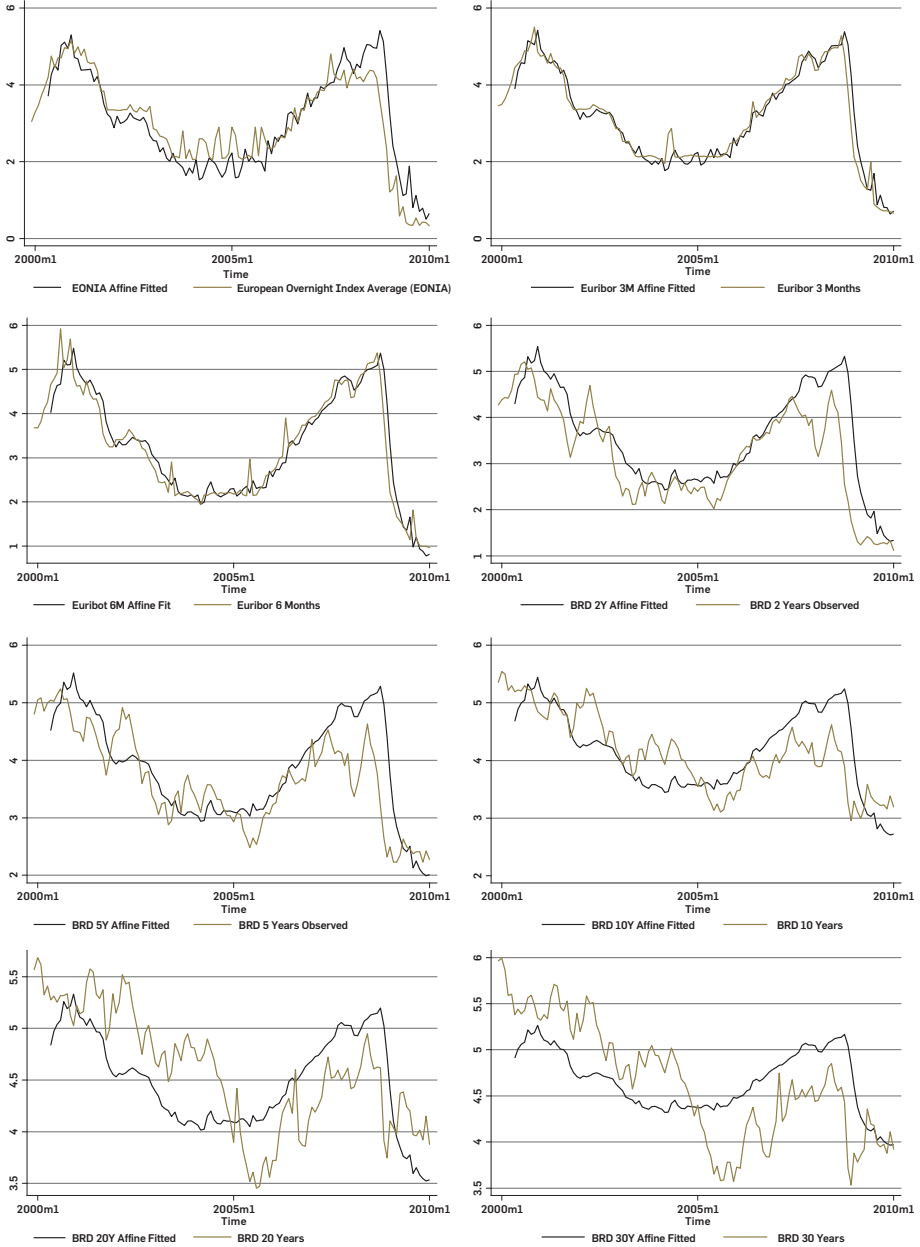


Figure 10. Yield curve fitted with an affine term structure model using latent factors as state variables

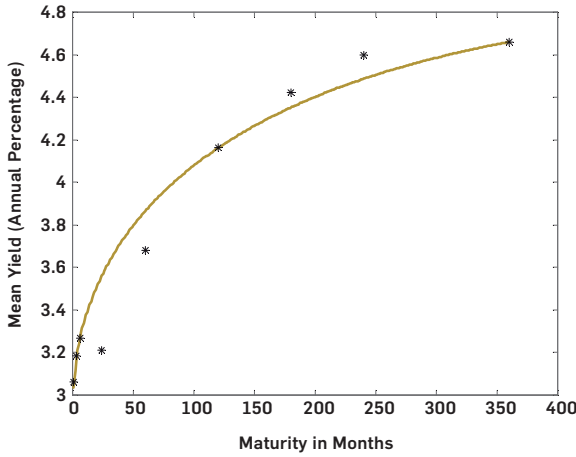
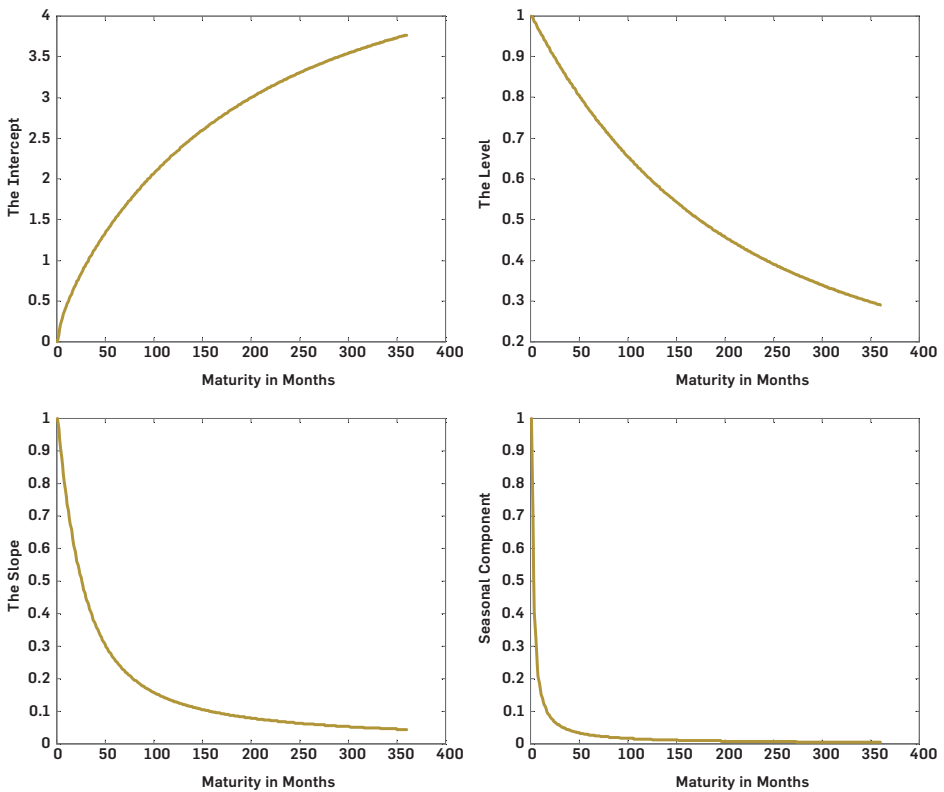


Figure 11. Fitted coefficients $A(N)_i/N$ and $B(N)_i/N$ as in equation (17)



■ 5. Policy implications

In light of these results, it appears that capital and money markets as well as the ECB seem to react to changes in macroeconomic variables which in turn result in movements in the level, the slope and the seasonal components of yields. However, the data give the impression that yields exhibit a rather limited or timid feedback on the wider economy, in support of the local level model with no feedback instead of a *VAR* model as seen in Diebold *et al.* (2006). Therefore the ECB, in terms of its Policy Rate and monetary aggregates, can only ensure interest rates are low in times when consumption growth is low in order to not make things worse, but according to the data and period analysed, there is no evidence that innovation in yields create a response from macro variables, similar to the results seen in Diebold, Rudebusch and Aruoba (2006). In addition, our results confirm that the level is the most important factor contributing to yield curve movements followed by the slope, and that this is the case for both the money as well as for the capital market yields. In times of high consumption growth, thus when consumer confidence is high and unemployment is low, the central bank is expected to increase interest rate levels in order to anchor long term inflation to low levels. However, the data does not support the existence of a significant feedback from yields' latent factors to *PPI*, but a rather one-way effect from *PPI* to yields' latent factors only. Therefore it appears that in times of low consumption growth, central banks can only limit their action to low interest rates in order to ensure that the economy does not deteriorate further, as there appears to be little evidence of interest rates influencing the wider economy.

■ 6. Conclusions and final remarks

In this paper a local level model with a stochastic slope and a stochastic seasonality have been calibrated using European yields. The analysis involved the use of the state space methodology to a structural equation model which, in state space terminology was to estimate an unobserved state or latent factors being the level, the slope and the seasonality to an observation or measurement equation linking the observed yields to the unobserved latent factors. The results confirm the views of Diebold *et al.* (2006) and, provide strong evidence of macroeconomic effects on yields however and, weaker evidence of yield curve effects on the macroeconomy. This essay has also explored the possibility of breaking the yield curve in two: the money market and the capital market yield curves. It has been shown that by doing so results are more encouraging than those seen in the no-arbitrage experiences. In addition, a discrete time affine term structure model (hence no-arbitrage) has been calibrated with the level, the slope and the seasonal component and both, the local



level model as well as the no-arbitrage term structure model performed quite well in explaining yield curve movements. However, similar to most of current literature, the explanatory power diminishes as maturities become larger.





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