# ON RENÉ THOM'S SIGNIFICANCE FOR MATHEMATICS AND PHILOSOPHY

Peter TSATSANIS

**Abstract**: We attempt to summarize some of the work of the mathematician René Thom with respect to his work in the mathematical and philosophical planes. A brief summary of his mathematical work is given followed by his work on structural stability and on morphogenesis which make up, in our opinion, the core of his philosophical work. A brief outline of Catastrophe Theory is given followed by a discussion of Thom's philosophical program to geometrize thought and linguistic activity (his semiophysics). Some of Thom's thoughts in biology, linguistics and semiotics is presented and we conclude with some observations on the present status of Catastrophe Theory.

**Keywords:** René Thom, Catastrophe Theory, Semiophysics, Natural philosophy.

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## Introduction

René Thom's immortality is assured in the annals of Mathematics and History. His neologisms *attractor*, *basin of attraction*, *catastrophe point* and *Semiophysics* are part of this. In 1958, he was awarded the Fields Medal, the highest honor a mathematician can attain. Jean-Luc Goddard made the film *René* about Thom and Salvador Dali created his last paintings based on Thom's Catastrophe Theory (CT).

In the annals of Mathematics, his name is associated with many concepts, theorems and conjectures. Among some of the concepts, we find *Thom class, Thom algebra, Thom space, Thom isomorphism, Thom homo-morphism, Thom spectra, Thom prespectra, Thom functor, Thom stratification, Thom polynomials, Thom complexes, Thom diagonal, Thom homology operations, Thom map,* and *Thom encoding.* Among the theorems, we find *Thom isomorphism theorem, Thom isotopy theorem (lemma), Thom manifold theorem, Thom cobordism theorem, Thom splitting lemma, Thom transversality theorem* and *Thom classification theorem (of elementary catastrophes).* Many of Thom's conjectures have been proven. Some of these include the *Genericity of Stability*, *Genera of surfaces in* $\mathbf{CP}^2$ and its generalizations, the *Kähler-Thom conjecture* and the *Symplectic Thom conjecture* and *Thom's conjecture on triangulation of maps*.

These concepts, theorems and conjectures are found in Thom's fields of interest which at one time or another included Algebraic Geometry, Algebraic Topology, Differential Topology, Singularity Theory, Bifurcation Theory, Dynamical Systems Theory and CT.

His work on Cobordism, for which he received the Fields Medal, is adequately covered in [Hopf, 1960/2003], [Milnor, 1957 and 1958], [Basu et al, 2003] and the *Bulletin of the AMS*, **41**(3), 2004. It is



ISSN 2258 - 3335

instructive to note that [Hopf, 2003: 75], in discussing Thom's work at the Fields Medal ceremony in 1958, remarked that "only few events have so strongly influenced Topology and, through topology, other branches of mathematics as the advent of this work." On page 76, he writes, "One of the, by no means trivial, insights which Thom had obviously from the beginning was that the notion of cobordism is particularly suited for the study of differential manifolds." He concludes on page 77 with: "These ideas [on cobordism] have significantly enriched mathematics, and everything seems to indicate that the impact of Thom's ideas — whether they find their expression in the already known or in forth-coming works — is not exhausted by far." May, in [May, 1975: 215], says that "Thom's discovery that one can classify smooth closed n-manifolds up to a weaker equivalence relation of "cobordism" is one of the most beautiful advances of twentieth century mathematics." Although it is acknowledged that the modern theory of the topology of manifolds began with H. Whitney's work in the 1930's, James, in [James, 1999: 876], writes: "However, its real development began after Thom's work on cobordism theory in 1954…".

Thom's work in Differential Topology and Singularity Theory is adequately covered in [Haefliger, 1988] and [Teissier, 1988]. In [Seade, 2006: 6], we read that "Thom, in 1964, gave interesting ideas for the use of Morse Theory to study foliations on smooth manifolds." Nitecki, in [Nitecki, 1971: 28] writes about Thom's work on transversality. He says: "The main reason for introducing transversality is its usefulness in finding generic properties of maps. This utility stems from genericity, in very general circumstances, of the property of transversality itself. The prototype of such genericity theorems, due to Thom, says that for highly differentiable maps, transversality to a fixed submanifold is generic." Bruce and Mond, in [Bruce and Mond, 1999: ix], write: "Thom was led to the study of singularities while considering the question whether it is possible to represent homology classes in smooth manifolds by embedded sub-manifolds. With his Transversality Theorem (1956), he gave the subject a push towards a kind of Modern Platonism." On page x, they state: "Thom saw the jet bundle as a version of the Platonic world of disembodied ideas, partitioned into attributes (the orbits of the various groups which act naturally on jets) as yet unattached to objects (functions and mappings) which embody them." Thom defined a "jet bundle" as the space of Taylor polynomials (of a specific degree) of germs of maps from one smooth manifold to another smooth manifold. They continue on page xi with: "Thom also contributed to the idea of versal unfolding. […] The term 'versal' is the intersection of 'universal' and 'transversal', and one of Thom's insights was that the singularities of members of families of functions or mappings are versally unfolded if the corresponding family of jet extension maps is transverse to their orbits (equivalence classes) in jet space." They conclude on the same page with: "This insight, and Thom's Platonist leanings, led him to Catastrophe Theory [so named by Christopher Zeeman]. He identified and described the seven orbits of function singularities which can be met transversally in families of four or fewer parameters: these were his seven elementary catastrophes, which were meant to underlie all abrupt changes (bifurcations) in generic four-parameter families of gradient dynamical systems. […] Many of Thom's ideas in bifurcation theory and gradient dynamical systems have provided the basis for later development, and the controversy surrounding CT should not mask the importance of his contribution to the subject."

Much has been written about CT and some references are found at the end of this paper. What follows is a brief outline of the theory.

## § 1.— An outline of Catastrophe Theory

First and foremost, CT is a mathematical theory. Its fundamental theme is the classification of critical points of smooth functions. The essential characteristics of a smooth function can be recognized by studying its embedding in a smooth family of functions. As Thom pointed out in his first book, *Structural Stability and Morphogenesis* (*SSM*), this fact is of extreme importance for applications since natural phenomena are always subject to perturbations. CT has as its goal to classify systems according to their behavior under perturbation. When a natural system is described by a function of state variables, then the perturbations are represented by control parameters on which the function depends. This is how a smooth family of functions arises in the study of natural phenomena. An unfolding of a function is such a family: it is a smooth function of the state variables with the parameters satisfying a specific condition. Catastrophe Theory's aim is to detect properties of a function by studying its unfoldings.

In effect then, CT provides a framework for describing and classifying systems and events where significant qualitative changes of behavior in the system are caused by small continuous changes in parameters. Within this framework, it is possible to identify the essential variables in a problem, and provide a brief (and often to the point) universal description of that behavior.

In general, CT is used to classify how stable equilibria change when parameters are varied. The points in parameter space, at which qualitative changes in behavior occur, are examples of catastrophe points. Near these points, CT can provide a canonical form for the potential, which depends only upon the number of state variables and control parameters.

The theory should apply to *any gradient system where the force can be written as the negative gradient of a potential*. The points where the gradient vanishes are the critical points and CT is concerned with the degenerate points. At these points, the Hessian (matrix of second derivatives) plays an important role.

Thom showed that near the degenerate critical points, the function can be written as a sum of a quadratic form, defining a nondegenerate subspace (a Morse part), and a degenerate subspace (the non-Morse part). The non-Morse part of the function can be represented by a canonical form called a catastrophe function. This function is the sum of a catastrophe germ, containing the non-Morse point, and a universal unfolding, which removes the degeneracy of the critical point and makes the potential structurally stable.

Thom's classification theorem (for elementary catastrophes) lists these catastrophe germs and their unfoldings for functions whose codimension is at most four. There are only seven different types of degenerate critical points for such functions: what Thom called the seven elementary catastrophes. (This list has been expanded by Arnold's Russian school.) Thom used transversality as the main tool to prove the existence of universal unfoldings. He showed that any family of potentials depending on at most five parameters is structurally stable and equivalent around any point to one of these catastrophes. (See Bogdan and Wales for a fuller description. Much of the above, some *verbatim*, was taken from their article.)

Thom created a mathematically rigorous theory that showed "the true complementary nature of the seemingly irreconcilable notions of versality and stability, that is, *preserving identity in spite of development*. [Castrigiano and Hayes, 2004: xv, emphasis added.]

Thom recognized that it was this feature that would be of great importance for a theory of cognition as discussed in his *SSM.*

For Thom, CT was a methodology, and as the subtitle of his first book *Structural Stability and Morphogenesis* states that it is *An outline of a general theory of models*. These models range from theoretical biology to semiotics. In his Forward to the book *Catastrophe Theory* by [Castrigiano and Hayes, 2004], Thom writes, on page ix, that mathematicians should see CT as "just a part of the theory of local singularities of smooth morphisms, or, if they are interested in the wider ambitions of this theory, as a dubious methodology concerning the stability (or instability) of natural systems." Castrigiano and Hayes call CT "an intriguing, beautiful field of pure mathematics […]. It is a natural introduction to bifurcation theory and to the rapidly growing and very popular field of dynamical systems." (page xi) And as Thom says on page x of the Forward: "the whole of qualitative dynamics, all the 'chaos' theories talked about so much today, depend more or less on it."

## § 2.— Natural Philosophy

And what of morphogenesis? Thom discusses some aspects of morphogenesis in Chapter 4 of *SSM*. The birth and destruction of forms was the main thread in *SSM*. Louk Fleischhacker in [Fleischhacker, 1992: 248], writes that "Thom describes in an impressive way the possibility of grasping the development of individuals of a higher form of life." This was accomplished via his principles of morphogenesis discussed in [Lu, 1976:166-180]. The first of these principles is the assertion "that the stability of any morphogenetic phenomenon [defined mathematically – ed.], whether represented by a gradient system or not, is determined by



ISSN 2258 - 3335

the attractor set of a certain vector field." [Lu, l976: 171] Stability for Thom is a "natural condition to place upon mathematical models for processes in nature because the conditions under which such processes take place can never be duplicated; therefore, what is observed must be invariant under small perturbations and hence stable." [Wasserman, 1974: v] Thom's second principle of morphogenesis states that "what is interesting about morphogenesis, locally, is the transition, as the parameter varies, from a stable state of a vector field to an unstable state and back to a stable state by means of a process which we use to model the system's local morphogenesis." [Lu, l976: 175] Wasserman, on page 157, writes that "The models given by Thom are only intended to be local models for natural processes anyway; a global description is obtained by piecing together a large number of such local descriptions." Thom's third principle of morphogenesis states that "What is observed in a process undergoing morphogenesis is precisely the shock wave and resulting configuration of chreods [zones of stability – ed.] separated by the strata of the shockwave, at each instant of time (in general) and over intervals of observation time." [Lu, 1976: 179]. It then follows "that to classify an observed phenomenon or to support a hypothesis about the local underlying dynamic, we need in principle only observe the process, study the observed 'catastrophe (discontinuity) set' and try to relate it to one of the finitely many universal catastrophe sets, which would then become our main object of interest." [Lu, 1976: 180]. Even if a "process depends on a large number of physical parameters (as is often the case in applications), as long as it is described by a gradient model, its description will involve one of seven elementary catastrophes; *in particular, one can give a relatively simple mathematical description of such apparently complicated processes even if one does not know what the relevant physical parameters are or what the physical mechanism of the process is.* And the number of parameters which are involved in the physical mechanism plays no role in the description." [Wasserman, 1974: 161, emphasis added.] In Thom's words: "if we consider an unfolding, we can obtain a 'qualitative' intelligence about the behaviors of a system in the neighborhood of an unstable equilibrium point." [Castrigiano and Hayes, 2004: ix]. According to Thom, it was this idea that was not accepted widely and was criticized by some applied mathematicians "because for them only numerical exactness allows prediction and therefore efficient action." [Castrigiano and Hayes, 2004: ix]. Since the exactness of laws rests on analytic continua-tion which alone permits a reliable extrapolation of a numerical function, how can the theory of structural stability of differential systems on a manifold help here?

After the work of A. Grothendieck, it is known that the theory of singularity unfolding is a particular case of a general category — the theory of flat deformations of an analytic set and for flat local deformations of an analytic set only the hypersurface case has a smooth unfolding of finite dimension. For Thom, this meant that "if we wanted to continue the scientific domain of calculable exact laws, we would be justified in considering the instance where an analytic process leads to a singularity of codimension one (in internal variables). Might we not then expect that the process (defined, for example, as the propagation of an action) be diffused (in external variable) and subsequently propagated in the unfolding according to a mode that is to be defined? Such an argument allows one to think that the Wignerian domain of exact laws can be extended into a region where physical processes are no longer calculable but where analytic continuation remains 'qualitatively' valid." [Castrigiano and Hayes, 2004: ix]

The philosophical program Thom had in mind for CT was the geometrization of thought and linguistic activity. His Natural Philosophy aspirations were centered "on the necessity of restoring by appropriate minimal metaphysics some kind of intelligibility to our world." [*Semiophysics,* p. ix] On pages 218-220 of his *Semiophysics,* he writes: "Modern science has made the mistake of foregoing all ontology by reducing the criteria of truth to pragmatic success. True, pragmatic success is a source of pregnance and so of signification. But this is an immediate, purely local meaning. Pragmatism, in a way, is hardly more than the conceptualized form of a certain return to animal nature. Positivism battened on the fear of ontological involvement. But as soon as we recognize the existence of others and accept a dialogue with them, we are in fact ontologically involved. Why, then, should we not accept the entities suggested to us by language? Even though we would have to keep a check on abusive hypostasis, this seems the only way to bring a certain intelligibility to our environment. Only some realist metaphysics can give back meaning to this world of ours."

Thom is the first human being to give the first rigorously monistic model of the living being, and to reduce the paradox of the soul and the body to a single geometrical object. This is one of his greatest accomplishments. Even if some aspects of his model are incomplete or wrong, he has opened up a conceptual universe by this. As he says on page 320 of his *SSM*: "But in a subject like mankind itself, one can only see the surface of things. Heraclitus said, 'you could not discover the limits of the soul, even if you traveled every road to do so; such is the depth of its form.'" And so it is with Thom's work. Its importance is, as C. H. Waddington says, "the introduction, in a massive and thorough way, of topological thinking as a framework for theoretical biology. As this branch of science gathers momentum, it will never in the future be able to neglect the



ISSN 2258 - 3335

topological approach of which Thom has been the first significant advocate." [*SSM*: xxxi–xxxii]

## § 3.— Interdisciplinarity

Thom's thoughts ventured in many domains including biology, linguistics and semiotics. I conclude this article with a comment about each of these areas and a further comment about the current state of affairs.

In his *SSM*, pages 290-291, Thom discusses the malignity of the human attractor. In a preamble on evolution, he writes: "Let us start with the very basic objection of the finalists to a mechanist theory of evolution: if evolution is governed by chance, and mutations are controlled only by natural selection, then how has this process produced more and more complex structures, leading up to man and the extraordinary exploits of human intelligence? I think that this question has only a single partial answer, and this answer [that there is a mathematical structure guaranteeing stability- P.T.] will be criticized as idealistic. […] I think that likewise there are formal structures, in fact geometric objects, in biology which prescribe the only possible forms capable of having a self-reproducing dynamic in a given environment. […] Attraction of forms is probably one of the essential factors of evolution. Each eigenform (one might even say each archetype if the word did not have a finalist connotation) aspires to exist and attracts the wave front of existence when it reaches topologically neighboring eigenforms; there will be competition between these attractors, and *we can speak of the power of attraction of a form over neighboring forms, or its malignity. From this point of view it is tempting, with the present apparent halt in evolution, to think that the human attractor is too malignant*. Of the theoretically



ISSN 2258 - 3335

possible living forms only very few are touched by the wave front and actually come into being". [Emphasis added.] We should heed this insightful warning and recognize that the best chance for the survival of mankind is to slow down, in any possible meaningful way, the malignity of the human attractor. The comments of [Pérez Herranz, 2000] are very insightful on this topic.

In a discussion on language, Thom writes: "Thought is then a veritable conception, putting form on the dummy actant arising from the death of the verb, just as the egg puts flesh on the spermatozoid; thus thought is a kind of permanent orgasm. There is a duality between thought and language reminiscent of that which I have described between dreaming and play: thought is a virtual capture of concepts with a virtual, inhibited, emission of words, a process analogous to dreaming, while in language this emission actually takes place, as in play." [*SSM*, p. 313]

Thom's semiotics is a huge domain and the reader is referred to his *Semiophysics*, and the works of Jean Petitot, Wolfgang Wildgen and Laurent Mottron. Mottron puts it succinctly, in [Mottron, 1989: 92], when he writes, "Thom conceives the human mind as a tracing, a simulation, an 'exfoliation' of the outside world, constrained by the same *a priori* laws as the world. Thom uses ontogenetic examples to show that *a priori* semiotics and its psychological realizations overlap, supporting a realist philosophical position. This analogy cannot be reduced to a linear causality, in the sense that the human mind should be governed by the same laws as the world because it comes out of the world; rather, it is explained by the universality of laws governing abstract and concrete dynamic conflicts. In catastrophe-theoretic tradition, a classification is justified by the generality of its application, not by quantitative validation."



ISSN 2258 - 3335

## § 4.— Final remarks

Many have written and continue to write that the mathematics community became disillusioned with CT – the latest being George Szpiro who, in *Poincare's Prize* [Szpiro, 2007: 158-159], writes: "By and large, criticism was not directed against the mathematical underpinnings of Thom's work. Rather it centered on the indiscriminate use of the theory, […], for purported applications. Soon a backlash developed, and catastrophe theory, which had promised so much but produced so little outside of pure mathematics, sank into disrepute. Nowadays one hears little of it." Authors continue to write about this myth or remain ignorant about the import of Thom's work. Very few know that he was the one who introduced the 'attractor' concept which plays such a major role in so many areas. One has to search the literature to find many applications of Catastrophe Theory – from developmental biology [Striedter, 1998], fetal heart rates [Kikuchi et al, 2000], gravitational lensing [Petters et al, 2001] to recent results for phase transitions [Bogdan and Wales, 2004], energy landscapes [Wales, 2001, 2003], biological function [Viret, 2006] and biological systems theory [Gunawardena, 2010]. Simply, if one accepts the comments that Thom made in his Forward to [Castrigiano and Hayes, 2004: ix] (pages 7-8 of this article) about extending the Wignerian domain of exact laws, CT will remain useful for a long time.

For the interested reader, the books by [Woodcock and Davis, 1978], [Saunders, 1980], [Postle, 1980] and [Arnol'd, 1992] are a good beginning although Arnol'd is unnecessarily harsh on Thom because Arnol'd could not accept Thom's comments about extending the Wignerian domain to regions "Where physical processes are no longer calculable but where analytic continuation remains 'qualitatively valid'". The books by [Gilmore, 1981], [Poston and



ISSN 2258 - 3335

Stewart, 1978] and [Castrigiano and Hayes, 1993/2004] are geared to mathematicians and physical scientists. The books by [Zeeman, 1977] and [Thom, 1975/1989] can be read by all scientists and interested readers. They are not easy but as Thom says: "I will not deny that communication will be difficult, […], but my excuse is an infinite confidence in the resources of the human brain!" [*SSM*: xxxiii].

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ISSN 2258 - 3335

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ISSN 2258 - 3335

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Peter Tsatsanis
E-mail: petertsa@hotmail.com



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