

Are Portfolio Holdings Affected by Parameter Uncertainty and Ambiguity?

Proelss, Juliane; Schweizer, Denis

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Abstract

Multiple empirical surveys show that the asset allocation of institutional investors in alternative investments is small and considerably below the theoretical optimal, even after accounting for investment restrictions. This means investors are giving up diversification potential and holding inefficient portfolios. The question is why would otherwise rational investors behave like this? We argue that investors find it difficult to hard to infer reliable return (distribution) expectations for alternative asset classes, for many reasons. For example, alternative investments generally exhibit a lower level of transparency, identifying appropriate benchmarks can be difficult, and those available often have data biases. In short, alternative investment asset class return distributions have a higher uncertainty than traditional investments. Institutional investors may exhibit ambiguity toward alternative assets, and may be penalizing them, and hence underinvest in this asset class. We research our hypothesis that alternative investments (private equity, hedge funds, and real estate) are more ambiguous than traditional asset classes (bonds and stocks). We consider representative, empirically observed institutional investor portfolios. In our model we determine the different levels of ambiguity toward the marginal return distribution for each asset class in the investment portfolio. We find that investor uncertainty aversion reduces allocations to the risky and ambiguous portfolio. We also find that institutional investors consider all alternative investments to be highly ambiguous, and bonds as somewhat ambiguous. However, institutional investors tend to systematically overinvest in stocks. Based on our results, we suggest that the alternative investment industry, e.g., increase transparency, which is expected to decrease investors' ambiguity aversion and increase investment allocation.

Keywords: Alternative investments, Ambiguity aversion, Asset allocation, and Model misspecification.

JEL classification: G11, G15.

Proelss, Juliane. WHU – Otto Beisheim School of Management, Chair of Empirical Market Research, Burgplatz 2, 56179 Vallendar, Germany. E-mail: Juliane.Proelss@gmx.de.

Prof. Dr. Schweizer, Denis. ✉ WHU – Otto Beisheim School of Management, Assistant Professor of Alternative Investments, Burgplatz 2, 56179 Vallendar, Germany, ☎ +49 261 - 6509 724, Fax: +49 261 - 6509 729. E-mail: Denis.Schweizer@whu.edu.

■ 1. Introduction

Alternative investments such as real estate, private equity, and hedge funds have been included in the portfolios of large corporate and private investors, pension plan sponsors, foundations, insurance companies, and other institutional investors for many years now. However, empirically, we observe distinct differences in how these alternative assets are used by investor type, investor origin etc. (see, e.g., Greenwich Associates, 2007, and Russell, 2007).

All of these investors have one thing in common: Their realized allocations to alternative investments are usually small, and considerably below the theoretical optimal allocation even after accounting for data biases (e.g., Terhaar *et al.*, 2003, and Cumming *et al.*, 2011). In particular, German in comparison to their international counterparts tend to invest less of their assets under management and less often in alternative investments, even controlled for tight regulatory standards (Funke *et al.* 2005, Consulting, 2006).

This phenomenon is somewhat puzzling. The diversification advantages of alternative investments, as well as their positive role in risk adjusted portfolio performance, are well documented in the literature and in practical implementation (see, for instance, Karavas, 2000, Terhaar *et al.*, 2003, NACUBO, 2006, Watson Wyatt Worldwide, 2007, Cumming *et al.*, 2011).

However, we can identify numerous reasons why portfolio decision makers may find it difficult to integrate alternatives into their portfolio allocations:

- 1) It can be more difficult to identify a suitable benchmark for alternative investments than for traditional investments. There may be small differences, but traditional indices are usually highly correlated. For alternative investments, there are in general numerous providers, and offered indices differ significantly in calculation methodology, data reporting standards, use of listed or unlisted underlyings, existence of an index committee, investability, origin, strategy classification, minimum investment volume, balancing etc. Often, alternative investment indices do not even fulfil the basic requirements necessary to serve as benchmarks¹. They may be prone to back-filling bias, survivorship bias, self-selection bias, liquidation bias, backdelete bias, period bias, and high attrition as well as stale pricing or appraisal smoothing that can systematically distort index returns, and result in an underestimation of volatility and a distortion of the covariance/correlation with other assets. Thus, determining a strategic asset allocation that includes alternative investments (and the subsequent

¹ The index must be unambiguous, verifiable, accountable, and representative (Bailey, 1992, Sharpe, 1992).

monitoring of the investment) is a challenge, especially for institutional investors who may be less familiar with these investments.

- 2) The illiquid nature of some of their investments also means that alternative investments may have long lock-up periods and notification times. And with private equity, for example, there may also be long investment periods. Furthermore, reselling an alternative investment in a secondary market transaction can be difficult, especially in times of crises. During those times investors typically have to accept substantial discounts.
- 3) Alternative investments tend to be much more opaque than traditional investments, which can make them more vulnerable to fraud.²
- 4) Also alternative investments are sometimes structured as offshore investments to avoid regulatory implications. These investments may not be protected by the Federal Deposit Insurance Corporation, making them riskier than traditional assets.
- 5) Finally, alternative investments often require a high minimum investment amount or commitment and feature substantially higher fees than passive investments or mutual funds.

All of these issues are not independent, but interact with and impact each other. It is nearly impossible to estimate separately their influence on or implications for asset allocation and the risk premium. However, they all increase uncertainty, and make it more difficult to implement reliable risk management and strategic asset allocations.

From these arguments, in this article we posit that the low allocation in alternative investments may be explained by an aversion of investors to invest in highly uncertain and opaque investments. Uncertainty in this context refers to the expected return distribution and the return-generating process, and is due to a lack of or unreliable information.

We believe this definition is general enough to cover the problems discussed above with regard to benchmark difficulties, data biases, and lack of transparency. That is, if investors are unsure whether the observed data (from, e.g., differing benchmarks) and the derived return model are representative of the expected return distribution or return-generating process, they may tend to penalize an investment in this asset class. This hypothesis requires that we distinguish between the overall risk and uncertainty of the portfolio (for example, the mixed portfolio the investors are willing to hold).

² Recent examples include the \$450 million fraud at Bayou or AA Capital partners, as well as Bernard Madoff's infamous \$50 billion+ Ponzi scheme, which was marketed as a hedge fund strategy. The latter case was only possible with a "black-box" investment strategy.

To test our hypothesis, we consider representative, empirically observed portfolios of traditional and alternative investments by German and international institutional and large private investors. We attempt to explain the observed asset allocation using risk aversion, uncertainty aversion, *and* differences in the uncertainty perception of different asset. In line with our prior arguments, we also attempt to explain the tendency of German investors to pursue lower allocations of alternative investments than their international counterparts. From our results, we draw conclusions about the implications for the alternative investment industry.

The remainder of this article is structured as follows. Section 2 gives a short introduction to uncertainty aversion. In section 3, we present a framework that allows us to estimate the uncertainty aversion of different institutional and large private investors. At the same time, we can account for differences in information availability and uncertainty in the single (traditional and alternative) asset classes in the mixed portfolio. Section 4 illustrates a representative empirical asset allocation of German and international investors based on survey responses. It also includes our data set, and demonstrates the model framework presented in section 3. Section 5 estimates the implied uncertainty/ambiguity of the different investors. We further analyze reasons for the uncertainty perception, and derive possible ways to reduce uncertainty in alternative investments. Section 6 concludes.

■ 2. Literature Review

Empirical evidence suggests that investors in general and German investors in particular tend to hold underdiversified portfolios. Investors seem to allocate only a few assets, typically those with which they are familiar (bonds and stocks), and they tend to stay away from alternative assets. The problem grows worse if portfolio decision makers feel less competent using (alternative) assets (Heath and Tversky, 1991). In a recent study, Funke *et al.*, 2007, showed that uncertainty aversion might play an important role in explaining the low allocation to alternative investments. Our aim here is to advance this idea by estimating the overall uncertainty aversion of different types of investors toward a risky and uncertain portfolio. We will also derive estimates for the ambiguity investors feel toward single traditional and alternative investments.

We distinguish between “risk” (in the common sense), and (Knightian) “uncertainty” or “ambiguity”. For clarification, bets with known odds are referred to as “roulette lotteries”. Those bets are characterized by “risk” in the common sense. Bets with unknown odds but sometimes known probability distributions are often referred to as “horse lotteries”. Those bets are characterized by “(Knightian) uncertainty,” or

“ambiguity” (Knight, 1921, Ellsberg *et al.*, 1988). Investors tend to prefer risk over uncertainty (“Ellsberg Paradox”) (Ellsberg, 1961).³

To account for uncertainty or ambiguity,⁴ the literature suggests two basic approaches: 1) a purely statistical approach as defined by Bayesian statistics (for a discussion, see Ellsberg, 1961), or 2) an expected utility-based approach (multiple-priors) (see Gilboa and Schmeidler, 1989, and Schmeidler, 1989).⁵

A growing literature has used the tenets of this theory to interpret financial market phenomena.⁶ Generally, we can show that, if there are uncertain/ambiguous assets, uncertainty or ambiguity aversion decreases (increases) the demand for the risky and ambiguous/uncertain (safe and unambiguous) asset, and raises the equity premium in the economy. We believe this will be magnified for alternative investments.

To identify a model framework that allows us to research the “(alternative) investments puzzle” with uncertainty aversion, we briefly discuss the two basic streams of thought in the literature: ambiguity, and robust control/model uncertainty. Based on the literature, we derive a traceable model to estimate the overall uncertainty aversion to the mixed risky portfolio (which includes traditional and alternative assets).

The first approach originated in the work of Epstein and Wang (1994). The authors used a recursive approach to develop an infinite horizon extension to Gilboa and Schmeidler’s (1989) multiple-prior⁷ model.⁸ They used this framework on an inter-temporal asset pricing model (suggested by Lucas, 1978), to show that asset prices in the equilibrium are not bijective but indeterminate (see also Dow and Werlang, 1992). Chen and Epstein (2002) extended the original model to account for continuous time. Their model allows estimation of the “market price of uncertainty,” the sum of a risk premium and an ambiguity premium. Agents tend to assume the worst-case scenario (pessimism), so they perceive returns to be lower than they actually are. This results in lower demand, and a higher overall premium. The authors also show that the premium is more likely to increase during negative news periods (see also Epstein and Schneider,

³ For more recent studies, see, e.g., Charness and Gneezy (2003), Bossaerts *et al.* (2010), and Ahn *et al.* (2008).

⁴ A great deal of effort has been put into understanding ambiguity aversion and formally defining the problem. A detailed analysis would be beyond the scope of this article. For an introduction to the most widespread definitions and approaches, as well as its further developments and generalizations, see, e.g., Schmeidler (1989), Hazen and Lee (1991), Chow and Sarin (2001), Ghirardato and Marinacci (2001), Klibanoff *et al.* (2003, 2010), Ghirardato *et al.* (2004), Maccheroni *et al.* (2004, 2006), and Gilboa *et al.* (2007). For a comparison, see also Skiadas (2007) and Siniscalchi (2008).

⁵ The main difference between the approaches is that the Bayesian investor imposes a prior over other possible models. Under the multiple-prior approach, the investor solves for the infimum over other alternative equivalent models, subject to a penalty for deviations from the reference model (see also Uppal and Wang, 2003).

⁶ For a more comprehensive introduction, see, e.g., Mukerji and Tallon (2003).

⁷ Note that multiple priors are not the only decision model for ambiguity. For a comparison with an alternative model setting, see Wang (2003).

⁸ See also Epstein and Schneider (2003).

2008, for information effects and ambiguity and Anderson, Hansen, and Sargent, 2003, for another model of intertemporal ambiguity aversion based on an alternative form of maximin expected utility).⁹

Hansen and Sargent (2001), Skiadas (2003), Maenhout (2004), and Hansen *et al.* (2006) introduce a related concept about robust control and model uncertainty. The intuition is similar to Gilboa and Schmeidler (1989): Agents know the equilibrium price function or the return distribution and the respective return-generating process (henceforth referred to as the *reference model*) of a finite data set. But because they know the data set is finite, they tend to overly fear any misspecification (uncertainty), and therefore consider *alternative models* as potentially true instead.

We can derive the potentially true models by disturbing the *reference model*. Agents then prefer a model that is robust to smaller specification errors. Here, the authors introduce the concept of “perturbation”, which accounts for the (model) uncertainty with a “penalty term.” They again show that the allocation to the risky and uncertain asset (e.g., a stock) is lower if the agent accounts for model uncertainty/robustness.¹⁰ The concept implies that an increase in the “penalty term” (aversion to uncertainty) will result in a high premium.¹¹

Both concepts (“ambiguity aversion” and “robust control/model uncertainty”) have been used to explain observed market phenomena that could not be explained satisfactorily under traditional frameworks. However, to the best of our knowledge, neither concept has yet been used to explain alternative investment allocation. The literature on home bias and underdiversification is a related problem that can help us to identify a traceable multi-asset, multi-uncertainty model setting to estimate uncertainty aversion in different (alternative) investments.

The problem of underdiversification, or rather overallocation to selected assets, is addressed by both the “ambiguity” aversion and the “robust control/model uncertainty” lines of the literature. Epstein and Miao (2003) use the multiple-prior (“ambiguity”) setting to explain the home bias puzzle for a two-agent example who exhibits extreme pessimism (the worst case) in an equilibrium setting. However, a disadvantage of this model is that it is *not* traceable, and it does *not* allow for different ambiguity aversions.

⁹ These results also hold for different preferences, for example, with intertemporal substitution as found in the Kreps-Porteus/Epstein-Zin preferences (Chen and Epstein, 2002, Veronesi, 2005). Note, however, that the authors also illustrate cases where ambiguity implies a discount on the equity premium. In other words, investors are the opposite of ambiguity-averse.

¹⁰ The authors show that portfolio choice can be broken down into three components: 1) standard myopic demand adjusted for robustness, which implies a lower weight in the respective asset, 2) Merton’s (1971) hedging demand, and 3) another hedging demand implied by the robustness preference. For further details on the calculation and derivation, see, e.g., Veronesi (2005).

¹¹ The results hold using the Epstein-Zin preferences. Veronesi (2005), for example, has shown that, under the worst case, the equity premium cannot be explained by robust control. And a decrease in the “penalty term” increases the misspecification and the equity premium aversions. This means robust preferences fail to explain the equity premium puzzle.

Xepapadeas and Vardas (2004) use the “robust control” setting in a multiple-asset case to derive the conditions under which more uncertain assets (e.g., foreign assets) have a lower allocation than less uncertain assets (e.g., domestic real estate assets).¹²

Uppal and Wang (2003) suggest a traceable model to study different aversions to ambiguity across different assets in a setting related to both the “multiple-prior” and the “robust control” settings. The agents in this example have multiple priors but do not exhibit extreme pessimism toward them (unlike in Epstein and Miao’s, 2003, classical ambiguity approach). Compared to other approaches, this model also allows for differences in uncertainty among the assets: It considers one single parameter to model overall uncertainty about the joint distribution of the risky and uncertain portfolio. And it accounts for differences in the level of ambiguity for each asset, and its marginal return distribution in the portfolio.

The authors show that uncertainty aversion results in an overallocation to assets where there is no or significantly less ambiguity (e.g., domestic real estate stocks). We find that the latter case is inversely correlated with the problem of underdiversification to alternative investments that has been observed with German and international investors. In other words, we observe a lower allocation to alternative investments, and a higher allocation to domestic assets.

We find that Uppal and Wang’s (2003) model allows us to derive a traceable setting from which we can research investor uncertainty aversion. It also allows us to discern different levels of ambiguity about the single assets in the risky and uncertain portfolio.

In summary, we hypothesize that investors exhibit aversion toward uncertain return distributions of financial assets. They believe that alternative investments involve greater ambiguity than traditional assets, perhaps because of their lack of reliable information and benchmarks, and thus their return generation process and distribution. Risk –and uncertainty– averse investors may try to account for this in their asset allocation choices. The empirical asset allocation in alternative investments for German investors is even lower than for their international counterparts. We try to explain this difference in ambiguity perceptions (see also Funke *et al.*, 2007).

To test our hypothesis, we use the model setting proposed by Uppal and Wang (2003), which is suitable for researching the varying uncertainty to multiple traditional and alternative ambiguous assets with different levels of uncertainty. The next section presents our general model setting (which follows the style of Uppal and Wang, 2003).

¹² This problem is also discussed in Boyle *et al.* (2004), who show that in the presence of uncertainty (e.g., in a model misspecification setting), investors prefer holding familiar assets. The desire to hold a familiar asset increases as ambiguity over holding an unfamiliar asset increases (due to, e.g., market returns), and the volatility of the familiar asset decreases.

■ 3. Methodology

To explain the low allocation to alternative investments, we first present Uppal and Wang’s (2003) model framework, which is a direct extension of the Merton (1971) model, and calibrate the model to solve for the optimal portfolio weights under uncertainty. As noted earlier, its advantages are: 1) it is traceable, 2) it allows us to estimate investor uncertainty aversion to the overall risky and uncertain portfolio and its joint return distribution, and 3) it allows us to discern different levels of ambiguity about the return processes and the marginal probability distributions of the subsets or asset classes.

For the sake of clarity, we refer henceforth to the different levels of uncertainty toward the single asset classes as AMBIGUITY (aversion); we use UNCERTAINTY (aversion) to refer to the overall portfolio or the joint distribution. After presenting the framework, we calibrate the model in order to derive the uncertainty and the ambiguity aversion in a mixed portfolio of traditional and alternative investments. This allows us to draw conclusions about the general uncertainty aversion of different investors, as well as the ambiguity aversion of traditional and alternative investors. Ultimately, we aim to draw conclusions about how (alternative) investment funds can reduce uncertainty, and assess to what extent they are dependent on investor risk aversion.

Note that this methodology implicitly assumes that institutional investors do not invest in inefficient portfolios. Rather, they have a utility that is not directly observable, as investors obtain “disutility” from uncertainty in the return generation process of the different asset classes. The “disutility” increases with uncertainty aversion. In Section 4, we use our data set to illustrate this relationship.

In line with the “robust control” setting, agents assume a *reference model* for the distribution of the return process of the overall (risky and uncertain) portfolio based on the observed return data. They recognize that this may not be the true model, and become concerned about model misspecification. To account for this, agents maximize an expected utility function with model misspecification (uncertainty) V_t .¹³ That is, investors consider *alternative models* with any return-generating process (and hence return probability distribution) that is equivalent but not equal to the “*reference*” model P .¹⁴ The alternative models Q are essentially alterations of the *reference model* P , where it is included as a special case.

¹³ The idea is that agents formulate objectives for a reference probability P equal to the “true” probability distribution of the uncertain asset (see also Duffie and Epstein, 1992a, 1992b). To incorporate “uncertainty”, the reference should probably be altered to an equivalent probability Q , which is the infimum of priors equivalent to the reference probability (see also Gilboa and Schmeidler, 1989; Chen and Epstein, 2002) propose the use of density generators to derive Q . The authors find a unique (minimum) solution for the multiple-prior, recursive utility, which shows that in the case of uncertainty there is a penalty imposed on the return process (in the form of a drift). The perturbation is often referred to as an e-contamination approach. Chen and Epstein (2002) show in the classic ambiguity framework that agents assume the worst-case scenario when faced with a decision under uncertainty.

¹⁴ P is usually the result of an estimation based on observed data, and may be prone to misspecification.

Mathematically, we can write this relationship as: $dQ^{\xi} = \xi(X_{t+1})dP$ where Q^{ξ} refers to an *alternative model*, or a perturbation of the *reference model* P the agent is willing to consider instead. $\xi(x)$ equals the density function of vector X_t of state variables (asset returns) that deviate from the *reference model*.

Because there is more than one *alternative model*, however, agents need to assess how the *alternative model* compares with the *reference model*. This is done with the penalty function $\phi L(\xi) = \phi E^{\xi}(\ln \xi)$, where E^{ξ} equals the expectation under Q^{ξ} equivalent (but unequal) to the *reference model* P . ϕ expresses the willingness of agents to assume an *alternative model*, or the degree to which they believe the *reference model* is *not* “true”. The greater (smaller) the penalty ϕ for the *alternative model* is, the more certain is the agent that the *reference model* is (not) the “true” model (assume $\phi \geq 0$).¹⁵

As mentioned above, the traditional intertemporal additive expected utility with rational expectations is a special case of the more general expression with model uncertainty. If agents are confident that the *reference model* is the “true” model, the penalty imposed for any *alternative model* would be extremely high, so that $\phi \rightarrow \infty$. In this case, the utility function V_t converges to the “traditional” utility without uncertainty, and P equals the “true” model. Another special case occurs if agents are extremely unsure about the *reference model*. Then they may impose (close to) no penalty $\phi \rightarrow 0$ on any *alternative model* with probability distribution Q^{ξ} , so they may have no information from or of the *reference model* P . In this case, the expected utility in $t+1$ is driven by the worst-case scenario and results in the minimum utility, or $1 \inf(V_{t+1})$.

However, following Uppal and Wang (2003), we would usually expect that neither extreme case would apply in mature markets. Agents should be willing to use available information ($\phi \neq 0$), despite being somewhat unsure about the true probability distribution ($\phi \neq \infty$). Note that this setting¹⁶ only allows for uncertainty about the joint distribution of the risky assets.

In order to incorporate different levels of ambiguity toward the single asset classes, the risky and uncertain portfolio can be decomposed in $i=\{1, \dots, n\}$ subsets (the single asset classes in the portfolio) with $X_t = (X_{1,t}, X_{2,t}, \dots, X_{n,t})$ vectors of state variables (the observed portfolio returns) with its respective marginal return probability distribution P_{j_i} under the *reference model*. Together the subsets determine the (uncertain) joint return distribution of the portfolio X_t . Note that agents have

¹⁵ Note, $\psi[E_t^{\xi}(V_{t+1})]$ equals a scaling function that ensures the penalty function is consistent with the units of utility of $E_t^{\xi}(V_{t+1})$. Maenhout (2004) shows how to rescale the “penalty term” with function ψ to obtain scale invariance in the robust control setting. ψ is usually chosen for mathematical convenience.

¹⁶ Uppal and Wang (2003) also provide a comparison with a Bayesian setting.

different sources of information about each subset, and thus consider variations in the perturbations of the different state variables X_{J_i} so that $dQ^{\xi_i} = \xi_i(X_{J_i,t+1})dP_{J_i}$.

Now agents assess the penalty function with respect to each P_{J_i} . Because of the differences in information about each subset, agents tend to penalize each sub-set differently $\phi_i L(\xi_i)$. We assume agents have K sources of additional (reliable or unreliable) information about the single marginal distributions of any subvector. Then we can assess the overall penalty (on the joint distribution) as the sum of the penalty on the subsets, so that $\phi_i L(\xi) = \sum_{i=1}^K \phi_i L(\xi_i)$.

In this setting, the utility function is a generalization of the original V_t with uncertainty and ambiguity. The only difference is that we distinguish among the information and penalties for each subset (or each marginal return distribution of each asset class) in the portfolio, and do not take a net penalty over all aggregated information (about the joint distribution) of the risky and uncertain portfolio. We can interpret the extended V_t that there are different levels of ambiguity toward the subsets. And the amount of reliable information about the respective sub-sets is positively correlated with the penalty for any *alternative model* for the respective subset, and hence the total penalty on an *alternative model* for the *reference model*. We also consider cases when agents assume the information is “negative” – for example, it may be fraudulent, unreliable, wrong, or incomplete. In this case, the penalty for an *alternative model* decreases, but it never goes below zero.

In order to solve the portfolio optimization, we need to convert this problem into continuous time. Uppal and Wang (2003) show that the continuous-time equivalent of the discrete utility function V_t equals

$$0 = \inf_v \left[-\rho V + u(c) + v^T V_x + \frac{1}{2} \psi(V) v^T \Phi v + A(V) \right],$$

where $A(V)$ is the differential operator associated with the diffusion process¹⁷ of the state variables X_t of the risky and ambiguous portfolio. $-\rho V + u(c) + A(V)$ is equal to the standard Hamilton-Jacobi-Bellman equation for the *reference model* with probability distribution P . $v = (v_1, \dots, v_n)$ is equivalent to the change in drift (perturbation) of the state variables X_t , where the *reference model* P changes to the *alternative model* with probability distribution Q^{ξ} ¹⁸. As a result, $v^T V_x = (v_1, \dots, v_n)^T V_x$ represents the change from the *reference model* to the *alternative model*. Finally, $\Phi = \sum_{i=1}^K \phi_i (\sigma_{J_i} \sigma_{J_i}^T)_{n_t}$ equals the penalty function.

¹⁷ Under the reference model with probability distribution P , the return generation process for the state variables is assumed to follow an Ito process, as follows: $dX_t = [\mu_x(X_t, t)]dt + \sigma_x(X_t, t)dw_t$. Note that this setting assumes the returns are normally distributed, however, which may not be the case for alternative investments.

¹⁸ Under the alternative model with probability Q^{ξ} , the return generation process for the state variables accounts for the drift and consequently changes to $dX_t = [\mu_x(X_t, t) + v_i]dt + \sigma_x(X_t, t)dw_t^{\xi}$.

The portfolio choice problem can now be written in the standard Merton (1971) model¹⁹ setting with the alternative utility function that allows for different levels of model uncertainty. Uppal and Wang (2003) show that the optimal portfolio (weights) can be derived as a direct extension of the standard Merton (1971) portfolio weights with (indirect) lifetime utility, dependent on the return processes of the risky and uncertain portfolio (taking into account a penalty function or drift on the original return processes or *reference model* to account for *alternative models*), as well as on the wealth W of the agent. The only difference to the standard model is the additional drift ν (penalty) of the state variables (ambiguity).

To derive the penalty Φ , assume that the price processes of all risky and uncertain assets (traditional and alternative) are normally distributed and follow geometric Brownian motions.

Obviously, as we noted earlier, alternative investments may not be normally distributed. However, the error in not accounting for higher moments is relatively small compared to an inappropriate benchmark choice and an hence an unsuitable proxy for the *reference model*. Furthermore, there is evidence that we can account for higher moments with the combination of two normal distributions, or via a return generation process with the latter (Morton *et al.*, 2006). Thus we believe the advantages of a flexible and intuitive model outweigh any concerns about overall errors.

Assume a constant risk-free rate, and $T \rightarrow \infty$ (which may be appropriate if the investor is an institution with a long-term timeline). Following Uppal and Wang (2003) and Maenhout (2004), we assume the investor has power utility²⁰ so that $U(c) = c^{1-\gamma}/(1-\gamma)$, and the scaling function equals $\psi(V) = V \cdot (1-\gamma)/\gamma$. Note the scaling function is usually chosen for mathematical convenience.

We approximate the implied risk aversion $\hat{\gamma}$ for German and institutional investors from the observed (empirical) portfolio weights²¹ so that $\hat{\gamma} = \frac{(\hat{\mu}_R - \hat{r})}{\hat{\pi}_R \hat{\sigma}_R^2}$, where $\hat{\pi}_R$ equals the empirically observed portfolio proportion allocated to the risky portfolio, $\hat{\sigma}_R^2$

¹⁹ Agents want to maximize intertemporal lifetime utility with uncertainty. They have wealth W_t , which they can invest in risky assets. Furthermore, they can borrow and lend money at the risk-free rate, and can consume a single good. The returns of the risky assets (R_p, Y_p) follow an Ito process. This implies that all the risky asset returns considered are normally distributed, which again may not be the case for alternative investments. Therefore, the only exogenous source for model misspecification is the state variable for the risky and uncertain assets that make up the overall portfolio. The wealth evolution process is endogenously subject to model misspecification (from the state variables), and can be written as an evolution of the return on the assets. Based on these assumptions, Uppal and Wang (2003) derive the *indirect* lifetime utility $V(W_p, R_p, Y_p, t)$, and solve the maximization problem to derive the optimal portfolio weights. For further details, see Merton (1971) and Uppal and Wang (2003).

²⁰ We assume the agent has constant relative risk aversion (CRRA) and declining absolute risk aversion. We believe this utility is a reasonable choice for the following reasons (and despite some minor drawbacks, such as the St. Petersburg paradox): 1) It is the standard choice in the model misspecification / robust control literature, so our results are more comparable with those in the literature, and 2) it is reasonable to assume that, independent of the amount of assets under management, institutional investors will invest in the same asset allocation choice. For a comparison of different utility function families and their advantages and disadvantages, see, e.g., Xie (2000) and Alpanda and Woglom (2007).

²¹ We derive the observed asset allocation weights from Funke *et al.*'s (2007) survey.

equals the implied portfolio variance, and $\hat{\mu}_R$ equals the respective portfolio return of the risky assets. \hat{r} equals the proxy of the risk-free rate.

To derive estimates from observed portfolio weights for implied (empirical) risk $\hat{\gamma}$, implied uncertainty aversion $\hat{\Phi}$ ²² and implied ambiguity for each investor we maximize expected utility as follows (see also step 4 for the general form)²³:

$$\hat{\pi}_R = \hat{B} \frac{1}{\hat{\gamma}} \left[\hat{\sigma}_R \hat{\sigma}_R^T \right]^{-1} (\hat{\mu}_R - \hat{r})$$

$$\text{with } \sum \hat{\pi}_R = 1, \hat{\pi}_R \geq 0 \text{ and } \hat{B} = \left(I + \left[\hat{\sigma}_R \hat{\sigma}_R^T \right]^{-1} \hat{\Phi}^{-1} \right)^{-1}$$

$$\text{where } \hat{\Phi} = \hat{\phi} \begin{pmatrix} \hat{\sigma}_{11} & \dots & \hat{\sigma}_{1n} \\ \vdots & \ddots & \vdots \\ \hat{\sigma}_{n1} & \dots & \hat{\sigma}_{nn} \end{pmatrix}^{-1} + \hat{\phi} \begin{pmatrix} m_{11} \hat{\sigma}_{11}^{-1} & \dots & m_{1n} \hat{\sigma}_{1n}^{-1} \\ \vdots & \ddots & \vdots \\ m_{n1} \hat{\sigma}_{n1}^{-1} & \dots & m_{nn} \hat{\sigma}_{nn}^{-1} \end{pmatrix}$$

$$\text{and } \hat{\pi} = \frac{\hat{\phi}}{(1 + \hat{\phi})} \cdot \frac{1}{\hat{\gamma}} \cdot \frac{(\hat{\mu}_R - \hat{r})}{\hat{\sigma}_R},$$

where $\hat{\pi}_R$ is the vector of empirically observed portfolio weights of the risky and ambiguous assets, $\hat{\sigma}_R \hat{\sigma}_R^T$ is the variance-covariance matrix, and $\hat{\mu}_R$ is the return vector of the respective assets. Furthermore, \hat{r} equals the risk-free rate, I is the identity matrix, $\hat{\phi}$ is the implied (empirical) uncertainty about the joint distribution of the risky and uncertain portfolio, and m_{ij} ²⁴ is the additional information or ambiguity about the marginal distribution of each $i = \{1, \dots, n\}$ risky asset, where $m_{ij} = m_{ji}$.

To improve interpretability and check for robustness of the estimates, we assume that investors first consider information only about the marginal distribution of each respective asset class, not the joint distribution among the single asset classes. This equals the following restriction: $m_{ij} = m_{ji} = 0$ for $i \neq j$. In a second case, we assume that investors also consider ambiguity among the joint marginal distribution of the single asset classes, or $m_{ij} = m_{ji}$ for $i \neq j$.

To ensure that the estimates are reasonable and economically justified, we impose the following additional restriction: The implied drift²⁵ adjustment may not exceed

²² Note that the perception of risk and thus the ambiguity risk for alternative investments may be further increased by non-normally distributed returns (skewness and kurtosis). Therefore, any increase in risk will be included in the estimated uncertainty/ambiguity parameter, and may artificially enlarge our estimators. In the following analysis, risk implied by higher moments is no longer distinguishable from ambiguity risk.

²³ For proof and further details, see Uppal and Wang (2003) and Maenhout (2004).

²⁴ We may interpret $\sum \hat{m}_i$ in line with step 2 for the overall penalty based in each subset ϕ_i .

²⁵ Model ambiguity can also be interpreted in terms of change in the expected return from $\hat{\mu}_R$ to the implied return $\hat{\mu}_R - (\hat{\mu}_R - \hat{r}) / (1 + \hat{\Phi})$ (drift adjustment). For a more detailed discussion and interpretation, see also Uppal and Wang (2003), Maenhout (2004), and Skiadas (2007). To obtain reasonable results, we restricted the implied drift adjustment to $\hat{\mu}_R \geq \hat{\mu}_R - (\hat{\mu}_R - \hat{r}) / (1 + \hat{\Phi}) \geq 0$, so that the drift is not negative and does not exceed $\hat{\mu}_R$.

100%, and may not be below zero ($\Phi \rightarrow 0$ and $\Phi^{-1} \rightarrow 0$) (see also Uppal and Wang, 2003, for a reasonable choice for m and ϕ). Furthermore, $\hat{\pi}$ is the implied (empirical) proportion invested in the risky and uncertain portfolio, and $1 - \hat{\pi}$ is the implied (empirical) proportion invested in the risk-free rate.

In this problem, the only unknowns are 1) the implied parameter \hat{m}_{ij} for ambiguity about the marginal distribution of the risky and ambiguous single assets, and 2) the implied overall uncertainty aversion $\hat{\phi}$ toward the joint distribution of the risky and uncertain portfolio. Note that \hat{m}_{ij} determines the optimal portfolio choice, and that $\hat{\phi}$ together with $\hat{\gamma}$ is the allocation to the risk-free and the risky and uncertain portfolio.

In general, uncertainty and ambiguity decrease the allocation to the respective (risky and) uncertain assets. But, compared to the classic ambiguity framework of Epstein and Wang (1994), investors do not assume the most pessimistic scenario. They assign penalties according to their opinions about the degree of ambiguity for each risky asset class, and tend to exhibit overall uncertainty aversion toward risky and uncertain portfolios.

Note that the Merton (1971) model portfolio weights are a special case of this more general specification, which accounts for uncertainty and ambiguity. If $B=I$ ($\Phi \rightarrow \infty \equiv \Phi^{-1} \rightarrow 0$) the weights of the risky portfolio will equal the Merton (1971) portfolio weights (tangential or market portfolio). This is the case if $\hat{\phi} \rightarrow \infty$ (the penalty for any *alternative model* is very large), and/or $\hat{\phi} \neq 0$, and $m_{i, \{i=1, \dots, n\}} \rightarrow \infty$ for each asset (assuming investors have the maximum information about each asset class).

If $B \rightarrow 0$ ($\Phi \rightarrow 0 \equiv \Phi^{-1} \rightarrow \infty$), the penalty for any *alternative model* will be close to zero, and the agent will not hesitate to use an *alternative model* (extreme pessimism). This is the worst-case scenario, where no observed information is believed to be true. Under this condition, investors will only invest in the risk-free asset.

In the above model setting, uncertainty basically represents another layer of “risk” (or what is referred to as “Knightian uncertainty”). Skiadas (2007) shows that the above model is equivalent to a recursive utility with source-dependent risk aversion.²⁶ This implies that model uncertainty can also be written in terms of an alteration (increase) of the risk aversion from $\hat{\gamma}$ to $\hat{\gamma} \cdot (1 + \hat{\phi}) / \hat{\phi}$.

However, note that if we do not distinguish between risk and ambiguity aversion, investors appear more risk-averse, although they may actually be “only” uncertain about the asset class(es). This has interesting implications for the investment industry.

²⁶ This is a special case of a “quasi-quadratic proportional aggregator”, as presented in Schroder and Skiadas (2003).

As we argued in the introduction, agents may exhibit high uncertainty aversion and low implicit risk aversion (or lower than assumed from the observed portfolio weights). In this case, additional information from a reliable source may reduce uncertainty and result in a higher allocation to the formerly uncertain asset class (see Epstein and Schneider, 2008) for more details on what kind of information may reduce/increase uncertainty. If we observe high risk aversion, a reduction in uncertainty may only be beneficial for low-risk (and formerly highly uncertain) asset classes.

■ 4. Model Calibration

4.1. Asset Allocation of German and International Investors

We base our analyses on Funke *et al.*'s (2007) database.²⁷ To the best of our knowledge, it is the most comprehensive survey on alternative investments in German institutional investor and large client asset allocations. Their survey obtained responses from 102 German and 17 international investors.^{28,29} The responses were detailed enough to provide good estimates for the average asset allocation of these investors.³⁰

To check for robustness and to better understand the results within an international context, we compare them further with the results obtained by Russell (2006, 2007) and Mercer (2007). The majority of participants were corporate investors, banks, and insurance companies.

Table 1 reports the results from three comparable (panel) studies that addressed institutional investor asset allocation in alternative investments. We found that the allocation of German investors was lower than that for international or European investors. This supports the results of Funke *et al.* (2005) and Consulting (2006). We believe that this low alternative investment allocation is typical for German investors, and the survey results are thus a good proxy for German investors.

We examine the different backgrounds of the respondents, their geographic dissimilarity, and the timing of the different surveys. We find that Funke *et al.*'s (2007) international subsample results fall within the expected deviation from the single asset class allocations. We also find a somewhat higher average allocation to private equity,

²⁷ We thank the authors, who provided us with their extensive data set.

²⁸ Participants included Banks 42%, Corporate Investors 20%, Insurance Companies 15%, Foundations 12%, Pension Funds 6%, and Other 5%. Assets under management include below 0.5 bn EUR 37%, 0.5 to 10 bn EUR 37%, and over 10 bn EUR 14%.

²⁹ Our used database was generated by a questionnaire-based telephone survey done by a specialized consulting company named Roland Berger Market Research. Further details are available from the authors upon request.

³⁰ Funke *et al.* (2007) interviewed a total of 162 German and 55 international investors. Our sample includes only responses from investors who were not fully invested in a single asset class, and whose asset allocations to a single alternative investment class (real estate, hedge funds, and private equity) or other investments did not exceed 50%. We excluded those responses from specialized funds or companies in order to ensure our asset allocation is representative of average investors.

a lower allocation to hedge funds, and a lower allocation to real estate compared to the results of Mercer (2007) and Russell (2006, 2007).

We do not, however, use the later surveys to calibrate our model, because: 1) the reports were not detailed enough to derive the full asset allocations of the investors surveyed, and 2) the Funke *et al.* (2007) survey is consistent within its sample. Thus, the international and German subsamples are more comparable with each other.

In summary, we strongly believe that the survey results are representative of alternative investment asset allocations. Even if survey results are comparable to broader (international orientated) surveys, the drawback from using excerpts is always that the basic population is not totally covered. This is especially relevant for the quite small international sample size. For that reason the derived results should be used carefully and might not be useful for a generalization.

● Table 1. Alternative Investment Allocation

This table reports the relative number of organisations investing in private equity, hedge funds, and real estate, as well as the average allocation to those assets. For the international subsample, Funke et al.'s (2007) results are based primarily on the responses of corporate investors (88%). The German subsample includes the responses of banks/corporate investors (57%), foundations (14%), and pension funds/insurance companies (23%). The Mercer (2007) survey results are based 100% on pension fund responses. The Russell (2006, 2007) surveys are also based on pension fund responses, but to a slightly lesser extent (80% to 90%).

	Private Equity		Hedge Funds		Real Estate	
	Investing in...	Average Allocation	Investing in...	Average Allocation	Investing in...	Average Allocation
Funke <i>et al.</i> (2007)						
2007 (German)	15.8%	0.8%	13.9%	0.7%	46.5%	4.6%
2007 (International)	31.3%	8.3%	6.3%	1.9%	37.5%	4.5%
2007 (Total)	17.9%	1.9%	12.8%	0.9%	45.3%	4.6%
Mercer – European Institutional Market Place Overview (~Pension funds)						
2006 Eur. ex. UK	4.4%	3.0%	13.0%	4.8%	61.7%	8.0%
2006 UK	5.9%	2.9%	7.3%	6.9%	24.9%	6.9%
Russell Investments – Survey on Alternative Investing (~80%-90% Pensions)						
2007 Europe	54%	4.6%	29%	7.4%	71%	8.9%
2005 Europe	63%	4.5%	35%	5.3%	66%	9.8%

Based on Funke *et al.* (2007), we calculate the average asset allocation for different types of German and international investors (see Figure 1), and we approximate the portfolio allocations of the investors surveyed (Table 2). We find that, compared to the international subsample, German investors tend to have a greater allocation to bonds. That is the allocation to alternative investments in general, and to hedge funds and private equity in particular, is smaller than for the international subsample.

Table 2 shows that the German subsample invests a greater percentage in the risky (and uncertain) portfolio.

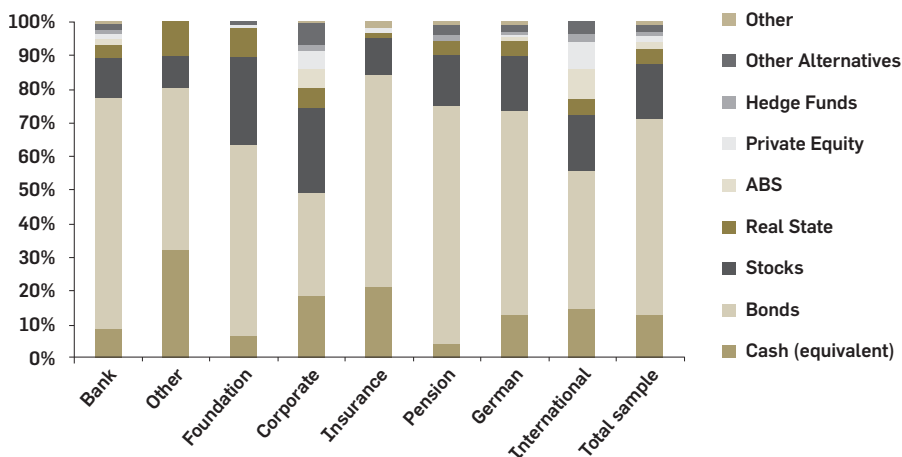
Table 2. Asset Allocation of German and International Investors

This table shows the capital-weighted portfolio allocation based on Funke et al. (2007). If the respondent answers about other investments and alternative investments were detailed enough, we reassigned the allocation to the respective asset class. We allocated “Other Alternative Investments” as follows: 25% to bonds (e.g., CDOs, convertibles, ABS), 25% to real estate (e.g., infrastructure, timber), 25% to private equity (infrastructure), and 25% to hedge funds (portable alpha, absolute return, commodity). This is in line with the results of Russell (2006, 2007) for the category “Other Alternatives”. For the sake of clarity, we increased each asset class in the remaining categories equally.

	Risk Free	Risky Portfolio	Stocks	Bonds	Hedge Funds	Real Estate	Private Equity
Total sample	13.8%	86.2%	19.4%	68.4%	2.2%	6.6%	3.4%
German subsample	13.4%	86.6%	18.9%	70.9%	1.8%	6.4%	2.0%
International subsample	16.2%	83.8%	22.0%	51.8%	5.3%	8.2%	12.8%

Figure 1. Asset Allocation According to Investor Type – 2007

This figure shows the capital-weighted portfolio allocation based on Funke et al. (2007). If the respondent answers were about other investments (e.g., company stocks) and the other alternative investments were detailed enough (CDOs, convertibles, infrastructure, absolute return, currency overlay, portable alpha, commodity), we reassigned the allocation to the respective asset classes (see also Table 2).



4.2. Data Set and Model Calibration

We now present the data set we use to estimate the tangential portfolio with and without ambiguity. We use two traditional asset classes,³¹ plus an estimation for the risk-free rate: risk-free (three-month Euro rate), stocks (MSCI Europe – Total Return), bonds (JPM Europe Government Bond – Total Return Index), and three alternative asset classes: hedge funds (HFRX Global Hedge Fund Index), real estate (FTSE NAREIT

³¹ We calibrate our model using survey data from Funke et al. (2007), which focuses on German and European investors. According to the literature, international equity portfolios are usually biased toward domestic stocks (see, e.g., French and Poterba, 1991, and Cooper and Kaplanis, 1994). Thus, we assume that European stock and bond indices will better account for the respondents’ asset allocation choices.

All REITs – Total Return Index), and private equity (CepreX – equally weighted index of the US Growth/Small Buyout and US Venture Capital single fund indices)³². We chose these indices because we assume that investors hold diversified portfolios of European stocks and government bonds.

For the alternative asset classes we choose the following indices: We use the HFRI Fund of Funds – Diversified Index as a proxy for hedge funds because it is an after-fee index that accounts for survivorship bias and does not backfill.³³ The index is representative of 95% of the fund of hedge fund market. And the attrition rate is only 2.2%, among the lowest of all data providers (Heidorn *et al.*, 2006). It is an equally weighted index, and therefore will not overweight large funds of funds.³⁴

We use after-fee data³⁵ from the CEPRES database to obtain a private equity benchmark. This database has the following advantages over other data providers such as Venture Economics or Cambridge Associates:³⁶ 1) The data vendor uses unlisted private equity to construct its indices; it is therefore a better benchmark for “real” (unlisted) private equity, 2) it is transaction-based, 3) the data vendor uses a methodology that avoids unrealistic and sudden book-value changes, and 4) it has no survival bias. The index we use is based on equally weighted single-fund index return data of after-fee unlisted US Growth/Small Buyout Index returns and US Venture Capital Index returns.

Just as with private equity, there are listed and unlisted indices. We would normally prefer an unlisted index, but there is no meaningful data on unlisted European real estate available. Hence, we use the FTSE NAREIT All REITs Index (henceforth referred to as real estate) as a proxy for a diversified real estate portfolio. The FTSE EPRS/NAREIT uses transaction data, and is representative of the market value of underlying properties and the leverage (about 50%, which is not unusual for real estate investments). Clayton and MacKinnon (2000) show that the relationship between REITs and direct real estate strengthened during the 1990s. Thus, we believe this index is a reasonable proxy for a mixed real estate portfolio.

³² The reported data for our private equity index are based on equally weighted index returns of after-fee unlisted US Growth/Small Buyout and US Venture Capital Index returns.

³³ The HFR database has the lowest survivorship bias (0.16%) of all hedge fund data providers (the range is up to 6.2%) (Ackermann *et al.*, 1999, Liang, 2000, Heidorn *et al.*, 2006).

³⁴ For more information, see www.hedgefundresearch.com. For an analysis of the representativeness of different hedge fund indices, see Amenc *et al.* (2005).

³⁵ To correct the single-fund buyout and venture capital indices for management fees, we apply the following procedure: 1) We reduce the single fund index returns for a single fund fee, and 2) we reduce the return data for a fund of funds fee based on a Monte Carlo model for the gross cash flows, and then reduce the gross cash flows for the fees (management fee, carry fee, etc.). On the basis of the obtained gross and net cash flows, we derive a function to correct the after-fee single fund index for fund of fund fees. The result is a synthetic fund of private equity fund investments. For further details, see <http://www.cepres.de>. The index is not investable.

³⁶ For more information on index construction, see www.cepres.de. See also Krohmer *et al.* (2010), Cumming *et al.* (2010), Schmidt (2004), and Schmidt *et al.* (2010).

All indices are on a monthly basis,³⁷ with a January 1999 inception date and a March 2008 end date. Our analyses include 111 observations. Alternative investments often exhibit autocorrelation, due to, e.g., spurious or stale pricing and appraisal smoothing, time-varying leverage, incentive-based fees, and accounting standards (see Budhraj and de Figueiredo, 2004). To obtain an unsmoothed data set we use the method suggested by Geltner (1991, 1993). Further details are available from the authors upon request. Table 3 shows the descriptive statistics for the unsmoothed data set.

Table 3. Descriptive Statistics of the Unsmoothed Monthly Return Distributions

This table reports the arithmetic mean, median, maximum, minimum, standard deviation (StD), skewness, kurtosis, and the Jarque-Bera test statistic (JB) of the monthly return distributions from January 1999 through March 2008. The descriptive statistics are calculated on after-cost return time series obtained from the data vendors. The private equity index is adjusted for first-order autocorrelation (Budhraj and de Figueiredo, 2004).

	Stocks	Bonds	Hedge Funds	Real Estate	Private Equity
Mean	0.0098	0.0038	0.0072	0.0111	0.0097
Median	0.0069	0.0056	0.0052	0.0162	0.0104
Maximum	0.1650	0.0289	0.0595	0.0971	0.1621
Minimum	-0.1518	-0.0190	-0.0385	-0.1526	-0.1372
StD	0.0594	0.0101	0.0172	0.0424	0.0340
Skewness	-0.0141	-0.3455	0.6844	-0.7621	0.0039
Kurtosis	3.2674	2.6473	4.7633	4.1810	9.5202
JB	0.33	2.78	23.05*	17.20*	196.62*

*Denotes rejection of the null hypothesis that the returns are normally distributed at the 1% and 5% levels.

Return: Table 3 shows that real estate has the highest mean (1.11%) and median (1.62%) monthly returns. Stocks and private equity are next, with means of 0.98% and 0.97%, respectively, and medians of 0.69% and 1.04%, respectively. The highest maximum monthly returns are for stocks (16.50%) and private equity (16.21%). Bonds have the lowest mean (0.38%), median (0.56%), and maximum (2.89%) monthly returns. Hedge funds are in-between, with 0.72% mean and 0.52% median monthly return.

Risk: Real estate (-15.26%) and stocks (-15.18%) have the highest risk as measured by the minimum monthly returns. They also exhibit the highest standard deviation (4.24% and 5.94%, respectively). Private equity has a slightly lower risk, with a -13.72% minimum monthly return and a 3.40% standard deviation.

³⁷ All data are in local currencies to obtain a pure exposure to the returns of the respective asset class and to avoid an artificial currency overlay exposure.

Bonds exhibit the lowest risk, with a standard deviation of 1.01% and a minimum monthly return of -1.90%. Hedge funds are again in-between, with 1.72% standard deviation, and -3.88% minimum monthly return.

Higher Moments: Additional sources of risk are found in the higher moment characteristics. Negative skewness and high kurtosis are usually considered unfavorable. We cannot reject the null hypothesis of normally distributed returns for all alternative investments at the 1% level. We also find that real estate (-0.76) and stocks (-0.35) exhibit the least favorable skewness characteristics. We find positive skewness for hedge funds (0.68), however, which is usually desirable for investors. We believe that using a fund of hedge fund index may explain this return distribution characteristic.

Private equity has the highest kurtosis (9.52), followed by real estate (4.18) and hedge funds (4.76). This leptokurtosis indicates a higher probability of extreme returns than the other return distributions (positive as well as negative), with comparable standard deviations. The positive skewness of hedge funds makes high positive returns more likely than normally distributed returns, and vice versa for private equity and real estate.

To assess the diversification potential of alternative investments, Table 4 reports the correlation for the unsmoothed return series. We find that all alternative asset classes can be interesting portfolio diversifiers, according to their low correlation with traditional investments. Furthermore, we find that the different alternative asset classes are not suitable substitutes for each other, as implied by their low correlation with other alternative investments.

● **Table 4. Correlation of Monthly Return Distributions**

This table reports the ordinary Pearson correlation coefficients of the monthly unsmoothed returns from January 1999 through October 2008, as well as the p-values for the hypothesis that a single correlation coefficient is zero.

Correlation p-value	Stocks	Bonds	Hedge Funds	Real Estate	Private Equity
Stocks	1.00 -----				
Bonds	-0.10 0.27	1.00 -----			
Hedge Funds	0.06 0.53	0.03 0.78	1.00 ----		
Real Estate	0.10 0.29	0.07 0.50	0.18 0.06	1.00 ----	
Private Equity	-0.01 0.89	-0.08 0.38	0.09 0.34	-0.02 0.80	1.00

***and *indicate that the null hypothesis that a single correlation coefficient is equal to zero cannot be rejected at the 1% and 5% levels, respectively.*

To better understand the asset classes' diversification potential, we calculate the efficient frontiers according to Markowitz (1952). The optimization is based on the historical observed return distribution characteristics from Table 3 and Table 4. We further derive the tangential portfolio(s) and the efficient Merton (1971) portfolios for different degrees of risk aversion under rational expectations.

As expected, Figure 2 confirms that the inclusion of all alternative investments can increase risk/return efficiency in the mean-variance framework. In the unrestricted case with alternative investments (All Asset Classes – Unrestricted), we note that the efficient frontier is superior to an optimization with no alternative investments (Traditional).

However, note that the unrestricted case is not feasible for all investor types. Pension plans and insurance companies, for example, may face portfolio allocation restrictions (see Table 8 in the appendix for an overview of German restrictions).

Figure 2 also plots the most restricted case (All Asset Classes – Restricted) and the respective portfolio weights (Restricted Model with Alternative Investments). However, only a small minority of investors surveyed in Funke *et al.* (2007) were subject to those restrictions. In our analysis, we focus on the unrestricted case, and add the restricted case for further comparison.

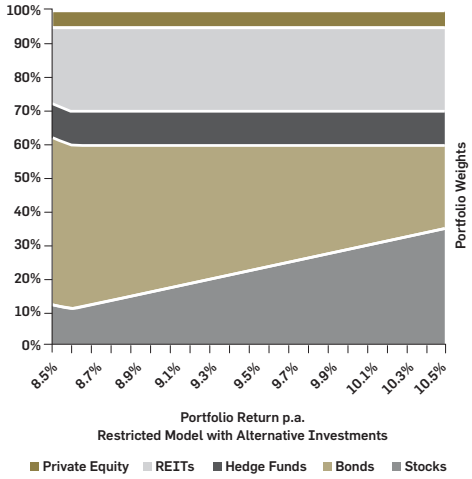
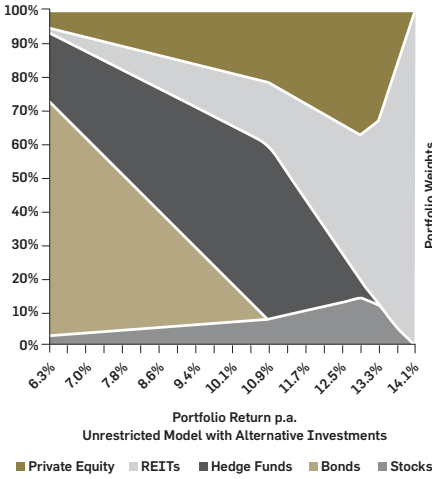
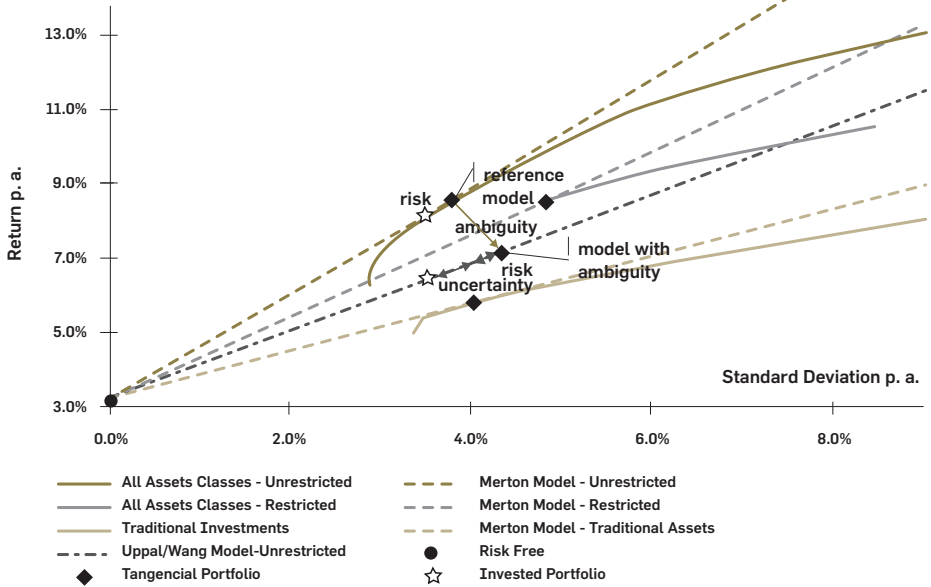
Again as expected, Figure 2 shows it is beneficial to include alternative investments in the unrestricted and the restricted cases. Note that the tangential portfolio under rational expectations in the restricted case holds a higher percentage of bonds and REITs as a substitute for hedge funds and private equity. The allocation to stocks does not increase significantly.

In any event, we find that the theoretical portfolio weights in alternative investments are well above the empirical portfolio weights observed by Russell (2006, 2007), Funke *et al.* (2007), and Watson Wyatt Worldwide (2007). In this article, we show that if we relax the rational expectations assumption, we can explain the lower allocation to alternative investments.

We argue that investors do not believe in the reference model derived from the observed data (e.g., Tangential Portfolio, All Asset Classes – Unrestricted) which is equal to the true model. Rather, they assume an alternative model. This model is in their opinion and to their best knowledge more robust and the least prone to model misspecification. It can be derived by disturbing the marginal return processes of the single asset classes (of the risky and uncertain portfolio). Thus, we see that ambiguity causes the *reference model* to move to the alternative model with ambiguity, according to the beliefs and information of the respective investor (see also Figure 2).

Figure 2. Efficient Frontier and Portfolio Weights without Restriction

This figure shows the results of a mean-variance optimization based on the historical return distribution as reported in Table 3 and Table 4 for different degrees of risk aversion, as well as the weights of the respective tangential portfolios.



	Stocks	Bonds	Hedge Funds	Real Estate	Private Equity
Tangential Portfolio (unrestricted)	5.5%	34.9%	36.7%	9.7%	13.2%
Tangential Portfolio (restricted)	12.7%	50.0%	10.0%	22.3%	5.0%
Typical Tangential Portfolio (with ambiguity)	16.6%	68.4%	2.2%	9.4%	3.4%

Investors do not normally invest in the tangential portfolio but in a mix of the risky (and uncertain) portfolio and the risk-free (Tobin's separation theorem). In the case of no-uncertainty (rational expectations), investors choose a portfolio in line with their level of risk aversion, which lies below the tangential portfolio.³⁸

If we drop the rational expectations assumption, investors may choose an alternative model that is more closely tied to their level of ambiguity and the available information. However, that does not imply agents are not uncertain about it. They choose to invest in a portfolio (of the risk-free rate and the *alternative model*) that not only accounts for their level of risk aversion, but also for uncertainty.

To test our hypothesis that we can explain the observed asset allocation with uncertainty and ambiguity aversion, we next derive the respective uncertainty/ ambiguity parameters for the risky and uncertain portfolio for different types of investors.

■ 5. Ambiguity Aversion

Similarly to the home bias example, or to the overallocation to domestic stocks presented by Uppal and Wang (2003), we use the suggested framework to explain the underdiversification in alternative investments. However, we extend and modify Uppal and Wang's (2003) example as follows: We consider two traditional and three alternative asset classes, thus giving the problem more dimensionality. We derive the risk aversion parameter empirically. We allow for information about the single marginal return distribution of *each* asset class (*for* $i \neq j$ $\hat{m}_{i,j} = \hat{m}_{j,i} = 0$) in the risky and ambiguous portfolio (henceforth referred to as the MODEL WITH AMBIGUITY, see also Figure 2). We also consider a second case where we allow for information among the marginal return distributions of the single asset classes ($\hat{m}_{i,j} = \hat{m}_{j,i} \neq 0$).³⁹ Because we observe underallocation, it is reasonable to assume a lack of information about alternative investments. Thus, we expect the ambiguity estimates for alternative investments $\hat{m}_{i,j}$ to be negative. The interpretation is: the lower $\hat{m}_{i,j}$, the higher the ambiguity.

In this article, we fit the empirical portfolio to the model with ambiguity and uncertainty. We also compare the implied risk parameters for risk, uncertainty, and ambiguity for different investors in order to explain international differences and the implications for the alternative investment industry. To obtain reasonable estimates, we use Uppal and Wang's (2003) results and impose restrictions on the implied drift adjustment. We do not allow for negative expected returns (see also section 3).

³⁸ We do not allow for leverage.

³⁹ This procedure also serves as a robustness check for the derived ambiguity parameter.

Based on Funke *et al.*'s (2007) survey results as reported in Table 2 and the empirical risk/return characteristics of the different asset classes reported in Table 3 and Table 4, we can derive the implied risk aversion $\hat{\gamma}$ for the different investor types (for further details, see also Table 5, and section 3, step 5).

● **Table 5. Portfolio Choice and Risk Aversion**

This table reports the risk/return characteristics for the empirical portfolio allocation for different investor types, as well as the implied risk aversion. The risk-free rate equals 0.27% p.m., or 3.24% p.a. based on the three-month Euro money rate. For the expected return estimates, we include only observations of respondents who have target portfolio return guidelines. We exclude all observations for which the target portfolio returns exceeded 50% or were equal to zero.

Same sample	Expected Return p.a. (Survey) ⁴⁰	Implied Portfolio Return p.a.	Implied Portfolio Volatility p.a.	Implied Risk Aversion $\hat{\gamma}$
Total sample	6.03%	6.58%	4.00%	19.9
German subsample	5.92%	6.44%	3.98%	19.4
International subsample	7.66%	7.49%	4.36%	21.3

Based on the reported asset allocation, we find that the implied risk aversion $\hat{\gamma}$ for German investors is below the risk aversion for the international subsample. At first, this is somewhat surprising. However, note that while international investors invest a higher percentage in the risk-free asset, they also invest in higher-risk portfolios, as measured by standard deviation. Furthermore, if we account for uncertainty, the “aversion” to the model with ambiguity increases. As we found in section 3, uncertainty aversion can be transformed into a modified and increased risk aversion. This would be in line with our assumption that uncertainty aversion is higher for the German subsample than for the international sample.

For the reported portfolio weights and the implied risk aversion, we can now derive the uncertainty aversion $\hat{\phi}$ for the joint distribution and the ambiguity \hat{m}_{ij} toward the marginal return distributions of the asset classes in the portfolio (see Table 6). We provide some explanatory notes before reporting the estimated implied parameters:

- 1) The higher (lower) the implied uncertainty parameter $\hat{\phi}$ the higher is the penalty for any *alternative model* or *model with ambiguity*, and hence the lower (higher) the uncertainty aversion of investors. That is, investors are more (less) willing to invest in a model with ambiguity instead of the risk-free (see also “Uncertainty” Figure 2). This occurs despite the fact that, based on the observed data, there will always be some information lacking about the respective model.

⁴⁰ To test whether the derived benchmarks are reasonable and conform with investor expectations, we calculate the portfolio return for the empirical asset allocation based on our data set. If we compare the expected implied return with the responses regarding expected returns for the respective portfolios from the Funke *et al.* (2007) survey, we find a high congruence. We assume that the benchmark indices used in this analysis (especially for alternative investments) are in line with investor expectations.

- 2) The higher (lower) the ambiguity parameter \hat{m}_{ij} the more (less) information has the investor about the marginal distribution of the n risky and ambiguous assets and/or accounts for it to derive a more robust *alternative model* (the model with ambiguity in Figure 2) as “substitute” for the *reference model*.

We could interpret positive \hat{m}_{ij} to mean that \hat{m}_{ij} is positively correlated with the amount of information investors believe they have about the single asset class and its return generation process. Therefore, the respective asset class seems less ambiguous, at least to investors.

We could interpret negative \hat{m}_{ij} to mean that investors are not using all the available information. It may be that they do not have sufficient knowledge or capacity to take advantage of all the information. They may also be unable to correctly interpret the available information, or to draw correct conclusions about the return process. Or, if they believe the information may not be full or even correct, they may place a penalty on incomplete or incorrect information to account for a lack of availability. Whatever the reason for negative \hat{m}_{ij} , however, the marginal return distributions of the *reference model* become more ambiguous, the implied drift adjustment on the single return generation processes increase, and the *model with ambiguity* moves further away from the *reference model* (see Figure 2).

Subsequently, we note that the higher \hat{m}_{ij} (in absolute terms), the less uncertain investors are regarding that asset class, and the higher their allocations to the respective assets.

We can now assess our hypothesis from section 1, that the low allocation to alternative investments can be explained by investor aversion to assets they are unsure about. The implied uncertainty $\hat{\phi}$ and the implied ambiguity \hat{m}_{ij} parameter for the different investors are reported in Table 6, the respective model fit in Table 7. We can also assess possible reasons that German investors have a lower alternative investment allocation than their international counterparts.

● **Table 6. Estimates for Ambiguity and Uncertainty Aversion**

This table reports the implied risk aversion $\hat{\gamma}$, the implied uncertainty about the joint distribution of the risky and uncertain portfolio $\hat{\phi}$, and the parameter \hat{m}_{ij} for the implied ambiguity of the marginal distribution of each risky asset class. Note \hat{m}_{ij} are scaled by $\hat{\phi}$, so that parameters can be interpreted relative to each other. Further details on the implied drift adjustment of the return process and on the full parameter matrix are available from the authors upon request.

	$\hat{\gamma}$	$\hat{\phi}$	$\sum_i \hat{m}_{i,Stocks}$	$\sum_i \hat{m}_{i,Bonds}$	$\sum_i \hat{m}_{i,Hedge Funds}$	$\sum_i \hat{m}_{i,Real Estate}$	$\sum_i \hat{m}_{i,Private Equity}$
Total sample	19.9	37.9	$\rightarrow\infty$	-0.55	-0.85	-0.84	-1.02
German subsample	19.9	21.9	$\rightarrow\infty$	-0.24	-0.69	-0.80	-0.98
German subsample (restricted)	19.9	39.4	$\rightarrow\infty$	1.24	0.03	-1.94	-1.44
International subsample	21.3	39.0	$\rightarrow\infty$	-0.64	-0.57	-0.82	-0.89
<i>for $i \neq j$ $\hat{m}_{ij} = \hat{m}_{ji} = 0$</i>							
Total sample	19.9	6.6	$\rightarrow\infty$	-0.55	-0.89	-0.66	-0.94
German subsample	19.9	4.8	$\rightarrow\infty$	-0.24	-0.89	-0.70	-0.92
German subsample (restricted)	19.9	14.8	$\rightarrow\infty$	20.0	-0.90	-0.77	-0.82
International subsample	21.3	5.0	$\rightarrow\infty$	-0.74	-0.90	-0.58	-0.87

The brown highlighted cells indicate a small deviation between the model with ambiguity and the empirical model. Cells that are not highlighted indicate a complete fit.

● **Table 7. Fit of the Observed and Implied Asset Allocation**

This table reports the implied asset allocation for the estimated implied ambiguity aversion.

	Fit	Risk Free	Stocks	Bonds	Hedge Funds	Real Estate	Private Equity
<i>$\hat{m}_{i,j} = \hat{m}_{j,i} \neq 0$</i>							
Total sample	0.8%	13.8%	16.3%	59.0%	1.9%	6.1%	2.9%
German subsample	1.1%	13.4%	15.9%	61.4%	1.6%	5.5%	2.2%
German subsample (restricted)	0.7%	13.4%	16.0%	61.4%	2.0%	5.5%	1.7%
International subsample	0.7%	16.2%	18.0%	43.4%	4.8%	6.9%	10.7%
<i>for $i \neq j$ $\hat{m}_{ij} = \hat{m}_{ji} = 0$</i>							
Total sample	4.8%	13.8%	14.3%	59.0%	1.9%	8.1%	2.9%
German subsample	4.6%	13.4%	14.1%	61.4%	1.6%	5.5%	4.0%
German subsample (restricted)	4.6%	13.4%	16.4%	59.1%	1.6%	5.5%	4.0%
International subsample	6.9%	16.2%	15.0%	43.4%	4.4%	10.3%	10.7%

The brown highlighted cells indicate a small deviation between the model with ambiguity and the empirical model. Cells that are not highlighted indicate a complete fit.

We find that uncertainty alone cannot explain the observed asset allocation of institutional and large private investors. However, if we consider additional information (or a lack thereof), and different levels of ambiguity for the single asset classes and marginal return distributions, we find that ambiguity/uncertainty aversion is a suitable explanation. However, we still cannot fully explain the overallocation to stocks. One reason may be that investors systematically overestimate the equity premium. For the following explanation of the implied uncertainty and ambiguity parameter, it is important to keep this in mind.

Uncertainty (choice between the risk-free model and the model with ambiguity): We find that uncertainty is higher (as measured by the lowest implied $\hat{\phi}$) for the German subsample than for the total sample and for the international subsample (Table 6). Note that German investors have a lower risk aversion than the total sample. If we consider investment restrictions (see also Table 8), uncertainty aversion decreases significantly. This is not surprising, however. The identical allocation in a model with ambiguity requires significantly more commitment (and less uncertainty aversion) than if investors are restricted in the portfolio choice.

Ambiguity for traditional asset classes (estimation of the model with ambiguity): As noted earlier, we find no ambiguity for stocks. Our estimates suggest that the market fully reflects all available information, and that all investors believe the information can be used to derive a “sound” reference model for this asset class. However, we also find that investors systematically overinvest in this asset class, and we conclude that they tend to be overconfident about this asset class.

We hypothesize that investors overestimate the equity premium based on their own beliefs (e.g., Black-Litterman approach), and not on directly observed data. This assumption is supported by the fact that 97% of investors surveyed in Funke *et al.* (2007) stated that past realized stock returns either met or exceeded their expectations. Stock returns from 2003 to 2007 were above long-term average, however, and investors may have assumed overly high priors. More recent market developments suggest that those high priors may be overly optimistic.⁴¹

From the implied ambiguity parameter for bonds, we find that investors consider bonds ambiguous. If we compare the German subsample with the international sample, we find that German investors are significantly less ambiguous about bonds. Considering the overall uncertainty aversion of German investors, this difference relative to the other ambiguous assets in the model with ambiguity becomes more profound. The difference is magnified further if we consider German investment restrictions. The reason for this lack of ambiguity may lie in cultural backgrounds, or the fact that German investors have a longer investment history in this asset class.

Investors may find bonds ambiguous because it can be so difficult to assess bond risk. The risks can be quite complex (e.g., credit risk, reinvestment risk, duration risk, legislative risk, liquidity risk, and interest rate risk). Hence, bond risk is also difficult to model and understand, and this implies ambiguity.

Ambiguity for alternative asset classes (estimation of the model with ambiguity): In general, we find that all the investor types exhibit very high levels of ambiguity toward all alternative investment asset classes (as measured by negative implied μ). In fact, the observed data is unreliable, and is not considered suitable to derive a robust *reference model* for this asset class. In other words, estimates for implied ambiguity reflect extreme pessimism. All investors are willing to consider *alternative models*. For this reason, we also find that the implied ambiguity of the subsample investors does not vary significantly if we account for fit, which is largely influenced by the overallocation to stocks that cannot be explained by ambiguity (see Table 6).

We also find that the alternative investment allocation differences between the international and German subsamples may be explained by a lower ambiguity toward bonds relative to the extreme pessimism toward alternative investments. This sentiment is amplified by a higher level of uncertainty in German investors, especially if we consider German investment restrictions.

Altogether, we find that the high level of ambiguity and the extreme pessimism toward alternative investments is supported by the fact that most investors surveyed in Funke *et al.* (2007) could not state whether past realized returns from hedge fund or private equity investments were acceptable. Furthermore, if we compare the risk comprehension of the surveyed investors with the observed risks of alternative investments, the figures do not match (Funke *et al.*, 2007, and Table 3). Investors are seemingly influenced more by the general opinion of the risk/return profile of alternative investments than by their own experiences with such investments.

In summary, our findings support our initial hypothesis that the low allocation to alternative investments can be explained by the uncertainty and ambiguity aversion.

⁴¹ To analyze the impact of the expected return on the results for our implied ambiguity parameters we refer to the case where the expected return of the alternative assets (real estate, private equity, and hedge funds) is reduced by values $\Delta\mu$ compared to the base case in Table 6. Even though we consider a range between zero and 0.04 for $\Delta\mu$; it is hard to believe that the historic returns exceed the *ex ante* expected returns by more than $\Delta\mu = 0.02$. However, to control for robustness we also compute the values for $\Delta\mu = 0.03$ and $\Delta\mu = 0.04$. Since the expected annual return of hedge funds is only $\mu = 0.0863$ in the standard case, we think that further reductions to a value below $\mu = 0.0863 - 0.04 = 0.0463$ would no longer result in any meaningful estimates because the (*ex ante*) expected return for investments into hedge funds is too close to the risk-free rate. We find that even for extremely low expected returns of the alternative investments our results are still valid. The main consequences of lower expected returns $\Delta\mu$ is that the ambiguity parameters m for alternative investments are higher and closer to those of traditional investments, but the order of the ambiguity magnitude remains (also for the order within the alternative investments). This effect is intuitive because the high implicit ambiguity parameters in absolute terms result from the fact that institutional investors have low holdings in the alternative investments relative to their attractiveness documented e.g. by their expected return. Once the expected return declines, the attractiveness of the alternative investments shrinks so that these assets must have a lower implicit ambiguity in order to explain the observed portfolio holdings for less favorable expected returns.

We further find that the lower allocation by German investors can be explained by a higher uncertainty aversion, combined with high ambiguity (extreme pessimism) toward alternative investments *and* lower ambiguity toward bonds.

We conclude this section with further robustness considerations, and by drawing some implications for the alternative investment industry. According to the more recent surveys of Russell (2006, 2007) and Funke *et al.* (2007), the most important reasons for not including alternative investments in portfolios are the lack of investor expertise, a lack of resources on the part of investors to monitor this complex asset class, uncertainty about expected returns, low transparency, non-quantifiable risks, and a lack of data or data access. Note that all these reasons increase uncertainty. Hence, it is reasonable to assume that investors believe the alternative investment asset classes are very ambiguous. In comparison, we found that the higher costs, complex investment restrictions, low liquidity, and low fee comparability of alternative investments are of lesser importance. And they do not tend to increase uncertainty.

What can the alternative investment industry do?

Investors have stated they are inexperienced with these types of investments. Therefore, it might be useful to offer smaller “starter” investments at lower costs that will require less investor commitment. In this way, investors could gain experience and increase their comfort level with this asset class. This may also increase understanding of these assets, and consequently reduce ambiguity. Better information policies and risk reports, as well as more education about the risk and returns of alternative asset classes, may also help. Finally, experienced and reliable consultants may offer a solution for the lack of resources that are available to monitor the complexity of these asset classes.

The problems of low transparency, lack of data and data access, and non-quantifiable risks can be solved if the industry is willing to improve data availability and transparency. This would also help to build trust, which is invaluable during market downturns. The industry could also begin reporting more frequent and comprehensive standards to databases in order to offer a more reliable benchmark. This could greatly decrease uncertainty, and increase interest and investment in alternative assets. Note also that the more reliable the information and its source, the lower the uncertainty and we assume the greater the effect on ambiguity aversion (Epstein and Schneider, 2008).

In the case of a lower risk aversion and a high uncertainty even the riskier alternative investments could benefit from these suggestions. Investors with lower risk aversion would be naturally more willing to invest in low risk alternative investments if ambiguity can be reduced.

■ 6. Conclusion

We find that the allocation to alternative investments of large institutional and private investors in general (and German investors in particular) is small and considerably below the theoretical optimal allocation even after accounting for investment restrictions. We argue that because of problems such as identifying a sound benchmark, data biases, smoothed returns of alternative investment indices, and a lack of transparency, it can be difficult to infer reasonable return (distribution) expectations. In short, the asset class becomes uncertain.

In this paper, we hypothesize that investor aversion against this uncertainty explains the low allocations. In particular, we researched the assumption that alternative investments are more ambiguous than traditional asset classes (such as bonds and stocks). We further addressed the lower allocations by German investors compared to their international counterparts.

To test our hypothesis, we considered representative, empirically observed portfolios of traditional and alternative investments (hedge funds, private equity, and real estate) of German and international institutional and large private investors. We used a model developed by Uppal and Wang (2003) that formalizes the problem of investors who are concerned about uncertainty and model misspecification. The framework allowed us to distinguish overall uncertainty to the *joint* distribution of the risky and uncertain portfolio, as well as different levels of ambiguity toward the marginal return distribution for each risky and ambiguous asset class in the portfolio.

We find that uncertainty alone cannot explain the observed asset allocations of institutional and large private investors. However, if we consider different levels of ambiguity for the single asset classes, we observe a high level of underdiversification relative to the Merton (1971) model. We thus explain the underdiversification as a result of ambiguity.

We find that all the investors exhibited an uncertainty *aversion*, but it was greatest for the German subsample. We found no ambiguity toward stocks, but we did find that the investors tended to systematically overinvest in this asset class, and that they may even overestimate the risk premium. We also found that international investors consider bonds ambiguous - more so than German investors. We theorize that the complexity of bond risks, which makes them harder to model, may be the reason.

Finally, we found that all the investors considered alternative investments highly ambiguous, and even exhibited extreme pessimism toward this asset class. Ultimately, the lower alternative investment allocation by the German subsample can be explained by higher uncertainty aversion, extreme pessimism, and lower ambiguity toward bonds.

To conclude, we believe the high level of ambiguity toward alternative investments is due to a lack of trust in this asset class, which is heightened by lower levels of investor experience, a lack of resources to monitor this complex asset class, and, most importantly, low transparency, lack of data and data access, and non-quantifiable risks.

We believe that creating smaller starter investments with lower costs could alleviate some of these problems. There is also a need for experienced and reliable consultants, as well as better information policies that include education about alternative investment risk and returns, greater data availability, and higher transparency from reliable sources.

These measures may serve to decrease ambiguity, and may lead to greater interest and investment in alternative investments. We believe, for example, that investors who exhibit low risk aversion with high uncertainty aversion would be particularly likely to begin investing in alternative investments if ambiguity could be decreased.

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■ 7. Appendix

● Table 8. Investment Restrictions According to Investor Type

This table illustrates investment regulations for German institutional investors. We imposed the tightest legal restrictions. Note that restrictions were applicable for about one-quarter of the survey respondents (Figure 2). For further details on legal Regulations see §66 VAG, and §54b VAG.

Type of Organization	Stocks	Bonds	Hedge Funds	Private Equity	Real Estate
Most regulated organizations (e.g., pension funds, insurance)	35%	50%	10%	5%	25%
Least regulated organizations (e.g., corporate clients)	None	None	None	None	None

