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# Low-Frequency Components and the Weekend Effect Revisited: Evidence from Spectral Analysis

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**Abstract**

We revisit the well-known weekend anomaly (Gibbons and Hess, 1981; Harris, 1986; Smirlock and Straks, 1986; Connolly, 1989; Giovanis, 2010) using an established macroeconometric technique known as spectral analysis (Granger, 1964; Sargent, 1987). Our findings show that using regression analysis with dichotomous variables, spectral analysis helps establishing the robustness of the estimated parameters based on a sample of the S&P500 for the 1972-1973 period. As further evidence of cycles in financial times series, we relate our application of spectral analysis to the recent literature on low-frequency components in asset returns (Barberis *et al.*, 2001; Grüne and Semmler, 2008; Semmler *et al.*, 2009). We suggest investment practitioners to consider using spectral analysis for establishing the 'stylized facts' of the financial time series under scrutiny and for regression models validation purposes.

**Keywords:**

Spectral analysis; Weekend anomaly; Financial cycles; Low-frequency Components; Asset returns.

***JEL classification:***

C1; G11; G17.

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## 1. Introduction

The main purpose of this article is to provide the finance community with an overview of a method known as spectral analysis that is well known among academic economists since the mid 60's (Granger, 1964). Thereafter, it has mainly gained popularity in the field of economic sciences among specialists in macroeconomics (Sargent, 1987[1]). As a matter of fact, it has also been a part of the basic curriculum of specialists in econometrics and macroeconometrics for quite long time (Dhrymes, 1970; Box and Jenkins, 1977; Hamilton, 1994); and more recently, in general applied econometrics (Greene, 2000). Lately, it has also appeared in the field of financial econometrics (Wang, 2003). However, there is not much research using this method that can be found in the applied finance literature; where it does not seem to have gained the same popularity as in the field of economic sciences.

We thus propose an application well known by financial academics and practitioners that is the weekend anomaly (Gibbons and Hess, 1981; Harris, 1986; Smirlock and Straks, 1986; Connolly, 1989[2]; Racicot and Théoret, 2001; Giovanis, 2010). As these authors have shown, the weekend anomaly can be simply tested by using a basic dichotomous regression of the index of the S&P500 for the time period 1970-1973. Moreover, it can be tested for the Monday anomaly by using a Student $t$ test or its associated $p$-value (the significance of that variable). According to Connolly (1989), this type of financial regression may suffer from several types of misspecifications (autocorrelation, conditional heteroskedasticity[3], etc.). Nevertheless, spectral analysis can be used as further evidence of a cycle in the time series under scrutiny; even if there is an apparently misspecified financial regression model that shows a significant variable related to the problem of the Monday anomaly, as shown in our application. Therefore, we propose using spectral analysis for regression model validation purposes.

In this article, we also discuss the new strand of literature related to the theory of low-frequency components in time series of asset returns. The presumption is that there are important low-frequencies in financial time series of returns (Barberis *et al.*, 2001; Grüne and Semmeler, 2008; Semmler *et al.*, 2009). We believe that the literature on this theory could benefit from a judicious use of spectral analysis due to the fact that it is specifically designed for discovering a priori cycles of unknown length.

The paper is organized as follows. Section 2 describes the methodology used to identify the weekend anomaly and presents an introduction on low-frequency components in fi-

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*[1] Paquin (1979) applied spectral analysis for identifying cycles in regional unemployment time series in Canada.*

*[2] For a discussion of the weekend anomaly and January anomaly and a good list of references on the subject, see Megginson (1997).*

*[3] For an introduction on ARCH modelling and other useful nonlinear specifications in finance, see Racicot (2000a, 2003a).*

nancial time series. In section 3, we present our regression results and spectral density representation. Finally, section 4 provides a conclusion.

## 2. Methodology

### 2.1. Regression Models for the Weekend Anomaly

To estimate the Weekend Anomaly, we follow Connolly (1989) and Racicot and Théoret (2001) and use dichotomous variables built on the S&P500 index. The method consists in estimating certain parameters related to the days of the week to evaluate the impact of those days that have the most significant influence on the returns of the index. This effect is also known as the day-of-the-week (DOW) effect (Smirlock and Starks, 1986). As it has been shown by several authors (Gibbons and Hess, 1981; Harris, 1986; Keim, 1983), stock returns tend systematically to fall on Monday, and that is mostly for the time period of 1970-1973. After that period, the effect presumably vanished due to the presence of arbitrageurs (Black, 1993). However, some evidence points towards the fact that there would be also a DOW anomaly in other time period (Giovanis, 2010)[4]. Considering this fact, our aim is to simply illustrate the use of spectral analysis on a well-known 'stylized fact', rather than to debate whether or not there would be a DOW anomaly in other recent financial time series[5]. In the jargon of macroeconomists, the 'stylized facts' are the basic empirical fact (Blanchard and Fisher, 1989) or the Granger (1964, 1966) 'typical spectral shape' of financial time series[6]. The following financial regression is used to estimate the Monday anomaly (e.g. Racicot and Théoret, 2001 or Giovanis, 2010)

$$r_t = \beta_1 m_t + \beta_2 tu_t + \beta_3 w_t + \beta_4 th_t + \beta_5 f_t + e_t \qquad (1)$$

where $r_t = ln\left(\frac{S_t}{S_{t-1}}\right)$ is the return computed using the daily observations of the S&P500 index for the year 72 and 73; $m_t$, $tu_t$, $w_t$, $th_t$, and $f_t$ are dichotomous variables which identifies, respectively, the days of the week: Monday, Tuesday, Wednesday, Thursday and Friday, and $e_t$ is, as usual, an error term. To generate our dichotomous variables, we used the EViews program that appears in Table 1.

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[4] *Giovanis (2010) uses STAR (Smooth Transition Autoregressive) models to test the DOW effect. Particularly, he estimates the following nonlinear regression:* $r_t = \pi_1 ' w_t + \beta_1 D_{MON} + \beta_2 D_{TUE} + \beta_3 D_{WED} + \beta_4 D_{THU} + \beta_5 D_{FRI} + (\pi_2 w_t + \gamma_1 D_{MON} + \gamma_2 D_{TUE} + \gamma_3 D_{WED} + \gamma_4 D_{THU} + \gamma_5 D_{FRI})F(r_{t-d};\gamma,c) + u_t$ *where the* $D$*'s are defined as in equation (1) and* $w_t = (r_{t-1} \dots r_{t-j})$ *is a vector of explanatory variables. He considers two transition functions: the logistic one,* $F(r_{t-d}) = (1 + exp[-\gamma(1/\sigma)(r_{t-d} - c)])^{-1}$ *and the exponential one:* $F(r_{t-d}) = (1 - exp[-\gamma(1/\sigma^2)(r_{t-d} - c)^2]$, $\gamma > 0$*; where* $r_{t-d}$ *is the transition variable,* $c$ *is the threshold, and* $\gamma$ *is the slope of the transition function. Franses and van Dijk (2000) provide an interesting introduction on this type of regime-switching models of returns. See also Racicot and Théoret (2001).*

[5] *Wilmott (2007, pp. 243-245) discusses some evidence of the DOW effect in the VIX volatility index ('a measure of the implied volatility of the ATM SPX'). That effect could be tested by means of the method suggested in this article.*

[6] *The Granger 'typical spectral shape' displays a power spectrum with the following characteristics: a smooth peak at low frequency and an exponential decay afterward (at higher frequencies). Using the words of Sargent (1987, pp. 279), 'the dominant feature of the spectrum of most economic time series is that it generally decreases drastically as frequency increases, with most of the power in the low frequency, high periodicity bands'.*

**Table 1. An example of EViews Code used to Generate the Dichotomous Variables of Equation (1)**

| | |
|---|---|
| smpl 1 503 | genr w72=0 |
| genr m72=0 | genr th72=0 |
| genr t72=0 | genr f72=0 |
| ' Loop until the last observation for !i=0 to 502 | else |
| if day72(!i+1)=1 then | genr w72(!i)=0 |
| genr m72(!i)=1 | endif |
| else | if day72(!i+1)=4 then |
| genr m72(!i)=0 | genr th72(!i)=1 |
| endif | else |
| if day72(!i+1)=2 then | genr th72(!i)=0 |
| genr t72(!i)=1 | endif |
| else | if day72(!i+1)=5 then |
| genr t72(!i)=0 | genr f72(!i)=1 |
| endif | else |
| if day72(!i+1)=3 then | genr f72(!i)=0 |
| genr w72(!i)=1 | endif |
| | next |

SOURCE: RACICOT AND THÉORET (2001)

## 2.2. The Geometric Brownian Motion (GBM) Model

The Geometric Brownian Motion (GBM) is one of the most popular models used in quantitative finance. This model is at the heart of the Black and Scholes (1973) option pricing model. It can be used as a data-generating process and should show the behaviour of the random walk model. In the following discussion, we briefly describe how to obtain a simulated time series of asset returns using this type of financial modelling[7].

Assuming that $S_t$ is the price of a stock $S$ observed at time $t$, the basic GBM model for the returns of that stock price is given by

$$\frac{dS}{S} = \mu\, dt + \sigma\, dz \qquad (2)$$

where $\mu$ and $\sigma$ are, respectively, the mean and standard deviation of $dS/S$. The element, $dz$, is the stochastic part of (2). It is known as the Wiener process and defined as

$$dz = \varepsilon \sqrt{dt} \qquad (3)$$

where $\varepsilon \sim N(0,1)$, is a standard normal distribution and $dt$, is an infinitesimal time step.

[7] *For more information on the subject, see Racicot and Théoret (2001, 2004, and 2006).*

To obtain the empirical counterpart the GBM model, (2) must be discretized in an efficient way. Applying Itô's lemma on (2) and after discretizing the resulting equation, one obtains an efficient model to be simulated. The following equations show how to proceed. For a function $G(x, t)$ that depends on a stochastic variable $x$, Itô's lemma is given by

$$dG=\frac{\partial G}{\partial x}dx+\frac{\partial G}{\partial t}dt+\frac{1}{2}\frac{\partial^2 G}{(\partial x)^2}\sigma^2 dt \tag{4}$$

Thus, applying (4) on a function $F=ln(S)$ which does not depend on time $t$ gives

$$dF=\frac{\partial F}{\partial S}dS+\frac{1}{2}\frac{\partial^2 F}{(\partial S)^2}\sigma^2 dt \tag{5}$$

Replacing the derivatives in equation (5) by their analytical results and $dS$ by (2), we obtain

$$dF=dlnS=\frac{dS}{S}=\left(\mu-\frac{1}{2}\sigma^2\right)dt+\sigma dz \tag{6}$$

Then, by integrating both sides of (6), we obtain the following exact discretized version of equation (2)

$$r_t=\frac{\Delta S_t}{S_{t-1}}=\left(\mu-\frac{1}{2}\sigma^2\right)\Delta t+\sigma\,\varepsilon\,\sqrt{\Delta t} \tag{7}$$

To perform our spectral analysis of the data generated by equation (7), we assume a risk neutral universe so that $\mu$ can be replaced by the risk-free rate $r_f$. The power spectrum resulting from the simulation of (7) is presented in Section 3.

### 2.3. Low-frequency Components in Asset Returns

In this section we briefly discuss some literature on the theory of low-frequency components in asset returns (Barberis *et al.*, 2001; Grüne and Semmler, 2008; Semmler *et al.*, 2009). The presumption of this approach is that there would be long cycles[8] in asset returns, as it is shown by the following model (Semmler *et al.*, 2009)[9]

$$r_t^e=\beta_0+\beta_1\,sin(w1t)+\beta_2\,cos(w1t)+\beta_3\,sin(w2t)+\beta_4\,cos(w2t) \tag{8}$$

where $w1=2\pi/5.2857$, $w2=1\pi/3.3636$. By using the Discrete Fourier Transform (DFT) filter to estimate the parameters of equation (8) on an annual sample of equity returns ($r_t^e$) for the time period of 1929-2000, Semmler *et al.* (2009) have found that

[8] *'A series may be said to possess a "cycle" if its covariogram is characterized by (damped) oscillations. The typical "length" of the cycle can be measured by $2\pi/w$, where $w$ is the angular frequency associated with the damped oscillations in the covariogram' (Sargent 1987, pp. 247).*

[9] *Kaufman (1984, chapter 15) presents a model that shares some similarities with equation (8), that is:* $y_t=a_1cos(w_1t)+b_1sin(w_1t)++a_2cos(w_2t)+b_2sin(w_2t)$.

those parameters are equal to: 0.0718, -0.0971, 0.0086, 0.0712 and 0.0135, respectively for: $\beta_0$, $\beta_1$, $\beta_2$, $\beta_3$ and $\beta_4$. They applied the same approach on the real interest rate time series for the same time period and frequency using the same model, which is given by

$$r_t^e = \alpha_0 + \alpha_1 sin(w1t) + \alpha_2\, cos(w1t) + \alpha_3\, sin(w2t) + \alpha_4\, cos(w2t) \tag{9}$$

where $w1 = 2\pi/24.667$, $w2 = 2\pi/5.2857$. They have found that $\alpha_0, \alpha_1, \alpha_2, \alpha_3$ and $\alpha_4$ are equal to, 0.01, 0.0182, -0.014, -0.0133 and -0.0042, respectively.

Our aim here is to make the investment practitioners realize that the studies on low-frequency components in financial time series are also evidence of potentially interesting application of spectral analysis, because this literature relates to portfolio management (Semmler *et al.*, 2009). By analogy with some of the empirical works done by specialists in macroeconomics, spectral analysis techniques could be used as supplementary tools for the works specialized in empirical finance to help to discover potentially underlying cycles in the data. In Section 3, we present the power spectrum of (8) as further evidence of the usefulness of spectral analysis for the investment practitioners.

## 3. Regression Results and Spectral Density Representation

### 3.1. Regression Results

By running the EViews code presented in Table 1 and applying ordinary least squares (OLS) to equation (1), we obtain the results displayed in Table 2.

**Table 2. OLS Estimation of Equation (1)**

| Variable | Coefficient | s.d. | *t*-stats | *p*-value |
|---|---|---|---|---|
| $m_t$ | -0.0028 | 0.0009 | -3.20 | 0.0015 |
| $tu_t$ | 0.0007 | 0.0008 | 0.89 | 0.3716 |
| $w_t$ | 0.0004 | 0.0008 | 0.54 | 0.5871 |
| $th_t$ | 0.0006 | 0.0008 | 0.73 | 0.4636 |
| $f_t$ | 0.0003 | 0.0008 | 0.32 | 0.7473 |

| | | | |
|---|---|---|---|
| $R^2$ | 0.02 | Akaike crit. | -6.76 |
| Adj-$R^2$ | 0.016 | Schwarz crit. | -6.72 |
| Sum sqr resi | 0.03 | Hannan-Quinn crit. | -6.74 |
| Log likeli | 1702.07 | | |
| D.W. stat | 1.56 | | |

This table shows that the only significant variable is $m_t$ with an estimated coefficient of -0.0028 (or -0.28 if scaled by 100 as often done in this literature). It is significant at 1% with a $p$-value of 0.0015 ($t$-statistic of -3.20). Thus, this regression analysis confirms the fact that Monday is the only day of the week that presents an anomaly. Taking into account this result, we can conclude that Stock returns would decline only on Monday– for that sample of data. It should be noted that the Durbin-Watson statistic gives a result that is not close to the 2.0 value. This might indicate the presence of autocorrelation in the residuals. Furthermore, by running other tests on the residuals, we observe that there seems to be a problem of conditional heteroskedasticity[10]. Nevertheless, spectral analysis helps in confirming our results when applied to this sample. A discussion on this topic is presented in the following section.

### 3.2. Spectral Analysis

To perform the power spectrum of our data, we use a parametric Yule-Walker algorithm which is based on an estimation of an autoregression of order p, AR(p). More precisely, the power spectrum can be represented by a Fourier transform of the autocovariance function, which is (Hamilton, 1994)

$$s_y(\omega)=\frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\gamma_j e^{-iwj} \tag{10}$$

where $\gamma_j=E[(y_t-\mu)(y_{t-j}-\mu)]$ is the autocovariance function, $e^{-iwj}$ is the Fourier transform[11], $\omega$ represents the frequency, $i=\sqrt{-1}$, a complex number. Applying De Moivre theorem, the Fourier Transform becomes

$$e^{-iwj}=cos(wj)-i\ sin(wj) \tag{11}$$

Thus, the power spectrum can be simplified to

$$s_y(\omega)=\frac{1}{2\pi}\left[\gamma_0+2\sum_{j=1}^{\infty}\gamma_j cos(wj)\right] \tag{12}$$

The power spectrum or the power spectral density ($PSD$) can be estimated using a parametric model which could be either a general ARMA($p$, $q$) model or, as in our case, using an AR(p) model. Thus, the power spectrum for an ARMA($p$, $q$),

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[10] *Giovanis (2010) provides evidence of that fact. Other type of misspecifications might arise when using financial regression models. Racicot (1993, 2000, 2003), Coën and Racicot (2007), Racicot and Théoret (2010a, b) provide a discussion of misspecification tests in similar contexts.*

[11] *The Fourier transform of a time series* $\{x_1, x_2, x_3, \ldots, x_n\}$ *can be written as:* $J(\lambda)=n^{-1/2}\sum_{t=1}^{n}x_t e^{-it\lambda}$, $-\infty<\lambda<\infty$ (Brockwell and *Davis, 1991). More precisely (Sargent, 1987), the Riesz-Fischer theorem states that for a sequence of complex numbers* $\{c_n\}_{n=-\infty}^{\infty}$, *there exists a complex-valued function* $f(\omega)$ *defined for real* $\omega$*'s belonging to the interval* $[-\pi, \pi]$ *such that* $f(\omega)=\sum_{j=-\infty}^{\infty} c_j e^{-i\omega j}$*. The function* $f(\omega)$ *is called the Fourier transform of the* $c_k$*. An important property of the Fourier transform is that it is an* isometric isomorphism *from* $l_2(-\infty, \infty)$ *to* $L_2\ [-\pi, \pi]$ *where* $l_2$*, is the space of square summable sequences* $\{x_k\}_{k=-\infty}^{\infty}$ *and* $L_2$*, the space of* square Lebesgue integrable functions. *Both are linear spaces. According to Sargent (1987), the Fourier transform "is a* one-to-one *transformation of points in* $l_2(-\infty, \infty)$ *into points in* $L_2\ [-\pi, \pi]$ *that preserves both linear structure (i.e. it is an* isomorphism*) and distance between "points" (i.e., it is an* isometric *mapping)". For more information on this subject, see Sargent (1987, chapter XI).*

$$y_t = c+\phi_1\, y_{t-1}+\phi_2\, y_{t-2}+\cdots+\phi_p\, y_{t-p}+\varepsilon_t+\theta_1\,\varepsilon_{t-1}+\theta_2\;\varepsilon_{t-2}+\cdots+\theta_q\varepsilon_{t-q} \tag{13}$$

is given by

$$s_y(\omega) = \frac{\sigma^2}{2\pi}\,\frac{(1+\theta_1 e^{-iw}+\theta_2 e^{-i2w}+\cdots+\theta_q e^{-iqw})}{(1+\theta_1 e^{-iw}+\theta_2 e^{-i2w}+\cdots+\theta_q e^{-ipw})} \tag{14}$$

The parameters of equation (13) can be estimated by the method of maximum likelihood (or by the two step least squares approach[12]) and the estimated values substituted in (14). But in our case, this equation is simplified because we are using the basic AR(p) process, which implies that all the $\theta$'s are null. A power spectrum is thus a representation of $s_y(\omega)$ as a function of the frequencies $\omega_1, \omega_2, \ldots \omega_M$ with $\omega_j = 2\pi j/n$ for a given time series of length $n$.

Figures 1a and 1b show the power spectrum for the sample of the observations used to estimate equation (1) using, an AR(12) and an AR(52), respectively.

## Figures 1a and 1b. **The Monday Anomaly**

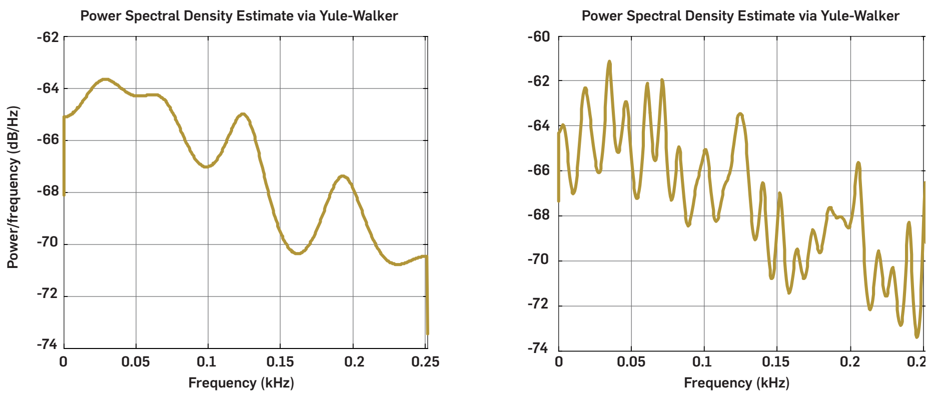



Table 3 displays the MATLAB®[13]commands to be run to obtain Figure 1a and 1b.

## Table 3. **MATLAB® Commands for Generating Figure 1**

```
>>load sp500-1970-1973.txt
>>hyulear=spectrum.yulear(12);
>>fs=503;
>>psd(hyulear,sp500_1970_1973, 'Fs', fs)
```

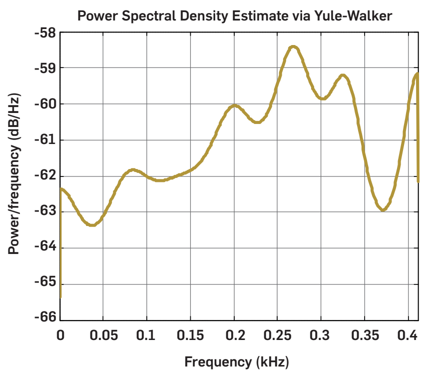
[12] *Gourieroux and Montfort (1990, pp. 228-229) describe a very simple method based on OLS that requires only two steps. Firstly, obtain the estimated residuals from applying OLS of $y_t$ on its lagged values: $\varepsilon_t = y_t - \sum_{j=1}^{p} \phi_j y_{t-j}$. Secondly, take the lagged values of these estimated residuals then to apply OLS on them and the lagged values of the $y_t$:*
$y_t = c+\phi_1 y_{t-1}+\phi_2\, y_{t-2}+\cdots+\phi_p\, y_{t-p}+\varepsilon_t+\theta_1\hat{\varepsilon}_{t-1}+\theta_2\,\hat{\varepsilon}_{t-2}+\cdots+\theta_{t-q}\hat{\varepsilon}_{t-q}$.

As it can be seen in Figure 1 (Figures 1a and 1b)[14], the smooth peak shown approximately at frequency 0.18/0.25 = 0.72[15] is probably an evidence of the Monday anomaly, since our regression analysis shows a significant coefficient precisely for that explanatory variable. Using the formula $\omega_j = 2\pi j/n$, we obtain the number of days at which a cycle might appear; that is: $n/j = 2\pi/\omega_j = 8.73$ where $\omega_j$ is approximately 0.72. This result can be interpreted as follows. At approximately every two stock markets effective weeks of five days (i.e. $2 \times 5 = 10 \approx 9$), there would be one Monday that shows a significant anomaly. From our regression analysis, we have found that there seems to be an anomaly on Monday. Spectral analysis would thus confirm an anomaly but for one Monday over three. By combining spectral analysis with our basic regression model, we are able to provide a more accurate picture of the behaviour of the presumed anomaly.

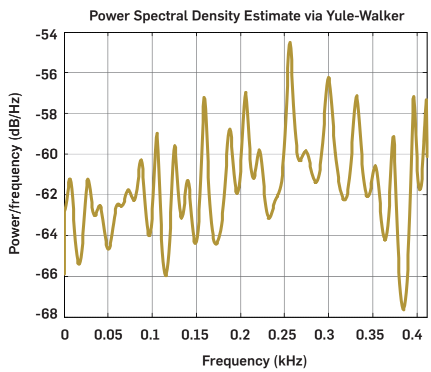
Figures 2a and 2b also show the power spectrum of the daily S&P500 using a different time period which range from January 2007 to April 2010.

Figures 2a and 2b. **Spectrum of the Daily S&P500 Returns January 2007 to April 2010**

**Estimation using an AR(12)**




**Estimation using an AR(52)**




What should be seen in Figures 2a and 2b, it is that spectral analysis seems to be able to capture the financial crisis that was recently raging in the U.S. A simple plot of the time series would show that there is a sine-wave with high amplitude starting in 2007 moving to mid 2009, which is not the Granger 'typical spectral shape' found in several economic time series.

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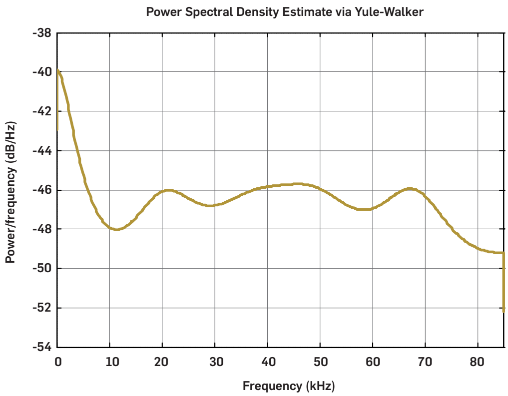
[13] *MATLAB® is a registered trademark of The MathWorks, Inc. Note that in MATLAB,* $\omega = 2\pi f/f_s$ *where* $f_s$ *is the sample size.*

[14] *It should be noted that EViews 7.0 also provide a simple procedure to perform spectral analysis. The power spectrum can be estimated by following the next three steps: 1) estimate an AR(p) process using Quick, Esitmate Equation; 2) in the estimated equation dialogue box, click on View and select ARMA Structure; 3) choose Frequency Spectrum with Graph selected in the Display option.*

[15] *We compute this ratio to obtain the relative frequency because we have specified in MATLAB the number of observations using* $fs = 503$. *This implies that the frequencies' axis is displayed in KHz. That is why we obtained* $0.25 = (503/2)/1000$. *Note that we have also used an AR(12) and an AR(52) to estimate our power spectrum, taking into account the fact that there is 12 months or 52 weeks in a year. When increasing the order of the AR(p), from* $p = 12$ *to* $p = 52$, *we observe a small shift in the spectrum (Figure 1a and Figure 1b), which might slightly change our conclusions.*

In order to help in establishing the stylized facts of that series of observations, we also performed the spectra of the S&P500 index using a different frequency; specifically, the monthly returns for the time period of January 1995 to February 2009. The result appears in Figure 3.

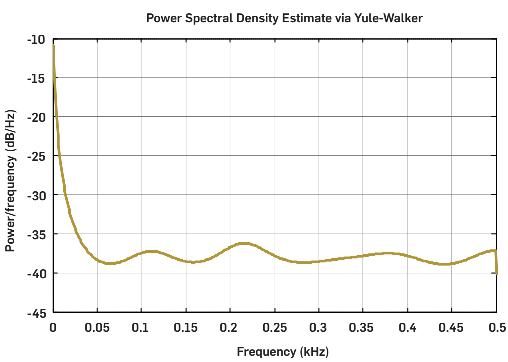
**Figure 3. Power Spectrum of the Montly Returns of the S&P500 January 1995 to February 2009. Spectrum Estimation using an AR(12)**




This figure displays much less slope changes in comparison to Figures 1a or 1b and it is very close to the strong white noise (as shown in the following figure). However, we can see a small hump at high frequency (approximately 69 Hz) that seems not to be significant but could have been related, if it was more pronounced, to the well-know January effect. We leave this interesting subject for further research.

The figures presented below are also used to help in establishing the stylized facts of our financial time series. The first of them (Figure 4) shows a stochastic process known as a strong white noise (Gourieroux and Jasiak, 2001), the second of them (Figure 5) shows the Gaussian white noise and the last one (Figure 6) represents the popular lognormal process used in most of the financial applications.

**Figure 4. Spectral Density of a Strong white Noise Series**



In Figure 4, we can see that the power spectrum of a strong white noise is linear with a null slope for all frequencies. We can consider this characteristic as reference in order to help in identifying the data generating process of financial time series; which could be assumed to be a strong white noise process in the presence of such spectrum. Thus, these financial time series might present the property of being unforecastable.

Figure 5. **Spectra of a Normal (0,1)**

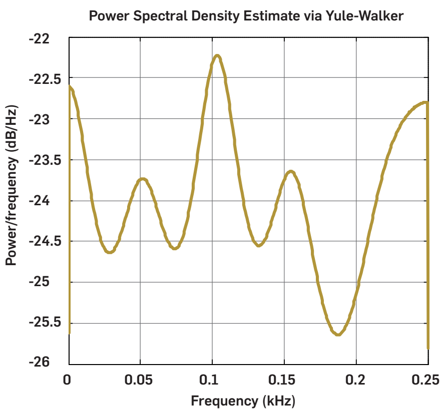
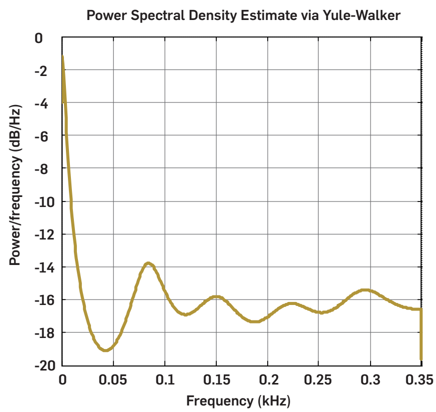



Figure 6. **Spectra of a Lognormal (0,1)**




Figures 5 and 6 represent the power spectrums of simulations with, a standard normal random number generator and a lognormal one, respectively. As seen in Figure 5, the spectrum of random number, generated from a normal random number generator, is quite wobbly and even showing some cycles. Since a more linear spectrum could have been expected, we could conclude that some statistical artifact might be detected by the PSD and that this aspect might be the result of an inappropriate random number generator. In fact, it is a simple question of scaling. The waves appearing in figure 5 are not significant at a two standard deviation level[16].

Upon a closer look at the method that is often used to generate normal random deviates, we see a nonlinear structure that might cause the generated variables to behave less smoothly than simple uniform random deviates. For instance, the Box-Muller (1958) transformation (Press *et al*., 1989 or Benninga, 2008) is a very efficient procedure often used to generate normal deviates based on uniform random variable generator. The following discussion gives a brief description of how to generate normal deviates based on $U(0,1)$ deviates. Assume that we want to generate two normal deviates $y_1$ and $y_2$ based on $x_1$ and $x_2$ .

*[16] Granger (1964, chapter 4) shows how to build a confidence interval and its chi-square test. He also provides a table to perform the test. The two standard deviations rule is a rough approximation that might reject peaks that are in fact significant.*

$$y_1=\sqrt{-2lnx_1}\; cos2\pi x_2 \qquad (15)$$
$$y_2=\sqrt{-2lnx_1}\; sin2\pi x_2 \qquad (16)$$

where $x_1$ and $x_2$ are two $U(0,1)$ deviates. As equation (15) and (16) show, the Box-Muller transform uses sine and cosine functions. These functions could explain the waves obtained in Figure 5 which are a priori unexpected. Finally, to obtain our normal deviates, based on (15) and (16), write $x_1$ and $x_2$ as a function of $y_1$ and $y_2$ and then apply a Jacobien transformation. For instance,

$$p(y_1)dy_1=\left|\frac{dx_1}{dy_1}\right| dy_1=\frac{1}{\sqrt{2\pi}}e^{-y_1^2/2}dy_1 \qquad (17)$$

this equation is the obtained probability density function (p.d.f.) of the well-known normal density based on $y_1$. Analogously to the Kuznets's transformation (Sargent, 1987), it is possible that the generated data could be some statistical artifact detected by the power spectrum. However, this is not the case here. In fact, it could be argued that this is a desirable property as it generates what is generally observed in financial time series.

In the same way, we might relate Figure 6 to the Kuznets's transformation due to the fact that it shows some similarities. Applying a Kuznets's transformation to a white noise process, one can generate a time series that shows large peaks at low frequencies and small peaks at high frequencies; hence, the time series under scrutiny would seem to be characterized by long swings. These swings might be statistical artifacts that are sometimes induced by the transformation and not a characteristic of the data. Here again, if we compute a two standard deviations band, the apparently large swings are not significant. It is just a question of scaling.

In addition, this power spectrum will be use as a base of comparison of any other type of financial time series. We provide the power spectrum as a base of comparison to a simulated Geometric Brownian Motion (GBM), since it is at the hearth of basic option pricing models (e.g. the Black and Scholes, 1973, formula's). The spectrum of this simulated stochastic process is shown in Figure 7.

In this figure, we can observe that the GBM generates lot of waves at different frequencies showing many cycles, resembling Figures 5 and 6. These cycles could also be tested using a two standard deviation band, the result might be that they are not significant. Note that the popularity of this type of modelling in applied finance is probably due to its ease of implementation. Moreover, it can be easily modified in order to account for other stylized facts like stochastic volatility, jumps, etc. These modifications of the GBM would consequently generate a power spectrum that might show some peaks at some frequencies that could correspond to the financial cycles observed in financial time series.

Figure 7. **Spectra of a Geometric Brownian Motion (GBM) – Equation (7)**

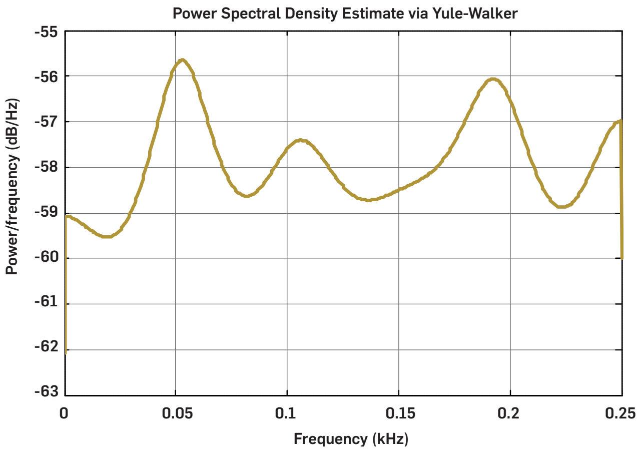



Figure 8. **Spectra of a Low-Frequency Model – Equation (8)**

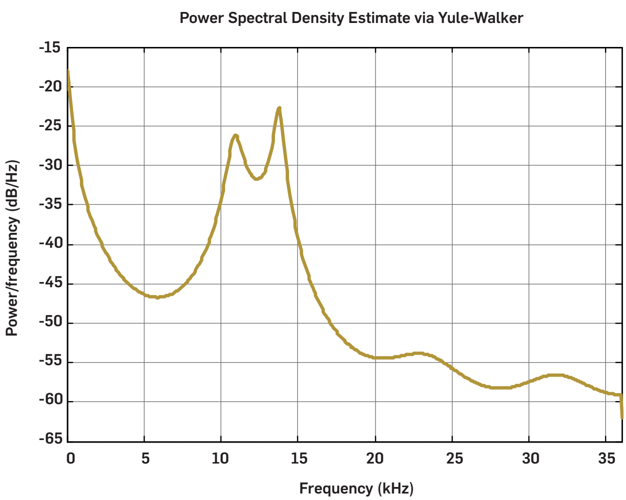
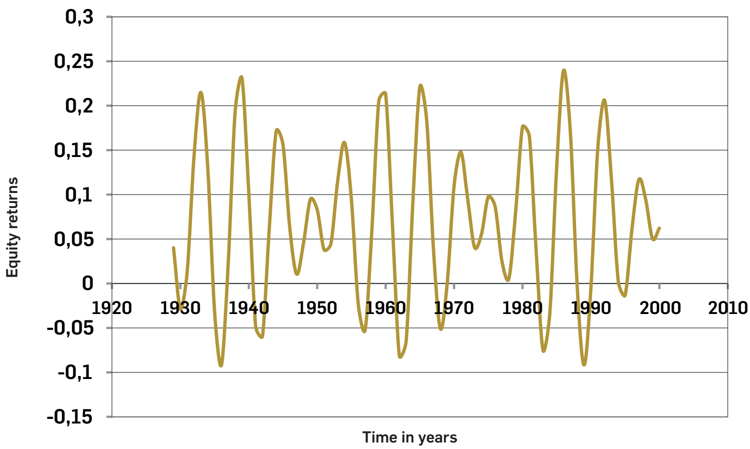



Figure 8 shows the power spectrum of equation (8). This exercise intends to help the financial practitioners to have an idea of the different shapes of spectrums that one can obtain from financial time series. This type of model generates returns with induced sine-waves, so the resulting spectrum should show some peaks at some frequencies, as we can observe in this figure.

The figure shows two pronounced peaks, one approximately at frequency 11 Hz and another one at 14 Hz. We can compute the numbers of years at which the cycle seems to appear using: ${}^{n}/_{j}={}^{2\pi}/_{\omega_j}=21$ where, $\omega_j$ is approximately 0.3 = 11/36 for the first peak and, ${}^{n}/_{j}={}^{2\pi}/_{\omega_j}=17$ where, $\omega_j$ is approximately 0.38 = 14/36 for the second one (36 =

72 observations divided by 2). This can be interpreted as follows. For the first peak, we obtain a cycle of approximately 21 years and for the second one, a cycle of approximately 17 years. The first of these cycles can be identified as one of the Kuznet's long wave (20 to 30 years) and, the second as one of the building cycle (15 to 20 years) (Granger, 1966). For this reason, we could conclude that the model proposed by Semmeler *et al*. (2009) generates some well-known stylized facts. Subsequently (see Figure 9), we observe similar patterns repeating themselves with the higher peaks at a higher frequency and then, the lower peaks at a lower frequency.

Figure 9. **Equity Returns Generated by Equation (8)**




In addition, Figure 9 shows the two peaks identified in Figure 8 which seem to be positioned at similar range of frequencies.

## 4. Conclusion

In this article, we try to show the usefulness of some of these techniques for the financial analysts and investment community by proceeding analogously as in Granger (1964, 1966) and Sargent (1987). We intent to establish the 'stylized facts' and the Granger 'typical spectral shape' using popular distributions as base of comparison to the standard financial time series. This exercise was conclusive. Consequently, by using the well-known Monday anomaly (Gibbons and Hess, 1981; Harris, 1986; Smirlock and Starks, 1986; Connolly, 1989; Racicot and Théoret, 2001; Giovanis, 2010), we could confirm that our result from a basic regression with dichotomous variables is in fact significant for the Monday anomaly even if the regression is in itself questionable. Thus, spectral analysis can be seen as supplementary tool for helping to confirm some results; for example, in our case, the presence of a cycle for a particular subsample of returns: the S&P500 index for the period 1970-1973. The January anomaly is another well-known stylized fact in the financial literature (Keim, 1983; Tinic and West, 1989; Maloney and Rogalski, 1989; Fama, 1991, Black, 1993). This anomaly could also be tested by the technique of spectral analysis using an approach similar to the one proposed in this paper. Furthermore, after studying the behaviour of the low-frequency components in equity returns (Semmler *et al.*, 2009), we found spectral analysis quite useful for detecting cycles in data generated by the estimated low-frequency model. Therefore, we conclude that this model is able to generate not only cycles of relevant frequencies, but also two cycles of different length.

Actually, some work has been done by researchers (Racicot and Théoret, 2008) to dynamize Jensen's alpha and beta, using the Kalman filter (Racicot and Théoret, 2007, 2010a) and the conditional models in the context of hedge fund returns. This work intends to improve the basic static models of returns frequently used by investment practitioners and academics to establish the performance of these funds. However, further research might be done on how to use spectral analysis to help identifying the cyclical behaviour of important performance and risk parameters, for instance, in the hedge funds industry.

Finally, another possible avenue of research could be based on the use of the *coherence* measure; which is the analogue of the correlation measure (e.g. Pearson or Spearman correlation coefficient) between pairs of financial time series. Indeed, coherence between hedge funds indices or coherence between the Gross Domestic Product (GDP) and hedge funds indices could be computed to help understanding the co-movements of these important indices. Investment practitioners, like portfolio managers who reallocate their portfolios based on sometime unreliable forecasts, could benefit from a better understanding of these measures in order to help establishing leading or lagging indicators of their financial time series.

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