

# Contagion between United States and European markets during the recent crises

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## Abstract

The main objective of this paper is to detect the existence of financial contagion between the North American and European markets during the recent crises. To accomplish this, the relationships between the US and the Euro zone stock markets are considered, taking the daily equity prices of the Standard and Poor's 500 as representative of the United States market and for the European market, the five most representative indexes. Time Series Factor Analysis (TSFA) procedure has allowed concentrating the information of the European indexes into a unique factor, which captures the underlying structure of the European return series. The relationship between the European factor and the US stock return series has been analyzed by means of the dynamic conditional correlation model (DCC). Once the DCC is estimated, the contagion between both markets is analyzed. Finally, in order to explain the sudden changes in dynamic US-EU correlation, a Markov switching model is fitted, using as input variables the macroeconomic ones associated with the monetary policies of the US as well as those related to uncertainty in the markets. The results show that there was contagion between the United States and European markets in the Subprime and Global Financial crises. The two-regime Markov switching model has helped to explain the variability of the pair-wise correlation. The first regime contains mostly the financially stable periods, and the dynamic correlations in this regime are explained by macroeconomic variables and other related with monetary policies in Europe and US. The second regime is explained mainly by the Federal Funds rate and the evolution of the Euro/US Exchange rate.

**Keywords:** Contagion, Dynamic Conditional Correlation, Financial Markets, Markov Switching Model, Time Series Factor Analysis, Macroeconomic variables.

**JEL classification:** C58, E44, G01, G15.

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## ■ 1. Introduction

The relationship between international financial markets is an issue of high interest that is related to the study of the correlation dynamics between markets. The last decade (2001-2010) is characterized by important facts in world economy and finance: the introduction of the European single currency, financial integration within the European Union, the increase in the price of raw materials and high inflation. But the Subprime Mortgage Crisis and the Global Financial Crisis have had a large effect on the relationship between the US market and most important European markets, e.g., Germany, France, United Kingdom, Spain and Italy.

The main goal of this paper is to study if there was financial contagion between the North American and European markets during the recent crises. However, when we analyze interrelationships across different financial markets it is necessary to distinguish between interdependence and contagion. There is now a reasonably large body of literature that attempts to distinguish the two. This literature has been reviewed by Dornbusch *et al.* (2000), Claessens *et al.* (2001), Pericoli and Sbracia (2003), Dungey *et al.* (2005), Bekaert *et al.* (2009) and Aslanidis *et al.* (2010). According to Forbes and Rigobon (2002), contagion is defined as a significant increase in cross-market comovements, while any continued market correlation at high levels is considered interdependency. Therefore, the existence of contagion must involve evidence of a dynamic increment in correlations.<sup>1</sup>

In this research, we first study the existence of common patterns in the return time series of European markets indexes<sup>2</sup>. Statistical methodology for reducing dimensionality allows us to capture the underlying structure of the European return time series. The empirical findings suggest one factor. Then, we analyze the relationship between the “European factor” and the US stock return time series by means of the dynamic conditional correlation model DCC-GARCH introduced by Engle (2002), which relaxes the excessive parameter constraints of the earlier GARCH models. Baele (2005) studies stock market integration between the U.S. market and European countries using a regime-switching GARCH model. Once the dynamic conditional correlation is estimated, we study the contagion in the crisis period. The methodology for testing contagion introduced by Chiang *et al.* (2007) corrects the problems of bias in the contagion test developed by Forbes and Rigobon (2002).

Finally, because of these sharp and unexpected correlation movements, the paper attempts to shed light on the macroeconomic variables that explain them, which in turn

<sup>1</sup> Allen and Gale (2001) and Pericoli and Sbracia (2003) provide an overview of the alternative definitions of contagion that have been proposed in the literature.

<sup>2</sup> Kim *et al.* (2005) investigate stock market integration among the European countries before and after the establishment of the European currency union.

leads to the use of Markov regime-switching (or simply regime-switching) models (Hamilton, 1989).

The remainder of this paper is organized as follows: Section 2 introduces the data. Section 3 analyses the methodology and evaluates the empirical findings. The paper concludes with a summary of the main results.

## ■ 2. Data

In order to study the relationships between the U.S. stock market and the Eurozone stock markets, we consider the daily equity index closing prices of the Standard and Poor's 500 (SP) for the North American market; and to represent the European market we use five daily stock indexes DAX (Germany), CAC40 (France), MIB30 (Italy), FTSE (United Kingdom) and IBEX35 (Spain)<sup>3</sup>. The sample period spans from December 31, 2001 to December 31, 2010.<sup>4</sup>

The evolution of daily time series indexes for the European market is plotted in Figure 1. All the time series exhibit the same behavior at different scales. In order to get a better comparison between indexes, all of them have been transformed, setting the index base of January 1, 2001 equal to 100. During this decade we can distinguish a first period of continuous decline from 2001 to 2003, followed by a rapid rise in prices until August 2007, which is the moment when the Subprime Crisis shakes the markets. Then, in 2008, the prices start a vertiginous fall through to 2009. Finally, the prices in the final year under study do not seem to follow any common pattern.

Following the usual practice, stock returns are calculated as first differences of the natural log of stock-price indexes, and they exhibit the typical features of financial time series. Table 1 shows descriptive statistics for the returns of the indexes. The unconditional correlation in Table 2 indicates a high correlation within the European index returns as well as between these and SP.

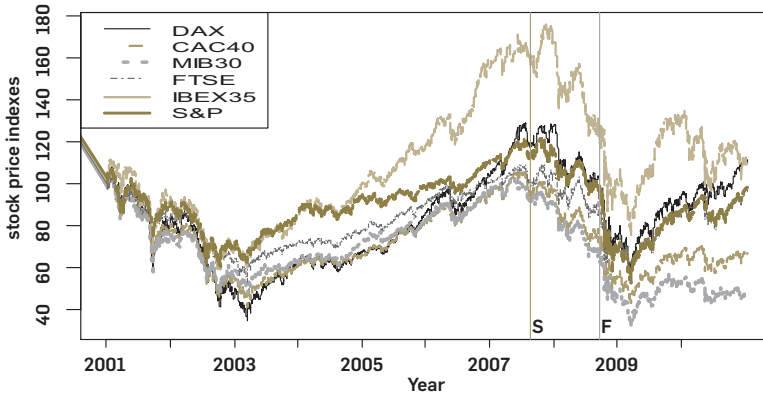
In order to find which variables are responsible for the changes in the estimated dynamical correlation, we consider a set of variables to explain the variation in the US-EU correlation. On the one hand we consider variables related to the monetary policies of the US, such as: the Federal Funds Rate (FFR), which in the United States is the interest rate at which private depository institutions (mostly banks) lend balances (federal funds) at the Federal Reserve to other depository institutions; the 10-Year Treasury Constant Ma-

<sup>3</sup> Although FTSE (UK) does not belong to the Eurozone, it has been considered in this group because the English market has a big influence in the continental markets.

<sup>4</sup> In case of national holidays in any country, the missing value is replaced by the last trading value.

turity Rate (DGS10); and the Euro/US exchange rate (EU.USD). On the other hand, we also consider variables related to uncertainty in the markets, such as the daily S&P Eurozone Government Bond Index return (SPGBI) and the Europe Brent spot price (Brent).

**Figure 1. Evolution of stock price indexes**



Note: The vertical solid lines are placed in the start of crisis periods. S stands for Subprime Crisis; and F stands for Global Financial crisis

**Table 1. Descriptive statistics for index returns**

	Mean	St. Dev	Skewness	Kurtosis	Minim	Maxim	Jarque-Bera test
SP	-0.001	1.349	-0.120**	8.675***	-9.470	10.957	8200.5***
DAX	0.004	1.628	0.058	4.708***	-7.433	10.797	2415.7***
CAC40	-0.016	1.562	0.082*	5.610***	-9.472	10.595	3430.5***
MIB30	-0.029	1.467	-0.143***	8.451***	-12.239	10.765	7785.6***
FTSE	-0.002	1.310	-0.102**	6.752***	-9.265	9.384	4969.9***
IBEX35	0.032	1.507	0.158***	6.983***	-9.586	13.483	5321.4***

Note: This table summarizes the descriptive statistics of the equity index returns. Statistics include mean, standard deviation, skewness, kurtosis, minimum, maximum and Jarque-Bera normality test. \* denotes statistical significance at the 10% level, \*\* at the 5% level and \*\*\* at the 1% level. The sample period includes 2601 observations.

**Table 2. Unconditional Correlation between stock index returns**

	SP	DAX	CAC40	MIB30	FTSE	IBEX
S.P	1					
DAX	<b>0.816</b>	1				
CAC40	<b>0.917</b>	<b>0.803</b>	1			
MIB30	<b>0.840</b>	0.546	<b>0.930</b>	1		
FTSE	<b>0.927</b>	<b>0.926</b>	<b>0.930</b>	<b>0.756</b>	1	
IBEX35	<b>0.842</b>	<b>0.902</b>	<b>0.771</b>	0.587	<b>0.871</b>	1

Note: This table shows unconditional correlations between stock index returns over the sample period

### ■ 3. Methodology and results

The clear interdependence among European markets, and the existence of the Economic and Monetary Union (EMU) in Europe, leads us to study the existence of common patterns in the returns time series of European markets indexes. In the first part of this section we use a Time Series Factor Analysis procedure to analyze common patterns in the returns time series. Next we estimate a dynamic conditional correlation model with a DCC-GARCH model to obtain the pair-wise correlations between factors and analyze the effect of the crises periods on conditional correlations, introducing a dummy variable for each crisis. In order to explain the movements in correlation, we introduce macroeconomic variables in a Markov regime-switching model.

#### 3.1. Time Series Factor Analysis (TSFA)

There are different methodologies for reducing dimensionality and capturing the underlying structure of the return time series. But the characteristics of the data, a set of multivariate series which exhibits serial correlation, implies that Principal Component Analysis (PCA) and Factor Analysis (FA) are not useful. In this case the alternative can be to use Dynamic Factor Analysis (DFA), introduced by Watson and Engle (1983), or Time Series Factor Analysis (TSFA), introduced by Gilbert and Meijer (2005), because the two methodologies allow observations to be dependent over time. The first methodology assumes a predetermined relationship between the factors at time  $t$  and the factors at time  $t-1$ . If this relationship is misspecified, the factors estimated by DFA can be biased. Whereas TSFA estimates a model for a time series with as few assumptions as possible about the dynamic process governing the factors, it does not particularly assume stationary covariance.

The relationship between the observed time series  $y_t$  ( $M$ -vector of length  $n$ ) and the unobserved factors  $\xi_t$  ( $k$ -vector with  $k \ll M$ ) is explained by the model:

$$y_t = \alpha_t + B\xi_t + e_t \quad (1)$$

where  $\alpha_t$  is the  $M$ -vector of intercept parameters,  $B$  is a  $M \times k$  matrix parameter of loadings and  $e_t$  is a random  $M$ -vector of measurement errors. The random vector  $e_t$  is assumed to be not correlated with the latent variable  $\xi_t$ .

In our case, and very often in the field of time series analysis, the log of stock price indexes  $y_t$  is an integrated time series of order 1. However, in order to achieve stationarity, stock returns are calculated as first differences of the natural log of stock-price indexes. In this case, defining  $D$  as the difference operator, (1) becomes:

$$Dy_t = (\alpha_t - \alpha_{t-1}) + B D\xi_t + De_t \quad (2)$$

Following the conditions assumed by Gilbert and Meijer (2005), this model may lead to consistent estimators obtained by maximum likelihood.

In order to choose the number of factors, the rule of thumb is that the number of factors should be equal to the number of eigenvalues that are larger than one. In this case the values of these eigenvalues are:

● **Table 3. Eigenvalues**

$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\lambda_5$
4.413	0.211	0.180	0.131	0.064

Thus, only one factor is considered. With the intention of reinforcing the selection of a one- or two-factor model, we introduce two statistics for measuring models with a varying number of factors (Wansbeek and Meijer (2000)): the Comparative Fit Index (CFI) and the Root Mean Square Error of Approximation (RMSEA).

Table 4 presents the loadings of the standardized solution for the two-factor and one-factor models, the communality estimations and the values of CFI and RMSEA. The Communality is the squared multiple correlation for the variables as dependent using the factors as a predictors, in other words, it is the proportion of variance of each return time series explained by the common factors. The values of the communality are very similar for one or two factors (see Table 4). The CFI is a pseudo- $R^2$ , based on  $\chi^2$  statistic that compares a model to the null model<sup>5</sup>. Its value is always between 0 and 1. A general rule is that CFI should be greater than 0.9 for the model containing all the factors, in our case for one-factor-model the CFI is 0.996.

● **Table 4. Factors obtained by TSFA**

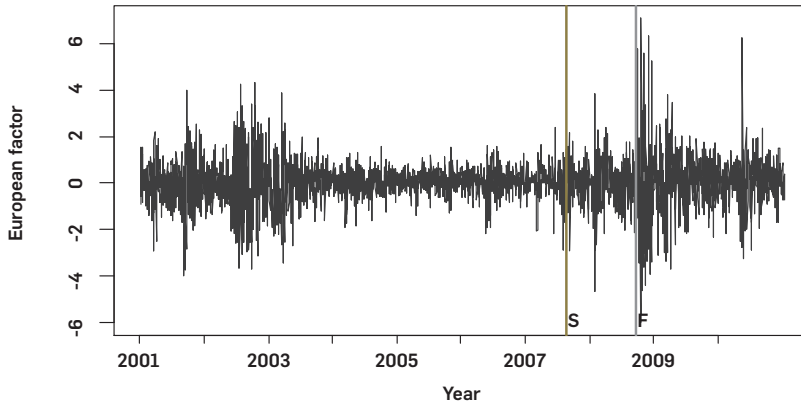
	Factor 1	Factor 2	Communality	Factor 1	Communality
DAX	1.472	0.070	<b>0.789</b>	1.450	<b>0.793</b>
CAC40	1.582	0.110	<b>0.976</b>	1.534	<b>0.964</b>
MIB30	1.167	-0.541	<b>0.995</b>	1.353	<b>0.850</b>
FTSE	1.225	0.082	<b>0.833</b>	1.198	<b>0.835</b>
IBEX35	1.336	-0.078	<b>0.825</b>	1.372	<b>0.829</b>
CFI	0.996	0.999			
RMSEA	0.076	0.037			

Note: This table shows the standardized loadings, CFI, RMSEA statistics for each factor and the communality of each stock market indexes return.

<sup>5</sup> In factor analysis, the usual null model is the same as the zero-factor model, i.e., the model that specifies that all observed variables are independently distributed.

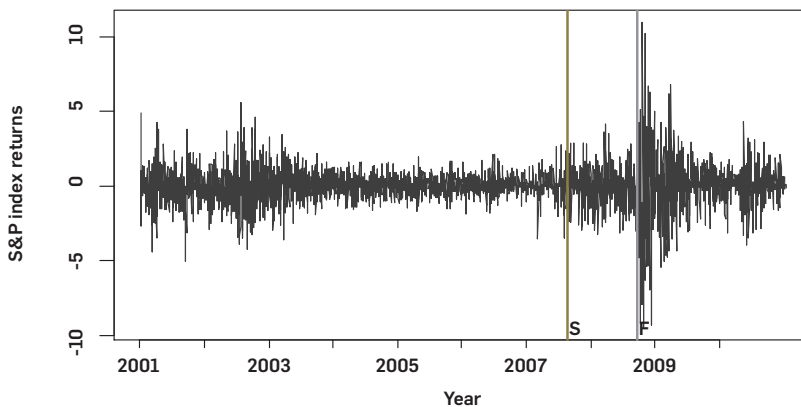
The RMSEA is a non-negative number, based on  $\chi^2$  statistic that measures the lack of fit per degree of freedom. Usually an RMSEA that is less than 0.05 for the model containing all the factors is considered a well-fitting model. Here, the one-factor model RMSA is equal to 0.076 and the two-factor model RMSEA is 0.037. Although the two-factor model RMSEA is 0.037, suggesting that this two-factor model could be better than only one, according to the previous analysis and for simplicity, we consider the convenience of the one-factor model. This point reinforces the idea of a global market for more developed European countries. Figures 2 and 3 show the evolution of daily return time series for the European factor and for S&P daily returns, high volatility is observed during crisis periods, especially during the Global Financial crisis.

**Figure 2. Evolution of Factor 1: European markets returns**



Note: The vertical solid lines are placed in the start of crisis periods. S stands for Subprime Crisis; and F stands for Global Financial crisis

**Figure 3. Evolution of S&P index returns**



Note: The vertical solid lines are placed in the start of crisis periods. S stands for Subprime Crisis; and F stands for Global Financial crisis

### 3.2. Dynamic conditional correlation

Before starting the topic of Dynamic Conditional Correlation (DCC\_GARCH), there is a question that concerns us: Did the crisis produce an increase in the mean and/or volatility of the indicators, European factor and SP returns? With the intention of solving that question, an autoregressive of order 1 for the mean equation with a GARCH(1,1) for volatility has been estimated for each indicator. Those models include two dummy variables that take the value 1 during the crisis period and 0 otherwise, one for the Subprime crisis ((8/15/2007 – 9/14/2007) and the other for the Global Financial crisis ((9/15/2008 – 10/14/2008)

The proposed models are described by Eq. (3) and Eq. (4):

$$y_{i,t} = \mu + \rho_1 y_{i,t-1} + \rho_2 y_{i,t-2} + g_{i,1} \text{cris1}_{i,t} + g_{i,2} \text{cris2}_{i,t} + \varepsilon_{i,t} \quad (3)$$

$$h_{i,t} = \alpha_0 + \alpha_1 \varepsilon_{i,t-1}^2 + b_1 h_{i,t-1} + e_{i,1} \text{cris1}_{i,t} + e_{i,2} \text{cris2}_{i,t} \quad (4)$$

where  $\varepsilon_{i,t} \sim N(0, \sigma)$ ,  $t=1, \dots, n$ .  $n$  is the sample size,  $i=1, 2$  refers to the European factor and SP returns, in that order;  $g_1$  and  $g_2$  to the Subprime and Financial crises, respectively. The estimation has been done using the *R* code described in Table 5.

● **Table 5. R code for a AR(2)-GARCH(1,1) model with dummy variables**

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#### Initialize parameters

---

```
ar2garch11_exo=function(par){
  N=length(rt)
  h=rep(1,N)
  at=rep(0,N)
  for (t in 3:N) {
    at[t]=rt[t]-mu-p1*rt[t-1]-p2*rt[t-2]-g1*cris1[t]-g2*cris2[t]
    h[t]=a0+a1*at[t-1]^2+b1*h[t-1]+e1*cris1[t]+e2*cris2[t]
  }
  eps=at/sqrt(h)
  garchln= -0.5*sum(log(h))-0.5*sum(eps^2)
  -garchln
}
```

---

#### Optimization

---

```
result=optim(par,ar2garch11_exo,method="CG",hessian=T,control=list(trace=3,maxit=2000))
result$par # parameters estimated
vc=solve(result$hessian) # var-cov matrix
se=sqrt(diag(vc)) # standard errors
result$par/se #t-values
```

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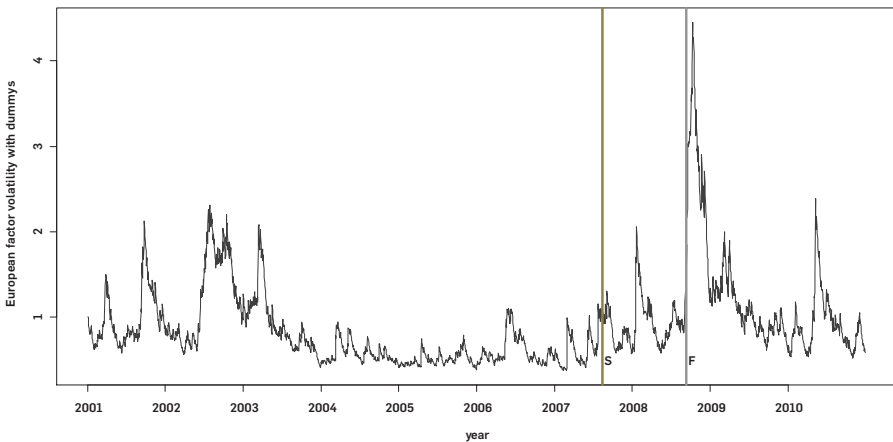
The results obtained from this procedure are in Table 6 and the estimated volatilities are in Figures 4 and 5.

● **Table 6. AR(2)-GARCH(1,1) estimation with Dummy variables**

		European factor returns		S&P returns	
		estimators	St. error	estimators	St. error
Mean equation:	$\mu$	0.039***	0.013	0.044***	0.017
	$p_1$	-0.044**	0.021	-0.060***	0.021
	$p_2$	-0.016	0.021	-0.049***	0.020
	$g_{i,1}$	-0.087	0.209	0.249	0.260
	$g_{i,2}$	-0.423	0.537	0.181	0.761
Variance equation:	$\alpha_0$	0.011***	0.002	0.011***	0.003
	$\alpha_1$	0.100**	0.012	0.062***	0.008
	$b_1$	0.887**	0.012	0.928***	0.009
	$e_{i,1}$	0.0265	0.039	0.026	0.046
	$e_{i,2}$	0.915**	0.404	0.980***	0.397
	$Q(20)$	20.333	15.233		
	$Q^2(20)$	24.060	31.37*		

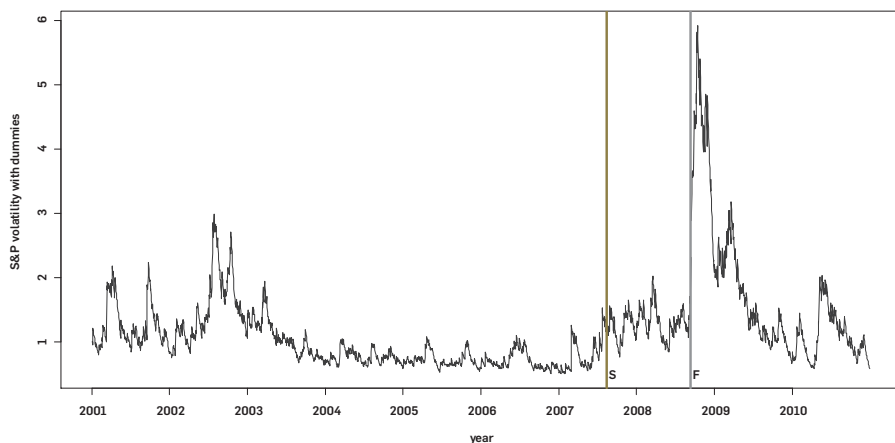
Note: The *t*-statistics are in parenthesis. \*\*\*, \*\* and \* denote statistical significance at the 1%, 5% and 10% level.  $Q(20)$  is the Ljung-Box statistic up to 20 days for testing the independency of the residuals and  $Q^2(20)$  is the Ljung-Box statistic up to 20 days for the squared residuals in order to test the heteroskedasticity of them.

■ **Figure 4. Volatility estimated for the European factor by means of the AR(2)-GARCH(1,1) with dummy variable for the crises**



Note: The vertical solid lines are placed in the start of crisis periods. S stands for Subprime Crisis; and F stands for Global Financial crisis

**Figure 5. Volatility estimated for the SP returns by means of the AR(2)-GARCH(1,1) with dummy variable for the crises**



Note: The vertical solid lines are placed in the start of crisis periods. S stands for Subprime Crisis; and F stands for Global Financial crisis

Results show that the model for the mean is different for the European factor than for the S&P returns series. The European factor mean equation exhibits the no significant parameter  $p_2$  while for the S&P returns series,  $p_2$  parameter is significant. The parameters associated with the volatility equation are highly significant for both indicators, showing the existence of volatility in both series. But, most importantly, the parameters related with the mean equation for the Subprime and Global financial crises ( $g_1$  and  $g_2$ ), are not significant in either of the two series whereas the parameters associated with the volatility equation ( $e_1$  and  $e_2$ ) are only significant for the Global financial crises in both series. They represent an increase of more than 0.9 in the estimated volatility in both cases. This result is reflected in the huge augment of volatility for both series around the time of this crisis (see Figures 4 and 5).

These previous findings reinforce our intuitive idea that there is contagion between both markets. We will test it by verifying whether there is a sudden increase in the dynamical correlation which coincides with the last two crises, using the methodology proposed by Chiang et al (2007). To accomplish this, a dynamic conditional correlation model with symmetric GARCH (DCC-GARCH) is estimated for obtaining the pair-wise correlations between S&P stock return indexes and the European factor.

The DCC-GARCH model, proposed by Engle (2002), estimates conditional variance and correlations in two steps. In the first step, a univariate GARCH model for each variable is estimated; the univariate variance estimates are subsequently introduced as inputs in the second step of the estimation process.

In order to capture the interrelations in mean and in variance across the different markets, an econometric VAR-GARCH model has been estimated for the European factor and SP returns. The selected order for the VAR model is 2. Table 7 displays the estimated VAR(2) model Eq. (5), together with the estimated GARCH(1,4) model of Eq. (7).

$$Y_t = \mu_t + d_1 Y_{t-1} + d_2 Y_{t-2} + \varepsilon_t \quad (5)$$

where  $Y_t$ ,  $Y_{t-1}$  and  $Y_{t-2}$  are matrices (2x1),  $d_i$ ,  $i=1,2$  matrices (2x2) and  $\varepsilon_t$  a matrix (2x1).

In this case  $Y_t = \begin{bmatrix} FACT\_EUR \\ SP\_D1 \end{bmatrix}_t$

and we can express it as:

$$\begin{aligned} Y_{1,t} &= \mu_1 + \sum_{p=1}^2 d_{11,p} Y_{1,t-p} + \sum_{p=1}^2 d_{12,p} Y_{2,t-p} + \varepsilon_{1,t} \\ Y_{2,t} &= \mu_2 + \sum_{p=1}^2 d_{21,p} Y_{1,t-p} + \sum_{p=1}^2 d_{22,p} Y_{2,t-p} + \varepsilon_{2,t} \end{aligned} \quad (6)$$

where  $FACT\_EUR$  is the European factor,  $SP\_D1$  is the SP return series,  $t=1, \dots, n$ ,  $i=1, 2$ , and  $\varepsilon_t | F_{t-1} \sim N(0, H_t)$ .  $F_t = \{Y_{i,1}, \dots, Y_{i,t-1}\}$  is the set of the observations of  $Y_i$  until time  $t-1$ .  $H_t$  is the conditional variance matrix.

The DCC-GARCH model allows us to obtain the conditional variance matrix as  $H_t = D_t R_t D_t$ , where  $R_t$  is the (2x2) time-varying correlations matrix and  $D_t$  is a (2x2) diagonal matrix of conditional standard deviations  $\sqrt{h_{ii,t}}$ . In other words:

$$H_t = \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 \\ \cdot & \sigma_2^2 \end{bmatrix}_t, R_t = \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix}_t \text{ and } D_t = \begin{bmatrix} \sqrt{h_{11}} & 0 \\ 0 & \sqrt{h_{22}} \end{bmatrix}_t$$

or equivalently,

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 \\ \cdot & \sigma_2^2 \end{bmatrix}_t = \begin{bmatrix} \sqrt{h_{11}} & 0 \\ 0 & \sqrt{h_{22}} \end{bmatrix}_t \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix}_t \begin{bmatrix} \sqrt{h_{11}} & 0 \\ 0 & \sqrt{h_{22}} \end{bmatrix}_t$$

where  $h_{ii,t}$ ,  $i=1,2$ , follows an univariate GARCH(1,4) defined as:

$$h_{ii,t} = C_i + \sum_{q=1}^4 A_{iq} \varepsilon_{i,t-q}^2 + B_i h_{ii,t-1} \quad (7)$$

and the elements of  $H_t$  are:

$$H_{ij,t} = \sqrt{h_{ii,t}} h_{jj,t} \rho_{ij}, \quad i, j=1,2$$

and

$$\rho_{ii} = 1, \quad i=1,2$$

In the second stage, the vector of the standardized residuals,  $u_{i,t} = \varepsilon_{i,t} / \sqrt{h_{ii,t}}$  is employed to develop the DCC correlation specification:

$$Q_t = (1 - \alpha - \beta) \bar{Q} + \alpha u_{t-1} u'_{t-1} + \beta Q_{t-1} \quad (8)$$

and

$$R_t = (\text{diag}(Q_t))^{-1/2} Q_t (\text{diag}(Q_t))^{-1/2} \quad (9)$$

where  $\bar{Q} = E[u_t u'_t]$  is the unconditional covariance of the standardized residuals.  $Q_t = (q_{ij,t})$  is the time-varying covariance matrix of the standardized residuals.

In Eq. (8),  $\alpha$  and  $\beta$  are scalar parameters,  $u_t = D_t^{-1} \varepsilon_t$  is the standardized residual matrix and  $\bar{Q}_t$  is the unconditional covariance matrix of  $u_t$ . The parameters  $\alpha$  and  $\beta$  capture the effects of previous shocks and previous dynamic conditional correlations on current dynamic conditional correlations.

The correlation estimators of Eq. (9) are, in general, of the form:

$$\rho_{ij,t} = q_{ij,t} / \sqrt{q_{ii,t} q_{jj,t}}, \quad i, j = 1, 2, \dots, n, \text{ and } i \neq j \quad (10)$$

The time-varying correlation coefficient for a bivariate case can be written as:

$$\rho_{ij,t} = \frac{(1 - \alpha - \beta) \bar{q}_{ij} + \alpha u_{i,t-1} u_{j,t-1} + \beta q_{ij,t-1}}{\sqrt{[(1 - \alpha - \beta) \bar{q}_{ii} + \alpha u_{i,t-1}^2 + \beta q_{ii,t-1}] [(1 - \alpha - \beta) \bar{q}_{jj} + \alpha u_{j,t-1}^2 + \beta q_{jj,t-1}]}} \quad (11)$$

The DCC model is estimated by maximization of the following log-likelihood function:

$$L = -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + 2 \log|D_t| + \log|R_t| + u'_t R_t^{-1} u_t) \quad (12)$$

The results of applying the DCC-GARCH model are reported in Table 7. The results show that the whole parameters are significant. Ljung-Box statistics for the residuals and for squared residuals prove that both do not exhibit serial correlation.

## Table 7. DCC Estimation Results

### Mean equation

$$\begin{bmatrix} FACT\_EUR \\ SP\_D1 \end{bmatrix}_t = \begin{bmatrix} 0.039^{***} \\ (0.010) \\ 0.065^{***} \\ (0.013) \end{bmatrix}_t + \begin{bmatrix} -0.271^{***} & 0.292^{***} \\ (0.019) & (0.014) \\ 0.097^{***} & -0.109^{***} \\ (0.026) & (0.018) \end{bmatrix} \begin{bmatrix} FACT\_EUR \\ SP\_D1 \end{bmatrix}_{t-1} + \begin{bmatrix} -0.084^{***} & 0.079^{***} \\ (0.018) & (0.014) \\ 0.056^{**} & -0.090^{***} \\ (0.026) & (0.020) \end{bmatrix} \begin{bmatrix} FACT\_EUR \\ SP\_D1 \end{bmatrix}_{t-2} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}_t$$

In parenthesis, the standard deviation

(\*\*), (\*\*\*): significant at 0.05 and at 0.01 level respectively

## Volatility equation

Variables	Constant	ARCH estimators				GARCH estimator
	C	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	B
FACT_EUR	0.020*** (0.002)	0.021*** (0.005)	0.043*** (0.009)	0.058*** (0.006)	0.053** 0.013	0.803*** (0.010)
SP_D1	0.022*** (0.003)	0.009 (0.007)	0.025*** (0.009)	0.024*** (0.007)	0.055*** (0.006)	0.869*** 0.006

In parenthesis, the standard deviation

(\*\*), (\*\*\*): Mean significant at 0.05 and at 0.01 level respectively

## Conditional correlation equation

Conditional correlation equation	
$\alpha$	$\beta$
0.018*** (0.004)	0.977*** (0.005)

In parenthesis, the standard deviation

(\*\*\*): Means significant at 0.01 level

### 3.4. The effect of crises on the dynamics of conditional correlations

To assess the effect of the crises periods on the dynamics of conditional correlations, we introduce two dummy variables, one for each crisis. According to our definition (Muñoz et al. 2010) there is contagion between markets when the dummy variable is both significant and positive in the mean and/or variance of the pair-wise correlation coefficients. Thus, contagion exists when pair-wise correlations increase during crisis times relative to correlations during peaceful times and/or they are more volatile.

Crisis variables are defined as dummy variables, indicators that take the value 1 during the crisis period and 0 otherwise. Crisis<sub>k</sub> for k=1, 2 is a dummy variable for the Sub-prime crisis (8/15/2007–9/14/2007) and the Global Financial crisis (9/15/2008 – 10/14/2008), respectively. The applied equations system is described as:

$$\rho_{ij,t} = \mu + \phi_1 \rho_{ij,t-1} + \sum_{k=1,2} \alpha_k \text{Crisis}_{k,t} + \epsilon_{ij,t} \quad (13)$$

$$h_{ij,t} = \omega_0 + \omega_1 \epsilon_{ij,t-1} + \beta_1 h_{ij,t-1} + \sum_{k=1,2} \delta_k \text{Crisis}_{k,t} \quad (14)$$

where  $i, j=1, 2$  and  $t=1, \dots, n$

A significant estimated coefficient for the dummy variable will be interpreted as a structural change in the mean and/or variance that produces a shift in the mean and/or vari-

ance of the conditional correlation. The order  $p=1$ , in Eq. (13), has been chosen by means of the AIC criterion. This analysis will enable us to detect if conditional correlations are different and/or more volatile before, during or after the crises.

Table 8 shows the tests for detecting changes in correlations. We observe that the goodness of fit is quite correct as indicated by the values of the Ljung-Box  $Q(20)$  and  $Q^2(20)$ . The coefficients associated with the GARCH(1,1) model, for the variance equation, are all significant, indicating that it is necessary to correct the dynamic correlations by heteroskedasticity. When we look at the coefficients related to the crisis variables, we can conclude that during the Subprime Crisis the correlation only increased in level but not in volatility. The Global Financial crisis increases both: the level of correlation as well as the volatility of the pair-wise correlations between American Markets and European Markets.

**Table 8. Contagion detection using Dynamic Conditional Correlation**

	Mean Equation				Variance Equation		
	$\mu$	$\phi_1$	$\alpha_k$	$\varpi_0$	$\varpi_1$	$\beta_1$	$\delta_k$
Crisis	0.005***	0.992***	0.007*	0.000***	0.139***	0.803***	-0.000
1	(0.001)	(0.002)	(0.004)	(0.005)	(0.008)	(0.009)	(0.000)
Crisis			0.009*				0.0001**
2			(0.005)				(0.000)

$Q(20) = 34.962^{**}$

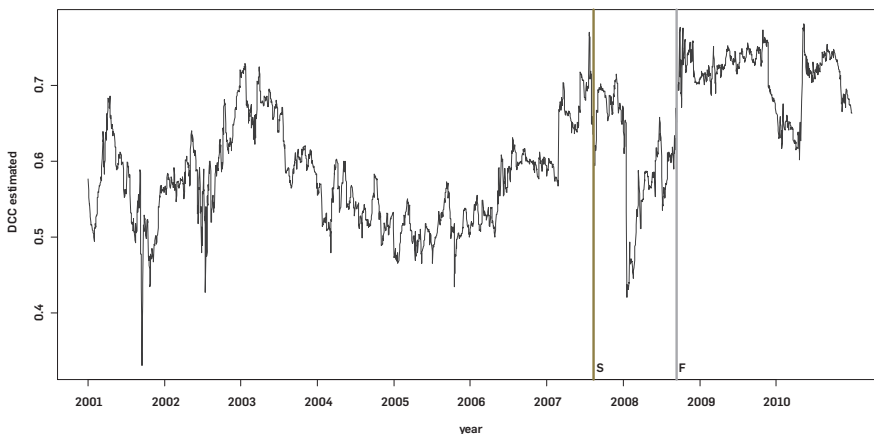
$Q^2(20) = 3.888$

$Q(20)$  is the Ljung-Box statistic up to 20 days for testing the independency of the residuals and  $Q^2(20)$  is the Ljung-Box statistic up to 20 days for the squared residuals in order to test the heteroskedasticity of them.

In parenthesis, the standard deviation

(\*), (\*\*) and (\*\*\*): Mean significant at 0.1, 0.05, and 0.01 level respectively.

**Figure 6. DCC representation with crisis indicators**



Note: The vertical solid lines are placed in the start of crisis periods. S stands for Subprime Crisis; and F stands for Global Financial crisis

### 3.5. Regression with Markov Switching Model

At this point, the relationships between the dynamical correlation estimated in the previous point and the macroeconomics variables (Federal Funds Rates (FFR), the 10-Year Treasury Constant Maturity Rate (DGS10), and Euro/US exchange rate (EU.USD), the daily S&P Eurozone Government Bond Index return (SPGBI), and the Europe Brent spot price (Brent) have been studied. The aim of this is to detect if changes in the DCC have been produced by changes in the macroeconomic indicators. One of the suitable procedures for detecting them is to apply a Markov switching model.

Markov switching models have been introduced by Hamilton (1989) for detecting the periods of recession and growth in the US economy, exploring the changes in the GDP. Fontdecaba *et al.* (2009) applied this methodology to the electricity prices and Sanchez *et al.* (2009) have developed a library on the *R* package for estimating this kind of model.

Regression Markov switching models are appropriated for finding out the cause of jumps in a time series, as for example our estimated DCC. Those models would allow us to break the series down into several states or regimes, characterized by different underlying processes. A jump in the response variable, in our case the estimated DCC, can be considered as the moment in which the series switches from one regime to another. This causes us to assume that the DCC series is influenced by a non-observable random variable  $S_p$ , called *state* or *regime*. If  $S_t=1$ , the process (our DCC) is in regime 1 while if  $S_t=2$ , the process is in regime 2. Those regimens will be characterized by the influence of the explicative variables. The next step is to calculate the probability of being in the same state at time  $t+1$  or of changing to the other.

The best assumption for this case is to assume that the process is Markov in the sense that the state at time  $t$ ,  $S_p$ , depends on the past only through the most recent value of the state  $S_{t-1}$ . So that the transition probability can be defined by  $P(S_t=j | S_{t-1}=i)=p_{ij}$  ( $i,j=1,2$ ).

It is useful to pick the transition probabilities up in the transition matrix  $P$ :

$$P = \begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}$$

Thus, depending on the selected explicative variables, the model can be written as:

$$y_t = \begin{cases} \beta_0^{(1)} + \beta_1^{(1)}X_{1,t} + \dots + \beta_k^{(1)}X_{k,t} + \beta_{k+1}X_{k+1,t} + \dots + \beta_jX_{j,t} + \varepsilon_{t,1} & S_t=1 \\ \beta_0^{(2)} + \beta_1^{(2)}X_{1,t} + \dots + \beta_k^{(2)}X_{k,t} + \beta_{k+1}X_{k+1,t} + \dots + \beta_jX_{j,t} + \varepsilon_{t,2} & S_t=2 \end{cases}$$

where the variables with parameters  $\beta_{i,t}^{(j)}$ ,  $i=1,2$ , are variables with switching effect, while variables with parameters  $\beta_{m,t}$  are variables without switching effect.  $y_t$  is the observation of the DCC estimated at time  $t$ , and the parameters of the model to es-

estimate are: the deviations of the states:  $\sigma^{(1)}, \sigma^{(2)}$ ; the coefficients with switching effect:  $\beta_0^{(1)}, \dots, \beta_k^{(1)}; \beta_0^{(2)}, \dots, \beta_k^{(2)}$ ; the regression coefficients without switching effect:  $\beta_{k+1}, \dots, \beta_j$  and the transition probabilities:  $p_{11}$  and  $p_{22}$ .

The parameters' estimation has been carried out by maximizing the likelihood function with the estimation algorithm EM (Dempster *et al.*, 1977). This algorithm alternates two steps:

- Expectation Step (E): The expectation of the non-observable variables is calculated from the pre-fixed parameters. In this case the hidden variables are the states.
- Maximization Step (M): the likelihood function obtained in the E-step is maximized by means of an optimization routine, assuming that the states are known from E-step. In other words, the maximization has been performed conditional to the S state. In this case, the functional dependence between the response variable  $y$  and the explicative variables  $X$  correspond to a linear model and the estimation has been carried out by means of Ordinary Least Squares (OLS). Conditional to the S state means a different set of parameters for each state. Transition probabilities are estimated from the observed change frequencies.
- Steps E and M are repeated until convergence.

### Markov switching estimation results

As the Global financial crisis started in the US, our proposal is to find the mainly US macroeconomic indicators that can explain the changes in the DCC as well as its evolution. First of all, a linear regression between DCC and the US macroeconomic indicators is estimated, in order to obtain preliminary estimation values as initial values in the estimation of the Markov switching regression. The results are in Table 9.

● **Table 9. Linear regression estimation**

	estimators	St. error
Intercept	5.86e-01***	2.83e-02
FFR	-2.20e-03*	1.09e-03
DGS10	-2.93e-02***	3.66e-03
EU.USD	-1.21e-01***	1.42e-02
Brent	6.88e-05	9.21e-05
SPGBI	2.03e-03***	1.39e-04
R <sup>2</sup>	0.346	
Residual St.error	0.066	

Note: \*\*\*, \*\* and \* denote statistical significance at the 1%, 5% and 10% level.



All of the estimated values are significant except for those associated with the Brent variable, and the  $R^2$  is equal to 0.3462. The analysis of residuals shows the convenience of a model which captures the changes in DCC evolution. In order to improve the model, we consider a two-regime Markov Switching Model<sup>6</sup>. The estimated parameters are in Table 10.

● **Table 10. Markov switching estimation**

	Regime 1		Regime 2	
	estimators	St. error	estimators	St. error
intercept	0.444***	0.029	0.559***	0.023
FFR	-0.033***	0.001	-0.019***	0.001
DGS10	0.026***	0.004	0.003*	0.001
EU.USD	-0.146***	0.011	0.125***	0.011
Brent	0.001***	0.000	0.000*	0.000
SPGBI	0.134***	0.010	-0.001	0.027
$R^2$	0.615		0.708	
Residual st. error	0.034		0.033	

Note: \*\*\*, \*\* and \* denote statistical significance at the 1%, 5% and 10% level.

Table 11 shows that the probability of remaining in Regime 1 at time  $t$ , given that the DCC is in Regime 1 at time  $t-1$ , is very high, and the same is true for Regime 2. The probability of remaining in the first state is 0.995 and 0.996 in the second one, showing that each regime is very persistent. This indicates that DCC exhibits a small number of break points, many of them explained by the movements of some macroeconomic indicators, as we will see below.

● **Table 11. Transition probabilities**

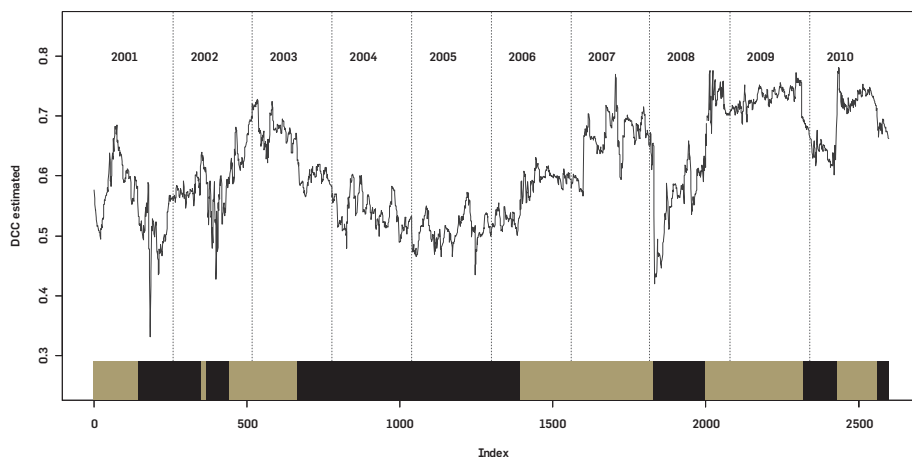
	Regime 1	Regime 2
Regime 1	0.995	0.004
Regime 2	0.004	0.996

Regime 1 is associated, in general, with lower values of the correlation while the highest correlations are in Regime 2. The Subprime crisis and the Global Financial Crisis are located in the Regime 2. There is a switch from Regime 1 (low correlation) to the Regime 2 (high correlation) at the exact moment when the Global Financial crisis

<sup>6</sup>We also considered a Markov switching model with three states but the model was highly instable.

starts (observation number 2000). These changes of regimes support the idea of discontinuities in the volatility propagation mechanisms (Billio and Caporin 2005).

**Figure 7. DCC representation and Markov switching regimes**

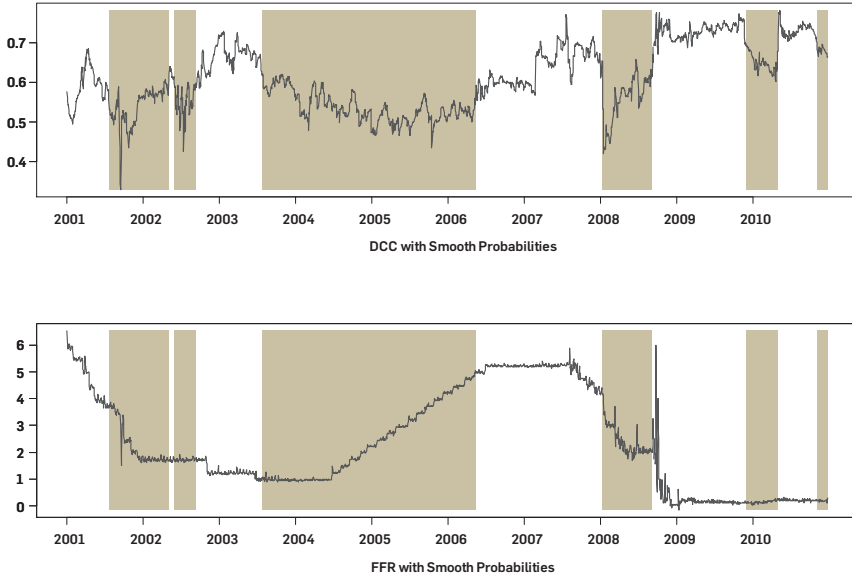


Note: Regime 1 in black and Regime 2 in brown

Regarding the relationships between the estimated DCC and the macroeconomic indicators, we point out that DCC follows approximately the same evolution as Federal Funds Rates (FFR) until the beginning of 2008, but with an inverse relationship because the regression coefficient is negative. Boivin and Giannoni (2007) indicates that a change in the Federal Funds rate have a smaller impact on the US economy now than it used to. There is a huge increase in FFR, coinciding with the start of the Global Financial crisis and, after that, it decreases suddenly while the dynamic correlation between Europe and the US reaches the highest levels. Regime 1 mostly picks up the largest discrepancies between DCC and FFR, whereas Regime 2 is associated with lower correlation between DCC and FFR.

10-year Treasury constant maturity rate (DGS10) is a measure of risk and, according to Nippani and Smith (2010), it is not viewed by the market as “default risk-free,” especially during the financial crisis. This could be the reason why this indicator is generally constant throughout the study period, with the exception of sudden drops in the middle of 2007 and at the end of 2008, which were perhaps caused by the Lehman Brothers and the Global Financial crisis, respectively. The regression coefficient between DCC and DGS10 is positive for the first regime in the Markov switching model and approximately 10 times higher than in the second regime. This could be explained by the fact that the first regime contains mostly stable financial stable periods. The dynamical correlations between Europe and US in those periods are not generally the highest.

**Figure 8. DCC representation and FFR (Federal Funds Rate) with Markov switching regimes**



Note: Regime 1 in brown and Regime 2 in white

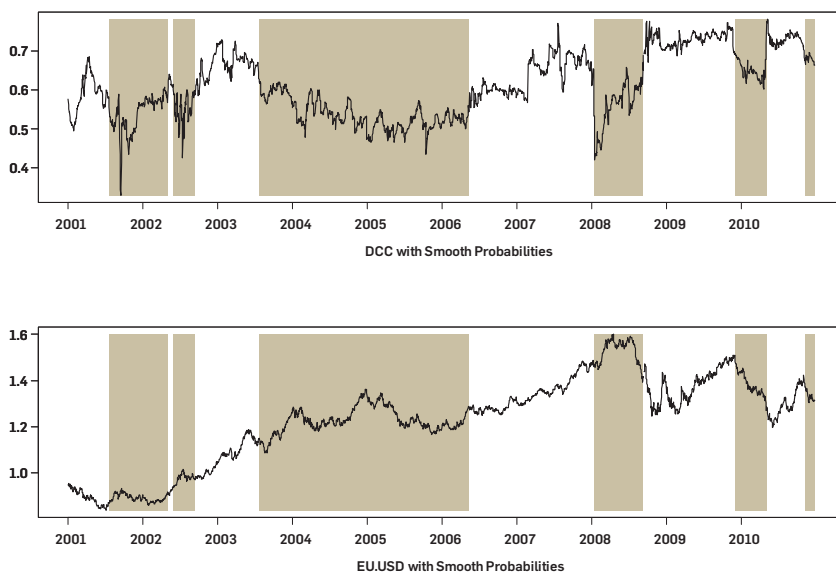
**Figure 9. DCC representation and DGS10 (10-Year Treasury Constant Maturity Rate) with Markov switching regimes**



Note: Regime 1 in brown and Regime 2 in white

The other three series –the S&P Eurozone Government Bond Index Prices (SPGBI), the European Brent Spot Price (Brent) and the evolution of the Euro/US Exchange rate (EU.USD)- exhibit a tendency of almost always increasing, with the exception of the first period (2001-2002) for the Brent and the EU.USD. In terms of the EU.USD, this increasing tendency shows the strength of the Euro against the US Dollar and therefore the power of the European economy against the North American economy. In the fitted Markov switching model, regime 1 picks up periods that the EU.USD basically increases and the dynamical correlation decreases, or vice versa, while both variables in regime 2 follow the same direction increase/increase or decrease/decrease. One possible explanation is that, in periods with financial stability (Regime 1), a high Euro value against the US Dollar supposes greater security in the European market, thus a lower dependency on the North American market; therefore, the DCC decreases. By contrast, in periods of crisis, the value of the Euro decreases in comparison with the US Dollar, because European markets follow the U.S. market, which gives them greater security.

**Figure 10. DCC representation and EU.USD (Euro/US exchange rate) with Markov switching regimes**



Note: Regime 1 in brown and Regime 2 in white

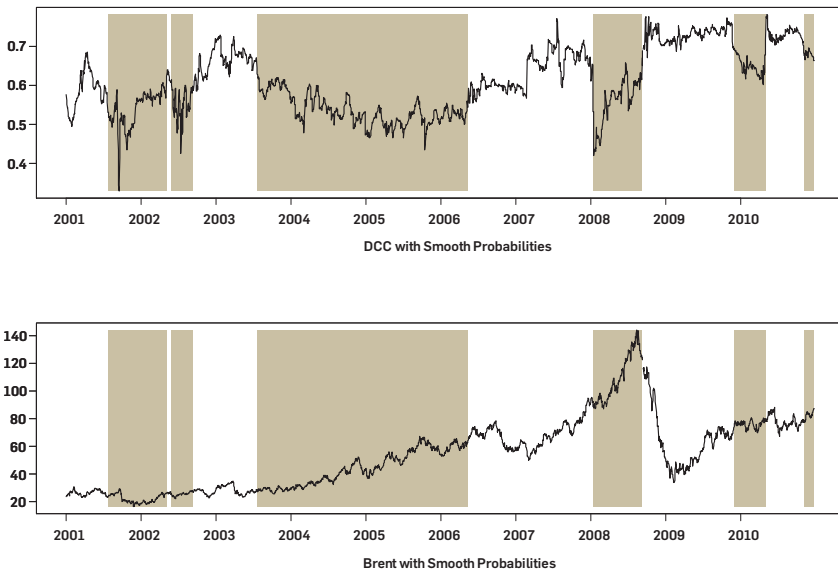
Brent displays a huge volatility, starting with the global financial crisis (2008) and finishing at the end of 2009. Additionally Rahman and Serletis (2010) say that oil prices are one of the major determinants of the economy in the United States, reducing the growth of oil prices more in high volatility periods than in low volatility periods. This could be an explanation of why the coefficient of the Markov switching regression is much higher in regime 1 (0.0010) than in regime 2 (0.0002). Remember that regime

2 contains the Global financial crisis and there was a breakpoint for the Brent variable associated with the starting point of the Global Financial crisis (Muñoz and Dickey, 2010). The different oil price shocks are considered to be possible causes of the economic crises (Hamilton, 2009). In fact, in July 2008, the European Brent Spot Price reached a record high of 143.95 Dollars per Barrel, which was followed two months later by the beginning of the Global Financial Crisis.

In addition, the Brent and EU/ USD have related patterns and we can observe a growing tendency, interrupted only in the period from August, 2008 until February, 2009. We can observe a high volatility for SPGBI.

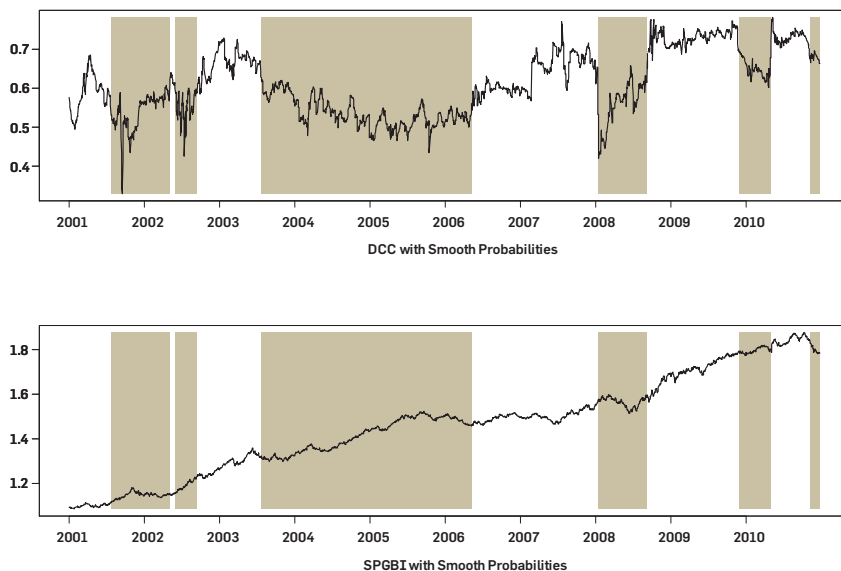
The same reasoning that was applied to the Brent could be applied to the SPGBI, but more notable, because the coefficient for regime 1 is positive. However, it is not significant for regime 2, showing that there is a strong relationship between SPGBI in regime 1 but not in regime 2. Regime 1 contains mostly the financially stable periods, so in this situation markets and fixed income go in the same direction.

**Figure 11. DCC representation and the Brent (Europe Brent Spot price) with Markov switching regimes**



Note: Regime 1 in brown and Regime 2 in white

**Figure 12. DCC representation and the SPGBI (S&P Eurozone Government Bond Index return) with Markov switching regimes**



Note: Regime 1 in brown and Regime 2 in white

## 4. Conclusions

This paper analyzes the relationship between the US and European Markets in the last decade (2001-2010) and the existence of contagion in crises periods. The results of the TSFA procedure reveal a significant relationship among the analyzed European stock markets (Germany, France, United Kingdom, Spain and Italy) and therefore it leads as to group them into one factor. The estimation of the conditional correlations between the S&P stock return indices and the European return factor shows empirically the existence of contagion in the Subprime and Global Financial Crisis. The results further reveal that, because the contagion effects are different in each case the correlation increased its level and its volatility. However, during the Subprime Crisis the correlation increased only in its level. This concurs with the opinion of Bartram and Bonard (2009): “the current financial crisis differs from many of the previously studied crises in that it is both, severe and global”. Finally, a two-regime Markov Switching Model allows as to explain the variability of the pair-wise correlation by means of introducing macroeconomic variables, thus providing further evidence of the existence of changes in the correlation dynamics. The first regime contains mostly the financially stable periods. The dynamical correlations between the European and US markets are explained by the Federal Funds rate (FFR), 10-year Treasury constant

maturity rate (DGS10), S&P Eurozone Government Bond Index Prices (SPGBI), the European Brent Spot Price (Brent) and the evolution of the Euro/US Exchange rate (EU.USD). However, the second regime is explained mainly by the Federal Funds rate (FFR) and the evolution of the Euro/US Exchange rate (EU.USD).

In conclusion, this research has provided evidence for the existence of contagion during the Subprime and the Global Financial Crisis between the US and European stock markets. This implies that the benefits of risk diversification diminish during crisis periods. The Markov switching model identified two regimes, and allows us an economic interpretation of each regime, which is explained through macroeconomic variables and others related with monetary policies in Europe and the US. The estimated Markov switching model supports the discontinuity in the dynamic of the correlation which supposes discontinuity in the volatility propagation mechanisms between markets.

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