Conservation of momentum – walking the boat

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Abstract

We present an apparatus for checking the law of conservation of momentum in the student lab. We can easily construct it at a low cost and it yields good results.

Keywords: Conservation of momentum, Frictional force.

Resumen

Se presenta un aparato para revisar la ley de conservación del momento en el laboratorio para estudiantes. Es fácil de construir a bajo costo y produce buenos resultados.

Palabras clave: Conservación de momento, Fuerza de fricción.

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I. INTRODUCCIÓN

Many texts [1, 2, 3] contain problems similar to the following:

A man of mass 60 kg is standing on a flat boat at a point 10 m from shore as shown in Fig.1.

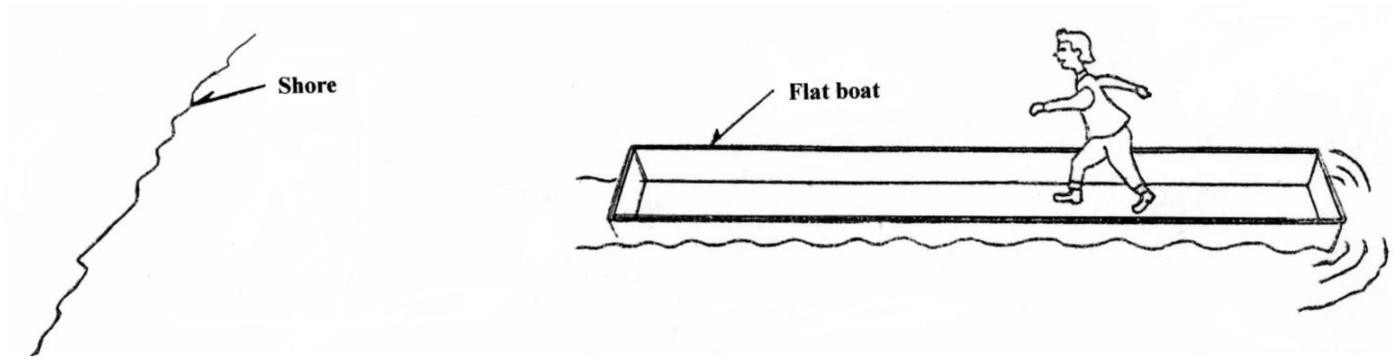

FIGURE 1. If a man walks on a boat toward shore, the boat moves in opposite direction.

He walks 2.4 m relative to the boat toward shore and then stops. If the mass of the boat is 300 kg and it is assumed there is no friction between the boat and the water, how far is the man from shore when he stops?

Conservation of momentum predicts that the center of mass of the boat-man system does not move. Application of this concept predicts that, while the man walked 2.4 m on the boat, the boat moves away from shore 0.4 m and the man is now 8 m from shore.

The apparatus described here allows checking the above calculation in the student laboratory. Construction cost is low and the results agree well with theory.

Figure 2 shows a runway made of acrylic sheet which can roll freely over another acrylic plate separated from it by many steel balls of diameter 0.475 cm, which greatly reduce any friction. The lower plate must be carefully leveled.

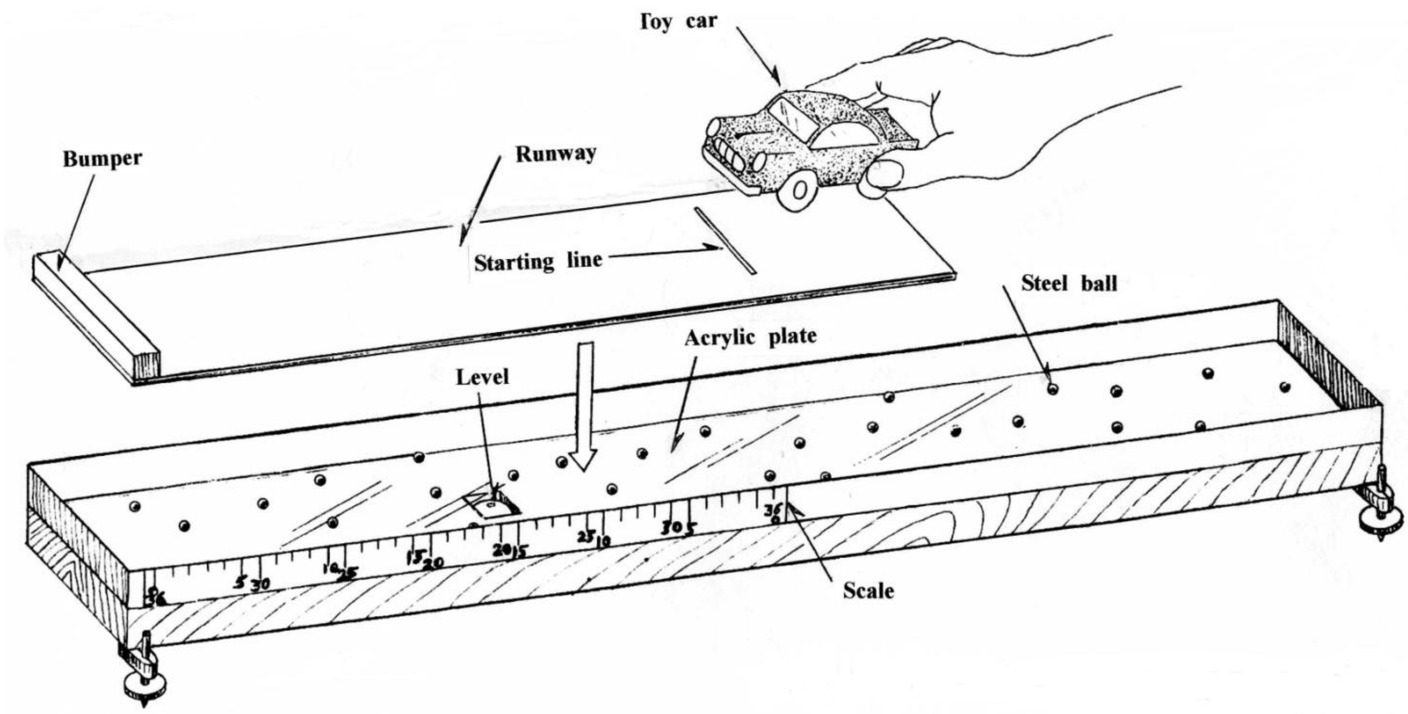

FIGURA 2. An apparatus for checking conservation of momentum.

A spring-wound toy car placed at one end of the runway is released and the car and runway begin to move in opposite directions. When the car strikes a bumper at the end, both car and runway stop simultaneously. The ratio of the distances moved by car and runway as well as their respective velocities, relative to the fixed lower plate, are in the inverse ratio of their masses.

Using this apparatus, the two experiments shown in Fig. 3 were performed.

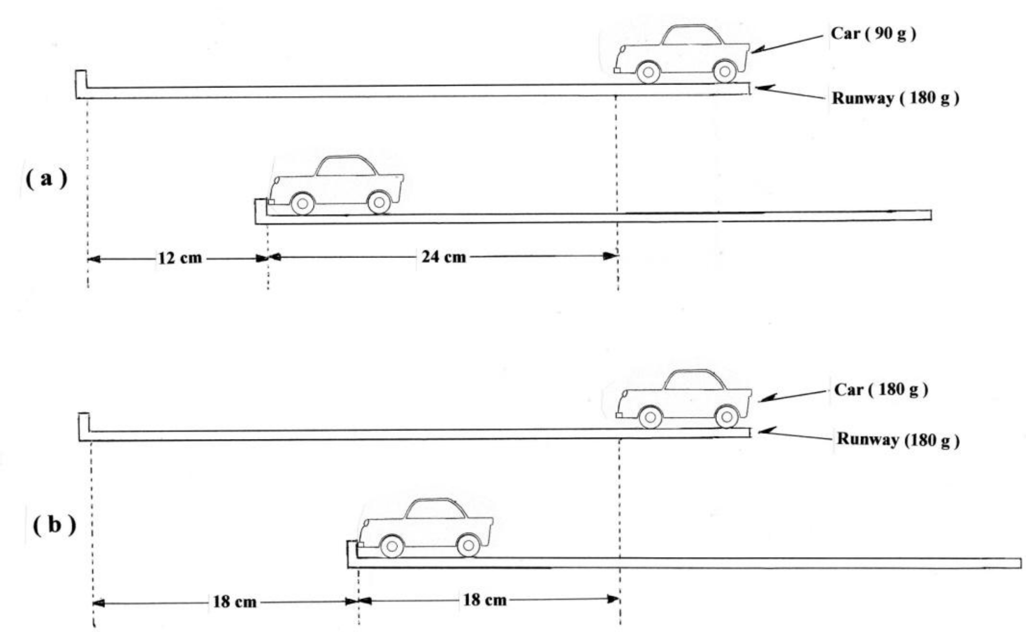

FIGURA 3. (a) The car moves 24 cm, and the runway moves 12 cm. (b) Both car and runway move the same distance, 18 cm.

In Fig. 3a, the car has a mass of 90 g and moves 24 cm while the runway has a mass of 180 g and should move 12 cm in the opposite direction. In Fig. 3b, the mass of car and runway are 180 g each and each should move 18 cm. Experimental results were in good agreement with these predictions.

An analysis of the role of the frictional forces between the runway are the lower plate reveals the reasons for the validity of the assumption that friction is negligible. Figure 4 is the side view of the car, runway, and lower plate, showing the forces on each.

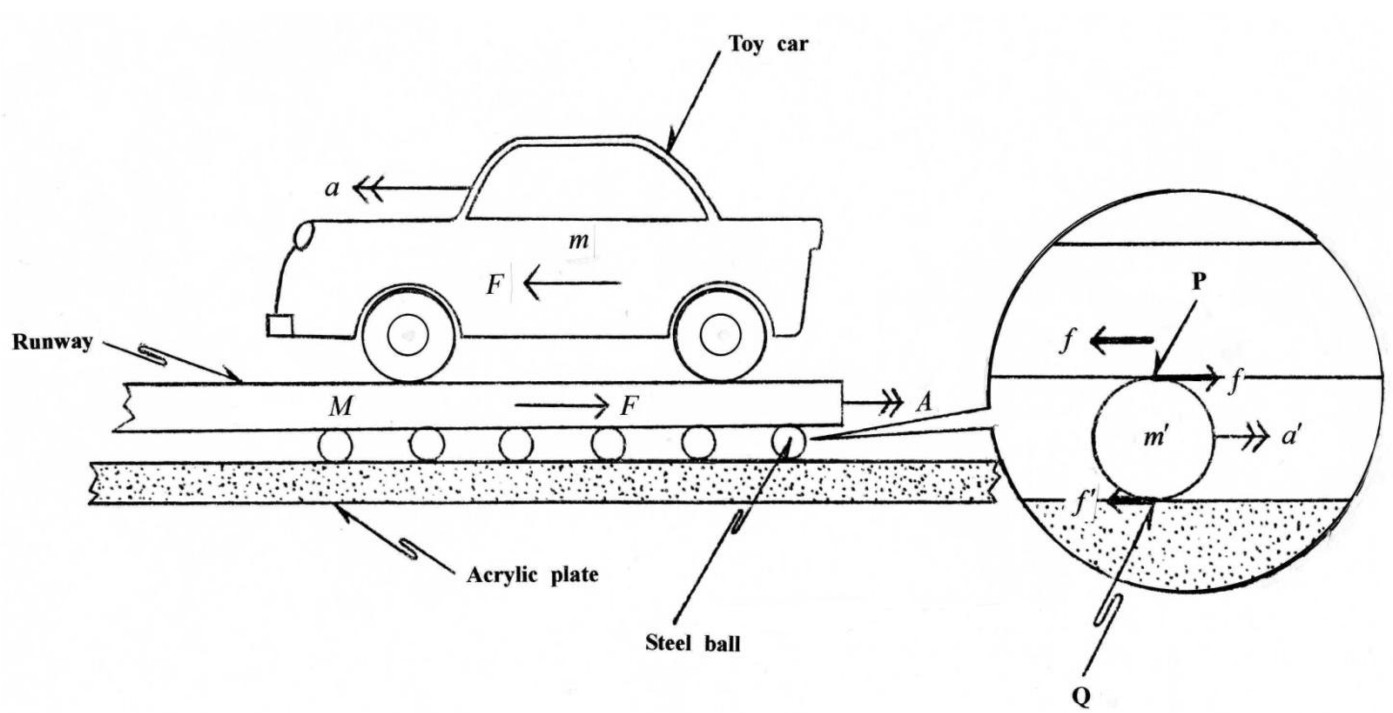

FIGURA 4. The side view of the apparatus. Arrow shows a force and double-arrow shows an acceleration.

If the spring in the car provides a force F of the wheels on the runway which is assumed to be constant over these short travel distances, then a constant acceleration a results and the distance S traveled by the car can be found from

$$S = \frac{1}{2}at^2 = \frac{F}{2m}t^2. \qquad (1)$$

where t is the time the car is in motion on the runway and m is the mass of the car.

There are 30 balls of mass m' and radius r. In Fig. 4, f and f' are the forces of friction between runway and ball, and acrylic plate and ball respectively. These forces act horizontally, tangent to the ball, at the points of contact, P and Q, as shown. It is assumed that the forces producing translational motion of the ball act at the center of mass such that

$$f - f' = m'a', \qquad (2)$$

where a' is the acceleration of the ball. The rotational motion of the ball about their center of mass follows from

$$(f + f')r = I\alpha, \qquad (3)$$

where I is the moment of inertia, $\frac{2}{5}m'r^2$, and α is the angular acceleration, $\frac{a'}{r}$ Similarly, for the runway of mass M,

$$F - nf = MA. \qquad (4)$$

Where n is the number of balls. The acceleration of the runway is

$$A = 2r\alpha. \qquad (5)$$

The distance moved by the runway can be found using

$$X = \frac{1}{2}At^2. \qquad (6)$$

Combining Eqs. (1), (4), and (6), we obtain

$$\frac{S}{X}=\frac{M}{m}\left(\frac{F}{F-nf}\right). \qquad (7)$$

Equation (7) shows that the rolling friction of the balls can be neglected if the value of nf is very small compared to F. Let us estimate the ratio of nf and F for this arrangement using n=30, m□ =0.44lg, and M=180g. From Eqs. (2) and (3)

$$f=0.7m'r\alpha \qquad (8)$$

and from Eqs. (4) and (5)

$$F-nf=2Mr\alpha \qquad (9)$$

which can be combined to get

$$\frac{nf}{F}=\frac{7nm'}{20M+7nm'}. \qquad (10)$$

Using the above numerical values, $\frac{nf}{F}=0.025$. Then, from Eq. (7) $\frac{S}{X}=1.03\frac{M}{m}$ which explains the excellent experimental result.

Using the apparatus described above, the experimental results were in good agreement with the law of conservation of momentum.

REFRENCES

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