AN INTEGRATED INVENTORY POLICY WITH DETERIORATION FOR A SINGLE VENDOR AND MULTIPLE BUYERS IN SUPPLY CHAIN WHEN DEMAND IS QUADRATIC

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ABSTRACT
In this paper, an integrated inventory policy for deteriorating item is developed considering single vendor and multi buyers in supply chain system, when demand of a product is increasing function with time. The model assumed constant rate of deterioration at each level of supply chain. A numerical example is shown to validate the results of the proposed development. The sensitivity analysis is carried out for certain model parameters. It is observed that the integrated decision for inventory policy between the players of supply chain lowers the total joint cost compared with an independent decision made by the supplier and all N buyers.

KEYWORDS: Supply chain, Single supplier and multi buyers, Integrated inventory policy, Deterioration, Quadratic demand

MSC : 90B05

RESUMEN
En este trabajo se considera una política de inventario integrado de deterioro teniendo en cuenta solo el vendedor y los compradores múltiples en el sistema de la cadena de suministro, cuando la demanda de un producto se incrementa en función del tiempo. El modelo asume constante la velocidad de deterioro en cada nivel de la cadena de suministro. Un ejemplo numérico se utiliza para validar los resultados del desarrollo propuesto. El análisis de sensibilidad se realiza con los parámetros del modelo determinado. Se observa que la decisión de la política de inventario integrado entre los actores de la cadena de suministro reduce el costo total de la articulación en comparación con una decisión independiente realizado por el proveedor y los compradores N.

1. INTRODUCTION

In a number of articles available in the literature, several inventory modeling problems have been discussed extensively. Most of these research articles considered the different sub system in supply chain independently and derived inventory policy either from vendor's or buyer's point views. But often the inventory policy derived for one player may not be acceptable to the other players of the supply chain. If the inventory policy is derived in cooperation with the all the players of the supply chain, then it can be beneficial to all the players of the supply chain. Also, integrated inventory system minimizes the overall integrated cost of the entire channel.

Goyal(1977) and Benerjee(1986) developed a research study to derive joint economic lot size policy from a vendor to a buyer. The main objective of the study was to minimize the total relevant costs for the vendor and the buyer both. Goyal(1988) generalized Banerjee's model by relaxing the assumption of the lot for lot policy of the vendor. Goyal and Gupta(1989) and Sarmah, Acharya and Goyal(2006) discussed the importance of coordination between vendor and buyer in the supply chain management.

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A large number of noticeable studies related to deteriorating inventory system can be found in the literature. Deterioration is defined as decay, decomposition, evaporation of the product and thereby loosing 100 % utility of it's efficiency. Blood components, medical products, fruits and vegetables are some of the examples of deteriorating commodities. Ghare and Schrader (1963) first consider deterioration of an item in the inventory system. Afterwards, Raafat(1991) , Heng et al (1991), Shah and Shah(2000) and Goyal and Giri (2001) discussed deteriorating inventory system. Yang and Wee(2005) derived a win-win strategy for an integrated system of vendor-buyer when units in inventory are subject to deteriorate at a constant rate.

In the most of the above articles either constant or linearly trended demand was assumed. The inventory models derived with the assumption of linearly trended demand by Silver and Meal (1969), Silver (1979), Xu and Wang (1991), Chung and Ting (1993, 1994), Bose et al. (1995), Hariga (1995), Giri and Chaudhuri (1997), Lin et al. (2000) consider that demand changes uniformly over time, which is not seen in the market of some products like electronic commodities, blood, fashion goods etc. Mehta and Shah (2003, 2004) consider the demand to be exponentially time varying which is unrealistic for newly introduced product. In order to have alternative demand pattern, quadratic demand is considered. This type of demand is partially constant, partially varies linearly and partially varies exponentially with time.

It can be seen that in most of the article discussed above inventory system is derived considering single vendor and single buyer. But in practice, one vendor-one buyer situation can be seen very rarely. In fact, in most of the cases structure of supply chain involves single vendor and multi buyers or multi vendor and multi buyers. There are certain issues need to be considered for single vendor – multi buyers system such as joint replacement of items, production inventory policy etc. An integrated single vendor and multiple buyers inventory model was studied by Woo et al (2001) and Yang and wee (2001).This study will address the issue of joint policy of a single vendor and multiple buyers inventory system when demand of an item is quadratic.

2. ASSUMPTIONS AND NOTATIONS

2.1 Assumptions

The model is developed under the following assumptions:

1. A supply chain system of single – vendor and multi – buyers is considered.
2. A single item with constant deterioration rate is considered. Deterioration is proportional to on hand stock in inventory. Deteriorated items can not be repaired or replaced during a cycle time.
3. The replenishment rate is instantaneous.
4. Shortages at any stage are not allowed.
5. Buyer orders the same order quantity whenever order is placed.
6. It is assumed that items can be deteriorate only after they have been received in to inventory.
7. The demand of an item is quadratically increasing with the time.

2.2 Notations

The proposed model is derived using the following notations:

N	Number of buyers
$R_i(t)$	Annual demand rate of the i^{th} buyer (units/unit time) = $a_i(1 + b_i t + c_i t^2)$, where a_i is constant demand, b_i and c_i are linear and exponential rate of change of demand with respect to time . Also, $a>0$, $a >> b, c$ and $0 < b, c < 1$, where $i = 1,2....N$.
T	The length of vendor's cycle time (decision variable)
n_i	Number. of delivery per order cycle time T for the i^{th} buyer (decision variable)
Θ	The deterioration rate
$I_v(t)$	Inventory level for vendor's at any time t, $0 \le t \le T$
$I_{bi}(t)$	Inventory level for i^{th} buyer at any time t, $0 \le t \le \dfrac{T}{n_i}$

I_{mv} The maximum inventory level of vendor
I_{mbi} The maximum inventory level of i^{th} buyer
C_v Vendor's per unit purchase cost ($/unit)
C_b Buyer's per unit purchase cost ($/unit)
A_v Vendor's ordering cost per order cycle ($/cycle)
A_b Buyer's ordering cost per order cycle ($/cycle)
I_v Vendor's inventory carrying charge fraction per unit per time unit ($/annum)
I_b Inventory carrying charge fraction per unit per time unit for buyer ($/annum)
TC_b Total cost of all Buyer's per time unit
TC_v Total cost of vendor per time unit
TC Total cost for vendor-buyer inventory system when they take decision jointly

3. MATHEMATICAL MODEL

In this section, mathematical model of an integrated inventory system for single vendor and *N* buyers is developed. In the supply chain system, vendor observed different demands from all *N* buyers during a cycle time *T*, which are satisfied from stock available in the inventory. The objective of this research study is to minimize total joint cost of single vendor and all *N* buyers. Here, vendor's total cost per time unit is a composite of ordering cost, inventory holding cost and deterioration cost. Figure 1 shows the stock trend for vendor inventory system.

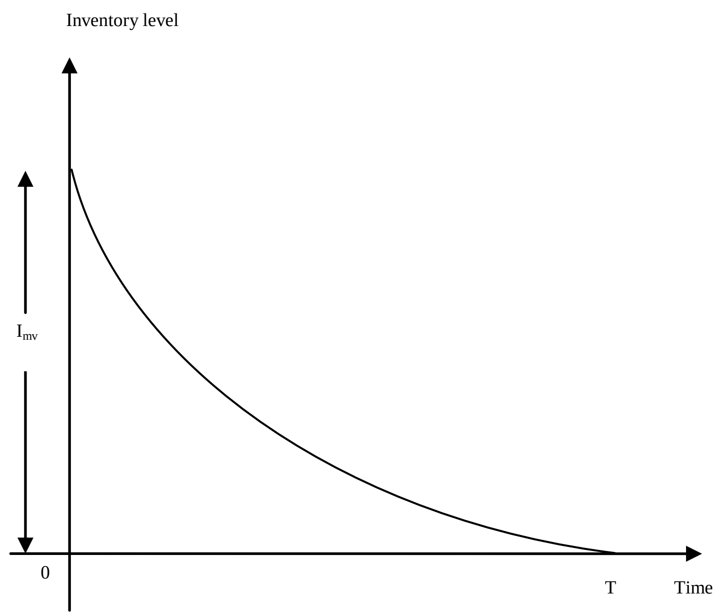

Figure 1: Vendor's Inventory system at any time t.

From Figure 1, vendor's inventory status can be represented by the following differential equation.

$$\frac{dI_v(t)}{dt} + \theta I_v(t) = -\sum_{i=1}^{N} -a_i(1+b_i t+c_i t^2) \qquad , \; 0 \le t \le T \qquad (1)$$

During any cycle time *T*, each buyer place different order quantity and received replenishment quantity over an interval of time. Hence, for different buyers in supply chain order quantity *Imb* and the number of delivery *n* are different. In a given cycle *T*, the inventory status for i^{th} buyer is depicted in figure 2.

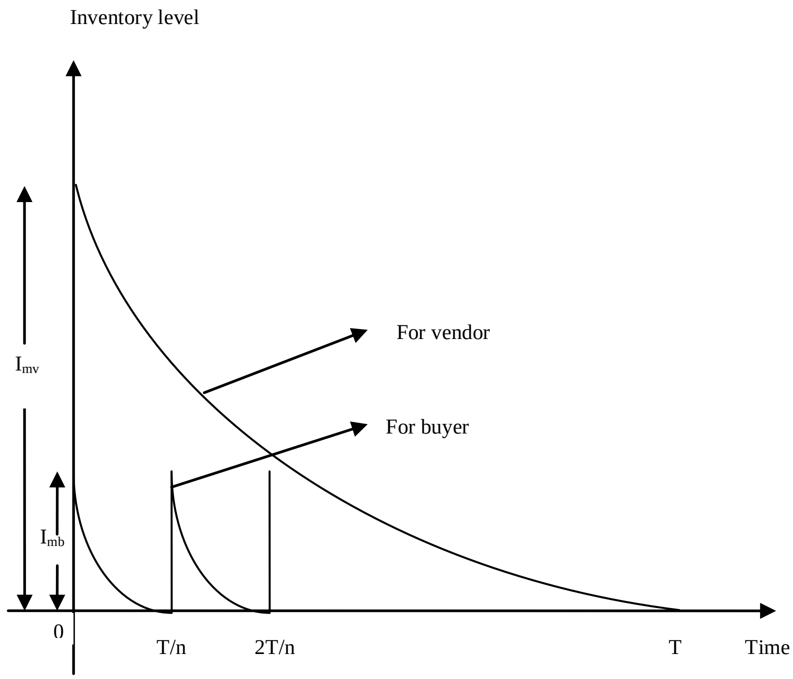

Figure 2: Vendor-Buyers Inventory system at any time t

From the Figure 2, inventory system for i^{th} buyer can be expressed in the mathematical form using the following differential equation

$$\frac{dI_{bi}(t)}{dt} + \theta I_{bi}(t) = -a_i(1 + b_i t + c_i t^2), \quad 0 \le t \le \frac{T}{n_i} \quad \text{where } i = 1,2\ldots N \tag{2}$$

Using various boundary conditions $I_v(T) = 0$, $I_{bi}(T_{bi}) = 0$ and $T_{bi} = \frac{T}{n_i}$, the solutions of the above differential equations are

$$I_v(t) = \sum_{i=1}^{N} a_i \left[\frac{e^{\theta(T-t)}}{\theta}(1 + b_i T + c_i T^2) - \frac{e^{\theta(T-t)}}{\theta^2}(b_i + 2c_i T - \frac{2c_i}{\theta}) + \frac{1}{\theta^2}(b_i + 2c_i t - \frac{2c_i}{\theta}) - \frac{1}{\theta}(1 + b_i t + c_i t^2) \right]$$

$$, \quad 0 \le t \le T \tag{3}$$

$$I_{b_i}(t) = a_i \left[\frac{e^{\theta(\frac{T}{n_i}-t)}}{\theta}(1 + \frac{b_i T}{n_i} + \frac{c_i T^2}{n_i^2}) - \frac{e^{\theta(\frac{T}{n_i}-t)}}{\theta^2}(b_i + \frac{2c_i T}{n_i} - \frac{2c_i}{\theta}) + \frac{1}{\theta^2}(b_i + 2c_i t - \frac{2c_i}{\theta}) - \frac{1}{\theta}(1 + b_i t + c_i t^2) \right]$$

$$, 0 \le t \le \frac{T}{n_i} \quad \text{where } i=1,2\ldots N$$

(4)

Using $I_v(0) = I_{mv}$ and $I_{bi}(0) = I_{mbi}$, the purchase quantities for the vendor and the i^{th} buyer are

$$I_{mv} = \sum_{i=1}^{N} a_i \left[\frac{e^{\theta(T-t)}}{\theta}(1 + b_i T + c_i T^2) - \frac{e^{\theta(T-t)}}{\theta^2}(b_i + 2c_i T - \frac{2c_i}{\theta}) + \frac{1}{\theta^2}(b_i + 2c_i t - \frac{2c_i}{\theta} - \frac{1}{\theta}(1 + b_i t + c_i t^2) \right] \tag{5}$$

$$Imb_i = a_i\left[\frac{e^{\frac{\theta T}{n_i}}}{\theta}\left(1+\frac{b_iT}{n_i}+\frac{c_iT^2}{n_i^2}\right)-\frac{e^{\frac{\theta T}{n_i}}}{\theta^2}\left(b_i+\frac{2c_iT}{n_i}-\frac{2c_i}{\theta}\right)+\frac{1}{\theta^2}\left(b_i-\frac{2c_i}{\theta}\right)-\frac{1}{\theta}\right] \quad \text{where } i=1,2...N \tag{6}$$

During the cycle time *[0, T]*, the i^{th} buyer's inventory level is $\int_0^{\frac{T}{n_i}} I_{bi}(t)\,dt$. Hence, the average inventory level for all *N* buyers per time unit is $\frac{1}{T}\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} I_{bi}(t)\,dt$. Therefore, the inventory holding cost for all *N* buyers is

$$IHC_b = C_b I_b \frac{1}{T}\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} I_{bi}(t)\,dt \tag{7}$$

The ordering cost for the i^{th} buyer is n_iA_b. Hence, the ordering cost for all *N* buyers per time unit is

$$OC_b = \frac{1}{T}\sum_{i=1}^{N} n_i A_b . \tag{8}$$

During cycle time *T*, the number of units deteriorated for the i^{th} buyer is $n_i\left(I_{mb} - \frac{R(\frac{T}{n_i})T}{n_i}\right)$. Therefore, deterioration cost for all *N* buyers is

$$DC_b = \frac{C_b}{T}\sum_{i=1}^{N} n_i\left(Imb_i - \frac{T\,R(\frac{T}{n_i})}{n_i}\right). \tag{9}$$

Hence, the buyer's total cost, TC_b per time unit is

$$TC_b = IHC_b + OC_b + DC_b \tag{10}$$

During the cycle time *T*, the vendor's average inventory level per time unit is $\frac{1}{T}\int_0^T I_v(t)\,dt$. The vendor's inventory in the joint two-echelon inventory model is the difference between the vendor-buyers combined inventory and all buyer's inventory. Therefore, vendor's holding cost per time unit is

$$IHC_v = C_v I_v \left[\frac{1}{T}\int_0^T I_v(t)\,dt - \frac{1}{T}\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} I_{bi}(t)\,dt\right] \tag{11}$$

the vendor's ordering cost per time unit is

$$OC_v = \frac{A_v}{T}. \tag{12}$$

The units deteriorated at the vendor's inventory system is $\left(I_{mv} - \sum_{i=1}^{N} n_i I_{mbi}\right)$. Hence, cost due to deterioration of units is

$$DC_v = \frac{C_v}{T}\left(I_{mv} - \sum_{i=1}^{N} n_i I_{mbi}\right). \tag{13}$$

The vendor's total cost TC_v per time unit is

$$TC_v = IHC_v + OC_v + DC_v \tag{14}$$

The joint total cost; TC is the sum of TC_b and TC_v.

$$TC = TC_b + TC_v. \tag{15}$$

Here, joint total cost TC is the function of discrete variable n and continuous variable T, where $i = 1, 2....N$.

4. COMPUTATION PROCEDURE

Here, the objective is to determine the value of n_i, which minimizes the joint total cost TC, where $i = 1, 2, ..., N$. Since the number of delivery n_i per order cycle T is a discrete variable, the following steps can be carried out to determine value of n_i.

(i) For different values of n_i, differentiate the total cost function TC from (15) with respect to decision variable T and set it equal to zero. ie. $\frac{\partial TC}{\partial T} = 0$. For each n_i, denote order cycle T by notation $T(n_i)$, where $i = 1, 2, ..., N$.

(ii) Find the optimal solution of n_i and T such that, the following condition must satisfy:

$$TC\,(n_i^* - 1, T(n_i^* - 1)) \geq TC\,(n_i^*, T(n_i^*)) \leq TC\,(n_i^* + 1, T(n_i^* + 1)) \tag{16}$$

Here, $(n_1^*, n_2^*, n_3^*, ..., n_N^*)$ and $T(n_1^*, n_2^*, n_3^*, ..., n_N^*)$ produce the optimal solution.

5. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

To illustrate proposed model in the simplest manner, the supply chain system with single vendor and two buyers is assumed. For numerical analysis, following parameter values are considered in proper units:

N =*No of buyers* = 2
a_1 = *Annual rate of constant demand of* 1^{st} *buyer* =80,000
a_2 = *Annual rate of constant demand of* 2^{nd} *buyer* =90,000
b_1 = *Linear rate of change of demand of* 1^{st} *buyer* =0.05
b_2 = *Linear rate of change of demand of* 2^{nd} *buyer* =0.05
c_1 = *Exponential rate of change of demand of* 1^{st} *buyer* =0.10
c_2 = *Exponential rate of change of demand of* 2^{nd} *buyer* =0.10
$C_v = 10$, $C_b = 13$, $I_v = 0.15$, $I_b = 0.30$, $A_v = 2000$, $A_b = 200$, $\theta = 0.10$

In table 1 and table 2, the optimal solution is exhibited for independent and integrated decision. If both the buyers follow independent policy then ordering policy is $(n_1 = 3, n_2 = 3)$ with total cost of *$ 62417*. If both the buyers agree to join in the integrated system with ordering policy of $(n_1^* = 2, n_2^* = 2)$ then the total integrated cost is significantly reduced to *$ 60176*. The graph of the total cost for independent and integrated inventory policy is shown in Figure 3. The buyer's cost increase when both the buyers and the vendor agree for making a joint decision. In the integrated policy, vendor benefits *$3761* and buyer looses *$1520*. Since, integrated strategy is beneficial to vendor, buyers do not agree for joint decision. To encourage and attract the buyers to cooperate in the integrated system, vendor should offer the buyer a permissible delay in payment or some proportion of sharing of extra benefits. This integrated policy reduces the integrated total cost defined as $PICR = \frac{TC(n_1, n_2) - TC(n_1^*, n_2^*)}{TC(n_1, n_2)}$ by *3.59 %*.

Table 1: Optimal solution of n_1 and n_2

n_1	n_2	T	BC	VC	TC
1	1	0.0807	35430	24766	60196
1	2	0.0868	31070	29120	60190
	3	0.0913	31100	30400	61500
	4	0.0953	32160	30980	63140
2	1	0.0865	31960	28497	60457
2*	**2***	**0.0927**	**26310**	**33866**	**60176**
2	3	0.0973	25580	35710	61290
2	4	0.0101	26060	36710	62770
3	1	0.0909	32350	29510	61860
3	2	0.0972	25930	35480	61410
3#	**3#**	**0.1017**	**24790**	**37627**	**62417**
3	4	0.1056	24950	38820	63770
4	1	0.0947	33680	29910	63590
4	2	0.1009	26640	36332	62972
4	3	0.1054	25140	38670	63810
4	4	0.1093	25080	40030	65110
4	5	0.1127	25630	40920	66550

Note that * indicates the integrated optimal solution of n_1 and n_2 which minimizes *TC* and # indicates the buyer's optimal solution of n_1 and n_2 which minimizes *BC*

Table 2: Comparison of Independent and Integrated Policy.

	Independent $n_1^* = 3$, $n_2^* = 3$	Integrated $n_1^* = 2$, $n_2^* = 2$	Change in Cost
Buyers			
Ordering cost	11800	8627	-3173
Holding Cost	11270	15420	4150
	1720	2263	543
Total cost of Buyers	**24790**	**26310**	1520
Vendor			
Ordering cost	19667	21567	1900
Holding Cost	8740	5980	-2760
	9220	6319	-2901
Total cost of Vendor	**37627**	**33866**	-3761
Total cost of System	**62417**	**60176**	-2241

Hence, the adoption of the integrated policy, instead of the independent policy is significantly decreasing the channel cost.

The results obtained in the numerical analysis indicate to perform sensitivity analysis for the model parameter. Its aim is to identify the parameters that are more relevant to the performance of the system. The sensitivity analysis is carried out by increasing or decreasing model parameters by *20%*. The PICR value is obtained for change in the model parameter. Here, *PICR* = $\frac{TC(n_1, n_2) - TC(n_1^*, n_2^*)}{TC(n_1, n_2)}$ where

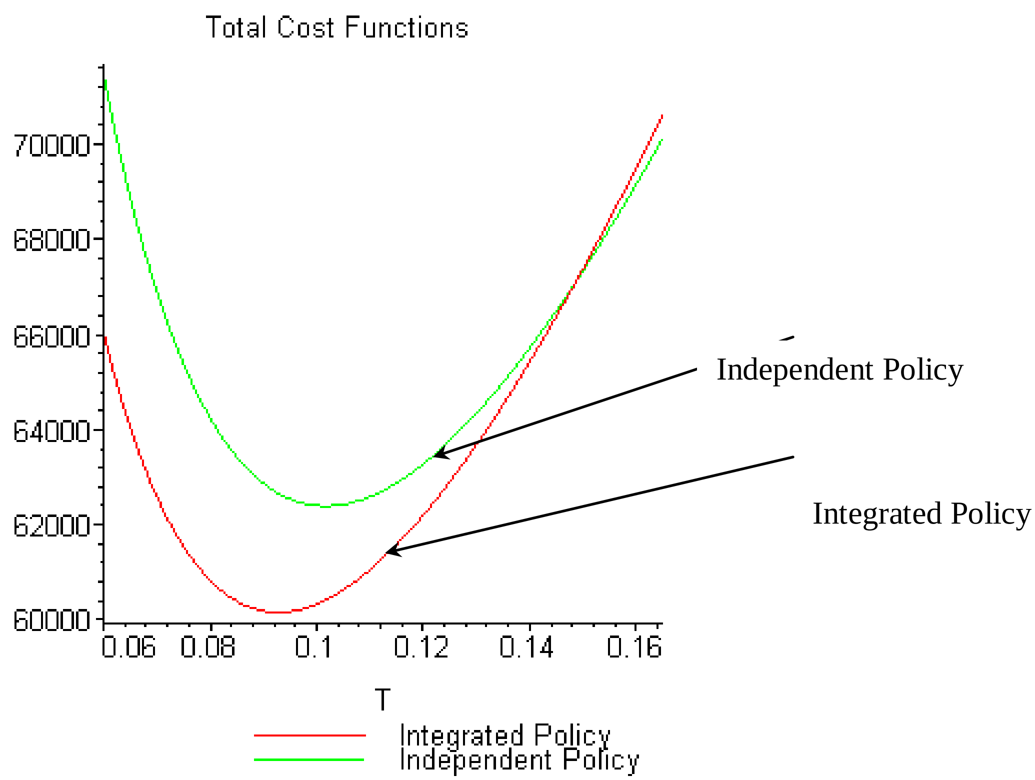

Figure 3: Total cost for integrated Vs independent Vendor-Buyers Inventory system

$TC(n_1^*, n_2^*)$ represents the optimal value of the total integrated cost *TC* and n_1^* and n_2^* are the optimal values of n_1 and n_2 represent optimal number of delivery for both the buyers. The results of sensitivity analysis for a constant rate of the demand; a_i, the linear rate of change of the demand; b_i, the exponential rate of change of the demand; c_i unit cost; C_v and C_b , ordering cost; A_v and A_b , inventory holding cost; I_v and I_b and deterioration rate; θ, is given in Tables 3 to 9. From the Tables 3 to 9, it is observed that the range of *PICR* is from *2.47* to *5.39*. The value of *PICR* is very sensitive to all the model parameters a_i , b_i , c_i ,C_v and C_b , A_v and A_b , I_v and I_b and θ ,where as (n_1^*, n_2^*) are not sensitive to all the parameters.

From Table 3, the increase in the fixed demand; a_i changes *PICR* in the range *(3.691, 3.415).*

Table 3: Sensitivity analysis for constant demand a_i

a_1	a_2	n_1^*	n_2^*	TC^*	n_1^*	n_2^*	$TC^{\#}$	***PICR (%)***
48000	*54000*	*2*	*2*	*46652*	*3*	*3*	*48440*	*3.691*
64000	*72000*	*2*	*2*	*53830*	*3*	*3*	*55860*	*3.634*
80000	***90000***	***2***	***2***	***60176***	***3***	***3***	***62417***	***3.590***
96000	*108000*	*2*	*2*	*65900*	*3*	*3*	*68270*	*3.472*
112000	*126000*	*2*	*2*	*71183*	*3*	*3*	*73700*	*3.415*

From table 4, it is observed that increase in linear rate of change of the demand; b_i, changes in the value of *PICR* observe in the range *2.583* to *4.6.* It is observed that increase in linear rate of change of the demand; b_i, increases the percentage of integrated cost reduction *(PICR)* very significantly.

Table 4: Sensitivity analysis for rate of change of linear demand b_i

b_1	b_2	n_1^*	n_2^*	TC^*	n_1^*	n_2^*	$TC^\#$	PICR (%)
0.03	0.03	2	2	60340	3	3	61940	2.583
0.04	0.04	2	2	60260	3	3	62200	3.119
0.05	**0.05**	**2**	**2**	**60176**	**3**	**3**	**62417**	**3.590**
0.06	0.06	2	2	60060	3	3	62540	3.965
0.07	0.07	2	2	59940	3	3	62830	4.600

From table 5, it is observed that *PICR* changes in the range *(3.389, 3.804)*, when the exponential rate of change of d the demand; c_i, is changed.

Table 5: Sensitivity analysis for rate of change of exponential demand c_i

c_1	c_2	n_1^*	n_2^*	TC^*	n_1^*	n_2^*	$TC^\#$	PICR (%)
0.06	0.06	2	2	60150	3	3	62260	3.389
0.08	0.08	2	2	60140	3	3	62300	3.467
0.10	**0.10**	**2**	**2**	**60176**	**3**	**3**	**62417**	**3.590**
0.12	0.12	2	2	60170	3	3	62470	3.682
0.14	0.14	2	2	60180	3	3	62560	3.804

From table 6, it is observed that when the unit costs C_v and C_b are increased, the *PICR* value decreases in the range *(3.32, 3.713).*

Table 6: Sensitivity analysis for unit cost C_v and C_b

C_v	C_b	n_1^*	n_2^*	TC^*	n_1^*	n_2^*	$TC^\#$	PICR (%)
6	7.8	2	2	46651	3	3	48450	3.713
8	10.4	2	2	53820	3	3	55880	3.686
10	**13**	**2**	**2**	**60176**	**3**	**3**	**62417**	**3.590**
12	15.6	2	2	65910	3	3	68260	3.443
14	18.2	2	2	71190	3	3	73640	3.327

From table 7, the *PICR* value increases from *3.46* to *3.651* with increases in the ordering costs A_v and A_b.

Table 7: Sensitivity analysis for ordering cost A_v and A_b

A_v	A_b	n_1^*	n_2^*	TC^*	n_1^*	n_2^*	$TC^\#$	PICR (%)
1200	*120*	*2*	*2*	*46560*	*3*	*3*	*48230*	*3.463*
1600	*160*	*2*	*2*	*53790*	*3*	*3*	*55740*	*3.498*
2000	***200***	***2***	***2***	***60176***	***3***	***3***	***62417***	***3.590***
2400	*240*	*2*	*2*	*65926*	*3*	*3*	*68400*	*3.617*
2800	*280*	*2*	*2*	*71250*	*3*	*3*	*73950*	*3.651*

From table 8, when the holding cost is increased, the *PICR* value becomes smaller. The increase in the holding costs I_v and I_b decrease the *PICR* value from *5.391* to *2.475*.

Table 8: Sensitivity analysis for I_v and I_b

I_v	I_b	n_1^*	n_2^*	TC^*	n_1^*	n_2^*	$TC^\#$	PICR (%)
0.09	*0.18*	*2*	*2*	*50890*	*3*	*3*	*53790*	*5.391*
0.12	*0.24*	*2*	*2*	*55710*	*3*	*3*	*58230*	*4.328*
0.15	***0.30***	***2***	***2***	***60176***	***3***	***3***	***62417***	***3.590***
0.18	*0.36*	*2*	*2*	*64320*	*3*	*3*	*66250*	*2.913*
0.21	*0.42*	*2*	*2*	*68180*	*3*	*3*	*69910*	*2.475*

From table 9, it is observed that *PICR* changes in the range *(3.507, 3.624)*, for changes in the value of deterioration rate; θ. From the sensitivity analysis, we observe that the value of *PICR* is more sensitive to holding costs I_v and I_b and the linear rate of change of the demand; b_i.

Table 9: Sensitivity analysis for rate of deterioration θ

θ	θ	n_1^*	n_2^*	TC^*	n_1^*	n_2^*	$TC^\#$	PICR (%)
0.06	*0.06*	*2*	*2*	*56410*	*3*	*3*	*58460*	*3.507*
0.08	*0.08*	*2*	*2*	*58270*	*3*	*3*	*60410*	*3.542*
0.10	***0.10***	***2***	***2***	***60176***	***3***	***3***	***62417***	***3.590***
0.12	*0.12*	*2*	*2*	*61970*	*3*	*3*	*64280*	*3.594*
0.14	*0.14*	*2*	*2*	*63753*	*3*	*3*	*66150*	*3.624*

6. CONCLUSION

In this study, a mathematical model for deteriorating items is developed to study an optimal integrated strategy of a single vendor-multi buyers inventory system when the demand is quadratic. A numerical analysis reveals that the joint policy lowers the total cost of an inventory system, even though the total cost of all the buyers increases significantly. To entice the buyer for cooperating, a promotional incentive in terms of trade credit should be offered by vendor to the buyers, which also helps to maintain long term contract between all players of supply chain.

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