URBAN GROWTH:
TRENDS VS. NOISE

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Abstract: *This paper carries out a comparative analysis of the so-called classical urban growth models and ramdon urban growth models in order to explain their explanatory capabilities about urban growth and cities size distribution. The process of innovation through experimentation embedded in the classical urban growth models has shed new ligth to explain the coexistence of both diversified and specialized cities and role that diversified cities play as "nursery cities" by facilitating experimentation (Duranton and Puga, 2001). Classical urban growth models do not naturally generate the Zipf's law (the rank-size rule for cities), whereas ramdon urban models provide a number of explanations for this key stylized fact. The theoretical foundations of both kind of models and their degree of compatibility are also examined. An exact statement of the conditions under which both type of models may be compatible is also needed.*
Keywords: *Classical urban growth / Ramdon urban growth / Innovation / Diversified cities / Specialized cities / Zipf's law.*

CRECIMIENTO URBANO: TENDENCIAS VS. RUIDO

Resumen: *En este artículo se lleva a cabo un análisis comparativo de los modelos de crecimiento urbano clásico y aleatorio para comentar su capacidad explicativa de los fenómenos de crecimiento urbano y de la distribución por tamaño de las ciudades. El proceso de innovación basado en la experimentación incorporado a los modelos de crecimiento urbano clásico aporta nueva luz sobre la coexistencia de ciudades diversificadas y especializadas y sobre el papel de las ciudades diversificadas como ciudades vivero para facilitar la experimentación (Duranton y Puga, 2001). Los modelos de crecimiento urbano clásico no generan de forma natural la ley de Zipf (la regla rango-tamaño para las ciudades), mientras que los modelos de crecimiento aleatorio nos aportan un conjunto de explicaciones para este hecho estilizado. Se examinan también los fundamentos teóricos y el grado de compatibilidad de ambos tipos de modelos. Se concluye la necesidad de una formulación exacta de las condiciones bajo las que ambos tipos de modelos pueden ser compatibles.*
Palabras clave: *Crecimiento urbano clásico / Crecimiento urbano aleatorio / Innovación / Ciudades diversificadas / Ciudades especializadas / Ley de Zipf.*

1. INTRODUCTION

Cities grow in population and in size over time. While those facts are well documented (e.g., Black and Henderson, 2003, Henderson, 2005), it is still unclear what really drives this growth. Cities might grow because they have a more favourable industrial structure, better amenities, or a more educated population. Alternatively, cities that grow might do so because they are 'lucky'. For at least some places, the role of historical accidents in urban growth is well documented. Silicon Valley and the rise of Dalton in Georgia as the us capital of the carpet industry (Saxenian, 1994, Krugman, 1991) are two examples among many that spring to mind.

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When investigating the causes of urban growth, empirical research is usually unconcerned with historical accidents and the role of luck. In a regression, 'accidents' usually enter the error term and are treated as noise. However, we argue here that the relationship between trends and accidents is not so simple. To do so, we provide a comparative analysis of the 'classical' urban growth literature, which focuses on trends, and 'random' urban growth models, which focus on accidents. A complete survey of the 'classical' urban growth models and related empirical work is beyond the scope of this paper. Instead of browsing through a large literature, we instead focus of one particular model which is representative of a broad class of models. The random growth literature is much smaller and more novel. We give it a more comprehensive treatment.

Classical urban growth models provide consistent explanations for urban growth that rely on one set of factor or another, depending on the model. They are also consistent with other stylised facts about cities such as those regarding their sectoral composition for instance. Finally, many of those models have strong microeconomic foundations and provide a justification for the existence of cities. However, the approach taken by classical urban growth model does not square well with a well-known regularity about the size distribution of cities. Namely, the size of cities appears to be well approximated by a Pareto distribution with exponent minus one.

Random urban growth models often start from very different assumptions and provide a fundamentally different explanation regarding the nature of urban growth. They highlight randomness and granularity in the urban growth process whereas the classical literature views urban growth as smooth and deterministic. Furthermore, random urban growth models also naturally generateskewed distributions of city sizes. Under plausible conditions, they can also generate an exact Pareto distribution with exponent minus one.

Given their abilities to shed light on different aspects of cities, classical and random growth models might be viewed as complements. However, as argued in the last part of the paper, there is a strong tension between them. Basically, random growth models require there to be no trend for a Pareto distribution to occur in steady state. This incompatibility is not insurmountable but runs deep enough so that these two classes of models can only be consistent with one another under specific conditions

2. CLASSICAL URBAN GROWTH MODELS: NURSERY CITIES AS AN EXAMPLE

As an example of a classical urban growth model, we use the 'nursery cities' model of Duranton and Puga (2001). This model attempts to make a connection between the literature on growth and innovation and urban economics. More preci-

sely, it uses a model of process innovation through experimentation to derive a number of implications about the urban landscape. The insights delivered by this model shed light on a variety of stylised facts about cities and how they link with economic growth.

This model can be summarised as follows. Entrepreneurs can introduce new products by paying a fixed cost of entry. At first, entrepreneurs do not fully master the production process for their products and can only produce 'prototypes' (to use the jargon of the model). Mass-production of a product requires process innovation. Mass-production is desirable because it allows entrepreneurs to produce with higher productivity.

Process innovation, which in the real world is enormously complex, is modelled in a simple way and tailored to deal with urban issues. There is a finite set of inputs in the economy. Among them, one is the 'ideal' set of inputs that each entrepreneur needs for mass-production. That is, process innovation is synonymous with discovering one's own ideal set of inputs for a new product. To do this, each entrepreneur need to engage in sampling. At each period, an entrepreneur can only sample at most one new set of inputs and use them for prototype production. As soon as an entrepreneur samples her ideal set of inputs, she knows this is it and can start mass-production.

The use of a particular set of inputs, either for prototype production or mass-production (if it is the ideal one), requires physical proximity with its producers. A possibility would be for input producers to be dispersed and for entrepreneurs to change location every time they want to sample a new set of inputs. There is a problem with this learning strategy: Moving is costly. As a result entrepreneurs would like to be able to sample different sets of inputs at the same location.

Besides, input producers benefit from agglomeration economies. Having more input producers of the same kind in the same location increases their efficiency. This assumption reflects a fundamental fact about cities: the increased concentration of firms and particularly firms from the same sector raises their efficiency. This fact was noted first by Alfred Marshall back in 1890. Modern econometric studies have confirmed it over and over again (see Rosenthal and Strange, 2004, for a review). In practice, as well as in the model, this tendency for producers to concentrate is limited by the existence of urban costs.

That moving is costly and that input producers want to be together to increase their productivity create an interesting tension. Learning entrepreneurs who try to discover their ideal set of inputs would like to sample everything at the same place. That is, entrepreneurs who have not yet discovered their ideal set of inputs want to locate in a very diversified local economy. However, producers of a particular type of inputs would like to locate together with producers of the same type of inputs. This pushes towards the existence of specialised cities.

Provided moving costs are neither too high nor too low, an interesting equilibrium emerges. It reconciles the needs for specialisation and diversity along the life-

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cycle of firms. Entrepreneurs develop new products in cities with a diversified production structure. It allows them to sample easily and discover their ideal set of inputs. After discovering this ideal set of inputs, entrepreneurs are no longer interested in urban diversity. Because, input producers in different sectors do not benefit from each other directly, industrial diversity makes cities bigger and thus more costly. As a result, entrepreneurs who know what their ideal inputs are would like to be in a city that is specialised only in the production of those inputs[1]. That is, provided moving is not prohibitively costly, entrepreneurs who have discovered their ideal set of inputs will want to move away from diversified city and be in specialised cities to benefit from agglomeration effects in their sectors. In this sense, we can think of diversified cities as 'nursery cities' where learning takes place and specialised cities as the places where the production of mature goods occurs.

To summarise, the model of Duranton and Puga (2001) proposes a set of predictions about how the process of growth and innovation will take place spatially. Beyond this, it rationalises a number of stylised facts about cities. First, there is a key new prediction originating from the model. Firms that relocate will predominantly relocate away from diversified cities to specialised cities in their sector of activity. The evidence presented in the introduction of Duranton and Puga (2001) is highly supportive of this prediction. This model also predicts the coexistence in equilibrium of specialised and diversified cities, a prominent feature of the urban landscape of advanced countries (Duranton and Puga, 2000). Consistent with the growth in cities literature initiated by Glaeser, Kallal, Scheinkman, and Schleifer (1992) and more particularly with the work of Henderson, Kuncoro, and Turner (1995), specialised cities seem to be bring benefits to firms in mature industries whereas firms in high-tech industries appear to benefit more from local diversity. The patterns of entry and exit predicted by the nursery city model are also consistent with empirical results from firm-level studies (e.g. Dumais, Ellison, and Glaeser, 2002, Bernard and Jensen, 2007).

Although it is not strictly speaking a model of endogenous growth, the model of Duranton and Puga (2001) fits well within the literature that extends urban models to consider economic growth. Let us call this class of models the classical urban growth models. While this is not the place to discuss this literature in depth, a number of papers are worth mentioning[2]. Eaton and Eckstein (1997) consider a model where there are agglomeration effects in the accumulation of human capital. They formalise the suggestion of Lucas (1988) that human capital accumulation takes place primarily in cities. Interestingly, the dynamic human capital externality at the core of Eaton and Eckstein's model is at the root of both economic growth and of the existence of cities. Glaeser (1999) proposes a different form of dynamic externality through direct interactions. His argument is that learning can only occur

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[1] A proportion of firms die every period to insure that new firms keep entering and learning is never exhausted.

[2] See Berliant and Wang (2005) for a review of this literature.

through the teaching of 'young unskilled' workers by 'old skilled' workers. Cities favour learning by providing more opportunities for young workers to meet old workers. While obviously very stylised, this model captures the idea that the greater possibilities for direct interactions between workers in cities may be at the origin of the accumulation and diffusion of knowledge. Black and Henderson (1999) propose a model with a static human capital externality in cities. Larger cities make workers more productive. In turn, workers spend part of their time accumulating human capital. This accumulation of human capital reinforces the human externality that takes place within cities. In turn, that makes cities more attractive. They grow in population, and this reinforces again the human capital externality. A particularly nice feature of Black and Henderson (1999) is that human capital accumulation, output growth, and population growth in cities all go hand in hand.

Despite their emphasis on different aspects of how the growth process and urban development interact, these models share a number of common elements. First, they follow primarily Lucas' (1988) pioneering work on human capital externalities and growth. The framework of Romer (1990) in which growth occurs through new innovations that are patented and increase the general stock of knowledge has arguably less relevance when one is interested in the spatial aspects of growth[3]. The discussion of Schumpeterian insights for growth as modelled in Aghion and Howitt (1992) is postponed until below.

Second, cities are viewed as an equilibrium outcome between agglomerations forces that make larger cities more productive and urban costs such as increased land scarcity and congestion. Both sets of forces are usually modelled in a detailed fashion with a particular focus on the micro-economic foundations of agglomeration. A fundamental property of this class of models is the existence of a bell-shaped curve for earnings net of urban costs as a function of population size. As a city grows, both agglomeration economies and urban costs increase. The increase in agglomeration economies initially dominate for small cities while higher urban costs eventually take over to limit the growth of large cities.

Third, these models are 'smooth' in the sense that growth proceeds smoothly through atomistic agents. At each period, a fraction of prototype producers learn about their ideal production process (Duranton and Puga, 2001), a fraction of workers become skilled after being taught by others (Glaeser, 1999), or existing residents increase their human capital by some fraction (Eaton and Eckstein, 1997, Black and Henderson, 1999).

Fourth, and related to the previous point, these models are deterministic. Structural characteristics of cities predict their growth. For instance, in Duranton and Puga (2001), the sectoral composition of activities in cities predicts how much

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[3] There is an interesting literature on the spatial dimension of patents as represented for instance by Jaffe, Trajtenberg, and Henderson (1993) or, more recently, Agrawal, Cockburn, and McHale (2006). As shown below, Romer (1990) nevertheless serves as the basis for a couple of urban growth models.

learning by firms will take place[4]. In Glaeser (1999), learning by workers is predicted by the composition of cities both in terms of skills and demographics. In Eaton and Eckstein (1997) and Black and Henderson (1999), it is the initial level of human capital of cities, their initial size, and their sectoral activity that predict how much growth will take place. Introducing a stochastic element in the model would obviously attenuate this determinism at the city level[5]. For instance, random city effects at each period could influence human capital accumulation. However, what these models are telling us is that some urban characteristics like the average level of human capital in Black and Henderson (1999) positively map into urban growth. In Black and Henderson (1999), a higher level of human capital leads to faster population growth in cities and this is of first-order importance in the sense that what is left unexplained is a residual. While this point may seem obvious (most models in applied theory are about deriving some comparative statics that can be brought into a regression), random urban growth models work very differently.

Before going deeper into that point, let me highlight the main limitation of this class of models. It lies in its inability to generate naturally a plausible distribution of city sizes. In Duranton and Puga (2001), the model in its simplest form predicts that in equilibrium all cities are of the same size. We can easily relax this prediction by considering that agglomeration effects in cities have different intensities for different sectors. This is a well established empirical fact (Henderson, 2003, Rosenthal and Strange, 2004). It implies that specialised cities each achieve a particular size depending on their sector of specialisation. This is because the balance between agglomeration economies and urban costs differs across sectors of specialisation. It is also possible to assume that agglomeration effects weaken at the margin as a sector grows locally. This would immediately imply diversified cities being much larger than specialised cities, a well-established stylised fact in the urban literature (Duranton and Puga, 2000)[6]. However, it remains that the size of cities is determined by their 'type' and this does naturally map into the observed distribution of city sizes. This feature is not specific to Duranton and Puga (2001) but common to this entire class of models since Henderson (1974) (with one exception to be discussed below). Eaton and Eckstein (1997) and Black and Henderson (1999) also have several types of cities that all grow in parallel and thus do not have much to say about the size distribution of cities.

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[4] All the learning takes place in diversified cities. However process innovations are implemented in specialised cities. This is where TFP growth is then recorded. In this model, more innovation implies employment growth in diversified cities and more TFP growth in specialised cities. This prediction is consistent with the empirical findings of Cingano and Schivardi (2004).

[5] Proper modelling of microeconomic foundations for these shocks would be needed. We return to this issue below.

[6] Under some conditions, equilibrium city size is such that marginal agglomeration economies are equal to marginal urban costs. If marginal agglomeration economies are constant, the sectoral composition of cities does not matter to determine their size and all cities, regardless of what they do, reach the same size. With marginal agglomeration economies decreasing with size, a city with only one sector has lower marginal agglomeration economies than another city of the same size whose activity is split across many sectors. As a result, we expect diversified cities to be larger than specialised cities

To summarise, the literature that stems from Henderson (1974) and to which Duranton and Puga (2001) belongs to offers a theory of why there are cities (an equilibrium between agglomeration and dispersion forces), a theory (or a set of related theories) of what those cities do and their production structure, and a theory of their population size and growth (which depends on their type). The evidence of tension between agglomeration and dispersion forces seems incontrovertible. The insights delivered by this literature about what cities do and their production structure are also convincing and backed by a large body of evidence. As a set of theories about city size(s), this literature seems much weaker. A point we now turn to.

3. ZIPF'S LAW AND RANDOM GROWTH MODELS

Academic interest in the size distribution of cities predates the approach just described. Since Auerbach (1913), many have approximated the distribution of city sizes with a Pareto distribution. In a nutshell, the idea is to rank cities in a country from the largest to the smallest and then to correlate this ranking against their population in the following manner:

$$\log Rank = \text{Constant} - \zeta \log Size \tag{1}$$

The estimated coefficient ξ is the exponent of the Pareto distribution. Zipf's law (after Zipf, 1949) corresponds to the statement that $\xi = 1$. This implies that the expected size of the second largest city is half the size of that of the largest, that of the third largest is a third of that of the largest, etc[7].

The empirical validity of Zipf's law is hotly debated. The classic cross-country assessment of Rosen and Resnick (1980) is ambiguous because their average Pareto exponent of 1.14 for 44 countries has been interpreted as evidence both for and against Zipf's law. Follow up work by Soo (2005) broadly confirms these results, albeit with a more negative tone and a claim that Zipf's law is rejected for a majority of countries. This evidence should however be interpreted with care because countries differ in their definition of what is a city and quality of data[8].

The fact that the Zipf coefficient in large majority of countries is between 0.8 and 1.2 suggests that there is 'something' in the data and it would be hard to argue against any regularity in the size distribution of cities altogether. Zipf's law is both an important stylised fact and a useful benchmark. But no awe about it is justified. Whether Zipf's law should take primacy over other stylised facts about cities is also debatable.

Let us now explore the statistical processes that lead to Zipf's law. There are two (related) avenues: multiplicative and additive processes. These processes do

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[7] The deterministic reformulation of Zipf's law is usually referred to as the Rank Size Rule.

[8] For more about these issues, see the excellent survey of Gabaix and Ioannides (2004).

not tell us much about the underlying economic forces behind urban growth. An important goal of the recent literature has been to embed them in well-articulated economic models. For expositional reasons, let us follow the same path and start with the 'mechanics' of Zipf's law before turning to its 'economics'.

Following Gabaix (1999*a*) and Gabaix (1999*b*), multiplicative processes have attracted a lot of attention. These processes are referred to as Kesten processes (after Kesten, 1973). Some formal modelling is now needed. We borrow from Gabaix and Ioannides (2004) and consider an economy with fixed population size. Between *t* and *t*+1, city *i* grows according to $S_{it+1} = (1 + \tilde{\gamma}_{it+1})S_{it}$.

We impose Gibrat's law. The $\tilde{\gamma}$s are independently and identically distributed with density $f(\gamma)$.

After *T* periods the size of city *i* is:

$$\log S_{iT} = \log S_{i0} + \sum_{t=1}^{t=T} \log\left(1 + \gamma_{it}\right) \approx \log S_{i0} + \sum_{t=1}^{t=T} \gamma_{it} \tag{2}$$

We note that the approximation in this equation holds only when the shocks are small enough. By the central limit theorem, $\log S_{iT}$ is normally distributed and the distribution of S_{iT} is thus log normal. This distribution of city sizes does not admit a steady state and its variance keeps increasing. To obtain a steady state, one needs to impose a lower bound for city sizes (Gabaix, 1999*a*). Without a lower bound on city sizes, their distribution is single-peaked with thin tails at both ends as made clear above. This is because very few cities consistently get positive or negative shocks. With a lower bound on city sizes, things change dramatically because the thin lower tail disappears and there is instead a maximum of the density function at the lower bound. Preventing cities from becoming too small also allows the upper tail to be fed by more cities. As a result, it is fatter. This lower bound also allows for the existence of a steady state instead of an ever widening distribution. Interestingly this steady state implies a Pareto distribution[9]. The main alternative to the multiplicative process described above was originally proposed by Simon (1955). In essence, Simon's model assumes that aggregate population grows over time by discrete increments. With some probability, a new lump goes to form a new city. Otherwise it is added to an existing city. The probability that any particular city gets it is proportional to its population. This mechanism generates a Pareto distribution for city sizes. The Pareto exponent falls to one at the limit as the probability of new cities being created goes to zero.

Despite important differences between them, both multiplicative and additive processes have some version of Gibrat's law at their core, either directly though multiplicative shocks or through increases of fixed size that occur proportionately to population.

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[9] See Gabaix (1999*a*) for a complete proof.

Among existing models of random growth with an economic content, that proposed by Eeckhout (2004) is the simplest. There is a continuum of cities. City *i* at period *t* enjoys productivity *Ait* for labour, the only factor of production. Agglomeration economies increase the productivity of labour by a factor S_{it}^{θ}, and congestion costs reduce it by a factor where S_{it} is the population of city *i* at time *t*. Hence, output per worker in this city is $A_{it}S_{it}^{\theta-\sigma}$. To avoid complete concentration into a single city, we need $\theta<\sigma$. Free mobility across cities then implies the equalisation of output per worker across all cities.

Even though each city faces shocks, the law of large numbers applies in aggregate so that output per worker is deterministic. After normalising it to unity, the equilibrium size of city *i* is given by:

$$S_{it} = A_{it}^{\frac{1}{\sigma-\theta}} \tag{3}$$

With small i.i.d. shocks productivity evolves according to:

$$A_{it+1} = \left(1+\gamma_{it+1}\right)A_{it}$$

it is easy to see that after *T* periods, we have:

$$\log S_{iT} \approx \log S_{i0} + \frac{1}{\sigma-\theta}\sum_{t=1}^{t=T}\gamma_{it} \tag{4}$$

Equation (4) is derived in the same way as (2). The main difference is that instead of imposing 'arbitrary' population shocks, the model assumes cumulative productivity shocks. In a setting where free mobility implies that population is a power function of productivity (equation 3), the log normal distribution of city productivity maps into a log normal distribution of city population. As argued above, adding a lower bound for city size would imply Zipf's law instead[10].

The model of Rossi-Hansberg and Wright (2007) also relies on cumulative productivity shocks[11]. The main difference between this model and that of Eeckhout (2004) is that it treats cities like the classical urban growth literature as an equilibrium between agglomeration and dispersion forces.

It is important to show that random growth models can accommodate a standard modelling of cities. The main difference between random and classical urban

[10] Zipf's law is not desired by Eeckhout (2004). The empirical part of his paper makes the case for a log normal distribution for city sizes.

[11] Zipf's law is obtained in two cases by Rossi-Hansberg and Wright (2007). The first is the case described here with permanent shocks. The second is a situation with temporary shocks which affect factor accumulation. For alternative ways to generate Zipf's law with cumulative shocks see also Córdoba (2008).

growth models is not in the static modelling of cities but in what drives their dynamics.

Gabaix (1999*a*) considers a model where workers are mobile only at the beginning of their life when they need to pick a city. Workers derive (multiplicatively) separable utility from consumption and a local amenity. The level of amenity in a city is i.i.d. and drawn every period. With such shocks, the location problem of young workers boils down to the static maximisation of the product of the local amenity and the local wage. At the steady state equilibrium, this product for young workers is equalised across cities.

The production function is homogenous of degree one between young workers and incumbent residents (a fraction of survivors from previous period population). An interesting part of Gabaix' model is to show how temporary shocks have permanent effects. This arises through workers becoming immobile after their original choice and the production function which is homogenous of degree one so that the wage of young workers depends only on the ratio of young mobile workers to immobile incumbents. In this context, amenity shocks that in fact 'multiply the wage' in the utility function lead to Gibrat's law. Following the argument developed above, adding a lower bound leads to Zipf's law in steady-state. There are two differences with the previous two models. First, the shocks apply to amenities and not technology. Second, the shocks are temporary and not permanent.

The models of Gabaix (1999*a*), Eeckhout (2004), and Rossi-Hansberg and Wright (2007) are the three main multiplicative random growth models. Duranton (2006, 2007) proposes two related economic mechanisms that lead to additive random growth.

Duranton (2006) builds on Romer's (1990) endogenous growth model. Research is tied to production though local spillovers. As a result, research activity in one location is proportional to the number of local products. With mobile workers and no cost nor benefits from cities, city population is proportional to the number of local products. In equilibrium, small discrete new innovations occur in cities proportionately to their population size. Innovations need to be discrete to avoid the law of large numbers from applying and leading to parallel growth for all cities. Newly invented products are either produced where they were developed or alternatively some natural resource forces them to be produced at a new location. The latter leads to the creation of a new city. After each innovation in a city, there is an increase in labour demand to produce the new product. In turn, this implies population growth. In essence, this models puts a geographical structure on a discrete version of Romer (1990). As shown by Duranton (2006), this maps directly into Simon (1955) and generates Zipf's law as a limit case when the probability of a new city tends to zero[12].

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[12] It also avoids some pitfalls of Simon (1955) which converges slowly. The cumulative and exponential nature of the growth process in Romer (1990) ensures that shocks, although additive, occur more frequently as time passes which leads to much faster convergence.

Duranton (2007) uses a related model which builds instead on the Schumpeterian growth model of Grossman and Helpman (1991). In this framework, profit-driven research tries to develop the next generation of a product up a quality ladder. A success gives it a monopoly which lapses when the next innovation on the same product occurs. Products are discrete to ensure the necessary granularity for shocks to affects cities. Again, local spillovers tie research on a given product to the location of its production. The core of the model is that research might succeed in improving the products it seeks to improve (same-product innovation) or, sometimes, because of serendipity in the research process, it might succeed in improving another product (cross-product innovation).

With same-product innovation, the location of activity is unchanged by innovation and successful new innovators only replace incumbent producers in the same city. With cross-product innovation, the old version of the improved product stops being produced where it used to be and starts being produced in the city where the innovation took place. This typically leads to a relocation of production with a population gain for the innovating city and a loss for the city of the incumbent producer. An example of cross-product innovation is the xerography.

In the late 1950s a Rochester (NY) firm, Haloid Company, was attempting to improve on Eastman Kodak's technology in the photographic industry. Its innovation was instead an improvement in the reprographic industry. As a result, the reprographic industry moved from New York where it was originally located to Rochester where Haloid and the photographic industry were located.

To prevent cities from disappearing forever, the model also assumes that there is a core product in each city that cannot move. Symmetry and the absence of other costs and benefits from cities also ensure that city population is proportional to the number of products manufactured locally.

In steady-state, this model does not quite lead to Zipf's law because new innovations are not exactly proportional to city size. Because they already have more products, large cities have fewer of them to capture from elsewhere. On the other hand, the smallest cities with only one fixed product can only grow. Hence, growth is less than proportional to city size and this leads to a distribution of city sizes that is less skewed than Zipf's law. This distribution does well at replicating the us city size distribution. Unlike other models of random growth, it does not focus exclusively on the size distribution of cities. It also replicates the fast churning of industries across cities, a well documented fact (Simon, 2004, Duranton, 2007, Findeisen and Südekum, 2008).

4. TWO MUTUALLY EXCLUSIVE APPROACHES TO URBAN GROWTH?

Classical urban growth models and random urban growth models both appear to contain a grain of truth. They address different aspects of the urban growth process

and are able to replicate different stylised facts. At first sight they seem to complement each other. The key question is whether they are compatible. There are two main differences between these two classes of models. First, in classical urban growth models growth is smooth whereas in random growth models it is granular as growth proceeds through discrete shocks[13]. With infinitesimal shocks, the law of large numbers would apply within each city and the interesting results of random growth models would disappear. Even though random growth models cannot be smoothed, the smoothness of classical urban growth models can easily be roughened. In fact, classical urban growth models are smooth for tractability and esthetic reasons. Adding shocks or some other form of granularity would be conceptually easy but would make solving these models much more complicated. This suggests that granularity is not an issue from the theoretical perspective and that there is no real opposition here between the two classes of model.

The second main difference between classical and random growth models of cities regards the role of shocks. Classical urban growth models follow the traditional approach where growth is driven by city characteristics and what is left unexplained is treated as a residual. In random growth models, the 'residual' is everything.

To understand this point better, consider a simple urban growth regression:

$$\log S_{it+1} - \log S_{it} = a_1 \log S_{it} + a_2 \log X_{it} + \varepsilon_{it+1} \tag{5}$$

where the growth of city i between t and $t+1$ depends on its population size in t, a set of characteristics X, and a random term. as starting point, it is useful to consider that classical urban growth models focus on S and X whereas random growth models focus on t. The issue is whether Zipf's law is compatible with $a_1 \neq 0$ or $a_2 \neq 0$.

While there is some disagreement in the literature about the importance of mean reversion in city population data (e.g., Black and Henderson, 2003, vs. Eeckhout, 2004), past city population is more often than not a significant determinant of city growth and its coefficient appears with a negative sign in urban growth regressions. However, mean reversion is not sufficient to invalidate random growth models. As made clear by Gabaix and Ioannides (2004), what matters is not mean reversion in itself but the existence of a unit root in the urban growth process. That is, random growth models rely on a 'weak' version of Gibrat's law not on its strong version. To understand this point more precisely, let us follow Gabaix and Ioannides (2004) and assume the following error structure:

$$\varepsilon_{it} = \gamma_{it} + \mu_{it} - \mu_{it-1}$$

where γ_{it} is i.i.d. and μ_{it} is stationary. In that case, there is mean reversion since growth between t and $t+1$ is negatively correlated with size in t. On the other hand,

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[13] Although random growth models can be specified in continuous time like Gabaix (1999*a*), some form of granularity is needed.

absent other determinants of urban growth, one can easily show that this error structure implies:

$$\log S_{iT} = \log S_{i0} + \sum_{t=1}^{t=T} \gamma_{it} + \mu_{iT} - \mu_{i0} \qquad (6)$$

This equation has much in common with (2) where the summation of the γ shocks combined with a lower bound for city size leads to Zipf's law. The main difference is the contemporaneous error term μT. The heuristic developed by Gabaix and Ioannides (2004) argues that if the tail of the summation in γ is fatter than that of μ, Zipf's law should still occur in steady state. Intuitively, mean reversion does not matter provided it is 'dominated' by the cumulated 'Gibrat's shocks'. While insightful, this example remains very particular. Much remains to be done in this area. We need to know what is the weakest version of Gibrat's law compatible with Zipf's law.

Turning to the other determinants of urban growth, let us return to equation (5), assume $a_1=0$, allow for a_2 to be time varying, and consider that $\varepsilon_{it}=\gamma_{it}$ which is i.i.d. After simplification, we obtain:

$$\log S_{iT} = \log S_{i0} + \sum_{t=1}^{t=T} \gamma_{it} + \sum_{t=1}^{t=T} a_{2t} X_{it} \qquad (7)$$

It is now easy to understand that any term $a2\ X$ that is constant over time and differs across cities would lead to a distribution that differs from Zipf's law. Simply put, when cities experience city-specific trends, there is divergence in the long-run and no steady state distribution.

This basic incompatibility between classical and random urban growth models should not be exaggerated. First, the upper tail of the city size distribution may remain Pareto despite different growth trends. To understand this point, consider two groups of cities, fast -and slow- growing cities (corresponding to the case where X is an indicator variable in equation 5). Provided the lower bound city size for each group of cities grows with its trend, there is a Pareto distribution emerging for each group of cities and divergence between the two groups[14]. If this divergence is slow, at any point in time the overall distribution will be a mixture of two Pareto distributions with coefficient minus one but different lower bounds. Above the largest of the two lower bounds, this distribution will be Pareto. Over one century, a difference of 1 percent per year in the trend implies a factor of only 2.7 for city size differences. With a lower bound for slow growing cities of, say, 10,000 people, the

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[14] The lower bound needs to increase with the trend, otherwise cities of smaller relative size would occur over time which would weaken the reflection leading to a Pareto distribution. In the extreme case of a fast receding lower bound, it is easy to see that one would return to a log normal distribution.

corresponding lower bound for fast growing cities will be 27,000 after one century. Above 27,000 the size distribution of cities is Pareto. Slow divergence is thus difficult to observe in the data.

If divergence between groups is fast, the slow-growth group will quickly become vanishingly small. For instance, it can be that contemporaneous distributions of city sizes which are typically truncated at some threshold between 10,000 and 100,000 may only contain 'good sites'. 'Bad sites' have not developed into large cities and only led to small settlements. The evidence of a Pareto distribution in the lower tail of the distribution is much weaker than in the upper tail (Eeckhout, 2004, Michaels, Rauch, and Redding, 2008, Rozenfeld, Rybski, Gabaix, and Maske, 2009). This is consistent with this argument. Put differently, fast divergence is also difficult to observe in the data. Only 'intermediate' divergence will be easily observed.

Second, classical and random urban growth models are also compatible when the effects of $a_{2t}X_{it}$ are short lived, that is when there is mean reversion in a_2 or in X. Mean reversion in a_2 corresponds to the situation where a permanent characteristic has a positive effect over a period of time and negative effect over another. In the us for instance, it is possible that hot summers were conducive to population growth after the development of air-conditioning but not before. Proximity to coal and iron was arguably a factor of growth during the late 19th and early 20th century. It became irrelevant after.

Mean reversion in X corresponds instead to the situation where the determinants of growth are temporary in cities. For instance, it could be that receiving roads is a factor of urban growth as suggested by Duranton and Turner (2008) and that the growth of roads is proportional to population[15]. In that case, what growth regressions and classical urban models treat as explanatory variables need to be thought of as the shocks in random growth models. This observation suggests that shocks in the context random growth models need not be equated with residuals in urban growth regressions. It also highlights the need for more microfoundations for the shocks in random growth models.

These remarks suggest that different time horizons between classical and random growth models may go a long way towards making them compatible with each other. Classical urban growth models, which constitute the theoretical underpinning of standard urban growth regressions, may be looking at the growth of cities around a particular period whereas random growth models may have a much longer time horizon. In that case, classical urban growth models help us uncover short run proximate factors of urban growth whereas random growth models help us understand the fundamental mechanics that drive urban growth in the long run.

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[15] Duranton and Turner (2008) reject this second condition for the last quarter of the 20th century but not for the 25 years prior to this which saw a major expansion in the us road system.

5. CONCLUSION

Classical urban growth models deliver important insights about the growth cities and illuminate a number of other issues such as the respective roles of diversity and specialisation in urban development. These models also make some suggestions about the relative sizes of cities. However they do not naturally generate a key stylised fact, Zipf's law.

Random urban growth models have recently proposed a number of explanations for this stylised fact. They are also a challenge for classical urban growth models because they work according to radically different principles. In a nutshell, classical growth models are all about trends whereas random growth models are all about shocks. As shown above, these frameworks can coexist but only under fairly restrictive conditions. An exact statement of these conditions is still needed. A more systematic empirical exploration of random growth models is also needed. To do this, urban economists will need to use techniques which are outside their normal toolbox.

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