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NK DSGE MODEL FOR TWO LARGE ECONOMIES:  
THE EU AND THE US**

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# RICARDIAN EQUIVALENCE PROPOSITION IN A NK DSGE MODEL FOR TWO LARGE ECONOMIES: THE EU AND THE US

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## Abstract

This paper examines the macroeconomic effects of active fiscal policy management coupled with a monetary policy that follows the Taylor principle. The objective is to investigate the relevance of the Ricardian Equivalence Proposition (REP) in a framework where two large open economies interact and a fraction of the consumers is financially constrained. According to an estimated vector autoregressive model, a positive shock in government expenditure leads to an increase in private consumption (at odds with the permanent income hypothesis). The channels are studied in a fully microfounded dynamic stochastic general equilibrium model economy calibrated for the Euro Area (EU-12) and for the United States. The crucial parameter that drives the break of the REP is the share of financially constrained consumers. Firms produce tradable varieties in a monopolistic competition framework and pricing is à la Calvo, which leads to nominal price stickiness. Labor varieties are immobile across countries and are demanded in an aggregated fashion by firms. Fiscal policy is specified as a time-consistent rule. We simulate through impulse-response functions parameterizations that yield results consistent with the REP, and estimate a subset of deep parameters employing Bayesian techniques.

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# LA PROPOSICIÓN DE EQUIVALENCIA RICARDIANA EN UN MODELO DSGE NK PARA DOS ECONOMÍAS GRANDES: UE Y EEUU

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## Resumen

Este artículo examina los efectos macroeconómicos de un manejo activo de la política fiscal junto con una política monetaria fundada en el principio de Taylor. El objetivo es investigar la relevancia de la proposición de equivalencia Ricardiana (REP por sus siglas en inglés) en un marco donde dos economías grandes y abiertas interactúan, y donde una fracción de los consumidores se encuentra financieramente limitada. De acuerdo con un modelo de vectores autorregresivo estimado, un impulso positivo del gasto de gobierno conlleva un aumento en el consumo real privado (en contradicción con la hipótesis del ingreso permanente). Los canales son estudiados con un modelo estocástico y dinámico de equilibrio general microfundado, calibrado para la Unión Europea (EU-12) y para Estados Unidos. El resultado de la REP depende crucialmente de la participación de consumidores que se comportan de un modo no-Ricardiano. Las firmas producen variedades de productos transables internacionalmente en un mercado de competencia monopolística. La fijación de precios es *à la Calvo*, lo que implica rigidez nominal de precios. La fuerza de trabajo no migra y las variedades de habilidades son demandadas en forma agregada por las firmas. La política fiscal sigue una regla consistente en el tiempo. Se realizan funciones impulso-respuesta con parametrizaciones del modelo que generan respuestas coherentes con la REP, y se estima un conjunto de parámetros estructurales empleando técnicas Bayesianas.

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# 1 Introduction

Fiscal policy (FP) management has re-emerged as an interesting theme in recent studies. As the world economy has become increasingly interlinked due to the deepening of the integration process, natural actors to cope with or internalize spillover effects are today supranational institutions. This is specially true for European supranational institutions which were born as a result of a broad and long-lasting political consensus.<sup>1</sup> Under such a consensus European Monetary Union (EMU) members formalized policy delegations with special mandate towards policy coordination. However, looking back into the economic and political integration process, it does not display a straight path. Often, EMU members have learned policy lessons in a rude way. Just to mention one example, the just elected President F. Mitterrand, embarked alone on a Keynesian expansionary programme at a time of world recession (1981) and when France's partners were pursuing restrictive economic policies. The experiment failed, the Franc heavily depreciated, reserves vanished and the measures had to be reverted. Implementing uncoordinated policies became more and more costly. As similar experiences occurred in Europe, the countries eventually signed an agreement on limiting their abilities to set independent fiscal policies. Hence, they built consensus on bounding rules, such as the Maastricht Treaty which specified a limit on the budget deficit to output ratio of three per cent. This limit was complemented with a balanced-budget guideline known as Stability and Growth Pact (SGP) in 1997 (Breuss (2007)). As a result, these institutions served as explicit pillars for the EMU, which once settled meant a firm background for the common currency definitively introduced in 2002.

In 2005, Italy and Germany found it hard to meet the requirements of the SGP because their economic growth slowed down during the period 2000-2004 making difficult to roll-over debt services. In order to break these tendencies in the GDP growth rates, several EMU members instrumented budgetary expansions.<sup>2</sup> At those times, several organizations such as the Organization for Economic Cooperation and Development (OECD), the International Monetary Fund (IMF), the European Commission, *inter alia*, were concerned with tax cuts because consumers could not consider the corresponding issuance of bonds as net wealth in their portfolios. This effect is commonly known as the Ricardian Equivalence Proposition (REP) and would imply that a today's tax cut would not affect agents' permanent income because they are *exactly* compensated by the discounted (higher) future tax liabilities that the government would need in order to honor its outstanding debt. In other words, they feel equally rich with and without the policy change.

Similarly, there are important implications if the REP holds in the U.S. because the huge Federal budget deficit that resulted from the fiscal package sent to the U.S. Congress in the beginning of 2009 will be reverted sooner or later.

In rigor, the REP could be regarded as a direct implication of the permanent hypothesis, though it hinges on quite strong assumptions, which are briefly summarized in what follows:

1. the economic agent has a horizon that extends to the *infinite*. Finite life durations, in addition, require: (a) parents who care about the utility of their children in overlapping models, saving for bequest motives, see Barro (1974); (b) an individual who faces uncertainty about how long her life will last, see Blanchard (1985);
2. there is no uncertainty of the future income streams, meaning that insurance markets exist and effectively cover bad states of nature, (see, Feldstein (1988));
3. the output —also population, in the case of pay-as-you-go social security system— does not grow enough to enable the government to rollover the debt continuously;
4. individuals are fully rational, i.e., bounded rationality is neglected;
5. borrowing differential rates is insignificant in terms of the required information to distinguish good from bad investment projects;
6. the new debt is allocated entirely in portfolios owned by home consumers (as long as the share of the debt stock held by foreigners remains constant);
7. taxes are non-distortionary, i.e., the marginal tax rate does not change during the relevant horizon.

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<sup>1</sup>A list of supranational European institutions includes: the European Commission, the European Parliament, the European Court of Justice, the European Central Bank and the Court of Auditors.

<sup>2</sup>In a currency area, member countries count with fiscal policy as a feasible policy instrument to mitigate recessions (or decelerations) of economic activity.

The objective of this paper is to investigate under which conditions the REP holds employing dynamic stochastic general equilibrium (DSGE) models and to examine the short-run interaction of two large open economies, where a fraction of the consumers are financially constrained (and therefore acting in a non-Ricardian way) in the framework of a fully microfounded DSGE model. The modeling strategy followed forces the separation of consumers in two types: (i) "unconstrained" consumers which have access to financial markets and able to smooth consumption; and (ii) remaining consumers face financial constraints, who in practice have no access to borrowing/lending arrangements. As a result, their ability to consume is fully determined by their disposable income (defined as total taxes subtracted from the wage income) as proposed by Campbell & Mankiw (1989). Savings, commonly defined as the residual of what is not consumed, are maintained in bonds and cash solely by unconstrained consumers.<sup>3</sup> This separation of consumers' types has important consequences in open economies which so far, in our opinion, have not been sufficiently analyzed in the literature. This is our contribution.

Briefly, the rest of the model contains firms that produce tradable varieties in a monopolistic competition framework with nominal price stickiness. Labor's varieties are immobile across countries and demanded by firms in an aggregated fashion. Fiscal policy is specified as a time consistent rule.

The DSGE model is suited to deal with business cycle fluctuations, but it contains parameters that must be recovered from long run datasets. In this aspect, some hint to properly calibrate the model come from the estimation of an unrestricted vector autoregression (VAR) model. We proceed to estimate a simple VAR model with classical methods employing macroeconomic aggregate data of the EU-12 it proxies the Euro area.<sup>4</sup> In particular, we uncover the implied dynamic multipliers in order to calibrate key (long run) parameters of a DSGE model that match developments resulting from a fiscal stimulus. The crucial parameter that drives the break of the REP is the share of non-Ricardian consumers. We plot impulse response functions (IRFs) of key macroeconomic variables to make clearer the point considering several shocks. Finally, we estimate a subset of parameters employing Bayesian techniques.

The paper's structure is as follows: in Section 2 we briefly review the literature on REP; in Section 3 we present the results of a VAR specification for the Euro area (EU-12); in Section 4 we lay out a fully specified theoretical model that includes internationally tradable goods and two (large) open economies presenting consumers and firms problems and we close the model imposing equilibrium conditions; in Section 5 we present both monetary and FP rules to which the CB and the fiscal authority are committed, respectively. Section 6 calibrates the theoretical model following the literature and assuming plausible real and nominal shocks IRFs are drawn. Section 7 estimates a subset of deep parameters of the DSGE model with Bayesian techniques. Finally, Section 8 concludes.

## 2 A brief literature review

Sargent & Wallace (1981) were among the first to jointly address intertemporal theoretical aspects of monetary and fiscal policies; however, it is often assumed in the literature that FP "adjusts" by an appropriate selection of lump-sum transfers to neutralize the effects of (non-distortionary) taxes. In particular, it is quite standard in DSGE models to assume that FP is flexible in the aforementioned way and monetary policy reacts to inflation and the output gap in a way that is consistent with the so-called Taylor principle.

Leeper (1991) emphasizes that it is crucial, to distinguish between active versus passive FP. The former refers to the case where one policy authority pursues its objective unconstrained by the other's behavior, while the latter is consistent with a constrained behavior. So, it is clear that to study FP impulses that are non-monetarist and non-Ricardian in nature monetary and FP must be active. Leith & von Thadden (2006) reports that when both policies are active, conditions that assure the existence of a unique equilibrium sequence (i.e., its solution is determinate in the sense of Blanchard & Kahn (1980)) become stricter. Intuitively, this is because the solution must be compatible with non-explosive sequences for both the price level and the public debt.

The early empirical literature seems to be inconclusive in favor of the validity of the REP. Becker (1995), Seater (1993), Bernheim (1988) surveyed the evidence from aggregate data samples and came up with contradictory conclusions. Further evidence that appeared in the 1990s employing VARs have reached a sort of consensus, giving support to basically three main findings: (i) disregarding the particular identification strategy utilized, there is *no study* suggesting that consumption responds negatively to an expansionary budgetary policy (a prediction that would be against the permanent income hypothesis); (ii) the real wage responds positively when a government

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<sup>3</sup>We acknowledge, however, that to some extent the presence of the so-called rule-of-thumb consumers are merely a shortcut that may disregard other interesting avenues of research (e.g. learning). The advantage is that it allow us to keep our model within a medium scale meanwhile it leads to valuable output in order to derive policy recommendations.

<sup>4</sup>The EU-12 aggregate comprises the following countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Portugal, Spain and The Netherlands. The data employed is detailed in the Data Appendix.

impulse takes place; and (iii) that fiscal spillovers to neighbor countries occur via trade, a phenomenon not found in the former literature. The idea is that a fiscal expansion stimulates home output because of the shift of aggregate demand, leading to more domestic imports (and thus more exports and output of foreign countries). This evidence points to spillover effects solely since government imports seem to be insignificant, Giuliadori & Beetsma (2005), Fatas & Mihov (2001a), Fatas & Mihov (2001b), *inter alia*.

In contrast, studies that test the REP/permanent income hypothesis using samples with individual/household data, seem to offer support to the REP, e.g., De Juan & Seater (1999), Campbell & Cocco (2007), *inter alia*.

Which modification for the DSGE model would be necessary to reproduce the VAR evidence? It is now quite known that a RBC model predicts a fall in consumption and in real wages as a result of a FP expansion (Baxter & Crucini (1993)). Linnemann & Schabert (2003) add nominal rigidities in an otherwise standard RBC model and report that a positive fiscal impulse yields a drop in consumption and an increase in real wages. In view of this result, recently the standard NK DSGE model with separable utility was extended with consumer heterogeneity: some of them are fully rational and optimizers, whereas others are financially constrained and decide consumption with a pattern characterized as hand-to-mouth consumption. In particular, Mankiw (2000) assumes that the latter type of agents behave in a myopic way because they do not have access to financial markets to smooth consumption. Thereby, their consumption depends on disposable income rather than on permanent income. The main reference we follow in this paper is Galí *et al.* (2007b). While these authors focus on a closed economy, we extend their framework to two (large) open economies, that we further take to EU-12 and U.S. data. This extension is promising since it will allow us to characterize the international spillover effect as well as relative price fluctuations that would explain the pattern of spillovers, which is a novelty in the literature. Galí *et al.* (2007b)'s model predicts that if rule of thumb agents are relatively many w.r.t. the total, aggregate consumption responds positively to the government impulse (because the effect is dominated by constrained consumers).

A second strand of the real business cycle (RBC) literature assumes homogeneity of consumers but these present non-separable preferences. This literature employs preferences of the types KPR King *et al.* (1988) and GHH Greenwood *et al.* (1988). Linnemann (2006) presents a model where consumption increases after a shock to government. The model has flexible prices and assumes that hours and consumption are complements. Other results of the model, though logically consistent seem to be counterintuitive because the utility function considers consumption as an inferior good along with a downward sloping labor supply. In contrast, Monacelli & Perotti (2008) compare KPR and GHH preferences and conclude that in both consumption is a normal good, thereby a shock in government expenditure increase consumption only if real wages and hours go up.

Recent evidence for the U.S. employing a DSGE model suggests that fiscal multipliers are small in normal times but sharply increase when the interest rate hits the zero lower bound, Christiano *et al.* (2009). The reason for this is that an increase in government spending lowers desired national savings and breaks the effects of Keynes (1936)'s paradox of thrift (a mechanism by which Keynes explains why depressions exacerbates). It is beyond the scope of this paper to appropriately treat the zero lower bound and its effects on fiscal multipliers; instead, we focus on normal times (as Cogan *et al.* (2009) do).

### 3 Assessing the effects of an expansionary FP in the closed economy

In this section we focus on the effects on consumption resulting from an expansionary FP, particularly on fiscal multipliers. The main hypothesis is that the essential macroeconomic variables can be represented by a VAR model (this method is valid if we take for granted assumptions set by Hamilton (1994), p. 261).

In a recent paper, Galí *et al.* (2007b) present evidence for the U.S., suggesting that expansionary FP, say an increase of public expenditures, leads to an expansion of private consumption of nondurable goods. As we referred in the introductory section, practically all studies so far have found a positive consumption dynamic multiplier of the FP on impact and also at different (future) time horizons. Hinging on these results, we estimate a VAR using key aggregate macroeconomic series for the EU-12 aggregate. The estimation exercise is constrained to consider a very parsimonious VAR model that is fit with data from 1991Q1 to 2006Q4.<sup>5</sup> The estimation follows closely Galí *et al.* (2007b), to make our results fully comparable with evidence from what they call “small” VAR model, which is just a stylized closed-economy model for the U.S.

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<sup>5</sup>Notice that to be fully comparable with the model developed in Section 4 the model has to add foreign economy variables for the rest of the world. However, technical limitations prevent us to conduct such an application. There are aggregation concerns for data on the EU-12 entity such as the unification process that took place in Germany in 1991, the implementation of the ECU (a basket of currencies of the European Community member states during the convergence process to adopt the Euro, *inter alia*). Consequently, adding more than four variables in the VAR becomes a risky enterprise for identification.

We consider the following VAR (4) specification:<sup>6</sup>

$$\mathbf{y}_t = \Gamma_0 + \Gamma_1 \mathbf{y}_{t-1} + \Gamma_2 \mathbf{y}_{t-2} + \Gamma_3 \mathbf{y}_{t-3} + \Gamma_4 \mathbf{y}_{t-4} + \varepsilon_t \quad (1)$$

where  $\mathbf{y}_t \equiv (G_t, Y_t, C_t, PD_t)'$  and  $\varepsilon_t$  are  $4 \times 1$  vectors. The former includes the following endogenous variables: government expenditure, GDP, private consumption and primary deficit, while  $\varepsilon_t$  is a disturbance vector with mean the null vector and variance-covariance matrix  $\Sigma$ . The ordering of  $\mathbf{y}_t$  reflects that government spending is assumed predetermined relative to the other variables included in the VAR since we assume shocks' identification employ a Cholesky decomposition.

In the framework of difference equations, a dynamic multiplier of (1) for one-quarter-ahead effect of variable  $j$  to variable  $i$  can be defined as:

$$\frac{\partial \mathbf{y}_{t+1}^{(i)}}{\partial \mathbf{y}_t^{(j)}} \equiv \Gamma_1^{(i,j)}. \quad (2)$$

As in Galí *et al.* (2007b), we consider EU-12 aggregates of general government spending (general government spending net of military expenditures), gross domestic product, private consumption and general government budget deficit. Table 1 summarizes our findings for EU-12 aggregates which are in accordance with those reported by Galí *et al.* (2007b). The dynamic multipliers of consumption and GDP are positive when expansionary FP takes place within a two years horizon. This evidence suggests that consumers in the Euro area react to increases in public expenditure increasing their consumption as well, response that is at odds with the well known REP and the neoclassical model. If the latter model would apply, consumers would have behaved differently, taking for granted that future tax slips will increase to cover the current budgetary deficit (that equals the amount in bonds that the government needs to sell today).

Our results are also comparable with other studies. Perotti (2005) focuses on the evidence of five OECD countries including the U.S. He finds that expansionary FP triggers GDP increases that are smaller than 1-to-1, except for the U.S. in the pre-1980 period. The fiscal multipliers have become substantially weaker over time. Moreover, Blanchard & Perotti (2002) finds that multipliers are smaller in Europe compared to the U.S. A recent survey on fiscal multipliers is Spilimbergo *et al.* (2009). There is a difference in the sense that we find FP multipliers that have hump-shaped form, that contrast with the monotonically decreasing multipliers reported by Galí *et al.* (2007b).

Table 1. Fiscal multipliers of public expenditure in the EMU

Quarter	Private Consumption		GDP	
	Full government spending	Government spending excluding military	Full government spending	Government spending excluding military
1 <sup>st</sup>	0.061 (0.04)	0.059 (-0.11)	0.047 (0.51)	0.044 (0.15)
2 <sup>nd</sup>	0.148	0.143	0.139	0.132
3 <sup>rd</sup>	0.177	0.168	0.206	0.196
4 <sup>th</sup>	0.252 (0.09)	0.237 (0.24)	0.274 (0.31)	0.261 (-0.12)
5 <sup>th</sup>	0.299	0.280	0.344	0.329
6 <sup>th</sup>	0.278	0.255	0.374	0.359
7 <sup>th</sup>	0.249	0.221	0.413	0.398
8 <sup>th</sup>	0.198 (0.19)	0.166 (0.32)	0.434 (0.28)	0.417 (0.34)

**Note:** Authors' calculations for the EU-12 aggregates in a closed economy model. Comparable FP multipliers for the U.S. in a similar model are estimated by Galí *et al.* (2007a) on Table 1, p. 233, which are reported in brackets to compare them.

## 4 The model

To begin with, we assume that there are two regions in the world economy. Each region of the world economy is populated by a continuum of economic agents “consumers” that live infinitely and that are normalized to one. Home consumer  $j$  is indexed by  $j \in [0, 1]$ .<sup>7</sup> Likewise, foreign consumers are denoted by  $j^*$  and indexed by  $j^* \in [0, 1]$ . Moreover, each region has an administrative authority—the national government—, which levies taxes and issues bonds with which it can purchase goods (or transfer money).

<sup>6</sup>Alternative estimates with different lags lengths provided us with the information to determine that an adequate  $l$  is 4 quarters. Relevant information criteria considered were Akaike information criterion and Schwarz criterion.

<sup>7</sup>We can think about dynasties of individuals that continue living through their children owing to intergenerational solidarity, to relax the problem of choosing a discrete living period. Alternatively, we may consider adding a probability of death of the representative individual, as Blanchard (1985).



There are two types of rational consumers in both economies: (i) financially constrained consumers (myopic though fully rational) and (ii) those that can access financial markets. Since we focus on the short run, we assume that these types contain a fixed number of agents (leading to constant shares), i.e., those constrained agents do not learn how to overcome the constraint.

#### 4.1 Consumers' intratemporal problem

All goods varieties (home and foreign produced) are tradable, have world markets and are indexed with  $h$  and  $f$  indices that belong to the  $[0, 1]$ .<sup>8</sup> Agent  $j$ 's consumption is devoted to purchase home and foreign goods. Prices are denominated in home currency; thus, if the law of one price (LOOP) holds, then  $P_{F,t}(f) = \mathcal{E}_t P_{F,t}^*(f)$  and foreign goods prices are converted by the nominal exchange rate  $\mathcal{E}_t$ .<sup>9</sup> We assume that the representative agent takes varieties' prices as given when choosing quantities. Thus, agent  $j$ 's nominal consumption is,

$$P_t C_t^j = \int_0^1 P_{H,t}(h) C_{H,t}^j(h) dh + \int_0^1 P_{F,t}(f) C_{F,t}^j(f) df. \quad (3)$$

Individual aggregate consumption is represented by the index  $C_t^j$ , which is specified as a Constant Elasticity of Substitution (CES) function, with relevant elasticity of substitution ( $\eta_c > 1$ ) of home and imported goods ( $H$  and  $F$ , respectively):

$$C_t^j \equiv \left[ \varphi^{\frac{1}{\eta_c}} \left( C_{H,t}^j \right)^{\frac{\eta_c-1}{\eta_c}} + (1-\varphi)^{\frac{1}{\eta_c}} \left( C_{F,t}^j \right)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}, \quad (4)$$

where  $\varphi$  stands for the share in the consumption of home goods in terms of the total tradable goods,  $T$ . The counterpart  $(1-\varphi)$  refers to the share of foreign produced goods,  $F$ , in terms of  $T$ .<sup>10</sup>

Notice that aggregate consumption (4) involves the consumption indices of home and foreign produced varieties. In particular, we assume that these indices are Dixit-Stiglitz aggregators of all consumed varieties with elasticity of substitution  $\theta_h$  and  $\theta_f$ , greater than one. The consumption indices for home and foreign tradable goods are given by:

$$C_{H,t}^j \equiv \left[ \int_0^1 C_{H,t}^j(h)^{\frac{\theta_h-1}{\theta_h}} dh \right]^{\frac{\theta_h}{\theta_h-1}} \quad \text{and} \quad (5)$$

$$C_{F,t}^j \equiv \left[ \int_0^1 C_{F,t}^j(f)^{\frac{\theta_f-1}{\theta_f}} df \right]^{\frac{\theta_f}{\theta_f-1}}. \quad (6)$$

The intratemporal problem solved by agent  $j$  is to minimize  $P_t C_t^j$  subject to Equation (4) choosing  $C_{H,t}^j$  and  $C_{F,t}^j$ . Optimality conditions yield optimal consumption of home produced and imported goods:

$$C_{H,t}^j = \varphi \left[ \frac{P_{H,t}}{P_t} \right]^{-\eta_c} C_t^j, \quad (7)$$

$$C_{F,t}^j = (1-\varphi) \left[ \frac{P_{F,t}}{P_t} \right]^{-\eta_c} C_t^j, \quad (8)$$

which, combined would yield  $C_{H,t}^j = \frac{\varphi}{(1-\varphi)} \left[ \frac{P_{F,t}}{P_{H,t}} \right]^{\eta_c} C_{F,t}^j$ . Notice that prices are denominated in home currency (again if the LOOP holds, then  $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ ) and the "true" CPI that results is:

$$P_t = \left[ \varphi P_{H,t}^{1-\eta_c} + (1-\varphi) P_{F,t}^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}. \quad (9)$$

At the variety level, there are two additional intratemporal problems to be solved. Given the solutions of  $C_{H,t}^j$  and  $C_{F,t}^j$  obtained above, optimal demand functions of both home and imported varieties can be analogously derived.

<sup>8</sup>A more general model would define an additional index for varieties such that  $h \in [0, \kappa]$  along with  $f \in [\kappa, 1]$ . For simplicity, we assume that the 'segment' of varieties is equal to population shares of the two countries, which does not imply loss of generality (it is just a normalization applied to a continuous variable's range).

<sup>9</sup> $\mathcal{E}_s$  is defined as the price of a unit of foreign currency in terms of the home currency. Notice that this definition is the inverse of the financial nominal exchange rate.

<sup>10</sup>When  $\eta_c \rightarrow 1$ ,  $C_s \equiv \frac{C_{H,s}^\varphi C_{F,s}^{1-\varphi}}{\varphi^\varphi (1-\varphi)^{1-\varphi}}$  and the aggregate price index is  $P_s = P_{H,s}^\varphi P_{F,s}^{1-\varphi}$ .

Intratemporal problems involve the minimization of the expenditure spent on: (i) home varieties,  $P_{H,t}C_{H,t}^j$ , subject to Equation (5) by choosing  $C_{H,t}^j(h)$ ; and (ii) foreign varieties  $P_{F,t}C_{F,t}^j$  subject to Equation (13) choosing  $C_{F,t}^j(f)$ . Optimal varieties' demands result:

$$C_{H,t}^j(h) = \left[ \frac{P_{H,t}(h)}{P_{H,t}} \right]^{-\theta_h} C_{H,t}^j, \quad (10)$$

$$C_{F,t}^j(f) = \left[ \frac{P_{F,t}(f)}{P_{F,t}} \right]^{-\theta_f} C_{F,t}^j, \quad (11)$$

where again under LOOP  $P_{F,t}(f) = \mathcal{E}_t P_{F,t}^*(f)$ . The associated home and imported tradable goods price indices (components of the CPI) are as following:

$$P_{H,t} = \left[ \int_0^1 P_{H,t}(h)^{1-\theta_h} dh \right]^{\frac{1}{1-\theta_h}}, \quad (12)$$

$$P_{F,t} = \left[ \int_0^1 (P_{F,t}(f))^{1-\theta_f} df \right]^{\frac{1}{1-\theta_f}}. \quad (13)$$

These prices (identified at the optimum with Lagrange multipliers) can be interpreted as minimum prices to buy one bundle of  $C_{H,t}^j$  and  $C_{F,t}^j$ , respectively.

Following Benigno (2004), the government sector also demands home tradable varieties,  $G_{H,t}(h)$ . We assume that the government faces the same elasticity of substitution and relative prices as agent  $j$  does, so that the relevant demand is:

$$G_{H,t}(h) = \left[ \frac{P_{H,t}(h)}{P_{H,t}} \right]^{-\theta_h} G_{H,t}, \quad (14)$$

where aggregated government purchases,  $G_{H,t}$ , is defined similarly as Equation (5), with a relationship between  $G_{H,t}$  and  $G_t$  given by an equation similar to Equation (7), though the government home bias parameter,  $\varphi_G$ , is assumed to be one. This is justified since as we mentioned in Section 2, the government expenditure on imported goods seems to have a small impact on the foreign economy, see Giuliodori & Beetsma (2005).  $G_t$  is taken as exogenous.<sup>11</sup>

For the foreign economy, a similar set of demands holds both for the representative consumer  $j^*$  and for the foreign government.

## 4.2 Consumer's intertemporal problem

### 4.2.1 Unconstrained consumers

Agent  $j$  seeks to maximize the present value of her expected lifetime utility,  $\check{U}_t^j$ :

$$\check{U}_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ U(C_s^j, bC_{s-1}^j, \varepsilon_{U_s}) + L\left(\frac{M_s^j}{P_s}\right) - V\left(N_{H,s}^j\right) \right], \quad (15)$$

which depends on aggregate (current and lagged) consumption,  $C_s^j$ , real money balances,  $\frac{M_s^j}{P_s}$ , and work effort in terms of hours worked in the home tradable good sector,  $N_{H,s}^j$ . Lagged consumption is the simplest way to introduce internal habit formation in consumption.<sup>12</sup> In addition, liquidity services provided by real balances of money generate utility  $L(\cdot)$ . What is more, disutility  $V(\cdot)$  is derived from work effort—in terms of worked hours,  $N_{H,s}^j$ —devoted in the production of  $H$  goods.<sup>13</sup> Finally,  $\varepsilon_{U_s}$  is a disturbance that is *iid* with mean one and standard error  $\sigma_{\varepsilon_U}$ .

<sup>11</sup>The parliament and the administration get to a consensus based on political motives about its level.

<sup>12</sup>The introduction of internal habit formation rather than external habit formation is motivated by the study of Grishchenko (2007). Using long-horizon aggregate stock market returns, she found that there is strong support for internal habit formation preferences, which decays slowly over time. In addition, this feature adds realism to the model's predictions (IRFs) and help explaining asset pricing puzzles. In particular, IRFs are much alike than those obtained with an unrestricted VAR. See Fuhrer (2000).

<sup>13</sup> $\check{U}_t^j$  is a real-valued function, additive and separable. We assume that its components,  $U(\cdot)$ ,  $L(\cdot)$  and  $V(\cdot)$ , are all increasing in their arguments. Moreover, under usual assumptions,  $U(\cdot)$  and  $L(\cdot)$  are concave in consumption and  $\frac{M_s^j}{P_s}$  ( $U_{CC} < 0$  and  $L_{\frac{M^j}{P}} \frac{M^j}{P} < 0$ ), while  $V(\cdot)$  is convex in  $N_{H,s}^j$ ,  $V_{N_H^j N_H^j} > 0$ .

It is rational for the agent to maximize its expected future utility (15) conditional upon the information available in period  $t$ , subject to the sequence of dynamic budget constraint in real terms ( $\forall s : t, t + 1, \dots$ ):

$$\begin{aligned} & \frac{(1 - \tau_w)W_{H,s}^j N_{H,s}^j}{P_s} + TR_s^j + \frac{(1 - \tau_D)(D_{H,s}^j + D_{MF,s}^j)}{P_s} + \\ & \cong C_s^j + \frac{M_s^j - M_{s-1}^j}{P_s} + \frac{1}{P_s} \left( \frac{B_{s+1}^j}{1 + I_s} - B_s^j \right) + \frac{1}{P_s} \left( \frac{\mathcal{E}_s B_{s+1}^{*j}}{\varepsilon_{UIP_s} (1 + I_s^*)} - \mathcal{E}_s B_s^{*j} \right). \end{aligned} \quad (16)$$

In the LHS of the budget constraint the sources of agent's real income can be found: net of tax wage income from work supplied to the home tradable goods sector,  $N_{H,s}^j$ , real dividends net of tax,  $(1 - \tau_D) (D_{H,s}^j + D_{MF,s}^j) / P_s$ , expected real returns on home-issued bonds holdings and real transfers from the government. Furthermore, the uses of resources are in the RHS: consumption, variation in the stock of money holdings in real terms and variation in the stock of riskless home and foreign bonds (without any subscript).

Following Woodford (2003), we assume that home asset (securities) markets are complete, thus we treat different assets comprising the portfolio as one, see Plasmans *et al.* (2007).  $B_s^j$  denotes the portfolio's value at the beginning of the period, which includes bonds and shares. This greatly simplifies the algebra and it is supported by the known envelope theorem that assures that all investment alternatives should produce the same real return at the optimum.  $B_s^j$  is like a world-traded bond and it is denominated in issuer's currency, in which worldwide agents take positions to finance domestic consumption (indirectly trade deficits). The riskless (non-contingent) nominal return is denoted by  $I_s$  with one-period-maturity. Notice that  $\varepsilon_{UIP_s}$  is a shock that is *iid.* with mean one and standard error  $\sigma_{\varepsilon_{UIP}}$  that accounts for a shock in the UIP condition.

Other home financial assets are per capita nominal dividend-coupons  $D_{H,s}^j$ ,  $D_{MF,s}^j$  and money. The government claims a tax rate  $\tau_D$  to period  $s$  per capita nominal dividends.<sup>14</sup> Aggregate money demand,  $M_s = \int_0^1 M_s^j dj$ , equalized money supply which is under control of the CB, though indirectly, since the operating instrument is the nominal interest rate.

We impose that both home and foreign agents at the beginning are not indebted, i.e.:<sup>15</sup>

$$B_{-1}^j = \mathcal{E}_{-1} B_{-1}^{*j} = 0 \quad B_{-1}^{j*} = \mathcal{E}_{-1} B_{-1}^{*j*} = 0. \quad (17)$$

To inquire about the implications of Equation (17), first, let us define the stock of wealth at time  $s$  in terms of each currency of home agent as  $\mathcal{W}_{H,s} \equiv \frac{M_{s-1}^j + B_s^j + \mathcal{E}_s B_s^{*j}}{P_s}$  and  $\mathcal{W}_{F,s} \equiv \frac{M_{s-1}^{j*} + \frac{B_s^j}{\mathcal{E}_s} + B_s^{*j*}}{P_s^*}$  for foreign. Note that those assets are maintained in the portfolio for different reasons; while money facilitates transactions, bonds are used to store value and are issued to finance foreign current account deficits.<sup>16</sup> Initially, we can easily check that the wealth is identical across members of the different regions.<sup>17</sup> Moreover, recalling that asset markets are complete within the regions, thus we can predict perfect risk sharing in consumption. Consequently, we state that the problem of the agent is fully described maximizing the utility, Equation (15) subject to the budget constraint (16), given the initial conditions (17), the sequences of prices and incomes and the transversality condition. See Section 4.5.

In order to solve the model, the lifetime utility function features a constant relative elasticity risk aversion (CRRA),

$$\check{U}_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{\varepsilon_{U_s} (C_s^j)^{1-\sigma}}{(1-\sigma) (C_{s-1}^j)^{b(1-\sigma)}} + \chi_M \left( \frac{M_s^j}{P_s} \right)^\varepsilon - \frac{\chi_N (N_{H,s}^j)^{1+\iota}}{1+\iota} \right], \quad (18)$$

where  $\sigma > 0$  is a parameter that measures agent's disposition to take risks —the greater when the agent is more risk averse— and the inverse of the intertemporal elasticity of substitution in consumption, while the parameter  $b \in [0, 1]$  stands for the persistence in consumption or internal habit formation. As was defined above,  $\varepsilon_{U_s}$  stands for a shock to preferences which specifically affects consumption decisions. Moreover,  $\chi_M$  and  $\chi_N$  are scale parameters. Finally,  $\varepsilon$  is the elasticity of demand of money and  $\iota$  is the inverse of the elasticity of substitution in exert work effort respect to real wage.

<sup>14</sup>Real home per capita dividends are given by  $\frac{1}{P_s} \int_0^1 (D_{H,s}^j + D_{MF,s}^j) dj$ .

<sup>15</sup>If home country has a positive stock of debt, it must add to the riskless interest rate a premium, which is a non-decreasing function of debt position; see Schmitt-Grohe & Uribe (2001).

<sup>16</sup>Shares also are used to store value; however, it likely delivers more volatility and capital gains could be non-positive. These aspects lead us to consider risk aversion of agents as crucial.

<sup>17</sup>To confirm that, plug Eq. (17) into  $\mathcal{W}_{H,s}$  and  $\mathcal{W}_{F,s}$ .

The resulting optimality conditions of the consumer problem are sufficient when budget constraint (16) is exhausted and the solution is "interior".<sup>18</sup> First, representative home agents would like to smooth consumption along time as much as they can, thus initially at time  $s$ , for every positive time interval  $\tau - s$ , consumers equalize the marginal rate of substitution between consumption at  $\tau$  and present consumption, i.e.,  $\beta^{\tau-s} E_s [U_C(C_\tau^j)] = U_C(C_s^j)$ .<sup>19</sup>

$$\beta^{\tau-s} E_s \left[ \frac{\varepsilon_{U\tau} (C_\tau^j)^{-\sigma}}{(C_{\tau-1}^j)^{b(1-\sigma)}} - b\beta \frac{\varepsilon_{U\tau+1} (C_{\tau+1}^j)^{1-\sigma}}{(C_\tau^j)^{b(1-\sigma)+1}} \right] = \frac{\varepsilon_{Us} (C_s^j)^{-\sigma}}{(C_{s-1}^j)^{b(1-\sigma)}} - b\beta E_s \left[ \frac{\varepsilon_{Us+1} (C_{s+1}^j)^{1-\sigma}}{(C_s^j)^{b(1-\sigma)+1}} \right]. \quad (19)$$

They decide consumption *within*-period such that marginal utility of real consumption equals the marginal utility of real income:

$$\Lambda_s^j = \frac{\varepsilon_{Us} (C_s^j)^{-\sigma}}{(C_{s-1}^j)^{b(1-\sigma)}} - b\beta E_s \left[ \frac{\varepsilon_{Us+1} (C_{s+1}^j)^{1-\sigma}}{(C_s^j)^{b(1-\sigma)+1}} \right]. \quad (20)$$

Second, agent  $j$ 's money demand is given by:

$$P_s \frac{\varepsilon_{\chi M} \left( \frac{M_s^j}{P_s} \right)^{\varepsilon-1}}{\Lambda_s^j} = \frac{I_s}{1+I_s} E_s \left[ \frac{P_s}{P_{s+1}} \right], \quad (21)$$

as real money balances increases utility; however, given the nominal interest rate  $I_s$ , having money in pockets imply an opportunity cost represented by the "expected" real interest rate of bonds forgone.

Third, given information up to time  $s$  the home optimality conditions w.r.t.  $B_s^j$  allow us to price the internationally traded bond (in foreign currency) obtaining at any time for home (likewise for foreign bonds):

$$\frac{1}{P_s} E_s \left[ \frac{\Lambda_s^j}{1+I_s} - \beta \Lambda_{s+1}^j \right] = \frac{1}{P_s} E_s \left[ \frac{\mathcal{E}_s \Lambda_s^j}{\varepsilon_{UIP_s} (1+I_s^*)} - \beta \mathcal{E}_{s+1} \Lambda_{s+1}^j \right] = 0, \quad (22)$$

which result in home and foreign Euler equations:

$$\frac{\Lambda_s^j}{P_s} = \beta (1+I_s) E_s \left[ \frac{\Lambda_{s+1}^j}{P_{s+1}} \right], \quad (23)$$

$$\frac{\Lambda_s^{j*}}{P_s^*} = \beta \varepsilon_{UIP_s} (1+I_s^*) E_s \left[ \frac{\Lambda_{s+1}^{j*}}{P_{s+1}^*} \right]. \quad (24)$$

Optimum domestic holding of foreign bonds is:

$$\Lambda_s^j = \beta \varepsilon_{UIP_s} (1+I_s^*) E_s \left[ \Lambda_{s+1}^j \frac{P_s}{P_{s+1}} \frac{\mathcal{E}_{s+1}}{\mathcal{E}_s} \right]. \quad (25)$$

The uncovered interest rate parity (UIP) condition results from dividing (23) by (25),

$$\varepsilon_{UIP_s} (1+I_s^*) E_s \left[ \frac{\mathcal{E}_{s+1}}{\mathcal{E}_s} \right] = (1+I_s), \quad (26)$$

which conveys, at any period  $s$ , the same information as the non-arbitrage across financial markets condition:

$$E_s \left[ \frac{\Lambda_{s+1}^j}{\Lambda_s^j} \right] = E_s \left[ \frac{RER_s}{RER_{s+1}} \frac{\Lambda_{s+1}^{j*}}{\Lambda_s^{j*}} \right], \quad (27)$$

<sup>18</sup>Optimality conditions for our implicit utility function (15) follow. First, Eq. (20) reads as  $U_C(C_s^j) = \Lambda_s^j$ . Second, Eq. (21) is  $P_s \frac{L_M \left( \frac{M_s^j}{P_s} \right)}{\Lambda_s^j} = \frac{I_s}{1+I_s} E_s \left[ \frac{P_s}{P_{s+1}} \right]$ . The other FOCs remain the same.

<sup>19</sup>If consumption were specified to be state dependent,  $C(S_s, s)$ , consumers would equalize the marginal rate of substitution between consumption at  $s = \tau$  and present consumption to the appropriate discount factor,  $\Psi_{H,s,\tau}(S_\tau)$ , i.e.,  $\beta^{\tau-s} E_s \left[ \frac{U_C^j(C_\tau(s_\tau))}{U_C^j(C_s)} \right] = \Psi_{H,s,\tau}(s_\tau)$ . Moreover, the budget constraint is binding at any time and for all histories  $h_s$ . Ascari (2004) suggests that models where the duration of the relevant contracts is state-dependent may characterize more accurately observed data developments. At least for the Belgium economy; however, Aucremanne & Dhyne (2005) find that state-dependent contracts seems to be empirically irrelevant for explaining price rigidities.

which results from combining (23) and (24) and where the real exchange rate is defined as  $RER_s \equiv \frac{\mathcal{E}_s P_s^*}{P_s}$ . As in Chari *et al.* (2002), solving (27) backwards yields:

$$RER_s = k \frac{\Lambda_s^j}{\Lambda_s^{j^*}}, \quad (28)$$

where  $k$  depends on initial conditions.<sup>20</sup> We follow the literature and assume that  $k = 1$ , meaning that the initial indebtedness is zero and also that consumers have similar consumption bundles.

#### 4.2.2 Constrained consumers

Regarding the share of the households that are financially constrained,  $\lambda^r \in (0, 1)$ , it is important to notice that they consume as much as they get in their disposable income. A representative consumer of this type,  $j^r$ , behaves as a rule-of-thumb consumer, i.e., optimizing intratemporally, but he does not so intertemporally. Technically, it means that the Lagrangean for the consumer can be broken down for each period, where utility function as Equation (18) is maximized.

An expansionary FP will shift aggregate demand and output, as disposable income rises rule-of-thumb consumers will consume more, disregarding any future tax liability. If and only if the higher consumption of rule-of-thumb consumers offset the downward shift of optimizers' consumption, then aggregate consumption will go up. As a result, we would be able to replicate with our model the evidence reported in Section 3. In the following section we explore the aggregation.

Following Coenen & Straub (2005), Di Bartolomeo & Manzo (2007), Di Bartolomeo *et al.* (2007), Galí *et al.* (2007b), the binding budget constraint of the representative rule-of-thumb consumer,  $j^r$  (we omit it to simplify notation) can be stated as:

$$C_t^r = \frac{(1 - \tau_w) W_{H,t} N_{H,t}^r}{P_t} + TR_t^r, \quad (29)$$

where, a simple comparison with Equation (16) reveals that these consumers do not save: (i) there are no dividends proceeds; and (ii) they are not able smooth consumption by keeping money or bonds. As in the case of optimizing households, hours  $N_{H,t}^r$  are determined by firms' labor demand and are not chosen optimally by each household given the wage  $W_{H,t}$  (see Section 4.4). Finally,  $TR_t^r$  are transfers (if  $T_t^r < 0$ ) or taxes paid in a lump-sum fashion (if  $TR_t^r > 0$ ).

#### 4.2.3 Aggregation of consumers' choice variables

As we stated in the previous section, the economy embrace both types of consumers: optimizers and rule-of-thumb. The share of the formers in the total consumers is  $1 - \lambda^r$ . Therefore, aggregated consumption,  $C_t^{agg}$ , is obtained as the weighted average of the respective aggregated consumptions:

$$C_t^{agg} \equiv \lambda^r \int_0^1 (C_t^r)^{j^r} dj^r + (1 - \lambda^r) \int_0^1 C_t^j dj. \quad (30)$$

Likewise, for the number of hours worked,

$$N_{H,t}^{agg} \equiv \lambda^r \int_0^1 (N_{H,t}^r)^{j^r} dj^r + (1 - \lambda^r) \int_0^1 N_{H,t}^j dj. \quad (31)$$

Notice that we assume that each firm decides how much labor to hire (given the wage, see Section 4.4), and allocates its labor demand uniformly across households (the type does not signal any difference in the marginal productivity of labor). As a result,  $N_{H,t}^r = N_{H,t}^j$ , and Equation (31) reduces to  $N_{H,t}^{agg} = N_{H,t}^r = N_{H,t}^j$ .

### 4.3 Producers and importers

Home tradable goods are produced by a large amount of firms in the home economy. Part of this production is sold at the home market and the remaining abroad as exports. Suppose that there is a continuum of independent (producers) firms indexed in the  $(0, 1)$  interval, each of them enjoying monopolistic power on varieties produced. In addition, there are a continuum of importers, also indexed in the  $(0, 1)$  interval. A common characteristic is that producers and importers enjoy monopolistic power, being able to set prices that maximize their profits.

<sup>20</sup>Specifically,  $k = \mathcal{E}_0 P_0^* U_{C0} / P_0 U_{C^*0}$ .

### 4.3.1 Technology and marginal costs

Final goods producers at home are indexed by  $i \in (0, 1)$  and have access to the following technology:<sup>21</sup>

$$Y_t(i) = A_t(i) N_{H,t}(i)^{\psi_H} - FC(i), \quad (32)$$

where  $\psi_H$  stands for the mean (constant) productivity,  $N_{H,t}(i)$  is the number of hours hired  $FC(i)$  is a (constant) fixed cost of managing the firm and  $A_t(i)$  denotes an exogenous technological process:

$$A_t(i) = \rho_A A_{t-1}(i) + \varepsilon_{y,t}, \quad (33)$$

where  $\rho_A \in (0, 1)$  measures the persistency and  $\varepsilon_{y,t}$  is an *iid.* disturbance with mean zero and constant variance  $\sigma_{\varepsilon_y}^2$ .

We assume that import firms simply repackage and give a domestic brand to otherwise standardized goods, which they finally sell in the domestic market.

The firm's problem is to minimize the total cost which is given by the wage bill,  $W_{H,t} N_{H,t}(i)$ , subject to (32). The optimal labor demand is obtained by inversion of (32):

$$N_{H,t}(i) = \left[ \frac{Y_t(i) - FC(i)}{A_t(i)} \right]^{\frac{1}{\psi_H}}, \quad (34)$$

which plugged into the objective,  $W_{H,t} N_{H,t}(i)$ , leads to the total cost function  $TC_H = W_{H,t} \left[ \frac{Y_t(i) - FC(i)}{A_t(i)} \right]^{\frac{1}{\psi_H}}$ . Differentiation of  $TC_H$  w.r.t.  $Y_t(i)$  yields the domestic marginal cost:

$$MC_H = \frac{W_{H,t}}{\psi_H} [Y_t(i) - FC(i)]^{\frac{1-\psi_H}{\psi_H}} \left[ \frac{1}{A_t(i)} \right]^{\frac{1}{\psi_H}}, \quad (35)$$

similarly for the foreign firm we obtain  $N_{F,t}(i^*)$ ,  $TC_F$  and  $MC_F$ .

The domestic (foreign) firm faces wages set by unions,  $W_{H,t}^{\mathcal{H}}(i)$ , where there are a continuum of competitive unions  $\mathcal{H}(\mathcal{F}) \in [0, 1]$ . The firm  $i$  chooses the specific labor varieties so that it minimizes the cost  $W_{H,t}^{\mathcal{H}} N_{H,t}^{\mathcal{H}}(i)$  subject to the bundle definition:

$$N_{H,t}(i) \equiv \left[ \int_0^1 (N_{H,t}^{\mathcal{H}}(i))^{\frac{\varrho_L - 1}{\varrho_L}} d\mathcal{H} \right]^{\frac{\varrho_L}{\varrho_L - 1}}, \quad (36)$$

Cost minimizations yields firms  $i$ 's labor demand:

$$N_{H,t}^{\mathcal{H}}(i) = \left( \frac{W_{H,t}^{\mathcal{H}}}{W_{H,t}} \right)^{-\varrho_L} N_{H,t}(i), \quad (37)$$

where  $\varrho_L > 1$  is the (constant) elasticity of substitution among any domestic labor varieties and average domestic wages is<sup>22</sup>

$$W_{H,t}^{1-\varrho_L} = \int_0^1 (W_{H,t}^{\mathcal{H}})^{1-\varrho_L} d\mathcal{H}. \quad (38)$$

The foreign firm  $i^*$  solves a similar problem with relevant elasticity given by  $\varrho_L^*$ .

### 4.3.2 Pricing

In the empirical literature, e.g., Aucremanne & Dhyne (2005), price movements reveal different degrees of stickiness. In particular, for those varieties that are effectively traded, either exported or imported, one key determinant of the price (besides the marginal cost) is the nominal exchange rate, which easily propagates with imperfect pass-through. Stickiness may be the result of multiple causes; however, the implied effect is that propagation takes place imperfectly to both real and nominal variables.

We model price stickiness following Calvo (1983), who assume that domestic firms adjust their price infrequently and in such an event, they reset prices according to 'price signals', which follow an exogenous *iid.* Poisson process

<sup>21</sup>The  $i$ -index links firms and varieties since we assume full specialization of production.

<sup>22</sup>By definition,  $N_{H,t}^{\mathcal{H}} \equiv \int_0^1 N_{H,t}^{\mathcal{H}}(i) di$ .

with constant probability. Hence, firms set prices in staggered 'contracts' of random duration. For instance, this probability in the home tradable goods market is  $1 - \varphi_H^{(i)}$ , meaning that firm  $i$  would be able to announce a new price with probability  $1 - \varphi_H^{(i)}$ ; otherwise, the old price, remains in effect (e.g., instrumented in a contract). Hence, this firm  $i$  will not be able to adjust its price on its market with probability  $\varphi_H^{(i)}$ . This probability is the so-called Calvo price parameter.<sup>23</sup>

To analyze the maximization problem of the producer, notice that the law of large numbers can be applied since the number of firms is large, so that we drop the Calvo price parameter's upper index  $i$ . If firm  $i$  of type  $m = \{H, MF\}$  gets to announce a new contract in period  $t$ , at that time it chooses a price to maximize the value of its discounted profit stream over states of nature in which that price holds.<sup>24</sup> Thus, domestic firm  $i$  of type  $m$  solves:

$$\max_{\{\mathbf{P}_{m,t}(i)\}} E_t \left\{ \sum_{a=0}^{\infty} \Delta_{t,t+a}(j)(\varphi_m)^a \left[ \left[ \check{\mathbf{P}}_{m,t}(i) \right]' \mathbf{Y}_{m,t+a}(i) - TC_{m,t+a}(i) \left( [\mathbf{Y}_{m,t+a}(i)]' \iota_m \right) \right] \right\}, \quad (39)$$

subject to relevant demand functions.<sup>25</sup> In Equation (39), the (nominal) discount factor from  $t$  to  $t+a$ , applied by firm  $i$  to the stream of future profits, results from (22) for home assets as  $\Delta_{t,t+a}^j \equiv \beta^a \frac{E_t[\Lambda_{t+a}^j]}{\Lambda_t^j}$  with  $\beta$  the households' discount factor and  $\frac{E_t[\Lambda_{t+a}^j]}{\Lambda_t^j}$  the household  $j$ 's marginal utility of nominal wealth, which indeed does not differ across agents because of complete assets markets ( $\Lambda_t^j = \Lambda_t$ ). In addition,  $\mathbf{P}_{m,t}(i)$  is the appropriate vector of relevant prices of home produced goods in sector  $m$  and  $TC_{m,t+a}(i)(\cdot)$  is the (nominal) total cost of production at period  $t+a$  of firm  $i$  of domestic type  $m$ , which is a function of firm  $i$ 's total output during period  $t$ . Moreover,  $\iota_m$  is the unity vector consisting of an appropriate number of ones which equals the number of markets and  $(\varphi_m)^a$  is the vector of Calvo probabilities for price vector  $\check{\mathbf{P}}_{m,t}(i)$  remaining unchanged for producer  $i$  of domestic type  $m$ . Entries of this vector correspond to elements of relevant prices  $\check{P}_{H,t}(i)$  and  $\check{P}_{MF,t}(i)$  and  $\varphi_m \equiv [\varphi_H, \varphi_{MF}]'$ .

Solving Equation (39) subject to relevant demands, we obtain the following optimality condition (see Plasmans *et al.* (2007), Subsection 8.1.1 and Appendix I):

$$\check{\mathbf{P}}_{m,t}(i) = \begin{bmatrix} \frac{\theta_h}{(\theta_h - 1)} \frac{E_t \left[ \sum_{a=0}^{\infty} (\varphi_H \beta)^a \frac{\Lambda_{t+a}^j}{\Lambda_t^j} MC_{H,t+a}(Y_{H,t+a}(i))(P_{H,t+a})^{\theta_h} Y_{H,t+a} \right]}{E_t \left[ \sum_{a=0}^{\infty} (\varphi_H \beta)^a \frac{\Lambda_{t+a}^j}{\Lambda_t^j} Y_{H,t+a} (P_{H,t+a})^{\theta_h} \right]} \\ \frac{\theta_f}{(\theta_f - 1)} \frac{E_t \left[ \sum_{a=0}^{\infty} (\varphi_{MF} \beta)^a \frac{\Lambda_{t+a}^j}{\Lambda_t^j} MC_{MF,t+a}(C_{F,t+a}(i))(P_{F,t+a})^{\theta_f} Y_{F,t+a} \right]}{E_t \left[ \sum_{a=0}^{\infty} (\varphi_{MF} \beta)^a \frac{\Lambda_{t+a}^j}{\Lambda_t^j} C_{F,t+a} (P_{F,t+a})^{\theta_f} \right]} \end{bmatrix}, \quad (40)$$

where firm  $j$ 's prices  $\check{\mathbf{P}}_{m,t}(j, i)$  of domestic type  $m$  are aggregated over consumers as it is done for wages in Plasmans *et al.* (2007), Equation (102) in Appendix B.2, resulting in vectors  $\check{\mathbf{P}}_{m,t}(i)$ .<sup>26</sup>

Since any domestic price at period  $t$ ,  $P_{m,t}(i)$ , is assumed to be a CES aggregator of the predetermined price  $\{P_{m,t-1}(i)\}$  and the newly set price  $\check{P}_{m,t}(i)$  according to Calvo in (40), this domestic price index for a typical domestic company  $i$  of type  $m$  can be written as:

$$(P_{m,t}(i))^{1-\theta_n} = \varphi_m (P_{m,t-1}(i))^{1-\theta_n} + (1 - \varphi_m) \left( \check{P}_{m,t}(i) \right)^{1-\theta_n}, \quad (41)$$

where the  $n$ -subscript stands for  $h$  and  $f$ . It can be shown that three different Phillip curves can be derived if Equation (40) is rewritten in terms of appropriate inflation rates.<sup>27</sup>

<sup>23</sup>So that the average duration of a price contract, i.e. the average duration between two subsequent price adjustments is  $\left(1 - \varphi_H^{(i)}\right)^{-1}$  periods, since  $0 < \varphi_H^{(i)} < 1$ . For example, a Calvo price parameter equal to 0.75 implies an average duration of 4 periods.

As  $\varphi_H^{(i)} \rightarrow 0$ , firm  $j$  in the final goods sector sets its prices each period, which is the flexible price case.

<sup>24</sup>Notice that prices quoted by consumption importers are invoiced in the domestic currency and exporters in the foreign currency.

<sup>25</sup>For example, under equilibrium, the domestic aggregate optimal demand of good  $i$ , becomes:

$$y_{H,t+a}(i) = \left( \frac{p_{H,t}(i)}{P_{H,t+a}} \right)^{-\theta_h} Y_{H,t+a},$$

taking equilibrium conditions into account,  $Y_{H,t+a} = C_{H,t+a} + G_{t+a}$ .

<sup>26</sup>Notice that as  $\varphi_m \rightarrow 0$ , the relevant firms reset their prices each period (the flexible price case) and a particular firm  $j$  of type  $m$  sets its price as a (monopolistic) markup over its marginal cost, i.e. then  $\check{P}_{m,t}(i) \rightarrow \frac{\theta_m}{(\theta_m - 1)} MC_{m,t}(i)$  with  $MC_{m,t}(j) \equiv MC_{m,t}(Y_{m,t}(j))$ .

<sup>27</sup>By exploiting the recursive form of the infinite summations and log-linearizing w.r.t. the steady state values.

## 4.4 Staggered wage setting

Labor is immobile across countries. Each country's labor market presents monopolistic competition where firms are wage takers.<sup>28</sup> It is assumed that labor suppliers have learned that they possess special abilities that can be imperfectly substituted by firms which is delegated to wage unions. Regardless the consumer type, constrained or unconstrained, the centralization by union makes labor efforts equal in equilibrium. This is an artifice that allows us to treat the wage setting problem aside from the consumer problem.<sup>29</sup> The same strategy is considered in Galí *et al.* (2007b), Furlanetto (2007), Forni *et al.* (2007), Coenen & Straub (2005), among others.

Following Calvo (1983), there is a known probability that gives unions the opportunity to reset home and foreign wages,  $(1 - \varphi_W)$  and  $(1 - \varphi_W^*)$ , respectively.<sup>30</sup> Each draw of the 'signal' is exogenous and independent of past realizations and assumed to follow a Bernoulli distribution. Unions that are not signaled, a share  $\varphi_W$  of them, keep the nominal wage at the level observed in the previous period. In the event that the union is signaled, it will maximize the present value utility of *any* employee-member by choosing an optimal wage,  $\check{W}_{\mathcal{H},t}$ , subject to the domestic labor demand:

$$N_{H,t}^{\mathcal{H}} = \left( \frac{W_{H,t}^{\mathcal{H}}}{\check{W}_{\mathcal{H},t}} \right)^{-\varrho_L} N_{H,t},$$

that results from integration of (37) over firms  $i$ . The firm  $i$ 's problem yields the following optimality condition:

$$E_t \left[ \sum_{a=0}^{\infty} (\beta \varphi_W)^a N_{H,t+a}^{\mathcal{H}} \left[ \Lambda_{t+a}^j (1 - \tau_w) \frac{\check{W}_{\mathcal{H},t}}{P_{t+a}} + \mu_W \chi_N (N_{H,t+a}^{\mathcal{H}})^{\iota} \right] \right] = 0 \quad (42)$$

where  $\mu_W \equiv \frac{\varrho_L}{\varrho_L - 1}$  stands for the desired *gross* mark-up over the marginal rate of substitution,  $MRS_{t+a} \equiv -\frac{U_{N_H}}{U_C} = \frac{\chi_N (N_{H,t+a}^{\mathcal{H}})^{\iota}}{\Lambda_{t+a}}$ .

Since all signaled agents set the same wage  $\check{W}_{\mathcal{H},t} = \check{W}_t$ , the aggregate wage of the home economy explained by Equation (38), is the weighted sum of the newly set wage and the previous one:

$$W_t^{1-\varrho_L} = \varphi_W W_{\mathcal{H},t-1}^{1-\varrho_L} + (1 - \varphi_W) \check{W}_{\mathcal{H},t}^{1-\varrho_L}. \quad (43)$$

## 4.5 Equilibrium conditions

The labor market equilibrium conditions are simple since there is no migration (similarly for the foreign country):

$$N_{H,t}(i) = \int_0^1 N_{H,t}^{aggr}(i, j) dj, \quad \forall t \quad (44)$$

Regarding the bonds issued by the government, those denominated in home currency are used to finance public expenditures, while those denominated in foreign currency is the counterpart amount of the accumulated previous net trade balances,  $\sum_{k=0}^t NX_{t-k}$ , where  $NX_t$  is defined as exports minus imports at period  $t$ .

We can state the bond equilibrium condition as follows:

$$B_t = (1 - \lambda^r) \left( \int_0^1 B_t^j dj + \mathcal{E}_t \int_0^1 B_t^{*j*} dj^* \right), \quad \forall t. \quad (45)$$

Resource constraints for the domestic and the foreign countries are:

$$Y_{H,t} = \left[ \frac{P_{H,t}}{P_t} \right]^{-\eta_c} (\varphi C_t^{aggr} + G_t) + (1 - \varphi) \left[ \frac{P_{H,t}^*}{P_t^*} \right]^{-\eta_c} C_t^{aggr*}, \quad (46)$$

$$Y_{F,t} = \left[ \frac{P_{F,t}^*}{P_t^*} \right]^{-\eta_c} (\varphi C_t^{aggr*} + G_t^*) + (1 - \varphi) \left[ \frac{P_{F,t}}{P_t} \right]^{-\eta_c} C_t^{aggr}. \quad (47)$$

<sup>28</sup> Alternatively, if labor were homogeneous, the supply of working hours in the home economy would be coming from a consumers  $j$ 's FOC:  $\chi_N (N_{H,s}^j(i))^{\iota} = \Lambda_s W_{H,s}$ , where  $N_{H,s}^j(i)$  denote hours supplied of the individual  $j$  to home representative firm  $i \in [0, 1]$  and the clearing-market nominal wage is  $W_{H,s}$ .

<sup>29</sup> In a setup where all consumer are unconstrained, the wage contract is set so that it maximizes the expected discounted sum of agent  $j$ 's utility flows, while supplying all the labor requested. For instance, under Calvo wage schedule, we refer to Equation (80) in Subsection 8.2.2 and Appendix K in Plasmans *et al.* (2007).

<sup>30</sup> In applying Calvo wage setting, we follow Coenen & Straub (2005). Furlanetto (2007) shows that similar results can be obtained employing quadratic adjustment costs. Schmitt-Grohe & Uribe (2006) show that, up to second order approximation, there are negligible differences between the Erceg *et al.* (2000) model, i.e., where each household is the monopolistic supplier of a differentiated type of labor input and the one in which households supply a homogenous labor input that is transformed by monopolistically competitive labor unions into a differentiated labor input (Schmitt-Grohe & Uribe (2005)).



## 5 Fiscal and Monetary policy

So far the model is a simplified version of Plasmans *et al.* (2007) extended with the two types of consumers and with active FP. In this section, we comment on the assumed fiscal and monetary policies followed by the home country. Similarly, the foreign economy is subject to same restrictions and rules.

### 5.1 Fiscal policy

In this section, we present a simplified structure of the government of the home economy. The government levies taxes from dividends,  $\tau_D$ , and from the wage bill,  $\tau_w$ . However, it is not bounded by genuine resources: it can issue bonds and sell them to the agents and provide money for transactions. In any period  $t$ , the outstanding bonds stock or/and money increase (decrease) if expenditures are higher than tax proceeds. Expenditures of the government are explained by purchases of goods, which are sold by firms. The government faces the following nominal budget constraint (GBC):

$$\begin{aligned} \tau_D \left[ \int_0^1 \left( D_{H,t}^j + D_{MF,t}^j \right) dj \right] + \tau_w W_{H,t} N_{H,t} + \int_0^1 \left( M_t^j - M_{t-1}^j \right) dj + \int_0^1 \frac{B_{t+1}^j}{(1+I_t)} dj - \int_0^1 B_t^j dj \\ + \int_0^1 \frac{B_{t+1}^{j*}}{(1+I_t)} dj^* - \int_0^1 B_t^{j*} dj^* \geq P_t \int_0^1 TR_t^j dj + P_t G_t. \end{aligned} \quad (48)$$

Equation (48) includes on the left hand side labor revenues, money creation and net domestic and foreign borrowing, while on the right hand side outlays of government revenues (transfers and goods purchases) are considered. In particular, we assume that the government do not disfavor any type of consumers, so we assume the transfers are the same,  $TR_t^j = (TR_t^r)^{j^r} \Rightarrow TR_t = TR_t^r$  (by aggregation, see Equation (30)).

Abstracting from different government levels, we assume that lump-sum transfers (taxes are negative) are set according to the following rule:

$$TR_t = G_t^{\phi_1} Y_{H,t}^{\phi_2} e^{\varepsilon_{TRt}}, \quad (49)$$

where transfers are tied to real domestic public expenditure and (possibly) also to domestic output volumes when  $\phi_2 \neq 0$ . Besides,  $\varepsilon_{TRt}$  stands for an exogenous shock to transfers that is *iid.* with mean zero and standard error  $\sigma_{\varepsilon_{TR}}^2$ . Of course, this is an arbitrarily simple manner to endogenize transfers. We assume  $\phi_1 > 0$ , which intuitively means that transfers are positively correlated with public purchases. Likewise, a similar explanation justifies that  $\phi_2 > 0$ .

The real public expenditure  $G$  is assumed to be exogenous. We construct the transformed stream:  $g_t \equiv \left( \frac{G_t - G}{Y} \right)$ , that accounts for deviations of the real expenditure from its steady state, normalized by the steady state real GDP. It evolves according to the following AR(1) process:

$$g_t = \rho_g g_{t-1} + \varepsilon_{gt}, \quad (50)$$

where  $0 < \rho_g < 1$  and  $\varepsilon_{gt}$  represents an *iid.* shock with mean zero and constant variance  $\sigma_{\varepsilon_g}^2$ .<sup>31</sup>

### 5.2 Monetary policy

Designing monetary policy rules concerns the choice of (a) the monetary policy instruments, (b) the variables to be targeted and (c) their targeted values. In theory, a Central Bank (CB) can define different monetary policy instruments to be targeted as, e.g., (i) interest rate targeting, (ii) exchange rate targeting and (iii) money supply targeting. In the literature, variables that are often targeted are: (1) real output (gap), (2) (changes in) prices, (3) (changes in) exchange rates, (4) (changes in) interest rates, (5) a combination of real output and prices in the form of nominal GDP.

Kydland & Prescott (1977) claim that monetary policy effectiveness depends, not only on policy actions undertaken, but also on the public perception about these actions and its expectations about future actions. Consequently, policy is more effective when future actions are predictable, so that a monetary authority can commit itself to a certain course of policies. As Atoian *et al.* (2004) argue, commitment permits the CB to distribute 'policy medicine' over time. For example, when the CB wishes to offset inflation that will result from a supply shock, under commitment, it can raise interest rates moderately provided that it maintains higher rates for a period of time. In contrast, in the case of lack of commitment, a higher initial rate increase will be necessary because of the public doubts that the CB will sustain this interest rate increase.

<sup>31</sup>Alternatively, an endogenous expenditure rule could be considered, for instance, one that includes cyclical GDP in Equation (50).

Atoian *et al.* (2004) also argue that optimal commitment does not need to take the form of a reaction function with fixed coefficients. In general, an optimal commitment rule has the form of a state-contingent plan that presents the instrument setting as a function of the history of exogenous shocks. However, optimal commitment is not practical because, first, as noted by Woodford (2003), it is not feasible to provide an advance listing of all relevant contingencies and, second, it is difficult for the public to distinguish between discretion and a complicated contingency rule. Both problems are avoided when the CB commits to a rule with fixed coefficients.

Which form should such a rule with fixed coefficients take? Since most CBs use a short-term interest rate as their control variable, we are focusing on rules that relate this short-term interest rate to economic conditions. The most famous and widely used examples of simple (short-term) interest rate rules are those proposed by John Taylor. The **standard Taylor rule** (see Taylor (1993b)), which relates the interest rate target to inflation and output (gap) in a log-linearized form, is:

$$i_{t,t+1} = \lambda_0 + \lambda_1 \pi_t^{(4)} + \lambda_2 y_{H,t} + \varepsilon_{it}, \quad (51)$$

where  $\pi_t^{(4)} \equiv \sum_{j=0}^3 \pi_{t-j}$  and  $y_{H,t}$  are annualized domestic inflation and (logarithmic) deviations of domestic output w.r.t. their respective steady state values, which are assumed to be the target variables of the home monetary authority. Moreover,  $\lambda_0$  is a constant defined as  $\left(\frac{1+\pi}{\beta}\right)^4$ . Taylor (1993a) assigns coefficient values consistent with an accurate description of Federal Reserve policy for quarterly data and domestic annualized inflation:  $\lambda_1 = 1.5$  and  $\lambda_2 = 0.5/4 = 0.15$ . The intuition for the value of the former reaction parameter is that the CB must raise the interest rate by more than any increase in inflation in order to raise the real rate of interest, cool the economy and move inflation back toward its target. This refers to the so-called “lean against the wind” policy advocated by Taylor.

Moreover, Taylor (1999) suggests an alternative that allows for interest-rate smoothing:

$$i_{t,t+1} = (1 - \lambda_3) \left[ \lambda_0 + \lambda_1 \pi_t^{(4)} + \lambda_2 y_{H,t} \right] + \lambda_3 i_{t-1,t} + \varepsilon_{it}, \quad (52)$$

where we assume that the smoothing procedure follows an  $AR(1)$  process with smoothing parameter  $\lambda_3$  and what is inside the braces is the CB’s desired interest rate that comes from the standard rule (51).<sup>32</sup>

McCallum (1997) argues that the policymakers’ reaction is more accurate if it is based on lagged and not on current values of output and inflation. In response, Taylor (1999) suggests an alternative form of his rules where lagged values of output and inflation replace the current values in (51). In contrast, Clarida *et al.* (2000), *inter alia*, argue that rules in which the CB reacts to forward-looking variables are optimal in the case of a quadratic objective function of the monetary authorities, which will also be utilized in this paper. The difference between backward-looking, contemporaneous and forward-looking monetary rules relates primarily to the information set of the monetary policymakers. For instance, in the case of a contemporaneous rule, the current inflation rate, on which the CB is assumed to have adequate information, is targeted.

### 5.3 Shocks driving the economy

We consider several shocks, all of them characterized as exogenous processes. They have similar structure as Equation (50) and their introduction in the model is motivated because they are useful to drive the set of endogenous variables of the economy. We take into account the following shocks:

1. UIP shock,  $\varepsilon_{UIP_t}$ , interpreted as shift in the foreign asset’s return;
2. domestic (foreign) productivity shocks,  $\varepsilon_{yt}(\varepsilon_{yt}^*)$ , understood as a shift in the production function;
3. domestic (foreign) MP rule shock,  $\varepsilon_{it}(\varepsilon_{it}^*)$ ;
4. domestic (foreign) transfers shock,  $\varepsilon_{TRt}(\varepsilon_{TRt}^*)$ , interpreted as an increase of transfers to the public;
5. domestic (foreign) expansionary budgetary policy shock,  $\varepsilon_{gt}(\varepsilon_{gt}^*)$ ;
6. domestic (foreign) preference shock,  $\varepsilon_{Ut}(\varepsilon_{Ut}^*)$ , interpreted as a upward shift of full the preference map.

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<sup>32</sup>We calibrated  $\lambda_3 = 0.75$  throughout all exercises. This assumption allows us to interpret that the CB’s interest rate that actually prevails now will have no-effect in the 4-quarters ahead interest rate.

These shocks are considered as unexpected by the agents. Moreover, in general, they have AR(1) structures that turn simple the modeling of the degree of persistency. For example, whilst a permanent shock is consistent with a  $\rho$ -parameter equal to one, a purely temporary shock is with  $\rho = 0$ .

Formally, exogenous processes are gathered in the vector  $\xi_t$ , that follows the following AR(1) process:

$$\xi_t = \underset{(11 \times 1)}{\rho} \underset{(11 \times 11)}{\xi_t} + \underset{(11 \times 1)}{\mathbf{v}_t}, \quad (53)$$

where  $\mathbf{v}_t$  are innovations, i.e., *iid* with mean zero vector and diagonal variance-covariance matrix  $\Sigma_{\mathbf{v}}$ . Formally,  $\xi_t \equiv (\varepsilon_{UIP_t}, \varepsilon_{y_t}, \varepsilon_{y_t}^*, \varepsilon_{i_t}, \varepsilon_{i_t}^*, \varepsilon_{TR_t}, \varepsilon_{TR_t}^*, \varepsilon_{g_t}, \varepsilon_{g_t}^*, \varepsilon_{U_t}, \varepsilon_{U_t}^*)'$ ,  $\rho \equiv \mathbf{diag}(\rho_{UIP}, \rho_A, \rho_A^*, \rho_i, \rho_i^*, \rho_{TR}, \rho_{TR}^*, \rho_g, \rho_g^*, \rho_U, \rho_U^*)$ , and  $\mathbf{v}_t \equiv (v_{\varepsilon_{UIP}}, v_{\varepsilon_y}, v_{\varepsilon_y^*}, v_{\varepsilon_i}, v_{\varepsilon_i^*}, v_{\varepsilon_{TR}}, v_{\varepsilon_{TR}^*}, v_{\varepsilon_g}, v_{\varepsilon_g^*}, v_{\varepsilon_U}, v_{\varepsilon_U^*})'$ .

## 6 Calibration and simulation methodology

A RE equilibrium is then a set of processes of the endogenous variables that satisfy both first order conditions (from corresponding optimal problems) and equilibrium conditions at all dates  $t \geq 0$  given the exogenous processes included in the vector  $\xi_{t+t}$ .

The non linear DSGE model containing both economies can be specified as an implicit multivariate function as the most compact manner:

$$E_t [F_\theta(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{v}_t)] = \mathbf{0}, \quad (54)$$

where  $\mathbf{y}_t \in \Lambda \subseteq \mathbb{R}^n$  is the set of endogenous variables, while  $\mathbf{v}_t$  are structural innovations defined above. The function  $F_\theta : \Lambda^3 \times \mathbb{R}^{11} \rightarrow \Lambda$  is real in  $\mathbb{C}^2$  parameterized by the real vector  $\theta \in \Theta \subseteq \mathbb{R}^p$  where  $p$  is the dimension of the parameter space that include deep parameters.

Assuming the existence of a non linear stochastic difference equation (unique, stable and invariant) of the form:

$$\mathbf{y}_t = \mathbf{H}_\theta(\mathbf{y}_{t-1}, \mathbf{v}_t), \quad (55)$$

that solves (54) where  $\mathbf{H}_\theta$  is a collection of policy and transition functions. Repeated substitution of Equation (55) into (54) provides a system where  $\mathbf{y}$  and  $\mathbf{v}$  are included in the information set at time  $t$ . Given that we know the exact form (our hypothesized model) of  $F_\theta$ , our unknown is  $\mathbf{H}_\theta$ .

The model (54) has a solution at a fixed point that is known as the deterministic ( $E_t[\mathbf{v}_t] = \mathbf{0}$ ) steady state. Formally,

$$F_\theta(\mathbf{y}^*(\theta), \mathbf{y}^*(\theta), \mathbf{y}^*(\theta), \mathbf{0}) = \mathbf{0}, \quad (56)$$

where  $\mathbf{y}^*(\theta) = \mathbf{H}_\theta(\mathbf{y}^*(\theta), \mathbf{0})$ . The steady state is a crucial element to solve our model given the fact that we use local approximation methods. That means that Jacobians and Hessians, etc. that arise because of the Taylor expansion of (54) are evaluated at  $\mathbf{y}^*(\theta)$ .

The model is log-linearized around the steady state (first order approximation, i.e.,  $\hat{\mathbf{y}}_t \equiv \mathbf{y}_t - \mathbf{y}^*(\theta)$ ) and reshuffled in a linear state space system as suggested by Sims (2002) using a guess policy function:

$$\mathbf{B}_1 \hat{\mathbf{y}}_t + \mathbf{B}_2 \hat{\mathbf{y}}_{t-1} + \mathbf{C} \mathbf{v}_t + \mathbf{D} \eta_t = \mathbf{0}, \quad (57)$$

since we solve (57) with his RE algorithm. It is primarily based on the systematic perturbation of the policy function around the steady state. Note that  $\eta_t$  contains expectational errors (so that we drop the expectation operator).

Second, Equation (57) is rewritten after applying the QZ factorization as:<sup>33</sup>

$$\mathbf{Q}' \mathbf{\Lambda} \mathbf{Z}' \hat{\mathbf{y}}_t + \mathbf{Q}' \mathbf{\Omega} \mathbf{Z}' \hat{\mathbf{y}}_{t-1} + \mathbf{C} \mathbf{v}_t + \mathbf{D} \eta_t = \mathbf{0}.$$

Third, generalized eigenvalues of  $\mathbf{B}_1$  and  $\mathbf{B}_2$  are reorganized in  $\mathbf{\Lambda}$  and  $\mathbf{\Omega}$  in increasing order from the left to the right ( $\mathbf{Q}'$  and  $\mathbf{Z}'$  are reorganized accordingly). Redefining transformed variables as  $\check{\mathbf{y}}_t \equiv \mathbf{Z}' \hat{\mathbf{y}}_t$  and premultiplying the system by  $\mathbf{Q}$  results in the following upper triangular system:

$$\mathbf{\Lambda} \check{\mathbf{y}}_t + \mathbf{\Omega} \check{\mathbf{y}}_{t-1} + \mathbf{Q} \mathbf{C} \mathbf{v}_t + \mathbf{Q} \mathbf{D} \eta_t = \mathbf{0},$$

where  $\check{\mathbf{y}}_t \equiv (\check{\mathbf{y}}_{1t}, \check{\mathbf{y}}_{2t})'$  (similarly for other matrices), with the block  $\mathbf{\Lambda}_{12}$  has been zeroed out (corresponding to forward-looking variables  $\check{\mathbf{y}}_{2t}$ ). We refer to Sims (2002) for the details in solving the following step, the fourth, where  $\check{\mathbf{y}}_{2t}$  is solved iterating forwardly, and then (once  $\check{\mathbf{y}}_{2t}$  is known)  $\check{\mathbf{y}}_{1t}$  is solved iterating backwardly. The

<sup>33</sup>Matrices  $\mathbf{Q}'$  and  $\mathbf{Z}'$  are unitary matrices (with  $\mathbb{R}$  or  $\mathbb{C}$  numbers), while  $\mathbf{\Omega}$  and  $\mathbf{\Lambda}$  are upper triangular.

critical issue arises in solving  $\check{\mathbf{y}}_{1t}$  because it involves expectational errors  $\mathbf{Q}\mathbf{D}\eta_t$  and exogenous shocks errors  $\mathbf{Q}\mathbf{C}\mathbf{v}_t$ . Uniqueness of the solution requires the following necessary and sufficient condition:  $\mathbf{Q}_1\mathbf{D} = \Phi\mathbf{Q}_2\mathbf{D}$ , which if satisfied means that expectational errors that work as loading factors are neutralized, yielding the following solution:

$$\hat{\mathbf{y}}_t = \Xi_0\hat{\mathbf{y}}_{t-1} + \Xi_1\mathbf{v}_t, \quad (58)$$

where:

$$\begin{aligned} \Xi_0 &\equiv \mathbf{Z}\mathbf{\Lambda}_{11}^{-1} [\mathbf{\Omega}_{11}(\mathbf{\Omega}_{12} - \Phi\mathbf{\Omega}_{22})] \mathbf{Z}', \text{ and} \\ \Xi_1 &\equiv \begin{bmatrix} \mathbf{\Lambda}_{11}^{-1} & -\mathbf{\Lambda}_{11}^{-1}(\mathbf{\Lambda}_{12} - \Phi\mathbf{\Lambda}_{22}) \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} (\mathbf{Q}_1 - \Phi\mathbf{Q}_2) \mathbf{C} \\ \mathbf{0} \end{bmatrix}. \end{aligned}$$

Note that Equation (58) is nothing more than an SVAR, which allow us to calculate IRFs as well as variance decompositions for shocks  $\mathbf{v}_t$ , *conditional* on a standard calibration.<sup>34</sup> The next section describes chosen parameters. Then, we analyze IRFs when shocks placed in rows 1, 2 and 4 in Equation (53) hit the economy.

## 6.1 Calibration

The simulation exercise is conditional on the calibration assumed. In particular, we take as usual in the literature a quarter as the unit of time in which decisions are made. Beginning with the share of rule-of-thumb consumers, we assume that  $\lambda^r = \lambda^{r*} = 0.6$  a value that seems reasonable in light of the evidence suggested by Mankiw (2000) (we also tried  $\lambda^r = \lambda^{r*} = 0.1$  and  $\lambda^r = \lambda^{r*} = 0.8$ ). Regarding parameters that affect the utility function, Equation (18), we calibrate the inverse of the Frisch elasticity,  $\iota$ , to 3. The literature suggests values for the Frisch elasticity of around 0.20 for the U.S. (Rotemberg & Woodford (1998), Galí *et al.* (2007b)) and higher values, around 0.3 for the Euro Area (EA) (Forni *et al.* (2007) suggest 0.33). Moreover, the risk aversion parameter  $\sigma$ , is assumed equal to 1.5 as is standard in the literature. The habit persistence parameter,  $b$ , is assumed equal to 0.5, though Smets & Wouters (2007) suggest higher values (0.7). Furthermore, the elasticity of money demand,  $\varepsilon$ , equals 2/3, while the willingness to postpone consumption or the so-called discount factor,  $\beta$ , is set to 0.99 in accordance with an annual nominal interest rate of 4 percent. Technology is assumed to display decreasing returns to scale with  $\psi_H$  set equal to 0.98. As price and wage stickiness depend on the respective Calvo parameters, we assume that on average all wages and prices are reviewed once a year, so we equally calibrate Calvo probability's parameters  $\varphi_H = \varphi_{MF} = \varphi_W = 0.75$ . Regarding the elasticity of substitution of home and foreign goods,  $\eta_c$ , it equalizes 1.8, while we assume no home bias in the consumption bundle,  $\varphi = 0.5$ . The persistency coefficients of AR(1) processes,  $\rho \equiv \mathbf{diag}(\rho_{UIP}, \rho_A, \rho_A^*, \rho_i, \rho_i^*, \rho_{TR}, \rho_{TR}^*, \rho_g, \rho_g^*, \rho_U, \rho_U^*)$ , are calibrated as  $\rho \equiv \mathbf{diag}(0, 0.95, 0.95, 0, 0, 0, 0, 0.85, 0.85, 0, 0)$ . This means that it is introduced persistency only in technology and public expenditure processes, whereas the remaining ones are purely temporary shocks. These assumptions are dictated by common sense.

Monetary policy rules are assumed to be based on the Taylor principle. Therefore, the reaction parameter to inflation,  $\lambda_1$ , is set equal to 1.5, with no inertia, i.e.,  $\lambda_0 = 0$  (Canzoneri *et al.* (2005) estimated a value of 0.22 for the U.S.) and a reaction parameter to output gap,  $\lambda_2$ , set equal to 0.125. The smoothing parameter,  $\lambda_3$ , is assumed 0.75, consistent with a monetary policy that has full effect after four quarters.

Fiscal rule parameters are set as follows,  $\phi_1 = 0.1$  and  $\phi_2 = 0.05$ . Regarding taxes, we calibrate the wage tax rates for the EA and U.S. following Coenen & Straub (2005)'s proposal:  $\tau_w^{EA} = 0.45$  and  $\tau_w^{EA} = 0.15$ . Such tax rates will have sizable effects on disposal income of non-Ricardian agents.

Consistent with EA data, steady-state parameters  $\frac{WN^r}{C} = \frac{Y}{C}$  that enter in the log-linearized form of Equation (29) are set to equal to  $\frac{1}{0.77}$ .

## 6.2 Numerical simulations

Why the IRFs are so important for policymaking? Because they reveal to policymakers the propagation mechanism working after the occurrence of a shock: we can assess variables' responses signs and convergence patters towards their steady-state values. Likewise, variance decompositions deliver the relative size of fluctuations in (observed and unobserved) variables' variability. Playing with draws of structural shocks in the parameterized model, policy makers can figure out how policy interventions affect key variables and even calculate their confidence intervals.

In following subsections we will replicate results consistent with positive multipliers of consumption for expansionary FP illustrated reported in Table 1. We explored a wide range of values for  $\lambda^r$  that yields determinate solutions, but simply we display three representative cases.

<sup>34</sup>The model is solved with the set of routines called DYNARE, see Juillard (2005a).

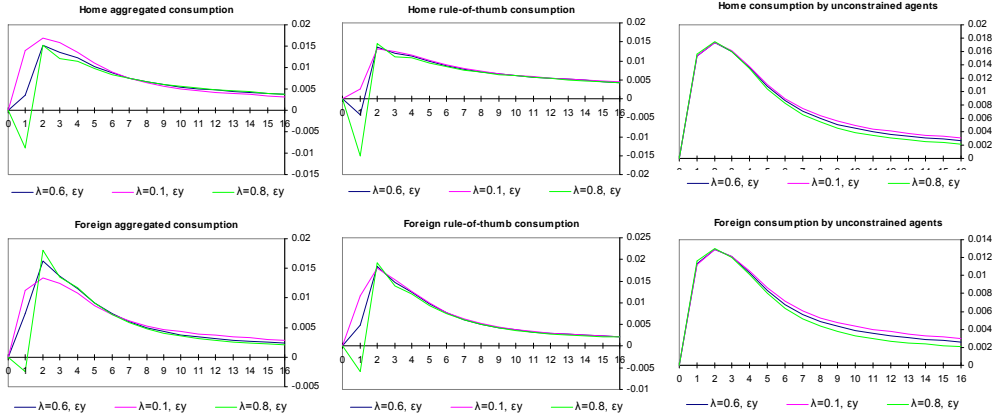


Figure 1: Productivity improvement shock (Taylor rule with smoothing)

### 6.2.1 IRF of a productivity shock

The simulation of an unexpected shock in productivity of one standard deviation (SD) will raise consumption of fully rational consumers, while decreasing consumption of rule-of-thumb consumers, see Figure 1. Given that the latter goes down more deeply, it leads to a negative response of aggregated consumption. In opposition, foreign aggregated consumption is boosted because of the spillover effects. This result corresponds with the unambiguous rise of foreign unconstrained consumption. There is a rise of consumption of foreign rule-of-thumb consumers, however it does not maintain for all consumer type shares. The positive spillover effect in the foreign country takes place because of the rise in the real wage that is full when measured w.r.t. imported goods prices (substantially cheaper). Notice that this expenditure switching effect is so strong in both economies because all goods are fully tradable, while the labor force is locked in the corresponding country. For an analysis of the adjustment process in the presence of non-tradable goods, see Plasmans *et al.* (2007).

### 6.2.2 IRF of a monetary policy shock

In Figure 2 we report a negative, unexpected and purely temporary shock in MP and its effects on consumption.<sup>35</sup> The immediate effect occurs in the money market as both the nominal and the real interest rate go down since at period zero the CB does not react. Of course, the CB reacts in subsequent periods following the rule (52).

As a result, aggregated consumption goes temporarily up, and also it is the case for home constrained consumers. For unconstrained consumers it becomes more attractive to consumers postpone saving and to consume more at the present. However, the effect that prevails in unconstrained consumers' behavior depends on how large is the share of rule-of-thumb consumers in the economy. For example, a positive consumption response is verified for  $\lambda^r = 0.6$  but after two periods, consumption goes down because they expect the full reaction of the CB increasing the interest rate to neutralize the shock (recall that  $\lambda_3 = 0.75$ , so that a monetary policy has full effect after four quarters).

Given the tradability of the goods, foreign financially unconstrained consumers foresee a temporary opportunity to consume more because of the expansionary effect of the foreign output resulting from higher exports to the home country. This expansion in foreign consumption is quite weak and is full if all consumers were unconstrained. Foreign rule-of-thumb agents consume less on impact because of the drop in the disposable income due to higher foreign taxes to back the issuance of bonds that is employed to make sustainable the trade deficit (i.e., to balance the net foreign asset position).

### 6.2.3 IRF of a public expenditure shock

In our opinion, the most interesting case to analyze concerns expansionary FP in the home country which is illustrated by IRFs of Figure 3 under a monetary regime where rule (52) applies. Conditional on the calibration we proposed, in Figure 3 we observe that a positive impact on consumption occurs as a result of expansionary policy. This result lasts a few quarters (approximately two quarters or more for  $\lambda^r > 0.6$ ) becoming negative afterward. This behavior is the result of the relatively large weight of rule-of-thumb consumers in the economy that overturn

<sup>35</sup>The shock is rationalized as resulting from an exogenous shift of the money demand to the left.

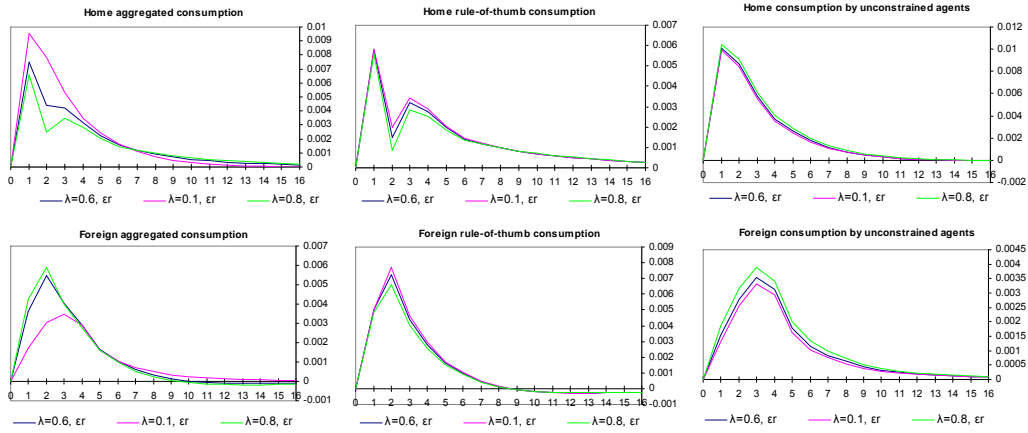


Figure 2: Temporary negative monetary shock (Taylor rule with smoothing)

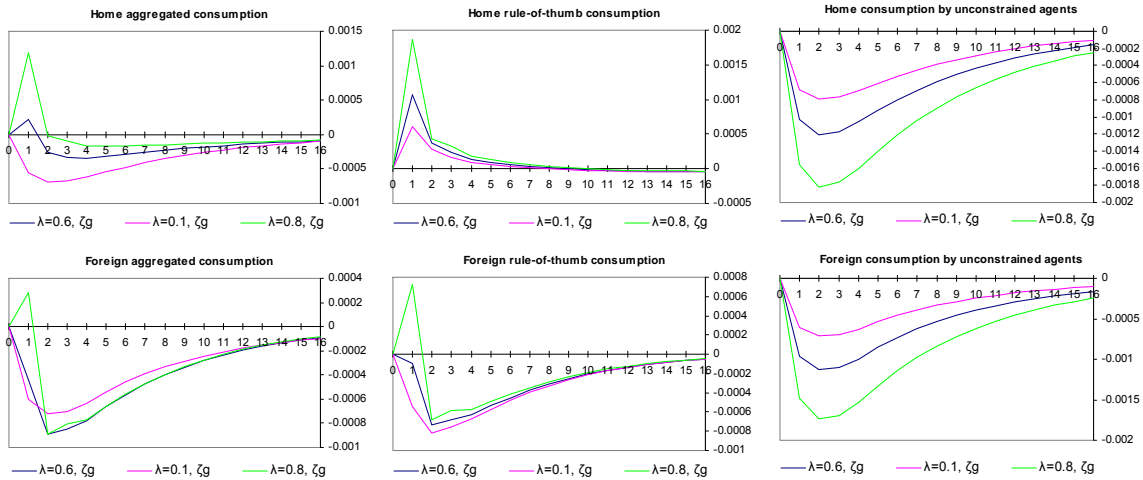


Figure 3: Government spending shock in the home country (Taylor rule with smoothing)

the negative adjustment of consumption (according the REP) in response of future liabilities of the government. Such an example shows under which conditions it is likely that the argument of Mankiw (2000) could have empirical relevance. For a parameterization where  $\lambda^r < 0.45$ , the response of home aggregate consumption remains negative within four years. On the other hand, large values of  $\lambda^r$  lead to model indeterminacy (Blanchard & Kahn (1980) conditions do not hold). Therefore, the critical question that arises is which estimate of  $\lambda^r$  is supported by the data of the U.S. and of the EA.

## 7 Estimation

In this section, we describe the data considered for the construction of observable variables. Besides, in next subsection, we estimate the deep parameters of the model using Bayesian techniques.<sup>36</sup>

### 7.1 Data

We estimate our model for the EU-12 and for the U.S. economies. According to very recent figures, it is quite reasonable to consider these economies as symmetric in terms of GDP and openness.

<sup>36</sup>All computations are performed with DYNARE set of routines, Juillard (2005b).

Euro area data are proxied by relevant EU-12 aggregates (Eurostat), available on a quarterly basis. To obtain a dataset with homogeneous frequency we interpolated military expenses series (SIPRI database) with Chow & Lin (1971)'s method. All series are corrected from seasonality, deflated by the corresponding CPI index and translated into per capita terms dividing by total population in the working age (16 to 65 years old) from OECD statistical compendium. Finally, all series are taken in terms of deviations from their respective trends, as obtained by applying the Hodrick-Prescott filter with smoothing parameter  $\lambda$  equal to 1600. Our sample runs from 1991Q1 to 2006Q4.

Variables used for estimation are per capita private consumption, per capita government expenditure and per capita GDP. Per capita government expenditure net of military expenditure gave similar estimation results. Table A.1 in the Data appendix provides details on variables' construction. The same variables are calculated with U.S. data. Further details are summarized in Table A.2 in the Data appendix.

## 7.2 Data into the DSGE model and the likelihood function

Given the solution of the model from Equation (58) a direct estimation approach would maximize its likelihood function with respect to  $\theta$  and  $vech(\Sigma_v)$ ; however, we must acknowledge that not all variables included into  $\hat{\mathbf{y}}_t$  are observed. To include data, a partition of the vector  $\hat{\mathbf{y}}_t$  into observed and unobserved variables is needed, so that  $\hat{\mathbf{y}}_t \equiv (\hat{\mathbf{y}}_t^o, \hat{\mathbf{y}}_t^{uno})'$ . In our model,  $\hat{\mathbf{y}}_t^{o'}$  is  $6 \times 1$ . Then, a state-space representation is derived from (58) which includes a measurement equation:

$$\begin{aligned}\hat{\mathbf{y}}_t &= \Xi_0 \hat{\mathbf{y}}_{t-1} + \Xi_1 \mathbf{v}_t, \\ \hat{\mathbf{y}}_t^o &= \Upsilon \hat{\mathbf{y}}_t + \epsilon_t,\end{aligned}\tag{59}$$

where  $\Upsilon$  is a  $6 \times n$  binary matrix that selects the observed variables from  $\hat{\mathbf{y}}_t$ ,  $\epsilon_t$  is a measurement error that is assumed to be *iid* with mean zero vector and variance  $\Sigma_\epsilon$ . More explicitly, our specific measurement equation is:

$$\begin{pmatrix} Y_{H,t}^o \\ C_t^{aggr,o} \\ G_t^o \\ i_t^o \\ \pi_t^o \\ Y_{F,t}^o \\ C_t^{aggr*,o} \\ G_t^{*o} \\ \pi_t^{*o} \end{pmatrix} = [ \dots \quad \mathbf{I}_9 \quad \dots ]_{(9 \times n)} \begin{pmatrix} \vdots \\ Y_{H,t}^{uno} \\ C_t^{aggr,uno} \\ G_t^{uno} \\ i_t^{uno} \\ \pi_t^{uno} \\ Y_{F,t}^{uno} \\ C_t^{aggr*,uno} \\ G_t^{*,uno} \\ \pi_t^{*,uno} \\ \vdots \end{pmatrix}_{n \times 1} + \begin{matrix} \epsilon_t \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{matrix}_{9 \times 1},\tag{60}$$

where the first entry of  $\hat{\mathbf{y}}_t^o$  in (60) correspond to the LHS of the GDP identity, or domestic resource constraint, Equation (46); while the second and third entries link observed (aggregate) home tradable consumption and observed government expenditures to RHS of (46). Symmetrically, 6th to 9th entries of  $\hat{\mathbf{y}}_t^o$  correspond the variables included in the foreign resource constraint (47).

Denoting the sample as  $\hat{Y}_T^o \equiv \{\hat{\mathbf{y}}_1^o, \hat{\mathbf{y}}_2^o, \hat{\mathbf{y}}_3^o, \dots, \hat{\mathbf{y}}_T^o\}$ , the density of  $\hat{Y}_T^o$  conditional on the parameters (likelihood) can be written as:

$$\begin{aligned}\mathcal{L}(\theta, vech(\Sigma_v), vech(\Sigma_\epsilon); \hat{Y}_T^o) &= p(\hat{Y}_T^o | \theta, vech(\Sigma_v), vech(\Sigma_\epsilon)), \\ &= p(\hat{\mathbf{y}}_0^o | \theta, vech(\Sigma_v), vech(\Sigma_\epsilon)) \\ &\times \prod_{t=1}^T p(\hat{\mathbf{y}}_t^o | Y_{t-1}^o, \theta, vech(\Sigma_v), vech(\Sigma_\epsilon)),\end{aligned}\tag{61}$$

which includes a marginal density (involving the distribution of the initial condition)  $p(\hat{\mathbf{y}}_0^o | \theta, vech(\Sigma_v), vech(\Sigma_\epsilon))$  and a conditional density. Given our linearized model (59) and our definition of  $\epsilon_t$ , it follows that  $\hat{\mathbf{y}}_0^o \sim \mathbf{N}(E_\infty[\hat{\mathbf{y}}_t^o], V_\infty[\hat{\mathbf{y}}_t^o])$ .<sup>37</sup> Concerning the second factor, the conditional density involves the evaluation of  $\hat{\mathbf{y}}_t^o | Y_{t-1}^o$  which is not directly observable since  $\hat{\mathbf{y}}_t^o$  depends on other unobserved endogenous variables given by the model; however, it is at hand the following identity:

$$p(\hat{\mathbf{y}}_t^o | Y_{t-1}^o, \theta, vech(\Sigma_v), vech(\Sigma_\epsilon)) \equiv \int_{\Lambda} \left[ \begin{matrix} p(\hat{\mathbf{y}}_t^o | \hat{\mathbf{y}}_t, \theta, vech(\Sigma_v), vech(\Sigma_\epsilon)) \\ \times p(\hat{\mathbf{y}}_t | Y_{t-1}^o, \theta, vech(\Sigma_v), vech(\Sigma_\epsilon)) d\hat{\mathbf{y}}_t \end{matrix} \right],$$

<sup>37</sup> Construction of the likelihood for an AR(1) and AR(p) processes are derived in Hamilton (1994) Ch.5, Sections 2 and 3, respectively. In case  $\hat{\mathbf{y}}_0^o$  contains variables with unit roots, the initialization assumes an infinite  $V_\infty[\hat{\mathbf{y}}_0^o]$ , which is known as diffuse Kalman filter.

where the density of  $\hat{\mathbf{y}}_t^o | Y_{t-1}^o$  depends on the mean of the density of  $\hat{\mathbf{y}}_t^o | \hat{\mathbf{y}}_t$  where the relevant weight is the density of  $\hat{\mathbf{y}}_t | Y_{t-1}^o$ . The former density is directly given by the measurement Equation (59), while  $\hat{\mathbf{y}}_t | Y_{t-1}^o$  is computed by the Kalman filter.

### 7.3 Bayesian estimation: the likelihood meets prior densities

Bayesian estimation and evaluation techniques have been particularly successful in estimation of not only small DSGE models but also medium to large-scale New Keynesian models. The estimation procedure combines a likelihood function (61) derived from our model with the specification of a prior distribution for  $\theta \equiv (\theta, \text{vech}(\Sigma_v), \text{vech}(\Sigma_\epsilon))'$ . As a result, the state-space representation can be translated to form the posterior distribution.

The idea behind the Bayesian principle is to look for a parameter vector which maximizes the posterior density, given the prior and the likelihood based on the data. Formally, the posterior density  $p(\theta | \hat{\mathbf{y}}^o)$  is related to the prior and the likelihood as follows:

$$p(\theta | \hat{\mathbf{y}}^o) = \frac{p(\hat{\mathbf{y}}^o | \theta) p(\theta)}{p(\hat{\mathbf{y}}^o)} \propto p(\hat{\mathbf{y}}^o | \theta) p(\theta) = L(\theta | \hat{\mathbf{y}}^o) p(\theta), \quad (62)$$

where  $p(\theta)$  is the prior density of the parameter vector,  $\mathcal{L}(\theta | \hat{\mathbf{y}}^o)$  is the likelihood of the data and  $p(\hat{\mathbf{y}}^o) = \int_{\Theta} p(\hat{\mathbf{y}}^o | \theta) p(\theta) d\theta$  is the unconditional data density, which, since it does not depend on the parameter vector to be estimated, can be treated as a proportionality factor and accordingly can be disregarded in the estimation process. Assuming *iid* priors, the logarithm of the posterior is given by the sum of the log likelihood of the data and the sum of the logarithms of the prior distributions:

$$\ln(p(\theta | \hat{\mathbf{y}}^o)) = \ln(\mathcal{L}(\theta | \hat{\mathbf{y}}^o)) + \sum_{i=1}^N \ln(p(\theta_i)). \quad (63)$$

The latter term can be directly calculated from the specified prior distributions of the estimated parameters. For the computation of the log likelihood of the data the Kalman filter is applied to the DSGE model solution (the state-state representation) for the number of periods,  $T$ , provided by the data  $\hat{\mathbf{y}}^o$ .

The (multivariate) posterior distribution for our DSGE model would not exist in closed form; however, it can be approximated through a Gaussian density providing the sample size grows.<sup>38</sup> Following Tierney & Kadane (1986), the posterior is understood as a kernel of unknown form,  $\mathcal{K}(\theta) \equiv \mathcal{K}(\theta, Y_T^o)$ , given that (one of) its mode is assumed to be known,  $\theta^*$ , taking logs and approximating the kernel using a  $2^{nd}$  order Taylor expansion, yields:

$$\log \mathcal{K}(\theta) \approx \log \mathcal{K}(\theta^*) - \frac{1}{2}(\theta - \theta^*)' [H(\theta^*)]^{-1} (\theta - \theta^*),$$

where  $H(\theta^*)$  is minus the inverse of the Hessian of the model evaluated at the posterior mode. Consequently, the Gaussian posterior would be:

$$p(\theta^*) \approx (2\pi)^{T/2} |H(\theta^*)|^{-1/2} \exp \left\{ -\frac{1}{2}(\theta - \theta^*)' [H(\theta^*)]^{-1} (\theta - \theta^*) \right\},$$

which enables us to approximate posterior moments, as derived by Kass *et al.* (1989) and Tierney *et al.* (1989).

The whole point is that the asymptotic approximation ( $T \rightarrow \infty$ ) makes sense if and only if the true posterior does not differ from the hypothesized Gaussian. More exact results for our sample range can be derived via simulation given its non-standard shape, employing an approximation method around the optimum that generates a (large) sample of draws using the Markov-Chain Monte Carlo (MCMC) algorithm. This is useful to characterize the shape of the posterior distribution, from which inference can be drawn. The Metropolis-Hastings algorithm is implemented using a jumping distribution to visit areas that are not at the tails of the posterior. The validity of the "jump" is assessed via acceptance-rejection instrumented with the Metropolis-Hastings algorithm, where proposal draws that are accepted (rejected) are included (excluded) in Markov chain. The researcher establishes the ratio of acceptance. The simulation is considered large enough when pooled moments converge to within moments of the chain, see Brooks (1998).

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<sup>38</sup>As the sample enlarges, the choice of the prior density would not affect the posterior.



## 7.4 Estimation results

This section provides Bayesian estimates of deep parameters of our two-country model that are supported by data. Leith & von Thadden (2006) studied local determinacy conditions to effectively solve a model with interaction of fiscal and monetary policies. Therefore, it could be potentially interesting to estimate these parameters for the U.S. and for the EA.

The estimation strategy employed starts with reduced dimensions of the parameter space (the beginning is the extreme case where all but one parameter is calibrated) and, subsequently, it is expanded. In subsequent trials, where the dimension parameter space was enlarged, we found that some parameters were poorly identified. Finally, we keep those parameters that are identified (by exploring the neighborhood of the mode) thus the information contained in the data is correctly mapped to parameters. Of course, the researcher aims at obtaining estimates of the parameter vector with the largest dimension, and this can be done adding more data series or a longer data span (which can only be added if the number of shocks is higher, otherwise we could not solve it due to stochastic singularity). Notice though, that in Section 5.3 we specified exogenous shocks, which we judged adequate. Adding more shocks could be an interesting extension, though this is potentially troublesome if they are not motivated by economic theory (shocks should help to identify data series), see the warnings posed by Chari *et al.* (2009) in this respect.

Bearing these considerations in mind, we report in Table 2 Bayesian estimates arising from the maximization of the posterior distribution of our DSGE model. Relevant information we specified includes:

1. prior means, SDs and density types;
2. prior lower and upper bounds (if prior distributions are truncated); and
3. posterior means and SDs and 90 percent confidence interval for the posterior mean.

Table 2 reports Bayesian estimates for our two-country model.

There are three density types chosen for the priors: (i) beta densities for parameters bounded within the (0,1) interval; (ii) inverted Gamma densities for those parameters whose expected values are strictly positive (and possibly unbounded from above) such as standard errors; and (iii) normal densities. Furthermore, prior means and SDs were set considering economic theory as well as estimates from related previous studies.

Next we describe estimates of posterior means and briefly we interpret them. Beginning with estimates of the Frisch elasticity,  $1/\iota$ , estimates are 0.27 and 0.38 for the EA and the U.S., respectively. Moreover, estimates of the risk aversion parameters are 2.62 for the EA and 5.31 for the U.S., the latter seems to be larger than what the literature report. Estimates of internal habit formation indicate quite similar degrees of persistency; while for the EA it is 0.85 the one for the U.S. is 0.88, indicating that internal habit in consumption is quite persistent.

Technology processes are very persistent with estimates of  $\rho_A$  and  $\rho_A^*$  close to one, we observe values of 0.998 and 0.997 for the EA and the U.S., respectively. Estimates of persistency in government expenditure are 0.67 and 0.79 for the EA and the U.S., respectively.

Nominal rigidities are modeled as Calvo contracts where probabilities of updating prices and wages may reveal the average duration of contracts. We observe that price contract durations last on average 1.5 and 1.33 quarters for the EA and the U.S., respectively. As for wages, we observe values of 1.28 and 1.3 quarters, respectively. These estimates indicate more price flexibility than Bils & Klenow (2004).

Further, FP estimates  $\phi_1$  and  $\phi_1^*$  are 0.13 and 0.07, respectively. Moreover, estimates  $\phi_2$  and  $\phi_2^*$  are nearly the same. Regarding estimates of the MP rule, reaction to inflation parameters are 1.47 and 1.54, while those to output gap are 0.79 and 0.14, for the EA and the U.S., respectively. The measure of smoothing implicit in the MP rules, suggest that the policy is fully in effect after 4.56 and 5.67 quarters for the EA and the U.S., respectively. Elasticities of substitution of home a foreign bundles as well as home bias in consumption parameters are poorly identified, so we do not estimate them.

Table 2 Bayesian estimation of deep parameters

Log data density: 2049.53.

Parameters	Density	Prior mean	Prior SD	Post mean	Post mean 90% interval	
					lower	upper
$\iota$	$N$	3	0.5	3.7391	2.9984	4.4587
$\iota^*$	$N$	3	0.5	2.622	1.8414	3.3766
$\sigma$	$N$	2	0.5	2.9029	2.2068	3.5825
$\sigma^*$	$N$	2	0.5	5.3166	4.7523	5.9106
$b$	$N$	0.5	0.2	0.8501	0.7767	0.9309
$b^*$	$\beta$	0.5	0.2	0.878	0.783	0.977
$\varphi_H$	$\beta$	0.5	0.15	0.3329	0.223	0.4447
$\varphi_H^*$	$\beta$	0.5	0.15	0.2488	0.1504	0.3369
$\varphi_W$	$\beta$	0.5	0.15	0.2185	0.1314	0.3083
$\varphi_W^*$	$\beta$	0.5	0.15	0.2309	0.1796	0.2827
$\rho_A$	$\beta$	0.75	0.1	0.9981	0.9969	0.9994
$\rho_A^*$	$\beta$	0.75	0.1	0.9976	0.9958	0.9995
$\rho_g$	$\beta$	0.75	0.1	0.6682	0.5421	0.8062
$\rho_g^*$	$\beta$	0.75	0.1	0.7931	0.6654	0.9308
$\phi_1$	$\beta$	0.25	0.15	0.1337	0.0101	0.2544
$\phi_1^*$	$\beta$	0.25	0.15	0.0687	0.01	0.134
$\phi_2$	$\beta$	0.05	0.02	0.0487	0.0193	0.0784
$\phi_2^*$	$\beta$	0.05	0.02	0.0494	0.019	0.0793
$\lambda_1$	$N$	1.5	0.125	1.4678	1.2574	1.6568
$\lambda_2$	$N$	0.125	0.05	0.0786	-0.0007	0.1599
$\lambda_3$	$\beta$	0.75	0.1	0.7806	0.7305	0.8297
$\lambda_1^*$	$N$	1.5	0.125	1.5424	1.3522	1.7334
$\lambda_2^*$	$N$	0.125	0.05	0.1431	0.0695	0.221
$\lambda_3^*$	$\beta$	0.75	0.1	0.8235	0.7743	0.8736
Standard errors						
Parameters	Density	Prior mean	d.f.	Post mean	lower	upper
$\sigma_{\varepsilon_{UIP}}$	inv $\Gamma$	0.01	2	0.0535	0.045	0.0623
$\sigma_{\varepsilon_y}$	inv $\Gamma$	0.01	2	0.0053	0.0043	0.0063
$\sigma_{\varepsilon_y^*}$	inv $\Gamma$	0.01	2	0.0027	0.0022	0.0033
$\sigma_{\varepsilon_i}$	inv $\Gamma$	0.01	2	0.0046	0.0039	0.0054
$\sigma_{\varepsilon_i^*}$	inv $\Gamma$	0.01	2	0.0015	0.0012	0.0017
$\sigma_{\varepsilon_{TR}}$	inv $\Gamma$	0.01	2	0.0085	0.0023	0.0153
$\sigma_{\varepsilon_{TR}^*}$	inv $\Gamma$	0.01	2	0.0089	0.0024	0.0167
$\sigma_{\varepsilon_g}$	inv $\Gamma$	0.005	1	0.0158	0.0135	0.0181
$\sigma_{\varepsilon_g^*}$	inv $\Gamma$	0.005	1	0.0318	0.0269	0.0363
$\sigma_{\varepsilon_U}$	inv $\Gamma$	0.01	2	0.011	0.0069	0.015
$\sigma_{\varepsilon_U^*}$	inv $\Gamma$	0.01	2	0.039	0.0326	0.045

## 8 Conclusions

This paper investigates fiscal and monetary policies and how they can interact in order to better stabilize large economies. As stabilization power of policies is potential and depends on the specific model setup, we examine a fully microfounded model focusing attention on the short-run interaction of two large open economies, where a fraction of the consumers are financially constrained (and therefore acting in a non-Ricardian way). We argued that the separation of consumers' types has important consequences in open economies which have not been sufficiently analyzed in the literature.

Firstly, we estimated a small VAR model specification with minimum structure and capture in a parsimonious way, interesting results for the EU-12 aggregate. The fiscal multipliers obtained are positive and comparable in size to those found by Galí *et al.* (2007b) for the U.S., the only qualitative difference if we plot the sequence of fiscal multipliers the graph would have shown a hump-shaped form, whereas Galí *et al.* (2007b) findings suggest that fiscal multipliers decrease along time monotonically. Results are at odds with the suggested prediction from RBC models based on the permanent income hypothesis.

Taking into account these consumption developments, we accomplished a classical numerical simulation analysis with our model, where parameters calibrated resemble the EU-12 and the U.S. economies and we ask under which conditions we are able to generate developments of consumption as those predicted by the VAR. We find that we need more than 50 percent of rule-of-thumb consumers in both economies to reproduce the IRFs of VAR, a similar figure was proposed by Mankiw (2000). Different shares of non-Ricardian consumers were considered and its implications for stability analyzed, confirming that if the share of rule-of-thumb consumers increase to such an extent that become dominant, the model's solution becomes indeterminate, an issue also shown by Galí *et al.* (2007b).

The analysis of IRFs allows us to conclude that the monetary policy design may have little influence in the channel of transmission that matters for consumption fluctuations and by them to consumption multipliers. Critically, active FP will shape aggregate consumption fluctuations through the transfer's channel together with disposable income fluctuations, regardless the shock we considered.

We estimated a subset of deep parameters conditional on a the rule-of-thumb share that is 0.6 to obtain similar IRFs as from the estimated VAR employing Bayesian techniques. These estimates seem to be in accordance with the literature.

For future research agenda, we acknowledge that the current model needs to be extended to take into account the stock of physical capital to generalize our conclusions. Moreover, different taxation regimes should be analyzed focusing more on how the disposable income is generated and varies with changes in transfers or in tax proceedings. Meanwhile, the presented model assumed that these channels were shut down.

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## A Data Appendix

Table A.1

Variables for EU 12 (BE, DE, IE, GR, ES, FR, IT, LU, NL, AT, PT, FI)	
Description	Source and series' name
Government expenditure divided by lagged GDP trend	Eurostat: "na-p3_s13"
Government expenditure net of military expenses divided by lagged GDP trend	Eurostat: "na-p3_s13"; SIPRI database
Government revenues divided by lagged GDP trend	Eurostat: "gov_q_ggufa", SA adj
Government deficit divided by lagged GDP trend	Constructed
GDP over working age population, in logs	Eurostat, "na-blgm", OECD: "POPT"
Private consumption over working age population, in logs	Eurostat: "na-p3"; OECD: "POPT"

Note: Countries abbreviations are Belgium, Germany, Ireland, Greece, Spain, France, Italy, Luxemburg, The Netherlands, Austria, Portugal, Finland, respectively.

Table A.2

Variables for U.S.	
Description	Source and series' name
Government expenditure divided by lagged GDP trend	FRED II: "GCEC1"
Government expenditure net of military expenses divided by lagged GDP trend	FRED II: "GCEC1"; SIPRI database
Government revenues divided by lagged GDP trend	Constructed
Government deficit divided by lagged GDP trend	FRED II: "TGDEF"
GDP over population older than 16 years old, in logs	FRED II: "GDP", "CNP16OV"
Private consumption over working age population, in logs	FRED II: "PCECC96", "CNP16OV"



## B Steady state

Evaluating the Euler equation (23) and the UIP condition (26) at the steady state with zero inflation and zero depreciation rate yields:

$$\frac{1}{\beta} - 1 = I = I^*. \quad (64)$$

In addition, the steady state home marginal cost is derived from Equation (40):

$$P_H = \frac{\theta_h}{(\theta_h - 1)} MC_H \Rightarrow \frac{mc_H}{p_H} = \frac{\theta_h - 1}{\theta_h}, \quad (65)$$

where  $mc_H \equiv \frac{MC_H}{P}$  and  $p_H \equiv \frac{P_H}{P}$ . Similarly, for imported goods marginal cost:

$$P_F = \frac{\theta_f}{(\theta_f - 1)} \frac{MC_{MF} P_F^{\theta_f} C_F \sum_{a=0}^{\infty} (\varphi_{MF} \beta)^a}{C_F (P_F)^{\theta_f} \sum_{a=0}^{\infty} (\varphi_{MF} \beta)^a} = \frac{\theta_f}{(\theta_f - 1)} MC_{MF} \Rightarrow \frac{mc_{MF}}{p_F} = \frac{\theta_f - 1}{\theta_f},$$

where  $mc_{MF} \equiv \frac{MC_{MF}}{P}$  and  $p_F = \frac{P_F}{P}$ . Given the fact that there is no transportation cost,  $MC_{MF} = \mathcal{E} MC_F$ . In real terms,  $mc_{MF} = RER mc_F$ , where  $RER \equiv \frac{\mathcal{E} P^*}{P}$  and  $mc_F \equiv \frac{MC_F}{P^*}$ .

Consider the foreign economy version of Equation (40):

$$P_F^* = \frac{\theta_f^*}{(\theta_f^* - 1)} MC_F \Rightarrow \frac{mc_F}{p_F^*} = \frac{\theta_f^* - 1}{\theta_f^*}, \quad (66)$$

where  $mc_F \equiv \frac{MC_F}{P^*}$  and  $p_F^* \equiv \frac{P_F^*}{P^*}$ . Similarly, the foreign importer has real marginal cost given by  $\frac{mc_{MH}}{p_H^*} = \frac{\theta_h^* - 1}{\theta_h^*}$ .

Since  $MC_{MH} = \frac{MC_H}{\mathcal{E}} = \frac{\theta_h - 1}{\theta_h} P_H$  or in real terms  $mc_{MH} = \frac{mc_H}{RER} = \frac{\theta_h - 1}{\theta_h} \frac{p_H}{RER}$ .

To sum up, we obtain the following expressions for marginal costs:

$$\begin{aligned} \text{home: } mc_H &= \frac{\theta_h - 1}{\theta_h} p_H, & mc_{MF} &= RER mc_F = \frac{\theta_f^* - 1}{\theta_f^*} RER p_F^*, \\ \text{foreign: } mc_F &= \frac{\theta_f^* - 1}{\theta_f^*} p_F^*, & mc_{MH} &= \frac{mc_H}{RER} = \frac{\theta_h - 1}{\theta_h} \frac{p_H}{RER}. \end{aligned}$$

The real wage could be unambiguously determined equalizing  $mc_H$  from (65) with the real version of Equation (35) if  $\psi_H \rightarrow 1$  holds (i.e. under constant returns to scale (CRS)):

$$w_H = \frac{\psi_H \left( \frac{\theta_h - 1}{\theta_h} \right)}{[Y_H(i) - FC(i)]^{\frac{1 - \psi_H}{\psi_H}} \left[ \frac{1}{A} \right]^{\frac{1}{\psi_H}}}, \quad (67)$$

otherwise, if  $\psi_H \neq 1$  the real wage at the steady state comes from Equation (42):

$$\begin{aligned} N_H^{\mathcal{H}} \left[ \Lambda^j (1 - \tau_w) \frac{\check{W}_{\mathcal{H}}}{P} + \mu_W z (N_H^{\mathcal{H}})^{\iota} \right] \sum_{a=0}^{\infty} (\beta \varphi_W)^a &= 0, \\ \frac{\check{W}_{\mathcal{H}}}{P} &= \frac{\mu_W}{(1 - \tau_w)} MRS, \end{aligned} \quad (68)$$

where  $z = 1$  and  $MRS \equiv -\frac{\chi_N U_{N_H}}{U_C} = \frac{(N_H^{\mathcal{H}})^{\iota}}{\Lambda}$ . To get the prevailing real wage, plug (68) into the real version of Equation (43):

$$\begin{aligned} \left( \frac{W_H}{P} \right)^{1 - \varepsilon_L} &= \varphi_W \left( \frac{W_{\mathcal{H}}}{P} \right)^{1 - \varepsilon_L} + (1 - \varphi_W) \left( \frac{\mu_W}{(1 - \tau_w)} MRS \right)^{1 - \varepsilon_L}, \\ w_H &= \frac{\mu_W}{(1 - \tau_w)} MRS. \end{aligned}$$

In what follows, we assume CRS so that Equation (67) determines the real wage:

$$w_H = A \left( \frac{\theta_h - 1}{\theta_h} \right), \quad (69)$$

where  $A$  is an scale parameter that measures technology at the steady state, if  $A = 1$ , then  $w_H$  equals  $\frac{m_{CH}}{p_H}$ .

We assume that  $N_H^{aggr} = N_H^r = N_H$  and  $TR^{aggr} = TR^r = TR$  in the remaining to obtain real steady state aggregates. First, notice that the consumption of constrained households, Equation (29), can be written as:

$$C^r = (1 - \tau_w)w_H N_H + TR, \quad (70)$$

where transfers  $TR$  are determined by the binding steady state *real* GBC:

$$\tau_D (d_H + d_{MF}) + \tau_w w_H N_H + \frac{I}{(1+I)} (b^{hp} + b^{fp}) - \frac{I^*}{(1+I^*)} RER b^{*hp} = TR + G.$$

where  $d_H \equiv \frac{D_H}{P}$ ,  $d_{MF} \equiv \frac{D_{MF}}{P}$ ,  $b^{hp} \equiv \frac{B^{hp}}{P}$  and  $b^{fp} \equiv \frac{B^{fp}}{P}$  (recall that  $\varepsilon_{UIP} = 1$ ). In addition,  $B^{hp}$  ( $B^{fp}$ ) stands for the home bonds that are invested in home (foreign) portfolios. Notice that the *gross* total stock of debt denominated in home currency is  $b \equiv b^{hp} + b^{fp}$ . Similarly,  $b^* \equiv \frac{B^*}{P^*}$  which can be expanded into foreign and home holdings of foreign denominated bonds:  $B^* = B^{*fp} + b^{*hp}$ . To convert into home real terms we multiply by  $RER \equiv \frac{\varepsilon P^*}{P}$ .

To determine the amount of tax levied on domestic producers and importers dividends,  $\tau_D (d_H + d_{MF})$ , we ought to expand  $d_H = \frac{1}{\theta_h} (C_H + G_H)$  and  $d_{MF} = \frac{1}{\theta_f} (C_F + G_F)$ .<sup>39</sup> To do so, note that domestic and import demands evaluated at the steady state are:

$$C_H = \varphi [h(T)]^{\eta_c} C^{aggr}, \quad C_F = (1 - \varphi) \left[ \frac{h(T)}{T} \right]^{\eta_c} C^{aggr}, \quad (71)$$

$$C_H^* = (1 - \varphi^*) [h(T^*)]^{\eta_c} C^{aggr*}, \quad C_F^* = \varphi^* \left[ \frac{h(T^*)}{T^*} \right]^{\eta_c} C^{aggr*}, \quad (72)$$

where  $\frac{P}{P_H} \equiv h(T) = h\left(\frac{P_F}{P_H}\right) = \frac{[\varphi P_H^{1-\eta_c} + (1-\varphi)P_F^{1-\eta_c}]^{\frac{1}{1-\eta_c}}}{P_H} = [\varphi + (1-\varphi)T^{1-\eta_c}]^{\frac{1}{1-\eta_c}}$ ,  $\frac{P}{P_F} = \frac{h(T)}{T}$  and  $C^{aggr}$  comes from Equation (30). With this information, we can substitute demands into dividends equations:

$$d_H = \frac{1}{\theta_h} [h(T)]^{\eta_c} (\varphi C^{aggr} + G),$$

$$d_{MF} = \frac{1}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1 - \varphi) C^{aggr}.$$

As a result, we can rewrite steady state real transfers as:

$$TR = \tau_D \left[ \frac{1}{\theta_h} [h(T)]^{\eta_c} [\varphi (\lambda^r C^r + (1 - \lambda^r) C) + G] + \frac{1}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1 - \varphi) [\lambda^r C^r + (1 - \lambda^r) C] \right]$$

$$+ \tau_w w_H N_H + \frac{I (b^{hp} + b^{fp})}{(1+I)} - \frac{I^* RER b^{*hp}}{(1+I^*)} - G, \quad (73)$$

where we replaced  $C^{aggr}$  by  $\lambda^r C^r + (1 - \lambda^r) C$ .

Replacing the latter expression into (70) yields,

$$C^r = \tau_D \left[ \frac{1}{\theta_h} [h(T)]^{\eta_c} \left( \varphi \frac{\lambda^r C^r + (1 - \lambda^r) C}{Y_H} AN_H + \frac{G}{Y_H} AN_H \right) \right]$$

$$+ \frac{1}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1 - \varphi) \frac{\lambda^r C^r + (1 - \lambda^r) C}{Y_H} AN_H$$

$$+ (1 - \tau_w) w_H N_H + \tau_w w_H N_H + \frac{I}{(1+I)} \frac{b}{Y_H} AN_H - \frac{G}{Y_H} AN_H,$$

<sup>39</sup>Note also that  $d_H = \frac{1}{\theta_h} (AN_H - C_H^* - G_H^*)$  and  $d_{MF} = \frac{1}{\theta_f} (AN_F - C_F^* - G_F^*)$ .

$$C^r = F_1 \left\{ \begin{array}{c} \tau_D \left[ \frac{1}{\theta_h} [h(T)]^{\eta_c} \left( \varphi \frac{(1-\lambda^r)C}{Y_H} AN_H + \frac{G}{Y_H} AN_H \right) \right. \\ \left. + \frac{1}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1-\varphi) \frac{(1-\lambda^r)C}{Y_H} AN_H \right] \\ + (1-\tau_w)w_H N_H + \tau_w w_H N_H + \frac{(1-\beta)(b^{hp}+b^{fp})}{Y_H} AN_H - \frac{I^*}{(1+I^*)} \frac{RERb^{*hp}}{Y_H} AN_H - \frac{G}{Y_H} AN_H \end{array} \right\},$$

where  $F_1 \equiv \left\{ 1 - \tau_D \lambda^r \left[ \frac{1}{\theta_h} [h(T)]^{\eta_c} \varphi + \frac{1}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1-\varphi) \right] \right\}^{-1}$  and with the steady state value of the nominal interest rate from (64) that  $\frac{I}{(1+I)} = \frac{I^*}{(1+I^*)} = I\beta = 1 - \beta$ . Rearranging, it yields,

$$C^r = N_H F_1 \left\{ \begin{array}{c} \tau_D A \left[ \frac{1}{\theta_h} [h(T)]^{\eta_c} \left( \varphi \frac{(1-\lambda^r)C}{Y_H} + \frac{G}{Y_H} \right) + \frac{1}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1-\varphi) \frac{(1-\lambda^r)C}{Y_H} \right] \\ + A \left( \frac{\theta_h - 1}{\theta_h} \right) + (1-\beta) \left[ \frac{b^{hp}}{Y_H} + \frac{b^{fp} - RERb^{*hp}}{Y_H} \right] A - \frac{G}{Y_H} A \end{array} \right\},$$

where  $b^{fp} - RERb^{*hp}$  is the real *net* foreign assets position (domestic real in the sense that is measured w.r.t. a home composite good). Therefore what appears into braces is the total real debt position in terms of GDP. These steady state ratios along with  $\frac{(1-\lambda^r)C}{Y_H}$  and  $\frac{G}{Y_H}$  are estimated from long data series. Therefore, using the following definition,

$$\Phi_1 \equiv AF_1 \left\{ \begin{array}{c} \tau_D \left[ \frac{1}{\theta_h} [h(T)]^{\eta_c} \left( \varphi \frac{(1-\lambda^r)C}{Y_H} + \frac{G}{Y_H} \right) + \frac{1}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1-\varphi) \frac{(1-\lambda^r)C}{Y_H} \right] \\ + \frac{\theta_h - 1}{\theta_h} + (1-\beta) \left[ \frac{b^{hp}}{Y_H} + \frac{b^{fp} - RERb^{*hp}}{Y_H} \right] - \frac{G}{Y_H} \end{array} \right\},$$

we can write  $C^r = \Phi_1 N_H$ , which partially explains the steady state aggregate consumption level in Equation (30); thus:

$$C^{aggr} \equiv \lambda^r \Phi_1 N_H + (1 - \lambda^r) C.$$

It remains to solve for  $C$  to get finally  $C^{aggr}$ . To do so, we write the CBC at the steady state as:

$$\begin{aligned} (1 - \tau_w) (1 - \lambda^r) w_H N_H^r + (1 - \lambda^r) TR^r + (1 - \tau_D)(d_H + d_{MF}) + \\ = (1 - \lambda^r) C + (1 - \beta) [b^{hp} + b^{fp} - RERb^{*hp}], \end{aligned}$$

Substituting  $TR$  from (73) into the latter expression yields:

$$\begin{aligned} (1 - \tau_w) (1 - \lambda^r) w_H N_H + \underbrace{[(1 - \lambda^r) \tau_D + (1 - \tau_D)]}_{=1 - \tau_D \lambda^r} AN_H \left[ \begin{array}{c} \frac{1}{\theta_h} [h(T)]^{\eta_c} \left( \varphi \frac{\lambda^r C^r + (1 - \lambda^r) C}{Y_H} + \frac{G}{Y_H} \right) \\ + \frac{1}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1 - \varphi) \frac{\lambda^r C^r + (1 - \lambda^r) C}{Y_H} \end{array} \right] + (1 - \lambda^r) \tau_w w_H N_H \\ + (1 - \lambda^r) (1 - \beta) [b^{hp} + b^{fp} - RERb^{*hp}] - (1 - \lambda^r) G = (1 - \lambda^r) C + (1 - \beta) [b^{hp} + b^{fp} - RERb^{*hp}], \end{aligned}$$

simplifying and solving for  $C$  yields:

$$\begin{aligned} C = N_H \left\{ \begin{array}{c} w_H + \frac{(1 - \tau_D \lambda^r)}{(1 - \lambda^r)} \left[ \frac{A}{\theta_h} [h(T)]^{\eta_c} \left( \varphi \frac{\lambda^r C^r}{Y_H} + \frac{G}{Y_H} \right) + \frac{A}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1 - \varphi) \frac{\lambda^r C^r + (1 - \lambda^r) C}{Y_H} \right] \\ + \frac{(1 - \tau_D \lambda^r)}{(1 - \lambda^r)} \left\{ \frac{A}{\theta_h} [h(T)]^{\eta_c} \varphi + \frac{A}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1 - \varphi) \right\} (1 - \lambda^r) \frac{C}{Y_H} - \frac{\lambda^r}{(1 - \lambda^r)} \frac{(1 - \beta) [b^{hp} + b^{fp} - RERb^{*hp}]}{Y_H} A - \frac{G}{Y_H} A \end{array} \right\}, \\ = F_2 N_H \left\{ \begin{array}{c} w_H + \frac{(1 - \tau_D \lambda^r)}{(1 - \lambda^r)} \left[ \frac{A}{\theta_h} [h(T)]^{\eta_c} \left( \varphi \frac{\lambda^r C^r}{Y_H} + \frac{G}{Y_H} \right) + \frac{A}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1 - \varphi) \frac{\lambda^r C^r}{Y_H} \right] \\ - \frac{\lambda^r}{(1 - \lambda^r)} \frac{(1 - \beta) [b^{hp} + b^{fp} - RERb^{*hp}]}{Y_H} A - \frac{G}{Y_H} A \end{array} \right\}, \end{aligned}$$

where  $F_2 \equiv \left\{ 1 - (1 - \tau_D \lambda^r) \left[ \frac{1}{\theta_h} [h(T)]^{\eta_c} \varphi + \frac{1}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1 - \varphi) \right] \right\}^{-1}$ . Thus, it can be written in a short form as  $C = \Phi_2 N_H$ , where  $\Phi_2$  is defined as follows:

$$\Phi_2 \equiv F_2 \left\{ \begin{array}{c} w_H + \frac{(1 - \tau_D \lambda^r) A}{(1 - \lambda^r)} \left[ \begin{array}{c} \frac{1}{\theta_h} [h(T)]^{\eta_c} \left( \varphi \frac{C}{Y_H} + \frac{G}{Y_H} \right) \\ + \frac{1}{\theta_f} \left[ \frac{h(T)}{T} \right]^{\eta_c} (1 - \varphi) \frac{C}{Y_H} \end{array} \right] \\ - \frac{\lambda^r}{(1 - \lambda^r)} \frac{(1 - \beta) [b^{hp} + b^{fp} - RERb^{*hp}]}{Y_H} A - \frac{G}{Y_H} A \end{array} \right\}.$$

Therefore, the steady state version of (30) is:

$$C^{aggr} \equiv [\lambda^r \Phi_1 + (1 - \lambda^r) \Phi_2] N_H,$$

where it remains to find an expression for  $N_H$ . Consider the following identity:  $MRS \equiv -\frac{U_{N_H}}{U_C} = \frac{\chi_N (N_H^\eta)^{\iota}}{\Lambda^j}$ , where  $\Lambda^j = C^{-\sigma-b+b\sigma}(1-\beta b)$  and taking into account the real version of Equation (43),  $w_H = \frac{\mu_W}{(1-\tau_w)} MRS$ , it follows that,

$$w_H = \frac{\mu_W}{(1-\tau_w)} \frac{\chi_N (N_H)^\iota}{C^{-\sigma-b+b\sigma}(1-\beta b)},$$

which equalized to  $w_H = A \left( \frac{\theta_h - 1}{\theta_h} \right)$  implies:

$$\left[ A \left( \frac{\theta_h - 1}{\theta_h} \right) \frac{(1-\tau_w)}{\mu_W \chi_N} C^{-\sigma-b+b\sigma}(1-\beta b) \right]^{\frac{1}{\iota}} = N_H.$$

Since the consumption of unconstrained consumers depends on  $N_H$ , replacing  $C = \Phi_2 N_H$  into expression above it yields a solution for  $N_H$ :

$$\begin{aligned} \left[ A \left( \frac{\theta_h - 1}{\theta_h} \right) \frac{(1-\tau_w)}{\mu_W \chi_N} (\Phi_2 N_H)^{-\sigma-b+b\sigma} (1-\beta b) \right]^{\frac{1}{\iota}} &= N_H, \\ \left[ A \left( \frac{\theta_h - 1}{\theta_h} \right) \frac{(1-\tau_w)}{\mu_W \chi_N} (1-\beta b) (\Phi_2)^{-\sigma-b+b\sigma} \right]^{\frac{1}{\iota}} (N_H)^{\frac{-\sigma-b+b\sigma}{\iota}} &= N_H, \\ \left[ A \left( \frac{\theta_h - 1}{\theta_h} \right) \frac{(1-\tau_w)}{\mu_W \chi_N} (1-\beta b) (\Phi_2)^{-\sigma-b+b\sigma} \right] &= \frac{N_H^\iota}{(N_H)^{-\sigma-b+b\sigma}}, \\ \left[ A \left( \frac{\theta_h - 1}{\theta_h} \right) \frac{(1-\tau_w)}{\mu_W \chi_N} (1-\beta b) (\Phi_2)^{-\sigma-b+b\sigma} \right]^{\frac{1}{\iota+\sigma+b-b\sigma}} &= N_H. \end{aligned} \quad (74)$$

Now we derive the domestic and foreign resource constraints:

$$\begin{aligned} Y_H &= \left[ \frac{P_H}{P} \right]^{-\eta_c} (\varphi C^{aggr} + G) + \left[ \frac{P_H^*}{P^*} \right]^{-\eta_c} (1-\varphi^*) C^{aggr*}, \\ &= [h(T)]^{\eta_c} (\varphi C^{aggr} + G) + \left[ \frac{\frac{\theta_h^*}{\theta_h^*-1} \frac{\theta_h - 1}{\theta_h} P_H / \mathcal{E}}{P^*} \right]^{-\eta_c} (1-\varphi^*) C^{aggr*}, \\ &= [h(T)]^{\eta_c} \left\{ (\varphi C^{aggr} + G) + \left[ \frac{P}{P_H} \frac{\theta_h^*}{\theta_h^*-1} \frac{\theta_h - 1}{\theta_h} \frac{P_H}{\mathcal{E} P^*} \right]^{-\eta_c} (1-\varphi^*) C^{aggr*} \right\}, \\ &= [h(T)]^{\eta_c} \left\{ (\varphi C^{aggr} + G) + \left[ \frac{P}{\mathcal{E} P^*} \frac{\theta_h^*}{\theta_h^*-1} \frac{\theta_h - 1}{\theta_h} \right]^{-\eta_c} (1-\varphi^*) C^{aggr*} \right\}, \end{aligned}$$

under symmetry, i.e. assuming:  $\theta_h = \theta_h^*$ ,  $\varphi = \varphi$ , previous expression simplifies to:

$$Y_H = [h(T)]^{\eta_c} (\varphi C^{aggr} + G) + [RER h(T)]^{\eta_c} (1-\varphi) C^{aggr*}.$$

and from Equation (28), evaluated at the steady state ( $RER = \Lambda^j / \Lambda^{j*} = \frac{C^{-\sigma-b+b\sigma}}{(C^*)^{-\sigma-b+b\sigma}}$ ) the relationship between consumption levels of unconstrained consumers is:

$$C = C^* RER^{\frac{1}{-\sigma-b+b\sigma}},$$

since Equation (30) also holds in the foreign country, it follows that:

$$\begin{aligned} Y_H &= [h(T)]^{\eta_c} (\varphi C^{aggr} + G) + [RER h(T)]^{\eta_c} (1-\varphi) [\lambda^r C^{r*} + (1-\lambda^r) C RER^{-\sigma-b+b\sigma}], \\ &= [h(T)]^{\eta_c} \left\{ \varphi C^{aggr} + G + (1-\varphi) \underbrace{[RER^{\eta_c} \lambda^r C^{r*} + RER^{\eta_c - \sigma - b + b\sigma} (1-\lambda^r) C]}_{\text{foreign aggregate consumption}} \right\}, \end{aligned}$$

following Galí (2008), from the risk sharing condition we find that  $C = C^* RER \frac{1}{-\sigma-b+b\sigma}$ , and at the steady state can be restated as  $C = C^* q(T)^{-\sigma-b+b\sigma}$ , where  $RER = \frac{P_F}{P} = \frac{T}{h(T)} \equiv q(T)$ . Note that  $C^{r*} = \Phi_1^* N_F$ , which leads to  $Y_H(T, T^*, N_F)$ :

$$Y_H = [h(T)]^{\eta_c} (\varphi C^{aggrr} + G) + T^{\eta_c} \left[ (1 - \varphi) \left( \lambda^r \Phi_1^* N_F + (1 - \lambda^r) C \left[ \frac{T}{h(T)} \right]^{-\sigma-b+b\sigma} \right) \right]. \quad (75)$$

The equilibrium condition that holds in the foreign country:

$$\begin{aligned} Y_F &= \left[ \frac{P_F^*}{P^*} \right]^{-\eta_c} (\varphi^* C^{aggrr^*} + G^*) + \left[ \frac{P_F}{P} \right]^{-\eta_c} (1 - \varphi) C^{aggrr}, \\ &= \left[ \frac{P_F^*}{P^*} \right]^{-\eta_c} (\varphi^* C^{aggrr^*} + G^*) + \left[ \frac{\frac{\theta_f}{\theta_f-1} \frac{\theta_f^*-1}{\theta_f^*} \mathcal{E} P_F^*}{P} \right]^{-\eta_c} (1 - \varphi) C^{aggrr}, \\ &= \left[ \frac{P_F^*}{P^*} \right]^{-\eta_c} \left\{ (\varphi^* C^{aggrr^*} + G^*) + \left[ \frac{P^* \frac{\theta_f}{\theta_f-1} \frac{\theta_f^*-1}{\theta_f^*} \mathcal{E} P_F^*}{P} \right]^{-\eta_c} (1 - \varphi) C^{aggrr} \right\}, \\ &= \left[ \frac{P_F^*}{P^*} \right]^{-\eta_c} \left\{ (\varphi^* C^{aggrr^*} + G^*) + \left[ \frac{\mathcal{E} P^*}{P} \right]^{-\eta_c} (1 - \varphi) C^{aggrr} \right\}, \\ &= (h(T^*))^{\eta_c} \left[ \varphi (\lambda^r C^{r*} + (1 - \lambda^r) C RER^{-\sigma-b+b\sigma}) + G \right] + \left( \frac{h(T^*)}{RER} \right)^{\eta_c} (1 - \varphi) C^{aggrr}, \\ &= (h(T^*))^{\eta_c} \left[ \varphi \left( \lambda^r C^{r*} + (1 - \lambda^r) C \left[ \frac{T}{h(T)} \right]^{-\sigma-b+b\sigma} \right) + G \right] + \left( \frac{h(T^*)}{\frac{T}{h(T)}} \right)^{\eta_c} (1 - \varphi) C^{aggrr}. \end{aligned} \quad (76)$$

To determine the steady home (foreign) state terms of trade, we need to solve Equations (75), (76), (74) and its foreign counterpart. Unknowns are  $T$  ( $T^* = \frac{1}{T}$ ),  $Y_H$  ( $= Y_F$ ),  $N_H$  and  $N_F$ .

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