OPTIMAL LINEAR INCOME TAX WITH HOUSING DEDUCTION

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ABSTRACT

This paper generalizes the standard model of the optimal linear income tax to include housing deduction. Unlike previous literature, we start from a dynamic equilibrium model and examine the steady state equilibrium. We then analyze first order conditions for our linear tax structure. The discussion suggest that as the economy we introduce in our model is more complex than in the standard model, new efficiency effects from increasing the marginal tax rate or the housing deduction appear that we should take in account. Obviously the importance of all those efficiency effects depend on their compensated elasticity but we should not a priori discard them.

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1. INTRODUCTION

Housing deduction, nowadays effective in the Spanish income tax, has criticized by several authors that defend its removal. Camarero et al. (1993) consider that housing deduction generates horizontal inequality. Zubiri (1990) sees little efficiency gains from it since the fiscal subsidy is completely transferred from buyers to sellers through increases in prices as the price elasticity of supply is very low. He also considers that the housing subsidy does not induce the acquisition of a first necessity good properly since it is a discriminatory protection against those that have lower income and consequently need to be helped. L∴pez (1996) shows with a dynamic model that in the long run housing deduction has contributed with other factors to the increase of housing prices during the 1980's in Spain. Despite all those opinions against housing deduction the present Spanish government has announced a reform of the Spanish income tax that will maintain this deduction.

Analyzing the existing literature we can see two different approaches to the problem of designing an income tax. On one hand we have the theory of optimal taxation that has formalized the design of a tax system that maximizes the social welfare, see Mirlees (1971), Sheshinski (1972), Atkinson (1973), Atkinson and Stiglitz (1980) and Slemrod (1994). The resulting optimal tax system takes account of the trade-off between efficiency and a more equal distribution of wealth. Unfortunately those models which work with heterogeneous agents represent a highly simplified economy. On the other hand we have dynamic macroeconomics models that examine the effect of a certain tax change on the steady state utility of one representative agent, see Turnovsky and Okuyama (1994).

Instead of the classical static optimal tax model we use an infinite horizon model following Turnovsky and Okuyama (1994) but we modify it in a suitable way to introduce equity objectives.

The remainder of the paper proceeds as follows. In section 2 we lay out the representation of the economy and we analyze the steady state equilibrium. In section 3 we eventually establish the optimal linear income tax and the optimal housing deduction. Section 4 offers some conclusions.

2. THE ANALYTICAL FRAMEWORK

2.1 Structure of the economy

The production side is as simple as possible. Labour is used in this economy to produce a good, considered the numeraire commodity, that may be used for consumption or alternatively as a stock of housing to produce housing services. We assume that all the stock of housing is residential and owner occupied. The economy consist of a number of infinitely lived individuals that are equal in all regards except for their ability represented by w. The utility function has the usual concavity properties. In a first stage individuals decide the number of hours they want to work, their consumption of both the numeraire good and the housing services and their accumulation of housing stock and government bonds to maximize their utility subject to the budget constrain. In a second stage the government decides the structure of the linear progressive tax that maximizes a social welfare function subject to the revenue constrain. This structure consist of three elements: the minimum income guaranteed, the marginal tax rate and the housing deduction.

Each individual maximizes:

$$\sum_{0}^{00} \mathrm{b}^{t} U(C_t\ ,\ L_t\ ,\ H_t\ ) \quad U_C > 0,\ U_L < 0\ ,\ U_H > 0, (1a)$$

Where

C = per capita consumption of the non-housing good,

L = per capita labor supply,

H = per capita consumption of housing services,

🙰 = consumer rate of time preference.

subject to

$$P_t \;\; H_t \; + \; b_{t+1} \; + \; C_t \; + \; (1 - \mathrm{d}\,)\,(h_{t+1} - \; h_t\,) \; = \; \left[w \;\; L_t \; + \; r_{h_t} \;\; h_t \; + \; r_t \;\; b_t \right] (1 \; - \; \mathrm{t}\;) \; + \; G_t$$

$$C_t \;\; \geq \;\; 0, \quad 1 \;\; \geq \;\; L_t \;\; \geq 0, (1b)$$

and initial conditions

$$b(0) = b_0 \qquad h(0) = h_0\,, (1c)$$

where

P = (imputed) price of housing services, expressed in terms of the numeraire good,

b = per capita stock of Government bonds, assumed to be denominated in terms of the numeraire good,

✉ = housing deduction,

h = per capita stock of housing,

w = individual ability,

r_h = real rental rate on housing,

r = real rate of return of government bonds,

☺ = marginal tax rate,

G = minimum income guaranteed.

Equation (1b) states that individuals spend their after tax income from labor, bonds, housing stock and the Governmentэs minimum income guaranteed in housing services, consumption, accumulation of bonds and housing stock.

Solving the intertemporal optimization problem defined in equations (1a) to (1c) we obtain the following optimality conditions:

$$U_C \;\; = \;\; \mathrm{l}\;, (2a)$$

$$U_L + w(1-\tau)U_C = 0, (2b)$$

$$\frac{U_H}{U_C} = P, (2c)$$

$$r_{h_t} = r_t\,(1-\delta)\,, (2d)$$

where λ, the costate variable associated with the accumulation eq. (1b), is the marginal utility of wealth, measured in terms of the numeraire commodity. Eq. (2a) equates the marginal utility of consumption of the numeraire good to the marginal utility of wealth. Eq. (2b) defines the labor supply where w is the ability of the individual. There is a critical salary w_0 such that

$$L_t > 0 \text{ when } w > w_0$$
$$L_t = 0 \text{ when } w \leq w_0, (2e)$$

Eq (2c) can be interpreted as defining the price of housing services as the marginal rate of substitution between housing services and consumption. Eq. (2d) equates the rate of return of the housing stock to the interest rate of bonds taking into account the housing deduction.

We assume that H and h are proportionally related by: $H_t=\alpha h_t$ then P and r_h are related by $\alpha P_t=r_{ht}$ (marginal income of housing services= marginal cost of housing services). Without loss of generality we take $\alpha=1$ so that

$$H_t = h_t, (2f)$$

$$P_t = r_{h_t}, (2g)$$

$$P_t\ H_t = r_{h_t}\ h_t, (2h)$$

In addition, the following transversality conditions must be met:

$$\lim_{t\to\infty} \beta^t\ U_c\ h_{t+1} = \lim_{t\to\infty} \beta^t\ U_c\ b_{t+1} = 0, (2i)$$

In the production side we assume constant prices and the absence of benefits. The government revenue per individual is R_0 then the production restriction is

$$\int_w^\infty C_t dF + R_0 + \int_w^\infty (h_{t+1} - h_t)dF = \int_{w_0}^\infty wL_t dF, (2j)$$

We assume $\int_{w}^{\infty} dF = 1$. Using restriction (1b), the previous restriction can be rewritten as

$$R_0 + G + \int_{w}^{\infty} \left[\delta (h_{t+1} - h_t) + r_t b_t \right] dF = \int_{w}^{\infty} \left[\tau (wL_t + r_{h_t} h_t + r_t b_t) + (b_{t+1} - b_t) \right] dF \, , (2k)$$

which is the usual government restriction, where incomes from taxes and public bonds must be equal to expenses. Finally we assume a continuously balanced budget

$$b_{t+1} - b_t (2l)$$

and the government restriction becomes

$$R_0 + G + \int_{w}^{\infty} [\delta (h_{t+1} - h_t) + r_t b_t] dF = \int_{w}^{\infty} [\tau (wL_t + r_{h_t} h_t + r_t b_t)] dF \, , (2m)$$

2.2 The Macroeconomic equilibrium and steady state equilibrium

The macroeconomic equilibrium

The macroeconomic equilibrium consists of the following set of equations that hold at all points of time and jointly determine C_t, H_t, L_t, h_{t+1}, P_t, r_t, r_{ht} and λ_t.

$$U_C = \lambda_t \, , (3a)$$

$$U_L + w(1-\tau) U_C = 0 \, , (3b)$$

$$\frac{U_H}{U_C} = P_t \, (3c)$$

$$r_{h_t} = r_t (1-\delta) \, , (3d)$$

$$-\lambda_t + \beta \lambda_{t+1} [1 + r_{t+1}(1-\tau)] = 0 (3e)$$

$$P_t H_t + C_t + (1-\delta)(h_{t+1} - h_t) = [w L_t + r_{h_t} h_t + r_t b] (1 - \tau) + G \, , (3f)$$

$$P_t = r_{h_t} \quad (3g)$$

$$H_t = h_t, \quad (3h)$$

The steady state equilibrium

This equilibrium is attained when: $\lambda_t = \lambda_{t+1}$ and $h_t = h_{t+1}$ and it implies the following relations

$$\tilde{r} = \frac{(1-\beta)}{\beta}\frac{1}{(1-\tau)} \quad (4a)$$

$$\tilde{r}_h = \tilde{r}\,(1-\delta), \quad (4b)$$

$$U_L + w(1-\tau)U_C = 0, \quad (4c)$$

$$\frac{U_H}{U_C} = \tilde{P}, \quad (4d)$$

$$\tilde{P}\tilde{H} + \tilde{C} = [w\tilde{L} + \tilde{r}_h\tilde{h} + \tilde{r}\,b]\,(1-\tau) + G, \quad (4e)$$

$$\tilde{P} = \tilde{r}_h, \quad (4f)$$

$$\tilde{H} = \tilde{h}, \quad (4g)$$

$$\tilde{U}_C = \tilde{\lambda}, \quad (4h)$$

These equations jointly determine the long-run equilibrium values of: $\tilde{C}$, $\tilde{H}$, $\tilde{L}$, $\tilde{h}$, $\tilde{P}$, $\tilde{r}$, $\tilde{r}_h$, $\tilde{\lambda}$. We can solve the system in a very simple recursive manner. First, eq. (4a) determines the value of $\tilde{r}$ in terms of β and τ. An increase in the marginal tax rate will increase the rate of return of government bonds. Eq. (4b) yields $\tilde{r}_h$ in terms of β, τ, and δ. Once $\tilde{r}_h$ is obtained, equation (4f) will determine $\tilde{P}$. As before, a higher marginal tax rate raises the rate of return of the per capita stock of housing. Equations (4c), (4d) and (4e) may be solved jointly to determine the value of $\tilde{C}$, $\tilde{H}$, $\tilde{L}$. Substituting $\tilde{C}$ in (4h) we can deduce the value of $\tilde{\lambda}$. Knowing $\tilde{H}$, then $\tilde{h}$ immediately follows from (4g).

3.GOVERNMENT TAXING DECISIONS

In this section the government decides the structure of a linear progressive tax. That is, a minimum guaranteed income, a marginal tax rate and the housing deduction. Therefore we are generalizing the nowadays standard model of optimal linear income tax to include the decision of fixing a certain housing deduction. Moreover we have extended the standard model to include the effect that the marginal tax rate may have on the real rate of return of government bonds and the real rate of return of per capita stock of housing.

We assume the government maximizes the social welfare function

$$\int_w^\infty \tilde{\Psi}(V)dF,(5a)$$

where V is the indirect utility function, subject to

$$R_0 + G + \tilde{r}\int_w^\infty bdF = \tau\int_w^\infty [w\tilde{L} + \tilde{r}(1-\delta)\tilde{H} + \tilde{r}b]dF,(5b)$$

Forming the Lagrangean

$$L = \int_w^\infty \left[\tilde{\Psi}(V) + \mu\left(\tau\left[w\tilde{L} + \tilde{r}(1-\delta)\tilde{H} + \tilde{r}b\right] - R_0 - G - \tilde{r}b\right)\right]dF,(5c)$$

We may derive the first-order conditions with respect to G, t and ⊠.

$$\int_w^\infty \left[\tilde{\Psi}'\frac{\partial V}{\partial G} + \mu\left(\tau\left[w\frac{\partial L}{\partial G} + \tilde{r}(1-\delta)\frac{\partial H}{\partial G}\right] - 1\right)\right]dF = 0,(6a)$$

$$\int_w^\infty \left[\tilde{\Psi}'\frac{\partial V}{\partial \tau} + \mu\left(\left[w\tilde{L} + \tilde{r}(1-\delta)\tilde{H} + \tilde{r}b\right] + \tau\left[w\frac{\partial L}{\partial \tau} + \tilde{r}(1-\delta)\frac{\partial H}{\partial \tau}\right] + \tau\frac{\partial r}{\partial \tau}\left[b + (1-\delta)\tilde{H}\right] - b\frac{\partial r}{\partial \tau}\right)\right]dF = 0,(6b)$$

$$L = \int_w^\infty \left[\tilde{\Psi}'\frac{\partial V}{\partial \delta} + \mu\left(\tau\left[w\frac{\partial L}{\partial \delta} + \tilde{r}(1-\delta)\frac{\partial H}{\partial \delta} - \tilde{r}\tilde{H}\right]\right)\right]dF = 0,(6c)$$

We try to simplify those expressions taking into account that

$$\frac{\partial V}{\partial G} = \frac{\partial V}{\partial M}\frac{\partial M}{\partial G} = \gamma,(7a)$$

$$\frac{\partial L}{\partial G} = \frac{\partial L}{\partial M}, (7b)$$

$$\frac{\partial H}{\partial G} = \frac{\partial H}{\partial M}, (7c)$$

We also use Roy's Identity

$$-\frac{\frac{\partial V}{\partial \tau}}{\frac{\partial V}{\partial M}} = w\tilde{L} + \tilde{r}\tilde{b} + \tilde{r}(1-\delta)\tilde{H}$$

We define Z as: $Z = w\tilde{L} + \tilde{r}\tilde{b} + \tilde{r}(1-\delta)\tilde{H}$ so that

$$-\frac{\partial V}{\partial \tau} = \gamma Z, (7d)$$

$$-\frac{\frac{\partial V}{\partial \tau(1-\delta)\tilde{r}}}{\frac{\partial V}{\partial M}} = H$$

$$\leftrightarrow \frac{\partial V}{\partial \delta} = \frac{\partial V}{\partial \tau(1-\delta)\tilde{r}} \frac{\partial \tau(1-\delta)\tilde{r}}{\partial \delta} = \tau\tilde{r}\gamma H, (7e)$$

And the Slutsky equations

$$\frac{\partial L}{\partial \tau} = -wS_{LL} - Z\frac{\partial L}{\partial M}, (7f)$$

$$\frac{\partial H}{\partial \tau} = \tilde{r}(1-\delta)S_{HH} - Z\frac{\partial H}{\partial M}, (7g)$$

where S_{LL},S_{HH} are the substitution terms. S_{LL} is the compensated response of labor to the net wage and S_{HH} is the compensated response of housing demand to the net price.

Those two equations allow us to rearrange (6a), (6b) and (6c) as follows

$$\int_{w}^{\infty}\left(\tilde{\Psi}'\frac{\gamma}{\mu}+\tau\left[w\frac{\partial L}{\partial M}+\tilde{r}(1-\delta)\frac{\partial H}{\partial M}\right]-1\right)dF = 0, (8a)$$

$$\int_{w}^{\infty}\left[\left[\tilde{\Psi}'\frac{\gamma}{\mu}+1\right]Z+\tau\left(w\left[-wS_{LL}-Z\frac{\partial L}{\partial M}\right]+\tilde{r}(1-\delta)\left[\tilde{r}(1-\delta)S_{HH}-Z\frac{\partial H}{\partial M}\right]+\frac{\partial r}{\partial \tau}b+(1-\delta)\tilde{H}\right)-b\frac{\partial r}{\partial \tau}\right]dF = 0,$$

$$\int_{w}^{\infty}\left[\tilde{\Psi}'\frac{\gamma}{\mu}\tau\tilde{r}\tilde{H}+\tau\left(w\frac{\partial L}{\partial \delta}+\tilde{r}(1-\delta)\frac{\partial H}{\partial \delta}-\tilde{r}\tilde{H}\right)\right]dF = 0, (8c)$$

We now define B to be the net social marginal valuation of income, measured in terms of government revenue and modified to include the effect in the increase in income on the housing stock. B measures the benefit from transferring one monetary unit to the household allowing for the marginal tax paid on this extra monetary unit.

$$B = \tilde{\Psi}'\frac{\gamma}{\mu} + \tau\left[w\frac{\partial L}{\partial M} + \tilde{r}(1-\delta)\frac{\partial H}{\partial M}\right], (9)$$

Using this condition aswell as the equations (8a), (8b) and (8c) we characterize the optimal tax policy as

$$\overline{B} = 1$$

where $\overline{B}$ is the mean value of B.

$$\int_{w}^{\infty}\left[(B-1)Z+\tau\left[\frac{wL}{1-\tau}\varepsilon_{LL}-\frac{\tilde{r}(1-\delta)}{1-\tau}\tilde{H}\varepsilon_{HH}-\frac{\tilde{r}}{1-\tau}\varepsilon_{r\tau}\left[b+(1-\delta)\tilde{H}\right]+b\frac{\partial r}{\partial \tau}\right]\right]dF = 0$$

where ε_{LL} is the compensated wage elasticity of labor $\left(\varepsilon_{LL} = S_{LL}\dfrac{(1-\tau)w}{L}\right)$, ε_{HH} is the compensated price elasticity of housing $\left(\varepsilon_{HH} = S_{HH}\dfrac{r(1-\delta)(1-\tau)}{H}\right)$ and $\varepsilon_{r\tau}$ is the tax rate elasticity of interest rate $\left(\varepsilon_{r\tau} = \dfrac{\partial r}{\partial(\tau)}\dfrac{(1-\tau)}{r}\right)$ all expressed in wage equivalent units.

which is equal to:

$$\frac{\tau}{1-\tau}=\frac{-COV(Z,B)-\int_{w}^{\infty} b\frac{\partial r}{\partial \tau}dF}{\int_{w}^{\infty}\left[w\tilde{L}\varepsilon_{LL}-\tilde{r}(1-\delta)\tilde{H}\varepsilon_{HH}-\tilde{r}\left[b+(1-\delta)\tilde{H}\right]\varepsilon_{r\tau}\right]dF},(10b)$$

and eventually

$$\int_{w}^{\infty}\left[\tilde{\Psi}'\frac{\gamma}{\mu}\tau\tilde{r}\tilde{H}+\tau\tilde{r}(1-\delta)\frac{\partial H}{\partial \delta}\right]dF=\tau\int_{w}^{\infty}\left(r\tilde{H}-w\frac{\partial L}{\partial \delta}\right)dF,(10c)$$

Condition (10a) can be interpreted as in the Atkinson-Stiglitz model (1980). It says that the minimum guaranteed income should be adjusted in such a way that the net social valuation of income (B) should, on average, be equal to the cost (one monetary unit). Condition (10b) establishes the optimal tax rate and can be compared to the expression found in the Atkinson-Stiglitz model.

$$\frac{\tau}{1-\tau}=\frac{-COV(w\tilde{L},B)}{\int_{w}^{\infty}[w\tilde{L}\varepsilon_{LL}]dF}$$

In our model $COV(w\tilde{L}+\tilde{r}\tilde{b}+\tilde{r}(1-\delta)\tilde{H}, B)$ subtitutes $COV(w\tilde{L},B)$ because $w\tilde{L}+\tilde{r}\tilde{b}+\tilde{r}(1-\delta)\tilde{H}$ and not $w\tilde{L}$ is pre-tax income, given that we have introduced government bonds and housing in the original model. We can interpret the covariance as a marginal measure of inequality. The greater is the inequality aversion the higher will the marginal tax rate be. When there is no aversion to inequality $COV(w\tilde{L}+\tilde{r}\tilde{b}+\tilde{r}(1-\delta)\tilde{H}, B)$ will be zero and the marginal tax rate will be zero aswell. The second term that appears in the numerator of equation (10b) is new. It can be interpreted as the disincentive the government has to increase marginal tax rates when the interest rate depends on the tax rate as in our model. When government raises the tax rate the interest rate and the government debt increases. The denominator is also modified. In the Atkinson-Stiglitz version it is the compensated labor supply elasticity, weighted by labor income. In our version we include the compensated price elasticity of housing weighted by housing expenditure and the tax rate elasticity of interest rate weighted by the expenditure in government bonds and housing expenditure. The compensated price elasticity of housing tends to reduce the marginal tax rate. An increase in the tax rate raises the price of housing reducing the investment in housing stock and consequently government's income. On the other hand the tax rate elasticity of interest rate will increase the marginal tax rate. This is due to the fact that an increase in the tax rate will increase the interest rate as well as the government revenue coming from income taxation. This equation shows that the decision about the optimal progressivity of the income tax must take into account efficiency effects others than the labor supply effect. An increase of the tax rate will probably have consequences in the interest rate that we can not ignore.

Finally condition (10c) tells us that the optimal value for the deduction δ is such that the social marginal benefit from increasing the housing deduction must be equal to its marginal cost. The marginal benefit appears at the left hand side of the equation and is made of two elements. The first one is the increase in consumer's utility due to a reduction of the price of per capita housing services. The second one is an extra revenue collected because individuals raise their per capita stock of housing and pay more income taxes on them. The marginal cost is also made of two elements. The decrease in government revenue because the real rate of return of per capita stock of housing has decreased and the decrease in government revenue due to the fact that per capita labor supply will be lower.

4. CONCLUSIONS

This paper has analyzed an optimal linear tax with housing deduction. Motivation for this analysis has been provided by the major controversy that has emerged in Spain about whether a housing deduction should be included in the income tax and its effects. The novelty of this analysis is the inclusion of dynamic effects that were absent on previous optimal linear income tax models. The discussion suggests that when a more complex economy is considered new efficiency effects from increasing the marginal tax rate or the housing deduction appear that should be taken into account. Obviously the importance of all those efficiency effects depends on their compensated elasticity but we should not a priori discard them.

Finally it goes without saying that the analysis is based on many restrictive assumptions. We have considered there is a numeraire commodity that can be used for consumption or alternatively as a stock of housing to produce housing services. A possible way for improving the model could be to introduce two different goods in the economy, a composite good, which we could call non-housing and housing and two productive sectors. The housing and the non- housing sector. This would probably allow us to see the effect of the housing deduction on the price of housing services, which is one of the most questioned effects of the housing deduction.

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