Is a Hedonic Approach Only Applicable to Small Scale Public Projects? A general equilibrium analysis

Noboru Hidano*
Tokyo Institute of Technology

Key words: hedonic price, benefit estimation, local public goods, capitalization hypothesis, cost benefit

Abstract

Despite the conventional proposal that the hedonic approach is limited to a marginal improvement of an amenity resulting from a public policy and project, the present study was performed to examine the accuracy of this method in a nonmarginal context. A numerical general equilibrium analysis was adopted to examine the overestimation theorem of capitalization in mixed land use. The overestimation ratios of most cases were small and maximum ranges of the ratios with respect to area and the degree of improvement in the amenity resulting from the policy and project were identified. The results were also compared with the ratios of an actual large-scale transport project in Japan estimated by a general equilibrium analysis and using national hedonic price functions.

*E-mail:hidano@wb3.so-net.ne.jp
nhidano@soc.titech.ac.jp
Web: www.soc.titech.ac.jp/~hidano
Postal address: Department of Social Engineering,
Tokyo Institute of Technology, 2-12-1-W9-89,
Ookayama, Meguro, Tokyo, 152-8552, Japan
Tel:+81-3-5734-3185
Fax:+81-3-5734-3582

1. Introduction

Although many previous studies have suggested that the hedonic price method is applicable only to small public projects (Freeman (1993), Palmquist (1991) among others), it is not yet clear how small a project should be to merit application of this method. There have been several attempts to determine the validity of the capitalization hypothesis on which the hedonic approach is based. For example, some studies examined whether the benefit of a public policy and project can be captured in land prices or rents and wages (Starrett (1981), Roback (1982), Kanemoto (1985,1988), Hidano *et al.* (1992), and Hidano (1997), Hidano and Hayashiyama (1997) among others). Kanemoto successfully examined the capitalization hypothesis in the context of cost-benefit analysis, taking into account the cost of the policy and project. He showed, for the first time, the condition of cross-sectional capitalization of the benefits of public policies and projects in terms of land value or land rent as well as the overestimation theorem of hedonic measure to examine the gross benefit of the policy and project for an *ex ante* evaluation. Scotchmer (1985, 1986) confirmed that, even in the case of a homogeneous consumer, hedonic measures could not depict the benefits of amenity changes correctly despite the argument put forward by Rosen (1974) that the hedonic price differentials equal the value of amenity in the case of a homogeneous consumer[1].It is generally accepted that identification of a bid price function or any form of utility function is very difficult from the viewpoint of econometrics (see Sheppard (1999)). Recently, Epple and Sieg (1999) demonstrated that utility functions of housing and amenities could be estimated by taking into account the heterogeneous taste and income of consumers provided we determine the functional form. Kanemoto's paper was groundbreaking because he demonstrated that it is not necessary to identify the bid price functions from the hedonic data to measure the benefits of a public project and policy. We showed that it is true in some cases of homogeneous and in a specific heterogeneous consumer case[2].

---

[1] Bartik (1988) discussed the overestimation of the hedonic price model in a different context.

[2] We have discussed the degree of overestimation in the context of heterogeneous households and found that it is not as large as we expected (Hidano (2002), Hidano (1997), Hidano *et al.* (1992)), although the results were justified in a specific case. However, it should be noted that if heterogeneous consumers live in all regions, the hedonic measures can depict the actual gross benefits of the policy or project (see Hidano (2002)).

Although Kanemoto demonstrated equality conditions and the overestimation theorem, he did not discuss the degree by which the hedonic measure overestimates the gross benefit when the project and policy is not marginal. The purpose of the present study was to examine overestimation theorem using a model that incorporates the land and amenity inputs of a factor of production in more general context, *i.e.*, mixed land use and to find the correct upper bound of the overestimation ratio. For practical use, it is necessary to know the upper range of overestimation of the hedonic measure. The purpose of the present study was to demonstrate comprehensively that the overestimation caused by the hedonic measure should be small enough to be able to use the hedonic approach in decision making. The structure of the paper is as follows: We present our model in section 2, discuss the overestimation theorem in section 3, and present, in section 4, the overestimation ratios calculated by numerical analysis assuming constant elasticity of substitution (CES) utility and production functions under nonmarginal conditions. We calculate the upper bound of the overestimation ratio despite the values of the parameters being unknown. Finally, in section 5, we demonstrate that the overestimation ratio of a large national project in Japan is consistent with the results of numerical analysis.

2. The Model

We adopt a two-region model and assume that homogeneous consumers (N) can relocate from one region (i) of area H_i to the other without cost. However, they cannot commute to the other region. They maximize their utility under the budget constraints. Non-wage income, *i.e.*, rent from land and dividends from firm profits, is distributed equally among consumers. A uniform national dividend scheme applies (Wildashin (1987)). The price of x (a composite good) is unity for normalization:

$$\max_{x,l} u\left(x_i, l_i^h, z_i\right)$$

$$\text{s.t. } w_i + s = x_i + r_i l_i^h \qquad (1)$$

$$s = (\pi^L + r_1 H_1 + r_2 H_2)/N \qquad (2)$$

where l^h is land, and z is an amenity, w is wage, r is land rent, s is a non-wage income and π^L is firm profits.

Firms maximize their profits under the constraints of a constant-return-to-scale technology, which requires workers n and land l^f for the firm. As we assume an open competitive economy, the number of firms can be assumed to be unity for each region and the profit of the firms becomes zero (it should be noted that the amenity z affects the firm's productivity as a local public good).

$$\begin{aligned} &max_{n,l} \pi^L{}_i = X_i - w_i n_i - r_i l_i^f \\ &\text{s.t.} \quad X_i = X\left(n_i, l_i^f, z_i\right) \end{aligned} \tag{3}$$

where X is the output of the firms. The difference from the Kanemoto model, assuming that land for production is determined beforehand, is that we allow mixed use of land. Mixed use increases the complexity of the problem, because it requires the model to consider the competition between households and firms for the use of land. However, it is important to examine this case because large-scale improvements inevitably change land use in the long-term. The rent of land for firms should be equal to that for residences because landlords will maximize their profits from their land[3]. We assume z_2 is larger than z_1. The equilibrium condition for mixed use land is:

$$H_i = n_i^o l_i^{h^o} + l_i^{f^o}. \tag{4}$$

Then, we introduce a policy or project to improve the level of the amenity in Region 1(z_1) up to the level of Region 2(z_2). Superscripts w and o indicate with and without the policy or project, respectively. Thus, with the policy or project, the level of amenity becomes the same in the two regions.

$$\begin{aligned} &z_1^o \rightarrow z_2^o \\ &z_1^w = z_2^o \end{aligned}$$

The cost of the policy or project is C, which is collected by the government as a lump sum tax. Thus, the non-wage income after tax with the policy or project is:

$$s = (\pi^{Lw} + r^w(H_1 + H_2)) / N - C / N. \tag{5}$$

The policy or project requires a composite good as an input.

Thus the quilibrium with the policy or project is:

[3] We can easily introduce zoning of land, under which conditions we use different rents for land used by firms and add firm rent to non-wage income as well as to the hedonic measure (Kanemoto (1985)).

$$n^{w}{}_{1}x^{w}{}_{1} + n^{w}{}_{2}x^{w}{}_{2} + C = X^{w}{}_{1} + X^{w}{}_{2}\,. \tag{6}$$

3. Theory of Cross-sectional Capitalization

Now, we can discuss the overestimation theorem and equality conditions. The cross-sectional hedonic measure is:

$$B = \left(r^{o}{}_{2} - r^{o}{}_{1}\right)H_{1}\,. \tag{7}$$

Kanemoto's overestimation theorem is:

$$B \geq C + V \tag{8}$$

where

$$V = N \cdot EV\quad . \tag{9}$$

Equivalent variation is defined by the expenditure function $E(\cdot,\cdot,\cdot,\cdot)$:

$$EV = E\left(1, r_2^o, z_2^o, u^w\right) - E\left(1, r_2^o, z_2^o, u^o\right).$$

It should be noted that the use of prices in *R*egion 2 avoids ambiguous results in the analysis (Kanemoto (1988)). Complete cross-sectional capitalization requires the equality of (8). Although Kanemoto argued that the equality holds when one of the following conditions is satisfied: 1) z_2-z_1 is sufficiently small; 2) H_1 is sufficiently small; and 3) Leontief utility and production functions are applied, the third condition is questionable (see Appendix 1). However, even several simple cases[4], where amenity can be substituted by other commodities in production, show that the results of the capitalization hypothesis are not sufficiently clear to allow us to determine the applicability of the hypothesis in reality. Therefore, it is worthwhile to discuss the degree of capitalization numerically. Thus, we define:

---

[4] For example, Proposition 1: The case of non-production: Even if wages are given to consumers as endowments, *i.e.*, wages with and without the project are the same, the benefits of the projects would be overestimated by the hedonic measure. Proof: Equality (8) holds only when the land rent of *R*egion 2, r_2, is not changed due to implementation of the project. This is only possible when equality condition 1) or 2) holds. Proposition 2: The case of production with only worker input: The benefits of the project are overestimated by the hedonic measure. Proof: The constant-return-to-scale assumption requires no difference between wage rates in both regions. Thus, this case is equivalent to Proposition 1.

$$B/(C+V). \tag{10}$$

This ratio shows an approximation of the hedonic measure used to estimate the exact gross benefit. If it is close to unity, then capitalization is perfectly achieved. In practical cost-benefit analysis, a ratio less than 1.2 or 1.3, although not nearly unity, still makes sense, because benefit figures estimated in conventional public and environmental studies sometimes differ by more than two or three times the amount (see Brookshire *et al.* (1982), Hiramatsu and Hidano (1989)). Although we have discussed several specific cases in which the overestimation ratios of a hedonic measure were small when mixed land use was permitted (Hidano (1997, 2000, 2002)), the extent of the theorem's applicability is not yet clear.

4. Numerical Analysis to Examine the Capitalization Hypothesis

Procedure

First, we specify the form of the functions of the model using CES functions as follows to obtain concrete results:

$$u_i = \left(\alpha x_i^{-\rho} + \beta\left(l_i^h\right)^{-\rho} + \gamma z_i^{-\rho}\right)^{-1/\rho}, \quad X_i = \left(\delta n_i^{-\sigma} + \varepsilon\left(l_i^f\right)^{-\sigma}\right)^{-1/\sigma} z_i^{\zeta}. \tag{11}$$

Previous studies have shown that marginal improvement could be measured by hedonic analysis, but these studies failed to identify the size limits of policies or projects in which the marginal improvement can still be measured quantitatively. We introduced two criteria to measure the size of policies or projects to correspond to Kanemoto's equality conditions 1) and 2), *viz.*, the degree of improvement by the policy or project and the size of *R*egion 1, which is directly affected by the policy or project. The former is measured by z_1/z_2, and H_1/H_2 indicates the relative size of the area affected directly. This study adopts an objective method to test the overestimation theorem of capitalization as follows:

i) For an area of a given size and degree of improvement, we calculate H_1/H_2 from 0.01 to 10.00 and z_1/z_2 ranging from 0.30 to 0.99. Thus, the value of z_1/z_2 shows the inverse of the improvement ratio. A small z_1/z_2 represents a drastic policy or a large project.

ii) We assume that the policy or project increases the level of amenity in *R*egion 1 from z_1 to z_2.

iii) To examine the overestimation ratio in comprehensive parameter space and to minimize the subjectivity of the parameter settings in the numerical analysis, the parameters are given in a systematic way.

iii)a. For utility function, we assume $\alpha+\beta+\gamma=1$

with each parameter changing from 0.1 to 0.8.

iii)b. The effects of the different values of elasticity $(1/(1+\rho))$ are examined using values from 0.1 to 2.0.

iii)c. For production functions, we then assume $\delta+\varepsilon=1$

where δ and ε are distributed over the parameter space. The power parameter of an amenity input ζ is tested from 0.1 to 0.9, and elasticity $(1/(1+\sigma))$ is tested from 0.1 to 2.0.

iii)d. The cost of the policy or project is fixed as 1% of the total production without the policy or project.

iv) Equilibrium values of the prices and other variables in the model developed in section 2, both with and without the policy or project, are calibrated.

Results

Spatial size and degree of improvement resulting from the policy or project and the overestimation ratio

Figure 1 shows the results in space $(H_1/H_2,\ z_1/z_2)$ at $\alpha=\beta=\gamma=1/3$, $\delta=\varepsilon=\zeta=1/2$, when the elasticities of both the utility and the production functions are 1.5. The overestimation ratio based on these data is less than 7%, even when the area directly improved by the policy or project covers half of the area of the society and when an improvement is as large as 100%, *i.e.*, $z_1=0.5\ z_2$ with the utility and production functions set at $\alpha=\beta=\gamma=1/3$ and $\delta=\varepsilon=\zeta=1/2$. It should be noted that when the area and degree of improvement are very small, the overestimation ratio seems to increase. This is because the cost of the project becomes much larger than the benefit, as we have fixed the cost of the policy or project at 1% of total production. If we reduce the cost according to the decrease in area and the degree of improvement by the policy or project, then the overestimation ratio declines rapidly (see Table 1).

Elasticities and overestimation ratio

Figure 2 shows the impacts of changes in the elasticities of the utility and the production functions on the overestimation ratio. We set $\zeta=0$ to avoid the impact of amenity on production. The overestimation ratios are less than 3% in this space.

Generally, decrease in the elasticities of both functions tends to decrease the overestimation ratio monotonically. However, it is interesting that this is true only down to specific values of the elasticities. In this figure, the value of elasticity of the utility function is around 0.5. The decrease in elasticity of production increases the overestimation ratio. This is especially true when elasticity in utility decreases down to about 0.1. As we proved in the previous section, even if the elasticities are zero, when the cost of the policy or project should not be converged to zero, the overestimation ratios inevitably have to be greater than unity. It also should be noted that in our utility function in (11), the gross benefit of a policy or project becomes zero when the elasticity in utility becomes zero. Even though we can observe these nonmonotonic characteristics[5] of the relationship between elasticity and overestimation ratios, the distortion is small enough to apply the hedonic measures to gross benefit estimation.

Impact of amenity on production

We can discuss the impact of the amenity on production by comparing ζ=0 and 0.5 in Table 2. First, this table shows that when the elasticities of production and utility functions are less than 0.2, the increase in ζ from 0 raises the overestimation ratio because the increase in ζ represents an increase in the degree of improvement by the policy or project. We should recall that the overestimation ratios of large improvement of amenity are not small.

We can see that when ζ=0.5 and the elasticity of production function decreases down to about 0.1, the overestimation ratios increase rapidly. This indicates that smaller elasticities in both production and utility functions are associated with higher overestimation ratios. This table also shows that the overestimation ratios are larger than 10% when the elasticity of the production function is less than or equal to 0.1 and the elasticity of the utility function is less than or equal to 0.2, if ζ=0.5. But in practice, high overestimation ratios are not problematic when the hedonic measure is nearly zero.

---

[5] This overall nonmonotonic nature of the changes of overestimation ratios with respect to elasticity is caused by the following factors. When the higher substitutability between land and labor in production and between land and consumer good in utility is assumed, the benefit can be capitalized in wages as well as land rent. Thus the overestimation ratio is high. But when the substitutability is small in production if the substitutability between land and consumer good in utility is so small, the increase of elasticity in production, *i.e.*, the increase in substitutability of production inputs expands the production range, and then the production increases and raises the demand of land for production and housing. The competition between the consumers and producers for land lessens the overestimation ratios. It is also assumed that the cost of the policy or project is positive in this section. These factors complicate the overestimation ratios with respect to elasticities.

Thus the gross benefit of the project is also zero thanks to the overestimation theorem.

In summary, in the CES case, we were not surprised that the benefits of a policy or project aiming at a 10% improvement in the amenity, which may offer more than 50% of the area, can be estimated by the hedonic measure with an overestimation ratio of less than 1.1 in the parameter set[6] used in the present study, unless the elasticity in production is extremely small, *i.e.*, 0.1.

Maximum overestimation ratio

Although the above results are interesting, caution must be taken in their interpretation due to the subjectivity of the adopted values of the parameters in the figures. Here, we will present the maximum overestimation ratios over the parameter space. As the values of the production and utility distributive parameters and power of the amenity in the production function are not known, we examined all possible combinations of values across the parameter space. Table 3 summarizes the maximum overestimation ratios across the parameter space by the degree of improvement and by the size of area affected directly by the policy or project at elasticities of 0.5 and 1.5. We change the constant cost assumption here. The cost of the policy or project is assumed to vary with respect to area (H_1) in *R*egion 1 and the degree of improvement, *i.e.*, $\{(z_2\text{-}z_1)\ H_1/10\}$ times production without the policy or project. From this table, we can judge the relevance and limitations of the hedonic analysis for estimating the gross benefits of various public policies and projects in terms that are more general. The results indicate that the hedonic measure can be utilized in most cases and that we should evaluate the benefits of drastic policies or large-scale projects providing local public goods.

5. Comparison of Overestimation Ratio of a Large-Scale Improvement Project with the Results of Numerical Analysis

Next, in order to discuss the applicability of overestimation ratios estimated in the previous section, we investigated the differences in overestimation ratios between the results of the numerical analysis and those described in Hidano (1997, 2002) for an actual large-scale interregional transport project in Japan. The overestimation ratios of

---

[6] We will discuss the precise parameter impacts on overestimation ratios in Appendix 2 for the case in which elasticities equal unity.

this project defined in (10) were derived using the hedonic measure in (7), calculated using two types of hedonic prices divided by the gross benefit, *i.e.*, the sum of benefits of the project in terms of equivalent variation and the cost of the project. These two hedonic measures were obtained from the hedonic prices calibrated using a general equilibrium model (see Appendix 3) and those calculated using the estimated nationwide hedonic functions. We examined the validity using the Hokuriku highway project (the cost (C) of the project is 1.438 billion yen), a major highway project in Japan. The highway connects 5 of the 47 prefectures in Japan and was designed to be a trunk road along the Sea of Japan. In our transport project estimation, there should be spillover effects of the project because of its large impact on the Japanese economy. We included this effect in both utility and production calculations by the introduction of an accessibility measure (ACC). This assumption was adopted to allow consideration of other modes of transport, such as airlines and high-speed interregional trains, and the fact that robustness of economic activities is based strongly upon the interactions among urban areas (see Appendix 3 equation A3-3).

Comparison between Actual and Estimated Overestimation Ratios

The equilibrium values of the variables with and without the project were calibrated and then the annual values were transformed into stock values with discount rate i (5%). For comparison of the hedonic measure $B=\left(r_2^o-r_1^o\right)H_1/i$ with the gross benefit of the project in terms of equivalent variation (EV), we introduced the formula below to adjust the transport accessibility (ACC) improvement because, even if implemented fully, the project could not bring the ACC of Hokuriku highway areas up to the ACC level of other regions:

$$\overline{B}=\left(r_2^o-r_1^o\right)H_1\left(ACC_1^w-ACC_1^o\right)/\left(ACC_2^o-ACC_1^o\right)/i. \tag{12}$$

The gross benefit of the project $V+C$ in (8) was calculated as the sum of the total population N times EV and C. From the calibration, $NEV+C$ was 3.84 billion yen. The adjusted hedonic measure was 4.29 billion yen. Thus, the overestimation ratio was 1.12. It should be noted that the results were not affected by changes in the value of the amenity variable parameter d in Appendix 3 equation A3-1 from 0.3 to 0.7 in increments of 0.1 (see Hidano (1997)). Next, we compared this value with that obtained by numerical analysis. We used the same elasticities and amenity input parameter in the production function, both of which are relevant in this comparison. As the value ρ of the utility function was -0.11, that of σ of the production function was -0.12, and the elasticities were 1.12 and 1.14, respectively. The maximum overestimation ratio estimated using these elasticities and the power of the amenity input in the production function ζ for the same area covered by the project in numerical analysis was 1.04. The discrepancy between 4% and 12% was due to the linear adjustment of the hedonic measure in (12) and distortion due to the increase in the number representing the factor of production and utility, *i.e.*, the inclusion of transport goods. The latter was justified by the observation that substitution effects among commodities increase the overestimation ratios. Thus, the number of goods that are substitutable increases, and consequently the overestimation ratio increases.

Finally, we examined the overestimation ratios using a real hedonic function rather than the equilibrium land prices (see Hidano (1997, 2002)). It was necessary to estimate a national land price function to calculate the benefits of large-scale projects. To depict the actual land market conditions, we estimated two functions for both land for housing and land for companies and commercial activities (see Appendix 4). Using these functions, we obtained the hedonic measure using the following formula:

$$\left[LP^h\left(ACC_1^w\right)-LP^h\left(ACC_1^o\right)\right]H_1^h+\left[LP^f\left(ACC_1^w\right)-LP^f\left(ACC_1^o\right)\right]H_1^f \tag{13}$$

where LP^h and LP^f represented residential and commercial land prices, respectively. The areas of land $H_1{}^h$ and $H_1{}^f$ were fixed for benefit estimation. This assumption was validated by the observation that the area for company and commercial use was not changed in the general equilibrium analysis described above. The hedonic measure was estimated as 3.96 billion yen, which exceeded the actual value by only 3%. It should be noted that this empirical study justified the parameter chosen in the simulation in section 4 and the results of the maximum overestimation ratio in Table 3 is a good benchmark for decision-making.

6. Concluding Remarks

In this study, we examined the validity of the hedonic approach not in marginal cases but in cases of drastic public policies or large-scale projects which extremely change the region. Our study was characterized by its strength in considering large price changes in the general equilibrium framework and in explicitly taking into account the costs of public policies or projects in numerical analysis. The results of the present study indicated that the hedonic measure based on cross-sectional capitalization theory can correctly approximate the benefits in unexpectedly large-scale cases. We investigated comprehensive sets of parameters in CES functions with homogeneous production technology and households. The results were also supported by our empirical study on a large-scale transport project in Japan. We have shown the maximum upper limits of the overestimation ratios in these sets of parameters. Although our results were based upon a specific functional form, it is advisable to change the prevailing conception of the limitations of the hedonic approach for benefit estimation, especially with regard to large-scale public policies or projects. As we have information related to the accuracy of the hedonic approach, we can apply the method to most cases to estimate the benefits of large-scale public projects and policies with characteristics of local public goods using *ex ante* hedonic price functions based upon land, housing, commerce, or office market price data.

Appendix 1

The Overestimation in the Case of Leontief Utility Function

Proposition: The hedonic measures cannot necessarily estimate the gross benefit of a policy or project correctly when the Leontief utility function is applied.

Proof: We will present a case in which the equality cannot be held. We assume the following utility function and a wage given exogenously.

$$u_i = (\min(x_i / \alpha, l_i^h / \beta))^{\gamma} z_i^{1-\gamma} \tag{A1-1}$$

The equilibrium without a policy or project:

$$x_i / \alpha = l_i^h / \beta = v_i \tag{A1-2}$$

$$w + s = x_i + r_i l_i^h, \qquad s = (r_1 H_1 + r_2 H_2)/N. \tag{A1-3}$$

Then

$$v_1 / v_2 = (z_1 / z_2)^{1-\gamma/\gamma},$$
$$H_1 / l^h{}_1 + H_2 / l^h{}_2 = N. \tag{A1-4}$$

Thus w should be $\alpha H/(\beta N)$.

We can get

$$r_1 = r_2 (z_1 / z_2)^{1-\gamma/\gamma} + (\alpha/\beta)((z_1/z_2)^{1-\gamma/\gamma} - 1),$$
$$x_i = (\alpha/(\beta N))(H_i z_i^{1-\gamma/\gamma} + H_j z_j^{1-\gamma/\gamma}) / z_i^{1-\gamma/\gamma},$$
$$l^h{}_i = (1/N)(H_i z_i^{1-\gamma/\gamma} + H_j z_j^{1-\gamma/\gamma}) / z_i^{1-\gamma/\gamma}. \tag{A1-5}$$

The expenditure function is

$$E^o = \min_{x, l^h} (x + r_2 l^h : (\min(x/\alpha, l^h/\beta))^{\gamma} z_2^{1-\gamma} \geq v_2^{\gamma} z_2^{1-\gamma})$$

$$= (1/N)(\alpha/\beta + r_2)\ (H_1 z_1^{1-\gamma/\gamma} + H_2 z_2^{1-\gamma/\gamma}) / z_2^{1-\gamma/\gamma}. \tag{A1-6}$$

The equilibrium with the policy or project is described as follows:

$$x = w - C/N, \tag{A1-7}$$

since $w = \alpha H/(\beta N)$,

then
$$x_m = \alpha H/(\beta N) - C/N < \alpha H/(\beta N) = \alpha l_m^h/\beta. \tag{A1-8}$$

m stands for the maximum available resource for each consumer.

Thus
$$\min(x_m/\alpha, l_m^h/\beta) = x_m/\alpha. \tag{A1-9}$$

The attainable utility with the policy or project is

$$u^w = ((w - C/N)/\alpha))^\gamma z_2^{1-\gamma} = ((\alpha H/(\beta N) - C/N)/\alpha)^\gamma z_2^{1-\gamma}. \tag{A1-10}$$

The expenditure function is

$$E^w = \min_{x,l^h}\ (x + r_2 l^h : (\min(x/\alpha, l^h/\beta))^\gamma z_2^{1-\gamma} \geq u^w)$$

$$= (\alpha H/(\beta N) - C/N)(1 + \beta r_2/\alpha) \tag{A1-11}$$

$$EV = E^w - E^o.$$

Then
$$(r_2 - r_1)H_1 - C - NEV = r_2 C\beta/\alpha \geq 0. \tag{A1-12}$$

The equality only holds when C is zero.

Appendix 2

The Case of Cobb-Douglas Functions

It is worthwhile discussing the case of the Cobb-Douglas function because analysis using the least number of parameters provides a clearer picture of the overestimation characteristics. The functions for utility and production used were:

$$u_i = x_i^{\alpha'} \left(l_i^h\right)^{\beta'} z_i^{\gamma'}, \tag{A2-1}$$

$$X_i = n_i^{a'} \left(l_i^f\right)^{1-a'} z_i^{b'}. \tag{A2-2}$$

Then, we set the parameters for numerical analysis for more general cases as follows:

Utility function

The sum of parameters is set to unity.

$$\alpha' + \beta' + \gamma' = 1$$

Production function

The value of a' ranges from 0.1 to 0.9, and b' is set under the same rule.

Area of land, and numbers of consumers and workers

The total area was assumed to be unity. Consumers and workers were identical and were normalized to unity.

The degree of improvement by the policy or project

We reviewed the cases of z_1 from 0.10 to 0.99 and z_2 equal to unity.

The size of the area affected directly by the policy or project

H_1/H_2 was from 0.01 to 10.00.

Results

Spatial size and degree of improvement resulting from the policy or project and the overestimation ratio

Figure A1 shows that even when *R*egion 1 was 10-fold larger than *R*egion 2, the overestimation ration was limited to below 30%. Our data indicate that in the case of a 20% increase in the amenity level, the overestimation ratio is limited to less than 3% with the utility and production functions set at $\alpha' = \beta' = \gamma'$ and $a' = b' = 0.5$.

The robustness of the results with different rates of substitution

Figures A2 and A3 show the robustness of the results when the rates of substitution in the utility and production functions were changed. With respect to the utility function, an increase of γ' inevitably increased the overestimation ratio because it decreased the weight of the land parameter (β') (Figure A2). As far as the utility function is concerned, capitalization of the amenity improvement into land does not occur when β' is zero. However, under the assumption of mixed land use, the amenity improvement increased the productivity of production, thus increasing land rent. This brings about the nonlinear relationship between the overestimation ratio, γ', and z_1/z_2 (Figure A3). It is interesting that even in this case, the overestimation ratio was lower than 15% when parameters of amenity and land in the production function were fixed at 0.5.

Examining the different rates of substitution between parameters in the production function also provides different overestimation ratios. Figure A4 shows the impacts of parameter changes in land and amenity on the overestimation ratio. When the influence of an amenity input on production (b') is large and the weight of land in production ($1-a'$) increases, then the overestimation ratio decreases as small as unity. This is reasonable, as capitalization is described by the cross-sectional hedonic measure, *i.e.*, differences in the land price or rent, particularly in the situation in which all goods are produced by land inputs. However, when b' becomes zero, *i.e.*, when an increase in the amenity does not increase productivity, the overestimation ratio increases owing to the small utility increment produced by the policy or project. In this case, the gross benefit of the policy or project itself converges to zero and the denominator of the equation (10) approaches zero. But the positive cost value of the policy or project remains in the numerator (see Appendix 1). Thus, the overestimation ratios increase.

Appendix 3

Model

The model has been discussed in Hidano (1997, 2000, 2002). This model excluded the agricultural, forest, and fishery industries because they are relatively minor in Japan. Production and utility functions were specified as follows:

$$u_i = \left(a x_i^{-\rho} + b \left(l_i^h \right)^{-\rho} + c \left(d_i^h \right)^{-\rho} + d ACC_i^{-\rho} \right)^{-1/\rho} \tag{A3-1}$$

$$X_i = \left(e n_i^{-\sigma} + f \left(l_i^f \right)^{-\sigma} + g \left(d_i^f \right)^{-\sigma} \right)^{-1/\sigma} ACC_i^{\varsigma} \tag{A3-2}$$

$$ACC_i = \left\{ DIDn_i \sum_k DIDn_k / \left(min_m \left(P_{ik}^m + W t_{ik}^m \right) \right)^2 \right\}^{\frac{1}{2}} \tag{A3-3}$$

where d_i^h was the number of consumer trips and d_i^f was the number of trips made by companies. ACC_i represented transport accessibility of region i, $DIDn_i$ was the population of a densely inhabited district in region i, P_{ik}^m was the cost (fares *etc.*) of transport mode m between regions i and k, W was the time value, and t_{ik}^m was travel time by mode m between i and k. This accessibility was interpreted as a potential transport amenity and defined as the inverse of generalized cost weighted by the size of the population. It should reflect actual economic interactions between two regions. The power parameter of generalized costs was chosen by the fitness criterion (R^2). Elasticities of utility and production functions in terms of commodities were:

$$\frac{1}{\rho+1} \quad \text{and}$$

$$\frac{1}{\sigma+1}, \text{ respectively.}$$

The model was described as follows:

Consumers maximize their utility subject to a budget constraint

$$w_i + s = x_i + r_i^h l_i^h + P_i\, d_i^h$$

$$s = (r_1 H_1 + r_2 H_2) / \sum_i n_i \tag{A3-4}$$

where P_i was the average transport cost per trip. The government was to provide actual transport services and amenities. We mainly considered the highway transport sector. Supply and demand for other modes of transport, such as airlines, trains, and ships, were assumed to be included in goods *x*. The production costs *VC* related to the transport services and maintenance costs of the stock were financed by fares paid by users, and new investment was funded by a lump sum tax T_i^T. The cost function was:

$$VC_i=\kappa\mu K_i+\nu S_i^{\nu} \tag{A3-5}$$

where *K* was the stock of infrastructure. Project infrastructure was determined by the present stock and investment *I*, which should decrease the time of travel. *S* was supply of transport services. κ was a constant for conversion of monetary into physical stock. The market clearing conditions with the project were:

$$\sum_i n_i x_i + I + \sum_i VC_i = \sum_i X_i . \tag{A3-6}$$

Governmental financial balance was:

$$\sum_i P_i \ (n_i d_i^h + d_i^f) = \sum_i VC_i , \tag{A3-7}$$

$$\sum_i n_i T_i^T = I. \tag{A3-8}$$

The quantities of transport services supplied and demanded should be equal.

$$S_i^{\nu}=n_i d_i^h+d_i^f \tag{A3-9}$$

The estimated values (see Appendix 4) were: a=0.251, b=0.273, c=0.811, ρ= -0.11, e=0.432, f=0.231, g=0.337, ζ= 0.103, σ= -0.121, λ= -1.155, μ= 0.034, ν= 1.666, and *d* was assigned as 0.5 (see Hidano (2002)).

Appendix 4

Methods of Estimation of Production and Utility Functions in a Drastic Policy or Large-Scale Project

The parameters of utility, production, and cost functions were estimated from 1985 annual data. To estimate the utility function in Japan, we used the following transport demand function, which can be derived by utility maximization under the budget constraints:

$$\ln\left(\left(w_i + s - P_i\ d_i^h\right)/\ d_i^h\right) = -1/(\rho - 1)\ln\left(c\,/\,P_i\ \right) + \ln\left(a^{\frac{1}{\rho+1}} + b^{\frac{1}{\rho+1}}\left(r_i^h\right)^{\frac{\rho}{\rho+1}}\right). \qquad \text{(A4-1)}$$

Although most parameters could be estimated in this formula, the parameter of the amenity variable *d* could not be estimated because of the lack of a market. In this study, we adopted the value of 0.5, and the validity of the value was examined by sensitivity analysis in general equilibrium calibration (Hidano (1997)). All data used for the estimation of production and utility functions were regional (prefectural level) and were mainly from 1985 cross-section regional statistics, with the exception of rent data which was not available nationwide. The study converted land price data into rent data using a 5% interest rate. Land price data were obtainable across the nation, although officially published average prices at the regional level were highly biased, being only averages of land assessments, not distributed at random either spatially or socioeconomically. To overcome this problem, we had to use what we felt was a sufficient number of samples in a region from the viewpoint of spatial distribution.

In this study, we adopted an alternative method of estimating normalized land prices across the nation based on hedonic price functions estimated by region and by the uses of land, *i.e.*, residential *vs*. firm or commercial use. We used the following procedure to derive normalized prices of land:

i) Selection of the capital or the largest city in each of the 47 regions.

ii) Estimation of 47 residential and 47 firm or commercial land hedonic price functions based on 1985 officially published land prices for each city by ordinary least square regression.

iii) Calculation of 47 normalized regional land prices for each use of land, assigning the same values for each attribute in the functions except for the distance from the city center, which varies widely because of differences in city size. We assumed that places 25 and 30 km from the city center were representative in Osaka and Tokyo, while we used a value of 3 km from the center for other cities.

It is not common to estimate nationwide hedonic functions. We examined the possibility of estimating the functions and the applicability of these functions for hedonic analysis. The estimations were performed based on the normalized residential and firm or commercial land prices determined as described above. The results are shown in Table A-1.

References

Bartik, T (1988), "Measuring the Benefits of Amenity; Improvements in Hedonic Price Models," Land Economics, 64 pp172-83.

Brookshire, DS, MA Thayer, WD Shulze, and RC d'Arge (1982), "Valuing Public Goods: A Comparison of Survey and Hedonic Approaches," American Economic Review, Vol.72, 165-193.

Epple, D, and H Sieg (1999), "Estimating Equilibrium in Models of Local Jurisdictions," Journal of Political Economy, Vol.107, No.4, pp645-681.

Freeman, MA III (1993), The Measurement of Environmental and Resource Value: Theory and Methods, Resource for the Future.

Hidano, N (1997), The Economic Measurement of Environmental and Infrastructure Project: A Hedonic Approach, Keiso Shobo.

Hidano, N (2000), "Is a Hedonic Approach Only Applicable to Small Scale Environmental Projects? A general equilibrium analysis," Discussion Paper No.117, Dep. of Land Economy, Cambridge University, pp1-30.

Hidano, N (2002), The Economic Valuation of the Environment and Public Policy: A Hedonic Approach, Edward Elgar.

Hidano, N, and Y Hayashiyama (1997), "On Validity of Capitalization Hypothesis in Two -Region Open Economy: Errata and Amendment," Discussion Paper No.97-1, Department of Social Engineering, Tokyo Institute of Technology.

Hidano, N, Y Hayashiyama, and Y Yamamura (1992), "The Measuring the Benefits of Interregional Transportation Improvement Projects: A Hedonic Approach," Proceeding of Japan Society of Civil Engineers, No.449, IV-17, pp67-76.

Hiramatsu, T, and N Hidano,(1989), " A Comparison of CVM and Hedonic Approach," Infrastructure Planning Review, No.7, pp104-114.

Kanemoto, Y (1985), "Estimating Benefits of Infrastructure Through the Analysis of Land Value," Discussion Paper No. UDD-82, Water Supply and Urban Development Department, The World Bank.

Kanemoto, Y (1988), "Hedonic Prices and the Benefits of Public Projects," Econometrica, Vol.56, pp981-989.

Palmquist, RD (1991), "Hedonic Models," in Measuring the Demand for Environmental Quality (Branden, JB, and CD Kolstad ed), North-Holland, pp77-120.

Roback, J (1982), "Wages, Rents, and Quality of Life," Journal of Political Economy, Vol.90, No.6, pp1257-1278.

Rosen, S (1974), "Hedonic Prices and Implicit Markets, Product Differentiation in Pure

Competition," Journal of Political Economy, Vol.82, pp34-55.

Scotchmer, S (1985), "Hedonic Prices and Cost/Benefit Analysis," Journal of Economic Theory, Vol.37, pp55-75.

Scotchmer, S (1986), "The Short Run and Long Run Benefits of Environmental Improvement," Journal of Public Economics, Vol.30, pp61-81.

Sheppard, S (1999), " Hedonic Analysis of Housing Markets," In Cheshire, P, E. Mills ed. Handbook of Regional and Urban Economics, Vol 3, Elsvier.

Starrett, DA (1981), "Land Value Capitalization in Local Public Finance," Journal of Political Economy, Vol.89, No.2, pp306-327.

Wildasin, DE (1987),"Theoretical Analysis of Local Public Economics," in Mills, ES. ed. Applied Urban Economics, Vol.2. Handbook of Regional and Urban Economics, North-Holland.

Table 1. Cost of the project and overestimation ratio at z_1=0.99z_2 and H_1=0.01

Cost as a proportion of the total production	0.01000	0.00100	0.00010
$B/(C+V)$	1.005127	1.00075	1.00009

α=β=γ=1/3, δ=ε=1/2, elasticities were 1.5

Table 2. Change in overestimation ratios with elasticity and power of amenity in production function

	Elasticity of production function	Elasticity of utility function 0.10	0.15	0.20	0.25	0.30
ζ=0	0.10	1.0051	1.0115	1.0131	1.0111	1.0092
	0.15	1.0072	1.0055	1.0084	1.0117	1.0150
	0.20	1.0115	1.0051	1.0060	1.0078	1.0097
	0.25	1.0147	1.0054	1.0055	1.0066	1.0080
	0.30	1.0166	1.0055	1.0055	1.0063	1.0074
ζ=0.5	0.10	1.2244	1.1916	1.1389	1.0889	1.0504
	0.15	1.0527	1.0424	1.0280	1.0158	1.0076
	0.20	1.0206	1.0163	1.0106	1.0059	1.0028
	0.25	1.0130	1.0101	1.0065	1.0035	1.0016
	0.30	1.0100	1.0076	1.0047	1.0025	1.0011

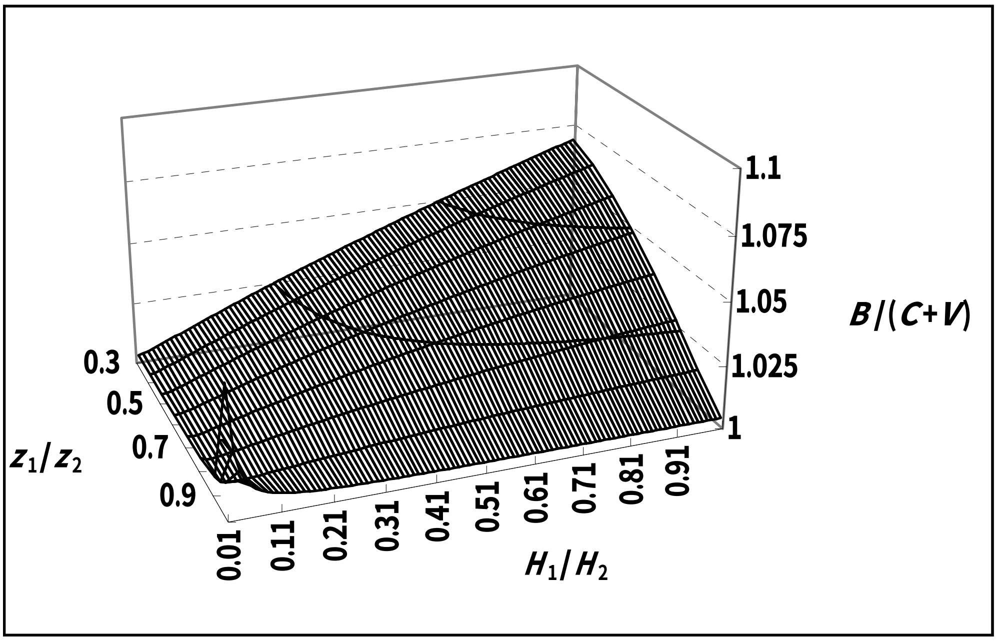
α=β=γ=1/3, δ=ε=1/2, H_1/H_2=1, and z_1/z_2=0.9

Table 3. Maximum overestimation ratios across parameter space

Elasticity		0.5							1.5						
		Area Size (H_1/H_2)													
		0.01	0.05	0.10	0.20	0.50	1.00	2.00	0.01	0.05	0.10	0.20	0.50	1.00	2.00
Degree of	0.99	1.0004	1.0019	1.0036	1.0067	1.0141	1.0222	1.0312	1.0014	1.0067	1.0129	1.0240	1.0494	1.0763	1.1034
Improvement	0.95	1.0010	1.0049	1.0096	1.0185	1.0418	1.0729	1.1184	1.0035	1.0175	1.0349	1.0693	1.1700	1.3222	1.5286
z_1/z_2	0.90	1.0013	1.0054	1.0106	1.0205	1.0469	1.0831	1.1376	1.0043	1.0215	1.0425	1.0821	1.1884	1.4058	1.8513
	0.80	1.0012	1.0054	1.0107	1.0206	1.0469	1.0826	1.1364	1.0052	1.0254	1.0520	1.0891	1.2994	1.4895	1.8713
	0.70	1.0011	1.0053	1.0105	1.0202	1.0459	1.0806	1.1322	1.0061	1.0312	1.0641	1.1311	1.3124	1.6106	1.8201

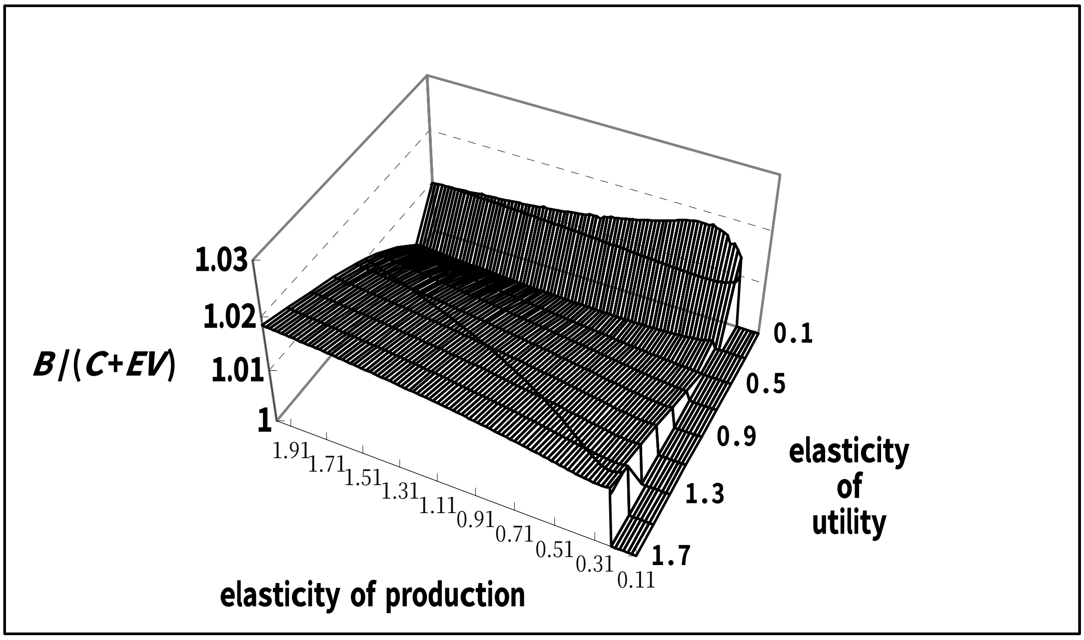
utility function parameter $\alpha \in \{0.1, 0.2, \ldots, 0.8\}$, $\beta \in \{0.1, 0.2, \ldots, 0.8\}$, $\gamma \in \{0.1, 0.2, \ldots, 0.8\}$

production function parameter $\delta \in \{0.1, 0.2, \ldots, 0.8, 0.9\}$, $\varepsilon \in \{0.1, 0.2, \ldots, 0.8, 0.9\}$, $\zeta \in \{0.1, 0.2 \ldots 0.8, 0.9\}$



α=β=γ=1/3, δ=ε=ζ=1/2, elasticities were 1.5

Figure 1. Overestimation ratio in area and degree of improvement space



$\alpha=\beta=\gamma=1/3$, $\delta=\varepsilon=1/2$, $\zeta=0$, $H_1/H_2=1$ and $z_1/z_2=0.9$

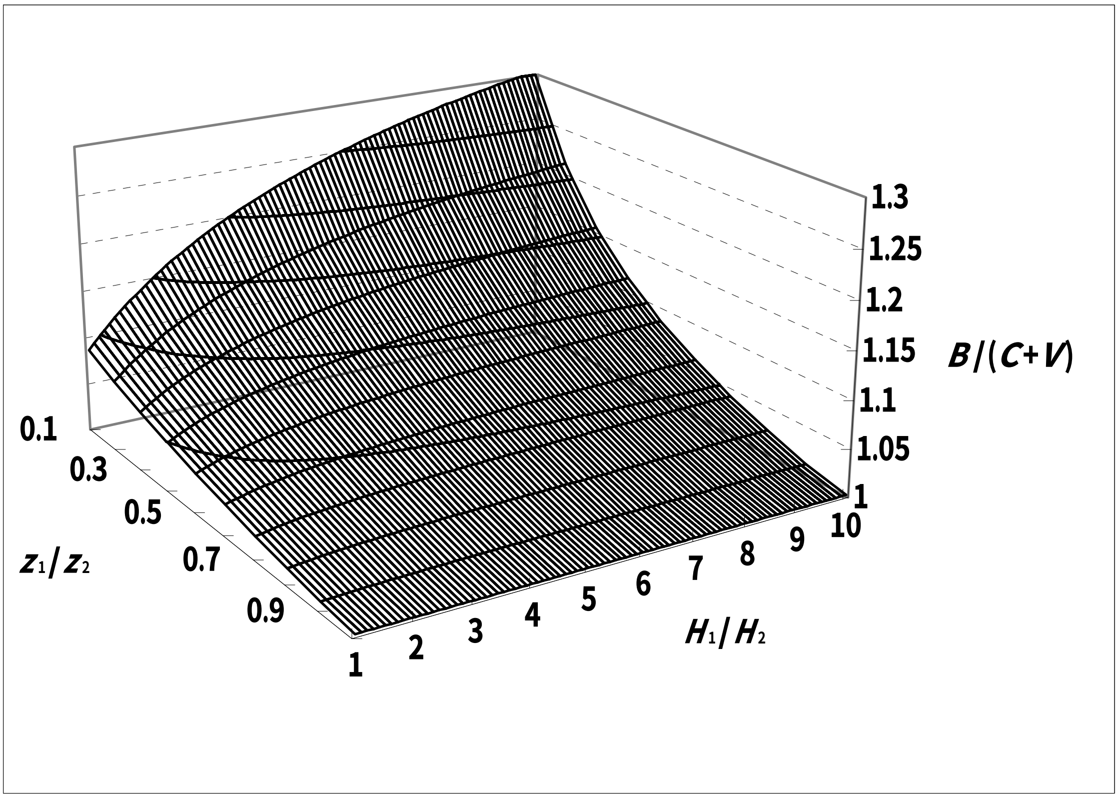
Figure 2. Overestimation ratio and elasticities

Table A-1　Nationwide hedonic price functions

Variables	Residence coefficient	Firm and Commercial coefficient
Annual wages	0.00068 (2.3)	0.00067 (1.9)
Snowy days dummy	-0.0722 (-1.9)	-
Sewerage dummy	-	0.182 (2.1)
Number of beds in hospitals	0.00000271 (2.0)	0.0000116 (3.0)
ACC (accessibility)	0.000138 (2.1)	0.000135 (2.0)
Constant	4.25	6.46
Sample size	47	47
R^2	0.68	0.72
MAPE	23.1	28.4

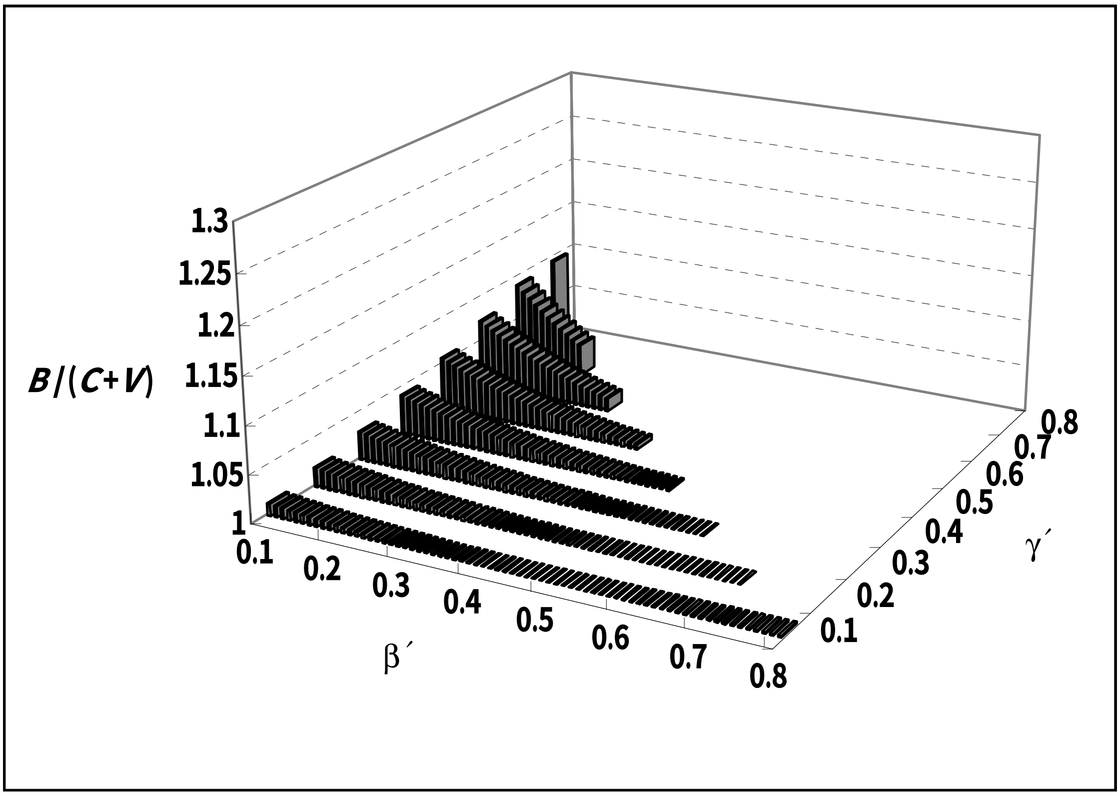
t-statistics are shown in parentheses.

Note: The dependent land price data were transformed into natural log values.



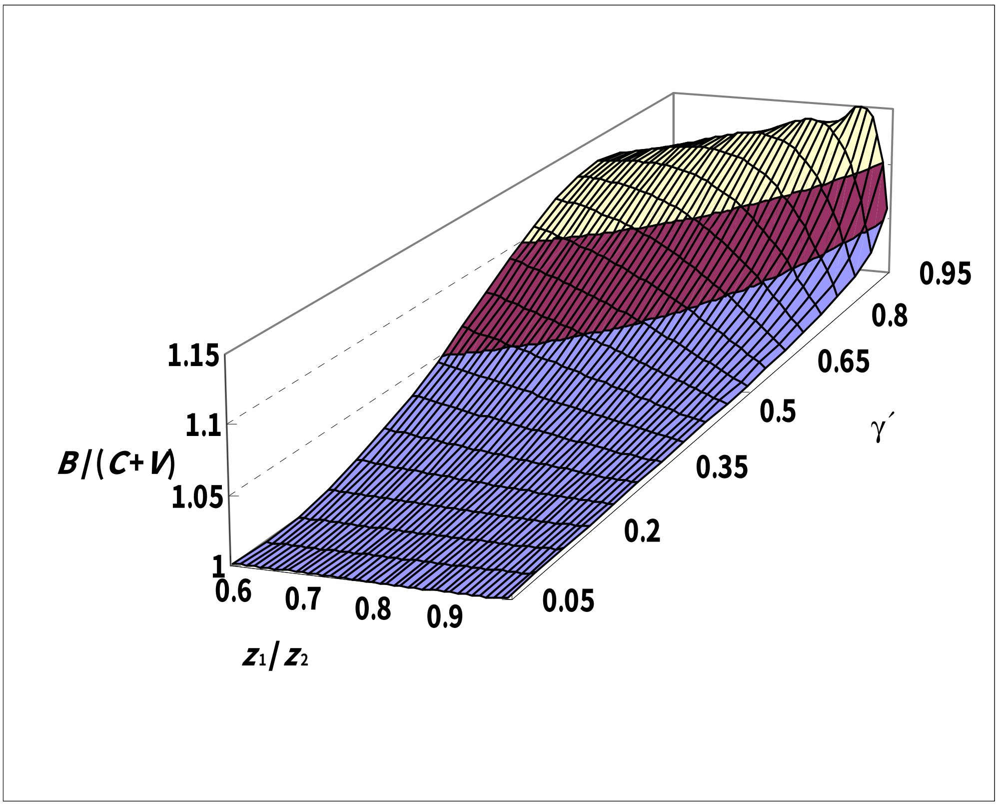
$\alpha' = \beta' = \gamma'$ and $a' = b' = 0.5$

Figure A1. Overestimation ratio when H_1/H_2 was more than unity and z_1/z_2



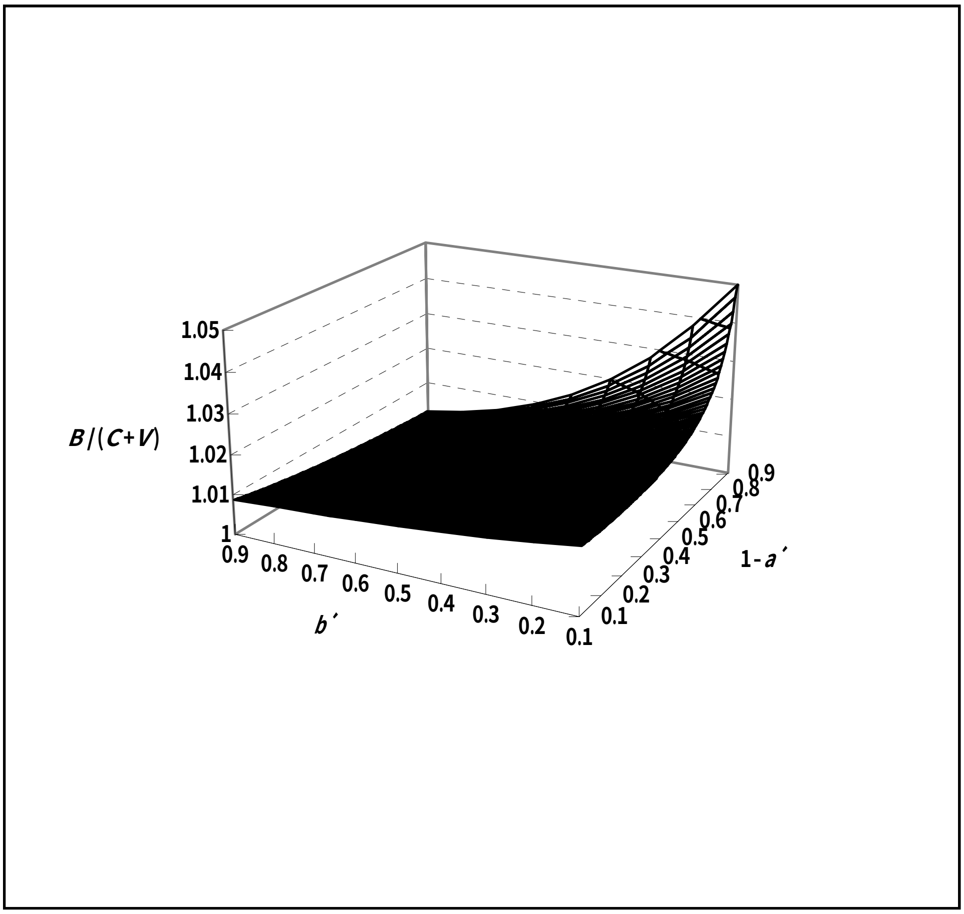
$\alpha' = 1 - \beta' - \gamma'$, $a' = b' = 0.5$, $H_1/H_2 = 1$, and $z_1/z_2 = 0.9$

Figure A2. Changes in overestimation ratio by different rates of substitution in utility



$\alpha' = \beta' = (1-\gamma')/2$, $H_1/H_2 = 1$, and $a' = b' = 0.5$

Figure A3. Nonlinear relationship between overestimation, weight of the amenity, and z_1/z_2



$\alpha' = \beta' = \gamma'$, $H_1/H_2=1$, and $z_1/z_2=0.9$

Figure A4. Overestimation ratio and parameters in production function