Macroeconomic Effects of an Indirect Taxation Reform under Imperfect Competition

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Abstract

This paper explores the effect of a tax reform which shifts from specific to value added taxation in a general equilibrium model with imperfect competition (Generalized Cournot and Free Entry Oligopoly). Such tax reform is characterized through a rate of substitution between taxes. This characterization allows us to find those rates of substitution between taxes which have an inflationary (deflationary) effect on price, as well as those rates which generate positive (negative) budget balance multiplier. Furthermore, the model captures the impact of the tax reform on welfare taking into account both government expenditure and profits, in contrast with the partial equilibrium approach.

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1 Introduction

Specific (or unit) taxes and ad valorem (or value-added) taxes exert different impacts on production under imperfect competition. Since most countries use both taxes to finance public spending it is natural to address the formal question of whether different schedules of indirect taxation can improve the efficiency of the economy, once imperfect competition is recognized in markets.

The literature on indirect taxation substitution has focused on the impact on consumer's surplus and production of different taxation schedules, under partial equilibrium. Suits and Musgrave (1955) and Bishop (1968) studied the effect of a substitution from specific to ad valorem taxation in the monopoly case, concluding that such tax reform yields higher tax revenues, and lower prices and profits. Dellipalla and Keen (1992), using a differential version of the Suits' and Musgrave´s tax reform, obtain similar conclusions with respect to prices, tax revenues, consumer´s surplus, profits and industry size, in a Generalized Cournot Oligopoly as well as in a Free Entry Oligopoly model. Therefore, the explicit comparison between ad valorem and specific taxation in partial equilibrium concludes that a tax reform which tilts the balance towards the ad valorem taxation yields to Pareto-improvements. Nevertheless, the partial equilibrium approach cannot address some interesting issues such as rent effects due to higher tax revenue and lower profits whereas a general equilibrium framework takes these into account.

The purpose of this paper is to study a tax reform which shifts from specific to value-added taxation, within a general equilibrium model and imperfect competition. In the model, a representative household is characterized through a particular utility function. The utility function depends on leisure, a composite good and government expenditure (which becomes in a quantity of publicly-provided composite good). The composite good is produced from labor by n identical firms under Cournot competition. These firms are burdened with both specific and value-added tax rates from where government expenditure is financed. Government´s policy consists in a tax reform which shifts from specific to value-added taxation. Such tax reform is characterized through a rate of substitution between taxes. This characterization allows us to find those rates of substitution between taxes which have an inflationary (deflationary) effect on price, as well as those rates which generate positive (negative) budget balance multiplier. Furthermore, the model captures the impact of the tax reform on welfare taking into account both government expenditure and profits, in contrast with the partial equilibrium case.

The paper is structured as follows. In the following section the model is presented and the equilibrium is defined. Section 3 studies tax reform in the Generalized Cournot Oligopoly model. Section 4 studies tax reform in the Free Entry Oligopoly model. Finally, section 5 summarizes the results.

2 Model and equilibrium

Let us consider an economy defined with two goods: leisure and a composite good produced from labor and a technology. And $n+2$ agents: a representative household, n non-competitive firms and a government, defined by the following assumptions:

i) Household preferences are represented by a separable utility function. On the one hand, a Cobb-Douglas sub-utility function over the quantity C of the composite produced good and the quantity L of leisure and, on the other, a sub-utility function over the quantity g publicly-provided of the composite produced good,

$$u(C,L,g) = \alpha \ln C + (1-\alpha) \ln L + \beta g, \tag{1}$$

where $\alpha \in (0,1), \beta > 0$. Let us denote by w the initial endowment of time (considered as the numèraire), p the price of the composite produced good and π the total profits of the firms. The household budget constraint is given by

$$pC = w - L + \pi. \tag{2}$$

ii) The industry is formed by n identical and non-competitive firms producing an amount q_j $(j = 1, 2, \ldots n)$ of composite output from labor using the following cost function

$$C(q_j) = k + cq_j,$$

which exhibits decreasing average cost. It is assumed that the labor market is competitive and firms´choices are independent of household's choice. It is also assumed that firms maximize profits and behave *à la Cournot.* Finally, the industry takes total expenditure in the economy $Y = p(C+g)$ as given,[1] thus the firms face with the unit isoelastic (inverse) demand function

$$p = Y/Q, \text{ with } Q = \sum_{j=1}^{n} q_j,$$

Finally, firms bear simultaneously a value added tax rate $t \geq 0$ and a specific tax rate $s \geq 0$ respectively. Therefore, the goal of the representative firm is to maximize

$$\left(\frac{Y}{(1+t)Q} - c - s \right) q_j - k,$$

[1] This assumption might be object because expenditure depends on industry profits. The model could be amended including a continuum of industries or considering, as in Caminal (1990), k symmetrical sectors (k high enough) with n firms each.

whose first order condition, using the symmetry, yields the supply side equilibrium

$$Q(t,s) = \frac{(n-1)}{n(1+t)(s+c)}Y, \tag{3}$$

$$p(t,s) = \frac{n(1+t)(s+c)}{n-1}, \tag{4}$$

$$\pi(t,s) = \frac{Y}{n(1+t)} - nk. \tag{5}$$

iii) Government uses the tax revenue to finance the amount g of government purchases. Thus, given the price p, the government budget constraint is

$$pg = G(t,s), \tag{6}$$

where

$$G(t,s) = \frac{t}{(1+t)}pQ + sQ, \tag{7}$$

is government tax revenue. Substituting equations (3) and (4) in equation (7), taking into account equation (6), government expenditure-revenue is given by

$$G(t,s) = \left(t + \frac{s}{(s+c)}\frac{(n-1)}{n}\right)\frac{Y}{(1+t)}. \tag{8}$$

Finally let us characterize the demand side equilibrium. Maximizing (1) subject to (2) household´s optimal choice is

$$pC = \alpha(w+\pi), \tag{9}$$

which represents household´s expenditure and

$$L = (1-\alpha)(w+\pi). \tag{10}$$

Therefore, taking into account equation (9) total expenditure in the economy is given by

$$Y = \alpha(w+\pi) + pg. \tag{11}$$

The particular shape of demand function allows us to interpret α as the marginal propensity to consume in equation (11) (see Mankiw, 1988). Substituting equations (5) and (8), taking into account (6), total expenditure in equilibrium is

$$Y(t,s) = \frac{\alpha n(1+t)(s+c)(w-nk)}{[(n-\alpha)c+(1-\alpha)s]}, \tag{12}$$

and using equations (3) and (4), taking into account (12), total output in equilibrium can be written as as

$$Q(t,s) = \frac{\alpha(n-1)(w-nk)}{[(n-\alpha)c+(1-\alpha)s]}, \tag{13}$$

it can be seen, given the preferences, total output does not depend on the value added tax rate. Finally, from (12) and (13) it is necessary that $w > nk$.

3 Tax reform in Generalized Cournot model

This section addresses the effect of a shift from specific to value-added taxation on prices, government expenditure, production and welfare. Starting from the initial situation given by the pair (t,s), the following tax reform is assumed,

$$ds = -\gamma dt \text{ with } \gamma > 0. \tag{14}$$

This type of reform generalizes the set of reforms that shift from specific to value-added taxation.[2] In particular γ represents the rate of substitution between both tax rates.

An immediate result is that the tax reform given by equation (14) increases total production and decreases total profit. In fact, starting from equation (13), the gradient of total output with respect to the vector of tax instruments is

$$\nabla Q(t,s) = \left(0, -\frac{\alpha(1-\alpha)(n-1)(w-nk)}{[(n-\alpha)c+(1-\alpha)s]^2}\right) = \left(0, -\frac{(1-\alpha)Q}{(n-\alpha)c+(1-\alpha)s}\right). \tag{15}$$

Notice that output level do not change whith respect to a change in value-added tax rate. This feature is due to the type of preferences assumed (with unit isoelastic elasticity) and the proportionality of such tax rate. This way, the increase in both government expenditure and price is balanced by a decrease in consumption yielding a total crowding-out effect. In consequence, the effect on total output of the tax reform given in equation (14) is

$$dQ^C = \frac{\partial Q}{\partial t}dt + \frac{\partial Q}{\partial s}ds = -\gamma\frac{\partial Q}{\partial s}dt > 0, \tag{16}$$

[2] For example, in Dellipalla and Keen (1992), the *P-shift* tax reform $\gamma = \frac{p}{(1+t)^2} = \frac{n(s+c)}{(n-1)(1+t)}$.

due to the value of the gradient given in (15). In advance the superindex C refers to the Generalized Cournot equilibrium and will be useful for comparison with Free Entry equilibrium. In relation with total profit, starting from equation (5), taking into account equation (12), the gradient with respect to the vector of tax instruments is

$$\nabla\pi(t,s) = \left(0, \frac{c\alpha(n-1)(w-nk)}{[(n-\alpha)c+(1-\alpha)s]^2}\right) = \left(0, \frac{cQ}{(n-\alpha)c+(1-\alpha)s}\right). \quad (17)$$

The total crowding-out effect under value-added taxation is reflected in total profit. Therefore the effect on total profit of the tax reform given in (14) is

$$d\pi = \frac{\partial\pi}{\partial t}dt + \frac{\partial\pi}{\partial s}ds = -\gamma\frac{\partial\pi}{\partial s}dt < 0,$$

Let us analyze the effect of such reform on prices, total expenditure and government expenditure-revenue. Calculating their gradients with respect to the vector of tax instruments starting from equations (4), (8) and (12) respectively, after operating and simplifying, we have

$$\nabla p(t,s) = \left(\frac{n(s+c)}{n-1}, \frac{n(1+t)}{n-1}\right), \quad (18)$$

$$\nabla Y(t,s) = \left(\frac{Y}{1+t}, \frac{c(n-1)}{[(n-\alpha)c+(1-\alpha)s]}\frac{Y}{(s+c)}\right), \quad (19)$$

$$\nabla G(t,s) = \left(\frac{Y}{1+t}, \frac{c(n-1)[n(1+t)-\alpha)]}{n[(n-\alpha)c+(1-\alpha)s]}\frac{Y}{(1+t)(s+c)}\right). \quad (20)$$

Those gradients show us that price, total expenditure and government expenditure-revenue in equilibrium increase monotonically with respect to the vector of tax instruments. This feature allows us to study the effect on these equilibrium values of any tax reform through level curves which start from any initial pair (t,s) of tax rates. Let us call this curves the iso-price (IP) curve , the iso-total expenditure (IY) curve and the iso-government expenditure-revenue (IG) curve.

The IP curve is given by the pairs (t,s) so that a tax reform as given in (14) holds the price unchanged. Taking into account equation (18) the slope of the IP curve is

$$\left.\frac{ds}{dt}\right|_p = -\frac{s+c}{1+t} < 0, \quad (21)$$

this way when in equation (14) $\gamma = \frac{s+c}{1+t}$ the tax reform becomes the iso-price tax reform. Therefore, taking into account (21), total differential of p with respect to the tax reform can be written as

$$dp^C = \left(\frac{\partial p}{\partial t} - \gamma\frac{\partial p}{\partial s}\right)dt = -\frac{n(1+t)}{n-1}\left(\gamma + \left.\frac{ds}{dt}\right|_p\right)dt, \quad (22)$$

which shows us that any tax reform γ such that $\gamma > \frac{s+c}{1+t}$ ($\gamma < \frac{s+c}{1+t}$) would decrease (increase) equilibrium price.

The IY curve is given by the pairs (t,s) so that a tax reform as given in (14) holds total expenditure unchanged. Taking into account equation (19) the slope of the IP curve is given by

$$\left.\frac{ds}{dt}\right|_Y = -\frac{(s+c)}{(1+t)}\frac{[(n-\alpha)c+(1-\alpha)s]}{(n-1)c} < 0. \tag{23}$$

Thus, total differential of Y with respect to the tax reform can be written as

$$dY^C = \left(\frac{\partial Y}{\partial t} - \gamma\frac{\partial Y}{\partial s}\right) dt = -\frac{c(n-1)Y}{(s+c)[(n-\alpha)c+(1-\alpha)s]}\left(\gamma + \left.\frac{ds}{dt}\right|_Y\right) dt, \tag{24}$$

therefore any tax reform γ so that $\gamma < -\left.\frac{ds}{dt}\right|_Y$ ($\gamma > -\left.\frac{ds}{dt}\right|_Y$) would increase (decrease) the total expenditure in equilibrium.

Finally, the IG curve is given by the pairs (t,s) so that a tax reform as given in (14) holds government expenditure-revenue unchanged. Taking into account equation (20) the slope of the IP curve is

$$\left.\frac{ds}{dt}\right|_G = -\frac{n(s+c)[(n-\alpha)c+(1-\alpha)s]}{(n-1)c[n(1+t)-\alpha]} < 0. \tag{25}$$

when $\gamma = -\left.\frac{ds}{dt}\right|_G$ becomes an iso-government expenditure-revenue tax reform. As in the former cases, total differential of G with respect to the tax reform given in (14) can be written as

$$dG^C = \left(\frac{\partial G}{\partial t} - \gamma\frac{\partial G}{\partial s}\right) dt = -\frac{c(n-1)[n(1+t)-\alpha)]}{n[(n-\alpha)c+(1-\alpha)s]}\frac{Y}{(1+t)(s+c)}\left(\gamma + \left.\frac{ds}{dt}\right|_G\right) dt, \tag{26}$$

showing that any tax reform γ so that $\gamma < -\left.\frac{ds}{dt}\right|_G$ ($\gamma > -\left.\frac{ds}{dt}\right|_G$) would increase (decrease) government expenditure-revenue in equilibrium.

Furthermore, according to equations (21), (23) and (25), the shape of the IP, IY and IG curves allow us to state the following property,

$$\left.\frac{ds}{dt}\right|_p > \left.\frac{ds}{dt}\right|_Y > \left.\frac{ds}{dt}\right|_G,$$

figure 1 depicts the shape of the IP, IY and IG curves.

As shown in figure 1, the tax reform defined in (14) can produce different effects on price, total expenditure and government expenditure-revenue in equilibrium. Firstly, if $\gamma < -\left.\frac{ds}{dt}\right|_p$ the tax reform would extend over the IP curve in figure 1 and therefore, price, total expenditure and government expenditure-revenue in equilibrium would be higher. Secondly, if $-\left.\frac{ds}{dt}\right|_p$ □ $\gamma < -\left.\frac{ds}{dt}\right|_Y$ the

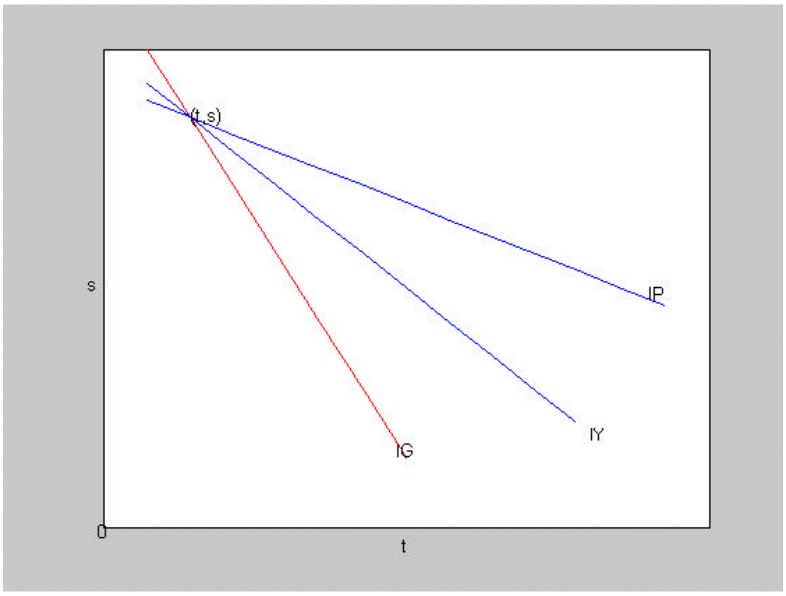

Figure 1: IP, IY and IG curves

tax reform would extend between the IP and the IY curves yielding an equilibrium in which price would not be higher, while total expenditure and government expenditure-revenue would increase. Thirdly, if $-\left.\frac{ds}{dt}\right|_Y \square \gamma < -\left.\frac{ds}{dt}\right|_G$ the tax reform would extend between the IY and the IG curves causing a decrease in price, a non-increase in total expenditure and an increase in government expenditure-revenue. Fourthly, if $-\left.\frac{ds}{dt}\right|_G \square \gamma$ the tax reform would extend down the IG curve yielding an equilibrium where both price and total expenditure would be lower while government´s expenditure-revenue would not be higher. For example, the Dellipalla and Keen´s (1992) *P-shift* tax reform extends down the IP curve, decreasing the price, but cross both the IY and the IG curves in such way that if $\frac{\alpha}{(1-\alpha)}\frac{ct}{(1+t)} < s < \frac{\alpha}{1-\alpha}c$ the budget balance multiplier $\frac{dY}{dG}$ is negative.[3]

Finally let us analyze the effect of the tax reform given in (14) on welfare. Substituting the equilibrium values given by equations (4), (9) and (10) on equation (1) the indirect utility function can be written as

$$V(t,s) = \alpha \ln \alpha + (1-\alpha)\ln(1-\alpha) + \ln(w + \pi(t,s)) - \alpha \ln p(t,s) + \beta \frac{G(t,s)}{p(t,s)}.$$

[3] A complete analisys of this case can be found in Torregrosa (1996).

Differentiating $V(t,s)$ with respect to (t,s),

$$\frac{\partial V(t,s)}{\partial t} = -\frac{\alpha}{p}\frac{\partial p}{\partial t} + \frac{\beta}{p^2}\left(p\frac{\partial G}{\partial t} - G\frac{\partial p}{\partial t}\right), \tag{27}$$

$$\frac{\partial V(t,s)}{\partial s} = \frac{1}{(w+\pi)}\frac{\partial \pi}{\partial s} - \frac{\alpha}{p}\frac{\partial p}{\partial s} + \frac{\beta}{p^2}\left(p\frac{\partial G}{\partial s} - G\frac{\partial p}{\partial s}\right). \tag{28}$$

Equations (27) and (28) show us how the changes on both tax rates cause distinct and ambiguous effects on welfare. The main difference between the effect on both tax rates arises from equation (17), where changes in the value-added tax rate do not change profit while an increase in the specific tax rate increases it. Other effects, concerned with changes in price and government expenditure-revenue, are similar. The total effect of the tax reform on welfare can be obtained through the total differential of $V(t,s)$, taking into account (14).

$$dV(t,s) = \left(\frac{\partial V(t,s)}{\partial t} - \gamma\frac{\partial V(t,s)}{\partial s}\right) dt, \tag{29}$$

substituting the partial derivatives given in equations (27) and (28) in equation (29) and operating

$$dV(t,s) = \left(\left(\frac{\alpha}{p} + \frac{\beta}{p^2}G\right)\left(\gamma\frac{\partial p}{\partial s} - \frac{\partial p}{\partial t}\right) + \frac{\beta}{p}\left(\frac{\partial G}{\partial t} - \gamma\frac{\partial G}{\partial s}\right) - \frac{\gamma}{(w+\pi)}\frac{\partial \pi}{\partial s}\right) dt,$$

and grouping terms, it can be expressed as

$$dV(t,s) = \left(\left(\frac{\alpha}{p} + \frac{\beta}{p^2}G\right)\left(\gamma - \left.\frac{ds}{dt}\right|_p\right)\frac{\partial p}{\partial s} - \frac{\beta}{p}\left(\gamma - \left.\frac{ds}{dt}\right|_G\right)\frac{\partial G}{\partial s} - \frac{\gamma}{(w+\pi)}\frac{\partial \pi}{\partial s}\right) dt. \tag{30}$$

Equation (30) resumes the total effect that the tax reform given in (14) causes on welfare in terms of the change in the specific tax rate. The initial part of equation, (30) made up of the first two terms placed between brackets, shows firstly the profitable effects on welfare of both prices and government´s purchases due to the fall in prices, the second term is strictly positive and is the sum of the currently shift from specific to value added taxation plus the iso-price tax reform. The second term of equation (30) is negative because the term placed between brackets, which represents the sum of the currently shift from specific to value added taxation plus the iso-government´s expenditure-revenue tax reform, is strictly positive. Finally, the third term of equation (30) shows the negative effect on welfare due to the fall in total profit. Although, like in partial equilibrium case, the tax reform raises welfare due to the profitable effect in prices. In contrats with what happens in partial equilibrium case two additional effects appear working in opposite ways.

4 Tax reform in Free Entry Oligopoly

This section analyzes how the industry size in equilibrium is affected by the tax reform given in (14) and its repercussion on total output, prices, expenditure and welfare. We follow the usual practice of treating $n \in (1, w/k)$[4] as a continuous variable. Thus substituting the total expenditure in equilibrium given by (12) into equation (5) imposing the zero profit condition and simplifying,

$$kcn^2 + ksn - \alpha(s+c)w = 0, \tag{31}$$

whose unique positive solution is

$$n(s) = \frac{1}{2kc}\left(\sqrt{k^2s^2 + 4kc\alpha(s+c)w} - ks\right). \tag{32}$$

As it can be seen in (32), as happens with total output and profit, the industry size in equilibrium does not depend on value added tax rate. Otherwise it is necessary to assume that $\alpha > k/w$ in order to guarantee that $n(s) > 1$ in equation (32). Let us calculate the variation of the industry size in equilibrium with respect to the specific tax rate.

$$\frac{\partial n}{\partial s} = \frac{1}{2c}\left(\frac{ks + 2c\alpha w}{\sqrt{k^2s^2 + 4kc\alpha(s+c)w}} - 1\right) > 0. \tag{33}$$

The fact that (33) is positive is due to the assumption that $\alpha > k/w$. This result is parallel to that achieved for profits in the previous section, and its insight is related to the fact that incipient profits attract entry (see Stern, 1987). Therefore taking into account that $\frac{\partial n}{\partial t} = 0$, the effect of the tax reform given in (14) on the industry size in equilibrium is

$$dn = \frac{\partial n}{\partial t}dt + \frac{\partial n}{\partial s}ds = -\gamma\frac{\partial n}{\partial s}dt < 0.$$

Total output, in accordance with the equation (13), depends on both industry size and specific tax rate (although is independent of the value-added tax rate). So, the effect of a change in specific tax rate on total output is

$$\frac{\partial Q}{\partial s} = -\frac{(1-\alpha)Q}{(n-\alpha)c + (1-\alpha)s} - \frac{k\left[(2\alpha-1)n - \alpha\right]}{(n-\alpha)c + (1-\alpha)s}\frac{\partial n}{\partial s}. \tag{34}$$

Equation (34) has two terms, the first one is equal to (15), which shows the variation on total output due to a variation in the specific tax rate in the Generalized Cournot equilibrium. The second term captures the net effect of the change in industry size on output as a consequence of change in the specific tax rate. Then, taking into account (16), the effect on total output of the tax reform given in equation (14) can be written as

[4] The left boundary of this interval is open because the unit isoelasticity of the demand function impedes the monopoly case. The openness of the right boundary is necessarily for the positiveness of the equilibria given in (12) and (13).

$$dQ^F = \left(\frac{\partial Q}{\partial t} - \gamma \frac{\partial Q}{\partial s} \right) dt = dQ^C + \frac{k[(2\alpha - 1)n - \alpha]}{(n-\alpha)c + (1-\alpha)s} \frac{\partial n}{\partial s} dt. \tag{35}$$

In advance the superindex F refers to the Free Entry equilibrium and it is useful for comparison with Generalized Cournot equilibrium. Secction A1 of the appendix shows that the effect of any tax reform on total output is higher under Free Entry equilibrium than under Generalized Cournot equilibrium, i.e. $dQ^F > dQ^C$, if $\alpha > \alpha^*$. Otherwise $dQ^F \square dQ^C$ when $\alpha \in (k/w, \alpha^*)$, where

$$\alpha^* = \frac{1}{2} + \frac{(2s+c)k + \sqrt{(2s+c)^2 k^2 + 8kwc(s+c)}}{8(s+c)w}.$$

In relation to the price the gradient of (14) with respect to the tax instrument vector is now

$$\nabla p(t,s) = \left(\frac{n(s+c)}{n-1}, \frac{1+t}{n-1}[n - \frac{s+c}{n-1}\frac{\partial n}{\partial s}] \right).$$

Notice that in this case, while the effect on price of a change in the value-added tax rate is the same as that produced by (18), the effect on price of changes in the specific tax rate now reflects the relationship with the industry size in equilibrium. Thus an increase in the specific tax rate has now two opposite effects on price. On the one hand, as in equation (18), an increase in the specific tax rate increases the price for a given industry size, and on the other hand, it decreases the price as a consequence of the increase in the numbers of firms in equilibrium. Therefore, taking into account (22), the effect on price of the tax reform given in (14) under Free entry equilibrium can be written as

$$dp^F = \left(\frac{\partial p}{\partial t} - \gamma \frac{\partial p}{\partial s} \right) dt = dp^C + \gamma \frac{(1+t)(s+c)}{(n-1)^2} \frac{\partial n}{\partial s} dt. \tag{36}$$

Thus $dp^F > dp^C$ as a consequence to the fall in the industry size in equilibrium due to the substitution of specific by value-added taxation. Equation (36) now has two parts: firstly the effect on price for a given industry size, and secondly the inflationary effect of a decrease in industry size. For example, while the iso-price tax reform ($\gamma = \frac{s+c}{1+t}$) does not change the price in the Generalized Cournot case ($dp^C = 0$), under Free Entry equilibrium the effect on price would be strictly positive.

As for total expenditure, the gradient of the equilibrium value given in (12) with respect to the vector of tax instruments is

$$\nabla Y(t,s) = \left(\frac{Y}{1+t}, \frac{c(n-1)}{[(n-\alpha)c + (1-\alpha)s]} \frac{Y}{(s+c)} + \frac{(1+t)nk(s - 2\alpha(s+c))}{(n-\alpha)c + (1-\alpha)s} \frac{\partial n}{\partial s} \right).$$

Therefore, taking into account (24), the effect of the tax reform given in (14) on total expenditure under Free Entry equilibrium can be written as

$$dY^F = \left(\frac{\partial Y}{\partial t} - \gamma \frac{\partial Y}{\partial s} \right) dt = dY^C - \gamma \frac{(1+t)nk(s - 2\alpha(s+c))}{(n-\alpha)c + (1-\alpha)s} \frac{\partial n}{\partial s} dt. \tag{37}$$

Thus the effect of the tax reform on total expenditure is higher under Free Entry equilibrium than under Generalized Cournot equilibrium, i.e. $dY^F > dY^C$, if $\alpha > s/2(s+c)$. Otherwise $dY^F \square\, dY^C$ when $\alpha \in (k/w, \frac{s}{2(s+c)})$.

In terms of government expenditure-revenue, the gradient of the equilibrium value given in (8) with respect to the tax instruments vector is

$$\nabla G(t,s) = \left(\frac{Y}{1+t}, \frac{c(n-1)[n(1+t)-\alpha)]}{n[(n-\alpha)c+(1-\alpha)s]} \frac{Y}{(1+t)(s+c)} + \frac{k\,[ns(1+t)(1-2\alpha)+\alpha s - 2\alpha ntc]}{(n-\alpha)c+(1-\alpha)s} \frac{\partial n}{\partial s} \right).$$

Therefore, taking into account (26), the effect of the tax reform given in (14) on government expenditure-revenue under Free Entry equilibrium can be written as

$$dG^F = \left(\frac{\partial G}{\partial t} - \gamma \frac{\partial G}{\partial s} \right) dt = dG^C - \gamma \frac{k\,[[s(1+t) - 2\alpha(s+t(s+c))]n + \alpha s]}{(n-\alpha)c+(1-\alpha)s} \frac{\partial n}{\partial s} dt, \tag{38}$$

As with equations (35), (36) and (37) this expression is formed by two parts, where the first is the effect of the tax reform for a given industry size, and the second reflects the effect of a change in industry size due to the change in the specific tax rate. According to (38), the effect of the tax reform on government expenditure-revenue is lower under Free Entry equilibrium than under Generalized Cournot equilibrium, i.e. $dG^F < dG^C$ for α such that $[[s(1+t) - 2\alpha(s+t(s+c))]n(\alpha) + \alpha s] > 0$, as $n(\alpha) > 0$ $\forall \alpha \in (k/w, 1]$, a more than sufficient condition for $dG^F < dG^C$ is $\alpha \square \frac{s(1+t)}{2[s(1+t)+tc]}$.

Therefore the fact that the tax reform generates differents changes in total output, total expenditure and government expenditure-revenue under Free Entry equilibrium than under Generalized Cournot equilibrium depends on marginal propensity to consume.

With regards to the effect of the tax reform given in (14) on welfare under Free Entry equilibrium, let us build the indirect utility function substituting the equilibrium values given by equations (4), (9) and (10) on equation (1) taking into account the zero profit condition

$$V(t,s) = \alpha \ln \alpha + (1-\alpha)\ln(1-\alpha) + \ln w - \alpha \ln p(t,s) + \beta \frac{G(t,s)}{p(t,s)}.$$

Differentiating $V(t,s)$ with respect to $r = t, s$

$$\frac{\partial V(t,s)}{\partial r} = -\frac{\alpha}{p}\frac{\partial p}{\partial r} + \frac{\beta}{p^2}\left(p\frac{\partial G}{\partial r} - G\frac{\partial p}{\partial r} \right), \tag{39}$$

thus, using (39), following the steps of equation (29), the effect of the tax reform given in (14) on welfare can be written as

$$dV(t,s) = \left(-\left(\frac{\alpha}{p} + \frac{\beta}{p^2}G \right) \left(\frac{\partial p}{\partial t} - \gamma \frac{\partial p}{\partial s} \right) + \frac{\beta}{p}\left(\frac{\partial G}{\partial t} - \gamma \frac{\partial G}{\partial s} \right) \right) dt,$$

or, taking into account (36) and (38)

$$dV(t,s) = -\frac{1}{p}(\alpha + \beta g)\, dp^F + \frac{\beta}{p} dG^F.$$

Note that as $dp^F > dp^C$ the profitable effect of the fall on prices due to the tax reform is less possible. For instance, the iso-price tax reform yields $dp^C = 0$, and $dp^F > 0$. Otherwise dG^F works increasing welfare, but it can be lower than dG^C depending on marginal propensity to consume. Therefore there is a wide range of values of the parameters such that the effect on welfare of the tax reform given in (14) can be lower in Free Entry Ologopoly than in Generalized Cournot Oligopoly.

5 Conclusions

This paper introduces two new perspectives to the classic treatment of the comparison between specific and value-added (ad valorem) taxation. On the one hand, the use of a general equilibrium model with imperfect competition and, on the other hand, the characterization of a generalized tax reform with shift from specific to value-added taxation. The first feature allows us to analyze how changes in both government revenue-expenditure and profit (as a consequence of the tax reform) can affect household´s indirect utility in a different way that occurs in the partial equilibrium case. Furthermore the model allows us to study the real effects, the nominal effects and the effects on welfare simultaneously. This permits the interpretation of the results in macroeconomic terms. With regards to the second feature (the characterization of a generalized tax reform with shift from specific to value-added taxation) the approach is different to those of Suits and Musgrave (1955) and Dellipalla and Keen (1992), which characterize particular tax reforms like the *matched pairs* or the *P-shift* respectively. In this paper however, a general concept of tax reform is used, one which defines a rate of substitution between specific and value-added taxes. This feature allows us to extract interesting conclusions related to the effect of the tax reform in prices and both real and nominal variables of the economy.

The main conclusions are related to the type of equilibrium considered. In Generalized Cournot equilibrium case one finds that any tax reform which shifts from specific to value-added taxation increases total output and decreases total profit. This conclusion is similar to that obtained in partial equilibrium for a particular characterization of the tax reform. Another result refers to the nominal variables, where a variety of effects on total expenditure, government expenditure-revenue and price in equilibrium, related to the substitution rate between taxes, can be found. This produces a range of situations in which all of the variables may rise, if the substitution rate is small enough, to the opposite if the substitution rate is high enough. An interesting case is that there exist substitution rates which decrease the price while increasing both total expenditure and government expenditure-revenue, a case which means that the tax reform generates a positive balanced budget multiplier. With respect to welfare, beyond the profitable effect that a fall in prices causes (as with the partial equilibrium case), here the decrease in both government expenditure and total profit can work to reduce the welfare generated.

In Free Entry Oligopoly case, one finds that any tax reform which shifts from specific to value-added taxation, decreases the number of firms in equilibrium. This adjustment in industry size increases the price with respect to the Generalized Cournot equilibrium. Furthermore, the adjustment in industry size has implications in the remaining variables. This way, if marginal propensity to consume is high enough, total output under Free Entry equilibrium increases beyond the value reached under Generalized Cournot equilibrium. Similarly results are found for total expenditure and government expenditure-revenue, as well as for welfare.

6 Appendix

A1) $dQ^F > dQ^C$ if $\alpha > \alpha^*$, and $dQ^F \leq dQ^C$ when $\alpha \in (k/w, \alpha^*)$, where

$$\alpha^* = \frac{1}{2} + \frac{(2s+c)k + \sqrt{(2s+c)^2k^2 + 8kwc(s+c)}}{8(s+c)w}.$$

D.- According with (35)

$$dQ^F - dQ^C = \frac{k[(2\alpha-1)n - \alpha]}{(n-\alpha)c + (1-\alpha)s} \frac{\partial n}{\partial s} dt,$$

thus the negativeness o positiveness of $dQ^F - dQ^C$ depends only of the sign of $\Gamma(\alpha) = (2\alpha-1)n - \alpha$. As $n(\alpha)$ is positive for $\alpha \in (k/w, 1]$ it is easy to see that for $\alpha \leq \frac{1}{2}$ $\Gamma(\alpha) < 0$ and therefore $dQ^F - dQ^C < 0$. When $\alpha > \frac{1}{2}$ the sign of $\Gamma(\alpha)$ is positive iff $n(\alpha) > \alpha/(2\alpha-1)$.Taking into account equation (32), and operating, this condition can be written as

$$(2\alpha-1)\left[\sqrt{k^2s^2 + 4kc\alpha(s+c)w} - ks\right] > 2kc\alpha,$$

or

$$(2\alpha-1)\sqrt{k^2s^2 + 4kc\alpha(s+c)w} > 2kc\alpha + (2\alpha-1)ks,$$

as both left and right side terms of the inequation are strictly positive it is true that

$$(2\alpha-1)^2\left[k^2s^2 + 4kc\alpha(s+c)w\right] > (2kc\alpha + (2\alpha-1)ks)^2,$$

developing terms and simplifying

$$(2\alpha-1)^2 w(s+c) > kc\alpha + (2\alpha-1)ks,$$

and developing $(2\alpha - 1)^2$ and grouping terms with respect to α this inequation can be written as

$$L(\alpha) = 4w(s+c)\alpha^2 - [(s+c)(k+4w)+sk]\,\alpha + (s+c)w + sk > 0.$$

$L(\alpha)$ is a convex parabola which reach its minimum at

$$\alpha_{\min} = \frac{1}{2} + \frac{(2s+c)k}{8(s+c)w},$$

and has two roots such that only one belongs to the interval $(\frac{1}{2}, 1)$ (to remark that we are analyzing the positiveness of $L(\alpha)$ for $\alpha > \frac{1}{2}$), this value is given by

$$\alpha^* = \frac{1}{2} + \frac{(2s+c)k + \sqrt{(2s+c)^2k^2 + 8kwc(s+c)}}{8(s+c)w}.$$

Therefore $L(\alpha) > 0$, i.e. $n(\alpha) > \alpha/(2\alpha - 1)$ for $\alpha > \alpha^*$. And $L(\alpha)$ □ 0 for $k/w < \alpha$ □ α^*.

7 References

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