# Uncertainty and Taxpayer Compliance[*]

## Jordi Caballé

Unitat de Fonaments de l'Anàlisi Econòmica and CODE

Universitat Autònoma de Barcelona

## Judith Panadés

Unitat de Fonaments de l'Anàlisi Econòmica

Universitat Autònoma de Barcelona

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### Abstract

The complexity of both tax code provisions and tax forms could induce taxpayers to commit errors when they fill their income reports. The existence of these involuntary mistakes affects the tax enforcement policy as tax auditors will face now two sources of uncertainty, namely, the typical one associated with taxpayers' income and that associated with report errors. Moreover, the inspection policy can be exposed to some randomness due to audit cost uncertainty. In this paper we provide an unified framework to analyze the effects of all these sources of uncertainty in a model of tax compliance where the interaction between auditors and taxpayers takes the form of a principal-agent relation. We show that more complexity in the tax code increases tax compliance. The effects of audit cost uncertainty are generally ambiguous. We also discuss the implications of our model for the regressive (or progressive) bias of the effective tax system.

*Key words:* Tax evasion, tax complexity, audit cost.

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[*] *Correspondence address:* Jordi Caballé. Universitat Autònoma de Barcelona. Departament d'Economia i d'Història Econòmica. Edifici B. 08193 Bellaterra (Barcelona). Spain. *Phone:* (34)-935.812.367. *Fax:* (34)-935.812.012. *E-mail:* Jordi.Caballe@uab.es

# 1. Introduction

The complexity of both tax code provisions and tax forms could induce taxpayers to commit errors when they fill their income reports. The existence of these involuntary mistakes affects the tax enforcement policy as tax auditors will face now two sources of uncertainty, namely, the typical one associated with taxpayers' income and that associated with report errors. Moreover, the inspection policy can be exposed to some randomness due to audit cost uncertainty. The aim of this paper is to provide an unified framework to analyze the effects of all these sources of uncertainty in a model of tax compliance where the interaction between auditors and taxpayers takes the form of a principal-agent relation.

The first models that analyzed the phenomenon of tax evasion through a portfolio selection approach (like Allingham and Sandmo, 1972; and Yitzhaki, 1974) assumed that all taxpayers were facing a constant and identical probability of being audited by the tax enforcement agency. However, consider a tax auditor that observes the amount of income reported by a taxpayer before conducting the corresponding audit. If this auditor wants to maximize the expected revenue from each taxpayer, there is no apparent reason why he should commit to an audit policy independent of the report he observes. An auditor using optimally all the relevant information at his disposal should make both the probability of inspection and the effort applied to a taxpayer contingent on the corresponding amount reported of income. One of the first attempts to analyze those contingent policies was made by Reinganum and Wilde (1985), who considered a model where the tax enforcement agency commits to follow a cut-off audit policy. According to this policy, taxpayers reporting less income than a given level are inspected, whereas the other taxpayers are not inspected. In a very influential paper the same authors (Reinganum and Wilde, 1986) considered an alternative scenario where a revenue-maximizing tax authority does not commit to an audit rule but selects an optimal policy given the realization of the taxpayers' reports. Moreover, in this new framework the probability of inspection is allowed to take all the possible values in the interval $[0, 1]$. Therefore, here the relationship between taxpayers and the tax enforcement agency mimics that of the typical principal-agent relationship with no commitment. The agency plays the role of the principal and follows a rule that gives the probability which a taxpayer is audited with. This probability rule ends up being a decreasing function of the amount reported of individual income. Taking as given the audit rule of the agency, taxpayers play the role of agents and choose their optimal reports in order to maximize the expected utility derived from their disposable income. Optimal reports turn out to be increasing functions of the true income.[1]

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[1] The previous basic models have been enriched in several directions. For instance, Border and Sobel (1994) allow for general objective functions for the principal; Mookherjee and P'ng (1989) study the implications of having risk averse agents; Sanchez and Sobel (1983) analyze the conditions under

The interaction between taxpayers and tax auditors is usually exposed to additional sources of randomness. The fact that tax codes are complex, vague and ambiguous has been recognized by several studies.[2] This aspect of tax codes makes difficult for the taxpayers to apply the law even if they want to do so. Scotchmer and Slemrod (1989) consider a model were the ambiguity of tax laws gives raise to an audit policy yielding random outcomes depending on the interpretation of the law made by the auditors. Such a randomness results in more compliance by taxpayers, since they want to reduce the risk of the penalties associated with a tough inspection that could reveal a large amount of evaded income.[3] In fact, Alm, Jackson and McKee (1992) have conducted experiments that confirm that audit randomness induces tax compliance.

Reinganum and Wilde (1988) consider another source of uncertainty faced by taxpayers, namely, that associated with the cost of conducting an audit. In their model the cut-off level of income that triggers an inspection is a function of an unknown audit cost. Therefore, taxpayers form non-degenerate beliefs about this cut-off income from the distribution of the audit cost. These authors conclude that some degree of induced uncertainty about the audit cost improves compliance and, thus, increases the revenue collected by the agency, but excessive uncertainty could decrease compliance.

The model we present in this paper considers sources of uncertainty similar to those appearing in the previous models. We will also model the interaction between the tax enforcement agency and taxpayers as a principal-agent relation so that tax auditors investigate taxpayers with an intensity that depends on the amount of reported income. The revenue accruing from the inspection is proportional to the effort made by the auditor and to the amount of evaded taxes. As in Reinganum and Wilde (1988), the audit cost is private information of each auditor and, thus, taxpayers do not know the exact response of auditors after reading their income reports. However, we depart from Reinganum and Wilde (1988) by considering general audit policies instead of cut-off ones. Another even more important departure is that we consider a fully rational expectations equilibrium. This means that taxpayers' beliefs about the audit costs coincide with the real distribution of these costs, whereas in the paper of Reinganum and Wilde the true distribution of these costs was degenerate and thus the confusion suffered by taxpayers about that cost was incompatible with the rational expectations equilibrium concept. In our model, the distribution of costs arises from the heterogenous quality of tax auditors due to different natural auditing skills or non-homogeneous formal training. Moreover, in our model we assume quadratic costs structures parametrized by the value of a coefficient parameter that is private

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which cut-off policies are optimal from the expected revenue viewpoint; and Erard and Feinstein (1994) introduce a fraction of honest taxpayers that always produce truthful reports. The principal-agent model has been tested by Alm, Bahl, and Murray (1993), who provide strong empirical support for this game-theoretical approach versus the alternative of random audit policies.

[2]See the abundant references in Section 9.1 of Andreoni, Erard and Feinstein (1998).

[3]Scotchmer (1989) and Jung (1991) consider instead models where tax complexity makes taxpayers uncertain about their true taxable income. Pestieau, Possen and Slutsky (1998) analyze the welfare implications of explicit randomization in tax laws.

information of each auditor. We will show with the help of a couple of examples that the effects of increasing the variance of that parameter value are very sensitive to the specific distribution under consideration.

We also allow for mistakes made by taxpayers when they fill their income report forms. As we have already said, the involuntary nature of these mistakes could be a consequence of the complexity of the tax law or of the tax form itself. Like in Rubinstein (1979), even a honest taxpayer can be exposed to a penalty by the tax authority since the income he reports does not coincide with his true income. As Scotchmer and Slemrod (1989), we show that an increase of tax complexity generates more revenue for the government. However, instead of making tax complexity a source of random audits, we make it responsible for income reports containing accidental imprecisions.

Our paper analyzes also other three questions. First, we show that a larger variance of the income distribution reduces (not surprisingly) tax compliance, since auditors face more uncertainty about a variable that is private information of taxpayers. Second, we evaluate the effects of the different sources of uncertainty on taxpayers welfare under the assumption that the government revenue is not used to provide goods or services entering in the taxpayers' utility function. Our analysis shows that expected utility responds ambiguously to changes in the variances of income and of report errors. Finally, we analyze the progressive (or regressive) bias of the audit policies followed by the tax auditors of our model. We show that the sign of this bias could be also ambiguous since a tax inspection could now serve as an instrument to correct for the involuntary mistakes leading to excessive tax contribution. This ambiguity concerning the effective progressiveness of the tax system is in stark contrast to what is obtained in the standard model of tax compliance with strategic interaction between auditors and taxpayers, where the resulting effective tax system is always more regressive than the statutory one (see Reinganum and Wilde, 1986; and Scotchmer, 1992).

The paper is organized as follows. Section 2 presents the model and derives the rational expectations equilibrium. Section 3 discusses some properties of the equilibrium. Section 4 contains the analysis of the potential progressive bias of the effective tax system. Section 5 discusses the implications of changes in the variance of the audit cost. Section 6 concludes the paper. All the proofs appear in the appendix.

## 2. The model

Let us consider an economy with a continuum of taxpayers distributed on the interval $[0, 1]$. Assume that the income $\tilde{y}$ of each taxpayer is a normally distributed random variable with mean $\overline{y}$ and variance $V_y \geq 0$ . The income of each taxpayer is independent of that of the others. Therefore, assuming that the strong law of large numbers applies in this situation, the empirical average income is $\overline{y}$. The tax law establishes a statutory tax rate $\tau \in (0, 1)$ on income. After observing the realization $y$ of his

income, a taxpayer optimally decides the amount $x$ of declared income.[4] However, individuals commit involuntary errors during the process of filling the corresponding income report forms. These errors take the form of a normally distributed random variable $\tilde{\varepsilon}$ having zero mean and variance $V_\varepsilon$. Therefore, the income report received by the tax enforcement agency will be the realization of the random variable $\tilde{z} = \tilde{x} + \tilde{\varepsilon}$.

In order to understand the nature of the discrepancy between the variables $\tilde{x}$ and $\tilde{z}$, note that a taxpayer could mistakenly think that some sources of income are tax-exempt while others are taxable, so that the report on taxable income sent to the agency collects these involuntary mistakes. Obviously, the complexity of the tax code is a natural source of the aforementioned errors. An even more direct source of mistakes arises from the design of the income report form that, in many circumstances, induces taxpayer confusion. For instance, if the sources of income are diverse and, thus, the report has to contain multiple components (as in Rhoades, 1999), then the final report could easily contain some imprecisions. Therefore, even if a taxpayer wants to declare an income of $x$ dollars, the final report $z$ submitted to the tax enforcement agency ends up being a noisy transformation of the intended report $x$.

The tax enforcement agency has a pool of tax auditors and each income report is assigned randomly to one auditor. The auditor chooses the audit intensity $p$ applied to a each taxpayer in order to maximize the expected net revenue (tax and penalty revenue, less audit cost) per taxpayer. Note that, due to the strong law of large numbers, this objective implies the maximization of the aggregate net revenue collected by the tax authority. The audit intensity is contingent upon the report $z$ observed by the tax auditor. We can interpret the audit intensity $p$ as a variable proportional to the effort $e$ applied to the inspection and to the penalty rate $f > 1$ on the amount of evaded taxes, i.e., $p = ef$. Moreover, the resources that can be exacted by these audits are assumed to be proportional to the audit intensity and to the amount of evaded taxes. Thus, the penalty revenue is

$$p\tau(y - z) = ef\tau(y - z).$$

Therefore, if the reported income $z$ coincides with the true taxable income $y$ of a taxpayer, then no new revenues will arise from an inspection. Moreover, no additional revenues are obtained by a tax auditor when either no effort is devoted to the inspection of potential tax evaders ($e = 0$) or no penalties are imposed on the amount of evaded taxes ($f = 0$). Finally, note that a taxpayer who wanted to be honest and selects $x = y$ could end up paying a penalty due to the involuntary errors summarized by the variable $\tilde{\varepsilon}$.

We assume that audit costs are quadratic in the effort devoted to auditing, $\frac{1}{2}\hat{c}e^2$ with $\hat{c} > 0$. This costs include all the resources spent by the tax auditor in the process of inspection. Note that, by making $c = \hat{c}/f^2 > 0$, the previous cost function becomes $\frac{1}{2}cp^2$. The value of the cost parameter $\hat{c}$, and thus of $c$, varies across auditors according

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[4]We suppress the tilde to denote the realization of a random variable.

to an exogenously given distribution. The value of that parameter could depend, for instance, on the natural skills and the previous training of the tax auditor. The exact value of his cost parameter $c$ is observable by each auditor but is not observable by taxpayers, so that the cost parameter is a random variable $\tilde{c}$ with a known distribution from the taxpayers' viewpoint. The relevant realization of the random variable $\tilde{c}$ for a given taxpayer corresponds to the value $c$ of the auditor assigned to him.

For the rest of the paper we will take as given the tax rate $\tau$ and the penalty rate $f$. Therefore, we will use the audit intensity $p$ as the decision variable of tax auditors. Finally, we assume that the random variables $\tilde{y}$, $\tilde{\varepsilon}$ and $\tilde{c}$ are mutually independent. The joint distribution of these random variables is common knowledge.

Let $p(z;c)$ be the audit intensity strategy of an auditor with a value $c$ of his cost parameter. This strategy assigns to each report an audit intensity. Since taxpayers do not observe the realization of the random variable $\tilde{c}$, they are uncertain about the audit policy that auditors will apply in their respective cases. Taxpayers are risk neutral and want to maximize the expected amount of their disposable income after the inspection has taken place. The expected disposable income of a taxpayer with initial income $y$ will be $E\left[y - \tau\tilde{z} - p\left(\tilde{z};\tilde{c}\right)\tau(y - \tilde{z})\right]$. Note that $y - \tau z$ is the disposable income after the amount $\tau z$ of taxes has been voluntarily paid and before the inspection has taken place. The amount $p(z;c)\tau(y - z)$ is the additional revenue that the auditor collects through the inspection.

Taxpayers form rational expectations about the strategies followed by tax auditors. Since they observe their true income, taxpayers follow a report strategy satisfying

$$x(y) = \arg\max_{x} E\left[y - \tau\left(x + \tilde{\varepsilon}\right) - p\left(x + \tilde{\varepsilon};\tilde{c}\right)\tau\left(y - x - \tilde{\varepsilon}\right)\right]. \tag{2.1}$$

The first order condition of this problem is

$$-1 - E\left[\frac{\partial p\left(x + \tilde{\varepsilon};\tilde{c}\right)}{\partial\left(x + \tilde{\varepsilon}\right)}\left(y - x - \tilde{\varepsilon}\right) - p\left(x + \tilde{\varepsilon};\tilde{c}\right)\right] = 0. \tag{2.2}$$

The second order condition is

$$-E\left[\frac{\partial^2 p\left(x + \tilde{\varepsilon};\tilde{c}\right)}{\partial^2\left(x + \tilde{\varepsilon}\right)}\left(y - x - \tilde{\varepsilon}\right) - 2\left(\frac{\partial p\left(x + \tilde{\varepsilon};\tilde{c}\right)}{\partial\left(x + \tilde{\varepsilon}\right)}\right)\right] < 0. \tag{2.3}$$

We see that, unlike the seminal papers of Allingham and Sandmo (1972) and Yitzhaki (1974), the taxpayer does not take as given the audit intensity but takes into account the effect of his report on the effort that the tax auditor will devote to enforce the tax law. Note also that we consider the audit intensity (or the audit effort) as the variable selected by auditors, whereas in previous models the relevant variable used to be the probability of inspection.

Tax auditors want to maximize the net revenue from each taxpayer they audit. Therefore, an auditor with a value of the audit cost parameter equal to $c$ chooses the

audit intensity $p$ to be applied to a taxpayer declaring the income level $z$, according to an audit strategy satisfying

$$p(z; c) = \arg\max_p E\left[\tau z + p\tau\left(\tilde{y} - z\right) - \frac{1}{2}cp^2 \,\Big|\, z\right], \tag{2.4}$$

where $z$ is the realization of the random variable $\tilde{z} = x\left(\tilde{y}\right) + \tilde{\varepsilon}$. The first order condition of this problem is

$$E\left[\tau\left(\tilde{y} - z\right) - cp \,\middle|\, z\right] = 0.$$

The sufficient second order condition is simply $c > 0$, which is satisfied by assumption. Therefore, the audit intensity is given by

$$p = \frac{\tau}{c}\left[E\left(\tilde{y}\,\middle|\, z\right) - z\right]. \tag{2.5}$$

An equilibrium with rational expectation is thus a report strategy $x(y)$ and an audit strategy $p(z; c)$ satisfying simultaneously $(2.1)$ and $(2.4)$. We will restrict our attention to linear report strategies, $x(y) = \alpha + \beta y$, and to audit strategies that are linear in the observed income reports, $p(z; c) = \delta(c) + \gamma(c)z$. Note that for these linear audit strategies the sufficient second order condition $(2.3)$ of the taxpayer problem becomes simply $E\left[\gamma(\tilde{c})\right] < 0$. The next proposition gives the unique equilibrium belonging to this class:

**Proposition 2.1.** *Assume that $V_\varepsilon > V_y / 4$. Then, there exists a unique equilibrium with rational expectations where $x(\cdot)$ is linear and $p(\cdot; \cdot)$ is linear in the amount $z$ of reported income. This equilibrium is given by*

$$x(y) = \alpha + \beta y,$$

*where*

$$\alpha = \frac{1}{2}\left\{\bar{y} - \frac{1}{\tau E\left(1/\tilde{c}\right)}\left[1 + \frac{V_y}{4V_\varepsilon}\right]\right\}, \tag{2.6}$$

$$\beta = \frac{1}{2}; \tag{2.7}$$

*and*

$$p(z; c) = \delta(c) + \gamma(c)z,$$

*where*

$$\delta(c) = \frac{1}{c}\left\{\frac{1}{E\left(1/\tilde{c}\right)}\left(\frac{V_y}{V_\varepsilon}\right) - \tau\bar{y}\left(\frac{V_y - 4V_\varepsilon}{V_y + 4V_\varepsilon}\right)\right\}, \tag{2.8}$$

$$\gamma(c) = \frac{\tau}{c}\left(\frac{V_y - 4V_\varepsilon}{V_y + 4V_\varepsilon}\right). \tag{2.9}$$

For the rest of the paper we will maintain the assumption $V_\varepsilon > V_y/4$, which is necessary and sufficient for the second order condition (2.3) of the taxpayer problem. This condition requires in fact that the audit intensity be decreasing in the amount of reported income. If the previous assumption were not imposed, the audit intensity could be increasing in reported income, so that taxpayers would find optimal to report an infinite negative income level. In this respect, note that when $V_\varepsilon \leq V_y/4$ taxpayers make moderate mistakes and, thus, the reports they submit are very informative about their true income. In this case, since $\beta = 1/2$, the amount of voluntarily evaded income $y - x(y)$ rises with the true income $y$ and, hence, tax auditors maximize the penalty revenue by inspecting more intensively the taxpayers who submit high-income reports. On the contrary, if the variance of errors is sufficiently high relative to that of income, as assumed in Proposition 2.1, the reports are not so informative and, hence, auditors attribute high-income reports to involuntary mistakes committed by taxpayers. Moreover, since in this case the dispersion of income is small, the optimal audit policy consists on inspecting more intensively the low-income reports, which are the ones that have a higher probability of being submitted by taxpayers who (involuntarily) underreport their true income.

Note that, using the equilibrium values of the coefficients $\alpha$, $\beta$, $\delta(c)$ and $\gamma(c)$, the previous equilibrium pair of strategies can be written as

$$x(y) = \frac{1}{2}\left\{y + \overline{y} - \frac{1}{\tau E\left(1/\tilde{c}\right)}\left[1 + \frac{V_y}{4V_\varepsilon}\right]\right\} \tag{2.10}$$

and

$$p(z;c) = \frac{1}{c}\left[\tau\left(\frac{V_y - 4V_\varepsilon}{V_y + 4V_\varepsilon}\right)(z - \overline{y}) + \frac{1}{E\left(1/\tilde{c}\right)}\left(\frac{V_y}{4V_\varepsilon}\right)\right]. \tag{2.11}$$

We see that, on the one hand, the intended report $x(y)$ is increasing in the true individual income $y$ and in the tax rate $\tau$.[5] Moreover, for a taxpayer with a given income level $y$, his report $x$ increases with the variance $V_\varepsilon$ of involuntary errors, whereas it is decreasing in the variance $V_y$ of income. Finally, the intended report is increasing in the expectation $E\left(1/\tilde{c}\right)$. On the other hand, the inspection intensity $p(z;c)$ applied to a taxpayer is decreasing in his income report $z$, as required by the second order condition (2.3), and decreasing in the cost parameter $c$. Finally, for a given report $z$ and a given realization of the value $c$ of the cost parameter, the inspection intensity $p$ is increasing in the variance $V_y$ of income, decreasing in the variance $V_\varepsilon$ of report errors, and decreasing in the expectation $E\left(1/\tilde{c}\right)$.

Let us discuss the previous properties of the equilibrium strategies. Consider a taxpayer with a given income level $y$. Clearly, as $V_\varepsilon$ increases the taxpayer knows that the probability of committing important mistakes by accident becomes larger.

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[5]Reported income increases with the tax rate as in the model of Yitzhaki (1974). Some authors claim that this comparative statics is at odds with empirical evidence and this has generated a strand of the literature aimed at obtaining a negative relation between reported income and tax rates (see Yaniv, 1994; Panadés, 2001; and Lee, 2001, among others).

Since the audit intensity is decreasing in reported income, taxpayers know that low reports will be heavily inspected, while high reports will not be exposed to so severe inspections. This bias in the audit policy induces taxpayers to minimize the probability of a rigorous audit. Hence, when $V_\varepsilon$ rises, the intended amount $x$ of reported income increases in order to raise the probability of generating a sufficiently large income report. The report $x$ decreases with the income variance $V_y$, which is consistent with the fact that tax auditors are facing more uncertainty about the true income of taxpayers. Finally, if taxpayers believe that the expected audit cost is high (that amounts "ceteris paribus" to a low value of $E\left(1/\tilde{c}\right)$), then they will expect a low audit intensity by the tax auditors. Therefore, optimal reports must be increasing in $E\left(1/\tilde{c}\right)$.

Concerning the audit intensity for given values of $z$ and $c$, we see that, as the variance $V_y$ of income increases, tax auditors face more uncertainty about a variable that is private information of taxpayers and, thus, more resources must be devoted to audit activities. The variance $V_\varepsilon$ of report errors affects negatively the inspection intensity. This is consistent with the fact that taxpayers raise the amount of income they report when $V_\varepsilon$ increases and, hence, less effort should be devoted to audit taxpayers that underreport less income on average. Moreover, the audit intensity is obviously decreasing in the cost parameter $c$ and is also decreasing in $E\left(1/\tilde{c}\right)$. Note that, if taxpayers expect a high value of the random variable $\tilde{c}$, then $E\left(1/\tilde{c}\right)$ will tend to be low. In this case they will underreport more income, since they think that the auditors will not be very aggressive in their inspection policy. The best response to this taxpayer strategy is to conduct an audit policy more aggressive than the one expected by taxpayers.

## 3. Properties of the equilibrium

In this section we study some indicators of the performance of the tax compliance policy in equilibrium. From $(2.10)$ we can compute first the expected reported income per capita in the economy,

$$E(\tilde{z}) = E\left[x(\tilde{y}) + \tilde{\varepsilon}\right] = \overline{y} - \frac{1}{\tau E\left(1/\tilde{c}\right)}\left[1 + \frac{V_y}{4V_\varepsilon}\right].$$

Note that, as occurs with the intended reports $x$, the expected reported income is increasing in both $E\left(1/\tilde{c}\right)$ and $V_\varepsilon$, whereas is decreasing in $V_y$. Moreover, $\frac{\partial E(\tilde{z})}{\partial \overline{y}} = 1$, so that an increase in the average income results in an increase of reported income of identical amount.

We can compute now the expected audit intensity to see how the different sources of uncertainty affect the audit policy of the tax enforcement agency on average. To this end we compute the unconditional expectation of $(2.11)$, which will give us the expected intensity before observing the realization of the cost parameter $\tilde{c}$ of each auditor,

$$E\left(\tilde{p}\right) = E\left[p\left(x(\tilde{y}) + \tilde{\varepsilon}; \tilde{c}\right)\right] = \frac{1}{2}\left(1 + \frac{V_y}{4V_\varepsilon}\right).$$

It is obvious that the expected audit intensity $E\left(\tilde{p}\right)$ is increasing in $V_y$, decreasing in $V_\varepsilon$ and independent of both $c$ and $E\left(1/\tilde{c}\right)$. Clearly, as $V_\varepsilon$ increases the reports become less reliable signals of the true income. Recall that high values of the variance of involuntary errors induce larger amounts of reported income. In this case tax auditors should reduce the average intensity in order to lower the probability of applying to much effort in inspecting honest taxpayers. Again, more income uncertainty, parametrized by the variance $V_y$, requires more effort by the auditors. Finally, observe that, when computing the unconditional expectation, we are eliminating the asymmetry referred to the audit cost between the agency and the taxpayer. When both the agency and the taxpayer face the same priors about the cost parameter $c$, the opposite effects of the distribution of $\tilde{c}$ on the reporting and inspection strategies cancel out on average.

We can now look at the expected revenue net of audit costs raised by the tax enforcement agency and see also how is affected by the different sources of uncertainty. The (random) net revenue per taxpayer is

$$\tilde{R} = \tau\left(x(\tilde{y}) + \tilde{\varepsilon}\right) + p\left(x(\tilde{y}) + \tilde{\varepsilon}; \tilde{c}\right)\tau\left(\tilde{y} - x(\tilde{y}) - \tilde{\varepsilon}\right) - \frac{1}{2}\tilde{c}\left[p\left(x(\tilde{y}) + \tilde{\varepsilon}; \tilde{c}\right)\right]^2. \qquad (3.1)$$

As we have already said, since there is a continuum of ex-ante identical taxpayers distributed uniformly on the interval $[0, 1]$, the expected net resources extracted from a taxpayer coincide with the aggregate net revenue raised by the agency.

**Corollary 3.1.** *The expected net revenue $E\left(\tilde{R}\right)$ is increasing in $V_\varepsilon$ and decreasing in $V_y$. Moreover, $E\left(\tilde{R}\right)$ is increasing in $E\left(1/\tilde{c}\right)$.*

A larger value of the variance of income means a larger disadvantage of tax auditors with respect to taxpayers and, hence, tax auditors end up putting to much effort on low income taxpayers, who are the ones that pay less fines. When $V_\varepsilon$ increases, taxpayers commit more errors so that the agency will raise more revenues both from the penalties imposed on involuntary evaded taxes and from the taxes on the larger amount of voluntarily reported income. Therefore, the tax authority benefits from taxpayer confusion and, hence, it has no incentives to reduce the complexity of either tax laws or tax forms. Finally, a low value of $E\left(1/\tilde{c}\right)$ is typically associated with a large expected cost. Hence, the expected net revenue will be low since taxpayers anticipate that it is very costly for the auditors to conduct an audit.

Note that we have just looked at the unconditional expected net revenue raised by the tax authority. However, we could look at the expected net revenue conditional to a given realization of the cost parameter $E\left(\tilde{R}|\tilde{c} = c\right)$. In this case, the effects of

changes in $V_\varepsilon$ and $V_y$ are the same as in the unconditional case. However, the effects of $E\left(1/\tilde{c}\right)$ are generally ambiguous. To see this, consider the case where there is no income uncertainty, that is $V_y = 0$ or, equivalently, $\tilde{y} = y$. This is in fact a situation very similar to that considered by Reinganum and Wilde (1988), where the agency knows the realization of its (homogeneous) audit cost parameter and taxpayers view this cost parameter as a random variable $\tilde{c}$ with a given distribution. More precisely, these authors consider a cut-off policy where a taxpayer with a given income $y$ is only inspected if his level of underreporting is so large that the penalty revenue outweighs the audit cost. They also assume a constant cost per inspection that the taxpayer views as if it were drawn from a uniform distribution. Coming back to our scenario, we can compute from (3.1) the following conditional expectation:

$$
E\left(\tilde{R}\,|\tilde{c} = c, \tilde{y} = y\right) = \tau y - \frac{1}{2E\left(1/\tilde{c}\right)} + \frac{1}{8c\left[E\left(1/\tilde{c}\right)\right]^2} + \frac{1}{2}\frac{\tau^2}{c}V_\varepsilon,
$$

which is obviously decreasing in the value of the cost parameter $c$ and increasing in the error variance $V_\varepsilon$. However, it is immediate to obtain that

$$
\frac{\partial E\left(\tilde{R}\,|\tilde{c} = c, \tilde{y} = y\right)}{\partial E\left(1/\tilde{c}\right)} \gtreqless 0 \text{ if and only if } E\left(1/\tilde{c}\right) \lesseqgtr \frac{1}{2c}. \tag{3.2}
$$

Therefore, if the tax authority can affect the beliefs of taxpayers through its disclosure (or secrecy) policy about its audit cost, then the expected revenue is maximized when taxpayers are induced to think that $E\left(1/\tilde{c}\right) = 1/2c$. Usually, a disclosure policy about the audit costs faced by the tax authority affects the variance of the distribution of $\tilde{c}$ as perceived by taxpayers. In the next section we will make explicit the relation between $Var(\tilde{c})$ and $E\left(1/\tilde{c}\right)$ through a couple of examples.

We discuss next the comparative statics concerning taxpayers' welfare. We assume that the revenue raised by the government is not spent in activities that affect the individuals' utility. Therefore, since taxpayers are risk neutral, we only have to compute the expected net income $E\left(\tilde{n}\right)$ of a taxpayer. Recall that the (random) net income of a taxpayer is

$$
\tilde{n} = \tilde{y} - \tau\left(x(\tilde{y}) + \tilde{\varepsilon}\right) - p\left(x(\tilde{y}) + \tilde{\varepsilon}; \tilde{c}\right)\tau\left(\tilde{y} - x(\tilde{y}) - \tilde{\varepsilon}\right).
$$

**Corollary 3.2.** (a) The expected net income $E\left(\tilde{n}\right)$ of a taxpayer is decreasing in $E\left(1/\tilde{c}\right)$.
  (b) If $V_y = 0$, then $E\left(\tilde{n}\right)$ is decreasing in $V_\varepsilon$.
  (c) The effects of changes in $V_\varepsilon$ and $V_y$ on $E\left(\tilde{n}\right)$ are ambiguous when $V_y > 0$.

Obviously, a low expected value of the parameter $c$ (i.e., a large value of $E\left(1/\tilde{c}\right)$) makes the expected net income small, since the tax auditor is expected to use a quite aggressive policy to fight tax evasion. To understand part (b) of the previous corollary, we just have to remind that reported income increases with the variance of

errors and that the auditor does not longer have an informative disadvantage when $V_y = 0$. Concerning the effects of the variances of income and of errors when $V_y > 0$, the results of the comparative statics are ambiguous according to part (c). Recall that an increase in the variance of income (errors) triggers more (less) underreporting of income and more (less) intensive audits. In principle these two effects on taxpayers' net income go in opposite directions. The dominating effect will thus depend on the particular parameter values.

## 4. The bias of the effective tax system

Another question that can be analyzed in the present context is the degree of effective progressiveness exhibited by the tax system in equilibrium. It is a well established result in the literature that the effective tax rate displays less progressiveness than the statutory one when the relationship between auditors and taxpayers is strategic (see Reinganum and Wilde, 1986; and Scotchmer, 1992). This is so because the agency will audit individuals reporting low income with more intensity than individuals producing high income reports. Therefore, even if the optimal amount of reported income is increasing in true income, high-income individuals find more attractive to underreport a larger proportion of their income. This generates a regressive bias in the effective tax structure once we take into account the penalty payments.[6]

In order to analyze whether the effective tax structure of our model is progressive or regressive, we should compute the average expected tax rate faced by a taxpayer and see how this rate changes with the true income $y$. The expected payment (including taxes and penalties) of a taxpayer having a level $y$ of income is

$$g(y) = E\left[\tau\left(x(y) + \tilde{\varepsilon}\right) + p\left(x(y) + \tilde{\varepsilon}; \tilde{c}\right)\tau\left(y - x(y) - \tilde{\varepsilon}\right)\right].$$

Note that in the previous expression we have to compute the expectation just with respect to the random variables $\tilde{\varepsilon}$ and $\tilde{c}$. The average expected tax rate is thus

$$\hat{\tau}(y) = \frac{g(y)}{y}.$$

Under effective proportionality $\hat{\tau}(y)$ should be independent of $y$, while under effective progressiveness (regressiveness) $\hat{\tau}(y)$ should be increasing (decreasing). The following corollary tells that, unlike the previous papers, the function $\hat{\tau}(y)$ could be non-monotonic:

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[6]Scochmer (1987) and Galmarini (1997) analyze the size of the regressive bias under cut-off audit policies when taxpayers are sorted into income classes and when taxpayers differ in terms of their risk aversion, respectively. These two modifications imply a reduction in the size of the regressive bias.

**Corollary 4.1.** *There exists an income level $\hat{y}$ such that the derivative of the average expected tax rate satisfies*

$$\hat{\tau}'(y) < 0 \quad \text{for all } y > \hat{y}.$$

*Moreover, the function $\hat{\tau}(y)$ could be either*
*(a) decreasing both on the interval $(-\infty, 0)$ and on the interval $(0, \infty)$.*
*or*
*(b) U-shaped on the interval $(-\infty, 0)$ and inverted U-shaped on the interval $(0, \infty)$.*

According to the first part of the corollary, the effective tax system is always locally regressive for sufficiently high income levels. Concerning the second part, the potential inverted U-shape of the average expected tax rate for positive income levels means that the effective tax system could display local regressiveness for sufficiently high levels of income, whereas it could display local progressiveness on a lower interval of positive income levels. In order to illustrate Corollary 4.1, Figure 1 displays the function $\hat{\tau}(y)$ for the following configuration of parameters values: $E\left(1/\tilde{c}\right) = 100/3$, $V_y = 1$, $V_\varepsilon = 2.5$, $\overline{y} = 3$ and $\tau = 0.2$. Figure 2 uses the same parameter values except that $V_\varepsilon = 1$. We see that the average expected tax rate can be monotonically decreasing (i.e., the tax system can be uniformly regressive), as in Figure 1, or inverted U-shaped on the interval $(0, \infty)$, as in Figure 2.

[Insert Figures 1 and 2]

To understand the potential non-monotonic behavior of the average expected tax rate, we should bear in mind that individuals suffering an inspection might end up receiving a tax refund. This is so because they could have declared an amount of income larger than the true one due to the involuntary mistakes in the process of filling the tax form. Note also that the existence of these report errors makes taxpayers to declare a larger amount of income. Therefore, audits could detect accidental excessive tax contributions. Since the audit intensity is decreasing in the amount of reported income and reports are decreasing in true income, low-income individuals are more intensively inspected and, thus, they are more likely to get tax refunds. Note that this feature of the audit policy induces a progressive bias in the tax system that could outweigh the aforementioned regressive bias present in strategic models of tax compliance. The potential non-monotonic behavior of $\hat{\tau}(y)$ just captures the trade-off between these two biases.

## 5. The effects of the variance of the audit cost

The comparative statics exercises of the previous section have been performed in terms of the expectation $E\left(1/\tilde{c}\right)$. In this section we analyze how this expectation could be affected by the moments of the distribution of $\tilde{c}$. In order to motivate this exercise,

assume that the tax enforcement agency has a given budget to provide some training to its inspectors. Let us assume that the amount of resources available per auditor is equal to $\hat{b}$. There is a stochastic training technology that relates the value of the cost parameter $c$ of a tax auditor with the amount $b$ invested in his training,

$$\tilde{c} = h(b; \tilde{\xi}),$$

where $h$ is strictly decreasing in $b$ and $\tilde{\xi}$ is a random variable independent of the amount $b$. As we already know, if the tax authority wants to maximize its aggregate revenue, then it has to maximize the expected revenue per taxpayer. According to Corollary 3.1, it is obvious that the agency should try to reach the largest possible value for $E\left(1/\tilde{c}\right)$.

The following natural question arising in this context is whether the tax agency should give identical training to all the auditors, or should allow for some non-homogeneous training that will give rise in turn to some dispersion in the idiosyncratic values of the audit cost parameter. We are thus implicitly assuming that the tax enforcement agency can control, at some extent, some statistical properties of the random variable $\tilde{\xi}$ at zero cost. To answer the previous question we analyze how the value of $E\left(1/\tilde{c}\right)$ is affected by the variance of the distribution of $\tilde{c}$ in two particular cases, namely, when the random variable $\tilde{c}$ is uniformly distributed and when it is log-normal. The choice of these two distributions allows us to be consistent with the second order condition of the tax auditor problem requiring that the value $c$ of his cost parameter be strictly positive.

Assume first that $\tilde{c}$ has a uniform density. In particular, let

$$h(b; \tilde{\xi}) = \hat{h}(b) + \tilde{\xi},$$

where $\tilde{\xi}$ has a uniform density with zero mean and $\hat{h}(b)$ is a positive valued and strictly decreasing mapping. Therefore, the mean of $\tilde{c}$ is

$$E\left(\tilde{c}\right) = \hat{h}(b) \tag{5.1}$$

and the variance is

$$Var\left(\tilde{c}\right) = Var\left(\tilde{\xi}\right). \tag{5.2}$$

The density of $\tilde{c}$ can be thus written as,

$$f(c) = \begin{cases} \dfrac{1}{2\eta} & \text{for } c \in (\bar{c} - \eta, \bar{c} + \eta) \\ \\ 0 & \text{otherwise.} \end{cases}$$

with $\eta > 0$ and $\bar{c} - \eta > 0$, so that $\tilde{c}$ takes always on positive values. Therefore, it holds that $E\left(\tilde{c}\right) = \bar{c}$ and $Var\left(\tilde{c}\right) = \eta^2/3$. It is then clear that $Var\left(\tilde{c}\right)$ is a strictly increasing function of $\eta$. Then,

$$E\left(1/\tilde{c}\right) = \int_{\bar{c}-\eta}^{\bar{c}+\eta} \left(\frac{1}{2\eta}\right)\left(\frac{1}{c}\right) dc = \frac{\ln\left(\bar{c}+\eta\right) - \ln\left(\bar{c}-\eta\right)}{2\eta}.$$

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After some simplification we obtain the following derivatives:

$$\frac{\partial E\left(1/\tilde{c}\right)}{\partial \bar{c}} = -\frac{1}{\bar{c}^2 - \eta^2} < 0, \tag{5.3}$$

$$\frac{\partial E\left(1/\tilde{c}\right)}{\partial \eta} = -\frac{1}{\bar{c}^2 - \eta^2} - \frac{ln\left(\bar{c}+\eta\right) - ln\left(\bar{c}-\eta\right)}{2\eta^2} < 0.$$

Hence, we have that

$$\frac{\partial E\left(1/\tilde{c}\right)}{\partial Var(\tilde{c})} < 0. \tag{5.4}$$

Since the audit cost of an auditor is strictly decreasing in the amount of resources devoted to his training, the derivative $(5.3)$ implies that the expected value of $\tilde{c}$ should be minimized and, thus, the agency should select $b = \hat{b}$ so that $E\left(\tilde{c}\right) = \hat{h}(\hat{b})$, as follows from $(5.1)$. This means that the agency should exhaust all the resources for training. Moreover, according to $(5.4)$, if the randomization device of the training technology generates a uniform distribution of the cost parameter $\tilde{c}$, a tax enforcement agency aiming at the maximization of its net revenue should try to minimize the variance of $\tilde{c}$. Obviously, this is achieved by minimizing the variance of the random variable $\tilde{\xi}$ (see $(5.2)$).

Assume now that the cost parameter $\tilde{c}$ is log-normally distributed. More precisely, assume that

$$h(b; \tilde{\xi}) = \hat{h}(b)\tilde{\xi}, \tag{5.5}$$

where $\tilde{\xi}$ is log-normal with $E(\tilde{\xi}) = 1$ and $\hat{h}(b)$ has the same properties as before. Therefore, the mean of the random variable $\tilde{c}$ is

$$E(\tilde{c}) = \hat{h}(b), \tag{5.6}$$

and its variance is

$$Var\left(\tilde{c}\right) = \left[\hat{h}(b)\right]^2 Var(\tilde{\xi}). \tag{5.7}$$

Let $E(ln\tilde{\xi}) = \mu$ and $Var\left(ln\tilde{\xi}\right) = \sigma^2$. Therefore, the mean of $\tilde{\xi}$ is

$$E(\tilde{\xi}) = \exp\left(\mu + \frac{\sigma^2}{2}\right) = 1,$$

so that $\mu = -\sigma^2/2$. Moreover,

$$Var(\tilde{\xi}) = \left[E(\tilde{\xi})\right]^2 \left[\exp\left(\sigma^2\right) - 1\right] = \exp\left(\sigma^2\right) - 1. \tag{5.8}$$

Since $\tilde{c}$ is log-normal, the random variable $ln\left(\tilde{c}\right)$ is normally distributed. Therefore, from $(5.5)$, we have that

$$E\left[ln\left(\tilde{c}\right)\right]) = ln\left(\hat{h}(b)\right) + \mu = ln\left(\hat{h}(b)\right) - \frac{\sigma^2}{2},$$

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$$Var\left[ln\left(\tilde{c}\right)\right] = \sigma^2.$$

Similarly, the random variable $1/\tilde{c}$ is log-normal as $ln\left(1/\tilde{c}\right)$ is normally distributed. Since $ln\left(1/\tilde{c}\right) = -ln\left(\tilde{c}\right)$, we get

$$E\left(ln\left(1/\tilde{c}\right)\right) = -ln\left(\hat{h}(b)\right) + \frac{\sigma^2}{2} \tag{5.9}$$

and

$$Var\left(ln\left(1/\tilde{c}\right)\right) = \sigma^2. \tag{5.10}$$

Therefore, using $(5.9)$ and $(5.10)$, we can obtain the mean of the random variable $1/\tilde{c}$,

$$E\left(1/\tilde{c}\right) = \exp\left[E\left(ln\left(1/\tilde{c}\right)\right) + \frac{Var(ln(1/\tilde{c}))}{2}\right] = \exp\left[-ln\left(\hat{h}(b)\right) + \sigma^2\right]. \tag{5.11}$$

A revenue-maximizing tax enforcement agency should select the largest feasible value of $E\left(1/\tilde{c}\right)$ (see Corollary 3.1), and it is obvious from $(5.11)$ that this is achieved by choosing simultaneously the lowest feasible value for $ln\left(\hat{h}(b)\right)$ and the largest feasible value for $\sigma^2$. The minimization of $ln\left(\hat{h}(b)\right)$ is accomplished again by selecting $b = \hat{b}$ so that $E\left(\tilde{c}\right) = \hat{h}(\hat{b})$, as follows from $(5.6)$. Having picked optimally the value of $E\left(\tilde{c}\right)$, note from $(5.8)$ that the maximization of the variance $\sigma^2$ means that the variance of $\tilde{\xi}$ has to reach its largest feasible value. Moreover, the previous policy implies that, for a given value of resources per auditor $\hat{b}$, the variance of $\tilde{c}$ must be set as large as possible by the tax enforcement agency (see $(5.7)$).

We see that the effect of the variance of the cost parameter $\tilde{c}$ on the expectation $E\left(1/\tilde{c}\right)$ under a log-normal distribution is the opposite to that obtained under a uniform distribution. Thus, if the results contained in Corollaries 3.1 and 3.2, and in expression $(3.2)$ were written in terms of the variance of $\tilde{c}$, the corresponding comparative statics exercises would be extremely dependent on the specific distribution of $\tilde{c}$ under consideration.

## 6. Conclusion

In the context of a model of strategic interaction between tax auditors and taxpayers, we have analyzed the effects of different sources of uncertainty on the performance of the tax compliance policy. Besides the typical uncertainty faced by tax auditors associated with the income of taxpayers, we add two additional sources of uncertainty. The first one refers to the involuntary errors committed by taxpayers when they fill their income reports. The variance of these errors is usually increasing in the complexity of both tax laws and report forms. The second source of uncertainty refers

to the fact that the cost of conducting an audit is private information of the tax auditors so that the inspection policy is viewed as random by the taxpayers. Our main results can be summarized as follows:

- Larger variance of involuntary errors results in more average income reported, less average audit intensity, and more net revenue for the government.

- Larger variance of the income distribution results in less average income reported, more average audit intensity, and less net revenue for the government.

- Larger average audit costs typically result in less average income reported, less net revenue for the government, and more disposable income for the taxpayers on average.

- The relation between the average expected tax rate and true income could be non-monotonic. Therefore, the tax system could be locally progressive on some range of income levels and locally regressive on some other range.

# A. Appendix

**Proof of Proposition 2.1.** The tax auditor observes the reported income $z$ and the value $c$ of the cost parameter and chooses the audit intensity $p$ in order to solve (2.4). Therefore, the optimal audit intensity is given by (2.5). The auditor conjectures that taxpayers follow linear report strategies, i.e., $x = \alpha + \beta y$, and thus,

$$\tilde{z} = x(\tilde{y}) + \tilde{\varepsilon} = \alpha + \beta\tilde{y} + \tilde{\varepsilon}.$$

Note that observing a realization of the random variable $\tilde{z}$ is informationally equivalent to observing a realization of the random variable

$$\frac{\tilde{z} - \alpha}{\beta} = \tilde{y} + \frac{\tilde{\varepsilon}}{\beta},$$

which has mean equal to $\bar{y}$ and variance equal to $V_\varepsilon / \beta^2$. Therefore,

$$E\left(\tilde{y}\,|z\right) = E\left(\tilde{y}\,\bigg|\,\frac{z - \alpha}{\beta}\right) = E\left(\tilde{y}\,\bigg|\,\tilde{y} + \frac{\tilde{\varepsilon}}{\beta}\right).$$

Since $\tilde{y}$ and $\tilde{\varepsilon}$ are mutually independent, we can apply the projection theorem for normally distributed random variables to get

$$E\left(\tilde{y}\,|z\right) = \bar{y} + \frac{V_y}{V_y + (V_\varepsilon / \beta^2)}\left(\frac{z - \alpha}{\beta} - \bar{y}\right). \tag{A.1}$$

Plugging (A.1) in (2.5) and collecting terms we obtain

$$p = \frac{\tau}{c}\left\{\left[1 - \frac{V_y}{V_y + (V_\varepsilon / \beta^2)}\right]\bar{y} - \left[\frac{V_y}{V_y + (V_\varepsilon / \beta^2)}\right]\frac{\alpha}{\beta}\right\} + \frac{\tau}{c}\left\{\frac{V_y}{[V_y + (V_\varepsilon / \beta^2)]\,\beta} - 1\right\}z.$$

The previous expression confirms that the audit strategy is linear in the observed report $z$. Therefore, letting $p(z; c) = \delta(c) + \gamma(c)z$ and equating coefficients, we get

$$\delta(c) = \frac{\tau}{c}\left\{\left[1 - \frac{V_y}{V_y + (V_\varepsilon / \beta^2)}\right]\bar{y} - \left[\frac{V_y}{V_y + (V_\varepsilon / \beta^2)}\right]\frac{\alpha}{\beta}\right\}, \tag{A.2}$$

and

$$\gamma(c) = \frac{\tau}{c}\left\{\frac{V_y}{[V_y + (V_\varepsilon / \beta^2)]\,\beta} - 1\right\}. \tag{A.3}$$

A taxpayer observes his true income $y$ and conjectures that the tax auditor follows an audit strategy that is linear in $z$, $p(z; c) = \delta(c) + \gamma(c)z$. Therefore, the objective of the taxpayer is to maximize

$$E\left\{y - \tau\left(x + \tilde{\varepsilon}\right) - \left[\delta(\tilde{c}) + \gamma(\tilde{c})(x + \tilde{\varepsilon})\right]\tau\left(y - x - \tilde{\varepsilon}\right)\right\}.$$

The optimal intended report $x$ must satisfy the following first order condition (see (2.2)):

$$-1 - E\left[\gamma(\tilde{c})\left(y - x - \tilde{\varepsilon}\right) - \delta(\tilde{c}) - \gamma(\tilde{c})\left(x + \tilde{\varepsilon}\right)\right] = 0.$$

Using the fact that $\tilde{c}$ and $\tilde{\varepsilon}$ are mutually independent, we can solve the previous equation for $x$,

$$x = \frac{1}{2}\left[y + \frac{\left(1 - \overline{\delta}\right)}{\overline{\gamma}}\right], \tag{A.4}$$

where $\overline{\delta} = E\left[\delta(\tilde{c})\right]$ and $\overline{\gamma} = E\left[\gamma(\tilde{c})\right]$. The second order condition (2.3) becomes simply $\overline{\gamma} < 0$. Note that (A.4) confirms that the report strategies used by taxpayers are linear in their income, that is, $x = \alpha + \beta y$. Therefore, equating coefficients we obtain,

$$\alpha = \frac{1}{2}\left(\frac{1 - \overline{\delta}}{\overline{\gamma}}\right), \tag{A.5}$$

and

$$\beta = \frac{1}{2}. \tag{A.6}$$

We must compute now the expected values of the coefficients $\delta(\tilde{c})$ and $\gamma(\tilde{c})$. To this end, we compute the expectation of (A.2) and (A.3) to obtain

$$\overline{\delta} = \tau E\left(1/\tilde{c}\right)\left\{\left[1 - \frac{V_y}{V_y + (V_\varepsilon/\beta^2)}\right]\overline{y} - \left[\frac{V_y}{V_y + (V_\varepsilon/\beta^2)}\right]\frac{\alpha}{\beta}\right\} \tag{A.7}$$

and

$$\overline{\gamma} = \tau E\left(1/\tilde{c}\right)\left\{\frac{V_y}{[V_y + (V_\varepsilon/\beta^2)]\beta} - 1\right\}. \tag{A.8}$$

We can find the values of $\alpha, \beta, \overline{\delta}, \overline{\gamma}$ solving the system of equations (A.5), (A.6), (A.7) and (A.8). After some tedious algebra we obtain the values of $\alpha$ and $\beta$ given in (2.6) and (2.7) and

$$\overline{\delta} = \frac{V_y}{4V_\varepsilon} - \tau E\left(1/\tilde{c}\right)\overline{y}\left(\frac{V_y - 4V_\varepsilon}{V_y + 4V_\varepsilon}\right),$$

$$\overline{\gamma} = \tau E\left(1/\tilde{c}\right)\left(\frac{V_y - 4V_\varepsilon}{V_y + 4V_\varepsilon}\right).$$

Note that the second order condition $\overline{\gamma} < 0$ is satisfied since $4V_\varepsilon > V_y$ holds by assumption

We can now find the coefficients $\delta$ and $\gamma$ defining the audit strategy. To this end we only have to plug the values of $\alpha$ and $\beta$ we have just obtained into (A.2) and (A.3). Some additional algebra yields the values of $\delta(c)$ and $\gamma(c)$ given in (2.8) and (2.9). ∎

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**Proof of Corollary 3.1.** The expected net revenue raised by a tax auditor before observing the realization of the cost $\tilde{c}$ and the report $\tilde{z}$ is

$$E\left(\tilde{R}\right) = E\left\{\tau\left(x(\tilde{y})+\tilde{\varepsilon}\right)+p\left(x(\tilde{y})+\tilde{\varepsilon};\tilde{c}\right)\tau\left(\tilde{y}-x(\tilde{y})-\tilde{\varepsilon}\right)-\tfrac{1}{2}c\left[p\left(x(\tilde{y})+\tilde{\varepsilon};\tilde{c}\right)\right]^2\right\}$$

$$= E\left\{\tau\left(\alpha+\beta\tilde{y}+\tilde{\varepsilon}\right)+\left[\delta(c)+\gamma(c)\left(\alpha+\beta\tilde{y}+\tilde{\varepsilon}\right)\right]\tau\left(\tilde{y}-\alpha-\beta\tilde{y}-\tilde{\varepsilon}\right)\right.$$

$$\left.-\tfrac{1}{2}c\left[\delta(c)+\gamma(c)\left(\alpha+\beta\tilde{y}+\tilde{\varepsilon}\right)\right]^2\right\}.$$

Using the equilibrium values of $\alpha$, $\beta$, $\gamma(c)$ and $\delta(c)$ obtained in Proposition 2.1, and after some cumbersome algebra, we obtain

$$E\left(\tilde{R}\right) = \frac{1}{128V_\varepsilon^2\left(V_y+4V_\varepsilon\right)E\left(1/\tilde{c}\right)}\left[V_y^3-4V_y^2V_\varepsilon-80V_yV_\varepsilon^2-192V_\varepsilon^3\right.$$

$$+128\tau\overline{y}V_yV_\varepsilon^2E\left(1/\tilde{c}\right)+512\tau\overline{y}V_\varepsilon^3E\left(1/\tilde{c}\right)-128\tau^2V_yV_\varepsilon^3\left[E\left(1/\tilde{c}\right)\right]^2$$

$$\left.+256\tau^2V_\varepsilon^4\left[E\left(1/\tilde{c}\right)\right]^2+16\tau^2V_y^2V_\varepsilon^2\left[E\left(1/\tilde{c}\right)\right]^2\right].$$

We can compute now the following derivative:

$$\frac{\partial E\left(\tilde{R}\right)}{\partial V_\varepsilon} = \frac{1}{64V_\varepsilon^3\left(V_y+4V_\varepsilon\right)^2E\left(1/\tilde{c}\right)}\left[16V_y^2V_\varepsilon^2+64V_yV_\varepsilon^3-V_y^4-4V_y^3V_\varepsilon\right.$$

$$\left.-96\tau^2V_y^2V_\varepsilon^3\left[E\left(1/\tilde{c}\right)\right]^2+256\tau^2V_yV_\varepsilon^4\left[E\left(1/\tilde{c}\right)\right]^2+512\tau^2V_\varepsilon^5\left[E\left(1/\tilde{c}\right)\right]^2\right].$$

It can be shown that the previous derivative becomes equal to zero only when $4V_\varepsilon = V_y$, whereas it is positive whenever $0 < V_y < 4V_\varepsilon$ , which holds by assumption.

Concerning the effects of $V_y$, we compute

$$\frac{\partial E\left(\tilde{R}\right)}{\partial V_y} = \frac{1}{64V_\varepsilon^2\left(V_y+4V_\varepsilon\right)^2E\left(1/\tilde{c}\right)}\left[V_y^3+4V_y^2V_\varepsilon\right.$$

$$+8\tau^2V_y^2V_\varepsilon^2\left[E\left(1/\tilde{c}\right)\right]^2-16V_yV_\varepsilon^2-384\tau^2V_\varepsilon^4\left[E\left(1/\tilde{c}\right)\right]^2$$

$$\left.-64V_\varepsilon^3+64\tau^2V_yV_\varepsilon^3\left[E\left(1/\tilde{c}\right)\right]^2\right] \tag{A.9}$$

The previous derivative becomes equal to zero whenever

$$V_y = 4V_\varepsilon, \tag{A.10}$$

$$V_y = 4V_\varepsilon\left[-1-\tau^2V_\varepsilon\left[E\left(1/\tilde{c}\right)\right]^2+\tau E\left(1/\tilde{c}\right)\sqrt{V_\varepsilon\left(\tau^2V_\varepsilon\left[E\left(1/\tilde{c}\right)\right]^2-4\right)}\right], \tag{A.11}$$

or

$$V_y = 4V_\varepsilon \left[ -1 - \tau^2 V_\varepsilon \left[ E\left(1/\tilde{c}\right) \right]^2 - \tau E\left(1/\tilde{c}\right) \sqrt{V_\varepsilon \left(\tau^2 V_\varepsilon \left[ E\left(1/\tilde{c}\right) \right]^2 - 4 \right)} \right]. \qquad \text{(A.12)}$$

The roots (A.11) and (A.12) are imaginary when $V_\varepsilon \in \left(0, \left[\tau E\left(1/\tilde{c}\right)/2\right]^{-2}\right)$. In this case, the single real root is the one given by (A.10). If $V_\varepsilon \geq \left[\tau E\left(1/\tilde{c}\right)/2\right]^{-2}$, then the roots (A.11) and (A.12) are real. The root (A.12) is obviously negative. Concerning the root (A.11), it can be easily checked that it is also negative when $V_\varepsilon \geq \left[\tau E\left(1/\tilde{c}\right)/2\right]^{-2}$. Therefore, (A.9) does not change its sign in all the parameter region satisfying $0 < V_y < 4V_\varepsilon$, which holds by assumption. Then, we only have to check numerically that (A.9) is negative in that region.

Finally, we can compute the following derivative with respect to $E\left(1/\tilde{c}\right)$:

$$\frac{\partial E\left(\tilde{R}\right)}{\partial E\left(1/\tilde{c}\right)} = \frac{1}{128 V_\varepsilon^2 \left(V_y + 4V_\varepsilon\right) \left[ E\left(1/\tilde{c}\right) \right]^2} \left[ -V_y^3 + 16\tau^2 V_y^2 V_\varepsilon^2 \left[ E\left(1/\tilde{c}\right) \right]^2 + 4 V_y^2 V_\varepsilon \right.$$

$$\left. -128\tau^2 V_y V_\varepsilon^3 \left[ E\left(1/\tilde{c}\right) \right]^2 + 80 V_y V_\varepsilon^2 + 256\tau^2 V_\varepsilon^4 \left[ E\left(1/\tilde{c}\right) \right]^2 + 192 V_y^3 \right].$$

The previous derivative is always positive, as it can be shown by checking that it has only two imaginary roots for $E\left(1/\tilde{c}\right)$, so that never changes sign for all positive real values of $E\left(1/\tilde{c}\right)$. ∎

**Proof of Corollary 3.2.** (a) The expected disposable income of a taxpayer is

$$E\left(\tilde{n}\right) = E\left[\tilde{y} - \tau\left(x(\tilde{y}) + \tilde{\varepsilon}\right) - p\left(x(\tilde{y}) + \tilde{\varepsilon}; \tilde{c}\right) \tau\left(\tilde{y} - x(\tilde{y}) - \tilde{\varepsilon}\right)\right] =$$

$$E\left\{\tilde{y} - \tau\left(\alpha + \beta\tilde{y} + \tilde{\varepsilon}\right) - \left[\delta(\tilde{c}) + \gamma(\tilde{c})\left(\alpha + \beta\tilde{y} + \tilde{\varepsilon}\right)\right]\tau\left(\tilde{y} - \alpha - \beta\tilde{y} - \tilde{\varepsilon}\right)\right\}.$$

Using the equilibrium values of $\alpha$, $\beta$, $\delta(c)$ and $\gamma(c)$ given in (2.6)-(2.9) and simplifying, we obtain

$$E\left(\tilde{n}\right) = \frac{1}{64 E\left(1/\tilde{c}\right) V_\varepsilon^2 \left(V_y + 4V_\varepsilon\right)} \left[ 64\overline{y} V_y V_\varepsilon^2 \left[ E\left(1/\tilde{c}\right) \right]^2 + 256\overline{y} V_\varepsilon^3 \left[ E\left(1/\tilde{c}\right) \right] \right.$$

$$\left. -64\tau\overline{y} V_y V_\varepsilon^2 \left[ E\left(1/\tilde{c}\right) \right] - 4 V_y^2 V_\varepsilon - 256\tau\overline{y} V_\varepsilon^3 \left[ E\left(1/\tilde{c}\right) \right] + 16 V_y V_\varepsilon^2 \right.$$

$$\left. + 64 V_\varepsilon^3 - V_y^3 + 64\tau^2 V_y V_\varepsilon^3 \left[ E\left(1/\tilde{c}\right) \right]^2 - 256\tau^2 V_\varepsilon^4 \left[ E\left(1/\tilde{c}\right) \right]^2 \right].$$

We can compute then the following derivative:

$$\frac{\partial E\left(\tilde{n}\right)}{\partial E\left(1/\tilde{c}\right)} = \frac{1}{64 V_\varepsilon^2 \left(V_y + 4V_\varepsilon\right) \left[ E\left(1/\tilde{c}\right) \right]^2} \left[ 4 V_y^2 V_\varepsilon - 16 V_y V_\varepsilon^2 - 64 V_\varepsilon^3 + V_y^3 \right.$$

$$\left. + 64\tau^2 V_y V_\varepsilon^3 \left[ E\left(1/\tilde{c}\right) \right]^2 - 256\tau^2 V_\varepsilon^4 \left[ E\left(1/\tilde{c}\right) \right]^2 \right]. \qquad \text{(A.13)}$$

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It can be easily shown that (A.13) never becomes zero and takes always negative values whenever $0 < V_y < 4V_\varepsilon$.

(b) Computing the derivative of $E(\tilde{n})$ with respect to $V_\varepsilon$, we obtain

$$\frac{\partial E(\tilde{n})}{\partial V_\varepsilon} = \frac{1}{32V_\varepsilon^3 (V_y + 4V_\varepsilon)^2 E(1/\tilde{c})} \left[ -512\tau^2 V_\varepsilon^5 [E(1/\tilde{c})]^2 + 8V_y^3 V_\varepsilon + 16V_y^2 V_\varepsilon^2 \right.$$

$$\left. + V_y^4 + 32\tau^2 V_y^2 V_\varepsilon^3 [E(1/\tilde{c})]^2 - 256\tau^2 V_y V_\varepsilon^4 [E(1/\tilde{c})]^2 \right].$$

For $V_y = 0$, the previous derivative simplifies to

$$\frac{\partial E(\tilde{n})}{\partial V_\varepsilon} = -\tau E(1/\tilde{c}) < 0.$$

(c) Let

$$\rho = \frac{V_y (V_y + 4V_\varepsilon)}{8\tau V_\varepsilon} \left( \frac{2}{16V_\varepsilon^3 + 8V_\varepsilon^2 - V_y^2 V_\varepsilon} \right)^{1/2},$$

Note that $\rho > 0$ whenever $0 < V_y < 4V_\varepsilon$ holds. Then, it can be easily checked that $\frac{\partial E(\tilde{n})}{\partial V_\varepsilon} \gtrless 0$ for all $E(1/\tilde{c}) \lessgtr \rho$.

Similarly, for the effects of $V_y$ on $E(\tilde{n})$ we can compute

$$\frac{\partial E(\tilde{n})}{\partial V_y} = \frac{1}{32V_\varepsilon^2 (V_y + 4V_\varepsilon)^2 E(1/\tilde{c})} \left[ V_y^3 - 8V_y^2 V_\varepsilon - 16V_y V_\varepsilon^2 + 256\tau^2 V_\varepsilon^4 [E(1/\tilde{c})]^2 \right].$$

Let

$$\theta = \frac{(V_y)^{1/2} (V_y + 4V_\varepsilon)}{16\tau V_\varepsilon^2},$$

Note that $\theta > 0$. Then, it can be easily verified that

$$\frac{\partial E(\tilde{n})}{\partial V_y} \lessgtr 0 \quad \text{for all } E(1/\tilde{c}) \lessgtr \theta. \qquad \blacksquare$$

**Proof of Corollary 4.1.** The average expected tax rate is

$$\hat{\tau}(y) = \frac{g(y)}{y} = \frac{E\left[ \tau(x(y) + \tilde{\varepsilon}) + p(x(y) + \tilde{\varepsilon}; \tilde{c}) \tau(y - x(y) - \tilde{\varepsilon}) \right]}{y} =$$

$$\frac{E\left\{ \tau(\alpha + \beta y + \tilde{\varepsilon}) + [\delta(\tilde{c}) + \gamma(\tilde{c})(\alpha + \beta y + \tilde{\varepsilon})] \tau(y - \alpha - \beta y - \tilde{\varepsilon}) \right\}}{y}.$$

Using the equilibrium values of the parameters characterizing the audit and report strategies given in (2.6)-(2.9), we get an expression of the following type:

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$$\hat{\tau}(y) = \frac{my^2 + ny + q}{sy},$$

where $m$, $n$, $q$ and $s$ depend on the parameters of the model. In particular,

$$m = 16\tau^2 \left[E\left(1/\tilde{c}\right)\right]^2 V_\varepsilon^2 \left(V_y - 4V_\varepsilon\right),$$

and

$$s = 64yV_\varepsilon^2 \left(V_y + 4V_\varepsilon\right) E\left(1/\tilde{c}\right).$$

Note that $m < 0$ as $V_y < 4V_\varepsilon$, whereas $s > 0$. Therefore,

$$\lim_{y \to \infty} \hat{\tau}(y) = -\infty,$$

$$\lim_{y \to -\infty} \hat{\tau}(y) = \infty,$$

and the function $\hat{\tau}(y)$ is discontinuous at $y = 0$. Moreover, the equation $\hat{\tau}'(y) = 0$ has two conjugate solutions,

$$\frac{\pm\sqrt{\Delta}}{4\tau V_\varepsilon E\left(1/\tilde{c}\right)},$$

with

$$\Delta = V_y^2 - 8\tau\bar{y}V_yV_\varepsilon E\left(1/\tilde{c}\right) + 8V_yV_\varepsilon - 32\tau\bar{y}V_\varepsilon^2 E\left(1/\tilde{c}\right) - 64\tau^2 V_\varepsilon^3 \left[E\left(1/\tilde{c}\right)\right]^2$$

$$+16V_\varepsilon^2 + 16\tau^2\bar{y}^2V_\varepsilon^2 \left[E\left(1/\tilde{c}\right)\right]^2.$$

These two solutions are both real with opposite sign when the term $\Delta$ is positive. Otherwise, the two solutions are imaginary. Therefore, on the one hand, when $\Delta$ is negative, the function $\hat{\tau}(y)$ is decreasing on the interval $(-\infty, 0)$ and is also decreasing on the interval $(0, \infty)$. On the other hand, if $\Delta$ is positive then the function $\hat{\tau}(y)$ is U-shaped on the interval $(-\infty, 0)$ and inverted U-shaped on the interval $(0, \infty)$. Note that in both cases there exists an income level $\hat{y}$ such that $\hat{\tau}'(y) < 0$, for all $y > \hat{y}$. Finally, note that $\Delta$ can be positive or negative depending on the parameter values. For instance, let $E\left(1/\tilde{c}\right) = 100/3$, $V_y = 1$, $\bar{y} = 3$ and $\tau = 0.2$. In this case, if $V_\varepsilon = 1$, then $\Delta = 2780.6$. However, if $V_\varepsilon = 2.5$, then $\Delta = -8723.4$. ∎

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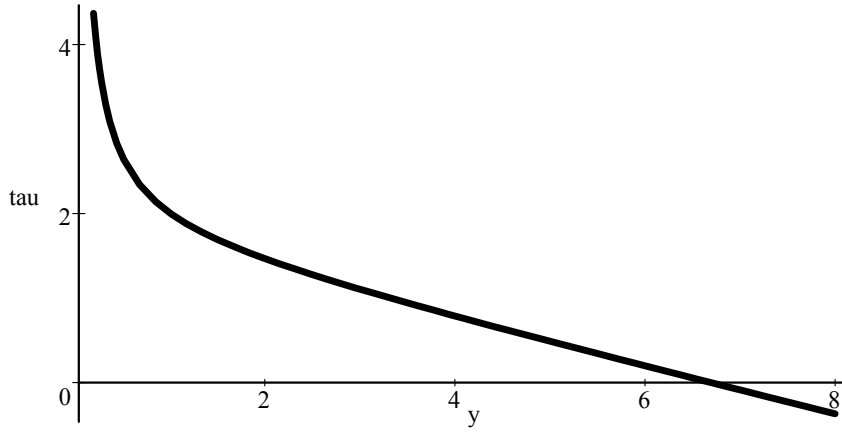
**Figure 1.** Average expected tax rate when $E\left(1/\tilde{c}\right) = 100/3$, $V_y = 1$, $V_\varepsilon = 2.5$, $\bar{y} = 3$ and $\tau = 0.2$
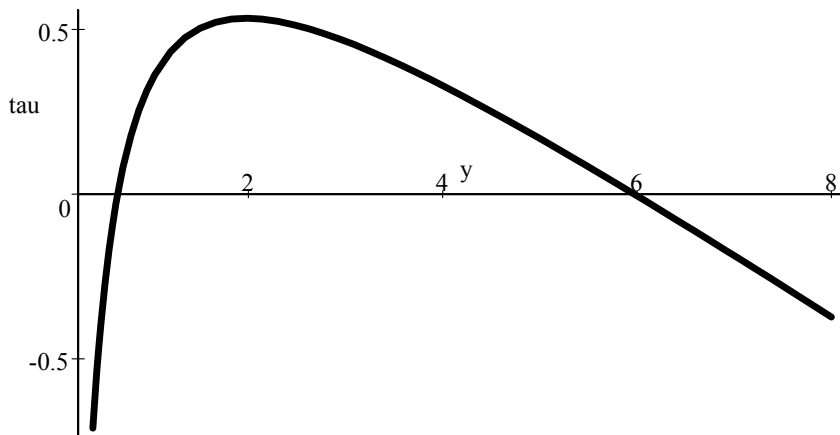


**Figure 2.** Average expected tax rate when $E\left(1/\tilde{c}\right) = 100/3$, $V_y = 1$, $V_\varepsilon = 1$, $\bar{y} = 3$ and $\tau = 0.2$