# INDIVIDUAL PREFERENCES REVEALED THROUGH EFFECTIVE MARGINAL TAX RATES: A NOTE ON PRELIMINARY RESULTS. 

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#### Abstract

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This paper inverts the usual logic of the applied optimal income taxation literature. Standard practice analyzes the shape of the optimal tax schedule that is consistent with a given social welfare function, a statistical distribution of individual productivities that fits available data on labor incomes and given preferences between consumption and leisure. In this paper, we go in the opposite direction. We start from the observed distribution of gross and disposable income within a population and from the observed marginal tax rates as computed in standard tax-benefit models. We then show that, under a set of simplifying assumptions, it is possible to identify the individual utility function that would make the observed marginal tax rate schedule optimal under some assumption about social welfare preferences. This provides an alternative way of reading marginal tax rates calculations routinely provided by tax-benefit models. In that framework, the issue of the optimality of an existing tax-benefit system may be analyzed by considering whether the individual utility function associated with that system satisfies elementary properties. A detailed application is given in the case of France.


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## Introduction

Several attempts were recently made at analyzing existing redistribution systems in several countries within the framework of optimal taxation theory. The basic question asked in that literature is whether it is possible to justify the most salient features of existing systems by some optimal tax argument. For instance, under what condition would it be optimal for the marginal tax rate curve to be U-shaped? (see Diamond (1998) and Saez (2001) for the US and Salanié (1998) for France). Or could it be optimal to have 100 per cent effective marginal tax rates at the bottom of the distribution as in some minimum income programs [Piketty (1997), Bourguignon and Spadaro (2000), Choné and Laroque (2001)]. Such questions were already addressed in the early optimal taxation literature and in particular in Mirrlees (1971) but the exercise may now seem more relevant because of the possibility of relying on large and well documented micro data sets and simulation models rather than on hypothetical statistical distributions. In all those papers the framework of the analysis is the basic optimal taxation model ${ }^{1}$. The solution of this model gives a rule that associate the tax schedule to the key ingredients of the model (i.e. the productivity distribution, the elasticity of agent's labor supply and the government's aversion to inequality implicit in the shape of the social welfare function) in a way that minimizes efficiency distortions due to redistribution. Starting from this framework, the papers cited above try to compute the shape of the optimal marginal tax rate using the distribution of observed hourly wages as proxy for the productivities, the values of the elasticity of labor supply estimated from observed hours of work and some given specification of the social welfare function. They compares, afterwards, the results of the computations with the shape of the observed (real) effective marginal tax rate trying to explain the eventual differences by optimal tax arguments.

An exception is Bourguignon and Spadaro (2000). In that paper they raise problems about the use of hours spent at work as measure of the agent's effort and about the use of hourly wages as proxy of the productivities. The reasons why they criticize this approach in optimal tax problems are basically four. First, labor supply may differ quite significantly from working hours when unobserved efforts are taken into account. Second, the econometric estimation of a labor supply model requires taking into account the non-linearity introduced by the taxbenefit system actually faced by individuals, and in particular the endogeneity of marginal tax rates. Econometric estimations of this type are now known to be little robust ${ }^{2}$. Third, econometric estimates of the elasticity of labor supply are known to differ substantially across various types of individuals. In particular, it is small for household heads and larger for spouses, young people and people close to retirement age. Under these conditions, it is complicate to choose the value for the elasticity of labor supply. Fourth, and more fundamentally, it seems natural to choose the household as the economic unit in a welfare analysis of taxes and benefits. But, then, the problem arises of aggregating at the household level concepts or measures that are valid essentially at the individual level. The approach proposed in Bourguignon and Spadaro (2000) is based on simple techniques of micro simulation. They start with a possible labor supply specification that leads to an analytically simple determination of the optimal redistribution scheme. Then they identify the "natural" distribution of the household work productivities from income data obtained from surveys. This is done by micro simulation, inverting the previous model under the arbitrary hypothesis of price elasticity of labor supply, and considering the budget constraints proper of the redistribution systems in force in the countries that we are studying. Their approach can be considered the dual of the econometric one. In the last one, we observe the income and

[^0]productivities of the agents, supposed to be identical to the gross wage rate. From this, one can (by estimation) deduces the parameters of labor supply behavior under certain functional form hypothesis. In this approach, one a priori asserts a functional form and (alternative) behavior parameters, and one deduces the implicit work productivity from the observed income. Whit the productivity distribution computed as explained and using the correspondent elasticity parameters, it is thus possible to analyze the form of the optimal redistributive scheme according to parameters describing the social aversion.

The results obtained when applying the standard optimal taxation framework to actual data depend very much on the shape of the social welfare function. Using the same methodology discussed above for recovering the productivity distribution, in a recent paper, Bourguignon and Spadaro (2002) propose a methodology that, making use of the "inversion of the optimal problem" techniques (see Kurz 1968), allows to reveal social preferences about inequality implicit in a given redistribution scheme. Instead of taking the social welfare function as given and deriving the optimal schedule of effective marginal tax rates along the income or ability scale, they do the reverse. They put the focus on the social welfare function that makes optimal the effective marginal tax rates schedule that corresponds to the redistribution system actually in place. This approach is the dual of the previous one. In the first case, wondering about the optimality of an actual redistribution system consists of comparing an optimal effective marginal tax rate schedule derived from some 'reasonable' social welfare function with the actual one. In the second case, it consists of checking whether the social welfare function implied by the actual redistribution schedule is in some sense 'reasonable', that is whether the marginal social welfare is everywhere positive and decreasing along the horizontal axis. The approach that is proposed is simply a way of "reading" the redistribution schedule, i.e. the average and marginal net tax curves that are commonly used to describe redistribution. This reading simply translates the observed shape of these curves into social welfare language. Comparing two redistribution systems or analyzing the reform of an existing system can thus be made directly in terms of social welfare. Instead of determining who is getting more out of redistribution and who is getting less, this reading of the marginal tax rate schedule informs directly on the differential implicit marginal social welfare weight given to one part of the distribution versus another. It is even conceivable that apparent anomalies in these preferences may be due to these assumptions being unsatisfactory. The observation of the effective average or marginal tax rate schedule may thus reveal more than social preferences. In some cases, it may suggest either that the tax schedule is inconsistent with optimality. But in others it may also reveal that some common assumptions on labor supply behavior or on the distribution of abilities are inconsistent. This seems equally useful information. They apply the methodology to the redistribution schemes and datasets of France, Spain and UK in 1995. The revealed marginal social welfare curves were found in agreement with standard optimal tax theory when the elasticity of labor supply was assumed to be low (0.1). Marginal social welfare was both positive and decreasing throughout the range of individual productivities, and therefore of individual utilities. However, marginal social welfare turned out to be negative at the very top of the distribution when the labor supply elasticity was assumed to be around the average of estimates available in the literature (0.5). This phenomenon was present in the three countries, although more pronounced in the case of France. The same result was also obtained with various specifications of household preferences between labor and consumption.
The correct specification of the subjective value assigned by the government to the elasticity of labor supply seems to be one of the major issues in applied optimal taxation. In this paper, we try to explore this issue. Starting from the observed distribution of gross and disposable income within a population and from the observed marginal tax rates as computed in standard
tax-benefit models, we show that, under a set of simplifying assumptions, it is possible to identify the individual utility function that would make the observed marginal tax rate schedule optimal under some assumption about social welfare preferences. This provides an alternative way of reading marginal tax rates calculations routinely provided by tax-benefit models. In that framework, the issue of the optimality of an existing tax-benefit system may be analyzed by considering whether the individual utility function associated with that system satisfies elementary properties. A detailed application is given in the case of France.
The structure of the paper is the following. Section 1 recalls the optimal taxation model and derives the duality relationship between the effective marginal tax rate schedule and the individual utility function in the simple case where social preferences about inequality are assumed to be of the Hyperbolic Absolute Risk Aversion (HARA) type. The second section discusses the empirical application of the preceding principle. In section 3, we characterize the individual utility function under a set of simple alternative assumptions about the degree of social aversion to inequality in the case of France. Last section is devoted to conclusions.

## 1. The Theoretical Framework

Mirless optimal income tax (or redistribution) model, in its canonical form, can be stated as follows.

$$
\begin{equation*}
\operatorname{Max} T() \int_{w_{0}}^{A} H[V[w, T()]] \cdot f(w) \cdot d w \tag{1.1}
\end{equation*}
$$

under contraints : $\quad\left(C^{*}, L^{*}\right)=\operatorname{Argmax}[U(C, L) ; C=w L-T(w L), L \geq 0]$

$$
\begin{align*}
& V[w, T()]=U\left(C^{*}, L^{*}\right)  \tag{1.3}\\
& A  \tag{1.4}\\
& \int_{w_{0}}^{A} T\left(w L^{*}\right) \cdot f(w) \cdot d w \geq B
\end{align*}
$$

In this optimization program, the function $U($ ), supposed increasing in consumption, decreasing in labor supply and quasi-concave in both arguments, represents the preferences of an agent between all the combinations of the real expenses of consumption (C) and work (L). The combination $\left(\mathrm{C}^{*}, \mathrm{~L}^{*}\right)$ is the preferred combination, under the budget constraint he/she confronts. W is the work unit income, that is to say the wage rate, if we suppose that L measures only the work duration or the " productivity" of an agent in a more general case. T ( ) is the tax paid. It is supposed to be only a function of the observed total income. V() is the utility level obtained effectively by the agent. Therefore it depends on his productivity and on the redistribution system $T()$. The distribution of productivities $f()$ in the population is defined within the interval ( $\mathrm{W}_{0}, \mathrm{~A}$ ). Finally, B is the budget that the government has to finance. From this point of view, the government is supposed to maximize the total social value of the individual utilities respect to the redistribution function T ( ). The relation between the private value and the social value of the individual utility is represented by the function H() , supposed to be concave.

The concavity of H() means that the government would like to redistribute part of the income of those who have a higher productivity and income to the people with low productivities. A way of obtaining this result is by increasing the tax T() according to income. But if it increases too quickly, the labor supply L* can decrease and the total amount to be redistributed can then being insufficient after considering the government budgetary
constraint. The trade off between efficiency - in other words a high level of labor supply and monetary income - and equity, or redistribution, constitute then the heart of the model. Under this general form, we can see that the optimal redistribution, represented by T() is a function of the individual labor supply behavior (as it proceeds from the preferences $U($ )), of the distribution of the productivities $f()$ and, finally, of the social welfare function $H()$.

The general solution of this problem is complex ${ }^{3}$. It is therefore rarely implemented without restrictions on individual preferences. A particular case, which has recently received a lot of attention, is the one where utility is separable with respect to consumption and work. The following function
$U(C, L)=C-B(L)$
where $\mathrm{B}(\mathrm{L})$ is a function decreasing and quasi-concave. It is easy to see that labor supply income elasticity is 0 .
With this particular specification of the preferences, we can easily show that the optimal marginal tax rate $t(\mathrm{w})$ of an agent whose productivity is w , is given by :

$$
\begin{equation*}
\frac{t(w)}{1-t(w)}=\left[1+\frac{B^{\prime \prime}(L) L}{B^{\prime}(L)}\right] \cdot\left[\frac{1-F(w)}{w \cdot f(w)}\right] \cdot\{1-S(w)\} \tag{3}
\end{equation*}
$$

with $S(w)=\int_{w}^{A} H^{\prime}(U) f(x) d x /[(1-F(w)]$
where $F()$ is the cumulative associated with $f()$ and $S(w)$ is the average marginal social value of the income of all the agents whose productivity is above w .
The interpretation of this equation is simple enough. Increasing the marginal tax rate of the agent with level of productivity w , the government both wins and loses income. It loses because the agents whose productivity is $w$ will decrease their labor supply. The corresponding loss is obtained by multiplying the left side of (3) by the term in $f(w)$ on the right - in other words the number of people who are at this level of productivity - and by the term $\mathrm{w} /\left(1+\frac{B^{\prime \prime}(L) L}{B^{\prime}(L)}\right)$ - in other words from how much the wage income decrease. The terms staying on the right could be interpreted as the additional income that the government obtains increasing the tax paid in the marginal bracket of the income corresponding to w by all those whose productivity is higher than $w$, that is to say $1-\mathrm{F}(\mathrm{w})$. This gain is corrected by the relative difference between the average marginal social value of the corresponding incomes and the average marginal social value of the income of those who effectively pay this supplementary tax.

Equation (3) is the starting point of our analysis. The key ingredients of the optimal tax schedule are the distribution of the productivities $f(w)$, the individual utility function $U(C, L)$

[^1]and the social welfare function $\mathrm{H}(\mathrm{U})$. The objective of our analysis is to use observations of effective marginal tax rate computed by micro simulation models as well as data on gross and disposable incomes of households in order to reveal the government subjective valuation of the individual utility functional form. To achieve this objective it is thus necessary to rewrite equation (3) in a way that can be directly estimated from data. To do it, three main problems have to be solved. Firstly we have to define the shape of social welfare function in a flexible way. Secondly, we have to transform the differential equation on w (that, as Bourguignon and Spadaro (2000) explain, it is not observable directly in data) in a differential equation on Y (that is immediately observable in data). Third, we have to define the theoretical framework of the inversion of the individual problem in order to get a condition taking into account the agent maximizing behavior implicit in observed gross and disposable income.

Concerning the specification of the social welfare function what has been done in this paper is to use an HARA specification. The social welfare function has been defines as:

$$
\begin{equation*}
H(U)=\frac{\left(U-U_{0}\right)^{\alpha}}{\alpha} \tag{5}
\end{equation*}
$$

where $U_{0}$ is an exogenous minimum level of welfare that government want guarantee and $\alpha$ is a parameter, taking values in the interval $(-\infty, 1]$, defining the concavity of the social welfare function and thus the level of aversion to inequality of the government. If $\alpha$ tends to one the government have is utilitarian; when $\alpha$ tends to $-\infty$ the government become rawlsian.

The second problem concerns mainly the term $\frac{1-F(w)}{w \cdot f(w)}$ in equation (3) (known as the inverse hazard ratio). As explained in the introduction there are various reasons for which the use of gross hourly wage as proxy of $w$ is inappropriate. A way to deal with this problem is by using the theoretical relation $(Y=w L)$ among gross labor income, effort and productivity implicit in agent individual utility maximization problem (1.2) and to derive an expression equivalent to the inverse hazard ratio, depending on Y.
In appendix 1 we demonstrate that the following identity holds:

$$
\begin{equation*}
\frac{1-F(w)}{w \cdot f(w)}=\frac{1-G(y)}{y \cdot g(y) \cdot\left[1+\varepsilon(L) \frac{1-t(y)+t^{\prime}(y) y}{1-t(y)-t^{\prime}(y) y \varepsilon(L)}\right]} \tag{6}
\end{equation*}
$$

where $g(y)$ is the distribution density of gross income, $G(y)$ is his cumulative, $t(y)$ is the effective marginal tax rate on gross income, $\mathrm{t}^{\prime}(\mathrm{y})$ his first derivative and $\varepsilon(\mathrm{L})$ is the compensated elasticity of labor supply. With the separability restriction imposed to the individual utility function (eq. 2) we have that:

$$
\begin{equation*}
\varepsilon(L)=\frac{U_{L}}{L U_{L L}}=\frac{B_{L}}{L B_{L L}} \tag{7}
\end{equation*}
$$

The condition on the inversion of the individual utility maximization problem (the third problem to deal with) is determinate in appendix 2. It give us a the following relation:

$$
\begin{equation*}
U(C, L)=C-B(L)=D Y-\int_{y \min }^{y}\left[\frac{1}{1-t(x)}\left(\frac{1}{\varepsilon(L)}+1+\frac{t(x) v(x)}{1-t(x)}\right)\right]^{-1} d x-\Theta \tag{8}
\end{equation*}
$$

where DY is the observed disposable income (used as proxy of consumption), $\Theta$ is a constant and $v()$ is the elasticity of effective marginal tax schedule with respect to gross income (i.e. $\left.v(y)=\frac{t^{\prime}(y) y}{t(y)}\right)$.

Using equation (6) and (8) we can rewrite equation (3) obtaining:

$$
\begin{align*}
& \frac{t(y)}{1-t(y)}=\left[1+\frac{1}{\varepsilon(L)}\right]\left[1+\varepsilon(L) \frac{1-t(y)+t^{\prime}(y) y}{1-t(y)-t^{\prime}(y) y \varepsilon(L)}\right]^{-1}\left[\frac{1-G(y)}{y g(y)}\right] * \\
& *\left[1-\frac{\left.\int_{y}^{y \max } H^{\prime}\left\{D Y-\int_{y \min }^{y}\left[\frac{1}{1-t(x)}\left(\frac{1}{\varepsilon(L)}+1+\frac{t(x) v(x)}{1-t(x)}\right)\right]^{-1} d x-\Theta\right\} g(z) d z\right]}{1-G(y)}\right] \tag{9}
\end{align*}
$$

that is a non linear equation in $\varepsilon(L)$ that can be solved numerically by fixed point algorithms starting from the observation of $t(y), v(y), g(y), G(y)$ and $D Y$. This equation is the consolidated form of two inversed optimal problems: the agent utility maximization and social welfare function utility maximization. The solution of this equation gives us the government subjective valuation of the analytical form of individual utility function. Its empirical implementation raises some technical problems that will be described in the next section.

## 2. Basic principles for empirical implementation

## a) Continuity and differentiability

The application of the modified optimal taxation formula (9), requires the knowledge of the continuous functions $f(w), t(w)$ and their derivatives. Unfortunately, what may be obtained from households data bases is a set of discrete observations of the gross income $Y_{i}$, the associated cumulative distribution function, $\mathrm{G}\left(Y_{i}\right)$ and the marginal tax rate function, $t\left(Y_{i}\right)$ The following operations permit to get an estimate of the derivatives of the function $\mathrm{g}(w)$ and $t(w)$.
(i) For any value of Y , obtain an estimate of the density function $\mathrm{g}(Y)$ and the effective marginal tax rate $t(Y)$ by kernel techniques defined over the whole sample of observations using a Gaussian kernel with an adaptive window ${ }^{4}$. These Kernel approximations are made necessary first by the need to switch from a discrete to a continuous representation of the distribution and the tax schedule and second by the heterogeneity of the population with

[^2]respect to some characteristics that may influence marginal tax rates and productivity estimates - household composition, for instance ${ }^{5}$.
(ii) Estimate the derivatives of $t(y)$ using again a kernel approximation computed over the whole sample. ${ }^{6}$
(iii) Compute the elasticity of $t(y)$ (i.e. the term $v(y)=y t^{\prime}(y) / t(y)$ ).
(iv) Solve the non linear equation (9) by fixed-point algorithms, computing $\varepsilon(L)$ for different specifications of parameters $\Theta, U_{0}$ and $\alpha$.

## b) Household size

It was assumed in the preceding section that all households had identical preferences and indirect utility functions. Practically, actual tax-benefit systems discriminate households according to various characteristics. Size and household composition are the main dimensions along which this discrimination is taking place. The issue thus arises of the way in which these characteristics can be implicitly or explicitly incorporated in the imputation of the social welfare function.
The results shown in the next sections are based on two extreme views. In the first one, the size of households is simply ignored in both the imputation of productivity and in tax optimisation. The implications of this choice are somewhat ambiguous. It may be seen in (13) that size affects productivity through two channels. On the one hand, a larger family - in terms of the number of potentially active adults - will generally have a greater gross labour income, which will contribute to a larger estimate of productivity. On the other hand, it will also face a different marginal tax rate. If the marginal tax rate is a decreasing function of household size for a given household income, as in most tax-benefit systems, then the preceding bias in the estimation of productivity will be attenuated.
The other extreme assumption consists of considering groups of households with the same size or the same composition as populations, which the redistribution authority seeks to maximize social welfare independently of each other. In other words, the optimal taxation problem involves finding an optimal tax-benefit schedule separately for each household group. This is implicitly done under some exogenous budget constraint, which makes the aggregate redistribution of income across the various groups of households exogenous.

## c) Households with zero income and households with apparently irrational behaviour

In presence of a guaranteed minimum income in a tax-benefit system, some households may find it optimal not to work at all. In the simple labor supply model above, this would correspond to a situation where the marginal tax rate is 100 percent. However, there is some ambiguity about these situations. Practically, some households are observed in pats of their budget constraint where the marginal tax rate is indeed 100 percent. There are two possible reasons for this. First, transitory situations may be observed where households have not yet converged towards their preferred consumption-labor combination. Second, transition periods are allowed by tax-benefit systems where beneficiaries of minimum income schemes may cumulate that transfer and labor income for some time so as to smoothen out the income path on return to activity.
The example of the French minimum income program (RMI) suggests the following way of handling the 100 marginal tax rate issues. People receiving the minimum income RMI and taking up a job lose only 50 percent of additional labor income during a so-called 'intéressement' period - 18 months. At the end of that period, however, they would lose all of

[^3]it if they wanted to keep benefiting from the RMI. Discounting over time, this means that the actual marginal tax rate on the labor income of an 'RMIste' is between 50 and 100 percent. Taking the middle of that interval, the budget constraint of that person thus writes:
$$
\mathrm{y}=\mathrm{RMI}+.25^{*} \mathrm{wL}
$$
if this person qualifies for the RMI -i.e. $\mathrm{wL}<$ RMI. But it is simply :
$$
y=w L \text { if } w L>\mathrm{RMI}^{7}
$$

This budget constraint is clearly convex. Therefore, there should be a range of labor incomes around the RMI where it would be irrational to be ${ }^{8}$. But, of course, some households are actually observed in that range, which is inconsistent with the model being used and/or the assumption made on the marginal tax rate associated with the RMI. One way of dealing with this inconsistency is to assume that all gross labor incomes are observed with some measurement error drawn from some arbitrary distribution. The measurement error is such that, without it, households would be rational and supply a quantity of labor outside the preceding range. This treatment of the data is analogous to the original econometric model describing the labor supply behavior of households facing a non-linear and possibly discontinuous budget constraint by Hausman (1985).

## Preliminary Results on a French Sample and Redistribution System.

The methodology, which has just been presented, has been applied to data from France. This application draws on a prototype version of the European tax-benefit model EUROMOD. This model simulates the tax-benefit systems of EU countries using representative household samples in each country ${ }^{9}$. To keep with the logic of the optimal taxation model, all households with zero income and with non-labor income, including pension and unemployment benefits above 10 per cent of total income were eliminated from the sample. Disposable income is computed with official rules for taxes and benefits instead of being taken directly from the data. Effective net marginal tax rates are calculated using the same rules. Following the last set of remarks in the preceding section, several applications have been run. They differ with respect to the value selected for the parameters $\Theta, U_{0}, \alpha$ and the choices made for handling household size.
The results obtained are presented in figure 1 and 2 . Figure 1 shows the revealed elasticity computed on the whole sample while figure 2 shows the same results for a sub sample of singles. In each figure there are three panels. Each of it contains the results obtained under some hypothesis on the level of minimum individual welfare that government want to guarantee to each individual (i.e. the terms $\mathrm{U}_{0}$ in equation 5 ) and for some given level of aversion to inequality (i.e the parameter $\alpha$ in equation 5) calibrated in order to get an average labor supply elasticity of 0.1 (red lines) and 0.5 (black lines). In each panel we shows the value of the parameters retained. It is important to remark that as $\alpha$ tends to one $(-\infty)$ the government becomes more utilitarian (rawlsian) The following table resumes the values of each parameter we presents in figure 1 and 2.

[^4]Table 1. Definition of different scenarios simulated and presented.

| $U_{0}=\Theta$ | Average $\varepsilon(\mathrm{L})$ | Singles | All Households |
| :---: | :---: | :---: | :---: |
| 0 | 0.1 | $\alpha=0.72$ | $\alpha=0.69$ |
|  | 0.5 | $\alpha=-0.22$ | $\alpha=-0.388$ |
| 5000 | 0.1 | $\alpha=0.75$ | $\alpha=0.71$ |
|  | 0.5 | $\alpha=-0.074$ | $\alpha=-0.31$ |
| 10000 | 0.1 | $\alpha=0.77$ | $\alpha=0.724$ |
|  | 0.5 | $\alpha=0.0 .08$ | $\alpha=-0.22$ |

We have calibrated the aversion to inequality in order to get an average elasticity of 0.1 and 0.5 because those values seems to be in the range of empirical estimations observed in econometric literature.
In figure 0 we show the main input of our analisys, i.e. the effective marginal tax rate in France in 1995, computed with the micro simulation model. The marginal tax rate curve has a U-shape. It is extremely high at the bottom of the distribution because of households facing marginal tax rates equal to 100 per cent due to the minimum guarantee ( $R M I$ ). Then, the marginal tax rate falls until a little after the median and then increases slowly with the progressivity of the income tax.

Figure 0. Effective marginal tax rate for France in 1995.


In general, the results obtained shows that the implicit government belief about the value of labor supply elasticity have an inverted U-shape. In all scenarios, it seems that French government assigns low values to the labor supply reactions of poor and rich and high values to the elasticity of the middle class. The maximum value is assigned to people around centile 10 (the elasticity takes values around 0.8 in the case of high aversion to inequality and 0.18 in the other one). The intuition is straightforward: government wants to redistribute income by using the labor income tax as instruments knowing that efficiency problems related to redistribution are higher when individual labor supply is highly sensitive to changes in net wage. If the observed effective marginal tax rate has an U-shape (it is the case for France) it follows that, ceteris paribus, efficiency problems are less important in the middle class range of incomes.

As expected, for a given redistribution system, if the government is supposed to be more inequality averse, the elasticity of labor supply implicit in the optimal income tax problem is higher. The intuition is the following. The observed marginal tax schedule, supposed to be the optimal one, is the best solution of the trade off between equity and efficiency concerns. If two governments, with different inequality aversion, solve the optimal tax problem in the same way, it means that they assign different weight to the efficiency problems. In particular, the more inequality averse is giving more importance to efficiency problems than the less averse; otherwise the optimal marginal tax rate would have been higher.
Another important issue to be stressed concerns the difference among the results for households treated as unit of analysis and the results for singles. Even if the shape of the elasticity curves do not changes, we observe that the revealed value of elasticity in the case of singles is slightly higher than for the case of the whole sample. This result may depend on the way government control for the redistribution among different types of households but this issue is not analyzed in our framework.

## Conclusion

This paper has explored an original side of applied optimal taxation. Instead of deriving the optimal marginal tax rate curve associated with some distribution of individual productivities, the analysis consists of retrieving the implicit government subjective valuations of the elasticity of labour supply that makes the observed marginal tax rates optimal under an arbitrary assumption about the social welfare function of the government.
The results show that, in the case of France, the government beliefs about the value of elasticity all along the income range is not uniform. Poor and rich people react less than middle class agents. This result is in line with the observation of an U-shaped marginal tax schedule with very high marginal rates at the bottom and at the top od the income distribution. An important lesson is the practical interest of reading actual tax-benefit systems through the government preferences that they reveal. It is customary to discuss and evaluate reforms in tax-benefit systems in terms of how they would affect some 'typical households' and more rarely what their implications are for the whole distribution, of disposable income. The instrument developed in this paper offers another interesting perspective. By drawing implicit elasticity curves consistent with a tax-benefit system before and after reforms, it is possible to characterize in a more precise way the distributional bias of the reform.

## APPENDIX 1: to be written.

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## APPENDIX 2: to be written. <br> $\qquad$

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Figure 1. Subjective Elasticity Curves: France, Total Sample

French Households; Uo = 0

centiles of gross income

French Households; Uo = 5000

centiles of gross income

French Households; Uo =10000


Figure 2. Subjective Elasticity Curves: France, Singles



[^0]:    ${ }^{1}$ See for instance Atkinson and Stiglitz (1980) or Tuomala (1990).
    ${ }^{2}$ See Blundell, Duncan and Meghir (1998).

[^1]:    ${ }^{3}$ See Atkinson and Stiglitz (1980).
    ${ }^{5}$ For the derivation of this equation see Atkinson and Stiglitz (1980) or Atkinson (1995), Diamond (1998), Piketty (1997). At the light of the previous note, this equation could simply be interpreted as a differential equation of the tax function $\mathrm{T}($ ). Its integration gives the redistribution function. The government budget constraint makes it possible to identify the constant of integration $T(0)$ that can be considered as a universal social contract tax (or a transfer if it is negative).

[^2]:    ${ }^{4}$ This choice was justified by the lack of observations and the increasing distance between them in the upper tail of the distribution. For technical details, see Hardle (1990).

[^3]:    ${ }^{5}$ Occupational status and home ownership are other sources of heterogeneity with respect to the tax system.
    ${ }^{6}$ The function $\Theta()$ itself may be approximated by Kernel techniques and then differentiated numerically. For technical details about the computations of kernel derivatives see Pagan and Ullah (1999, pag. 164).

[^4]:    ${ }^{7}$ All other benefits that may complement the RMI are ignored in this argument, but they are taken into account in the calculations made below.
    ${ }^{8}$ This interval may easily be computed using the preference function of households and the budget constraint described by the preceding conditional system. Note that it depends on the size and the socio-demographics characteristics of each household.
    ${ }^{9}$ See Immervoll et al. (2000).

