

Document de treball de l'IEB 2009/30

SUGGESTING AN ALTERNATIVE ELECTORAL PROPORTIONAL
SYSTEM. BLANK VOTES COUNT

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Fiscal Federalism

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treball de l'IEB

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ABSTRACT: I consider a two-party parliamentary election where parties compete on a quality (or valence) dimension. First I motivate why in such an election a voter may decide to cast a blank vote. Second I define a new voting system, inspired in the standard proportional representation system, where the percentage of blank votes is translated into vacant seats in the parliament. I analyze party competition assuming adapted versions of the models of “Bertand” and “Cournot”. I compare the equilibrium outcomes on parties’ quality and profits obtained with both the alternative proportional system and the standard one. I show that society and parties may have interests in conflict.

JEL Codes: D72

Keywords: electoral systems, blank voting, proportional representation systems, endogenous valence

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* I would like to thank Enriqueta Aragones for continuous support and guidance through this project.

1 Introduction

"A vote for David "None Of The Above" Gatchell is not a vote for David Gatchell – but rather a vote for "None Of The Above". A vote for a new election."

David Leroy Gatchell changed his middle name to "None of the Above" and ran in three elections in Tennessee as a candidate. Being a member of "None of the Above" he was dedicated that if winning he would declare a new election. "NOTA" as described in its manifesto is "A nonpartisan organization dedicated to enacting Voter Consent laws, giving voters the ballot option to reject all candidates for an office and to call for a new election with new candidates to fill that office". The idea is that a vote for "NOTA" is a protest vote that in this way is not a wasted vote.

In the same spirit and moving towards a proportional representation system, "Citizens for Blank Votes" compete in the Spanish elections with the slogan "For a Counting Blank Vote" and according to their manifesto they are dedicated in case of winning any seats in the parliament to leave them vacant. In the party's charter is stated that the party will stop existing when the Spanish electoral law will be reformed and will recognize blank votes by empty seats in the parliament.

More than thirty percent of the democratic countries around the world use as a system to elect their representatives in the parliament a proportional system. The system can vary across countries, with a typical example including a threshold that has to be achieved by a party in order to be represented. In its purest form such a system should translate the percentage of votes obtained by a party into the same percentage of representation in the parliament. In some systems there exists the choice of voting "None of The Above" or most common to cast a blank vote as way of expressing dissatisfaction towards the quality of the candidates. Although not considered as a party, blank votes are not represented in any parliamentary election and in most cases are actually just ignored and considered wasted votes.

The target of the paper is double. First to explain why someone would possibly cast a blank vote. Second, to examine the implications of an alternative proportional electoral system, with blank votes affecting the total number of seats in the parliament filled by the parties. The innovation of the system is the fact that the blank votes are not considered as "lost" but the percentage of blank votes is "represented" in the parliament by empty seats. In this way, the seats filled by the parties may be reduced. This can be considered as harmful for the parties although the legislation and governing procedure takes place, taking into account only the filled seats. In our case the parliament size might decrease.

We will begin the analysis with the simplest possible model that could give motivation to the voters to participate in an election and maybe decide to vote blank. We will assume expressive voting and we will consider a two parties election and a large electorate, where the probability of a voter being pivotal is zero. Each party will compete in the election announcing as a platform its ideal

policy which they will implement in case of winning. The competition of the parties will refer to valence characteristics and the most important feature of the model is that the number of seats occupied in the parliament will have a direct impact in the payoff functions of the parties. We will consider two versions of political competition which are adapted versions of the standard "Bertrand" and "Cournot" models of competition.

The parties' quality or valence characteristics were first introduced by Stokes (1963) and it refers to an exogenous non-policy characteristic of the candidate that is equally valued by all voters. Its interpretation may vary and it may refer to either candidate's charisma or intelligence, or incumbency advantage based on name recognition or reputation. We consider a party's quality as his valence characteristic. In our case the quality of the party will refer to the effort that the representatives of the parties in the parliament put. In terms of analyzing electoral systems similar interpretation of valence as quality has been followed by Iaryczower and Mattozzi (2008). They consider proportional representation and focus their analysis on the relationship between the parties' quality and the number of candidates.

In this paper the quality or valence characteristics of the parties is assumed to be endogenous. We follow the approach analyzed by Zakharov (2005). He was the first one to consider parties' quality or valence characteristics as endogenous, and he described a model in which candidates could increase their valence characteristics at a cost.

Regarding valence characteristic the literature has developed much considering this characteristic as exogenous and analyzing Downsian models of candidate location when one of the candidates is facing an exogenous valence advantage. The main works considering an exogenous valence advantage over one candidate have been done by Groseclose [2001] and Aragones and Palfrey [2002]. In our analysis though we will consider a fixed candidate location and an endogenous decision of valence characteristic.

In the following analysis after motivating the possibility of a voter to cast a blank vote we show the conditions under which the alternative system suggested in this paper can lead to higher level of quality of the parties considering two distinct versions of parties characteristics. The main results consider two parties facing the same cost for increasing their quality and we prove that if parties are office motivated ("Bertrand" competition) the standard proportional system is better for the society if parties do not cooperate. If we assume that parties cooperate then it would be better for the society to introduce the alternative electoral system. If we assume that parties are pure monetary profit maximizers ("Cournot" Competition) we identify again when the alternative system would be preferred by the society and the conditions we identify seem to be reflecting the reality. In both cases we show that when society benefits by introducing the alternative system, this is harmful for the parties since it leads to lower profits.

In section 2 we present the model and we define formally the alternative electoral system. Moreover we describe the parties and voters objectives. In Section 3 we analyze the voter's decision and we explain why a voter would cast a blank vote. In section 4 we analyze the parties' decision considering both

versions of competition, obtaining both general results and results related to specific further assumptions.

2 The Model

2.1 Electorate

Voters have four possible alternatives. Vote for one of the two parties (L, R), abstain (A) or cast a blank ballot (B). The policy space is assumed to be $X = [0, 1]$. Each voter is assumed to have an ideal policy x_i , where $x_i \in [0, 1]$

Voters will evaluate not only the candidates' proposed policy but also a non-policy envelope of the parties $\theta_j \in [0, 1]$. In the model we develop, this non-policy issue which is known as valence characteristic, is more intuitive if understood as quality of the party rather than a charisma or religion beliefs of the candidates, as sometimes referred in the literature.

Individuals that proceed to vote will be considered to have a utility function that is increasing in θ_j and is decreasing as a result of supporting a platform far from one's ideal point (this can be seen as a cost of supporting a party which is far from the individual's ideal point).

Since we have assumed a big electorate the possibility of affecting the election outcome will be zero. So following the idea introduced by Riker and Ordeshook (1968) the voters get utility by voting not only because of influencing the election outcome (which in our case is zero) but also because of a consumption benefit of voting d_i . The cost of voting associated to the act of voting will be c_i . In the model we assume that individuals are motivated by the need to make a statement. In other words they take in consideration their cost and social benefit of voting and while voting they are not policy oriented but just want to express themselves. The assumption of expressive voting is often considered as extreme but is compatible with the spirit of the paper, since in the suggested electoral system blank ballots will be used as a mean of dissatisfaction towards the political parties. In other words even voting blank, although not affecting the implemented policy is a way in which someone expresses his political beliefs.

In this way we can define the voters utility as follows:

$$U_i(s) = \begin{cases} \theta_L - |x_L - x_i|^2 + d_i - c_i & \text{if } s = L \\ \theta_R - |x_R - x_i|^2 + d_i - c_i & \text{if } s = R \\ d_i - c_i & \text{if } s = B \\ 0 & \text{if } s = A \end{cases}$$

Claim 1 *Given the above utility specification individual i with ideal point x_i participates in the election if and only if :*

- a) $d_i \geq c_i$
- b) $d_i < c_i$ and $\theta_L - |x_L - x_i|^2 \geq c_i - d_i$

We can describe the members of the electorate that participate in the election as consisting of two groups of individuals. The first part of the electorate that participates is that for which the consumption benefit of voting is larger than the cost of voting (i.e. $d_i \geq c_i$) and always participates in the election. So those are the people who have the need to express their political beliefs. In case they are not satisfied by none of the parties they will cast a blank vote but will not abstain.

The second motivation for people to vote even this is not the case (i.e. $d_i < c_i$), is if the quality of a party (θ_j) is high enough and the ideology of a party close enough to the voter's ideal point such that it exceeds the negative impact of the cost of voting (i.e. $\theta_L - |x_L - x_i|^2 \geq c_i - d_i$).

Or examining the same coin from the other side given the above utility specification individual i with ideal point x_i will abstain if and only if $d_i < c_i$ and $\theta_L - |x_L - x_i|^2 < c_i - d_i$.

From now on we will take in consideration the part of the electorate that participates in the election. So from now on the voting decision will consist of three alternatives and is as follows:

$$s = \begin{cases} L & \text{if } \theta_L - |x_L - x_i|^2 \geq \theta_R - |x_R - x_i|^2 \text{ and } \theta_L - |x_L - x_i|^2 > 0 \\ R & \text{if } \theta_R - |x_R - x_i|^2 > \theta_L - |x_L - x_i|^2 \text{ and } \theta_R - |x_R - x_i|^2 > 0 \\ B & \text{if } \theta_L - |x_L - x_i|^2 \leq 0, \theta_R - |x_R - x_i|^2 \leq 0 \end{cases}$$

So far and for the first results regarding voting there is no reason to add any assumptions regarding the distribution of ideal points of individuals. Moving from the whole electorate and the four alternatives to the part of the electorate that participates in the election and the three alternatives can be done without loss of generality. If we want to be more precise regarding distributions of whole electorate and of the members who proceed in the election we assume that whatever the distribution of ideal points of density one of the whole electorate, the ones who will decide to participate in the election have exactly the same distribution.

Notice that with our setup a voter that has an ideology closer to one of the parties may vote for the other if it is of higher quality. People will decide to vote blank if they are not satisfied by the combination of policy and non-policy characteristics of both parties (i.e. $\theta_j - |x_j - x_i|^2 \leq 0$ for both $j = L, R$). The difference of the above setup regarding the literature is that people don't vote for the party which is the best among the two if they consider both unattractive but they rather prefer to cast a blank ballot.

2.2 Electoral Outcome and Constitution of the Parliament

In the paper we are examining two distinct systems. The standard proportional representation system (SPR) and the alternative proportional system (APR) in which blank votes are represented in the parliament by vacant seats.

Regarding the vote share we will denote by v_j the vote share of alternative j . This will be the percentage of eligible voters that decide to participate in the election and vote for alternative j . Since now we are referring to the fraction of people who participate in the election and we do not take in consideration the people who decide to abstain as described above we can notice here that $v_L + v_R + v_B = 1$.

Regarding the seat share in the constitution of the parliament we will denote q_j the seat share of alternative $j = L, R, B$ independent of the system. This notation will be used when we are describing issues that are relevant under both systems.

To differentiate the seat share between the two systems we will denote by s_j the seat share of alternative j under the SPR system.

Finally, we will denote by b_j the seat share of alternative j under the APR system that takes in consideration the blank votes.

Given the above seat and vote share we can define formally the two systems:

Definition 2 *A SPR is a system that given the electoral result then the votes are represented into seats in the parliament as follows:*

$$s_L = \frac{v_L}{v_L + v_R} \quad s_R = \frac{v_R}{v_L + v_R} \quad s_B = 0$$

Definition 3 *An APR is a system that given the electoral result then the votes are represented into seats in the parliament as follows:*

$$b_j = \frac{v_j}{v_L + v_R + v_B} = v_j.$$

As defined above the two systems differ in the sense that for $v_B \neq 0$ in the SPR blank votes are considered as wasted since $s_B = 0$. On the contrary under the APR we have $b_B = v_B$. This difference is the reason why the SPR which is often called "full" is criticized in this paper as "fool" since a pure representation should take in consideration the blank votes.

If the parties were competing with the same characteristics under both electoral systems then they would get the same vote share but different seat share under the two systems and it would always hold that $b_j \leq s_j$ for $j = L, R$.

Claim 4 *The two systems are equivalent if and only if $v_B = 0$.*

2.3 Parties

We will consider two distinct versions of payoff functions for the parties. The common feature will be that in both cases the parties will obtain utility by the number of seats they occupy in the parliament. The two versions will determine whether the parties will compete in an adapted version of "Cournot" or "Bertrand" political competition.

In both cases, the two parties of the model have an ideal policy $x_L, x_R \in [0, 1]$ with $x_L \leq x_R$. In the analysis we will consider that the parties compete in the election by choosing as platforms their ideal policies. Their political competition will refer to their valence characteristics θ_j . We assume that it is costly for each party to increase its valence characteristic and that the parties are benefited by the number of seats they occupy in the parliament.

Definition 5 *The monetary profits of party j will be given by the following function:*

$$\pi_j(\theta_L, \theta_R, x_L, x_R) = f(q_j(\theta_L, \theta_R)) - c_j(\theta_j)$$

where the function $c_j(\theta_j)$ is the cost function of each party and will be satisfying $c_j(0) = 0, c'_j(\theta_j) > 0$ and $c''_j(\theta_j) \geq 0$. Notice that an increase in valence means an additional cost for the party (since $c'_j(\theta_j) > 0$) and the cost of one unit of valence is increasing in levels of valence because of the concavity of the cost function (i.e. $c''_j(\theta_j) \geq 0$). Moreover quality zero doesn't imply any cost.

Regarding $f(q_j(\theta_L, \theta_R))$ is a function reflecting the benefit the parties obtain through their members by representation in the parliament. Function $f_j(q_j)$ will be satisfying $f(0) = 0, f'(q_j) > 0$ and $f''(q_j) \leq 0$. So parties will get higher profits the more seats they fill in the parliament but the profit for each seat will be decreasing in the number of seats.

The monetary profits can be thought as the total benefit of the members in the parliament discounted by the cost of the effort they put. The benefit for the members can reflect the salaries or possibly other social benefits such as recognition that the representatives in the parliament enjoy. Regarding the effort they put it is costly and increases the quality of the party. If the monetary profits are equal to zero this can be understood as all members of the party elected in the parliament putting their highest effort possible for a high quality of the party (i.e. high level of valence) and in this way "reinvesting" through their effort all the benefits they get by occupying a seat in the parliament.

As stated above we will consider two payoff functions for the parties and the above defined monetary profits will be the common feature.

When referring to *Bertrand competition* the utility function of the parties will be given by:

$$U_j(\theta_L, \theta_R, x_L, x_R) = \begin{cases} W + f(q_j(\theta_L, \theta_R)) - c_j(\theta_j) & \text{if } q_j > q_{-j} \\ T + f(q_j(\theta_L, \theta_R)) - c_j(\theta_j) & \text{if } q_j = q_{-j} \\ f(q_j(\theta_L, \theta_R)) - c_j(\theta_j) & \text{if } q_j < q_{-j} \end{cases}$$

where $W > T > f(1/2)$

The above utility has to be maximized subject to a "budget constraint" which is the non-negative monetary profit condition. This means that party can not invest the possible value of holding office (W or T) in order to increase the quality of the party. The investment in order to increase the quality of the party has to be done by the members of the parliament . As described above this will be achieved through the members investing in costly effort the benefits they obtain by holding a seat in the parliament.

The above specification for the parties is a utility function that is affected in a separable way by the result of the election (through W and T) and the monetary profits.

The assumption $W > T > f(1/2)$ guarantees that the most important factor in the party's decision is the electoral outcome. So with the above utility specification both parties are trying to obtain the best electoral result possible by satisfying their budget constraint. Moreover the maximization of the above utility implies that after guarantying the best electoral result the parties are maximizing their monetary profits.

Notice that as we have specified the utility of the parties this can be related to the literature of duopoly when the two firms compete a la "Bertrand". In the standard Bertrand model of oligopoly (1883) we have two firms choosing simultaneously their prices and after committing to those prices they supply the quantity demanded by the market. Since the two firms are afraid of staying out of the market in case they set a price higher than their competitor their target is not to be left out of the market.

In the same sense in our model as we have specified the utility of the parties the parties have to choose a quality level that will guarantee them that they will not loose the election. They will first try to win or possibly tie, always taking in consideration the non-negative monetary profits condition. After they guarantee that their choice of quality gives them the best electoral result possible then they try to maximize their monetary profits.

When referring to *Cournot competition* the utility function of the parties will be given by:

$$U_j(\theta_L, \theta_R, x_L, x_R) = \pi_j(\theta_L, \theta_R, x_L, x_R) = f(q_j(\theta_L, \theta_R)) - c_j(\theta_j)$$

In this case the parties will be considered to be pure monetary profit maximizers. The result of the election now is not giving extra utility to the parties (since $W = T = 0$). The difference of the "Cournot" version of payoff function with the one we will refer as "Bertrand" is that with the latter utility specification parties are not afraid of loosing the election and staying out of the governing procedure. This can be considered as an adapted version of "Cournot" competition since both parties will coexist and as the results of the analysis later will show, both parties will be making positive monetary profits. As in standard "Cournot" competition the maximization of the utility will be taking in consideration the characteristics of the competitor.

2.4 The Game

We will consider a complete information setup and the election process will consist of the following two stages:

Stage 1: Both parties, having observed the ideologies and the cost function of the other party, choose simultaneously their level of the non-policy characteristics θ_L, θ_R .

Stage 2: All members of the electorate observe the ideologies and the levels of valence of both parties. They decide whether to vote and if they do so for which of the three alternatives to cast a vote.

Stage 3: The game ends and given the result of the election both parties obtain their profits (only monetary in case of "Cournot"- monetary and result of election in case of "Bertrand")

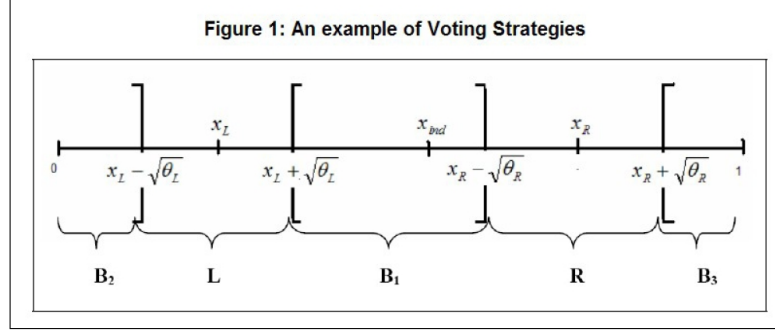
3 Voters' Decision

Beginning the analysis from the voters decision (stage 2) and after the individuals who proceed to vote have observed the parties' ideal policies and their valences we get the following conditions for the $[0,1]$ interval.

Let $\mathbf{x}_{ind} = \frac{1}{2}(\mathbf{x}_R + \mathbf{x}_L) + \frac{\theta_L - \theta_R}{2(x_R - x_L)}$ the indifferent voter. This is the voter that in case that voters would choose only between the two parties all individuals located at the left of him would vote for party L and all voters on the right would vote for R. Now that we have the third alternative to cast a blank ballot the strategies of the voters are as follows:

$$s(\mathbf{x}_R, \theta_R, \mathbf{x}_L, \theta_L) = \begin{cases} L & \text{if } \mathbf{x}_i \leq \mathbf{x}_{ind} \text{ and } \mathbf{x}_i \in (\mathbf{x}_L - \sqrt{\theta_L}, \mathbf{x}_L + \sqrt{\theta_L}) \\ R & \text{if } \mathbf{x}_i > \mathbf{x}_{ind} \text{ and } \mathbf{x}_i \in (\mathbf{x}_R - \sqrt{\theta_R}, \mathbf{x}_R + \sqrt{\theta_R}) \\ B & \mathbf{x}_i \in [0, \mathbf{x}_L - \sqrt{\theta_L}] \cup [\mathbf{x}_L + \sqrt{\theta_L}, \mathbf{x}_R - \sqrt{\theta_R}] \cup [\mathbf{x}_R + \sqrt{\theta_R}, 1] \end{cases}$$

An example of the above strategies is represented in figure 1.



For the example depicted in figure 1 and a uniformly distributed electorate we would have $v_L = 2\sqrt{\theta_L}$ and $v_R = 2\sqrt{\theta_R}$. the share of votes of each party. As it is graphically depicted the parties can increase their support by increasing their valence characteristic. In figure 1, voters that are not so close to the party's ideal point need higher quality of the party in order to be attracted and vote for them.

Proposition 6 *In a two party election, in the model described above, whatever the distribution of voters there will exist voters expressing their political beliefs by casting a blank vote if at least one of the three following conditions are satisfied:*

- a) $B_1 \neq 0$ if and only if $\sqrt{\theta_R} + \sqrt{\theta_L} < x_R - x_L$
- b) $B_2 \neq 0$ if and only if $\theta_L < x_L^2$ and $\theta_R < x_R^2$
- c) $B_3 \neq 0$ if and only if $\theta_L < (1 - x_L)^2$ and $\theta_R < (1 - x_R)^2$

Notice that in the figure 1 we have $B_k \neq 0$ for all $k = 1, 2, 3$

Proof:

First we show the voting strategies as stated and depicted in figure 1:

The first possibility we consider is for voter i to vote for party L . This will happen if $\theta_L - |x_L - x_i|^2 \geq \theta_R - |x_R - x_i|^2$, which means that party L is preferred by i to party R given the ideal point of the voter. Moreover it must hold that $\theta_L - |x_L - x_i|^2 > 0$ since if this was not the case then he would prefer to vote blank, since the quality and the ideology of the party would give the voter a negative utility.

We have that:

$$\theta_L - |x_L - x_i|^2 \geq \theta_R - |x_R - x_i|^2 \text{ which implies}$$

$$x_i \leq \frac{1}{2}(x_R + x_L) + \frac{\theta_L - \theta_R}{2(x_R - x_L)} = x_{ind}$$

Moreover $\theta_L - |x_L - x_i|^2 > 0$ which implies
 $x_i^2 + x_L^2 - 2x_Lx_i - \theta_L < 0$

Solving the above as an equation for x_i we get
 $x_i = \frac{2x_L \pm \sqrt{4x_L^2 - 4(x_L^2 - \theta_L)}}{2}$ which implies $x_i = x_L \pm \sqrt{\theta_L}$

Hence $\theta_L - |x_L - x_i|^2 > 0$ implies $x_i \in (x_L - \theta_L, x_L + \theta_L)$

In other words the voter will choose to vote for party L if he is on the left of the indifferent voter and he is located in the zone around the ideal point of party L that contains the individuals who are "satisfied" by the non-policy and policy characteristics of the party.

In the same way in order for voter i to vote for party R it must hold that:
 $x_i \in (x_R - \theta_R, x_R + \theta_R)$ and $x_i > \frac{1}{2}(\mathbf{x}_R + \mathbf{x}_L) + \frac{\theta_L - \theta_R}{2(x_R - x_L)} = x_{ind}$

Finally individual i will vote B if $\theta_L - |x_L - x_i|^2 \leq 0$ and $\theta_R - |x_R - x_i|^2 \leq 0$. This means that he is unsatisfied by both parties.

From the analysis above we have that $\theta_L - |x_L - x_i|^2 \leq 0$ implies $x_i \notin (x_L - \theta_L, x_L + \theta_L)$

Similarly $\theta_R - |x_R - x_i|^2 \leq 0$ implies $x_i \notin (x_R - \theta_R, x_R + \theta_R)$

And the above two conditions can be summarized as follows: Voter i casts a blank vote if $\mathbf{x}_i \in [0, \mathbf{x}_L - \sqrt{\theta_L}] \cup [\mathbf{x}_L + \sqrt{\theta_L}, \mathbf{x}_R - \sqrt{\theta_R}] \cup [\mathbf{x}_R + \sqrt{\theta_R}, 1]$ which in figure 1 are depicted as regions B_2, B_1, B_3 respectively.

We have shown why each voter would choose each one of the three alternatives.

Now in order to have some voters choosing a blank vote the three possibilities are:

- a) If $\mathbf{x}_R - \sqrt{\theta_R} > \mathbf{x}_L + \sqrt{\theta_L}$ which implies $\sqrt{\theta_R} + \sqrt{\theta_L} < \mathbf{x}_R - \mathbf{x}_L$ then $B_1 \neq 0$
- b) If $\mathbf{x}_L - \sqrt{\theta_L} > 0$ which implies $\theta_L < \mathbf{x}_L^2$ and $\mathbf{x}_R - \sqrt{\theta_R} > 0$ which implies $\theta_R < \mathbf{x}_R^2$ then $B_2 \neq 0$
- c) If $\mathbf{x}_L + \sqrt{\theta_L} < 1$ which implies $\theta_L < (1 - \mathbf{x}_L)^2$ and $\mathbf{x}_R + \sqrt{\theta_R} < 1$ which implies $\theta_R < (1 - \mathbf{x}_R)^2$ then $B_3 \neq 0$ ■

4 Parties' Decision

Now we proceed to the analysis of the first stage of the game. This is the stage when parties decide their levels of valence characteristics.

In the above game the actual players are the two parties. The voters just observe the decisions of the parties and decide whether to vote and if so for which of the alternatives. The political equilibrium will be the Nash equilibrium of the valence choosing game by the parties.

Definition 7 We say that (θ_L^*, θ_R^*) will consist a political equilibrium if:

$$\begin{aligned} U_L(\theta_L^*, \theta_R^*) &\geq U_L(\theta_L, \theta_R^*) \text{ for every } \theta_L \in [0, 1] \text{ and} \\ U_R(\theta_L^*, \theta_R^*) &\geq U_R(\theta_L^*, \theta_R) \text{ for every } \theta_R \in [0, 1] \end{aligned}$$

In other words, in equilibrium and given the distribution of the electorate no party will have any incentives to deviate from levels θ_L^* and θ_R^* respectively. Remember that the utility of each party consists of two elements. Their first target is the result of the election while their secondary is to maximize their profits.

4.1 Bertrand Competition

4.1.1 Full Symmetry

Definition 8 We will call full symmetry the case that both parties have the same cost function, both parties are symmetrically located around $1/2$ (i.e. $\mathbf{x}_R + \mathbf{x}_L = \mathbf{1}$) and the distribution of the individuals' ideal points is symmetric around $1/2$.

Proposition 9 In case of full symmetry, under both electoral systems there will exist a unique political equilibrium $\theta_L^* = \theta_R^*$ such that $\pi_L(\theta_L^*, \theta_R^*) = \pi_R(\theta_L^*, \theta_R^*) = 0$ and $U_L(\theta_L^*, \theta_R^*, x_L, x_R) = U_R(\theta_L^*, \theta_R^*, x_L, x_R) = T$.

Proof. The proof is intuitive. Since each party's first target is to win the election for every value of $\theta_j < \theta_j^*$ the other party will have incentives to set $\theta_{-j} = \theta_j + \varepsilon$ and win the election. The above pair (θ_L^*, θ_R^*) will consist the unique equilibrium since the budget constraint will be binding (i.e. monetary profits are zero) and none of the parties can keep increasing his level of valence.

■

So under both systems in case of full symmetry we will have both parties choosing the same level of valence and tying in the election. Moreover they will have zero monetary profits which means that all the positive gains they have by filling seats in the parliament is invested in increasing their quality characteristics. This result is of the same spirit as of the Bertrand paradox that arises in the two firms competing in a duopoly, when although the firms are only two they end up in zero profits equilibrium.

Comparing the two systems under Bertrand Competition and Full Symmetry

Proposition 10 *In the case of full symmetry let $(\theta_L^{*SPR} = \theta_R^{*SPR})$ and $(\theta_L^{*APR} = \theta_R^{*APR})$ be the equilibrium values of valence levels under the standard and the alternative proportional representation system respectively. Then $\theta_j^{*SPR} \geq \theta_j^{*APR}$ for $j = L, R$.*

Proof: Since in both systems parties will keep increasing the value of their valence characteristic at the level that will lead to profits equal to zero we will have:

$$\begin{aligned} \pi_j(\theta_L^{*SPR}, \theta_R^{*SPR}) &= \pi_j(\theta_L^{*APR}, \theta_R^{*APR}) = 0 \text{ which implies} \\ f(s_j(\theta_L^{*SPR}, \theta_R^{*SPR})) - c_j(\theta_j^{*SPR}) &= f(b_j(\theta_L^{*APR}, \theta_R^{*APR})) - c_j(\theta_j^{*APR}) \end{aligned}$$

Now in both systems the parties will share equally the votes so we have that in both cases $v_L = v_R$ and for the standard system $s_j = 1/2$ then we get that:

$$\begin{aligned} f(1/2) - c_j(\theta_j^{*SPR}) &= f(b_j(\theta_L^{*APR}, \theta_R^{*APR})) - c_j(\theta_j^{*APR}) \text{ which implies} \\ f(1/2) - f(b_j(\theta_L^{*APR}, \theta_R^{*APR})) &= c_j(\theta_j^{*SPR}) - c_j(\theta_j^{*APR}) \end{aligned}$$

By definition of our suggested system $b_j \leq s_j = 1/2$ and given that $f' > 0$ we have:

$$\begin{aligned} f(1/2) - f(b_j(\theta_L^{*APR}, \theta_R^{*APR})) &\geq 0 \text{ which implies } c_j(\theta_j^{*SPR}) - c_j(\theta_j^{*APR}) \geq 0 \\ \text{and given that } c'_j(\theta_j) > 0 &\text{ we get that } \theta_j^{*SPR} \geq \theta_j^{*APR} \blacksquare \end{aligned}$$

The implication of the above proposition is that in case of full symmetry and under SPR the quality of the two parties will be higher than under APR. This result by first look may seem contradicting the spirit of the paper. Notice that for the above to be true we have both parties competing and having always as a primary target not to loose the election even though their final monetary profits are zero. So in case this is true it makes sense that the more members they have in the parliament the more they will invest in quality.

It would be interesting though to see the implications of the alternative system in case we let the parties cooperate. In terms of the literature this can

be seen as moving from the Bertrand competition towards models of cartels. As we saw above in the unique equilibrium of the standard system the parties will decide to choose the same level of quality and this will finally lead to zero profits. Notice though that all values $\theta_L^{SPR} = \theta_R^{SPR} \in [0, \theta_j^{*SPR})$ may lead to positive profits for both parties. So if parties can cooperate and commit to a certain quality level they maximize their profits for the extreme values of valence equal to zero.

On the other hand this will not be the case by the alternative system. Notice that for zero values of valence all the seats in the parliament will be empty. Now the values of valence chosen will be a result of profit maximization that will satisfy the profit maximization first order condition:

$$\frac{\partial f}{\partial b_j} \frac{\partial b_j}{\partial \theta_j}(\theta_L^{*APR}, \theta_R^{*APR}) = c_j'(\theta_j^{*APR}) \text{ with } b_j \leq 1/2$$

This implies that the parties will agree to choose a level of valence such that a further increase in their quality will lead to an extra benefit that will be smaller than the cost they have to pay in order to increase their quality. Moreover they will stop increasing the quality in case they reach the maximum share of seats they can get $b_j = 1/2$.

To conclude we obtain a characterization of the political equilibrium for the case of full symmetry and two parties competing a la "Bertrand" that can be summarized as follows: In case the parties do not cooperate the quality of the parties will be higher under the existing system. On the other hand if we assume that parties cooperate the alternative system will lead to higher levels of quality.

4.1.2 Moving away from the full symmetry

In this part, we will assume that the parties face different cost functions. Without loss of generality we assume that each unit of quality is less costly for party L. So party L can increase his quality easier, or in other words can obtain the same quality with less effort (i.e. $c_L(\theta) < c_R(\theta)$ for each θ).

Proposition 11 *Under both systems assuming asymmetric costs that are in favor of party L if there exist a "Bertrand" equilibrium (θ_L^*, θ_R^*) it will be unique and must satisfy the following conditions:*

- a) $\frac{\partial f}{\partial q_L} \frac{\partial q_L}{\partial \theta_L}(\theta_L^*, \theta_R^*) = c_L'(\theta_L^*)$
- b) $\frac{\partial f}{\partial q_R} \frac{\partial q_R}{\partial \theta_R}(\theta_L^*, \theta_R^*) = c_R'(\theta_R^*)$
- c) $\theta_L^* \in (\theta_R^0, 1]$

Proof:

In equilibrium both parties must be maximizing the monetary profits. So the first order condition of each party has to be satisfied. Hence it must hold that:

$$\frac{\partial f}{\partial q_L} \frac{\partial q_L}{\partial \theta_L}(\theta_L^*, \theta_R^*) = c'_L(\theta_L^*) \text{ and } \frac{\partial f}{\partial q_R} \frac{\partial q_R}{\partial \theta_R}(\theta_L^*, \theta_R^*) = c'_R(\theta_R^*)$$

Profit maximization is not enough though. Because if $\theta_R = \theta_L^* + \varepsilon$ is affordable for party R then it has incentives to choose this level and win the election. This would lead to higher profits since the party would obtain by winning higher utility since $W + f(q_R(\theta_L^*, \theta_R)) - c_R(\theta_R) > f(q_R(\theta_L^*, \theta_R^*)) - c_R(\theta_R^*)$. So for an equilibrium to exist we need $\theta_L^* \in (\theta_R^0, 1]$ where θ_R^0 is the highest possible level of quality that is affordable to party R for level θ_L^* chosen by party L. ■

The intuition of this proposition is that when one party has an advantage regarding the cost of valence then this party can always win the election. So what we need in order for a "Bertrand" equilibrium to exist is that party R knows that he can not win the election (since $\theta_R = \theta_L^* + \varepsilon$ is not affordable) and both parties maximize the monetary profits, with party L having guaranteed that will win the election and party R knowing that can not afford a better result.

After having the above results for the case of full symmetry we will build an example in order to search for existence of "Bertrand" equilibrium for the two systems and allowing asymmetric costs of the two parties.

An example

In this section we will consider the simplest possible case that we can examine and characterize "Bertrand" equilibria under both systems. One important assumption we will consider is that under both systems there will exist a fraction of voters that will not be satisfied by the quality of the parties. Assuming that implies that quality is expensive enough for both parties such that they can not guarantee the satisfaction of all voters.

Regarding the voters distribution we will assume that the members that are deciding to participate in the election are uniformly distributed.

Regarding the parties we will assume:

$\mathbf{x}_L = 0, \mathbf{x}_R = 1$, which means that the parties are located at the extreme points of the line

$c_j(\theta_j) = a_j \theta_j, a_L < a_R$. In this example both parties have linear cost functions with lower cost per unit for the left party. Here without loss of generality we assume that the left party can increase its quality in lower cost.

$f_j(q_j) = q_j$, the function that evaluates the seat share of each party.

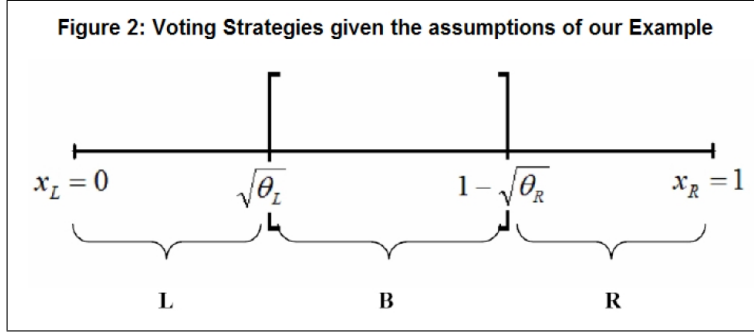
The monetary profits of the party will be given by the following function:

$$\pi_j(\theta_L, \theta_R) = q_j - a_j \theta_j$$

Under these assumptions we have that the indifferent voter is $\mathbf{x}_{ind} = \frac{1}{2} + \frac{\theta_L - \theta_R}{2}$ and we obtain the following voting strategies:

$$s(\mathbf{x}_R, \theta_R, \mathbf{x}_L, \theta_L) = \begin{cases} L & \text{if } \mathbf{x}_i < \mathbf{x}_{ind} \text{ and } \mathbf{x}_i \in [0, \sqrt{\theta_L}] \\ R & \text{if } \mathbf{x}_i > \mathbf{x}_{ind} \text{ and } \mathbf{x}_i \in (1 - \sqrt{\theta_R}, 1] \\ B & \mathbf{x}_i \in [\sqrt{\theta_L}, 1 - \sqrt{\theta_R}] \end{cases}$$

which are depicted in figure 2.



In figure 2 we have region B non empty since we are assuming $\sqrt{\theta_R} + \sqrt{\theta_L} < 1$. Notice that this coincides with region B_1 of the figure 1. Remember that in order to have $B_1 \neq \emptyset$ the condition needed was $\sqrt{\theta_R} + \sqrt{\theta_L} < \mathbf{x}_R - \mathbf{x}_L$ which in our case implies $\sqrt{\theta_R} + \sqrt{\theta_L} < 1$. This condition will be imposed since we are assuming that by nature the parties are not capable to satisfy all the voters.

In this example and assuming a uniformly distributed electorate we obtain that the share of votes of each party is $v_L = \sqrt{\theta_L}$ and $v_R = \sqrt{\theta_R}$.

That corresponds to the following seat shares under the two electoral systems considered:

$$\text{SPR: } s_L = \frac{\sqrt{\theta_L}}{\sqrt{\theta_L} + \sqrt{\theta_R}}, s_R = \frac{\sqrt{\theta_R}}{\sqrt{\theta_L} + \sqrt{\theta_R}} \text{ and } s_B = 0$$

$$\text{APR: } b_L = \sqrt{\theta_L}, b_R = \sqrt{\theta_R} \text{ and } b_B = 1 - \sqrt{\theta_L} - \sqrt{\theta_R}$$

Bertrand Equilibrium Under SPR

Here we consider that parties compete under SPR and as mentioned above since we are interested in comparing the two systems we impose that the cost of valence is expensive enough to guarantee blank votes. Under SPR we have seat shares as follows:

$s_L = \frac{\sqrt{\theta_L}}{\sqrt{\theta_L} + \sqrt{\theta_R}}$, $s_R = \frac{\sqrt{\theta_R}}{\sqrt{\theta_L} + \sqrt{\theta_R}}$ and $s_B = 0$
And the following profit functions:

$$\pi_L(\theta_L, \theta_R) = s_L - a_L \theta_L = \frac{\sqrt{\theta_L}}{\sqrt{\theta_L} + \sqrt{\theta_R}} - a_L \theta_L$$

$$\pi_R(\theta_L, \theta_R) = s_R - a_R \theta_R = \frac{\sqrt{\theta_R}}{\sqrt{\theta_L} + \sqrt{\theta_R}} - a_R \theta_R$$

Proposition 12 *For the above assumptions there is no equilibrium when parties are competing a la "Bertrand"*

Proof:

The computations can be found in the Appendix and are done by Mathematica. The sketch of the proof is as follows:

We prove the above by contradiction. Let $(\theta_L^{*SPR}, \theta_R^{*SPR})$ the equilibrium values. Then those values have to satisfy the three conditions of Proposition 11.

The two first conditions of the proposition are that first order conditions of each party have to be satisfied. This implies that it must hold

$$\frac{\sqrt{\theta_R}}{2\sqrt{\theta_L}(\sqrt{\theta_L} + \sqrt{\theta_R})^2} - a_L = 0 \text{ for party L}$$

$$\frac{\sqrt{\theta_L}}{2\sqrt{\theta_R}(\sqrt{\theta_L} + \sqrt{\theta_R})^2} - a_R = 0 \text{ for party R}$$

Solving the above two equations we get reaction functions:

$$\theta_L^{*SPR} = \frac{a_R}{2(2a_L a_R + \sqrt{a_L a_R (a_L + a_R)})^2} \text{ and } \theta_R^{*SPR} = \frac{a_L}{2(2a_L a_R + \sqrt{a_L a_R (a_L + a_R)})^2}$$

We impose the condition that $\sqrt{\theta_L^{*SPR}} + \sqrt{\theta_R^{*SPR}} < 1$, and we obtain that there the parties are not be able to satisfy all the voters if their unitary costs lie in the following regions:

$$0 < a_L \leq \frac{1}{2} \text{ and } a_R > \frac{1}{4a_L} \text{ or } a_L > \frac{1}{2} \text{ and } a_R > a_L$$

Now having found θ_L^{*SPR} we search for θ_R^{0SPR} by solving:

$$\pi_R = \frac{\sqrt{\theta_R^{0SPR}}}{\sqrt{\theta_L^{*SPR}} + \sqrt{\theta_R^{0SPR}}} - a_R \theta_R^{0SPR} = 0 \Rightarrow \theta_R^{0SPR} = A$$

Remember that θ_L^{0SPR} is the highest level of valence that can be afforded by party R given θ_L^{*SPR} .

For a Bertrand equilibrium to exist the third condition of Proposition 11 must be satisfied which implies that $\theta_L^{*SPR} > \theta_R^{0SPR}$ must hold. Having calculated the above two values we show that this can never hold. Hence, contradiction and there is no "Bertrand" equilibrium for the above assumptions. ■

The above proposition doesn't imply non-existence of Bertrand equilibrium. A Bertrand equilibrium can exist if the conditions of Proposition 11 are satisfied. What the above proposition implies is that under the specific assumptions it can not exist a Bertrand equilibrium. The reason is that θ_L^{*SPR} is smaller than θ_R^{0SPR} which means that party R can afford a valence level $\theta_R = \theta_L^{*SPR} + \varepsilon$ and win the election. Because of that there doesn't exist a Bertrand equilibrium.

Bertrand Equilibrium Under APR

Proposition 13 *In case of "Bertrand" Competition under APR we have a unique equilibrium $\theta_j^{*APR} = \frac{1}{4a_j^2}$ if the unitary costs satisfy: $\frac{1}{2} < a_L \leq 0.75$ and $a_R > \frac{a_L}{2a_L-1}$ or $a_L > 0.75$ and $a_R > 2a_L$*

Proof:

The computations can be found in the Appendix and are done by Mathematica. The sketch of the proof is as follows:

We have share of seats in the parliament given by:

$$b_L = \sqrt{\theta_L}, b_R = \sqrt{\theta_R} \text{ and } b_B = 1 - \sqrt{\theta_L} - \sqrt{\theta_R}$$

and the following profit functions:

$$\pi_L(\theta_L, \theta_R) = b_L - a_L \theta_L = \sqrt{\theta_L} - a_L \theta_L$$

$$\pi_R(\theta_L, \theta_R) = b_R - a_R \theta_R = \sqrt{\theta_R} - a_R \theta_R$$

which give the following first order conditions and optimal values respectively:

$$\frac{1}{2\sqrt{\theta_L}} - a_L = 0 \text{ which implies } \theta_L^{*APR} = \frac{1}{4a_L^2} \text{ for party L}$$

$$\frac{1}{2\sqrt{\theta_R}} - a_R = 0 \text{ which implies } \theta_R^{*APR} = \frac{1}{4a_R^2} \text{ for party R}$$

Imposing the condition $\sqrt{\theta_L^{*APR}} + \sqrt{\theta_R^{*APR}} < 1$ we obtain that the following conditions must hold:

$$0 < a_L \leq 1 \text{ and } a_R > \frac{a_L}{2a_L-1} \text{ or } a_L > 1 \text{ and } a_R > a_L$$

Having value θ_L^{*APR} we can solve for $\theta_R^{0,APR}$ from:

$$\pi_R = \sqrt{\theta_R^{0SPR}} - a_R \theta_R^{0SPR} = 0 \text{ which implies } \theta_R^{0SPR} = \frac{1}{a_R^2} \text{ which is the}$$

highest possible level of valence affordable by party R.

So finally in order to have an equilibrium it has to be the case that:

$\theta_L^{*APR} > \theta_R^{0,APR}$ which implies $\frac{1}{4a_L^2} > \frac{1}{a_R^2}$ which implies that $a_R > 2a_L$. So in order θ_L^{*APR} to be unaffordable by party R it has to be the case that the unitary cost of party R is higher than twice the unitary cost of party L.

Given that we have assumed that there exists people voting blank we have identified that it must hold:

$$0 < a_L \leq 1 \text{ and } a_R > \frac{a_L}{2a_L-1} \text{ or } a_L > 1 \text{ and } a_R > a_L$$

which combined with the fact that $a_R > 2a_L$ must hold we get that the intersection of the above two regions regarding unitary costs is:

$$\frac{1}{2} < a_L \leq 0.75 \text{ and } a_R > \frac{a_L}{2a_L-1} \text{ or } a_L > 0.75 \text{ and } a_R > 2a_L \blacksquare$$

On the contrary to the non-existence result of Bertrand equilibrium under SPR we identify the existence of Cournot equilibrium under APR with the assumption that party R has a unitary cost at least twice higher than the one of party L. These results are just an application of Proposition 11 and are not giving us any comparative intuition between the two systems.

4.2 Cournot Competition

Now we consider that the parties do not obtain any extra benefit if they win or tie the election. They are pure monetary profit maximizers. Remember that the utility function now is given by:

$$U_j(\theta_L, \theta_R, x_L, x_R) = f(q_j(\theta_L, \theta_R)) - c_j(\theta_j) = q_j - a_j \theta_j$$

Now we maintain the assumptions of the previous example, we consider the utility function corresponding to Cournot competition and we assume that $a_L \leq a_R$.

Beginning with *Cournot Competition Under SPR* as we have shown before we will have the following reaction functions that are the solution of the maximization problem of each party taking in consideration the first order condition of the competitor:

$$\theta_L^{*SPR} = \frac{a_R}{2(2a_L a_R + \sqrt{a_L a_R (a_L + a_R)^2})} \text{ and } \theta_R^{*SPR} = \frac{a_L}{2(2a_L a_R + \sqrt{a_L a_R (a_L + a_R)^2})}$$

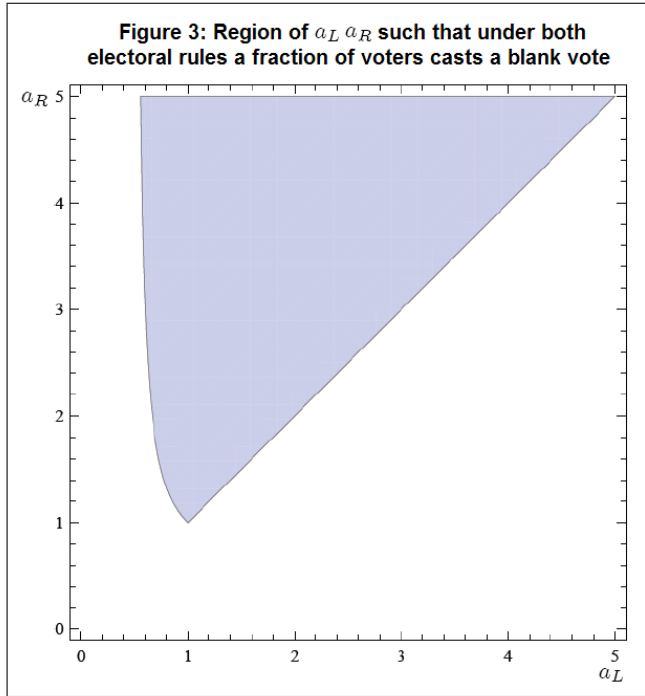
and having imposed the condition $\sqrt{\theta_L^{*SPR}} + \sqrt{\theta_R^{*SPR}} < 1$, we obtain that the following conditions regarding the unitary costs of each party that have to hold:

$$0 < a_L \leq \frac{1}{2} \text{ and } a_R > \frac{1}{4a_L} \text{ or } a_L > \frac{1}{2} \text{ and } a_R \geq a_L$$

Regarding *Cournot Competition Under APR* as we have shown before $\theta_j^{*APR} = \frac{1}{4a_j^2}$ and the unitary costs must satisfy: $\frac{1}{2} < a_L \leq 1$ and $a_R > \frac{a_L}{2a_L - 1}$ or $a_L > 1$ and $a_R \geq a_L$

Comparing the two systems under Cournot Competition

So now we can compare the two systems for assumptions regarding unitary costs that guarantee blank votes under both systems. This will be the intersection of case when $\frac{1}{2} < a_L \leq 1$ and $a_R > \frac{a_L}{2a_L - 1}$ or $a_L > 1$ and $a_R \geq a_L$. These values (restricted up to $a_R = a_L = 5$) can be seen in figure 3.



Having identified the above values of marginal costs we go a step further by comparing the valence levels of both parties and focusing in the region that APR will give higher levels of quality than the SPR.

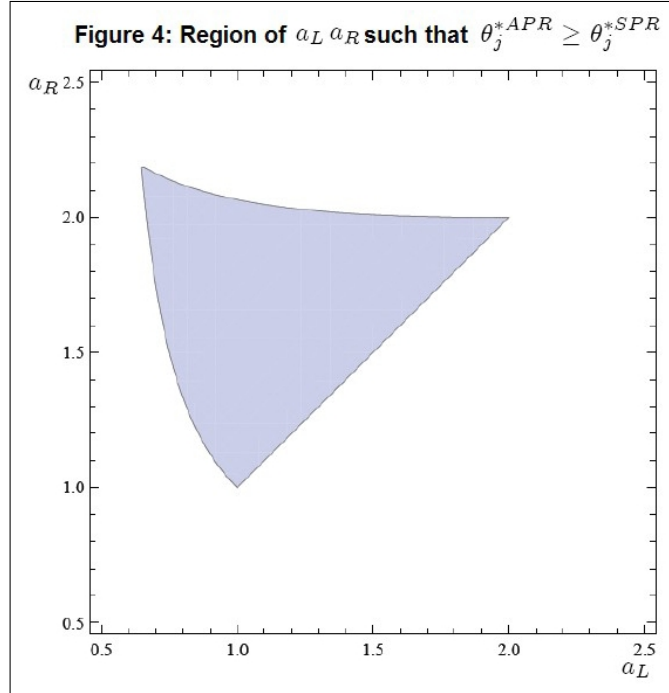
Proposition 14 *The quality of both parties will be higher under APR if and only if:*

- a) $\underline{a}_L < a_L \leq 1$
and $\frac{a_L}{2a_L-1} < a_R \leq \hat{a}_R$
- b) $1 < a_L < 2$ and $a_L \leq a_R \leq \bar{a}_R$
- c) $a_L = 2$ and $a_R = 2$

Proof: The computations can be found in the Appendix. The idea is that we are identifying the region that unitary costs a_L, a_R can lie in order to satisfy that $\theta_j^{*APR} \geq \theta_j^{*SPR}$ for $j = L, R$

Values $\underline{a}_L, \hat{a}_R, \bar{a}_R$ are real numbers and can be found in the Appendix.

The above region is depicted in figure 4.



The above proposition implies that it is not always the case that APR would lead to a better result for the society. The reason is that if the cost of valence for a party exceeds a specific amount then under APR they would be able to fill so few seats in the parliament that the means they would have to increase

the quality would be very restricted. As mentioned before the means that the parties have is the effort that the members of the parliament put. So if the cost is so high then it would be better to fill all the seats and have more members putting costly effort.

Proposition 15 *In the above identified region of unitary costs a_L, a_R that gives higher quality of both parties under APR, both parties make higher profits under SPR than APR*

Proof: The computations can be found in the Appendix. The idea is that we compare π_j^{*SPR} and π_j^{*APR} for $j = L, R$ in the region identified by Proposition 14 and we obtain that $\pi_j^{*SPR} > \pi_j^{*APR}$ for $j = L, R$ within all the region.

The result of the previous proposition is important. It implies that when society prefers APR to SPR, parties instead prefer SPR rather than APR since their profits are higher. This implies that if parties face costs as identified in Proposition 14 they would never have an interest in implementing the alternative electoral system that is discussed in the paper although it would lead to better results for the society.

The question now is how the region identified above could be understood. The intuition of the above propositions can be clarified if we consider symmetric costs $a_R = a_L$. By Proposition 14 in order to have higher quality under APR we have identified that it must hold that $a_R = a_L \in [1, 2]$. The interpretation of this region is as follows. The lower bound $a_R = a_L = 1$ guarantees that under both systems there will exist a fraction of dissatisfied people. Most interesting the upper bound $a_R = a_L = 2$ implies levels of valence $\theta_j^{*APR} = \theta_j^{*SPR} = \frac{1}{16}$ for $j = L, R$, which under both systems would give a vote share $v_j^{APR} = v_j^{SPR} = \frac{1}{4}$ for $j = L, R$. In other words, this means that half of the voters that participate in the election would cast a blank ballot ($v_B^{APR} = v_B^{SPR} = \frac{1}{2}$).

Would ever parties decide to change the electoral system and introduce the APR? The answer is no, if we consider that their marginal costs are not high enough, so they are not able to satisfy at least half of the voters. If we consider that this is the case and the parties have the means to choose a quality so that they are able to satisfy in terms of quality half of the voters (i.e. $a_R = a_L \in [1, 2]$) then the APR would benefit the society but would lead to lower profits for the parties.

5 Conclusion

In the first part of the paper we explain peoples incentives to cast a blank vote. We relate this to the fact that voters, for social reasons, have a need to make

a statement. A voter casts a blank vote expressing his dissatisfaction towards the policy and non-policy characteristics of both candidates. Even further we show that parties can increase their share of votes (or decrease the share of blank votes) by investing in a higher quality. So we relate directly quality of parties and share of blank votes. In the existing political systems though, given that the blank votes are neglected many of the voters either move away from expressing voting and choose the least bad candidate or decide to abstain, if voting is costly. Given that voting blank is perceived in the political system in the same way as abstaining this may not give any incentives to the voters to participate in the election.

Moving one step further we introduced an alternative electoral rule. Main characteristic is that voting blank is a real statement. So voters have indeed incentives to do so. As we have related quality and blank voting, we have shown that under quite realistic assumptions APR would benefit the society since the quality of both parties under APR is higher than under SPR. On the other hand, we have proved that when this is the case, the parties obtain lower profits. So the interests of the main players of the political game, namely parties and voters, are conflicting.

Would ever such an electoral rule be implemented? The answer is no, given that the ones who have the power of deciding on the electoral rules are the parties. A new electoral rule usually demands a constitutional change that should be voted by the parliament. So even if it is the case that the APR would benefit the society it would never be implemented. A possible case to implement such a system would be to allow for direct democracy when deciding on the choice of electoral rules.

The above paper is just a first attempt to motivate the existence of blank votes. Moreover we introduced a system that under quite realistic assumptions leads to higher quality of parties. The relationship between blank votes and abstention is still an open question. However in our setup it is easy to see that turnout would be higher under APR since the quality of the parties is higher. It would be of interest to investigate closer the relationship between turnout and blank votes under the two systems and possibly assume different voting behavior under the two systems, since under APR expressive voting does seem more reasonable than under SPR where voters express themselves but are not taken in consideration. Finally, we could extend the model further by including partisan voters, and allowing for further asymmetries such as asymmetric voter distributions and parties' locations.

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