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DYNAMIC DECENTRALIZED LEADERSHIP MODEL WITH SAVING
AND FREE MOBILITY**

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Fiscal Federalism

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ABSTRACT: This paper considers a simple dynamic decentralized leadership model with local borrowing and regional productivity enhancing investment. The central government is benevolent but cannot commit. The local governments strategically act while accounting for the ex post motive of the central government. We then investigate inefficiency in the subgame perfect equilibrium. We analyze the effect of central control on local borrowings. It is revealed that the central control is of no use. The model is extended to the case with residential mobility which gives different policy implications.

JEL Codes: H71,H72,H73,H77

Keywords: soft budget, local borrowing, local investment.

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1. Introduction

This paper highlights the soft budget problem in the context of intergovernmental grants, which *ex post* refers to the circumstance of “local government turning to the central government for fiscal relief” (Oates (2005)) and *ex ante* manipulating “its access to funds in undesirable ways” (Rodden et al. (2003)). The paper then addresses *ex post* bailouts induced by *ex ante* moral hazard or adverse incentive effects.

The modeling is analogous to the Samaritan’s dilemma in which altruistic individuals cannot commit not to assist the poor, whereas the latter *ex ante* anticipates *ex post* the motive of the former (Coate (1995)). If important local public services such as education and health are underprovided in a fiscally distressed region, this may be regarded as inequitable and thus unacceptable, leading to mounting pressure on the central government to assist such a region. The benevolence of the bailing out agent, however, is not necessary for this problem to occur. Goodspeed (2002) models the political economy of the soft budget in a two period setting with the central government providing interregional transfers for tactical purposes and local governments *ex ante* issuing bonds in a strategic manner.

The decentralized leadership literature considers the related issues, whereas addressing *ex ante* horizontal and reciprocal externalities, with the central government acting as a Stackelberg follower and local government as a leader (Caplan et al. (2000)). The present paper follows the literature and develops a simple dynamic model. With the two period setting, it contains regional productivity enhancing investment in the first

period as well as local public services in both periods. The local spending is financed by local taxes as well as local borrowing in the first period and intergovernmental transfers in the second period. The transfers are ex post optimized. We incorporate the ex ante central regulations on local borrowing. It is established that while the regional public investment turns to be efficient, ex ante the local governments are motivated to over-provide local public services in the anticipation that the burden is ultimately born nationwide. The upshot is that the ex ante central regulation on issuing local bonds is of no use to prevent such perverse incentives. In the extreme, the overspending in the sub-game perfect equilibrium is unchanged even if the balanced budget is imposed on the local governments ex ante prohibiting the borrowing. This occurs because the local governments can maneuver the residents' saving by the use of the ex ante local taxation, which lowers the second period disposal income of the residents. This in turn influences the ex post transfers. Interestingly, in the present model, the ex ante overspending does not necessarily imply over-borrowing as opposed to conventional models of the soft budget, but over-taxation.

The model is extended to the case of ex post residential mobility. It is then shown that the ex post transfers serve for different purpose than the basic model without mobility. The transfers are ex post designed to enhance efficiency of inter-regional allocation of population as is familiar in the fiscal federalism literature. The local government cannot attract ex post transfers by manipulating the residents' saving. This prevents the incentive of overspending in the first period, but discourage the productivity enhancing investment. The central regulation becomes effective to prevent otherwise over borrowing incentive of the local governments. The presence of the ex post mobility thus gives different policy implications.

The rest of this paper is organized as follows. Section 2 outlines the two period model. In Section 3, we consider the first best outcome of the model. Section 4 analyzes the ex post behavior of the central and local governments. In Section 5, we examine the ex ante behavior and derive the subgame perfect equilibrium. In Section 6, we consider the case of ex post free mobility across regions. Section 7 concludes this paper.

2. Basic Setting

The economy lasts two periods denoted by $t=1,2$ and contains J regions. Each region consists of a representative resident. We denote the population size in region i by n_i , with the total population given by $\sum_i n_i = N$. There are the central and local governments.

At $t=1$, local government in region i spends I_i^g public investment that enhances regional production $F_i(I_i^g, n_i)$ in the second period. Local public services are provided by the local government as well. Write the first and the second period provisions of the public services by g_i and G_i respectively. We assume that both the public services and $F_i(I_i^g, n_i)$ accrue to regional residents. The latter implies that the output is equally shared among the residents. Let $y_i = F_i(I_i^g, n_i)/n_i$ be resident's income in region i . There are no inter-regional spillovers associated with the public services and investments. In the second period, the central government is in charge of intergovernmental transfers, $\sigma_i (i = 1, \dots, J)$.

Resident's budget and utility

Consider the representative resident in region i . Denote by z_i his endowment in the first period. The regional production or $y_i \equiv F_i(I_i^g, n_i)/n_i$ realizes at $t=2$. His budget constraints at $t=1$ and 2 then are respectively expressed as:

$$c_i^1 = z_i - s_i - t_i^1, \quad c_i^2 = y_i + s_i - t_i^2. \quad (1)$$

c_i^t ($t=1,2$) and s_i are private consumption and saving. t_i^t is local tax level. In (1), interest rate of saving is assumed to be zero for the sake of simplicity.

The resident's utility is given by:

$$U(c_i^1, g_i, c_i^2, G_i) = u(c_i^1, g_i) + v(c_i^2, G_i) = u(z_i - t_i^1, g_i) + v(y_i - t_i^2, G_i). \quad (2)$$

The functions $u(\cdot)$ and $v(\cdot)$ are monotonically increasing and strictly concave. We also let $u_{cg} \geq 0$. The present model abstracts preference heterogeneity of residents in one region. Note that we instead account for inter-regional disparity in n_i and z_i . Also the production technology in the second period, $F_i(I_i^g, n_i)$, may be different among regions.

Government's budget constraint and intergovernmental transfer

In $t=1$, the local public spending is financed by local tax and borrowing. Its first period budget constraint of the local government is given by:

$$t_i^1 + b_i = g_i + I_i^g / n_i, \quad (3)$$

where $b_i \equiv B_i / n_i$ represents per capita local borrowing. The budget constraint in period 2 is

$$t_i^2 + \sigma_i / n_i - b_i = G_i, \quad (4)$$

where the interest rate for the borrowing is set to zero. Recall that σ_i denotes the subsidy from the central government to the region.

In the model, σ_i can be of either sign subject to the central budget constraint at $t=2$

$$\sum_{i=1}^J \sigma_i = 0 \quad (5)$$

A negative transfer implies that the central government taxes the local government. We omit the central tax on the residents for simplicity. Rather the critical presumption here is that the central government possesses full discretion over σ_i at $t=2$ so as to pursue its ex post objective.

The objectives of central and local governments

The central government decides the transfer level so as to maximize the utilitarian objective $\sum_i n_i U(c_i^1, g_i, c_i^2, G_i)$. In this regard, it is assumed to be benevolent. The present model can be easily extended to contain a political motive with the central government aiming to maximize expected votes in election as is in Goodspeed (2002). It would however add only minor complications. Instead we focus on the commitment problem that occurs without political considerations.

Turn to the local government. It decides the level of public services so as to maximize

the utility of its own region, $U(c_i^1, g_i, c_i^2, G_i)$. Substituting the local budgets (3) and (4), it becomes:

$$U = u(z_i - t_i - s_i, t_i - I_i^g / n_i + b_i) + v(y_i + s_i - t_i^2, t_i^2 + \sigma_i / n_i - b_i), \quad (6)$$

Timeline

Timing is critical in our model. The sequence of decision makings is summarized as follows:

First period (ex ante)	<ul style="list-style-type: none"> - Local governments choose g_i, t_i^1 and I_i^g - The residents select s_i and consumes
Second period (ex post)	<ul style="list-style-type: none"> - $y_i = F_i(I_i^g, n_i) / n_i$ realizes - The central government determines σ_i - The local governments choose G_i and t_i^2

The local borrowing at $t=1$ may be centrally regulated. In the second period, the central government acts, taking the ex ante decisions by local government as given. It then maximizes the social welfare from the ex post stand point, which gives rise to the commitment problem as we see below. Note, however that the central ex post optimization accounts for the local response of g_i and t_i . In the first period, the local government takes into account how its ex ante choices affect the ex post central government policy, namely the ex post design of the interregional transfers as well as the private saving decisions. In this regard, it is the Stackelberg leader, but behaves in a Nash manner toward other local governments in the same stage. The residents in

choosing s_i on the other hand act competitively given the policy instruments.

3. First Best Optimal Allocation

Before illustrating the subgame perfect equilibrium, as a reference, let us consider the first best allocation that is determined by maximizing social welfare W subject to the resource constraint:

$$\text{MAX}_{c_i^1, c_i^2, g_i, G_i, I_i^g} W = \sum_i n_i U(c_i^1, c_i^2, g_i, G_i),$$

$$\text{subject to } \sum_{i=1}^J n_i c_i^1 + \sum_{i=1}^J n_i g_i + \sum_{i=1}^J I_i^g + \sum_{i=1}^J n_i c_i^2 + \sum_{i=1}^J n_i G_i = \sum_{i=1}^J F_i(I_i^g, n_i) + \sum_{i=1}^J n_i z_i.$$

Due to strictly concavity of the sub-utility function, $u_1(\cdot)$ and $u_2(\cdot)$, the following proposition is immediate :

Proposition 1

The first best allocation is characterized by:

$$c_i^1 = c^{1**}, g_i = g^{**}, c_i^2 = c^{2**}, G_i = G^{**} \text{ for all } i \text{ and } I_i^g = I_i^{g**} \text{ with}$$

$$u_c = u_g = v_c = v_G \text{ and}$$

$$\frac{\partial}{\partial I_i^g} F_i = 1. \quad (7)$$

Equation (7) will be compared with the subgame perfect equilibrium conditions derived in the subsequent sections.

It is straightforward to see that the first best allocation would be achievable if the central government could commit to ex ante optimal transfer scheme such as:

$$\sigma_i^{**} / n_i = c^{1**} + c^{2**} + g^{**} + G^{**} + I^{g**} / n_i - z_i - F_i(I^{g**}, n_i) / n_i. \quad (8)$$

4. Ex Post Behavior of the Central and Local Governments

Local Government Ex post Behavior

We proceed backwards, starting from the second period local optimization. Given that y_i and b_i have realized and σ_i is transferred, the local government decides t_i^2 (and thus G_i through (4)) to maximize the second period regional utility:

$$MAX_{t_i^2} v(y_i + s_i^* - t_i^2, t_i^2 + \sigma_i / n_i - b_i).$$

The first order condition is given by:

$$v_c(c_i^2, G_i) = v_G(c_i^2, G_i) \quad (9)$$

Central Government Ex post Behavior

At the previous stage, the central government chooses σ_i to maximize ex post social welfare subject to the budget constraint (5):

$$MAX_{\tau_i, \sigma_i} \sum_{i=1}^I n_i \{v(y_i + s_i^* - t_i^2, t_i^2 + \sigma_i / n_i - b_i)\} \text{ subject to } \sum_{i=1}^J \sigma_i = 0.$$

given the ex ante regional decisions, but accounting for (9). The ex post optimization gives the first order condition as:

$$v_G(c_i^2, G_i) = v_G(c_j^2, G_j). \quad (10)$$

(10) along with (9) implies $c_i^2 = \bar{c}^2$ and $G_i = \bar{G}$ for all i . The ex post transfers fully equalize both the private consumption and the local public service at $t=2$.

Ex Post Resource Constraint

Combining the second period central and local budgets, we have ex post resource constraint:

$$N\bar{G} + N\bar{c}^2 = \sum_{i=1}^J (F_i(I_i^s, n_i) + n_i s_i^* - n_i b_i). \quad (11)$$

\bar{c}^2 and \bar{G} are determined by solving (9) and (11), and can be written as $\bar{c}^2(Z)$ and $\bar{G}(Z)$, where

$$Z \equiv \frac{1}{N} \sum_{i=1}^J (F_i(I_i^s, n_i) + n_i s_i^* - n_i b_i) = \frac{1}{N} \sum_{i=1}^J (F_i(I_i^s, n_i) + n_i Q_i) \quad (12)$$

In (12), we define regional net saving per capita $Q_i \equiv s_i^* - b_i$ for latter use. It is immediate to see that:

$$1 = \frac{d\bar{c}^2}{dZ} + \frac{d\bar{G}}{dZ}. \quad (13)$$

with

$$\frac{d\bar{G}}{dZ} = \frac{v_{cc} - v_{cG}}{v_{GG} - 2v_{cG} + v_{cc}}, \quad \frac{d\bar{c}^2}{dZ} = 1 - \frac{d\bar{G}}{dZ} = \frac{v_{GG} - v_{cG}}{v_{GG} - 2v_{cG} + v_{cc}}. \quad (14)$$

Ex Post Transfers

At this point, let us illustrate the features of the ex post optimal transfer function. Given that $c_i^2 = \bar{c}^2$ and $G_i = \bar{G}$ for all i , combining the local budget and the resident's budget constraints, we have:

$$\frac{\sigma_i}{n_i} = \bar{c}^2 + \bar{G} - y_i - Q_i \quad (15)$$

Taking into account (13), we can establish that ex post per capita transfers to region i are decreasing in own net saving and increasing when other regions save more (so borrow less).

$$\frac{\partial}{\partial Q_i} \left(\frac{\sigma_i}{n_i} \right) = \frac{n_i}{N} - 1 < 0 \quad (16.1)$$

$$\frac{\partial}{\partial Q_j} \left(\frac{\sigma_i}{n_i} \right) = \frac{n_i}{N} > 0 \quad \text{with } j \neq i \quad (16.2)$$

These results should be intuitive.

5. Ex Ante Behavior of the Local Government

Resident's saving optimization

In the first period, the representative resident decides the level of saving so as to maximize his utility:

$$\underset{s_i}{MAX} \quad u(z_i - t_i^1 - s_i, g_i) + v(y_i + s_i - t_i^2, G_i)$$

Note that the resident takes as given the local policy instruments. The first order condition is then given by:

$$u_c(z_i - t_i^1 - s_i^*, g_i) = v_c(y_i + s_i^* - t_i^2, G_i)$$

Once we incorporate the ex post optimization, this condition can be written as

$$u_c(z_i - t_i^1 - s_i^*, t_i^1 + b_i - I_i^g / n_i) = v_c(\bar{c}^2(Z), \bar{G}(Z)) \quad (17)$$

where Z is as defined by (12). The saving has the following feature:

$$\frac{\partial s_i^*}{\partial b_i} - \frac{\partial s_i^*}{\partial t_i^1} = 1 \quad (18)$$

For the proof, see Appendix 1, in which other characteristics of the saving function is described. Equation (18) will be used to illustrate the ex ante regional optimization.

Local Government's optimization

Accounting for the ex post central policy, which is summarized by $\bar{c}^2(Z)$ and $\bar{G}(Z)$, the local governments choose (t_i^1, I_i^g) to maximize the local utility in region i , that is:

$$MAX_{t_i^1, I_i^g} V_i = u(z_i - t_i^1 - s_i^*, t_i^1 + b_i - I_i^g / n_i) + v(\bar{c}^2(Z), \bar{G}(Z))$$

$$\text{where } Z \equiv \frac{1}{N} \sum_{j=1}^J (F_j(I_j^g, n_j) + n_j s_j^* - n_j b_j)$$

The first order conditions with respect to I_i^g and t_i^1 , become:

$$0 = n_i \frac{dV_i}{dI_i^g} = \frac{n_i}{N} F_i'(I_i^g) v_c - u_g^i + (v_c \frac{n_i}{N} - u_c^i) n_i \frac{ds_i^*}{dI_i^g} \quad (19)$$

$$0 = \frac{dV_i}{dt_i^1} = u_g^i - u_c^i + (v_c \frac{n_i}{N} - u_c^i) \frac{ds_i^*}{dt_i^1} \quad (20)$$

The last terms in (19) and (20) represent the policy induced change in the resident's saving. Note that the local government acting as a Stackelberg leader toward the ex post central policy assess the value of the saving differently from the resident. The saving aims to equate the marginal utilities of the private consumption between the two

periods as (15) (given that interest rate is zero). The local government however incorporates ex post equalization through the transfers. This lowers share of the saving accruing to the region to n_i / N , which leads to

$$v_c \frac{n_i}{N} - u_c^i = \left(\frac{n_i}{N} - 1 \right) v_c < 0 \quad (21)$$

The negative sign of (21) implies that each local government prefers to discourage own resident's saving in the first period.

Consider the local borrowing. The present model allows for the case that the central government regulates it. However, the following lemma established that such regulation does not constrain the ex ante local government's optimization since b_i is simply redundant.

Lemma 1

b_i and t_i are ex ante perfectly substitute policy instruments for the local government in the sense that:

$$\frac{dV_i}{dt_i^1} = \frac{dV_i}{db_i}$$

Proof : Using Equation (18) and (20), we have

$$\begin{aligned} \frac{dV_i}{dt_i^1} &= u_g^i - u_c^i + \left(v_c \frac{n_i}{N} - u_c^i \right) \left(\frac{ds_i^* (t_i^1, b_i, I_i^g)}{db_i} - 1 \right) \\ &= u_g^i - \frac{n_i}{N} v_c + \left(v_c \frac{n_i}{N} - u_c^i \right) \left(\frac{ds_i^* (t_i^1, b_i, I_i^g)}{db_i} \right) = \frac{dV_i}{db_i} \end{aligned}$$

Lemma 1 implies that the local borrowing is at the optimal level from the regional

standpoint when the local tax is optimized. In this regard the local government can undo the central regulation on b_i by manipulating t_i . Indeed, reducing the local bond by one dollar induces one dollar increase in local tax that in turn decrease the resident's saving by the same amount. By doing so, the net regional contribution to the second period resource $Q_i \equiv s_i^* - b_i$, remains the same. The following Proposition establishes these arguments.

Proposition 2

In the subgame perfect equilibrium, we have:

(1) The change of central regulation is perfectly absorbed by the change of the local

tax in the period 1, such that $\frac{dt_i^1}{db_i} = -1$.

(2) Through the change of the local tax, the change of central regulation is perfectly

absorbed by the change of private saving in the period 1, such that $\frac{ds_i^*}{db_i} = 1$.

(3) \hat{g}_i and \hat{c}_i^1 are independent from the regulation level, b_i^c .

(4) The investment level becomes efficient such that $\hat{I}^s = I^s$ for all i

Proof: See Appendix 2

The second statement in the Proposition implies the Ricardian equivalency with the private saving being adjusted to change in local borrowing. As in the third statement, therefore, the local borrowing exerts no impact on the first period regional resource allocation in the equilibrium. The investment level is at the first best. The cost of financing the investment either through local bond or local tax, the latter of which cuts

the savings, is nationwide shared in the second period, whereas its return is also pooled. That is, regional share of both the cost and the return on I^g is set equal to n_i / N . Thus, the incentive of overinvestment due to the ex post cost sharing is fully offset by the disincentive arising from the ex post income sharing in the present contest.

Social welfare:

Now we assess the social welfare in the equilibrium, which becomes

$$SW = \sum_{i=1}^I n_i \left\{ u(z_i - \hat{g}_i + \hat{I}^g - \hat{Q}_i, \hat{g}_i) + v(\bar{c}^2(\hat{Z}), \bar{G}(\hat{Z})) \right\}$$

Differentiating the welfare with respect to g_i and $Q_i \equiv s_i^* - b_i$ and evaluating the derivatives in the equilibrium yields the following:

$$\frac{dSW}{d\hat{g}_i} = n_i(u_g^i - u_c^i) < 0^2 \quad (21.1)$$

$$\frac{dSW}{d\hat{Q}_i} = -n_i u_c^i + \left(\sum_{i=1}^J n_i \right) \frac{n_i}{N} v_c = n_i(v_c - u_c^i) = 0 \quad (21.2)$$

From these, the following proposition can be established.

Proposition 3

In the equilibrium,

² From Equation (21) and Appendix 1, we have

$$u_g^i - u_c^i = v_c \left(1 - \frac{n_i}{N}\right) \frac{ds_i^*}{dt_i^1} = v_c \left(1 - \frac{n_i}{N}\right) \frac{1}{-\left(u_{cc}^i + \frac{n_i}{N} \frac{dv_c}{dZ}\right)} (u_{cc}^i - u_{cg}^i) \leq 0 \quad \text{since } u_{cg}^i \geq 0$$

- (a) \hat{g}_i (or the tax in the first period $\hat{t}_i^1 = \hat{g}_i + b_i - I_i^{g^{**}}$) is excessive irrespective of the size of local borrowing.
- (b) The level of the net saving becomes social optimum on margin.

Note that even when $b_i = 0$, that is, the ex ante local budget is set hard, the first result of the above proposition applies. The central regulation cannot prevent the local government from overspending ex ante therefore.

The intuition can be interpreted as follows. In anticipation of the ex post equalization, the local government has the motive to shift the burden of the first period public service to other residents. The local government ex ante succeeds in doing so by raising the local tax and lowering the saving, which ex post raises transfer from the central government. The ex post transfer compensates one dollar reduction in the per capita saving by $1 - n_i / N$, so the resident bears only n_i / N dollars.

To see it differently, combine the residents' life time budget with the local government one yielding the inter-temporal regional resource constraint as:

$$c_i^1 + \bar{c}^2 + g_i + \bar{G} + I_i / n_i = z_i + y_i + \sigma_i \quad (22)$$

In the present context, raising g_i financed by higher local tax at $t=1$ leads to increasing σ_i at $t=2$. In this regard, it is regional resource that is soften ex post.

Turn to net regional saving. Although it has first order welfare effect as (21.2), the ex ante overspending on g_i leaves less resources for the private consumption and the second period public service. Thus they are under-provided than the social optimum. Equation (22) just states that the remaining resource is optimally split between the two periods. This will be confirmed in the example of the logarithm utility function that

compares the equilibrium with the first best outcome, considering the individual values of Q_i .

To summarize, the ex post equalization that more benefits regions with less regional net saving leads to ex ante perverse behavior among the local governments. This cannot be resolved by the central control on the local bonds that aims to harden the ex ante local budgets. In the present context, it is the resident's saving instead of local borrowing that the local governments utilize to manipulate ex post transfers in their favor.

Log Example

In Proposition 3, social welfare implication of the net regional saving is assessed on margin. Instead, we can compare the equilibrium values of Q_i with the first best outcome. Consider logarithm utility function:

$$U(c_i^1, c_i^2, g_i, G_i) = \log(c_i^1) + \log(g_i) + \log(c_i^2) + \log(G_i) \quad (23)$$

Appendix 3 establishes the following corollary.

[Corollary to Proposition 2]

Comparing the first-best level of Q_i , we have the following result:

$$\hat{Q}_i \begin{matrix} < \\ > \end{matrix} Q_i^{**} \quad \text{if and only if} \quad \left[\begin{array}{c} \frac{N-1}{2 + \frac{n_i}{3}} \\ \frac{1}{2 + \frac{J-1}{12}} \end{array} \right] < \frac{2+J}{5+J}$$

where $\hat{Q}_i = s_i^* - b_i = z_i - \hat{c}_i^1 - \hat{g}_i$ and $Q_i^{**} = z_i - c_i^{**} - g_i^{**}$.

The inequality in the corollary holds with equality if $J=1$ and thus $n_i = N$.

Otherwise, given $J(>1)$, $\hat{Q}_i < Q^{**}$ is likely for a region with small n_i since the left hand side of the condition is increasing in n_i

6. Extension: Free mobility and the role of ex post transfer

As an extension, the present section considers the situation that the residents are ex post mobile across regions. In the basic case, the intergovernmental transfers is designed to equalize the residents' utilities ex post. With the residential mobility at $t=2$, however, their utilities will be equalized even in the absence of transfers. As addressed in the literature on fiscal federalism, transfers may rather be used to enhance efficiency of population allocation. It then seems that the ex post transfers give different consequences than the case without mobility.

6.1 Setting

Resident's budget and utility

For technical convenience, we assume that the second period public service takes a quasi linear form so that:

$$v(c_i^2, G_i) \equiv v(c_i^2 + \Psi(G_i)) = v(y_i + s_i - G_i + (\sigma_i - B_i)/n_i + \Psi(G_i)), \quad (24)$$

where B_i is assumed to be the total amount of the bond issued in the first period and is redeemed in the second period.³

This specification assures that the saving decision in the first period does not rely on where to reside ex post.

³ In this section, we consider the mobility of residents, so b_i used in sections above changes after residents move ex post.

6.2 Ex Post Behavior of the Central and Local Governments

Free Mobility and stability

Consider the individual who initially resides in region i , but may move to other regions in the second period. The mobility equilibrium implies that his utilities must be equalized between any of two regions, which corresponds to

$$F_j / n_j + s_i - G_j + (\sigma_j - B_j) / n_j + \Psi(G_j) = F_k / n_k + s_i - G_k + (\sigma_k - B_k) / n_k + \Psi(G_k) \quad (25)$$

for any k and j . In the present setting, s_i does not influence the individual's location decision because the individual carry his saving with him when moving to other regions. As addressed below, with the prefect mobility, this in turn implies that the first period saving is the same among residents born in the same region irrespective of their second period locations. Define $\phi_i \equiv F_i(n_i, I_i^s) / n_i - G_i + (\sigma_i - B_i) / n_i + \Psi(G_i)$. Then ϕ_i is equalized across regions. So we can let $\phi_i = \phi$ for all i :

$$\phi = F_i(n_i, I_i^s) / n_i - G_i + (\sigma_i - B_i) / n_i + \Psi(G_i) \quad (26)$$

Solving (25) for n_i gives the regional population function: $n_i = n_i(G_i, I_i^s, \sigma_i - B_i, \phi)$ with

$$\frac{\partial}{\partial G_i} n_i = \frac{\Psi'(G_i) - 1}{-D_i} \quad (27.1)$$

$$\frac{\partial}{\partial \sigma_i} n_i = \frac{1/n_i}{-D_i} \quad (27.2)$$

$$\frac{\partial}{\partial \phi} n_i = \frac{1}{D_i} \quad (27.3)$$

where

$$D_i \equiv \frac{\partial \left[\frac{F_i(n_i, I_i^s)}{n_i} \right]}{\partial n_i} - \frac{\sigma_i - B_i}{n_i^2} = \frac{\partial F_i / \partial n_i}{n_i} - \frac{F_i + \sigma_i - B_i}{n_i^2} \quad (28)$$

The stability of the mobility equilibrium implies $D_i < 0$. In the following the above inequality is always assumed. ϕ is determined by:

$$\sum_{i=1}^J n_i(G_i, I_i^s, \sigma_i - B_i, \phi) = N \quad (29)$$

Local Government Ex post Behavior

In the second period, one region contains heterogeneous residents in terms of the first period savings. The utility of the person born in region j at $t=1$ and residing in i at $t=2$ is given by $v(\phi_i + s_j) = v(\phi + s_j)$. His second period utility does not rely on where he resides. Then the regional optimization problem can be reduced to maximize ϕ , given that $F_i(I_i^s, n_i)$ and b_i are decided ex ante and σ_i is transferred. Making use of (27.1) and (29), we can establish

$$\frac{d}{dG_i} \phi = - \frac{\partial n_i / \partial G_i}{\sum_j \partial n_j / \partial \phi} = \frac{(\Psi'(G_i) - 1) / D_i}{\sum_j 1 / D_j} = 0 \quad (30)$$

In the optimum, the first order condition becomes:

$$\Psi'(G^*) = 1 \quad (30')$$

Central Government Ex post Behavior

As the regional optimization, the maximization of ex post welfare then becomes equivalent to maximizing ϕ . Note that the ex post welfare is given by $\sum_i \bar{n}_i v(\phi + s_i)$

where \bar{n}_i denote the first period population of region i . As the welfare weights, we use \bar{n}_i instead of n_i given that the second period utility of resident originated from region i becomes $v(\phi + s_i)$ no matter where he migrates ex post. Making use of (27.2) and (29), we have:

$$\frac{d}{d\sigma_i} \phi = - \frac{\partial n_i / \partial \sigma_i}{\sum_j \partial n_j / \partial \phi} = \frac{1/(n_i D_i)}{\sum_j 1/D_j} = \frac{1}{\sum_j 1/D_j} \frac{1}{\partial F_i / \partial n_i - (F_i + (\sigma_i - B_i))/n_i} \quad (31)$$

Given the central budget constraint (5), the above derivative must be equalized across regions in the ex post optimum. By (26) and (30') or $G_i = G^*$ for all i , the first order condition reduces to:

$$\frac{\partial F_i(n_i, I_i^g)}{\partial n_i} = \frac{\partial F_j(n_j, I_j^g)}{\partial n_j}, \quad (32)$$

which assures the efficient allocation of the population given I_i^g . Such allocation can be achieved by the use of the ex post transfer formula expressed as:

$$\sigma_i / n_i^* = \phi^* - \frac{F_i(n_i^*, I_i^g)}{n_i^*} + G^* + \frac{B_i}{n_i^*} - \Psi(G^*) \quad (33)$$

where

$$\phi^* = \frac{1}{N} \sum_{j=1}^I n_j^* \phi_j^* = \frac{1}{N} \sum_{j=1}^I \{F_j(n_j^*, I_j^g) - B_j\} - G^* + \Psi(G^*) \quad (34)$$

In the present context, the ex post transfers are used to assure efficiency of population allocation, whereas the basis model without mobility addresses ex post equity equalizing the private and public consumption. This difference has critical implications on the ex ante decisions of the local governments.

6.3 Ex Ante Behavior of residents and the Local Government

Resident's saving optimization

In the first period, the representative resident decides the level of saving to maximize his utility:

$$\underset{s_i}{MAX} u(z_i - t_i^1 - s_i, g_i) + v(\phi + s_i)$$

The first order condition is given by:

$$u_c(z_i - t_i^1 - s_i^*, g_i) = v_c(\phi + s_i^*) \quad (17')$$

It is immediate that the saving relies solely on the first period residence.

Local Government's optimization

The local governments choose (t_i^1, I_i^g) to maximize the local utility in region i , that is:

$$\underset{t_i^1, I_i^g}{MAX} V_i = u(z_i - t_i^1 - s_i^*, t_i^1 + (B_i - I_i^g) / \bar{n}_i) + v(\phi^* + s_i^*),$$

where \bar{n}_i is the population given in the first period.

Noting that $\frac{dV_i}{ds_i^*} = 0$, the first order conditions become:

$$0 = \frac{dV_i}{dI_i^g} = \frac{1}{N} \frac{\partial F_i(n_i^*, I_i^g)}{\partial I_i^g} v_c - \frac{1}{\bar{n}_i} u_g^i = 0 \quad (19')$$

$$0 = \frac{dV_i}{dt_i^1} = -u_c^i + u_g^i \quad (20')$$

Making use of (17') and (20'), equation (19') becomes

$$\frac{\bar{n}_i}{N} \frac{\partial F_i(n_i^*, I_i^g)}{\partial I_i^g} = 1 \quad (35)$$

In contrast with the basic case, the investment is underprovided. The intuition is the following. In the absence of the mobility, the ex post transfers serve to equalize both the savings and the regional outputs across regions. Ex ante, this motivates the local government to raise local tax to finance I_i^g (as well as g_i) that lowers the resident's saving, which in turn decreases the ex post equally shared resource Z . As such the local government can shift the burden of I_i^g . In the present case, however, the ex post transfers do not equalize the savings, which close the avenue for the local government to share the investment cost, whereas the return on I_i^g remains equalized. As consequence, the ex ante incentive of the investment is discouraged.

Consider the local borrowing, we have

$$\frac{dV_i}{db_i} = \frac{1}{\bar{n}_i} u_i^i - \frac{1}{N} v_c = \frac{u_c^i}{\bar{n}_i} \left(1 - \frac{\bar{n}_i}{N}\right) > 0 \quad (36)$$

The local government always prefers to issue the bond. The reason is straightforward. The ex ante borrowing to finance g_i benefits the native residents at $t=1$, whereas the repayment cost is nationwide shared. Note that the local borrowing in the present case does not increase the resident's saving, so the Ricardian Equivalence as stated in Proposition 2 does not apply.

Now the results above can be summarized as the following proposition.

Proposition 4

Suppose that residents are freely mobile across regions. Then in the subgame perfect

(time consistent) equilibrium, we have:

(a) $\tilde{I}_i^g < I_i^{g**}$.

(b) \tilde{I}_i^g takes a smaller value for the smaller region, namely, $\tilde{I}_1^g < \tilde{I}_2^g$ if $n_1 < n_2$.

(c) Maximizing the level of the local borrowing, b_i^* becomes optimal, which means that the regulation of local bonds is effective.

Proposition 4 is sharply in contrast with Proposition 3, the case without mobility. As addressed above, on one hand, the local investment becomes lower compared with the socially optimal level. On the other hand, the central regulation becomes effective since the local governments would issue bonds without limit otherwise.

Optimal Bond regulation

So what is the ex ante optimal level of the local borrowing? In the case that the central government regulates the borrowing, it will select B_i so as to maximize:

$$\sum_i \bar{n}_i V_i = \sum_i \bar{n}_i \left[u(z_i - \tilde{t}_i^1 - s_i^*, \tilde{t}_i^1 + (B_i - \tilde{I}_i^g)/\bar{n}_i) + v(\phi^* + s_i^*) \right] \quad (37)$$

where tilde denotes the local government's ex ante decisions. Incorporating (33), the optimization with respect to B_i yields:

$$\frac{d}{dB_i} \sum_j \bar{n}_j V_j = u_g^i - \sum_j \frac{\bar{n}_j}{N} v_c(\phi^* + s_j^*) = u_c^i - \sum_j \frac{\bar{n}_j}{N} u_c^j = 0 \quad (38)$$

In the last inequality, we use (17') and (20'). The above holds for all regions, which gives $u_c(\tilde{c}_i^1, \tilde{g}_i^1) = u_c(\tilde{c}_j^1, \tilde{g}_j^1)$ for all i and j. Alongside with (17') and (20'), we can establish the following proposition.

Proposition 5

Suppose that ex ante the central government can optimize the local bond regulation.

Then we have (i) $\tilde{c}_i^1 = \tilde{c}^1$, (ii) $\tilde{g}_i^1 = \tilde{g}^1$ and (iii) $s_i^* = \tilde{s}$ for all i .

The proposition shows that the ex ante regulation equalize the private consumption in both periods as well as the first period public service. This does not mean however that the first best can be restored since the regional investments remain too little.

SUMMARIZE

Finally we can summarize the levels of variables in the equilibrium as follows.

Table

	$I_i^{s^*}$	g_i^*	B_i
No-mobility	Optimal	Over-provision	redundant
Free-mobility	Under-provision	Optimal	Over-borrowing without regulations Optimal with regulations

7. Conclusion

This paper, by using a simple dynamic decentralized leadership model with local

borrowing and regional productivity enhancing investment, analyzes inefficiency of resource allocation induced by the ex post transfers and examine the effect of local borrowing regulation. We extend the model to allow the mobility of residents, in which the central regulation turns to be effective. The ex post transfer (Soft budget) is designed for achieving optimal allocation of population rather than assuring ex post equity.

The results obtained in this paper have addressed the following two points. First, consideration should be placed on dynamic strategic interaction between the local government and consumers through saving. Second, effects of the central regulation on local borrowing replies on whether or not residents are ex post mobile.

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Appendix 1: Effects on saving

From Equation (17), we have the following results of comparative statics.

$$\frac{\partial s_i^*}{\partial t_i^1} = \frac{1}{-\left(u_{cc}^i + \frac{n_i}{N} \frac{dv_c}{dZ}\right)} (u_{cc}^i - u_{cg}^i) < 0^4 \quad (\text{A.1})$$

$$\frac{\partial s_i^*}{\partial I_i^g} = \frac{1}{-\left(u_{cc}^i + \frac{n_i}{N} \frac{dv_c}{dZ}\right)} \left(\frac{1}{N} F'(I_i^g, n_i) \frac{dv_c}{dZ} + \frac{u_{cg}^i}{n_i} \right) \quad (\text{A.2})$$

$$\frac{\partial s_i^*}{\partial b_i} = \frac{-1}{-\left(u_{cc}^i + \frac{n_i}{N} \frac{dv_c}{dZ}\right)} \left(\frac{n_i}{N} \frac{dv_c}{dZ} + u_{cg}^i \right) \quad (\text{A.3})$$

Therefore we have

$$\frac{\partial s_i^*}{\partial b_i} + n_i \frac{\partial s_i^*}{\partial I_i^g} = \frac{1}{-\left(u_{cc}^i + \frac{n_i}{N} \frac{dv_c}{dZ}\right)} \frac{n_i}{N} \frac{dv_c}{dZ} (F'(I_i^g) - 1) \quad (\text{A.4})$$

$$\frac{\partial s_i^*}{\partial b_i} - \frac{\partial s_i^*}{\partial t_i^1} = \frac{-1}{-\left(u_{cc}^i + \frac{n_i}{N} \frac{dv_c}{dZ}\right)} \left(u_{cc}^i + \frac{n_i}{N} \frac{dv_c}{dZ} \right) = 1 \quad (\text{A.5})$$

$$\frac{dv_c}{dZ} = v_{cc} \frac{d\bar{c}^2(Z)}{dZ} + v_{cG} \frac{d\bar{G}(Z)}{dZ} = \frac{1}{v_{GG} - 2v_{cG} + v_{cc}} (v_{cc} (v_{GG} - v_{cG}) + v_{cG} (v_{cc} - v_{cG})) < 0$$

Appendix 2: Proof of Proposition 2

From Equation (19) and (20), we have

$$\begin{aligned}
0 &= n_i \frac{dV_i}{dI_i^g} + \frac{dV_i}{dt_i^1} \\
&= \frac{n_i}{N} \frac{\partial}{\partial n_i} F_i(I_i^g, n_i) v_c - u_g^i + (v_c \frac{n_i}{N} - u_c^i) n_i \frac{ds_i^*}{dI_i^g} + u_g^i - u_c^i + (v_c \frac{n_i}{N} - u_c^i) (\frac{ds_i^*}{db_i} - 1) \\
&= \frac{n_i}{N} \left(\frac{\partial}{\partial n_i} F(I_i^g, n_i) - 1 \right) v_c - \left(1 - \frac{n_i}{N} \right) v_c \left(n_i \frac{ds_i^*}{dI_i^g} + \frac{ds_i^*}{db_i} \right) \\
&= \left(\frac{\partial}{\partial n_i} F(I_i^g, n_i) - 1 \right) \frac{n_i}{N} v_c \left(1 - \frac{1 - \frac{n_i}{N}}{-\left(u_{cc}^i + \frac{n_i}{N} \frac{dv_c}{dZ} \right)} \frac{dv_c}{dZ} \right)
\end{aligned} \tag{A.6}$$

Since $1 - \frac{1 - \frac{n_i}{N}}{-\left(u_{cc}^i + \frac{n_i}{N} \frac{dv_c}{dZ} \right)} \frac{dv_c}{dZ} > 0$, we have

$$\frac{\partial}{\partial I_i^g} F(I_i^{g**}, n_i) = 1. \tag{A.6'}$$

Since Equation (21) is the function of g_i, I_i^g and Q_i , we have

$$\begin{aligned}
&u_g^i (\bar{c}^2(Z), \bar{G}(Z)) - u_c(z_i - g_i - I_i^g/n_i - Q_i, g_i) \\
&+ (v_c(\bar{c}^2(Z), \bar{G}(Z)) \frac{n_i}{N} - u_c(z_i - g_i - I_i^g/n_i - Q_i, g_i)) \frac{ds_i^*}{dt_i^1} = 0
\end{aligned} \tag{A.9}$$

where

$$Z \equiv \frac{1}{N} \sum_{j=1}^J (F_j(I_j^g, n_j) + n_j Q_j) \quad \text{and}$$

$$\frac{ds_i^*}{dt_i^1} = \frac{u_{cc}^i(z_i - g_i - I_i^g/n_i - Q_i, g_i) - u_{cg}^i(z_i - g_i - I_i^g/n_i - Q_i, g_i)}{-\left(u_{cc}^i(z_i - g_i - I_i^g - Q_i, g_i) + \frac{n_i}{N} \frac{dv_c(\bar{c}^2(Z), \bar{G}(Z))}{dZ}\right)}. \quad (\text{A.10})$$

The equilibrium $(\hat{g}_i, \hat{Q}_i, \hat{I}_i^g)$ can be characterized from Equations (A.6)-(A.7) and (A.9). Therefore we can confirm that $\hat{Q}_i = s_i^* - b_i$ and $\hat{g}_i = t_i^1 + b_i - \hat{I}_i^g$ are independent from the regulation level b_i^c , now we have $\frac{dt_i^1}{db_i} = -1$ and $\frac{ds_i^*}{db_i} = 1$. Also from $c_i^1 = z_i - g_i - I_i^g - Q_i$, \hat{c}_i^1 becomes independent from b_i^c

Appendix 3 : Example

In this appendix, we provide an example to compare the subgame-perfect solution with the first-best solution. We specify the utility function as follows.

$$U(c_i^1, c_i^2, g_i, G_i) = \log(c_i^1) + \log(g_i) + \log(c_i^2) + \log(G_i) \quad (\text{A.2.1})$$

$$c_i^1 = c^{1**}, g_i = g^{**}, c_i^2 = c^{2**}, G_i = G^{**} \text{ for all } i \text{ with } y'(I_i^{g**}) = 1.$$

The first-best allocation is characterized by $c^{1**} = g^{**} = c^{2**} = G^{**}$ with $y'(I_i^{g**}) = 1$. Overall resource constraint implies:

$$N(c^{1**} + g^{**} + c^{2**} + G^{**}) = \sum_{i=1}^I n_i y(I_i^{g**}) + \sum_{i=1}^I n_i z_i - \sum_{i=1}^I n_i I_i^{g**}, \quad (\text{A.2.2})$$

Now we have

$$c^{1**} = g^{**} = c^{2**} = G^{**} = \frac{1}{4}(\bar{y} + \bar{z} - \bar{I}^g) \quad (\text{A.2.3})$$

, where $\bar{y} \equiv \sum_{i=1}^I n_i y(I_i^g **)$, $\bar{z} \equiv \sum_{i=1}^I n_i z_i$, $\bar{I}^g \equiv \sum_{i=1}^I n_i I_i^g **$

From the ex post behavior of the central government and the local government in Stage 2, Equations (9) and (10) imply:

$$\bar{c}^2 = \bar{G}$$

Therefore, Equation (11) yields:

$$\bar{c}^2 = \bar{G} = \frac{1}{2N} \sum_{i=1}^J n_i (y(I_i^g) + s_i^* - b_i). \quad (\text{A.2.4})$$

Now we have

$$\bar{c}^2 = \frac{1}{2} (\bar{y} - \bar{b} - \bar{s}) \quad (\text{A.2.5})$$

, where $\bar{b} \equiv \sum_{i=1}^I n_i b_i$, $\bar{s} \equiv \sum_{i=1}^I n_i s_i^*$.

From Resident's saving optimization, Equation (17) implies:

$$c_i^1 = \bar{c}^1 = \bar{c}^2.$$

Then Equation (2) becomes

$$c_i^1 = z_i - s_i^* - t_i^1 = \bar{c}^2.$$

Therefore we have

$$\bar{c}^2 = \bar{z} - \bar{t}^1 - \bar{s}. \quad (\text{A.2.6})$$

Where $\bar{t}^1 \equiv \sum_{i=1}^I n_i t_i^1$

Combining (A.2.5) with (A.2.6), we have the equilibrium levels as follows.

$$\bar{c}^2 = \frac{1}{3}(\bar{z} - \bar{t}_i^1 + \bar{y} - \bar{b})$$

With Equation (3), we have

$$\bar{c}^2 = \frac{1}{3}(\bar{z} - \bar{t}_i^1 + \bar{y} - \bar{b}) = \frac{1}{3}(\bar{z} + \bar{y} - \bar{g} - \bar{I}^s) \quad (\text{A.2.7})$$

Using Equation (17), Equation (20) becomes

$$\frac{1}{\hat{g}^i} - \frac{1}{\bar{c}^2} - \frac{1}{\bar{c}^2} \left(1 - \frac{n_i}{N}\right) \left(\frac{1}{-\left(u_{cc}^i + \frac{n_i}{N} \frac{dv_c}{dZ}\right)} (u_{cc}^i - u_{cg}^i) \right) = 0.$$

Noting $\frac{dv_c}{dZ} = \frac{1}{v_{GG} - 2v_{cG} + v_{cc}} (v_{cc}(v_{GG} - v_{cG}) + v_{cG}(v_{cc} - v_{cG})) = \frac{v_{cc}}{2}$, we have

$$\frac{1}{\hat{g}^i} - \frac{1}{\bar{c}^2} - \frac{1}{\bar{c}^2} \left(1 - \frac{n_i}{N}\right) \left(\frac{v_{cc}}{-\left(v_{cc} + \frac{n_i}{N} \frac{v_{cc}}{2}\right)} \right) = 0$$

$$\frac{1}{\hat{g}^i} = \frac{1}{\bar{c}^2} \left(1 + \left(1 - \frac{n_i}{N}\right) \left(\frac{1}{-\left(1 + \frac{n_i}{N} \frac{1}{2}\right)} \right) \right) = \frac{1}{\bar{c}^2} \left(\frac{3n_i}{2N + n_i} \right),$$

$$\text{Finally we have } g^i = \left(\frac{2N + n_i}{3n_i} \right) \bar{c}^2 = \left(\frac{1}{3} + \frac{2}{3} \frac{N}{n_i} \right) \bar{c}^2 \quad (\text{A.2.8})$$

Inserting (A.2.7) into (A.2.8), we have

$$\hat{g}^i = \left(\frac{1}{3} + \frac{2}{3} \frac{N}{n_i} \right) \frac{1}{3} (\bar{z} + \bar{y} - \bar{g} - \bar{I}^g)$$

Average of g_i , which is defined by $\bar{g} \equiv \sum_{i=1}^J n_i \hat{g}_i / N$, can be calculated by

$$\bar{g} \equiv \sum_{i=1}^J \frac{n_i \hat{g}_i}{N} = \frac{1}{3} (\bar{z} + \bar{y} - \bar{g} - \bar{I}^g) \left(\frac{1+2J}{3} \right).$$

Therefore we have

$$\bar{g} = \frac{\frac{1}{3} \left(\frac{1+2J}{3} \right)}{1 + \frac{1}{3} \left(\frac{1+2J}{3} \right)} (\bar{z} + \bar{y} - \bar{I}^g) \quad (\text{A.2.9})$$

Inserting (A.2.9) into (A.2.7), we have

$$\bar{c}^2 = \frac{1}{3} (\bar{z} + \bar{y} - \bar{g} - \bar{I}^g) = \frac{1}{3} \left(1 - \frac{1}{3} \frac{\frac{1}{3} \left(\frac{1+2J}{3} \right)}{1 + \frac{1}{3} \left(\frac{1+2J}{3} \right)} \right) (\bar{z} + \bar{y} - \bar{I}^g) = \frac{1}{3} \frac{1}{1 + \frac{1}{3} \left(\frac{1+2J}{3} \right)} (\bar{z} + \bar{y} - \bar{I}^g)$$

Now we have

$$\begin{aligned}\hat{g}^i &= \left(\frac{1}{3} + \frac{2N}{3n_i}\right) \bar{c}^2 = \frac{1}{3} \frac{\left(\frac{1}{3} + \frac{2N}{3n_i}\right)}{1 + \frac{1}{3} \left(\frac{1+2J}{3}\right)} (\bar{z} + \bar{y} - \bar{I}^g) \\ &= \frac{1}{4} \frac{4 \left(1 + 2\frac{N}{n_i}\right)}{3(10+2J)} (\bar{z} + \bar{y} - \bar{I}^g) = \frac{1}{4} \frac{1 + \frac{2(\frac{N}{n_i} - 1)}{3}}{1 + \frac{2(J-1)}{12}} (\bar{z} + \bar{y} - \bar{I}^g)\end{aligned}$$

In addition, we have:

$$\bar{c}^1 = \bar{c}^2 = \bar{G} = \frac{1}{4} \frac{36}{30+6J} (\bar{y} + \bar{z} - \bar{I}^g) = \frac{1}{4} \frac{6}{5+J} (\bar{y} + \bar{z} - \bar{I}^g)$$

Combining these yields the following:

$$\hat{Q}_i = s_i^* - b_i = z_i - \bar{c}^1 - \hat{g}_i = z_i - \frac{1}{4} \left[\frac{1 + \frac{2(\frac{N}{n_i} - 1)}{3}}{1 + \frac{2(J-1)}{12}} + \frac{6}{5+J} \right] (\bar{z} + \bar{y} - \bar{I}^g)$$

$$\langle \rangle Q_i^{**} = z_i - c^{**} - g_i^{**} = z_i - \frac{1}{2} (\bar{y} + \bar{z} - \bar{I}^g)$$

implying

$$\left[\frac{2\left(\frac{N}{n_i}-1\right)}{1+\frac{3}{1+\frac{2(J-1)}{12}}} + \frac{6}{5+J} \right] \gg 2 \quad \text{or} \quad \left[\frac{2\left(\frac{N}{n_i}-1\right)}{1+\frac{3}{1+\frac{2(J-1)}{12}}} \right] \gg 2 - \frac{6}{5+J} = \frac{2(2+J)}{5+J}$$

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