COMPARISON OF OPTIMAL PORTFOLIOS SELECTED BY MULTICRITERIAL MODEL USING ABSOLUTE AND RELATIVE CRITERIA VALUES

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ABSTRACT
In this paper we select an optimal portfolio on the Croatian capital market by using the multicriterial programming. In accordance with the modern portfolio theory maximisation of returns at minimal risk should be the investment goal of any successful investor. However, contrary to the expectations of the modern portfolio theory, the tests carried out on a number of financial markets reveal the existence of other indicators important in portfolio selection. Considering the importance of variables other than return and risk, selection of the optimal portfolio becomes a multicriterial problem which should be solved by using the appropriate techniques.

In order to select an optimal portfolio, absolute values of criteria, like return, risk, price to earning value ratio (P/E), price to book value ratio (P/B) and price to sale value ratio (P/S) are included in our multicriterial model. However the problem might occur as the mean values of some criteria are significantly different for different sectors and because financial managers emphasize that comparison of the same criteria for different sectors could lead us to wrong conclusions. In the second part of the paper, relative values of previously stated criteria (in relation to mean value of sector) are included in model for selecting optimal portfolio.

Furthermore, the paper shows that if relative values of criteria are included in multicriterial model for selecting optimal portfolio, the return in the subsequent period is considerably higher than if absolute values of the same criteria were used.

KEY WORDS: portfolio selection, multicriterial model, relative criteria values, absolute criteria values

MSC 62P20, 90B50, 91B28, 90C29.

RESUMEN
En este trabajo seleccionamos un portafolio optimal del mercado de capitales Croata usando programación multicriterio. De acuerdo a la teoría moderna la maximización de los retornos a riesgo mínimo debe ser la meta de la inversión de cualquier inversionista exitoso. Sin embargo, por el contrario las esperanzas de la teoría moderna de portafolio, las pruebas llevadas a cabo sobre un número de mercados financieros revela la existencia de otros importantes indicadores en la selección de portafolios. Considerando la importancia de las diferentes variables del retorno y el riesgo, la selección del portafolio optimal se convierte en un problema multicriterial el que debe ser resuelto usando apropiadas técnicas.

Para seleccionar un portafolio óptimo, valores absolutos de criterios, como retorno, riesgo, valor del precio de la tasa del ingreso (P/E), tasa del valor del precio de registro (P/B) y tasa del valor del precio de venta (P/S) son incluidos en nuestro modelo multicriterio. Sin embargo el problema puede aparecer como el valor medio de algunos criterios que son significantemente diferentes para distintos sectores y porque los financieros enfatizan que las comparaciones de los mismos criterios para diferentes sectores pueden llevarnos a tomar incorrectas conclusiones. En la segunda parte del trabajo, valores relativos de criterios previamente fijados (en relación con el valor medio del sector) son incluidos en el modelo para seleccionar el portafolio optimal.

Además, este trabajo muestra que si los valores relativos de los criterios son incluidos en el modelo multicriterial para seleccionar el portafolio óptimo, el retorno en el subsiguiente periodo es considerablemente mayor que el valor absoluto de los mismos criterios fueran usados.

1. INTRODUCTION

In 1952 H. M. Markowitz developed the first model for portfolio optimization and with that model he placed the foundation of the modern portfolio theory. His model is based upon only two criteria: a return and a risk. The risk is measured by the variance of returns' distribution. Markowitz shows how to calculate portfolio which has the highest expected return for a given level of risk, or the lowest risk for a given level of expected return (so-called efficient portfolio). The problem of portfolio selection, according to this theory, is a simple problem of quadratic programming which consists in minimizing risk while keeping in mind an expected return which should be guaranteed.

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The importance of Markowiz's work is reflected in the fact that he won the Nobel Prize for economics in 1990. However, paralel to introducing the Markowiz's model in common usage its limitations and disadvantages were being noticed. The assumptions of the Markowitz model for portfolio optimization are the following:

- utility function which presents the investor's preferences is a quadratic function
- the returns have normal distribution

These assumptions were the starting point of many criticisms of this model. The former supposes that the utility function of an investor U whose wealth W is of the type: $U = a_0 + a_1 W + a_2 W^2$. This function cannot be consistent with economic theory unless a_1 is positive and a_2 is negative. These two restrictions are indispensable in order to find that the amount invested (absolute or relative) in the ventured portfolio increases with the increase in wealth[2].

The majority of the empirical tests on the capital markets resulted in assymetrical and (or) leptokurtic distribution [8]. At such distributions variance is not an adequate risk measure. Having recognised the disadvantages of variance as risk measure, new models for the selection of optimal portfolio which use alternative measure have been develeoped [12],[18].

However, contrary to the expectations of the modern portfolio theory, the tests carried out on a number of financial markets have revealed the existence of other indicators, beside return and risk, important in portfolio selection. Considering the importance of variables other than return and risk, selection of the optimal portfolio becomes a multicriterial problem which should be solved by using the appropriate techniques. The multi-criteria nature of the portfolio selection is well presented in the paper of Khoury et al. still 1993 [10], and many multi-criteria methods have already applied in portfolio selection [6],[14], [16].

There are a vast number of criteria that can be taken into consideration in portfolio selection and they are usually classified into two groups: accounting criteria and those based on market values. The accounting criteria are obtained analyzing audit reports, income statement, quarterly balance sheets, dividend records, sales records, etc. There are a large number of them like profitability indicators, liquidity and solvency indicators and indicators of financial structure of the company. They are used by the analysts (or managers) to give a synthesized and clear idea about the firm’s financial situation. Second criteria are market criteria and they contain all the information used by the stock analysts to appreciate a stock’s performance. The criteria used at this level are the mean return, total risk (variance), systematic risk (beta), the size measured by the stock capitalization, the PER (price earning ratio), stock liquidity and others. The use of the one criteria or the other depends on the manager’s attitude and objectives [1], [6].

Although all these criteria are important in selecting optimal portfolio the question arises whether stocks of different sector are comparable on all these criteria.

The problem might occur as the mean values of some criteria are significantly different for different sectors and because financial managers emphasize that comparison of the same criteria for stocks of different sectors could lead us to wrong conclusion.

To solve this problem we introduce relative criteria value defined as ratio of absolute value criteria for certain stock and mean criteria value of the corresponding sector. Furthermore we select optimal portfolio using absolute

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[2] For a certain level of wealth (*W*) and a utility function (*U*), we define the measure of absolute risk aversion (*ARA*) as: $ARA = -\frac{U''(W)}{U'(W)}$, and the measure of relative risk aversion (*RRA*) as $RRA = W \quad ARA$. A utility function must have decreasing *ARA* ($\frac{dARA}{dW} < 0$) and constant *RRA* ($\frac{dRRA}{dW} = 0$).

and relative criteria values respectively and then we compare optimal portfolios selected by absolute and relative criteria values.

This paper is organized as follows: following this introduction, Section 2 we define relative criteria values. In Section 3, we present the multi-criterion procedure. Section 4 presents the application to Croatian capital market. Section 5 summarizes the paper and indicates the possible directions for further research.

2. RELATIVE AND ABSOLUTE CRITERIA VALUES

In this paper we propose multicriterial model for portfolio selection. We are focused on transformation of criteria values used for selection of optimal portfolio which will exclude fault occurred when we compare stocks from different sectors. Namely, stocks of different sectors cannot be compared on those criteria which mean value is significantly different for different sectors. For such criteria we introduce relative criteria value defined as ratio of absolute value criteria for certain stock and mean criteria value of corresponding sector, i.e.

$$\text{Relative criteria value for certain stock} = \frac{\text{absolute value criteria for certain stock}}{\text{mean criteria value of corresponding sector}} \tag{1}$$

With the aim to prove that stock evaluation based on the relative values of “problematic” criteria gives better results that stock evaluation based on the absolute values of those criteria we select optimal portfolio by multicriterial model using absolute and relative criteria values respectively and then we compare optimal portfolios selected by absolute and relative criteria values.
The comparison of the obtained portfolios will be carried out by taking returns of pre-selected stocks in subsequent period and calculating return of each optimal portfolio in subsequent period.
The selected model we will be applied on Zagreb Stock Exchange (ZSE) as a real case. Zagreb Stock Exchange (ZSE) is a major stock market in Croatia and its market value is more than sixty billion dollars [24].

3. THE MULTICRITERIAL MODEL

The multi-criterial model applied in this paper is based on the PROMETHEE approach. In accordance with the PROMETHEE method each alternative *P* (in our case portfolios) are evaluated with two flows. Positive flow $\Phi^+(P)$ indicates how much the alternative is better than the others (on all criteria). Accordingly, the higher the $\Phi^+(P)$ the better the alternative. Negative flow $\Phi^-(P)$ indicates how much better than *P* the other alternatives are, i.e. how much *P* is dominated by the others. Accordingly, the lesser $\Phi^-(P)$ the better the alternative.

PROMETHEE II calculates the net flow Φ as the difference between the two flows, i.e.

$$\Phi(P) = \Phi^+(P) - \Phi^-(P), \tag{2}$$

so the higher the net flow $\Phi(P)$ the better the alternative. Positive and negative flows are calculated by comparison of all the pairs of alternatives.

Since the overall portfolios that can be made up from a set of pre-selected shares are infinite, it is impossible to compare all of the pairs of portfolios. Therefore, Khoury and Martel (1990) as well as Zmitri (1998) suggest a different procedure which evaluates each alternative (or rather its positive and negative flow) of the calculation by comparison with two fictions portfolios: one ideal ($\overline{P}$) and the other anti-ideal ($\underline{P}$). The positive flow $\Phi^+(P)$ is then obtained by comparison with anti-ideal, where the higher $\Phi^+(P)$ the better the alternative (it could be said the more distant from the anti-ideal it is). Accordingly, the closer to the ideal the better the alternative or the lesser the $\Phi^-(P)$ the better the alternative. According to the PROMETHEE II method the portfolio is better if it has a higher net flow Φ .

For criteria C_j to maximize we'll have

$$C_j(\overline{P}) = \max_i C_j(s_i), \tag{3}$$

where $S = \{s_1, s_2, ..., s_N\}$ is the set of N pre-selected shares. In the same way

$$C_j(\underline{P}) = \min_i C_j(s_i). \tag{4}$$

The set of possible solutions is the set of portfolios which can be formed from pre-selected shares. For any possible portfolio *P* has to be $\Phi(\underline{P}) \leq \Phi(P) \leq \Phi(\overline{P})$.

Positive and negative flows are calculated separately for each particular criteria, i.e. for each criteria C_j ($j = 1,2,...,n$) we have to calculate $\Phi_j^+(P)$ i $\Phi_j^-(P)$, and the net flow is obtained as a weighted sum of the difference between those flows, i.e.

$$\Phi(P) = \sum_{j=1}^{n} w_j \left(\Phi_j^+(P) - \Phi_j^-(P) \right) \tag{5}$$

where w_j are weights of the criteria obtained in agreement with the decision maker (or some of the methods, e.g. Analytical Hierarchy Process method – AHP method)

Value of criterion *j* for the portfolio P is obtained by multiplying the share of each stock s_i in the portfolio with the value of criterion *j* for the stock s_i :

$$C_j(P) = \sum_{i=1}^{N} x_i C_j(s_i) \tag{6}$$

Where x_i is the share invested in s_i in the portfolio *P*. Naturally, $\sum_{i=1}^{N} x_i = 1$.

For each criterion C_j ($j = 1,2,...,n$) preference functions are defined as in the PROMETHEE method where indifference (q) and preference (p) thresholds are certain numbers from the interval $\left[0, C_j(\overline{P}) - C_j(\underline{P})\right]$, i.e. the following is applicable $0 \leq q, p \leq C_j(\overline{P}) - C_j(\underline{P})$.

By analogy, if Gauss criterion is used the criterion is also applicable to the parameter *s*. Naturally, $q \leq p$ is always true. Let's assume, which is in line with the economic significance of such thresholds, that $p \in \left[q, \frac{C_j(\overline{P}) - C_j(\underline{P})}{2} \right]$, i.e. the highest value of preference threshold cannot exceed half the span between the lowest and the highest value according to that criterion.

In this paper we use the linear preference criterion which includes only one threshold.

The linear preference function from the PROMETHEE method has the following form:

$$\Psi(d)=\begin{cases}\dfrac{d}{p} & 0\le d<p\\ 1 & d\le p\end{cases} \tag{7}$$

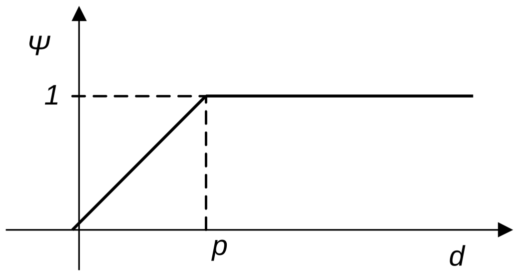

Figure 1. Linear function from PROMETHEE

Where *d* is the difference in evaluation of the two alternatives according to a certain criterion. In our case when portfolio *P* is compared to anti-ideal $(\underline{P})$, i.e. when we calculate $\Phi_j^+(P)$ difference *d* stands for the "distance" from anti-ideal (according to criterion *j*), i.e. the higher the difference the preference is closer to 1. Therefore, $d_j(P)=C_j(P)-C_j(\underline{P})=\sum_{i=1}^{N}x_iC_j(s_i)-C_j\left(\underline{P}\right)$. Since for every P $C_j(P)\ge C_j(\underline{P})$ is always true, it means that $d_j(P)\ge 0.$ is also always true. Therefore,

$$\Phi_j^+(P)=\Psi_j(P,\underline{P})=\begin{cases}\dfrac{C_j(P)-C_j(\underline{P})}{p} & 0\le C_j(P)-C_j(\underline{P})<p\\ 1 & C_j(P)-C_j(\underline{P})\ge p\end{cases} \tag{8}$$

The value of the positive flow will be higher if difference $d_j(P)=C_j(P)-C_j(\underline{P})$ is higher, i.e., when it exceeds *p* it will be 1. If we wish to present the positive flow as the portfolio P function we have:

$$\Phi_j^+(P)=\begin{cases}\dfrac{C_j(P)-C_j(\underline{P})}{p} & C_j(P)<C_j(\underline{P})+p\\ 1 & C_j(P)\ge C_j(\underline{P})+p\end{cases} \tag{9}$$

or graphically:

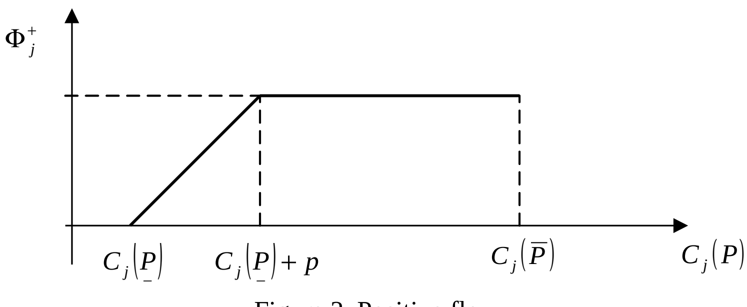

Figure 2. Positive flow

The figure clearly indicates that the positive flow is higher if $C_j(P)$ is higher (with the assumption that all criteria are maximized).

In analogy, the negative flow $\Phi_j^-(P)$ is considered for the difference of the portfolio *P* from the ideal $\left(\overline{P}\right)$. The smaller the distance $d_j(P)$ the better the portfolio. Since $C_j\left(\overline{P}\right) \geq C_j(P)$ is true for all possible portfolios *P* the difference $d_j(P)$ is in that case defined as: $d_j(P) = C_j\left(\overline{P}\right) - C_j(P)$, so that the negative flow has the form:

$$\Phi_j^-(P) = \begin{cases} \dfrac{C_j(P) - C_j\left(\underline{P}\right)}{p} & C_j(P) > C_j\left(\overline{P}\right) - p \\ 1 & C_j(P) \leq C_j\left(\overline{P}\right) - p \end{cases}. \qquad (10)$$

graphically:

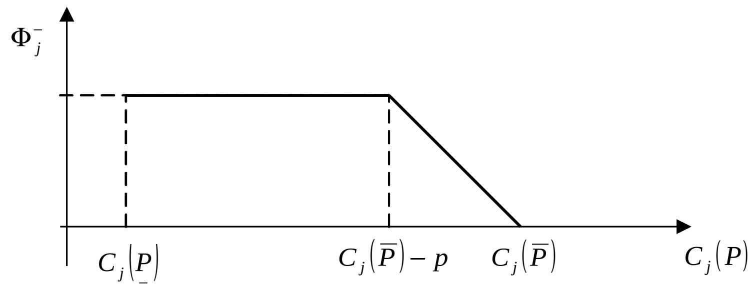

Figure 3. Negative flow

Therefore, the better the portfolio *P* according to criterion *j*, i.e. the higher $C_j(P)$ (where the criterion has to be maximized) the smaller $\Phi_j^-(P)$.

Finally, the net flow of the portfolio *P* is calculated as the difference, i.e. $\Phi_j(P) = \Phi_j^+(P) - \Phi_j^-(P)$, so we have:

$$\Phi_j(P) = \begin{cases} \dfrac{C_j(P) - C_j\left(\underline{P}\right)}{p_j^-} - 1 & C_j(P) < C_j\left(\underline{P}\right) + p_j^- \\ 1 - 1 \qquad C_j\left(\underline{P}\right) + p_j^- & C_j(P) < C_j\left(\overline{P}\right) - p_j^+ \\ 1 - \dfrac{C_j(P) - C_j\left(\underline{P}\right)}{p_j^+} & C_j(P) \geq C_j\left(\overline{P}\right) - p_j^+ \end{cases} \qquad \textbf{(11)}$$

$$\Phi_j(P) = \begin{cases} \dfrac{C_j(P) - C_j\left(\underline{P}\right) - p^-}{p^-}, & C_j(P) < C_j\left(\underline{P}\right) + p^- \\ 0, & C_j\left(\underline{P}\right) + p^- \leq C_j(P) < C_j\left(\overline{P}\right) - p^+ \\ \dfrac{p^+ + C_j(P) - C_j\left(\overline{P}\right)}{p^+}, & C_j(P) \geq C_j\left(\overline{P}\right) - p^+ \end{cases} \qquad (12)$$

The graphic presentation of the net flow according to criterion *j* is given in figure 4, where different threshold values for ”distance” from the ideal and anti-ideal are chosen.

Finally, the optimal portfolio is one that solves:

$$Max\,\Phi(P) \tag{13}$$

subject to;

$$\sum_{i=1}^{N} x_i = 1 \text{ i } 0 \le x_i \le x_{M\,i}\,, \tag{14}$$

where

$$\Phi(P) = \sum_{j=1}^{N} w_j \Phi_j(P)\,, \tag{15}$$

x_i : proportion invested in share $i\ (i = 1,2,...,N)$ in portfolio *P*,

$x_{M\,i}$ maximum proportion to invest in share *i* in portfolio *P*,

N is the number of pre-selected shares which can be included in portfolio *P*.

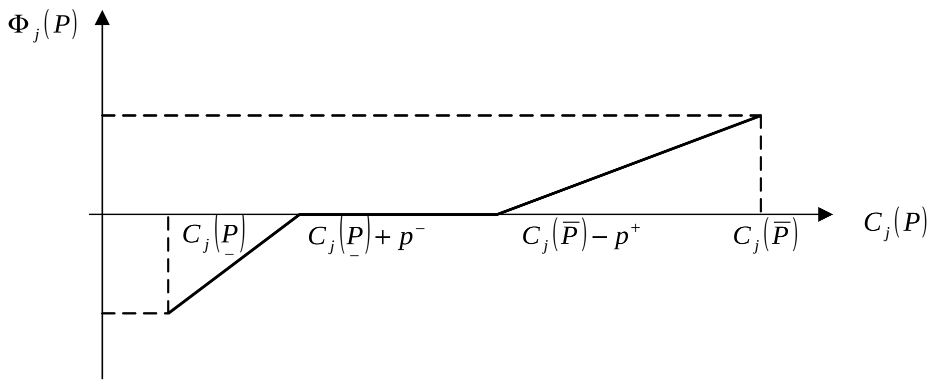

Figure 4. Net flow

4. APPLICATION TO CROATIAN CAPITAL MARKET

Table 1. Stock's absolute (Source: According to data on www.zse.hr)

	E(R)	Lover semi-variance	P/E	P/BV	P/S
	max	min	min	min	min
INA-R-A	-0,8635	3,0092	24,0000	2,2000	1,2000
ADRS-P-A	-0,2910	2,0545	14,0000	1,6000	3,1000
ATPL-R-A	3,2459	5,6079	23,3000	5,1000	3,9000
ERNT-R-A	-0,5970	2,4582	19,8000	3,4000	2,7000
PODR-R-A	-0,7195	1,8207	55,2000	1,8000	0,8000
IGH-R-A	3,6542	8,6027	38,8000	4,8000	3,0000
ZABA-R-A	-0,0817	3,1835	31,4000	3,4000	7,7000
DLKV-R-A	1,8973	4,3163	97,6000	12,2000	2,8000
CROS-R-A	0,5450	3,2167	39,6000	3,5000	2,1000
THNK-R-A	1,3197	6,4170	40,9000	4,6000	1,8000

Using the above model we subsequently calculate the optimal portfolios on the Croatian capital market. From the total number of securities quoted on the Zagreb stock exchange in 2007 a sample of ten shares has been separated. The shares sample contains the ten shares with the highest weights from CROBEX index in 2007: INA-R-A, ADRS-P-A, ATPL-R-A, ERNT-R-A, PODR-R-A, IGH-R-A, ZABA-R-A, DLKV-R-A, CROS-R-A, THNK-R-A.

First, absolute values of criteria, like return, risk, price to earning value ratio (P/E), price to book value ratio (P/B) and price to sale value ratio (P/S) are included in our multicriterial model for selecting optimal portfolio. The risk measure we used is the lower semi-variance (testing the yield distribution it turned out that the yields do not have normal distribution and therefore the variance is not an adequate risk measure)[12].

Table 2. Stock's relative values for the constructed criteria (Source: According to data on www.zse.hr)

	E(R)	Lover semi-variance	P/E	P/BV	P/S
	max	min	min	min	min
INA-R-A	-0,8635	3,0092	0,2051	0,7857	0,9131
ADRS-P-A	-0,2910	2,0545	0,1196	0,5714	2,3588
ATPL-R-A	3,2459	5,6079	0,2085	0,5167	0,4918
ERNT-R-A	-0,5970	2,4582	1,1092	1,1724	0,8438
PODR-R-A	-0,7195	1,8207	1,4568	0,7407	0,4167
IGH-R-A	3,6542	8,6027	0,4631	0,6877	1,5957
ZABA-R-A	-0,0817	3,1835	1,0680	1,1240	1,3750
DLKV-R-A	1,8973	4,3163	1,1650	1,7479	1,4894
CROS-R-A	0,5450	3,2167	0,7952	0,9859	1,0244
THNK-R-A	1,3197	6,4170	0,4882	0,6590	0,9574

In order to take into consideration the behaviour of investors, we proceed to the change of weights. Table 3 shows weights of six possible scenarios. We note, that this application is a simple illustration of the proposed approach. No real decision-maker is implied.

Table 3. Weight of each criterion

Criterion	Mean return	Lover semi-variance	P/E	P/B	P/S
Scenario 1	0.10	0.10	0.30	0.30	0.20
Scenario 2	0.10	0.10	0.20	0.30	0.30
Scenario 3	0.10	0.10	0.30	0.20	0.30
Scenario 4	0.10	0.10	0.40	0.20	0.20
Scenario 5	0.10	0.10	0.20	0.40	0.20
Scenario 6	0.20	0.20	0.20	0.20	0.20

Next we select optimal portfolio using relative value of P/E, P/B, P/S criteria. For criteria exclusively derived from data obtained from capital market like expected return, risk, stock liquidity there is no sense to use relative criteria values (for example, investor always prefer stock with higher stock liquidity no matter what the mean value of stock liquidity of the sector is).

Table 4. Optimal portfolios with relative criteria values

	INA-R-A	**ADRS-P-A**	**ATPL-R-A**	**ERNT-R-A**	**PODR-R-A**	**IGH-R-A**	**ZABA-R-A**	**DLKV-R-A**	**CROS-R-A**	**THNK-R-A**	**return in next period**
	Scenario 1										
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,0000	0,2000	0,0000	0,0000	0,0000	0,2000	1,2766
x_M=0,19	0,1900	0,1900	0,1900	0,0000	0,0000	0,1900	0,0000	0,0000	0,0500	0,1900	0,9967
x_M=0,18	0,1800	0,1800	0,1800	0,0000	0,0000	0,1800	0,0000	0,0000	0,1000	0,1800	0,7167
x_M=0,17	0,1700	0,1700	0,1700	0,0000	0,0000	0,1700	0,0000	0,0000	0,1500	0,1700	0,4368
x_M=0,16	0,1600	0,1600	0,1600	0,0000	0,0400	0,1600	0,0000	0,0000	0,1600	0,1600	0,2537
x_M=0,15	0,1500	0,1500	0,1500	0,0000	0,1000	0,1500	0,0000	0,0000	0,1500	0,1500	0,1191
	Scenario 2										
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,2000	0,0000	0,0000	0,0000	0,0000	0,2000	-0,8799
x_M=0,19	0,1900	0,1900	0,1900	0,0000	0,1900	0,0000	0,0000	0,0000	0,0500	0,1900	-1,0520
x_M=0,18	0,1800	0,1800	0,1800	0,0000	0,1800	0,0000	0,0000	0,0000	0,1000	0,1800	-1,2241
x_M=0,17	0,1700	0,1700	0,1700	0,0000	0,1700	0,0000	0,0000	0,0000	0,1500	0,1700	-1,3963
x_M=0,16	0,1600	0,1600	0,1600	0,0000	0,1600	0,0400	0,0000	0,0000	0,1600	0,1600	-1,0402
x_M=0,15	0,1500	0,1500	0,1500	0,0000	0,1500	0,1000	0,0000	0,0000	0,1500	0,1500	-0,4200
	Scenario 3										
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,0000	0,0000	0,0000	0,0000	0,2000	0,2000	-1,3644
x_M=0,19	0,1900	0,1900	0,1900	0,0000	0,0000	0,0500	0,0000	0,0000	0,1900	0,1900	-0,8521
x_M=0,18	0,1800	0,1800	0,1800	0,0000	0,0000	0,1000	0,0000	0,0000	0,1800	0,1800	-0,3397
x_M=0,17	0,1700	0,1700	0,1700	0,0000	0,0000	0,1500	0,0000	0,0000	0,1700	0,1700	0,1727
x_M=0,16	0,1600	0,1600	0,1600	0,0000	0,0400	0,1600	0,0000	0,0000	0,1600	0,1600	0,2537
x_M=0,15	0,1500	0,1500	0,1500	0,0000	0,1000	0,1500	0,0000	0,0000	0,1500	0,1500	0,1191
	Scenario 4										
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,0000	0,2000	0,0000	0,0000	0,0000	0,2000	1,2766
x_M=0,19	0,1900	0,1900	0,1900	0,0000	0,0000	0,1900	0,0000	0,0000	0,0500	0,1900	0,9967
x_M=0,18	0,1800	0,1800	0,1800	0,0000	0,0000	0,1800	0,0000	0,0000	0,1000	0,1800	0,7167
x_M=0,17	0,1700	0,1700	0,1700	0,0000	0,0000	0,1700	0,0000	0,0000	0,1500	0,1700	0,4368
x_M=0,16	0,1600	0,1600	0,1600	0,0400	0,0000	0,1600	0,0000	0,0000	0,1600	0,1600	0,2748
x_M=0,15	0,1500	0,1500	0,1500	0,1000	0,0000	0,1500	0,0000	0,0000	0,1500	0,1500	0,1718
	Scenario 5										
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,2000	0,0000	0,0000	0,0000	0,0000	0,2000	-0,8799
x_M=0,19	0,1900	0,1900	0,1900	0,0000	0,1900	0,0500	0,0000	0,0000	0,0000	0,1900	-0,3918
x_M=0,18	0,1800	0,1800	0,1800	0,0000	0,1800	0,1000	0,0000	0,0000	0,0000	0,1800	0,0964
x_M=0,17	0,1700	0,1700	0,1700	0,0000	0,1700	0,1500	0,0000	0,0000	0,0000	0,1700	0,5845
x_M=0,16	0,1600	0,1600	0,1600	0,0000	0,1600	0,1600	0,0000	0,0000	0,0400	0,1600	0,5445
x_M=0,15	0,1500	0,1500	0,1500	0,0000	0,1500	0,1500	0,0000	0,0000	0,1000	0,1500	0,2403
	Scenario 6										
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,2000	0,0000	0,0000	0,0000	0,0000	0,2000	-0,8799
x_M=0,19	0,1900	0,1900	0,1900	0,0000	0,1900	0,0000	0,0000	0,0000	0,0500	0,1900	-1,0520
x_M=0,18	0,1800	0,1800	0,1800	0,0000	0,1800	0,0000	0,0000	0,0000	0,1000	0,1800	-1,2241
x_M=0,17	0,1700	0,1700	0,1700	0,0000	0,1700	0,0000	0,0000	0,0000	0,1500	0,1700	-1,3963
x_M=0,16	0,1600	0,1600	0,1600	0,0310	0,1600	0,0090	0,0000	0,0000	0,1600	0,1600	-1,3584
x_M=0,15	0,1500	0,1500	0,1500	0,0751	0,1500	0,0249	0,0000	0,0000	0,1500	0,1500	-1,1898

By programming net flow and solving problem (13)-(15) in MATLAB we get solutions given in table 4 and 5. For calculation of an optimal portfolio we introduce constraints of maximal proportion because we want to avoid portfolio which are not diversified. Maximum proportion constraint are $x_M = 0.2,\ x_M = 0.19,\ \ x_M = 0.18,$ $x_M = 0.17,\ x_M = 0.16,\ x_M = 0.15.$

From the tables 4 and 5 we can see that portfolios obtained using absolute and relative criteria values in the multicriterial model are significantly different. Furthermore we took returns of pre-selected stocks in subsequent period and we calculated the return of each optimal portfolio in subsequent period. It is important to note that we have to make distinction between return in subsequently period which is measure of efficiency of the portfolio and expected return which is one of criteria used in multicriterial model for portfolio selection[3].

The results presented in the tables also indicate that every portfolio obtained using relative criteria values in model have higher return in the subsequent period than portfolios obtained using absolute criteria values[4].

Table 5. Optimal portfolios with absolute criteria values

	INA-R-A	**ADRS-P-A**	**ATPL-R-A**	**ERNT-R-A**	**PODR-R-A**	**IGH-R-A**	**ZABA-R-A**	**DLKV-R-A**	**CROS-R-A**	**THNK-R-A**	**return in next period**
	Scenario 1										
x_M=0,2	0,2000	0,2000	0,2000	0,2000	0,0000	0,0399	0,0000	0,0000	0,1601	0,0000	-2,2615
x_M=0,19	0,1900	0,1900	0,1900	0,1900	0,0822	0,0711	0,0000	0,0000	0,0867	0,0000	-1,7267
x_M=0,18	0,1800	0,1800	0,1800	0,1800	0,0459	0,0541	0,0000	0,0000	0,1800	0,0000	-2,1158
x_M=0,17	0,1700	0,1700	0,1700	0,1700	0,0830	0,0670	0,0000	0,0000	0,1700	0,0000	-1,9324
x_M=0,16	0,1600	0,1600	0,1600	0,1600	0,1201	0,0799	0,0000	0,0000	0,1600	0,0000	-1,7489
x_M=0,15	0,1500	0,1500	0,1500	0,1500	0,1500	0,0865	0,0000	0,0000	0,1500	0,0135	-1,5300
	Scenario 2										
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,2000	0,0342	0,0000	0,0000	0,1658	0,0000	-2,4422
x_M=0,19	0,1900	0,1900	0,1900	0,0035	0,1900	0,0465	0,0000	0,0000	0,1900	0,0000	-2,3408
x_M=0,18	0,1800	0,1800	0,1800	0,0421	0,1800	0,0579	0,0000	0,0000	0,1800	0,0000	-2,1473
x_M=0,17	0,1700	0,1700	0,1700	0,0805	0,1700	0,0695	0,0000	0,0000	0,1700	0,0000	-1,9524
x_M=0,16	0,1600	0,1600	0,1600	0,1190	0,1600	0,0810	0,0000	0,0000	0,1600	0,0000	-1,7580
x_M=0,15	0,1500	0,1500	0,1500	0,1500	0,1500	0,0859	0,0000	0,0000	0,1500	0,0141	-1,5317
	Scenario 3										
x_M=0,2	0,2000	0,2000	0,2000	0,2000	0,1139	0,0861	0,0000	0,0000	0,0000	0,0000	-1,3746
x_M=0,19	0,1900	0,1900	0,1900	0,1900	0,1439	0,0961	0,0000	0,0000	0,0000	0,0000	-1,2468
x_M=0,18	0,1800	0,1800	0,1800	0,1800	0,1738	0,1062	0,0000	0,0000	0,0000	0,0000	-1,1184
x_M=0,17	0,1700	0,1700	0,1700	0,1700	0,1700	0,1024	0,0000	0,0000	0,0476	0,0000	-1,2541
x_M=0,16	0,1600	0,1600	0,1600	0,1600	0,1600	0,0961	0,0000	0,0000	0,1039	0,0000	-1,4379
x_M=0,15	0,1500	0,1500	0,1500	0,1500	0,1500	0,0865	0,0000	0,0000	0,1500	0,0135	-1,5301
	Scenario 4										
x_M=0,2	0,2000	0,2000	0,2000	0,2000	0,0000	0,0398	0,0000	0,0000	0,1602	0,0000	-2,2619
x_M=0,19	0,1900	0,1900	0,1900	0,1900	0,0088	0,0412	0,0000	0,0000	0,1900	0,0000	-2,2990
x_M=0,18	0,1800	0,1800	0,1800	0,1800	0,0459	0,0541	0,0000	0,0000	0,1800	0,0000	-2,1156
x_M=0,17	0,1700	0,1700	0,1700	0,1700	0,0830	0,0670	0,0000	0,0000	0,1700	0,0000	-1,9322
x_M=0,16	0,1600	0,1600	0,1600	0,1600	0,1201	0,0799	0,0000	0,0000	0,1600	0,0000	-1,7488
x_M=0,15	0,1500	0,1500	0,1500	0,1500	0,1500	0,0865	0,0000	0,0000	0,1500	0,0135	-1,5300
	Scenario 5										
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,2000	0,0477	0,0000	0,0000	0,1523	0,0000	-2,2627
x_M=0,19	0,1900	0,1900	0,1900	0,0108	0,1900	0,0491	0,0000	0,0000	0,1802	0,0000	-2,2846
x_M=0,18	0,1800	0,1800	0,1800	0,0452	0,1800	0,0591	0,0000	0,0000	0,1757	0,0000	-2,1219
x_M=0,17	0,1700	0,1700	0,1700	0,0816	0,1700	0,0699	0,0000	0,0000	0,1685	0,0000	-1,9439
x_M=0,16	0,1600	0,1600	0,1600	0,1190	0,1600	0,0810	0,0000	0,0000	0,1600	0,0000	-1,7580
x_M=0,15	0,1500	0,1500	0,1500	0,1500	0,1500	0,0864	0,0000	0,0000	0,1500	0,0136	-1,5304
	Scenario 6										
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,1661	0,0339	0,0000	0,0000	0,2000	0,0000	-2,5273
x_M=0,19	0,1900	0,1900	0,1900	0,0036	0,1900	0,0464	0,0000	0,0000	0,1900	0,0000	-2,3408
x_M=0,18	0,1800	0,1800	0,1800	0,0420	0,1800	0,0580	0,0000	0,0000	0,1800	0,0000	-2,1466
x_M=0,17	0,1700	0,1700	0,1700	0,0805	0,1700	0,0695	0,0000	0,0000	0,1700	0,0000	-1,9524
x_M=0,16	0,1600	0,1600	0,1600	0,1190	0,1600	0,0810	0,0000	0,0000	0,1600	0,0000	-1,7580
x_M=0,15	0,1500	0,1500	0,1500	0,1500	0,1500	0,0865	0,0000	0,0000	0,1500	0,0135	-1,5300

5. SUMMARY AND CONCLUSIONS

[3] When we use the multicriterial model for selecting optimal portfolio proposed in this paper, we except that portfolio which have lower P/E, P/B, P/S, lower semi-variance and higher excepted return (which is calculated as mean return which stock realize in past periods) will achieve higher return in subsequent period.

[4] Base of portfolios could seem relatively small but for purposes of this paper is large enough to assure representative results and to inspire others scientists and financial experts for research on other capital market.

The aim of this paper was to point out one of the problems that arise when the multicriteria model is applied to selection of optimal portfolios. The problem researched in this paper refers to the comparability of stock from different sectors according to particular criteria. Namely, stocks of different sectors cannot be compared on those criteria the mean value of which is significantly different in different sectors. For such criteria we introduce relative criteria value defined as ratio of absolute value criteria for certain stock and mean criteria value of corresponding sector. Aiming at proving that stock evaluation based on the relative values of "problematic" criteria gives better results than stock evaluation based on the absolute values of those criteria we select optimal portfolio using absolute and relative criteria values respectively and then we compare optimal portfolios selected by absolute and relative criteria values.

The comparison of the obtained portfolios was carried out by taking the returns of pre-selected stocks in subsequent period and by calculating the return of all optimal portfolios in subsequent period. The results obtained form the Croatian capital market indicate that every portfolio obtained using relative criteria values in model has higher return in subsequent period than the portfolios obtained using absolute criteria values.

From all previously stated facts we can conclude that using relative values of criteria proposed in this paper in multicriterial models gives better results (in sense of return) than using absolute criteria values.

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