# Fiscal Issues in a Monetary Union<sup>\*</sup>

Guido Traficante<sup>†</sup>

First draft: November 2005 This draft: August 2007

#### Abstract

Constituting a Monetary Union requires not only the implementation of a common monetary policy, but also that countries coordinate their national fiscal policies. In a two country Monetary Union we explicitly show why fiscal stability, expressed as an intertemporal solvency condition, matters. The Monetary Union is assumed without breakup and is modeled as a dynamic game characterized by strategic interaction between the two players in the choice of the national fiscal policy the two countries intend to follow. It is the degree of symmetry between the idiosyncratic shocks which determines the state of nature and the desirability of a fiscal constraint over a more flexible regime. With perfectly asymmetric shocks, the two national governments will prefer to set independently their national fiscal policies in order to neutralize the effects of the shocks.

*Keywords*: Monetary Union; Fiscal Policy; Cross-country spillovers *JEL*: C7; E5

<sup>\*</sup>I wish to thank Francesco Lippi, Federico Ravenna and Carl Walsh for precious comments to my research proposal. I also thank Abhijit Sen Gupta, Peter Tillmann, Monika Thomas, Giorgio Di Giorgio, Gustavo Piga, Michele Bagella and seminar participants at the University of California, Santa Cruz and Tor Vergata University. Any remaining errors are, of course, my sole responsibility.

<sup>&</sup>lt;sup>†</sup>Tor Vergata University and LUISS-Guido Carli University. Address for correspondence: Department of Economics and Institutions, Tor Vergata University, Via Columbia 2, 00133 Rome, Italy. Email:gtraficante@luiss.it

### 1 Introduction

When two or more countries decide to constitute a Monetary Union (MU henceforth) they relinquish their autonomous monetary policy in favor of a common one. A traditional argument which supports the constitution of the MU deals with the non efficient nature of the non-cooperative policies taken by the national governments. In particular, the cost of giving up the autonomy in monetary policy is offset by the benefit arising from the impossibility of using the exchange rate *strategically*, e.g. with the aim of improving the balance of trade. Unfortunately, this strategic use of the exchange rate can lead to poor performances in improving the trade balance and, even worse, the result of a depreciation can be a high inflation level. In the latter case, the MU arrangement could be rationalized to prevent an improper use of the exchange rate.

In the last years, also because of the recent creation of the European Monetary Union (EMU), a huge literature has examined why some countries are prone to relinquish their independence in monetary policy in favor of a common central bank. While the traditional cost-benefit analysis<sup>1</sup> has been conducted with a very simplified dynamics to understand the incentives of constituting a MU, in a recent paper Fuchs and Lippi (2005), using a dynamic setup without an exogenous enforcement technology, show that a MU can be preferred to other forms of coordination among countries in the field of monetary economics. In particular, they prove that optimal policy in the MU must take account of individual incentives to leave the union. Furthermore, also the breakup is one of the optimal policies because there are some states of the world where the idiosyncratic shocks hitting the national economies are so asymmetric that countries find it too costly to give up the national monetary policy to react against them. It is the degree of symmetry in the shocks which determines if the MU is temporary or permanent.

<sup>&</sup>lt;sup>1</sup>See, for example, Persson and Tabellini (1995) and Dixit (2000).

In particular, the beauty of the methodology followed by Fuchs and Lippi is that it can be used in all the frameworks where two parties must coordinate their actions. The aim of this paper is to analyze how to coordinate national fiscal policies in a MU, evaluating which coordination mechanism is preferable. The analysis will be carried out in a dynamic setup with no commitment device available to the policy-makers. Differently from Fuchs and Lippi, we will model a MU without breakup, as we will discuss below.

Previous literature, in fact, shows that in a MU coordination is important not only to decide the common monetary policy to implement, but also in terms of the fiscal policies each national government runs. Woodford (1998) proves that a country who wants to share a common currency with another exposes itself to price-level instability if the partner is left to follow a *non-Ricardian* fiscal policy, even if the country itself is fiscally re $sponsible^2$ . Even worse, the existence of a common currency increases the temptation of reckless fiscal policies. That constitutes a rationale for the existence of explicit ceilings upon member countries' fiscal policies, e.g. the EMU Stability and Growth Pact<sup>3</sup>. In a couple of papers, however, Chari and Kehoe (1998 and 2002) show that the desirability of imposing fiscal constraints upon the countries members of a MU depends on the possibility the common central bank has to commit to its future policies. Without commitment, a benevolent monetary authority finds it optimal to set high inflation rates when the inherited debt levels of the member states are large. When a fiscal authority in a member state decides how much debt to issue, a freeriding problem arises: it recognizes the incentives of the monetary authority

<sup>&</sup>lt;sup>2</sup>According to Canzoneri et al. (2001), in a *Ricardian* regime the nominal anchor is monetary policy, whereas in a *non-Ricardian* regime fiscal policy provides the nominal anchor. The same authors assess the empirical plausibility of both regimes, concluding that the Ricardian regime fits better some episodes of the postwar US data.

<sup>&</sup>lt;sup>3</sup>In a recent paper, Leith and Wren-Lewis (2006) analyze a two country model of monetary union with nominal inertia and finitely lived consumers. They then evaluate the combinations of fiscal and monetary policies which guarantee price level determinacy and macroeconomic stability. In order to reach the latter objectives, there is need of an agreement between member countries on fiscal policy, but the required control of debt and/or deficit is less than what is prescribed by the Stability and Growth Pact.

to partially monetize its debt and it issues too much debt. In other words, when the commitment technology is not feasible to the monetary authority, fiscal constraints are desirable, otherwise there is no need for them to exist.

Beetsma and Bovenberg (1998) model a multi-country MU with national fiscal authorities acting as a Stackelberg leaders vis-à-vis the common central bank. The rationale for assuming a fiscal leadership is that in the EMU the European Central Bank is independent<sup>4</sup> but its policies can be affected indirectly through the effects of tax policies. Beetsma and Bovenberg find that monetary unification may discipline fiscal and monetary policy, by reducing inflation, taxes and public spending: This effect increases with the number of member countries, because in a larger union the strategic position of an individual government vis-'a-vis the common central bank weakens. On the other hand, fiscal coordination affects the main benefit which a MU generates: In absence of coordination, especially if the MU is large, the effect of unilateral change of tax policy on the common monetary policy is small, so that governments are discouraged from using taxes strategically. With coordination, each fiscal player internalizes the effects of a unilateral tax change on the other fiscal players and induce the central bank to change the inflation rate in the direction preferred by the fiscal players. Beetsma, Debrun and Klaassen (2001) use a two country MU to describe the strategic interaction among the two fiscal authorities and the common central bank. They find that fiscal coordination, realized by a supranational institution which minimizes the sum of the national governments loss functions, is desirable when the two member countries are hit by asymmetric disturbances. Without coordination, the country hit by a bad (good) shock would choose an excessively expansionary (contractionary) fiscal stance, in an attempt to offset the spillover effect of the fiscal contraction (expansion) in the other country. Under coordination, fiscal authorities internalize the fact that their mutual actions partially offset each other and they economize on the use of

<sup>&</sup>lt;sup>4</sup>This feature is modeled by assuming a central bank which does not take into account the governments' budget constraints.

their instruments. These findings, however, are at odds with conventional wisdom and a straightforward economic interpretation<sup>5</sup>, since they suggest to coordinate fiscal policies when in practice it is very difficult to achieve, i.e. when shocks are strongly asymmetric. The analysis we present in the following sections will emphasize the importance of the shocks correlation in order to decide the way in which coordinating national fiscal policies. Our conclusions, however, will be completely different from the ones in Beetsma, Debrun and Klaassen, and we think that a crucial element which justifies such a difference lies in the fact that our model is dynamic, imposing on the member countries to plan their policies also for the future periods. In a series of papers, Dixit and Lambertini (2001, 2003a and 2003b) draw an analogy between the inflation bias arising in the models when the central bank cannot commit and what happens in a MU where the central bank commits and the national governments are left free to follow discretionary fiscal policies. In this case, all the advantages arising from the commitment technology vanish, because the fiscal authority reaction function acts as a constraint on the monetary rule.

The evidence is, therefore, mixed about the need of fiscal constraints in a MU. It is clear that a coordination device among the members of the currency union is necessary, but the quoted literature analyzed the way of solving the problem of coordinating national fiscal policies in a very simplified way. In particular, the coordination among national fiscal policies seems very difficult and costly to implement and that leaves room to the presence of explicit fiscal constraints. Notice that the rationale behind this enforced cooperation (the fiscal constraints) among countries in the MU is similar to that which makes countries prefer an enforced arrangement, expressed by a common monetary policy, rather than trying to coordinate their independent monetary policies. The presence of fiscal constraints for member countries of a MU, however, adds to the loss of the monetary policy

<sup>&</sup>lt;sup>5</sup>See, for example, Buti and Sapir, 1998.

as an instrument to react against the shocks hitting the economy; in this scenario, fiscal policy is the only instrument available to the national governments, so that they could find it very costly to lose the possibility of not manoeuvring it freely. There is, therefore, a trade-off between the necessity of eliminating the incentive of running reckless fiscal policies by the part of the national governments and their willingness of using the fiscal policy as a tool to contrast the idiosyncratic shocks hitting the national economies.

Bearing that in mind, the traditional approach of assuming that the success of a MU is contingent upon the adherence to some time-invariant fiscal constraints is unsatisfactory under the theoretical viewpoint and the empirical side. Theoretically, the cost derived by the presence of fiscal constraints adds to the loss of autonomy in monetary policy. Empirically, several currency unions which preceded the EMU failed because of nasty fiscal shocks hitting one or more member countries<sup>6</sup>, usually due to war expenses.

In order to analyze this issue, we use the "limited commitment" technology, pioneered by Thomas and Worrall (1988) and Kocherlakota (1996), and recently applied by Fuchs and Lippi (2006) to the field of the currency unions. As we said above, the Fuchs and Lippi's setup is very stylized, so that it can be used for a variety of problems dealing with the coordination problem between agents' action. Before applying Fuchs and Lippi's methodology, in a two country MU model, we formally derived the cross-country fiscal spillovers between them. It emerges that the fiscal solvency in a MU must hold in aggregate terms, not necessarily for each country separately, so that the presence of a country which continuously runs reckless fiscal policy per se is not excluded if its counterpart complies with this kind of fiscal policy. A country in the MU, therefore, can have a debt which grows at an explosive rate if its counterpart's debt decreases proportionally. Technically, fiscal solvency requires that an intertemporal equilibrium condition must be satisfied: in particular, fiscal solvency in a MU calls for an inversely related

 $<sup>^{6}\</sup>mathrm{For}$  a detailed survey of the currency unions which preceded the EMU historically, see Cohen (1993) and Bordo and Jonung (1997).

relationship between the present value of the primary deficits run by each national governments.

This finding complicates the stylized setup present in Fuchs and Lippi (2006). In the latter paper, each period the two policy makers decide the policies to implement, and if they coincide, they constitute a MU, while the alternative regime is the one with independent monetary policies. In their setup, the dynamics of the game has just a strategic origin, but there do not exist any physical state variables; in other words, Fuchs and Lippi (2005) analyze a *repeated game*. Things are different in the case of fiscal policy, and in particular if we consider the deficit level each country runs time by time. The intertemporal equilibrium condition is a constraint to be satisfied, so that the game we are dealing with is a *dynamic game* characterized by: 1) the strategic interaction between the two players and 2) the presence of a physical state variable. The problem now becomes more complicated, however we analyze a particular case where it is possible to mimic the Fuchs and Lippi's methodology. In particular, we assume a MU without breakup where in each period the two countries should set simultaneously their fiscal policies. There can be also some states of the world in which the two countries do not agree on the national fiscal policies to implement and in the latter case we assume that they run a common fiscal policy such that aggregate fiscal solvency holds.

The paper is organized as follows. The cross-country fiscal spillovers in a MU are formally derived in the next section. Section 3 presents the behavior of the two countries which constitute the MU. Section 4 describes the dynamic game without breakup. Section 5 describes an example with an ad hoc objective function for the two national governments before concluding in section 6. Some appendices provide a derivation of some results.

### 2 MU and fiscal variables: a theoretical model

Here we present how the fiscal issue in a two-country MU arises considering an economic model where the economic agents are: a representative household and a national fiscal authority in each country, and a supranational central bank. For simplicity, we name the two countries Home and Foreign respectively and we let an asterisk denote Foreign variables.

#### 2.1 The household's problem

In this subsection we present the economic environment of the representative agent in our MU. Each country is populated by an infinitely-lived household receiving a stochastic endowment of  $y_t$ . Let us assume that the household maximizes the following utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U[c_t + g_t, M_t/p_t]$$
 (2.1)

s.t. 
$$M_t + B_t = p_t y_t - p_t c_t - p_t T_t + M_{t-1} + R_{t-1} B_{t-1}$$
 (2.2)

$$M_t, c_t \ge 0 \tag{2.3}$$

where  $U(c + g, m^7)$  is an increasing, concave function of both arguments, and  $\beta \in (0, 1)$  is the discount factor.  $R_t$  is the gross interest rate,  $T_t$  is the lump-sum taxation and  $B_t$  is the amount of risk-less bonds held by the household.

In the specification (2.1), we have assumed that real money balances enter the utility function. Government purchases  $g_t$  are perfect substitutes for private consumption expenditures  $c_t$ . This assumption allows us to focus on the effects of fiscal policy upon private budget constraint. In other words, government purchases have the same effect on the economy as transfers to households in order to finance private consumption of exactly the same amount. Consistently with this assumption, taxes are only lump-sum,

<sup>&</sup>lt;sup>7</sup>We define  $m_t \equiv \frac{M_t}{p_t}$ .

so that a tax increase has the same effect as a reduction in transfers to the households. The representative household faces the budget constraint, expressed by (2.2), which requires that the end-of-period financial wealth, i.e. money balances plus risk-less bonds, must be equal to the sum of the financial wealth at the beginning of the period and the net income, given by the difference between the stochastic endowment  $y_t$  and the consumption.<sup>8</sup>

Defining the financial wealth  $W_t \equiv R_{t-1}B_{t-1} + M_{t-1}$ , the no-Ponzi game condition implies the following transversality condition

$$\lim_{T \to \infty} \left( \prod_{j=t}^{T-1} R_j^{-1} \right) W_T = 0.$$
(2.4)

If we sum over the infinite horizon the single-period household budget constraint for each period and we impose the transversality condition (2.4), we get the following intertemporal budget constraint:

$$\frac{W_t}{p_t} = \sum_{s=t}^{\infty} \left( \prod_{j=t}^{s-1} r_j^{-1} \right) \left[ c_s + T_s - y_s + \frac{R_s - 1}{R_s} m_s \right]$$
(2.5)

where  $r_t = R_t \frac{p_t}{p_{t+1}}$  represents the real return of bonds.

The household maximization problem implies the usual conditions

$$U_{c,t} = \left(\frac{R_t}{R_{t-1}}\right) U_{m,t} \tag{2.6}$$

$$U_{c,t} = \beta R_t E_t \left(\frac{p_t}{p_{t+1}} U_{c,t+1}\right)$$
(2.7)

### 2.2 Countries' behavior

In each of the two countries, the national government does not control the creation of money, but they do control the evolution of the public expenditure and the lump-sum taxation. Continuing to analyze just the Home

<sup>&</sup>lt;sup>8</sup>The consumption we are referring to is given by the private expenditures plus public transfers, expressed simply by lump-sum taxation  $T_t$ , given our characterization of the utility function.

government, we can write its budget constraint (in real terms) as 9

$$g_t + \frac{R_{t-1}b_{t-1}^g}{1+\pi_t} = T_t + b_t^g$$
(2.8)

where  $\pi_t$  indicates the gross inflation rate and  $b_t^g$  is the debt issued by the fiscal authority, expressed in real terms. According to (2.8), each period the Home fiscal authority sustains public expenditure,  $g_t$ , plus interest payments on the outstanding debt,  $(R_{t-1}-1)b_{t-1}^g$ ; it has tax revenue  $T_t$  plus new issues of interest-bearing debts,  $b_t^g - b_{t-1}^g$  as financing sources.

The supranational central bank controls the issue of money,  $H_t$  and is supposed to equally divide the seigniorage revenue between the two countries, buying the debt issued by Home and Foreign:

$$B_t^{H(CB)} = R_{t-1}B_{t-1}^{H(CB)} + \frac{H_t - H_{t-1}}{2}$$
(2.9)

$$B_t^{F(CB)} = R_{t-1}B_{t-1}^{F(CB)} + \frac{H_t - H_{t-1}}{2}$$
(2.10)

In (2.9) and (2.10)  $B_t^{H(CB)}$  and  $B_t^{F(CB)}$  represent the amount of Home and Foreign's debt held by the central bank. The choice of modeling the central bank in this simplified way is because we intend to focus on the national fiscal policies. Furthermore, the need of fiscal coordination among countries belonging to a MU emerges also if the central bank's policy is formulated in a less *passive* way. Technically, as shown for instance by Woodford (1998), the fiscal issue in a MU consists in the presence of an aggregate fiscal solvency condition, which is derived below.

Furthermore, we have the following market clearing conditions:

$$c_t + c_t^* + g_t + g_t^* = y_t + y_t^* \tag{2.11}$$

$$M_t + M_t^* = H_t (2.12)$$

$$B_t^g + B_t^{g*} = B_t + B_t^* + B_t^M (2.13)$$

<sup>&</sup>lt;sup>9</sup>See Appendix A for details.

As shown in Woodford (1998) and Bergin (2000), a necessary condition for the existence of a rational expectation equilibrium<sup>10</sup> is that the *consolidated* no-Ponzi game condition holds, i.e.

$$\lim_{T \to \infty} \left( \prod_{j=t}^{T-1} R_j^{-1} \right) (W_t + W_t^*) = 0.$$
 (2.14)

This can be alternatively seen by looking at the following equation, showing that the sum between Home and Foreign's intertemporal budget constraints must equalize the sum of the intertemporal conditions for the three government entities. In particular, we have that<sup>11</sup>

$$\frac{R_{t-1}(B_{t-1} + B_{t-1}^*) + H_{t-1}}{p_t} = \sum_{s=t}^{\infty} \left( \prod_{j=t}^{s-1} r_j^{-1} \right) \left[ \Delta_s + \Delta_s^* + \frac{R_s - 1}{R_s} \frac{H_s}{p_s} \right]$$
(2.15)

where market clearing conditions have been used, and  $\Delta_t \equiv T_t - g_t$  represents the primary surplus of each country.

The last two equations show us what is the fiscal issue in a MU: the fact that (2.14) holds in aggregate terms does not necessarily imply that it holds for each country's debt level separately. Suppose Home's debt level grows at an explosive rate: this does not lead to the violation of equilibrium if Foreign's debt level decreases proportionally to offset Home's profligacy. This, of course, has a huge impact on the fiscal policy each country runs every period, i.e. on the level of  $\Delta_t$  and  $\Delta_t^*$ . In other words, Foreign is requested to purchase Home's increasing debt indefinitely, so that Foreign becomes a net creditor to Home. Notice that this is true independently on the responsible conduct of fiscal policy held by Foreign. Foreign's complicity to Home's reckless fiscal policy, in turn, means that each period a certain amount of wealth is subtracted to Foreign's citizens in favor of Home's ones.<sup>12</sup> This

 $<sup>^{10}</sup>$ Formally, a rational expectation equilibrium consists of a sequence of price level, interest rate, consumption and endowment streams, money balances, debt and transfers consistent with all the equilibrium conditions discussed above, both for Home and Foreign.

<sup>&</sup>lt;sup>11</sup>See Appendix B for details.

 $<sup>^{12}\</sup>mathrm{To}$  that extent, consider how the household's utility function was modeled and the absence of distortionary taxation.

agreement between the two countries is not painful if we assume perfect insurance between Home and Foreign, but this is not necessarily true because the households are ruled by two different governments, with distinct policy objectives.

The previous analysis, therefore, constitutes a rationale for the existence of some ceilings the public debt and the public deficit must be within, as envisioned by the EMU Stability and Growth Pact. Constituting a MU, in fact, requires not only to find a common monetary policy to follow, but also that the temptation each country has to run reckless fiscal policies must be contrasted with a harsh punishment. Nevertheless, it is worth noticing that in a MU the fiscal instrument is the only available instrument the national government has to react against adverse shocks hitting the national economy.

Given the household's behavior both in Home and Foreign, in the next section we describe the equilibrium of the dynamic game which is used to characterize the interactions between two governments belonging to a Monetary Union (MU). Before doing that, we try to simplify the notation to make the reasoning easier to follow.

Summing all the governmental entities' budget constraints and imposing equilibrium in the bonds' market, gives the following equation:

$$R_{t-1}\left(B_{t-1} + B_{t-1}^*\right) = B_t + B_t^* - p_t\left(\Delta_t + \Delta_t^*\right) + H_t - H_{t-1}$$
(2.16)

Equation (2.16) corresponds to the sum of the household's budget constraints in Home and Foreign. We assume that the MU is without breakups, hence equation (2.16) is always satisfied. Meeting condition (2.16) requires both countries to coordinate their national fiscal policies so that the transversality condition

$$\lim_{T \to \infty} \frac{B_{T+1} + B_{T+1}^*}{\prod_{j=t}^T R_j} = 0$$
(2.17)

holds.

Throughout the rest of the paper, to simplify the notation, we will ex-

press equation (2.16) as

$$A_{t+1} = R_t \left[ A_t + p_t \left( \Delta_t + \Delta_t^* \right) \right] - (R_t - 1) H_t$$
(2.18)

where  $A_t \equiv R_{t-1} (B_{t-1} + B_{t-1}^*) + H_{t-1}$ .

## 3 The environment

In this section we describe the behavior of the national governments in the two countries, Home and Foreign, belonging to a MU with the characteristics described in the previous section. The state of the world in period t is stochastic and is determined by the realization of a discrete i.i.d. random variable  $\theta_t$  with support  $S = \{s_1, s_2, \cdots, s_S\}$ ; the probability of  $\theta_t$  equalling s is denoted by  $\pi_s$ . The two governments have the same objective, represented by the following functions:

$$E_t \sum_{i=0}^{\infty} \beta^i U\left(\Delta_{t+i}, \theta_{t+i}\right) \tag{3.1}$$

$$E_t \sum_{i=0}^{\infty} \beta^i U^* \left( \Delta_{t+i}^*, \theta_{t+i} \right)$$
(3.2)

The function  $U(\Delta_t, \theta_t) (U^*(\Delta_t^*, \theta_t))$  is assumed to be bounded, concave and at least twice differentiable with respect to  $\Delta$  ( $\Delta^*$ ). Each allocation  $(\Delta_t, \Delta_t^*)_{t=1}^{\infty}$  must be such that equation (2.16) (or the equivalent formulation (2.18)) holds: if an allocation has these features, it is defined as a *feasible* allocation. The random variable  $\theta_t$  affects the utility function in terms of the optimal  $\Delta$  ( $\Delta^*$ ) to set. For example  $\theta_t$  determines a time-varying and country-specific policy target. We assume that the joint distribution of  $\theta_t$  is symmetric across the two countries.

A period-t history  $h_t$  is defined as the sequence of realizations for  $\theta_t$ ,  $[Bt, B_t^*]$  and  $[\Delta_t, \Delta_t^*]$ :

$$h_{t} = \left(\theta_{1}, \Delta_{1}, \Delta_{1}^{*}, B_{1}, B_{1}^{*}, \dots, \theta_{t-1}, \Delta_{t-1}, \Delta_{t-1}^{*}, B_{t-1}, B_{t-1}^{*}, \theta_{t}\right).$$

A strategy provides Home and Foreign with an action for each possible history. In other words, a strategy maps any possible history  $h_t$  into a policy choice  $[\Delta_t, \Delta_t^*]$ . In this dynamic game which models a MU without breakup, Home enters a contract with Foreign; a contract is a sequence of feasible history-dependent functions  $[\Delta_t, \Delta_t^*]$ . In absence of a commitment technology, at each point in time a player can renege the contract. A subgame perfect equilibrium specifies a strategy for each agent such that at every possible history a player is playing the best response to the other player's strategy. A policy pair  $[\Delta_t, \Delta_t^*]$  is subgame perfect if the player is worse off after deviating from the prescribed strategy  $[\Delta_t, \Delta_t^*]$ . Formally the following must hold (for all  $s \in S$  and  $\tau = 0, 1, 2, \ldots$ ):

$$U\left(\Delta_{\tau},\theta_{\tau}\right) + \beta E_{\tau} \left[\sum_{i=1}^{\infty} \beta^{i-1} U\left(\Delta_{\tau+i},\theta_{\tau+i}\right)\right] \ge U\left(\Delta_{\tau}^{d},\theta_{\tau}\right) + \beta \underline{v} \quad (3.3)$$

$$U^*\left(\Delta^*_{\tau}, \theta_{\tau}\right) + \beta E_{\tau} \left[\sum_{i=1}^{\infty} \beta^{i-1} U^*\left(\Delta^*_{\tau+i}, \theta_{\tau+i}\right)\right] \ge U\left(\Delta^{*d}_{\tau}, \theta_{\tau}\right) + \beta \underline{v} \quad (3.4)$$

where  $\Delta_{\tau}^{d}$  and  $\Delta_{\tau}^{*d}$  stand for the optimal deviation and  $\underline{v}$  is the lowest value attainable with a subgame perfect policy pair. Alternatively,  $\underline{v}$  can be defined as the worst sustainable value of the game.

#### 3.1 Domain of the value function

Let  $\Gamma$  denote the set of subgame perfect policy pairs, which is compact and convex. A contract  $[\Delta_t, \Delta_t^*]$  in  $\Gamma$  is efficient if there exists no other element in  $\Gamma$  which Pareto dominates it. The utility functions calculated for policy pairs in  $\Gamma$  constitute the set of subgame perfect payoffs, denoted by V.

Previously we introduced the variable  $\underline{v}$  saying that it is the lowest value attainable with a subgame perfect policy pair. In particular,  $\underline{v}$  solves the following problem:

$$\underline{v} \equiv \min_{\Delta_s^*, v_s^*} \sum_s \left[ U\left( \left( \frac{A_{t+1}}{p_t R_t} - \frac{A_t}{p_t} + \frac{(R_t - 1) H_t}{p_t R_t} - \Delta_s^* \right), \theta_s \right) + \beta V\left(v_s^*\right) \right] \pi_s$$
(3.5)

subject to

$$U^*\left(\Delta_s^*, \theta_s\right) + \beta v_s^* \ge U^*\left(\Delta_s^{*d}, \theta_s\right) + \beta \underline{v} \quad \forall s$$

$$U\left(\left(\frac{A_{t+1}}{p_t R_t} - \frac{A_t}{p_t} + \frac{(R_t - 1)H_t}{p_t R_t} - \Delta_s^*\right), \theta_s\right) + \beta V\left(v_s^*\right) \ge U\left(\Delta_s^d, \theta_s\right) + \beta \underline{v} \quad \forall s$$
$$(V\left(v_s^*\right), v_s^*) \in [\underline{v}, \overline{v}]$$

The first two constraints the minimization is subject to impose the policy pair to be subgame perfect, using  $\underline{v}$  as the punishment value in case of a deviation form the previously agreed strategy. In order to verify the last constraint, we need to know the best value attainable with a subgame perfect policy pair, indicated by  $\overline{v}$ . Bearing in mind that the subgame perfect equilibrium is self-rewarding,  $\overline{v}$  solves the following problem:

$$\overline{v} \equiv \max_{\Delta_s^*, v_s^*} \sum_s \left[ U\left( \left( \frac{A_{t+1}}{p_t R_t} - \frac{A_t}{p_t} - \frac{H_t}{p_t R_t} - \Delta_s^* \right), \theta_s \right) + \beta \overline{v} \right] \pi_s$$
(3.6)

 $subject \ to$ 

$$U^* \left( \Delta_s^*, \theta_s \right) + \beta \overline{v} \ge U^* \left( \Delta_s^{*d}, \theta_s \right) + \beta \underline{v} \quad \forall s$$
$$U \left( \left( \left( \frac{A_{t+1}}{p_t R_t} - \frac{A_t}{p_t} + \frac{(R_t - 1) H_t}{p_t R_t} - \Delta_s^* \right), \theta_s \right) + \beta \overline{v} \ge U \left( \Delta_s^d, \theta_s \right) + \beta \underline{v} \quad \forall s$$
$$\overline{v} = \sum_s \left[ U^* \left( \Delta_s^*, \theta_s \right) + \beta \overline{v} \right] \pi_s$$

where the last constraint imposes that  $\overline{v}$  is the maximum value available to reward adherence to the policy. It emerges that condition  $(V(v_s^*), v_s^*) \in$ V is to be verified by iteration: in fact, after computing a candidate for  $\underline{v}$ , we compute  $\overline{v}$  corresponding to the candidate  $\underline{v}$  and then check that  $(V(v_s^*), v_s^*) \in V$ .

# 4 The dynamic game without MU breakup

In the previous section we characterized the upper and lower bound of the value function, i.e. the minimum and maximum value both countries can get coordinating their national fiscal policies. We are not assuming MU breakup: in this setup, no breakup means that, for a given monetary policy run by a supranational central bank, Home and Foreign must guarantee that the aggregate solvency constraint (2.18) holds. In other words, we are ruling out any possible breakup due to *fiscal reasons*. Therefore, each period. Home and Foreign must jointly set the level of primary deficit to implement and, in contrast with a closed economy setup, the policy chosen by each country spills over into its counterpart in the MU. We can imagine that each period Home and Foreign observe how the realization of  $\theta$  affects the unconstrained value of primary deficit they want to run respectively. However, that unconstrained value can be implemented only if the counterpart cooperates, i.e. it moves away from its own unconstrained primary deficit in order to satisfy equation (2.18). Of course, the bigger the distance from the unconstrained deficit the more painful is the sacrifice in terms of utility; therefore, why should a country belonging to a MU (say Home) adjust its fiscal policy in favor of its counterpart (Foreign)? Except the assumption of no breakups, in a infinite-time horizon, Home adjusts its fiscal policy towards its counterpart's policy target, knowing that in the future Foreign could be asked to do the same in its favor. In other words, there exists a mechanism of cooperative insurance between Home and Foreign.

There can be also some states of the world in which Home and Foreign do not agree on the national fiscal policies to implement: in this case, given the assumption of no break-up, we assume that they run a common deficit value  $\frac{\tilde{\Delta}}{2}$  such that the aggregate deficit value  $\tilde{\Delta}$  guarantees that (2.18) holds. We are aware that there are other autarkic values harsher than the choice of a common deficit level, but considering that a common fiscal policy on two countries belonging to a MU adds to the loss of the national monetary policy,  $\frac{\tilde{\Delta}}{2}$  seems a harsh enough punishment to make Home and Foreign cooperate. Furthermore, the presence of a harsher and time dependent punishment value would be requested if we were to analyze not only the MU regime, but also the regime of independent national monetary policies (INMP), as in Fuchs-Lippi. In the latter case, in fact, a different autarkic value is deeply dependent on the sources of the breakup: if the MU breakup is consensual, Home and Foreign coordinate their policies in the most efficient way, while if the breakup occurs because of a unilateral deviation by one country, it is optimal to punish that country as harshly as possible.

It emerges that the problem of the policy pair  $[\Delta_t, \Delta_t^*]$  to choose can be interpreted as if there were a planner in the economy who assigns an expected utility  $V(v^*)$  to Home, conditional on having promised an expected utility level  $v^*$  to Foreign ("promise keeping" constraint, see equation (4.3) below). Furthermore, the previous analysis allows us to redefine the function  $V : [\underline{v}, \overline{v}] \rightarrow [\underline{v}, \overline{v}]$  and to characterize the Pareto frontier for the pair  $[\underline{v}, \overline{v}]$ with the following problem:

$$V(v_0^*) = \max_{\Delta,\Delta^*} E_0\left[\sum_{t=0}^{\infty} \beta^t U(\Delta_t, \theta_t)\right]$$
(4.1)

subject to

$$(\Delta, \Delta^*) \in \Gamma \tag{4.2}$$

$$E_0\left[\sum_{t=0}^{\infty} U^*\left(\Delta_t^*, \theta_t\right)\right] = v_0^* \tag{4.3}$$

The function V is the Pareto frontier; using arguments analogous to those of Fuchs and Lippi (2004, 2005), we can conclude that V is differentiable almost everywhere and that it satisfies the following functional equation:

Гас

$$V(v_0^*) = \max_{\{\Delta_s^*, v_s^*\}} \sum_s \pi_s \left[ U\left( \left( \frac{A_{s+1}}{p_s R_s} - \frac{A_s}{p_s} + \frac{(R_s - 1)H_s}{p_s R_s} - \Delta_s^* \right), \theta_s \right) + \beta V(v_s^*) \right]$$
(4.4)

 $subject \ to$ 

$$\sum_{s} \pi_s \left[ U^* \left( \Delta_s^*, \theta_s \right) + \beta v_s^* \right] = v_0^* \tag{4.5}$$

$$U^*\left(\Delta_s^*, \theta_s\right) + \beta v_s^* \ge U_C^*\left(\theta_s\right) + \beta V_C^* \tag{4.6}$$

$$U\left(\left(\frac{A_{s+1}}{p_s R_s} - \frac{A_s}{p_s} + \frac{(R_s - 1)H_s}{p_s R_s} - \Delta_s^*\right), \theta_s\right) + \beta V\left(v_s^*\right) \ge U_C\left(\theta_s\right) + \beta V_C$$

$$(4.7)$$

$$v_s^* \in [\underline{v}, \overline{v}] \tag{4.8}$$

According to the previous formulation, we can imagine that in a MU a social planner promises Foreign an amount  $v_0^*$  with corresponding utility (4.4). Taking the promise keeping constraint (equation (4.5)) as given, the planner maximizes Home's welfare subject to the participation constraints, respectively for Foreign and Home (equations (4.6) and (4.7)), requiring that both countries find it profitable not to follow the common fiscal policy  $\frac{\tilde{\Delta}}{2}$  which delivers an expected utility  $V_C$ . Finally, (4.8) imposes that promised continuation values lie in the domain of the value function.

For any feasible allocation providing Foreign with a utility level of  $v_0^*$ , we can divide the possible states of the world into the following four regions:

 $S_1 =$  states in which (4.6) binds

- $S_2 =$  states in which (4.7) binds
- $S_3$  = neither (4.6) and (4.7) binds

 $S_4 = \text{both } (4.6) \text{ and } (4.7) \text{ bind}$ 

The first-order conditions with respect to  $v_s^*$  in maximizing equation (4.4) subject to (4.5) - (4.8) give:

$$(\pi_s + \nu_s) V'(v_s^*) + \lambda \pi_s + \mu_s = 0 \quad \text{if} \quad v_s^* \in (\underline{v}, \overline{v})$$
  

$$\geq 0 \quad \text{if} \quad v_s^* = \overline{v}$$
  

$$\leq 0 \quad \text{if} \quad v_s^* = \underline{v} \quad (4.9)$$

where  $\lambda$  is the Lagrange multiplier on (18),  $\mu_s$  is the multiplier on (19), and  $\nu_s$  is the multiplier on (20).

The first-order condition with respect to  $\Delta_s^*$  gives:

$$(\pi_{s} + \nu_{s}) U' \left[ \left( \frac{A_{s+1}}{p_{s}R_{s}} - \frac{A_{s}}{p_{s}} + \frac{(R_{s} - 1)H_{s}}{p_{s}R_{s}} - \Delta_{s}^{*} \right), \theta_{s} \right] + (\lambda \pi_{s} + \mu_{s}) (U^{*})' (\Delta_{s}^{*}, \theta_{s}) = 0$$
(4.10)

An internal solution for  $v_s^*$  implies, via equations (22) and (4.10) the following condition:

$$\frac{U'\left(\Delta_s\left(\Delta_s^*\right), \theta_s\right)}{\left(U^*\right)'\left(\Delta_s^*, \theta_s\right)} = V'\left(v_s^*\right) \tag{4.11}$$

Equation (4.11) tells us that at an optimal point the countries' marginal rate of substitution is equal to the technical rate of transformation, expressed by the slope of the efficient frontier. Let us consider more specifically the properties of the four regions  $(S_1 - S_4)$ , bearing in mind also the envelope condition  $V'(v_0^*) = -\lambda$ .

**Region**  $S_1$ : Only Foreign's participation constraint binds, i.e.  $\nu_s = 0$ . This yields:

$$V'(v_s^*) = V'(v_0^*) - \frac{\mu_s}{\pi_s}$$
(4.12)

which implies that  $v_s^* > v_0^*$  because of the concavity of V and the fact that V' < 0. Hence, Foreign requires a larger prize in terms of fiscal freedom and future utility to remain in the prescribed policy without any deviation.

**Region**  $S_2$ : Only Home's participation constraint binds, i.e.  $\mu_s = 0$ . This yields the following condition:

$$V'(v_s^*) = V'(v_0^*) \frac{\pi_s}{\pi_s + \nu_s}$$
(4.13)

With a similar reasoning to that in Region  $S_1$ , we can conclude that  $v_s^* < v_0^*$ . In this region, the promised utility delivered to Foreign decreases; the result is, therefore, symmetric opposite to those obtained in region  $S_1$ .

**Region**  $S_3$ : Neither Home's nor Foreign's participation constraint binds, hence  $\mu_s = \nu_s = 0$ . This yields

$$V'(v_s^*) = V'(v_0^*) \tag{4.14}$$

and, by the strict concavity of V,  $v_s^* = v_0^*$ . When neither participation constraint binds, the expected utility to each country is the same one with which they entered the period s, i.e. the promised value is kept constant at  $v_0^*$  for Foreign and  $V(v_0^*)$  for Home. This, in turn, implies a constant ratio between the marginal utilities of Home and Foreign. In this region, the policy sequence  $(v_0^*, V(v_0^*))$  is subgame-perfect, so that there is no incentive to deviate from it. As shown in Fuchs and Lippi (2004, 2005), when neither participation constraint binds, we are in a situation which is isomorphic to a social planner's problem of maximizing the aggregate welfare, with constant and time-invariant Pareto weights.

### 5 An example economy

We assume that the utility functions reflects the cost of not hitting the desired policy target:

$$U\left(\Delta_t, \theta_t\right) \equiv \left(1 - \beta\right) \left[-\frac{1}{2} \left(\Delta_t - \varepsilon_t\right)^2\right]$$
(5.1)

$$U^*\left(\Delta_t^*, \theta_t\right) \equiv (1 - \beta) \left[ -\frac{1}{2} \left(\Delta_t^* - \varepsilon_t^*\right)^2 \right]$$
(5.2)

so that the state of the world is modeled as a time-varying target for the deficit. In this case, the first order condition with respect to  $\Delta^*$  is given by

$$(\pi_s + \nu_s) \left( \Delta_s - \varepsilon_s \right) = (\lambda \pi_s + \mu_s) \left( \Delta_s^* - \varepsilon_s^* \right) \tag{5.3}$$

which, combined with (2.18), gives the following policy for  $\Delta$ :

$$\Delta_s = \frac{\pi_s + \nu_s}{\pi_s(1+\lambda) + \nu_s + \mu_s} \varepsilon_s + \frac{\lambda \pi_s + \mu_s}{\pi_s(1+\lambda) + \nu_s + \mu_s} \left(\frac{A_{s+1}}{P_s R_s} - \frac{A_s}{P_s} + \frac{(R_s - 1)H_s}{P_s R_s} - \varepsilon_s^*\right)$$
(5.4)

From equation (5.4) it is evident how the policy for Home's deficit is a convex combination between its own policy target and Foreign target, taking as given the evolution of financial wealth and the monetary policy followed by the supranational central bank. The value of the weights will vary accordingly if either or both participation constraints do not bind. From the previous policy function it is not possible to draw analytical conclusions in terms of the preferences towards a common fiscal policy or autonomous fiscal policies. A special case, however, deserves attention, namely the case in which there exists perfectly negative correlation between the two shocks  $\varepsilon$ and  $\varepsilon^*$ . In this case it is straightforward to prove that there is a one-to-one response of the national fiscal policy to the idiosyncratic shock hitting the national economy. In particular, the optimal response of  $\Delta$  and  $\Delta^*$  will be

$$\Delta_s = \varepsilon_s + \frac{\lambda \pi_s + \mu_s}{\pi_s (1+\lambda) + \nu_s + \mu_s} \left( \frac{A_{s+1}}{p_s R_s} - \frac{A_s}{p_s} + \frac{(R_s - 1) H_s}{p_s R_s} \right)$$
(5.5)

$$\Delta_s^* = \varepsilon_s^* + \frac{\pi_s + \nu_s}{\pi_s(1+\lambda) + \nu_s + \mu_s} \left(\frac{A_{s+1}}{p_s R_s} - \frac{A_s}{p_s} + \frac{(R_s - 1)H_s}{p_s R_s}\right)$$
(5.6)

This is a crucial result of this paper: When the two countries belonging to a MU are hit by perfectly asymmetric shocks, there is no reason to coordinate their national fiscal policies according to an *ad hoc* common policy rule, since letting the two governments decide autonomously allow them to perfectly counteract the effect of the idiosyncratic shock hitting the economy. The rationale behind this finding is that when disturbances are country-specific, it is too costly for a government to lose the possibility of using the fiscal instrument after having delegated monetary policy to a supranational institution. How can we transfer this result to the conduct of fiscal policy in the relatively new EMU? We can think that in the first stages of the EMU the convergence process among the national economies was slow and, according to our analysis, the cost of relinquishing fiscal policy was really high for a EMU member country. In other words, similarly to what found in Canzoneri et al. (2001) and by Leith and Wren-Lewis (2006), the deficit cap imposed by the Maastricht Treaty is much stronger than necessary. However, new member countries are joining the EMU, hence the problem of having too harsh deficit constraints is still a hot issue.

As mentioned above, the case in which neither participation constraint binds is isomorphic to a social planner's maximization problem of maximizing the aggregate welfare, with constant and time-invariant Pareto weights. The result will be the so called *dictatorial outcome*, where a planner (or dictator) maximizes the aggregate welfare of Home and Foreign, with weights given by  $\omega$  and  $(1 - \omega)$  respectively.

In the latter case, the dictator solves the following problem:

$$\max_{\Delta,\Delta^*} \sum_{t=0}^{\infty} \beta^t \sum_{s \in \mathcal{S}} \pi_s \left\{ \omega U \left( \Delta_t, \theta_t \right) + (1 - \omega) U^* \left( \Delta_t^*, \theta_t \right) \right\}$$
(5.7)

$$s.t.$$
 (2.18) (5.8)

The optimality condition gives the following relationship between Home and Foreign's marginal utilities

$$\omega U'\left(\Delta_t, \theta_t\right) = \left(1 - \omega\right) \left(U^*\right)'\left(\Delta_t^*, \theta_t\right) \tag{5.9}$$

which, considering the shape of the utility functions in this example economy and equation (2.18) gives the following policies for  $\Delta_t$  and  $\Delta_t^*$  respectively:

$$\Delta_t = \omega \varepsilon_t + (1 - \omega) \left( \frac{A_{t+1}}{P_t R_t} - \frac{A_t}{P_t} + \frac{(R_t - 1) H_t}{P_t R_t} - \varepsilon_t^* \right)$$
(5.10)

$$\Delta_t^* = (1-\omega)\varepsilon_t^* + \omega \left(\frac{A_{t+1}}{P_t R_t} - \frac{A_t}{P_t} + \frac{(R_t - 1)H_t}{P_t R_t} - \varepsilon_t\right)$$
(5.11)

From the two previous equations it is straightforward that for  $\omega = 1$  the dictator only aims at Home's utility, whereas for  $\omega = 0$  he takes into account only Foreign's utility. In the case when neither participation constraint binds  $\omega = \frac{1}{1+\lambda}$ , but as the promise-keeping constraint always binds ( $\lambda$  is always strictly positive), the social planner cannot aim only at one country's utility.

### 6 Concluding remarks

The recent creation of the EMU has revived the interest in the field of currency unions. Previous literature showed how the choice of sharing a currency requires also to coordinate the national fiscal policies among the member countries of the MU. This paper, after deriving the fiscal issue present in a MU composed by two countries, extends the limited commitment technology approach to the choice of the deficit each country can run. In doing

that, we assume that the central bank is autonomous in its policy and the national governments act consequently to insure the union-wide fiscal solvency. In other words, we are excluding the possibility of a MU break-up due to fiscal reasons, as occurred in some historical episodes (Cohen, 1993). In particular, the fiscal solvency in a MU is shown to be guaranteed whenever an intertemporal solvency condition, relating the aggregate present value of the deficits with the aggregate level of debt, holds. This finding complicates the analysis with respect to the previous application (Fuchs and Lippi, 2005) of the limited commitment to the field of the MU, because in the latter case we deal with a repeated game, while the presence of a state variable like the deficit makes the game dynamic. Nevertheless, we choose to analyze a situation where the intertemporal solvency condition holds, but it is possible to apply the same methodology as in Fuchs and Lippi (2005). Namely, we consider the case in which each period countries must set its national fiscal policies to attain a union-wide deficit target, represented by an arbitrarily small value which guarantees the intertemporal fiscal solvency within the union. In particular, countries can simultaneously run deficit levels in accordance with the union-wide fiscal solvency, but if they fail to meet the aggregate solvency condition, they must follow a common fiscal policy. In this way, we depart from the previous literature according to which the success of a MU is contingent on the presence of an explicit time-invariant ceiling over the national fiscal policies.

The desirability of an explicit fiscal constraint is shown to be contingent on the state of nature and in particular on the degree of symmetry between idiosyncratic shocks. The main message deriving from our analysis is that the presence of strongly asymmetric shocks calls for independent national fiscal policies, without the need of having a global fiscal target within the MU. Similarly to what found in other papers which dealt the same topic using different models and methods, fiscal stabilization used in the EMU via the deficit rule established in the Maastricht Treaty can be considered too harsh.

While we focused on the way national fiscal policies must be coordinated between two countries belonging to a MU, we do not consider the possibility of a MU breakup caused by fiscal shocks. Furthermore, in analyzing fiscal policy, we do not consider the distortionary taxation. As shown in Duarte and Wolman (2005), there is evidence of the use of labor-income tax in response of inflation differentials among the countries in the EMU. We leave these tasks for future research.

# A Appendix: Derivation of the government budget constraint

Each period, the government budget constraint can be summarized by the following equation:

$$p_t g_t + R_{t-1} B_{t-1}^g = p_t T_t + B_t^g.$$
(A.1)

Indicating with lowercase real variables (e.g.  $b_t^g = \frac{B_t^g}{p_t}$ ), we can express the previous expression as

$$g_t + R_{t-1} \frac{b_{t-1}^g}{1 + \pi_t} = T_t + b_t^g \tag{A.2}$$

# **B** Appendix: Fiscal Solvency in the Monetary Union

The household's intertemporal budget constraints for Home and Foreign are respectively:

$$\frac{W_t}{p_t} = \sum_{s=t}^{\infty} \left( \prod_{j=t}^{s-1} r_j^{-1} \right) \left[ c_s + T_s - y_s + \frac{R_s - 1}{R_s} \frac{M_s}{p_s} \right]$$
(B.1)

$$\frac{W_t^*}{p_t} = \sum_{s=t}^{\infty} \left( \prod_{j=t}^{s-1} r_j^{-1} \right) \left[ c_s^* + T_s^* - y_s^* + \frac{R_s - 1}{R_s} \frac{M_s^*}{p_s} \right]$$
(B.2)

Therefore, the aggregate intertemporal budget constraint is

$$\frac{W_t + W_t^*}{p_t} = \sum_{s=t}^{\infty} \left( \prod_{j=t}^{s-1} r_j^{-1} \right) \left[ c_s + c_s^* + T_s + T_s^* - y_s - y_s^* + \frac{R_s - 1}{R_s} \frac{M_s + M_s^*}{p_s} \right]$$
(B.3)

Let us analyze, now, the governments' budget constraints. We focus on Home's government, the analysis for Foreign being symmetric. The government budget constraint for Home is

$$p_t g_t + B_{t-1}^g R_{t-1} = p_t T_t + B_t^g \tag{B.4}$$

Solving forward (B.4), we get the following intertemporal budget constraint for Home:

$$\frac{R_{t-1}}{p_t} B_{t-1}^g = \sum_{s=t} \infty \left( \prod_{j=t}^{s-1} r_j^{-1} \right) [T_s - g_s]$$
(B.5)

which corresponds, for the Foreign counterpart, to

$$\frac{R_{t-1}}{p_t} B_{t-1}^{*g} = \sum_{s=t} \infty \left( \prod_{j=t}^{s-1} r_j^{-1} \right) \left[ T_s^* - g_s^* \right]$$
(B.6)

Finally, let us consider the central bank in the Monetary Union. Aggregating (2.9) and (2.10), we get the following condition

$$R_{t-1}B_{t-1}^M - H_{t-1} = B_t^M - H_t, \qquad B_t^M \equiv B_t^{F(CB)} + B_t^{H(CB)}$$
(B.7)

whose forward solution is equal to

$$\frac{H_{t-1} - R_{t-1}B_{t-1}^M}{p_t} = \sum_{s=t}^{\infty} \left(\prod_{j=t}^{s-1} r_j^{-1}\right) \left[ \left(\frac{R_s - 1}{R_s}\right) \frac{H_s}{p_s} \right]$$
(B.8)

Imposing conditions (2.11)-(2.13), we are now able to show that the sum between Home and Foreign's budget constraints equalizes the sum of the intertemporal budget constraints for the three policy-makers (B.5), (B.6) and (B.8):

$$\frac{R_{t-1}(B_{t-1} + B_{t-1}^*) + H_{t-1}}{p_t} = \sum_{s=t}^{\infty} \left( \prod_{j=t}^{s-1} r_j^{-1} \right) \left[ \Delta_s + \Delta_s^* + \frac{R_s - 1}{R_s} \frac{H_s}{p_s} \right]$$
(B.9)

## References

- Abreu, Dilip, 1988. "On the Theory of Infinitely Repeated Games with Discounting", Econometrica, Vol. 56(2), pp. 383-396.
- [2] Abreu, Dilip, David Pearce and Ennio Stacchetti, 1990. "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring", Econometrica, Vol. 58(5), pp. 1041-1063.
- [3] Alesina, Alberto and Robert J. Barro, 2002. "Currency Unions", Quarterly Journal of Economics, CXVII, 409-436.
- [4] Beetsma, Roel M.W.J. and A. Lans Bovenberg, 1998. "Monetary Union without fiscal coordination may discipline policymakers", Journal of International Economics, Vol. 45, pp. 239-258.
- [5] Beetsma, Roel M.W.J., Xavier Debrun and Franc Klaassen, 2001. "Is Fiscal Policy Coordination in EMU Desirable?", CESIfo Working paper No. 599.
- [6] Beetsma, Roel M.W.J. and Henrik Jensen, 2004. "Mark-up Fluctuations and Fiscal Policy Stabilization in a Monetary Union", Journal of Macroeconomics, Vol. 26, pp. 357-376.
- [7] Benigno, Pierpaolo, 2004. "Optimal monetary policy in a currency area", Journal of International Economics, Vol. 63, pp. 293-320.
- [8] Bergin, Paul R., 2000. "Fiscal solvency and price level determination in a monetary union", Journal of Monetary Economics, Vol. 45(1), pp. 37-53.
- Bordo, Michael and Lars Jonung, 1997. "The history of monetary regimes
   Some lessons for Sweden and the EMU", Swedish Economic Policy Review, 4: 285-358.

- [10] Buti, Marco and André Sapir, 1998. "Economic Policy in EMU A Study by the European Commission Services", Clarendon Press, Oxford.
- [11] Canzoneri, Matthew B., Robert E. Cumby and Behzad T. Diba, 2001.
  "Is the Price Level Determined by the Needs of Fiscal Solvency?", American Economic Review, Vol. 91, pp. 1221-1238.
- [12] Canzoneri, Matthew B., Robert E. Cumby and Behzad T. Diba, 2002. "Should the European Central Bank and the Federal Reserve Be Concerned About Fiscal Policy?", in *Proceedings of a Conference on Rethinking Stabilization Policy*, Federal Reserve Bank of Kansas City.
- [13] Chari, V. V. and Patrick J. Kehoe, 1998. "On the Need for Fiscal Constraints in a Monetary Union", working paper 589, Federal Reserve Bank of Minneapolis.
- [14] Chari, V. V. and Patrick J. Kehoe, 2002. "Time Inconsistency and Free-Riding in a Monetary Union", mimeo, Federal Reserve Bank of Minneapolis.
- [15] Clarida, Richard, Jordi Galí and Mark Gertler, 2002. "A simple framework for international monetary policy analysis", Journal of Monetary Economics, Vol. 49, pp. 879-904.
- [16] Cohen, Benjamin J., 1993. "Beyond EMU: The problem of sustainability", Economics and Politics, Vol. 5(2): 187-203.
- [17] Dixit, Avinash, 2000. "A Repeated Game of Monetary Union", Economic Journal, Vol. 110:759-780.
- [18] Dixit, Avinash and Luisa Lambertini, 2001. "Monetary-Fiscal Policy Interactions and Commitment versus Discretion in a Monetary Union", European Economic Review, Vol.45, pp. 977-987.

- [19] Dixit, Avinash and Luisa Lambertini, 2003a. "Symbiosis of Monetary and Fiscal Policies in a Monetary Union", Journal of International Economics, Vol. 60(2), pp.235-247.
- [20] Dixit, Avinash and Luisa Lambertini, 2003b. "Interactions of Commitment and Discretion in Monetary and Fiscal Policies", American Economic Review, Vol.93(5), pp.1522-1542.
- [21] Duarte, Margarida and Alexander L. Wolman, 2005. "Fiscal Policy and Regional Inflation in a Currency Union", mimeo, Federal Reserve Bank of Richmond.
- [22] Fuchs, William and Francesco Lippi, 2004. "Monetary Union with Voluntary Participation", mimeo.
- [23] Fuchs, William and Francesco Lippi, 2005. "Monetary Union with Voluntary Participation", forthcoming in the Review of Economic Studies.
- [24] Kocherlakota, Narayana R., 1996. "Implications of Efficient Risk Sharing without Commitment", Review of Economic Studies, Vol. 63:595-609.
- [25] Koeppl, Thorsten, 2003. "Differentiability of the Efficient Frontier when Commitment to Risk Sharing is Limited", mimeo, European Central Bank.
- [26] Leith, Campbell and Simon Wren-Lewis, 2006. "Compatibility between monetary and fiscal policy under EMU", European Economic Review, Vol. 50, pp. 1529-1556.
- [27] Ljunqvist, Lars and Thomas J. Sargent, 2005. Recursive Macroeconomic Theory (MIT Press, Cambridge, MA).
- [28] Obstfeld, Maurice and Kenneth Rogoff, 1998. "Risk and Exchange Rates", NBER working paper 6694.

- [29] Persson, Torsten and Guido Tabellini, 1995. "Double edged incentives, institutions and policy coordination", in G. Grossman and K. Rogoff (Eds.) *Handbook of International Economics*, Volume III (North-Holland Amsterdam), 1973-2030.
- [30] Thomas, Jonathan and Tim Worrall, 1988. "Self-Enforcing Wage Contracts", Review of Economic Studies, Vol. 55, pp.541-554.
- [31] Uhlig, Harald, 2002. "One money, but many fiscal policies in Europe: what are the consequences?", mimeo.
- [32] Walsh, Carl E., 2003. Monetary Theory and Policy (MIT Press, Cambridge, MA).
- [33] Woodford, Michael, 1998. "Control of the public debt: a requirement for price stability?", in G. Calvo and M. King, eds., *The debt burden and its consequences for monetary policy* (St. Martin's Press, New York), pp. 117-154.
- [34] Woodford, Michael, 2001. "Fiscal Requirements for Price Stability", Journal of Money, Credit and Banking, Vol. 33(3), pp. 669-728.