Myopic Governments and Welfare enhancing Debt Limits

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Abstract

This paper studies myopic fiscal policy in an incomplete market setting. A shortsighted, but otherwise benevolent, fiscal policy maker who has a smaller discount factor than society runs deficits which in turn lead to a high stock of government debt in the long run. The high level of debt reduces welfare because distortionary labour income taxes have to be raised to finance the permanently higher interest payments. In two related settings, an economy without capital and one with capital, the paper shows how the introduction of a fee on excessive debt can almost completely restore the allocation of optimal policy under commitment. Thus, the paper demonstrates how the addition of a legal restriction to the policy maker’s problem can improve welfare.

JEL Codes: E62, E63, H63

Keywords: Optimal Fiscal Policy, Myopic Government; Fiscal Constraints; SGP; Social Welfare
1 Introduction

How do large amounts of sovereign debt affect welfare? And if they reduce welfare how can they be prevented? On the one hand, many models of optimal fiscal policy advise governments to accumulate assets to an extent where they can finance all future expenditures solely out of interest earnings. Such a policy is thought to maximize social welfare and is often derived from the application of the Ramsey approach to optimal taxation. On the other hand, this prescription is in contradiction with the behaviour of most real world governments. In many OECD countries we observe large and sustained levels of sovereign debt.\(^1\) There have been several positive theories trying to explain this discrepancy.\(^2\) If it is assumed that the (otherwise benevolent) fiscal planner has a smaller discount factor than society, then optimal fiscal policy leads to sustained and high levels of government debt. The high stock of debt in turn requires higher taxes to finance the higher interest payments. This only small modification of the benevolent planner paradigm allows to stay conceptually close to a standard normative approach and facilitates a direct comparison to it. Thus, in a first step I analyze how this difference between normative policy implication and actual policy implementation affects the allocation, and in particular whether it reduces welfare. In a second step I then introduce a fee on excessive debt to examine whether such a legal restriction can lead to welfare improvements. This is the main question this paper addresses. In two related settings it turns out that such a restriction to the myopic planner’s policy problem can restore a level of welfare (and the associated allocation) which is very close to the one of the fully benevolent planner. For example, the fee could be thought of as to reflect the 60% debt to GDP criterion from the Stability and Growth Pact (SGP). Thus, the paper demonstrates how the imposition of an additional constraint to a myopic government’s policy problem can indeed lead to welfare improvements.

The paper analyzes two model variants: an economy with capital and one without capital. The base model without capital is close to Aiyagari et al. (2002). They recover Barro’s (1979) result of optimal tax smoothing where tax rates resemble a random walk (independent of the nature of the stochastic process of the shock). The government has access to a flat rate tax on labour income and issues non-state-contingent bonds to finance its exogenously given and stochastic stream of government consumption. Kumhof and Yakadina (2007) extend this model by

\(^1\)See e.g. OECD Economic Outlook No. 82, December 2007.

introducing myopia. There the government has the same instantaneous utility function but a smaller discount factor than the households. The justification for the shorter planning horizon could be uncertainty about the government’s reelection as in Grossman and van Huyck (1988). The myopic fiscal policy maker finances a lower tax rate in the near future by issuing bonds. Without any other frictions to its maximization problem this policy will lead to an explosive path of government debt. But the inclusion of empirically reasonable small quadratic transaction costs on bond holdings will tend to raise the interest rate when the government wants to issue ever more bonds. This combination of myopia and transaction costs results in a sustainable and positive level of debt in equilibrium. Building on Kumhof and Yakadina’s analysis, the present paper first calculates the stochastic steady states for different degrees of myopia and then compares them to the outcome under the benevolent planner. It turns out that government myopia clearly leads to welfare reductions (approximated to second order). Hence, the paper introduces a fee on excessive debt. The proposed fee is proportional to the deviation of the actual stock of debt from some fixed reference value. The fine is assumed to be paid to a supranational institution which can also set the reference value and determine the proportionality. The fine therefore constitutes real costs to the economy and is not a mere market friction. The modeling of the fine follows Beetsma, Ribeiro and Schabert (2008). Depending on the choice of some policy parameters in the design of the fee, the government can completely be prevented from permanently overshooting the reference value. In the long run the government will stay just below the reference value of debt but in response to budgetary shocks it can still rely on temporary bond financing. The advantage of the proposed fee, as compared to a balanced budget rule, is that it still allows for (nearly) optimal dynamic policy in reaction to budget fluctuations. It just turns off the permanent distortion.

In the second set up I consider an economy with capital. Here, the government has the same instruments as in the base model. It can rely on taxes on labour and non-state contingent debt. It is not allowed to levy taxes on capital. It turns out that the effect of myopia on welfare is larger in this set up. In addition to a high stock of debt myopic policy now also decreases the stock of capital. A higher stock of outstanding debt implies an increase in the tax rate on labour which in turn lowers working time. A lower working time requires a lower stock of capital to keep the rate of return of capital at a level which is pinned down at steady state only by the discount factor of the households. But again,

\footnote{The fine is not on the deficit.}
the introduction of the fee, as specified in the first set up, can almost perfectly eliminate the distortions stemming from myopia and restore the truly benevolent planner’s solution.

2 An economy without capital

The resource constraint in this economy is given by

\[ y_t = c_t + g_t + f_t, \]

where \( f_t \) is the fee on debt accumulation which will be specified in detail below. The production function in this section is linear in its only input labour. In Section 5, I will consider the case of a Cobb-Douglas production function with labour and capital as inputs. Further, \( c_t \) denotes consumption and \( g_t \) is an exogenously given and stochastic stream of government consumption.

Households

There is no population growth and no technological progress. All agents have rational expectations. Households are identical, infinitely-lived, and of mass one. A representative household is characterized by time separable preferences and its objective is the expected sum of future discounted utility:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \]  

where \( n_t \) denotes working time and \( \beta \in (0, 1) \) is the discount factor. The household’s total amount of time is normalized to one and is divided between working time and leisure. It earns the wage \( w_t \) and has to pay a flat-rate labour income tax of \( \tau_t \). The household can invest in one-period risk-free government bonds, \( b_{t+1} \), at the period \( t \) price \( \frac{1}{R_t} \), where \( R_t \) is the gross rate of return. The budget constraint reads:

\[ c_t + \frac{b_{t+1}}{R_t} + \Phi_t \leq (1 - \tau_t) w_t n_t + b_t + \pi_t, \]

where

\[ \Phi_t = \frac{\phi}{2} y_t \left( \frac{b_{t+1}}{y_t} \right)^2. \]

\( \Phi_t \) are transaction costs which have to be paid to the financial intermediary when the household wants to enter the capital market. They are quadratic in the ratio of bond holdings, \( b_{t+1} \), over per capita output, \( y_t \), and proportional to \( y_t \), and \( y_t \) is taken as given, as in Kumhof and
Yakadina (2007). The firms’ and broker’s profits $\pi_t$ are redistributed to the household in a lump-sum way. The household maximizes (1) subject to (2) which yields the first order conditions for the consumption working time choice and for the consumption investment decision. Together with the transversality condition they are

\begin{align}
(1 - \tau_t) w_t u_{c,t} &= -u_{n,t} \\
\mu_c \left( \frac{1}{R_t} + \phi \frac{b_{t+1}}{y_t} \right) &= \beta E_t u_{c,t+1} \\
\lim_{t \to \infty} \beta^t E_0 \left[ u_{c,t} \left( \frac{1}{R_t} + \phi \frac{b_{t+1}}{y_t} \right) b_{t+1} \right] &= 0
\end{align}

Firms and financial intermediary

The firms produce with the linear production function $y_t = n_t$ and pay a wage equal to marginal productivity of labour $w_t = 1$. The financial intermediary has zero marginal and fixed costs and since firms make zero profits

$$\Phi_t = \pi_t.$$  

Government

The government has access to flat-rate taxes on labour income and can only issue risk free bonds. Markets are therefore incomplete in the sense of Aiyagari et al. (2002). They find that this asset market structure can recapture Barro’s (1979) result that tax rates follow a random walk. The government has a discount factor of $\gamma \beta$, where $\gamma \leq 1$. Myopia can for instance reflect uncertainty of the government in office about its probability of reelection. The lower discount factor makes the issuance of bonds relatively cheap for the government: At the steady state (and neglecting transaction costs for a moment) it would be willing to pay an interest rate on its outstanding debt of $\frac{1}{\gamma \beta}$ whereas the households only demand a rate of $\frac{1}{\beta}$. Depending on the degree of myopia this financing of government expenditures leads to a high and persistent stock of debt which will be shown to decrease welfare. Therefore I include a fee, $f_t$, on excessive debt which could, for example, be thought of as an institutional arrangement like the 60 percent criterion of the Maastricht treaty. Specifically, I assume that the government has to pay a proportional fee $\kappa$ whenever last period’s stock of debt $b_t$ exceeds some reference value.
The policy parameters $\kappa$ and $b_{ref}$ can be chosen freely by the supranational institution. The fee is

$$f_t = \kappa ( b_t - b_{ref} ) I [ b_t; b_{ref} ],$$  

(7)

where the indicator function

$$I [ b_t; b_{ref} ] = \begin{cases} 1 & \text{if } b_t > b_{ref} \\ 0 & \text{if } b_t \leq b_{ref} \end{cases}.$$  

(8)

Equation (8) states that a fee only has to be paid if the stock of debt overshoots the reference value $b_{ref}$. The government does not receive subsidies for staying below $b_{ref}$. The government budget constraint then reads

$$g_t + b_t = \frac{b_{t+1}}{R_t} + \tau_t w_t n_t - f_t.$$  

(9)

### 3 Fiscal policy problem

In this section I will first set up the policy problem before I then turn to a brief analysis of the general equilibrium. For the first part I now derive a sequence of implementability constraints from the household’s problem as in Aiyagari et al. (2002). To start with, substitute out prices $R_t$ and $\tau_t$ in the household’s budget constraint (2) by using the households first order conditions (3), (4) which gives

$$c_t + b_{t+1} \left[ \beta E_t \frac{u_{c,t+1}}{u_{c,t}} - \phi \frac{b_{t+1}}{y_t} \right] = -\frac{u_{n,t} n_t}{u_{c,t}} + b_t$$  

(10)

where (6) was used. Rewrite (10) as

$$b_t = c_t + \frac{u_{n,t} n_t}{u_{c,t}} - \phi \frac{b_{t+1}^2}{y_t} + \beta E_t \left( \frac{u_{c,t+1}}{u_{c,t}} \right) b_{t+1}.$$  

(11)

Note that $b_t$ in (11) is non-state contingent. Then replace $b_{t+1}$ in (11) by the right hand side of (11), with the time index adjusted one period ahead.\(^5\) This gives

\[
b_t = c_t + \frac{u_{n,t} n_t}{u_{c,t}} - \phi \frac{b_{t+1}^2}{y_t} + \beta E_t \left( \frac{u_{c,t+1}}{u_{c,t}} \right) \left[ c_{t+1} + \frac{u_{n,t+1} n_{t+1}}{u_{c,t+1}} - \phi \frac{b_{t+2}^2}{y_{t+1}} + \beta E_{t+1} \left( \frac{u_{c,t+2}}{u_{c,t+1}} \right) b_{t+2} \right].
\]

\(^4\)Obviously the specification of the fee also allows for more lags or non, but which does not change the results presented below.

\(^5\)For a detailed description of the substitution procedure see Appendix A.
Repeating this substitution for all occurrences of future bond holdings, applying the law of iterated expectations, and using the transversality condition (5) yields a sequence of implementability constraint for the incomplete market case:

$$u_{c,t}b_t = E_t \sum_{j=0}^{\infty} \beta^j u_{c,t+j} \left[ c_{t+j} + \frac{u_{n,t+j}n_{t+j}}{u_{c,t+j}} - \phi \frac{b_{t+j+1}^2}{y_{t+j}} \right].$$

Equ. (12) has to hold in every period $t$ for all realizations of $\varepsilon_t$ for a given $b_t$, i.e. $b_t$ is not state-contingent.

Besides (12), the resource constraint has to be satisfied by the fiscal planner. Accounting for (7) it reads

$$n_t = c_t + g_t + \kappa \left(b_t - b^{ref}\right) I \left[b_t; b^{ref}\right].$$

In (13) we can see how the fee $f_t$ constitutes real costs to the economy and is not just a market distortion. Since (12) and (13) imply (9) government solvency is ensured. The Lagrangian to the myopic planner’s policy problem then is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \left(\gamma \beta \right)^t \left\{ u_\left(c_t, n_t\right) + \eta_t \left(n_t - c_t - g_t - \kappa \left(b_t - b^{ref}\right) I \left[b_t; b^{ref}\right]\right) + \alpha_t \left(E_t \sum_{j=0}^{\infty} \beta^j u_{c,t+j} \left[ c_{t+j} + \frac{u_{n,t+j}n_{t+j}}{u_{c,t+j}} - \phi \frac{b_{t+j+1}^2}{y_{t+j}} \right] - u_{c,t}b_t \right) \right\}.$$  

Note that (a) the indicator function makes the problem non-continuous, and (b) the infinite double sum complicates the derivation of the Lagrangian. To address the first problem I rely on an approximation of the indicator function by the logistic function as in Beetsma, Ribeiro and Schabert (2008).

$$I \left[b_t; b^{ref}\right] \approx L_t \left(\delta, b_t, b^{ref}\right) = \frac{1}{1 + \exp(-\delta \left(b_t - b^{ref}\right))}, \quad \delta > 0,$$  

where

$$L_t \left(\delta, b_t, b^{ref}\right) \rightarrow I \left[b_t; b^{ref}\right] \quad \text{for} \quad \delta \rightarrow \infty.$$  

To overcome the second problem, I apply a recursive saddle point formulation based on Marcet and Marimon (1998). Therefore define a new stochastic multiplier $\mu_t = \frac{\mu_{t-1}}{\gamma} + \alpha_t$, where $\mu_{-1} = 0$. Then rewrite the infinite double sum in (14) recursively as

$$\sum_{t=0}^{\infty} \left(\gamma \beta \right)^t \alpha_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = \sum_{t=0}^{\infty} \left(\gamma \beta \right)^t \mu_t s_t,$$
where \( s_t = u_{c,t} \left( c_t + \frac{u_{n,t} n_t}{u_{c,t}} - \phi \frac{b_{t+1}^2}{n_t} \right) \). Using (15), (16), and \( y_t = n_t \) in (14) the transformed Lagrangian becomes a standard problem

\[
\mathcal{L} = E_0 \sum_{t=0}^{\infty} (\gamma \beta)^t \left\{ u(c_t, n_t) + \eta_t \left[ n_t - c_t - g_t - \kappa \left( b_t - b^{ref} \right) \right] L_t \right. \\
+ \left. \mu_t \left[ u_{c,t} c_t + u_{n,t} n_t - u_{c,t} \left( \phi \frac{b_{t+1}^2}{n_t} + b_t \right) \right] + \frac{\mu_t - 1}{\gamma} b_t u_{c,t} \right\}
\]

(17)

The first order conditions to (17) w.r.t. \( c_t, n_t, b_{t+1} \) are

\[
u_{c,t} + \frac{\mu_t - 1}{\gamma} b_t u_{c,t} = \eta_t - \mu_t \left[ +u_{c,t} - u_{c,t} \left( \phi \frac{b_{t+1}^2}{n_t} + b_t \right) \right]
\]

(18)

\[
u_{n,t} + \eta_t = -\mu_t \left( u_{n,n,t} n_t + u_{n,t} + u_{c,t} \phi \frac{b_{t+1}^2}{n_t} \right)
\]

(19)

\[
E_t \left( \frac{\mu_t}{\gamma} u_{c,t+1} - \mu_t u_{c,t+1} \right) = E_t \left( \eta_{t+1} \frac{\partial f_{t+1}^{appr}}{\partial b_{t+1}} \right) + \frac{\mu_t}{\gamma} \phi b_{t+1} \left( \frac{\gamma}{\beta} \right) c_t n_t
\]

(20)

where in (20) the derivative of the approximated fee

\[
\frac{\partial f_{t+1}^{appr}}{\partial b_{t+1}} = \frac{\partial \left( \kappa \left( b_{t+1} - b^{ref} \right) L_{t+1} \right)}{\partial b_{t+1}}.
\]

Equ. (18) - (20) together with (10), (13), and an exogenous process for \( g_t \) define the political equilibrium, given an initial value \( b_0 \).

Except of the last term on the LHS of (18), equ. (18) and (19) are exactly the same as the respective equations of the fully benevolent Ramsey planner problem. Exemplary, consider the case of log utility and \( \phi = 0 \). Then at steady state (18) becomes

\[
\eta = \left( \frac{1}{\gamma} - 1 \right) \frac{\mu b}{c^2} + \frac{1}{c}.
\]

(21)

For \( \gamma = 1 \), (21) collapses to \( \eta = \frac{1}{c} = u_c \). But for the myopic planner \( \gamma < 1 \), and thus \( \left( \frac{1}{\gamma} - 1 \right) > 0 \). He assigns more weight to temporary values of consumption in the intertemporal budget constraint than the households do. Or equivalently, it perceives the resource constraint as more binding.

In equ. (20) we can see how the policy maker equates the budget relaxing effect of new debt on the LHS to the associated higher expected

\footnote{For a detailed derivation of (16) see Appendix B.}
fee and transaction costs on the RHS. Equ. (20) exhibits Barro’s unit root component if we set $\gamma = 1$ and $\phi = \frac{\partial f_{apr}}{\partial b} = 0$. At steady state the relation reads

$$\frac{b}{y} = \frac{(1 - \gamma) \beta}{2\phi} - \frac{1}{2\phi} \left( \frac{\gamma \beta \eta}{\mu u} \right) \frac{\partial f_{apr}}{\partial b}.$$ (22)

Neglecting the second term on the RHS for a moment, we can see how increasing myopia (smaller values of $\gamma$) and smaller transaction costs (smaller values of $\phi$) lead to higher debt to GDP ratios. The second term on the RHS gives the effect of the fee on the debt to GDP ratio. A higher value of $\frac{\partial f}{\partial b}$, i.e., a higher $\kappa$, which is weighted by the positive ratio of Lagrange multipliers, lowers steady state debt, other things equal.

4 Results

4.1 Calibration

All parameters are calibrated for a quarterly frequency. The single period utility function is of the form

$$u(c_t, n_t) = c_t^{1-\sigma} - 1 - \mu(n_t^{1+\varphi} - 1).$$

The weight for working time in utility $\nu$ is calibrated to 4 and $\sigma$ and $\varphi$ are set to one. These values imply an equal division of the total time endowment into working time and leisure in the baseline business cycle model without taxes. The household’s discount factor $\beta = 0.99$. Multiplied by the proportional factor $\gamma$ it gives the government’s time preferences. I will vary $\gamma$ between 0.95 and 1 to demonstrate the effects of myopia on the allocation. When $\gamma = 1$ the model equals the benevolent Ramsey planner’s problem.

Government spending, $g_t$, is assumed to follow an AR(1) process

$$\ln g_t = (1 - \rho_g) \ln \bar{y} + \rho_g \ln g_{t-1} + \varepsilon_t^g, \quad \varepsilon_t^g \sim N(0, \sigma_{\varepsilon g}^2).$$

I assume that $(\rho_g, \sigma_{\varepsilon g}) = (0.9, 0.016)$, in line with Christiano and Eichenbaum (1992).

The policy parameter $\kappa$ is first set to zero to calculate the reference case without fees on debt. Then I increase $\kappa$ to show its effects on the allocation and in particular on welfare. The proportional factor $\phi$ in the transaction cost specification is assumed to equal 0.015, which corresponds to an increase of $R_t$ of about 6 basis points when the government increases $b_{t+1}$ by one percent. This value is at the upper end of the estimations of this relation as reported by Gale and Orszag (2003). The
model’s results are robust to a lower value of $\phi$ of about one order of magnitude. The parameter $\delta$ in the logistic function is set to 300 which gives a smooth approximation of the indicator function as Franses and Van Dijk (2000) show.

The ratios of government expenditures and the stock of debt to output, $g/y_t$ and $b^{ref}/y_t$, are set to about 0.2 and 0.6, respectively. Both are calibrated with regard to the baseline model without myopia. The former is the average of OECD countries as reported by Gali (1994) while the latter reflects the 60% criterion from the Maastricht treaty. The exact values of these ratios change since the absolute values of $g$ and $b^{ref}$ are fixed and, as shown below as one of the main results of the analysis, myopic governments reduce output and thereby increase the ratios of government consumption and debt to GDP. Table 1 summarizes the calibration of the economy.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Household discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>[0.95;1]</td>
<td>$\gamma \beta$ Government discount factor</td>
</tr>
<tr>
<td>$\nu$</td>
<td>4</td>
<td>Preference parameter $u(c,n) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \frac{\nu(n^{1+\rho}-1)}{1+\rho}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>Preference parameter</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>1</td>
<td>Preference parameter</td>
</tr>
<tr>
<td>$\rho^g$</td>
<td>0.9</td>
<td>Persistence of shock to government consumption</td>
</tr>
<tr>
<td>$\sigma_{e^g}$</td>
<td>0.016</td>
<td>Standard deviation of innovation to government consumption</td>
</tr>
<tr>
<td>$g/y$</td>
<td>~0.2</td>
<td>Share of government consumption to GDP</td>
</tr>
<tr>
<td>$b^{ref}/y$</td>
<td>~0.6</td>
<td>Share of debt to GDP</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>[0;1]</td>
<td>Fee on debt: $\kappa(b_t-b^{ref})I[b_t,b^{ref}]$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.015</td>
<td>Transaction costs</td>
</tr>
<tr>
<td>$\delta$</td>
<td>300</td>
<td>Smoothness of logistic function</td>
</tr>
</tbody>
</table>

Table 1: Parameter calibration for quarterly frequency.

### 4.2 Simulation

This subsection presents the results from the simulations of the model of section 3. Throughout this subsection $g = 0.1$ and $b^{ref} = 0.3$. As a starting point, I show how the model generates a unit root behaviour that many models of optimal policy exhibit.\(^7\) Then, I introduce transaction costs, myopia, and debt fees step by step to bring out the different effects of these features on the model. In particular it should become clear in what respects a myopic planner differs from the Ramsey planner and how the introduction of a fee on debt mitigates those, from a welfare point of view, detrimental deviations.

\(^7\)See e.g. Schmitt-Grohe and Uribe (2004).
Ramsey policy without transaction costs

To show how the myopic planner converges to the Ramsey planner I set $\gamma = 0.9999$, $\phi = 0$, and, $\kappa = 0$. This exercise serves to analyze in isolation optimal policy with incomplete markets and helps to bring out a feature which will also be apparent in all of the following specifications: With non-state contingent bonds, the government is not able to smooth distortions over states and therefore will try to smooth them over time. Variables therefore show more persistency in response to shocks. In other words, since the government cannot condition its debt on future states it cannot keep the tax rate constant and must adjust it in response to budgetary shocks.

Figure 1 displays the impulse response functions (IRF) for bonds, the tax rate, consumption, and labour to a positive innovation of one standard deviation to government consumption.\(^8\) We can see how the government uses both its instruments, bonds and taxes, to finance its temporarily higher expenditures. It adjusts the tax rate only once and for all to avoid welfare losses arising from a varying tax rate and the associated excess burden of taxation. The increase in the tax rate is just enough to finance the higher interest payments from the increasing and then permanently rolled over stock of debt. Consumption an labour will not recover from the lower level which is associated with higher taxes.

\(^8\)The IRFs are computed with the DYNARE software (Version 3), available at http://www.cepremap.cnrs.fr/juillard/mambo.
Myopic government, transaction costs, no fee on debt

Now I turn to the effects of myopia. Therefore I will include transaction costs. They ensure a stable and well defined equilibrium which is necessary to compare the different allocations of the myopic planner to the reference case of the Ramsey planner. Thus, for the remainder of the analysis $\phi = 0.015$. The IRF in Figure 2 show how all variables converge back to their steady states. Since households require the same effective interest rate as before, the transaction costs raise the gross interest rate the government has to offer and prevent the government from permanently rolling over a higher stock of debt. Accordingly, bonds now respond less and taxes more to a temporary increase of government consumption as compared to Figure 1. Because of the higher tax rate the impact response of labour is now more pronounced while consumption declines by more. Since consumption declines and labour rises welfare declines.
Having ensured convergence I now turn to the welfare analysis. The unconditional welfare measure is

$$v = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t).$$

(23)

But since I perform the welfare analysis at a well behaved steady state (23) can be rewritten recursively as $^9$

$$v_t = u(c_t, n_t) + \beta v_{t+1}.$$ 

(24)

Using (24), Table 2 now presents the effects of a lower government discount factor $\gamma$ on the allocation, and in particular on welfare. All values are second order approximations of the model at the stochastic steady.

$^9$See Schmitt-Grohe (2005)
The first row reports the reference case of the Ramsey planner, where $\gamma = 1$, and the government’s and households’ discount factors coincide. Descending, $\gamma$ is reduced to 0.95 which corresponds to a planning horizon of five years.\(^\text{10}\) As myopia increases, debt builds up increasing the interest rate. This in turn requires higher interest payments which are financed by higher labour taxes. Taxation decreases working time and thereby lowers consumption and welfare and increases the share of government consumption in GDP. The accumulation of assets in the first row in turn are the buffer stock savings of the Ramsey planner. Exemplarily for myopic fiscal policy the IRF for $\gamma = 0.97$ are reported in Appendix C. The only remarkable difference to the Ramsey case is in the first period. The myopic planner does not raise the labour tax on impact as much as the Ramsey planner and issues more bonds. Accordingly, consumption and labour respond less in this period.

The last row reports the percentage deviations of the myopic planner from the Ramsey planner. Welfare, $V$, decreases by about one percent, consumption and labour by about 4 percent. The ratio of debt to GDP, $b/n$, increases to about 165 percent, the share of government consumption by about four percent, and the tax rate by 32 percent. An intuition for these effects of myopia might lend the following: The myopic policy maker prefers nearby utility more than households. Thus it lowers taxes and increases consumption today, relying on debt. This will raise the interest rate as a result of the transaction costs. But as long as the gross interest rate is below some average of the public and private time preferences it will continue to accumulate debt.\(^\text{11}\) This process continues until the two effects of myopia and transaction costs have balanced resulting in the long run equilibrium.

Besides the higher interest payments and the associated excess burden of taxation, there exists a third effect which decreases welfare. It stems from the fact that, for the different degrees of myopia in Table 2,

\[ h = \frac{1}{4(1 - \gamma)}. \]

\[ \text{See Kumhof and Yakadina (2007), equ. (26).} \]

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<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$b$</th>
<th>$R$</th>
<th>$\tau$</th>
<th>$c$</th>
<th>$n$</th>
<th>$g/y$</th>
<th>$V$</th>
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<td>-3.7</td>
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Table 2: The effects of myopia (decreasing $\gamma$) on the allocation
(second order approximation)
I chose to fix the absolute value of government consumption, $g$, and not its share in GDP. Since working time decreases, so does output, $y = n$, which leads to a higher ratio $g/y$. Since $g$ are pure resource losses and do not generate any utility in this model welfare decreases as $g/y$ increases. Interpreting the model that way, welfare losses from myopia then also reflect the unnecessary output losses compared to potential output under the Ramsey planner. On the other hand, one could argue that it is rather effective output that determines the absolute size of $g$ in the long run. Therefore one should fix $g/y$ and look only at the first two sources of welfare losses. Nevertheless, even in this modified set up all remaining results of the model survive and from here on I will stick to the first interpretation that myopia decreases production and thereby crowds out private consumption.\footnote{The modified set up is available in an earlier working paper version, downloadable from the authors webpage.}

**Myopic government with debt limits**

Whereas in the previous analysis $\kappa$ was set to zero, I will now introduce the fee on debt, while $\gamma$ is held constant at 0.95. The results are reported in Table 3. The fee was specified in (7) and (15) and is repeated here for convenience

$$f_t = \kappa \left( b_t - b^{ref}\right) L_t.$$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\kappa$</th>
<th>$b$</th>
<th>$R$</th>
<th>$\tau$</th>
<th>$c$</th>
<th>$n$</th>
<th>$g/y$</th>
<th>$V$</th>
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<td>0.3999</td>
<td>0.5000</td>
<td>0.2001</td>
<td>-141.6491</td>
</tr>
</tbody>
</table>

**Table 3: Effects of an increasing fee on debt on the allocation.**

As a starting point, the first row in Table 3 repeats the last row from Table 2 (the myopic planner allocation for $\gamma = 0.95$) while the last row shows the Ramsey allocation I want to restore by imposing the continuous debt limit. In the second column I increased the proportional factor $\kappa$ from 0.001 to 1. This implies an increase of the fine from about two to 200
percent of GDP times the absolute deviation from the reference value.\textsuperscript{13} As we can see, all variables return towards the Ramsey allocation as $\kappa$ increases. This exercise demonstrates how imposing an additional borrowing constraint of the proposed type to the policy problem can improve welfare.

In principal there is no upper limit to the policy parameter $\kappa$. But when $\kappa$ takes on values of 10 or 100 the inaccuracy of the approximation of the indicator function can drive welfare above the Ramsey allocation. When $b_t$ approaches $b^{ref}$ from the left side $f_t$ becomes negative. It then depends on $\kappa$, the distance between $b$ and $b^{ref}$, and the smoothness parameter $\delta$ how large these additionally generated resources are. But for reasonable values of $\delta$ these effects only amount to a size of order $10^{-4}$. E.g. in the line before last $f_t = 1 * (0.2958 - 0.3) / (1 + \exp (-300 (0.2958 - 0.3))) = -0.00092$. Other than that, it will be shown below that it is the value of $b^{ref}$ which mainly affects welfare and that $\kappa$ can be set to values small enough such that the approximation does not affect the results.

Figure 3 displays the IRF of the model to an innovation to government consumption for the case of a myopic government, $\gamma = 0.97$, and an inclusion of the debt fee, $\kappa = 0.01$.

\textsuperscript{13}The specification of the fee in absolute terms complicates its interpretation a little. But it simplifies the first order conditions to the policy problem and their interpretation considerably. Further, all results are robust to a specification of the fee in relative terms.
We can see that the government still uses both its instruments, bonds and taxes. Thus the proposed fee allows for more flexibility than a balanced budget rule but still is able to prevent the government from accumulation an excessive stock of debt.\textsuperscript{14} Recall that the fee has to be paid with a lag of one period which explains its zero impact response. From the second period on \( f \) mirrors the path of \( b \). Although the fee is very small it suffices to let all variables converge faster. The primary deficit, \( d \), turns into a surplus from the second period on to pay back debt and reduce the fee. Due to myopia, the tax rate and consumption respond less on impact than under optimal policy (see also Figure A1).

There are three more questions that should be addressed at this point. The first one concerns the separation of first and second order welfare losses. In turns out that, even for high values of risk aversion, 99\% of the welfare losses are due to a distorted deterministic steady state. Thus, myopic fiscal policy mainly decreases welfare because of a suboptimal high stock of debt but not because of a suboptimal reaction

\textsuperscript{14}For \( \kappa \to \infty \) the proposed fee converges to a balanced budget rule.
to budget fluctuations. This leads to the second question which concerns the specification of the fee in terms of stock values. Since in this model the steady state stock of debt requires a permanent surplus it makes no sense to specify a deficit criterion to address the distortions stemming from a suboptimal steady state. This explains partly why the proposed fee works well. A third question then arises immediately: Which values of $\kappa$ and $b^{ref}$ are the optimal ones? For $\kappa$ we have already seen in Table 2 that values between 0.01 and 1 work reasonably well. From a technical point of view only care has to be taken of the accuracy of the approximation. More interesting is what to choose for $b^{ref}$, respectively $b^{ref}/y$. So far the analysis of the fee was in style of the SGP where for political reasons 60% was chosen. At the time of foundation of the SGP this ratio simply was about the average of the potential member states’ ratios. It should be clear by now that in this model any debt at steady state decreases welfare. Therefore it is natural to try a reference value of $b^{ref}/y = 0$. It results that such a value can almost completely restore the Ramsey allocation! For $\gamma = 0.95$, $b^{ref}/y = 0$, and $\kappa = 0.01$ the respective values are

$$b = 0.0011, \quad R = 1.0101, \quad \tau = 0.2002,$$

$$c = 0.3999, \quad n = 0.5, \quad g/y = 0.2,$$

and $$V = -141.6520.$$

5 An economy with capital

One of the limitations of the previous analysis is that it includes only one state variable, namely bonds. One could therefore think that the debt limit works only because it directly addresses the only intertemporal distortion there is. And that if a second state variable was added to the model, whose accumulation was also distorted, the fee could only mitigate the first distortion but would be ineffective regarding the second and would therefore loose overall effectiveness. This section will show that distortions arising from myopia are in deed larger in an economy with capital. They are due to a suboptimal low stock of capital. But it will also show that the proposed fee stays as effective as before. The reason is that by turning off the first distortion and thereby increasing working time, the fee also removes the second distortion by making capital more productive and thereby boosting its accumulation.

5.1 The Model

The resource constraint of the economy is

$$y_t = c_t + g_t + k_{t+1} - (1 - \delta) k_t + f_t,$$  \hspace{1cm} (25)
where investment \( i_t = k_{t+1} - (1 - \delta) k_t \) and \( \delta \) is the rate of depreciation.

Capital is owned by the households and rent by firms at a competitively determined rate \( r_t \) on a period by period basis. The representative firm is assumed to exist for only one period such that its problem is static. It produces with a Cobb-Douglas production function

\[
y_t = a_t k_t^\alpha n_t^{1-\alpha}
\]

with capital \( k_t \) and labour \( n_t \) as inputs and total factor productivity \( a_t \). The first-order conditions from the firm’s maximization problem are given by

\[
w_t = (1 - \alpha) a_t k_t^\alpha n_t^{1-\alpha}
\]

\[
r_t = \alpha a_t k_t^{\alpha-1} n_t^{1-\alpha}.
\]

Since producing firms still make zero profits, the household receives every period a lump-sum payment of \( \Phi_t = \pi_t \). The household is endowed with the stock of physical capital and undertakes investments in capital. Its budget constraint reads:

\[
c_t + b_{t+1} + k_{t+1} + \Phi_t \leq (1 - \tau_t) w_t n_t + R_{t+1}^e k_t + b_t + \pi_t
\]

where \( R_{t+1}^e = 1 + r_{t+1} - \delta \) is the return on capital. The household maximizes (1) subject to (28). This yields the first order conditions for \( c_t, n_t, b_{t+1}, \) and \( k_{t+1} \). We now have two Euler equations, one for bonds (30) and one for capital (31), and the respective transversality conditions.

\[
(1 - \tau_t) w_t u_{c,t} = -u_{n,t}
\]

\[
u_{c,t} \left( \frac{1}{R_t} + \phi \frac{b_{t+1}}{y_t} \right) = \beta E_t u_{c,t+1}
\]

\[
u_{c,t} = \beta E_t \left[ u_{c,t+1} R_{t+1}^e \right]
\]

\[
\lim_{t \to \infty} \beta^t E_0 \left[ u_{c,t} \left( \frac{1}{R_t} + \phi \frac{b_{t+1}}{y_t} \right) b_{t+1} \right] = 0
\]

\[
\lim_{t \to \infty} \beta^t E_0 u_{c,t+1} k_{t+1} = 0
\]

Comparing (30) and (31) we can see that the transaction costs will cause the interest rate on bonds to be higher than the return on capital if the government accumulates debt at steady state. The government has the same discount factor \( \gamma \beta \) as before and its budget constraint is the same

\[
g_t + b_t = \tau_t w_t n_t + \frac{b_{t+1}}{R_t} - f_t.
\]

A competitive equilibrium is a set of plans \( \{b_t, c_t, n_t, k_t, R_t, R_{t+1}^e, w_t\}_{t=0}^\infty \) satisfying (25) - (28) and (29) - (33), given a policy \( \{\tau_t\}_{t=0}^\infty \), an exogenous process \( \{g_t\}_{t=0}^\infty \), and initial conditions \( b_0 \) and \( k_0 \).
5.2 The policy problem

In this section I will proceed as before and derive a sequence of implementability constraints including capital and then set up the policy problem. To derive the household intertemporal budget constraint use (29) and (30) to substitute out prices and taxes, and rewrite (28) as

\[ b_t + R_t^e k_t = c_t + \frac{u_{n,t} m_t}{u_{c,t}} - \phi \frac{b_{t+1}^2}{y_t} + k_{t+1} + \beta E_t \left( \frac{u_{c,t+1}}{u_{c,t}} \right) b_{t+1}. \]  

(34)

To save notation define \( z_t = c_t + \frac{u_{n,t} m_t}{u_{c,t}} - \phi \frac{b_{t+1}^2}{y_t} \). Then use (34) again with the time index adjusted one period ahead to substitute for \( b_{t+1} \) in (34):

\[
\begin{align*}
  b_t + R_t^e k_t &= z_t + k_{t+1} + \beta E_t \frac{u_{c,t+1}}{u_{c,t}} b_{t+1} \\
  &= z_t + k_{t+1} + \beta E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} \left( z_{t+1} + k_{t+2} - R_{t+1}^e k_{t+1} + \beta E_{t+1} \frac{u_{c,t+2}}{u_{c,t+1}} b_{t+2} \right) \right] \\
  &= z_t + k_{t+1} \left( 1 - \beta E_t \frac{u_{c,t+1}}{u_{c,t}} R_{t+1}^e \right) + \beta E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} (z_{t+1} + k_{t+2}) \right] \\
  &\quad + \beta^2 E_t \left[ \frac{u_{c,t+2}}{u_{c,t}} b_{t+2} \right],
\end{align*}
\]

(35)

where the second equality used that \( b_{t+1} \) is known in period \( t \) and can therefore be included into the expectations operator. The last equality applied the law of iterated expectations and used that \( k_{t+1} \) is known in period \( t \) and can be extracted from the expectations operator. Now, iterate forward (35) by repeating the procedure for \( b_{t+2} \), use (31) to eliminate capital, and multiply (35) by \( u_{c,t} \)

\[
\begin{align*}
  u_{c,t} (b_t + R_t^e k_t) &= E_t \sum_{j=0}^{\infty} \beta^j u_{c,t+j} z_{t+j} + \lim_{j \to \infty} \beta^{j+1} E_t (u_{c,t+1+j} b_{t+1+j}) \\
  &\quad + \lim_{j \to \infty} \beta^j E_t (u_{c,t+j} k_{t+1+j}).
\end{align*}
\]

Apply the transversality conditions for bonds (32) and capital (33). This gives a sequence of implementability constraints

\[
\begin{align*}
  u_{c,t} (b_t + R_t^e k_t) &= E_t \sum_{j=0}^{\infty} \beta^j u_{c,t+j} \left( c_{t+j} + \frac{u_{n,t+j} m_{t+j}}{u_{c,t+j}} - \phi \frac{b_{t+1+j}^2}{y_{t+j}} \right).
\end{align*}
\]

(36)

Equ. (36) differs from (12) only in that it includes the end of period stock of capital on the LHS.
As before the government maximizes the household’s instantaneous utility function, but discounted by its own discount factor $\gamma \beta$. The policy problem then is

$$\max E_0 \sum_{t=0}^{\infty} (\gamma \beta)^t \left\{ u(c_t, n_t) \right\}$$

(37)

$$+ \eta_t \left( a_t k_t^{\alpha} n_t^{1-\alpha} + (1 - \delta) k_t - c_t - g_t - k_{t+1} - f_t^{apr} \right)$$

$$+ \psi_t \left( E_t \sum_{j=0}^{\infty} \beta^j u_{c,t+j} \left( c_{t+j} + \frac{u_{n,t+j} n_{t+j}}{u_{c,t+j}} - \phi \frac{b_{t+1+j}^2}{g_{t+j}} \right) \right.$$

$$- u_{c,t} (b_t - R_{t}^e k_t)$$

$$\left. + \chi_t \left( E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} \left( 1 - \delta + \alpha a_t k_t^{\alpha-1} n_t^{1-\alpha} \right) - \frac{1}{\beta} \right) \right] \right) .$$

Again approximate the fee and introduce a new stochastic multiplier $\mu_t = \frac{\mu_{t-1}}{+ \psi_t}$, with $\psi_0 = 0$. Further use that $y_t = a_t k_t^{\alpha} n_t^{1-\alpha}$, $R_{t+1}^e = 1 - \delta + \alpha a_t k_t^{\alpha-1} n_t^{1-\alpha}$, and rewrite (37) as

$$L = E_0 \sum_{t=0}^{\infty} (\gamma \beta)^t \left\{ u(c_t, n_t) \right\}$$

(38)

$$+ \eta_t \left( a_t k_t^{\alpha} n_t^{1-\alpha} + (1 - \delta) k_t - c_t - g_t - k_{t+1} - f_t^{apr} \right)$$

$$+ \mu_t \left( u_{c,t} c_t + u_{n,t} n_t - u_{c,t} \phi \frac{b_{t+1}^2}{a_t k_t^{\alpha} n_t^{1-\alpha}} \right)$$

$$- \psi_t u_{c,t} (b_t - (1 - \delta + \alpha a_t k_t^{\alpha-1} n_t^{1-\alpha}) k_t)$$

$$+ \chi_t \left( E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} \left( 1 - \delta + \alpha a_t k_t^{\alpha-1} n_t^{1-\alpha} \right) - \frac{1}{\beta} \right) \right) .$$

The first order conditions w.r.t. $c_t, n_t, k_{t+1}$, and $b_{t+1}$ are

$$\eta_t + \chi_t E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} R_{t+1}^e \right] + \psi_t u_{c,t} (b_t - (1 - \delta + \alpha a_t k_t^{\alpha-1} n_t^{1-\alpha}) k_t)$$

$$= u_{c,t} + \mu_t \left[ u_{c,t} c_t + u_{c,t} - u_{c,t} \phi \frac{b_{t+1}^2}{g_t} \right] + \chi_t \left[ E_t - \frac{1}{\beta} \right]$$

(39)

$$\psi_t u_{c,t} (1 - \alpha) \alpha a_t \left( \frac{k_t}{n_t} \right)^{\alpha} - u_{n,t} - \frac{\chi_t - 1}{\gamma \beta} (1 - \alpha) \alpha E_{t-1} \left[ \frac{u_{c,t}}{u_{c,t-1}} a_t k_t^{\alpha-1} n_t^{\alpha-1} \right]$$

$$= \eta_t (1 - \alpha) a_t \left( \frac{k_t}{n_t} \right)^{\alpha} + \mu_t \left[ u_{n,t} n_t + u_{n,t} + \frac{(1 - \alpha) u_{c,t} \phi b_{t+1}^{2 \alpha}}{a_t k_t^{2 \alpha-1} n_t^{\alpha-1}} \right]$$

(40)
\[ \eta_t - \gamma \beta \left\{ E_t \left[ \eta_{t+1} \left( \alpha a_{t+1} \left( \frac{n_{t+1}}{k_{t+1}} \right)^{1-\alpha} + 1 - \delta \right) \right] + \psi_{t+1} u_{c,t+1} \left( 1 - \delta + \alpha^2 a_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} \right) \right\} = \gamma \beta E_t \left[ \mu_{t+1} u_{c,t+1} \frac{\alpha \phi b_{t+1}^2}{a_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha}} \right] \] 
\[ + \chi_t (\alpha - 1) \alpha E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} a_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} \right] \] 
\[ = \gamma \beta E_t \left[ \phi b_{t+1} u_{c,t} \right] = \gamma \beta E_t \left[ \psi_{t+1} u_{c,t+1} + \eta_{t+1} \frac{\partial f_{apr}}{\partial b_{t+1}} \right] \]  

Equ. (39) - (42) together with (31), (34), and the definition of \( \mu_t \), and exogenous processes for \( a_t \) and \( g_t \) define the political equilibrium, given initial conditions \( b_0, k_0 \) and \( \psi_0 \).

5.3 Calibration and simulation

To be able to compare the results to the economy without capital everything is calibrated as before. In particular, \( g \) is set to 0.21, such that the share of government consumption \( g/y = 0.2 \) for \( \gamma = 1 \). As the government becomes increasingly myopic this share will rise to about 22% as \( y \) declines. The production elasticity parameter of capital, \( \alpha \), is set to 0.28 and the rate of depreciation equals 0.024. Total factor productivity, \( a_t \), is assumed to follow an AR(1) process in its logarithm

\[ \ln a_t = \rho a \ln a_{t-1} + \varepsilon_t^a, \quad \varepsilon_t^a \sim N \left( 0, \sigma_{\varepsilon^a}^2 \right). \]

I assume that \( (\rho a, \sigma_{\varepsilon^a}) = (0.9; 0.0064) \), in line with Christiano and Eichenbaum (1992).

To demonstrate the effects of myopia and of the fee, I will proceed as above. The results are presented in Table 4. I first decreased \( \gamma \) from 1 to 0.96, while \( \kappa \) is set to zero. All variables which were already present in the previous model, respond in the same way as before. The only new variable is capital which declines as expected. For \( \gamma = 0.96 \), the fourth row shows the deviations from the Ramsey allocation. Compared to the previous model, we can see that welfare losses have tripled, even though the debt to GDP ratio and the tax rate have not increased as much as before. But capital, working time, and consumption declined by more.
From row five on descending, I imposed a tightening fee on debt by increasing $\kappa$ while now $\gamma$ is held constant at $\gamma = 0.96$. Again all variables return towards the Ramsey allocation. The remaining differences are again due to the reference value of $b^{ref}/y = 0.6$ and are shown in the second last row. The last row reports the allocation for $b^{ref}/y = 0$. All variables almost exactly equal their Ramsey values and the loss in welfare is eliminated. By turning off the distortion on one state variable, the fee also eliminates the distortion on the other state variable. More precisely, it reduces taxes on labour and thereby boosts the accumulation of capital. The IRF to a shock to government consumption and productivity are relegated to the Appendix D and E, respectively.

### Conclusions

The standard normative Ramsey approach to optimal taxation cannot account for the high and persistent levels of government debt that we observe in many OECD countries. Twisting the standard Ramsey approach just slightly by assuming a smaller discount factor for the government and introducing convex transaction costs on bondholdings the presented model can generate high levels of debt. The paper first analyzes an economy without capital and then one with capital, leaving all other things unchanged. In both cases the allocation in terms of consumption, production, and tax rates which results from a higher stock of debt is clearly inferior to the Ramsey outcome. For both specifications it is shown that the detrimental departure from the Ramsey allocation can almost completely be reversed through the introduction of a fee on excessive debt accumulation. The fee resembles characteristics of the SGP but could be implemented at a national level as well. Thus, the

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15Second order approximation.
paper demonstrates how adding another constraint to a policy maker’s problem can enhance welfare in a second best world. The somewhat surprising result is that within the setting of optimal taxation constraining the planner’s problem even more can yield better outcomes. This paper thus supports the views of those who like to maintain or even strengthen the rules of the SGP. It also provides an argument for reformulating Article 115, I of the German Constitution, which tries to set an upper limit to the yearly deficit. It could also be used by the advocates in the "Föderalismuskommision II" of cutting down the power of the German Länder to issue own debt. Future research might include nominal variables and growth into the model and look for the best, or other, means to restrain excessive debt accumulation.
References


A Appendix

Consider the following utility function

\[ u(c_t, n_t) = \ln c_t - 2n_t^2. \]

For convenience define \( z_t = c_t (1 - \nu n_t^2) - \phi \frac{\beta^{t+1}}{\gamma t} \) in (11) which gives

\[ b_t = z_t + \beta E_t \left( \frac{c_t}{c_{t+1}} \right) b_{t+1}. \tag{43} \]

Notice that the constant term \( b_{t+1} \) in (43) has to be the same for all possible realizations of next period’s shock \( \varepsilon_{t+1} \). This can be seen by adjusting the time index in (43) one step forward:

\[ b_{t+1} = z_{t+1} + \beta E_{t+1} \left( \frac{c_{t+1}}{c_{t+2}} \right) b_{t+2}. \tag{44} \]

\( b_{t+1} \) in (44) is given at the beginning of the period. The government has to ensure that all possible allocations (depending on \( \varepsilon_{t+1} \)) of the RHS of (44) equal the LHS, since the bonds are risk free. Therefore, I can include \( b_{t+1} \) in (11) into the expectations operator and replace it by the RHS of (44)

\[
\begin{align*}
b_t &= z_t + \beta E_t \left( \frac{c_t}{c_{t+1}} \right) \left( z_{t+1} + \beta E_{t+1} \left( \frac{c_{t+1}}{c_{t+2}} \right) b_{t+2} \right) \\
&= z_t + \beta E_t \left( \frac{c_t}{c_{t+1}} \right) z_{t+1} + \beta \left( \frac{c_t}{c_{t+1}} \right) E_{t+1} \left( \frac{c_{t+1}b_{t+2}}{c_{t+2}} \right) \\
&= z_t + \beta E_t \left( \frac{c_t}{c_{t+1}} \right) z_{t+1} + \beta c_t E_{t+1} \left( \frac{b_{t+2}}{c_{t+2}} \right) \\
&= z_t + \beta E_t \left( \frac{c_t}{c_{t+1}} \right) z_{t+1} + \beta c_t E_{t+1} \left( \frac{b_{t+2}}{c_{t+2}} \right),
\end{align*}
\]

where the last equality used the law of iterated expectations. Repeating this substitution \( j \) times for future bondholdings \( b_{t+j} \) yields

\[ b_t = z_t + \beta E_t \left( \frac{c_t}{c_{t+1}} z_{t+1} \right) + \beta^2 E_t \left( \frac{c_t}{c_{t+2}} z_{t+2} \right) + \ldots + \beta^{j+1} E_t \left( \frac{c_t}{c_{t+j+1}} b_{t+j+1} \right). \tag{45} \]

Letting \( j \to \infty \) and dividing by \( c_t \) gives

\[
\frac{b_t}{c_t} = E_t \sum_{j=0}^{\infty} \beta^j \frac{z_{t+j}}{c_{t+j}} + \lim_{j \to \infty} \beta^{j+1} E_t \left( \frac{b_{t+j+1}}{c_{t+j+1}} \right).
\]
Using (4) and (5) the last term equals zero

\[
\lim_{j \to \infty} \beta^{j+1} E_t \left( \frac{b_{t+j+1}}{c_{t+j+1}} \right) = \lim_{j \to \infty} \beta^j \left[ \beta E_t \left( \frac{1}{c_{t+j+1}} \right) b_{t+j+1} \right] \\
= \lim_{j \to \infty} \beta^j E_t \left[ \frac{1}{c_{t+j}} \left( \frac{1}{R_{t+j}} + \phi \frac{b_{t+j+1}}{y_{t+j}} \right) b_{t+j+1} \right] \\
= 0.
\]
This appendix shows how the infinite double sum on the LHS of (16), which is repeated below for convenience, can be rewritten as

$$\sum_{t=0}^{\infty} (\gamma \beta)^t \alpha_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = \sum_{t=0}^{\infty} (\gamma \beta)^t \mu_t s_t$$

(46)

where $s_t = \frac{1}{c_t} \left( c_t (1 - \nu n_t^2) - \phi \frac{b_{t+1}^2}{n_t} \right)$, as before. Writing out the sums on the LHS yields

$$\text{LHS} = \alpha_0 \sum_{j=0}^{\infty} \beta^j s_{0+j} + (\gamma \beta) \alpha_1 \sum_{j=0}^{\infty} \beta^j s_{1+j} + (\gamma \beta)^2 \alpha_2 \sum_{j=0}^{\infty} \beta^j s_{2+j} + \ldots$$

$$= \alpha_0 \left[ s_0 + \beta s_1 + \beta^2 s_2 + \beta^3 s_3 + \ldots \right] + (\gamma \beta) \alpha_1 \left[ s_1 + \beta s_2 + \beta^2 s_3 + \beta^3 s_4 + \ldots \right] + (\gamma \beta)^2 \alpha_2 \left[ s_2 + \beta s_3 + \beta^2 s_4 + \beta^3 s_5 + \ldots \right] + \ldots$$

$$= \alpha_0 s_0 + \beta \alpha_0 s_1 + \beta^2 \alpha_0 s_2 + \beta^3 \alpha_0 s_3 + \ldots$$

$$+ \gamma \beta \alpha_1 s_1 + \gamma \beta^2 \alpha_1 s_2 + \gamma \beta^3 \alpha_1 s_3 + \gamma \beta^4 \alpha_1 s_4 + \ldots$$

$$+ (\gamma \beta)^2 \alpha_2 s_2 + \gamma^2 \beta^2 \alpha_2 s_3 + \gamma^2 \beta^3 \alpha_2 s_4 + \gamma^2 \beta^4 \alpha_2 s_5 + \ldots$$

Then factoring out the corresponding terms of $s_t$ gives

$$\text{LHS} = \alpha_0 s_0 + (\beta \alpha_0 + \gamma \beta \alpha_1) s_1 + (\beta^2 \alpha_0 + \gamma \beta^2 \alpha_1 + \gamma \beta^3 \alpha_2) s_2 + \ldots$$

$$= \alpha_0 s_0 + \beta \left( \alpha_0 + \gamma \alpha_1 \right) s_1 + \beta^2 \left( \alpha_0 + \gamma \alpha_1 + \gamma^2 \alpha_2 \right) s_2 + \ldots$$

$$= [\alpha_0] s_0 + \gamma \beta \left[ \frac{\alpha_0}{\gamma} + \alpha_1 \right] s_1 + (\gamma \beta)^2 \left[ \frac{\alpha_0}{\gamma^2} + \frac{\alpha_1}{\gamma} + \alpha_2 \right] s_2 + \ldots$$

Now, expressing the square brackets recursively through the sequence of $\mu_t = \frac{\mu_{t-1}}{\gamma} + \alpha_t$, with $\mu_{-1} = 0$ gives

$$\mu_0 = \frac{\mu_{-1}}{\gamma} + \alpha_0 = \alpha_0$$

$$\mu_1 = \frac{\mu_0}{\gamma} + \alpha_1 = \frac{\alpha_0}{\gamma} + \alpha_1$$

$$\mu_2 = \frac{\mu_1}{\gamma} + \alpha_2 = \frac{\alpha_0}{\gamma^2} + \frac{\alpha_1}{\gamma} + \alpha_2$$

$$\vdots$$
The LHS can then be written as

\[ LHS = \mu_0 s_0 + \gamma \beta \mu_1 s_1 + (\gamma \beta)^2 \mu_2 s_2 + (\gamma \beta)^3 \mu_3 s_3 + \ldots \]

\[ = \sum_{t=0}^{\infty} (\gamma \beta)^t \mu_t s_t \]

\[ = RHS. \]
C Appendix

IRF for the case of myopic fiscal policy with $\gamma = 0.97$, $\phi = 0.015$, and $\kappa = 0$. $d$ denotes the primary deficit and $R$ the gross interest rate.

![Figure A1: IRF to positive innovation of 1 sd to government consumption.](image)

D Appendix

IRF for the case of optimal fiscal policy with $\gamma = 1$, $\phi = 0.015$, and $\kappa = 0$ in an economy with capital.

Figure A2: IRF to positive innovation of 1 sd to government consumption.
E Appendix

IRF for the case of optimal fiscal policy with $\gamma = 1$, $\phi = 0.015$, and $\kappa = 0$ in an economy with capital.

Figure A3: IRF to positive innovation of 1 sd to total factor productivity.