Anti-evasion auditing policy in the presence of common income shocks*

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Abstract

When fairly homogeneous taxpayers are affected by common income shocks, a tax agency's optimal auditing strategy consists of auditing a low-income declarer with a probability that (weakly) increases with the other taxpayers' declarations. Such policy generates a coordination game among taxpayers, who then face both strategic uncertainty –about the equilibrium that will be selected– and fundamental uncertainty –about the type of agency they face. Thus the situation can be realistically modelled as a global game that yields a unique and usually interior equilibrium which is consistent with empirical evidence.

Results are also applicable to other areas like regulation or welfare benefit allocation.

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1 Introduction

It is common practice for tax agencies worldwide to use observable characteristics of taxpayers to partition the population into fairly homogeneous categories in order to better estimate their incomes: all other things being equal, those who declare well below the estimate are likely to be evaders and are audited, while those who declare about or above it are likely to be compliant taxpayers and are not inspected. But this "cut-off" auditing policy (Reinganum and Wilde (1985)) can lead to systematic mistargeting in the presence of common shocks: in good years the category would be under-audited (bars and pubs in a heat-wave); in bad years it would be over-audited (chicken-breeders in an avian-flu outbreak).

The present chapter focuses on the problem a tax agency faces when deciding its auditing policy within each audit category in such scenario. To avoid systematic mistargeting, the government needs *contemporaneous* data correlated with the common shock. I examine the possibility of using the profile of declarations of the taxpayers in a category as a signal of the shock experienced by them and show that, for a government facing a low-income declarer, the optimal auditing strategy is (weakly) increasing in the other taxpayers' declarations. Intuitively, the higher these declarations, the more likely the shock was a positive one, and so the more likely that someone who declares low income is an evader. Precisely this type of reasoning is presumed to be behind the method used by the IRS's "Discriminant Index Function" (DIF) to determine which taxpayers to audit.[1]

This policy introduces a *negative externality* among taxpayers, one that *would not exist otherwise*: if someone increases her declaration, everyone else's probability of detection is increased. This changes the nature of the evasion problem by creating a coordination game among agents: each one of them has incentives to evade if most other people evade as well, and prefers to comply if most of the rest are compliant. The resulting multiplicity of equilibria and its associated policy design problems are avoided by the presence of an information asymmetry in favour of the tax agency. A government's innate "toughness" with respect to evasion is a parameter that is its private information, enters its objective function and affects its optimal policy: *ceteris paribus*, tougher agencies will audit more intensively than softer ones. Since this parameter is an agency's private information, taxpayers need to estimate it in order to decide how much income to declare and they do it based on the information available to them, namely, their incomes and their signals. Each taxpayer's previous experiences, conversations with friends and colleagues and interpretation of media news constitute noisy signals of the government's type and are taxpayers' private information. The heterogeneity of signals makes different taxpayers perceive their situations as different from other taxpayers', and yet every one of them follows the same income declaration strategy. This

[1] On page 301, Alm and McKee (2004) say: "(...) a taxpayer's probability of audit is based not only upon his or her reporting choices, but also upon these choices relative to other taxpayers in the cohort. In short, there is a taxpayer-taxpayer game that determines each individual's chances of audit selection."

leads to the survival of only one equilibrium in which (usually) some people evade and others comply, a result that is empirically supported and yet unlikely to be predicted by other tax evasion models.

Previous research on the area (started by Allingham and Sandmo (1972) and surveyed by Cowell (1990) and Andreoni et al. (1998)) did analyse the effect of asymmetric information in the tax compliance game. Some only considered the presence of "strategic uncertainty" (i.e., the uncertainty that taxpayers face in coordination games about which equilibrium will be selected), usually generated by psychological and/or social externalities (Benjamini and Maital (1985), Fortin et al. (2004), etc.). Others restricted their attention to the "fundamental uncertainty" faced by the taxpayers with respect to the type of agency (Scotchmer and Slemrod (1989), Stella (1991), etc.). The present study, on the other hand, considers both types of uncertainty and thus models the situation as a global game (Carlsson and van Damme (1993), Morris and Shin (2002b)).

The closest references to the present article are Alm and McKee (2004), Basseto and Phelan (2004) and Kim (2005). The first one is a laboratory experiment where the (*ad hoc*) auditing policy is contingent on the distribution of income declarations, while the second and third ones use the global game technique to determine the optimal tax system and the auditing policy, respectively. This paper presents a theoretical analysis in which –unlike the laboratory experiment– the agency's optimal strategy is derived instead of assumed. The other two studies employ the same technique that I use here, but while Basseto and Phelan (2004) are concerned with the optimal tax system as designed by a government, this article focuses only on the targeting aspect of *one* of the agencies of the government. Finally, Kim (2005) generates the strategic interaction among taxpayers by adding a "stigma cost" to their utility functions, whereas in my case it is the result of a cunning tax agency that sets its auditing policy to maximise its objective function.

2 Model

The model focuses on the interaction between the tax agency (also referred to as "the government") and the taxpayers (or "agents") *within a given category.* For simplicity, I will use "population of taxpayers" and "common shocks" to indicate the members of the category and the shocks faced by them, and *not* those of the whole population (i.e., the set which is the union of all the categories), unless indicated otherwise.

2.1 Timing

The timing of the game is as follows:

1. Actors (tax agency and taxpayers) receive their pieces of private information (the agency its "type" λ, the taxpayers their incomes y and signals s).
2. Taxpayers submit their income declarations d and pay taxes accordingly.
3. Finally, the agency observes the vector of declarations $\mathbf{d}$ and undertakes audits and collects fines (if any).

2.2 Objective functions

TAXPAYERS

Taxpayers are uniformly distributed on the $[0,1]$ segment and are assumed to be risk-neutral, so that their utility is a linear function of their disposable income:

$$u\left(d_i, a_i, y_i\right) = y_i - td_i - a_i \cdot \Phi\left(d_i, y_i\right) \qquad \forall i \in [0,1] \tag{1}$$

where $y_i \in \{0,1\}$ is agent i's gross (taxable) income, $t \in (0,1)$ is the income tax rate,[2] $d_i \in \{0,1\}$ is agent i's income declaration, $a_i \in \{0,1\}$ is an indicator function defined as

$$a_i = \begin{cases} 1 & \text{if agent } i \text{ is audited} \\ 0 & \text{if agent } i \text{ is not audited} \end{cases} \tag{4}$$

and $\Phi\left(d_i, y_i\right)$ is the fine agent i should pay if audited, defined as

$$\Phi\left(d_i, y_i\right) = \begin{cases} f \cdot \left(y_i - d_i\right) & \text{if } d_i < y_i \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

[2] The tax system can be easily transformed into a progressive one by using the following change of variables

$$y := Y - B \tag{2}$$

where $Y \in \{B, B+1\}$ is a taxpayer's gross income, B is the exemption level and y is taxable income. The simplest progressive tax system is therefore

$$T = \begin{cases} t\left(D - B\right) & \text{if} \quad D \geqslant B \\ 0 & \text{if} \quad D < B \end{cases} \tag{3}$$

where $D \in \{B, B+1\}$ is the taxpayer's declaration. Thus, in bad years everyone in the class is exempt and in good ones everyone is liable to pay taxes.

where $f := (1+\varsigma)\,t$ and $\varsigma \in (0,1)$ is the surcharge rate that has to be paid by a caught evader on every dollar of evaded taxes).[3]

Tax agency

Narrowly defined, a tax agency's objective is to raise revenue. More generally, its problem consists of determining which citizens should be audited and which ones should not.

An agency, therefore, chooses its auditing strategy in order to minimise its targeting errors.[4]

These errors can be of two types: Negligence and Zeal. A negligence mistake occurs when a "profitable audit" is not undertaken. A zeal error takes place when an "unprofitable audit" is carried out.

An audit is defined as "profitable" if the fine obtained if undertaken more than compensates for the cost of carrying it out (formally, if $\Phi(d_i, y_i) > c$, where $\Phi(d_i, y_i)$ is the fine –as defined in equation 5– and $c \in (\varsigma t, (1+\varsigma)\,t)$ is the cost of the audit). It is assumed that an audit that discovers an evader is always profitable, while an audit that targets a compliant taxpayer is always unprofitable. Formally, if $\alpha_i = 1$ means that auditing agent i is profitable, then

$$\alpha_i := \begin{cases} 1 & \text{if } y_i = 1 \text{ and } d_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Hence, a negligence error (N_i) occurs when the audit is profitable ($\alpha_i = 1$) and it is not undertaken ($a_i = 0$). On the other hand, a zeal error (Z_i) occurs when the audit is not profitable ($\alpha_i = 0$) and yet it is undertaken ($a_i = 1$). Formally,

$$N_i := \begin{cases} 1 & \text{if } \alpha_i = 1 \text{ and } a_i = 0 \\ 0 & \text{otherwise} \end{cases} \qquad Z_i := \begin{cases} 1 & \text{if } \alpha_i = 0 \text{ and } a_i = 1 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

For the rest of the article, and due to the fact that they make the problem more tractable, I will use –without loss of generality– the following two error functions:

$$N_i := (1-a_i)\,(1-d_i)\,y_i \qquad Z_i := a_i\,[1-(1-d_i)\,y_i] \tag{8}$$

Different agencies can, however, value each kind of error differently. If λ is defined as the weight attached to negligence errors, the loss inflicted by agent i on an agency of type λ can be expressed as

$$L_i := \lambda N_i + (1-\lambda)\,Z_i \tag{9}$$

[3] The IRS applies rates between 20% (misconduct) and 75% (fraud) (Andreoni et al. (1998)), so $\varsigma \in (0,1)$ covers the relevant range. It is assumed that $(1+\varsigma)\,t < 1$, such that the fine if caught evading does not exhaust a high-income person's income.

[4] The analysis also holds if the the agency's objective function is based on expected net revenue.

Aggregating over all taxpayers, the loss function becomes

$$L(\mathbf{a}, \mathbf{d}, y) := \int_0^1 [\lambda N_i + (1 - \lambda) Z_i] \, di \tag{10}$$

since i (that indexes taxpayer i) is uniformly distributed on the $[0, 1]$ segment.

2.3 Strategy spaces

A taxpayer's strategy consists of choosing an income declaration $d \in \{0, 1\}$. A tax agency's strategy consists of choosing a vector of auditing decisions $\mathbf{a}$, such that its ith argument, $a_i \in \{0, 1\}$, indicates the auditing decision regarding taxpayer i.

2.4 Information sets

At the node where an actor A makes her decision, her information set, I_A, consists of the union of two sets: one that is common to all actors, I^c, and one that includes the actor's private information, I_A^p. Formally,

$$I_A = I^c \cup I_A^p \tag{11}$$

where $A \in \{TA, i\}$ stands for *actor*, TA for *tax agency* and i for *taxpayer* i, and superindices c and p identify the *common* and *private* sets, respectively.

Common set I^c

The common set I^c includes the exogeneous parameters of the problem (like the tax rate t and the surcharge rate ς) and the parameters of the probability distributions of the private information variables (income y_i, type of agency λ and signal s_i).

Incomes are assumed perfectly correlated to reflect the fact that common shocks affect similar agents in similar ways:[5]

$$y_i = y \ \forall i \in [0, 1] \tag{12}$$

"Good years" ($y = 1$) occur with probability $\gamma \in (0, 1)$ and "bad years" with probability $1 - \gamma$.[6]

The agency's type λ is a non-manipulable characteristic of the agency that affects the government's auditing policy. It is uniformly distributed on the $[\varepsilon, 1 - \varepsilon]$ segment $\left(0 < \varepsilon < \frac{1}{2}\right)$ and is independent of the income shock.

[5] The qualitative results hold when correlation is imperfect but sufficiently high.
[6] The qualitative results do not change significantly if income can take more than two values.

Taxpayers' signals s_i convey information about the government's type λ and are, on average, correct. They reflect the information about the agency's type that taxpayers get from all available sources: media news, previous experiences, conversations with colleagues and friends, etc. Formally,

$$s_i := \lambda + \varepsilon_i \qquad \forall i \in [0,1] \tag{13}$$

where ε_i is the error term, which is assumed to be white noise ($E(\varepsilon_i) = 0\ \forall i$), uniformly distributed on the $[-\varepsilon, \varepsilon]$ segment, and independent of income y_i, other taxpayers' errors $\varepsilon_{j \neq i}$ and the government's type λ.

TAXPAYER i'S PRIVATE SET I_i^p

Each taxpayer knows the realisation of her private information variables, namely, her income y_i and her private signal s_i. Furthermore, since all taxpayers know that the income distribution is degenerate, they know that every taxpayer has the same income y ($y = 0$ if $y_i = 0$ and $y = 1$ if $y_i = 1$).

TAX AGENCY$'$S PRIVATE SET I_{TA}^p

The agency knows the realisation of her private information variable, its type λ. Also, given the timing of the game, it observes the vector of income declarations $\mathbf{d}$, each argument $d_i \in \{0,1\}$ being the declaration of a taxpayer. Given the dichotomous nature of the declarations, the vector of income declarations can be summarised by a sufficient statistics, namely, the average declaration $D \in [0,1]$, which will be used henceforth.

2.5 Schematic representation of the game

Given the elements presented so far, the game can be represented as in tables 1 (for bad years) and 2 (for good years).

$y = 0$ $(1-\gamma)$		Agency's strategy	
		Audit $(a = 1)$	Do not audit $(a = 0)$
Taxpayer's	Declare low ($d = 0$)	$1-\lambda$ 0	0 0
strategy	Declare high ($d = 1$)	$1-\lambda$ $-t$	0 $-t$

Note: In each cell, Bottom-left element: taxpayer's utility; Top-right element: agency's loss.

Table 1: Compliance game in bad years (y=0).

$y = 1$ (γ)		Agency's strategy	
		Audit ($a = 1$)	Do not audit ($a = 0$)
Taxpayer's strategy	Declare low ($d = 0$)	0 / $1 - f$	λ / 1
	Declare high ($d = 1$)	$1 - \lambda$ / $1 - t$	0 / $1 - t$

Note: In each cell, Bottom-left element: taxpayer's utility; Top-right element: agency's loss.

Table 2: Compliance game in good years (y=1).

Taxpayers observe their income before making their declarations, so that they know which of the two games is being played. The tax agency, on the other hand, does not know the true value of y, and therefore does not know which of the two games is being played.

2.6 Equilibrium concept

A perfect Bayesian equilibrium must specify actors' posterior beliefs, taxpayers' income declaration strategies, the agency's auditing strategy and the average declaration in the category. Formally,

$$\lambda \quad | \quad s \sim U\left[s - \varepsilon, s + \varepsilon\right] \tag{14}$$

$$s \quad | \quad \lambda \sim U\left[\lambda - \varepsilon, \lambda + \varepsilon\right] \tag{15}$$

$$\Pr\left(y = 1 \mid D\right) = \begin{cases} 1 & \text{if} \quad D > 0 \\ \gamma & \text{if} \quad D = 0 \end{cases} \tag{16}$$

$$d_i^*\left(s, y\right) \in \underset{d_i \in \{0,1\}}{\arg\max} E\left\{u\left(d_i, a_i, y_i\right) \mid I_i\right\} \ \ \forall i \in [0, 1] \tag{17}$$

$$\mathbf{a}^*\left(D, \lambda\right) \in \underset{a_i \in \{0,1\} \ \forall i \in [0,1]}{\arg\min} E\left\{L\left(\mathbf{a}, \mathbf{d}, y\right) \mid I_{TA}\right\} \tag{18}$$

$$D\left(\mathbf{d}, \lambda\right) = \int_0^1 d_i^*\left(s, y\right) \ di \tag{19}$$

The first three lines indicate that all actors have Bayesian beliefs. In particular, taxpayers know that the posterior distribution of the type of the agency λ (conditional on the taxpayer's signal s) is uniform with support $[s - \varepsilon, s + \varepsilon]$ (equation 14). The tax agency, on the other hand, knows that the posterior distribution of signals (conditional on the agency's type λ) is uniform with support $[\lambda - \varepsilon, \lambda + \varepsilon]$ (equation 15). The agency also knows that the posterior distribution of income (conditional on the observed average declaration D) is such that equation 16 holds.

The following equations indicate that actors choose their actions optimally: taxpayers choose their declarations d in order to maximise their expected utility, conditional on the available information I_i (equation 17) and the tax agency chooses its auditing strategy $\mathbf{a}$ so as to minimise the expected losses due to targeting mistakes, conditional on its available information I_{TA} (equation 18). Finally, equation 19 aggregates the taxpayers' decisions to give the average declaration.

3 Solving the model

3.1 Preliminaries

In bad years ($y = 0$) taxpayers know that the game being played is the one depicted in table 1. The taxpayer's optimal strategy is therefore:

Proposition 1 *In bad years, every taxpayer declares low income. Formally, for all $i \in [0, 1]$*

$$d_i^* (s, 0) = 0 \tag{20}$$

Proof. From direct inspection of table 1. Declaring low income strictly dominates declaring high income: the payoff is 0 in the first case and $-t$ in the second one, irrespective of the other taxpayers' declarations and the agency's auditing decision. ■

It is therefore common knowledge that the game shown in table 1 simplifies to

$y = 0$ $(1 - \gamma)$		Agency's strategy: Audit ($a = 1$)	Agency's strategy: Do not audit ($a = 0$)
Taxpayer's strategy	Declare low ($d = 0$)	$1 - \lambda$ (agency's loss); 0 (taxpayer's utility)	0 (agency's loss); 0 (taxpayer's utility)

Note: In each cell, Bottom-left element: taxpayer's utility; Top-right element: agency's loss.

Table 3: Compliance game in bad years (y=0).

Thus, the compliance game consists of the ones shown in tables 2 (for good years) and 3 (for bad ones). Given the timing presented in section 2.1, the game is solved by backwards induction. Hence, I will analyse first the second stage and will solve the tax agency poblem

3.2 Tax agency problem

The agency's problem consists of choosing the audit vector **a** so as to minimise the expected losses, conditional on its information set I_{TA}. The expected loss function is therefore:

$$E\left\{L\left(\mathbf{a}, D, y\right) \mid I_{TA}\right\} = \Pr\left(y=0 \mid I_{TA}\right) \cdot L\left(\mathbf{a}, D, 0\right) + \Pr\left(y=1 \mid I_{TA}\right) \cdot L\left(\mathbf{a}, D, 1\right) \quad (21)$$

where $L(\mathbf{a}, D, 0)$ is the loss when income is low $(y=0)$ and $L(\mathbf{a}, D, 1)$ is the loss when income is high $(y=1)$. Define the probability of high income conditional on observing average declaration D as

$$\phi\left(D\right) := \Pr\left(y=1 \mid I_{TA}\right) \quad (22)$$

Note, also, that D represents the proportion of the population that declares high income. Hence, without loss of generality, we can assume that taxpayers on the $[0, 1-D]$ segment declare low income and those on the $(1-D, 1]$ segment declare high income. The loss function (equation 10) can therefore be re-written as

$$L\left(\mathbf{a}, D, y\right) = \lambda y \int_0^{1-D} \left(1-a_i\right) di + \left(1-\lambda\right)\left(1-y\right) \int_0^{1-D} a_i di + \left(1-\lambda\right) \int_{1-D}^{1} a_i di \quad (23)$$

where the first two terms correspond to the expected loss generated by those who declare low income and the last one corresponds to the loss generated by those who declare high income, and where the assumption of perfectly correlated incomes ($y_i = y \; \forall i \in [0, 1]$) allows us to take y out of the integral.

Furthermore, from the perspective of the agency, taxpayers who declare low income are indistinguishable from each other, and the same is true for those who declare high income. This means that the government will treat every person in each group in an identical way. This implies, therefore, that the agency has only two policy variables: a_0 and a_1, which are the audit decisions for taxpayers who declare low and high income, respectively. The loss function then becomes

$$L\left(\mathbf{a}, D, y\right) = \lambda y \left(1-a_0\right)\left(1-D\right) + \left(1-\lambda\right)\left(1-y\right) a_0 \left(1-D\right) + \left(1-\lambda\right) a_1 D \quad (24)$$

which takes the value

$$L\left(\mathbf{a}, D, 0\right) = \left(1-\lambda\right) a_0 \left(1-D\right) + \left(1-\lambda\right) a_1 D \quad (25)$$

when the income shock is negative $(y=0)$, and the value

$$L\left(\mathbf{a}, D, 1\right) = \lambda \left(1-a_0\right)\left(1-D\right) + \left(1-\lambda\right) a_1 D \quad (26)$$

Thus, from equations 22, 25 and 26, the expected loss function of equation 21 becomes

$$E_{TA}(L) = \{[1-\phi(D)](1-\lambda)a_0 + \phi(D)\lambda(1-a_0)\}(1-D) + (1-\lambda)a_1 D \tag{27}$$

where the subindex TA indicates that the expectation is conditional on the information set of the tax agency and the arguments of the loss function were omitted for simplicity.

The agency therefore minimises the expected loss, as indicated in equation 27. The results are summarised in the following proposition.

Proposition 2 *For every taxpayer, a λ-type agency's optimal auditing strategy is as follows:*

- *if a taxpayer declares high income ($d_i = 1$), do not audit her ($a_1 = 0$);*
- *if every taxpayer declares low income ($D = 0$) and the agency is "soft" (λ sufficiently low), do not audit anyone ($a_0 = 0$);*
- *if every taxpayer declares low income ($D = 0$) and the agency is "tough" (λ sufficiently high), audit everyone ($a_0 = 1$);*
- *if some taxpayers declare low income and others declare high income ($D > 0$), audit everyone who declares low income ($a_0 = 1$).*

Formally, for every taxpayer $i \in [0,1]$,

$$a_i^*(d_i, D, \lambda) = \begin{cases} 0 & \text{if } d_i = 1 \\ 0 & \text{if } d_i = 0,\ D = 0,\ \text{and } \lambda < \tilde{\lambda} \\ \in [0,1] & \text{if } d_i = 0,\ D = 0,\ \text{and } \lambda = \tilde{\lambda} \\ 1 & \text{if } d_i = 0,\ D = 0,\ \text{and } \lambda > \tilde{\lambda} \\ 1 & \text{if } d_i = 0,\ \text{and } D > 0 \end{cases} \tag{28}$$

where $\tilde{\lambda} := 1 - \gamma$ and $\gamma \in (0,1)$ is the probability of a good year.

Proof. In the appendix, page 24. ■

Intuitively, the proposition says that an agency's optimal auditing decision with respect to a given taxpayer i depends on the taxpayer's decision d_i, the declarations of all other taxpayers (summarised by the average declaration D) and the agency's type λ. When at least one person declares high income (and so $D > 0$), the government knows for sure – thanks to the perfect correlation assumption– that the shock was a positive one (it was a "good year"), and so the optimal strategy consists of auditing everyone who declares low income ($a_i^*(0, D > 0, \lambda) = 1$, since they are evaders) and not auditing anyone who declares high income ($a_i^*(1, D, \lambda) = 0$, since only "rich" taxpayers ever declare high income, and so their declarations are truthful). When everyone declares low income (so $D = 0$), the government cannot tell whether it faces a population of "poor" compliant taxpayers or one of "rich" evaders. The optimal policy therefore depends on how tough the government is (i.e., how high λ is) and how likely it is for the taxpayers to face a good year (i.e., the value of γ). If the agency is rather tough (λ is rather high), the optimal policy consists of auditing everyone (and the same is true if the probability of a good year, γ, is high). Otherwise (if the agency is rather soft or a bad year is very likely), it is better for the agency to audit no one.

These results are summarised in the following proposition:

Proposition 3 *For every taxpayer, a λ-type agency's optimal auditing strategy is:* (1) *(weakly) increasing in the agency's type λ, and* (2) *(weakly) increasing in the probability of a good year γ. Formally,*

$$(1) \quad \frac{\partial a_i^*(d_i, D, \lambda)}{\partial \lambda} \geqslant 0 \qquad\qquad (2) \quad \frac{\partial a_i^*(d_i, D, \lambda)}{\partial \gamma} \geqslant 0 \tag{29}$$

Proof. By direct inspection of equation 28. ■

Further characterizing the agency's optimal strategy, the next result describes how it depends on the taxpayer's own declaration as well as on every other taxpayer's declarations:

Proposition 4 *For every taxpayer, a λ-type agency's optimal auditing strategy is:* (1) *(weakly) increasing in every other taxpayers' declaration $d_{j \neq i}$, and* (2) *(weakly) decreasing in the taxpayer's own declaration d_i. Formally,*

$$(1) \quad \frac{\partial a_i^*(d_i, D, \lambda)}{\partial d_{j \neq i}} \geqslant 0 \qquad\qquad (2) \quad \frac{\partial a_i^*(d_i, D, \lambda)}{\partial d_i} \leqslant 0 \tag{30}$$

Proof. By direct inspection of equation 28. ■

Intuitively, this means that the agency audits individuals who declare high income with a lower probability than those who declare low income (as is standard in tax evasion models). The novelty of the present study is in the result of equation 30.1, which shows that a loss-minimising agency would use the information conveyed by the vector of income declarations (or the average declaration, which in this case is a sufficient statistics) when deciding its optimal policy. In particular, the declarations of other taxpayers provide *contemporaneous* information about the likelihood of a given income shock, improving the targeting proficiency of the agency that can thus perfectly distinguish between truthful and untruthful declarations when the average declaration is different from 0.

3.3 Taxpayer problem

Once the second stage game is solved, we can turn to the first stage and solve the taxpayer problem.

As shown in section 3.1, in bad years all taxpayers declare low income. Hence, here I will focus on the case when income is high.

In good years, each taxpayer i chooses her income declaration d_i so as to maximise her expected utility, conditional on her information set I_i. Her expected utility function is:

$$E\left\{u\left(d_i, a_i, y=1\right) \mid I_i\right\} = \Pr\left(a_i = 1 \mid I_i\right) \cdot u\left(d_i, 1, 1\right) + \Pr\left(a_i = 0 \mid I_i\right) \cdot u\left(d_i, 0, 1\right) \qquad (31)$$

Noting that the expected value of the audit decision simplifies to the probability of an audit:

$$E_i\left(a_i\right) = \Pr\left(a_i = 1 \mid I_i\right) \qquad (32)$$

and using equation 1, the expected utility function of equation 31 becomes

$$E_i\left(u\right) = 1 - td_i - f_i \cdot E_i\left(a_i\right) \qquad (33)$$

where the subindex i indicates that the expectation is conditional on the information set of taxpayer i and the arguments of the utility function were omitted for simplicity.

If the taxpayer evades $d_i = 0$, her expected utility equals gross income minus the expected fine:

$$E_i\left(u\left(evasion\right)\right) = 1 - f \cdot E_i\left(a_i\left(d_i = 0\right)\right) \qquad (34)$$

If the taxpayer complies, she gets utility

$$u(compliance) = 1 - t \qquad (35)$$

with certainty.

The taxpayer's optimal decision $d^*\left(y_i, E_i\left(a_i\left(d_i = 0\right)\right)\right)$ depends on the comparison between the two as follows

$$d^*\left(1, E_i\left(a_i\left(d_i = 0\right)\right)\right) = \begin{cases} 0 & \text{if } E_i\left(a_i\left(d_i = 0\right)\right) < P \\ \in [0,1] & \text{if } E_i\left(a_i\left(d_i = 0\right)\right) = P \\ 1 & \text{if } E_i\left(a_i\left(d_i = 0\right)\right) > P \end{cases} \qquad (36)$$

where $P := \frac{1}{(1+\varsigma)}$ is the probability of detection that eliminates evasion.

Intuitively, in good years taxpayers evade only if their subjective belief about the probability of being audited is not too high. This implies that an agent's declaration is (weakly) increasing in her expectation over the probability of detection.

Combining the results for bad and good years (proposition 1 and equation 36), the solution to the taxpayer problem is

$$d^*\left(y_i, E_i\left(a_i\left(d_i=0\right)\right)\right)=\begin{cases} 0 & \text{if } y_i=0 \\ 0 & \text{if } y_i=1 \text{ and } E_i\left(a_i\left(d_i=0\right)\right)<P \\ \in[0,1] & \text{if } y_i=1 \text{ and } E_i\left(a_i\left(d_i=0\right)\right)=P \\ 1 & \text{if } y_i=1 \text{ and } E_i\left(a_i\left(d_i=0\right)\right)>P \end{cases} \tag{37}$$

from which it is clear that an agent's declaration is (weakly) increasing in her gross income.

The latter results are summarised in the following proposition:

Proposition 5 *A taxpayer's optimal declaration strategy is:* (1) *(weakly) increasing in her (subjective) expectation over the probability of detection* $E_i\left(a_i\left(d_i=0\right)\right)$*, and* (2) *(weakly) increasing in her gross income* y_i*. Formally,*

$$(1) \quad \frac{\partial d^*\left(y_i, E_i\left(a_i\left(d_i=0\right)\right)\right)}{\partial E_i\left(a_i\left(d_i=0\right)\right)} \geqslant 0 \qquad\qquad (2) \quad \frac{\partial d^*\left(y_i, E_i\left(a_i\left(d_i=0\right)\right)\right)}{\partial y_i} \geqslant 0 \tag{38}$$

Proof. By direct inspection of equation 37. ■

Equation 30.1 and the first part of proposition 38 make taxpayer i's optimal declaration strategy a (weakly) increasing function of the other taxpayers' declarations:

Proposition 6 *Taxpayers' declarations are (weakly) strategic complements. Formally, for every* $j \neq i$*,*

$$\frac{\partial d_i^*\left(y_i, s_i\right)}{\partial d_{j \neq i}} \geqslant 0 \tag{39}$$

Proof. Directly from propositions 5 and 4. ■

This proposition opens a channel through which a higher signal leads to a higher declaration: a high signal means that other taxpayers are also likely to receive high signals –and to declare high income too– which increases the expected probability of detection and makes compliance relatively more attractive (i.e., provides incentives to (weakly) increase the amount of income declared).

Even more importantly, this result transforms the nature of the tax evasion problem, because it creates a *coordination game* among the taxpayers *on top of* the cat-and-mouse game that each one of them plays against the agency and that is usually the only one considered by the literature. The strategic complementarity between taxpayers' declarations, however, is not an inherent characteristic of the game, but rather one that is created by the agency in its attempt to minimise its targeting errors. Indeed, it is the fact that the auditing strategy is an increasing function of other taxpayers' declarations (Proposition 4) that *creates a negative externality* between taxpayers (proposition 6). That is, a cunning agency, willing to minimise its targeting-related losses, designs its optimal auditing strategy by introducing some strategic uncertainty (i.e., by creating a coordination game between taxpayers) that improves its ability to distinguish compliant from non-compliant agents and thus decreases the occurrence of targeting mistakes.

The taxpayer's optimal declaration strategy can be further characterised in terms of private signals, as shown in the next proposition:

Proposition 7 *In good years, a taxpayer's optimal declaration strategy:* (1) *is the same for all taxpayers, and* (2) *is (weakly) increasing in her private signal* s_i*. Formally,*

$$(1) \quad d^*(1, s_i) = \begin{cases} 0 & \text{if } s_i < \hat{s} \\ \in [0,1] & \text{if } s_i = \hat{s} \\ 1 & \text{if } s_i > \hat{s} \end{cases} \qquad (2) \quad \frac{\partial d^*(1,s_i)}{\partial s_i} \geqslant 0 \qquad (40)$$

where $\hat{s} := \tilde{\lambda} + \varepsilon(2P-1)$, $\tilde{\lambda} := 1 - \gamma$ *and* $P := \frac{1}{(1+\varsigma)}$.

Proof. The first part is the result of $\hat{s}$ being a constant that is independent of the identity of the taxpayer whose strategy is being studied. The determination of $\hat{s}$ is shown in the appendix, page 25. For the second part, by direct inspection of equation 40.1. ■

The intuition is straightforward: the higher the signal received ($s_i := \lambda + \varepsilon_i$ from equation 13), the higher is the taxpayer's (subjective) expectation over the government's type λ, meaning that the agent believes that, very likely, she faces a tough agency and, consequently, a high probability of detection. This decreases the (subjective) expected return of evasion and makes compliance more attractive, which leads the taxpayer to (weakly) increase her income declaration.

The first part of the proposition highlights the fact that, though having different private signals, all taxpayers agree on the "switching point" below which one should evade and above which one should comply. Note also that, as expected, each "type" of taxpayer (defining agent i's "type" as its private information pair (y_i, s_i)) has a unique optimal strategy: taxpayers with low income ($y_i = 0$) ignore their signals and always declare low income; taxpayers with high income ($y_i = 1$) do take into account the signals they receive and declare income as shown in equation 40.1.

4 Equilibrium

A priori, the generation of a coordination game among taxpayers does not look as a good idea for the agency because this kind of games present multiple equilibria, which make policy design a complicated matter. Nevertheless, this difficulty is overcome thanks to the presence of a second source of uncertainty (called "fundamental uncertainty") that allows for the tax evasion problem to be modelled as a "global game" (Carlsson and van Damme (1993), Morris and Shin (2002b)).[7]

This equilibrium-selection technique eliminates all but one equilibria owing to the introduction of some heterogeneity in taxpayers' information sets in the form of the noisy private signals they receive and that convey information about the government's private information parameter λ (the source of the "fundamental uncertainty"). Thus, taxpayers do not observe the true coordination game (as they would do if signals were 100% accurate), but slightly different versions of it. This is the case since taxpayers with different signals would work out different estimates of the agency's type λ and the average declaration D, and so of their probabilities of detection. The optimal declaration strategy, however, is one and the same for every "type" of taxpayer (propositions 1 and 7). The rationale for this result goes along the lines described in the paragraph immediately after the proof of proposition 6: my own signal gives me information about the possible signals that other taxpayers may have received and, more importantly, about the signals that they *cannot* have received, thus allowing me to discard some strategies that they cannot have followed. The application of this process iteratively by *every* taxpayer leads to the elimination of all strictly dominated strategies and leaves only one optimal strategy to be followed by every taxpayer (Morris and Shin (2002a)), namely, the ones in propositions 1 and 7.

As a consequence, once the private information variables (the agency's type λ and taxpayers' incomes and signals $(\mathbf{y}, \mathbf{s})$) are realised, the equilibrium will be *unique*.

However, depending on the value of λ, the equilibrium can present different features, as illustrated by the following proposition:[8]

Proposition 8 *In bad years ($y_i = 0\ \forall i \in [0,1]$), the average declaration is zero ($D = 0$), as is the level of evasion ($\kappa^* = 0$). In good years ($y_i = 1\ \forall i \in [0,1]$), the corresponding values*

[7] In other applications (bank runs, currency crises, etc), this technique has been criticised because of not taking into account the coordinating power of markets and prices (Atkeson (2000)). This criticism is greatly mitigated in the case of tax evasion, since there is no "insurance market against an audit" to aggregate information about the government's type (the "fundamental", in global games jargon).

[8] Since in bad years taxpayers declare low income in every scenario, the three cases are characterised (and labelled) according to the actions taken by taxpayers in good years.

are as follows:

	Full evasion ($\lambda < \hat{s} - \varepsilon$)	*Partial evasion* ($\hat{s} - \varepsilon < \lambda < \hat{s} + \varepsilon$)	*Full compliance* ($\hat{s} + \varepsilon < \lambda$)
Average declaration D	0	$\frac{\lambda+\varepsilon-\hat{s}}{2\varepsilon}$	1
Level of evasion κ^*	1	$1 - \frac{\lambda+\varepsilon-\hat{s}}{2\varepsilon}$	0

(41)

Proof. In the appendix, page 28. ■

This shows that, as expected, evasion is lower the tougher the government is.

Equilibrium strategies for each actor are shown in the following proposition:

Proposition 9 *The unique equilibrium of the tax evasion game looks like one of the following cases:* (1) Full evasion *($\lambda < \hat{s} - \varepsilon$): in good years, every taxpayer evades and nobody is audited,* (2) Partial evasion *($\hat{s} - \varepsilon < \lambda < \hat{s} + \varepsilon$): in good years, taxpayers with low signals ($s_i < \hat{s}$) evade and are audited with certainty while those with high signals ($s_i > \hat{s}$) comply and are not audited, and* (3) Full compliance *($\hat{s} + \varepsilon < \lambda$): in good years, every taxpayer complies and everyone who declares low income is audited. In bad years, taxpayers always declare truthfully. Formally,*

	Full evasion	*Partial evasion*	*Full compliance*
$d^*(0, s_i)$	0	0	0
$d^*(1, s_i)$	0	$\begin{cases} 0 & \text{if } s_i < \hat{s} \\ \in [0,1] & \text{if } s_i = \hat{s} \\ 1 & \text{if } s_i > \hat{s} \end{cases}$	1
$a_i^*(d_i, D, \lambda)$	0	$\begin{cases} 0 & \text{if } d_i = 1 \\ 1 & \text{if } d_i = 0 \end{cases}$	$\begin{cases} 0 & \text{if } d_i = 1 \\ 1 & \text{if } d_i = 0 \end{cases}$

(42)

Proof. Follows directly from the optimal strategies of the players (propositions 1 and 7 for the taxpayers, propostion 2 for the agency) and the characterisation of the equilibrium in terms of the average declaration (proposition 8). ■

The full evasion case occurs when the agency is so soft ($\lambda < \hat{s} - \varepsilon$) that all taxpayers know it will audit nobody who declares low income, and so everyone evades. The opposite occurs in the full compliance case, in which the agency is so tough ($\hat{s} + \varepsilon < \lambda$) that all taxpayers know it will audit everyone who declares low income, and so everyone complies. The partial evasion case occurs when the government is not too soft nor too tough ($\hat{s} - \varepsilon < \lambda < \hat{s} + \varepsilon$) and so taxpayers would like to do as most taxpayers do (*strategic complementarity*). They follow the optimal strategy described in proposition 7, which means that the vector of declarations will be different from zero. The agency, observing this, would know for sure that true income is high and so will audit everyone who declares 0 and nobody that declares 1.

Building on these results, one can further characterise the three cases:

Proposition 10 *The payoffs of the players in the three possible scenarios are as follows:*

	Full evasion	*Partial evasion*	*Full compliance*
Taxpayer/ Bad year	0	0	0
Taxpayer/ Good year	1	$\begin{cases} 1-t & \text{if } d_i = 1 \\ 1-(1+\varsigma)\,t & \text{if } d_i = 0 \end{cases}$	$1-t$
Tax Agency	$\gamma\lambda$	$\begin{cases} 0 & \text{if } \lambda < \tilde{\lambda} \\ (1-\gamma)(1-\lambda) & \text{if } \lambda > \tilde{\lambda} \end{cases}$	$(1-\gamma)(1-\lambda)$

(43)

Proof. Follows directly from the definition of the payoff functions of the players (equations 1 and 10), their optimal strategies (propositions 1, 2 and 7) and the characterisation of the equilibrium in terms of the average declaration (proposition 8). ■

In bad years a taxpayer's payoff is a direct consequence of her declaring truthfully her low income and getting no punishment or reward for doing so, regardless of the value of λ. The other two actors' payoffs, on the other hand, are different depending on the case under consideration. In good years, with full evasion, every taxpayer evades and, since no one is audited, they keep their gross incomes. In turn, since the agency audits no one, it suffers an expected loss of $\gamma\lambda$ because with probability γ the year is a good one and so everyone is an evader who is not caught (negligence errors) and with probability $1-\gamma$ the year is a bad one, everyone complies and nobody is audited (no zeal errors). Analogously, with full compliance, all taxpayers comply and so their disposable income is simply their gross income minus their voluntarily paid taxes, $1-t$. The expected loss of the agency is now $(1-\gamma)(1-\lambda)$ because with probability γ the year is a good one, everyone complies and nobody is audited (no negligence errors) and with probability $1-\gamma$ the year is a bad one and everyone complies but is audited anyway (zeal errors). The most interesting scenario is, however, the partial evasion one. Here, a soft agency ($\hat{s} - \varepsilon < \lambda < \tilde{\lambda}$) makes no targeting error whatsoever, thus reaching the best outcome it could aspire to. The rationale behind this result is that in good years some taxpayers will evade (those with low signals) while others will comply (those with high signals) and so the agency can perfectly distinguish evaders from compliant taxpayers, which implies that evaders are always caught (their payoffs are equal to gross income minus fine, $1-f$) while compliant taxpayers are never targeted (they get payoffs equal to gross income minus taxes $1-t$). In bad years, everyone declares zero and nobody is audited, so no mistake is made. A tough agency ($\tilde{\lambda} < \lambda < \hat{s}+\varepsilon$) will also catch every evader in good years, but will audit everyone in bad years, thus leading to the same expected loss than the "Full Evasion" case.

A surprising corollary can thus be stated: *"The relationship between the level of evasion and the agency's payoff is not monotonic."* Indeed, as we move from the right to the left column in equation 43 (i.e., as evasion increases), the agency's welfare first increases and then decreases. This means that the government is better off when it can create a *coordination game* among agents but, especially, when it in turn *makes taxpayers take different actions* (some evade, others comply), thus getting valuable information about the true income of the population and increasing its targeting accuracy.

To conclude the characterisation of the equilibrium, it is important to analyse how more accurate signals affect the level of evasion and the agency's payoff:

Proposition 11 *More precise information (formally, a lower ε) leads to:* (1) *(weakly) less compliance if the agency is soft ((weakly) more if it is tough), and* (2) *a (weakly) higher expected loss. Formally,*

$$(1) \quad \frac{\partial D^*}{\partial \varepsilon} \begin{cases} \geqslant 0 & \text{if } \lambda < \tilde{\lambda} \\ \leqslant 0 & \text{if } \lambda > \tilde{\lambda} \end{cases} \qquad (2) \quad \frac{\partial EL^*}{\partial \varepsilon} \leqslant 0 \tag{44}$$

where $\tilde{\lambda} := 1 - \gamma$.

Proof. In the appendix, page 29. ■

The first part of the proposition highlights the fact that the impact of better information on the level of evasion depends on the type of the agency. This is at odds with previous studies, which usually find that better information leads to more evasion, through the argument that it decreases the risk borne by taxpayers who, assumed to be risk averse, have therefore more incentives to evade.

Though compelling, this argument cannot be applied to the present case because here agents are assumed risk neutral. Yet, what matters is that the relationship between compliance and accuracy of information is not *intrinsically* (weakly) increasing or decreasing, but rather one whose shape depends on the type of the government. Intuitively, when an agency is soft (λ is low) it dislikes targeting compliant taxpayers and so would audit with a very low probability. For signals of a given precision $\varepsilon > 0$, agents will estimate the probability of detection and decide their income declarations accordingly. If signals became more precise (if ε decreased), agents would be more aware of the fact that the agency is soft (in the extreme case, when $\varepsilon = 0$, they would know it with certainty), and so would expect a lower probability of detection, which in turn makes evasion relatively more attractive and leads to lower compliance. An analogous story can be used when the agency is tough (λ is high).

The second part of the proposition, on the other hand, shows that more accurate information is *never* good for the tax agency. Though previous studies also found this relationship, they relied on the above mentioned risk aversion of taxpayers and on the monotonic relationship between the level of evasion and the tax agency's payoffs (debunked by proposition 10).

The channel used here, on the other hand, hinges on the new feature introduced by the agency's policy: the coordination game played by taxpayers. From the agency's perspective, and using proposition 10, the coordination scenarios ("Full Evasion" and "Full Compliance" cases) are (weakly) dominated by the coordination failure one ("Partial Evasion" case). Since more precise information decreases the likelihood of the latter scenario (because the probability of $\lambda \in (\hat{s} - \varepsilon, \hat{s} + \varepsilon)$ decreases), then agencies prefer low-precision signals over very accurate ones. Note, however, that this benefit is only available to soft agencies ($\hat{s} - \varepsilon < \lambda < \tilde{\lambda}$), because it increases an agency's ability to distinguish evaders from compliant taxpayers in good years and thus decreases the number of negligence errors it makes. On the other hand, tough agencies ($\tilde{\lambda} < \lambda < \hat{s} + \varepsilon$) cannot take advantage of it as the situation in bad years (which is the origin of such agencies' zeal errors) is unaffected by a change in the informativeness of signals.

Alternatively, a lower ε can be interpreted as an increase in the degree of aggregation of information (or information-sharing). That is, if taxpayers shared their signals, the effect would be equivalent to an increase in their precision, since the group's average signal is expected to be closer to the true value of λ than the individual ones. In the limit, if all signals were shared, taxpayers would know the government's type with certainty –this is exactly the same result as if all signals were perfectly accurate (i.e., if $\varepsilon \to 0$).

5 Discussion

As every other model, the one developed here is built around some simplifying assumptions that make it more tractable and elegant, but also more restrictive and unrealistic.

Indeed, it could be argued that tax agencies do not follow a "bang-bang" policy such that either everyone is audited or nobody is, but rather one where a fraction of the population is audited while the rest is not. The first approach is a direct consequence of the "*ex-post* horizontal equity" condition, while the second one would fit a situation that satisfies the condition of "horizontal equity in expectation". The former is a stronger version of the latter, but also leads to situations where those who declare equal amounts are *effectively* treated equally, a desirable feature of an optimal auditing policy in my view. However, if the second approach were used, the results would not be significantly different from the ones presented in the text, the only "major" difference being that a tough agency would not audit everyone, but rather just a fraction of the population sufficiently large as to eliminate all incentives to evade (with the added benefit that the enforcement costs will be lower due to the smaller number of audits undertaken).

Also unlikely to be found in the real world is the dichotomous character of income assumed here. When more than two levels of income are allowed, the auditing decision with respect to a given individual depends on the *relative position* of the taxpayer's declaration compared to the rest of the population's: if it is among the highest ones, then the taxpayer's probability of detection is still (weakly) increasing in the agency's type and, under mild assumptions, (weakly) decreasing in the amount declared; if it is not, the agency knows the taxpayer is lying and audits her with certainty. When only two levels of income are considered, this policy collapses to the one presented earlier in this chapter.[9]

Along similar lines, it is clear that the assumption of perfect correlation among the taxpayers' incomes is an implausible one. However, it is just intended to capture the fact that usually taxpayers that belong to the same category are homogeneous in most aspects, including income. Relaxing it will not change the (qualitative) results, as long as the common shocks are maintained as the *main source of income variability*. This ensures that there is still a significant degree of correlation among incomes and, therefore, that other taxpayers' declarations convey *useful information* about the common shock that affects the category. Also very important for the analysis is the fact that incomes within a class are more homogeneous than the signals received by its members, such that the differences among them are mainly due to disparate perceptions of the government's type. Thus, the assumption of perfect uniformity allows us to observe the effect of the fundamental uncertainty unadulterated by the presence of income heterogeneity, and so the analysis is greatly simplified.

Finally, the importance of the partitioning of the taxpayer population into fairly homogeneous categories is highlighted by the fact that the above mentioned "relatively high correlation" condition is achieved when the category consists of agents that are very similar to each other in terms of their "observables" (age, profession, gender, etc.), since in this case their idiosyncratic shocks will be relatively small compared to the category-wide ones.[10] However, since the partitioning problem is an issue this paper is not concerned with, the only related matter worth discussing here is the type of classes that favours the present model. And since the latter clearly relies on some degree of uniformity within the class, its predictions are more likely to fit the data from classes with a large number of rather homogeneous people (e.g., unskilled manufacture workers or non-executive public servants) than the ones from small and/or heterogeneous classes.

[9] Also, irrespective of the levels of income allowed, if they are bounded above (i.e., $y_i \leqslant y_{\max}\ \forall i \in [0,1]$), the agency would never audit those who declare $y_{\max}$. In the more realistic case of unbounded domain, the probability of detection simply decreases as the declaration increases, as is standard in the literature.

[10] These "observables" refer to variables that are exogenous to (or costly to manipulate by) the agents, and so do *not* include taxpayers' current declarations.

6 Conclusion

The question of a tax agency's optimal auditing strategy in the presence of common income shocks is relevant because it is not unusual for such shocks to be the main source of income variability for a group of fairly homogeneous taxpayers. Under these circumstances an agency's best policy consists of auditing those who declare low income with a probability that is (weakly) increasing in the declarations of the other taxpayers in the category. Intuitively, the higher these declarations, the more likely the shock was a positive one, and hence the more likely that someone who declares low income is an evader.

Implementing this policy does not require new information to be gathered by the agency, just using the available information better. Yet, it changes the nature of the problem for the taxpayers: on top of the standard cat-and-mouse game each one of them plays against the agency, they also play a coordination game against each other, a game in which a negative *externality* between them is *created*, a game taxpayers would *not* play if the policy were not contingent on the vector of declarations.

The heterogeneity in private signals eliminates the policy design difficulties that the multiplicity of equilibria appears to generate and paves the way for modelling the problem as a global game which not only is more realistic, but also predicts a unique equilibrium which is consistent with empirical evidence.

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A Appendix

Proof. Proposition 2

Derive the expected loss function (equation 27) with respect to the agency's two policy variables, namely, $a_0 := a(0, D, \lambda)$ and $a_1 := a(1, D, \lambda)$.

For the first part of the proposition, compute the derivative of the expected loss with respect to a_1:

$$\frac{\partial E_{TA}(L)}{\partial a_1} = (1 - \lambda) D \tag{45}$$

which is positive[11], so that the optimal strategy in order to minimise expected losses is to set

$$a_1^* = 0 \tag{46}$$

That is, the agency must not audit anyone who declares high income.

For the last three parts of the proposition, that determine the value of a_0, it is necessary to distinguish two cases: one when the average declaration is zero ($D = 0$, parts 2 and 3 of the proposition) and another when the average declaration is positive ($D > 0$, part 4 of the proposition).

Consider first the scenario in which the average declaration is positive ($D > 0$). Since it is common knowledge that declaring low income is the dominant strategy for taxpayers when income is low (proposition 1), the agency is able to infer that whoever declares high income says the truth, i.e., the posterior probability of the taxpayer having high income conditional on the taxpayer having declared high income is 1:

$$\Pr(y_i = 1 \mid d_i = 1) = 1 \tag{47}$$

Furthermore, given the perfect correlation between incomes, if the agency observes at least one high declaration (i.e., if $D > 0$), it is able to infer that the year is a good one with probability one: the posterior probability of a good year ($y = 1$) conditional on the average declaration being strictly positive is 1:

$$\Pr(y = 1 \mid D > 0) = 1 \tag{48}$$

(thus the first part of equation 16). From the definition of a negligence error (equation 8), therefore, the agency's best strategy is to audit everyone who declares low income ($d_i = 0$) when average income is strictly positive ($D > 0$). This proves the last part of the proposition.

Consider now the case when every taxpayer declares low income, so that the average dec-

[11] Variable a_1 is the audit decision regarding a taxpayer who declares high income. This means that at least one person declared high income, and so the average declaration D is strictly positive.

laration is zero ($D = 0$). In this scenario, the agency does not know if income is truly low ($y = 0$) or if it is high and every taxpayer evaded ($y = 1$ and $d_i = 0$ for every $i \in [0, 1]$). The agency's posterior belief over the probability of a good year conditional on the average declaration being equal to zero is therefore:

$$\Pr(y = 1 \mid D = 0) = \gamma \tag{49}$$

where γ is the prior probability of a good year (thus the second part of equation 16). Hence, deriving the expected loss function (equation 27) with respect to the audit decision a_0 yields

$$\frac{\partial E_{TA}(L)}{\partial a_0} = 1 - \lambda - \gamma \tag{50}$$

This expression is positive if $\lambda < 1 - \gamma$ and negative otherwise, so the agency will audit in the latter case ($\lambda > \tilde{\lambda} := 1 - \gamma$) and will not audit in the the first case ($\lambda < \tilde{\lambda} := 1 - \gamma$). This proves parts 2 and 3 of the proposition ■

Proof. Proposition 7

The proof is similar to that in Morris and Shin (1997), so I will concentrate on those elements specific to my model.

Propose a taxpayer strategy (for good years) of the following type:

$$d_i(s_i) = \begin{cases} 0 & \text{if} \quad s_i < \hat{s} \\ \in [0,1] & \text{if} \quad s_i = \hat{s} \\ 1 & \text{if} \quad \hat{s} < s_i \end{cases} \tag{51}$$

that is, a strategy according to which the taxpayer declares high income (comply) when the signal is sufficiently high and low income (evades) when it is sufficiently low.

To be optimal, such strategy must satisfy the following condition

$$\hat{s} \gtrless s_i \quad \text{if and only if} \quad E_i(u(\mathit{evasion})) \gtrless u(\mathit{compliance}) \tag{52}$$

where $E_i(u(\mathit{evasion}))$ and $u(\mathit{compliance})$ are given by equations 34 and 35.

The average declaration in the economy is therefore given by the proportion of taxpayers that receive signals greater than $\hat{s}$. Since signals are uniformly distributed around the true type of the agency λ (with support $[\lambda - \varepsilon, \lambda + \varepsilon]$), there are three cases to consider:

1. If $\hat{s} \leq \lambda - \varepsilon$, then $D = 1$;
2. If $\lambda - \varepsilon < \hat{s} < \lambda + \varepsilon$, then $D = \int_{\hat{s}}^{\lambda+\varepsilon} \frac{1}{2\varepsilon} ds = \frac{\lambda+\varepsilon-\hat{s}}{2\varepsilon}$; and
3. If $\lambda + \varepsilon \leq \hat{s}$, then $D = 0$;

That is, the average declaration in the economy D is a weakly increasing function of the type of the agency λ. In particular, if the agency is so tough (case 1 above) that even the person with the lowest signal $(\lambda - \varepsilon)$ complies (see equation 51), then everyone declares high income and $D = 1$. On the other hand, when the agency is so soft (case 3 above) that even the person with the highest signal $(\lambda + \varepsilon)$ evades, then everyone declares low income and $D = 0$. In intermediate cases, some people declare high income and others declare low income, so $D \in (0, 1)$. Formally,

$$D(\lambda) = \begin{cases} 0 & \text{if} \quad \lambda \leq \hat{s} - \varepsilon \\ \in [0,1] & \text{if} \quad \hat{s} - \varepsilon < \lambda < \hat{s} + \varepsilon \\ 1 & \text{if} \quad \hat{s} + \varepsilon \leq \lambda \end{cases} \tag{53}$$

Equation 28 gives the agency's optimal strategy. Such strategy depends on the individual taxpayer's declaration d_i, the declarations of every other taxpayer in the category D, and the type of agency λ. In particular, it states that the agency will audit a person who declares low income $(d_i = 0)$ only if one of the two following scenarios occur:

1. The average declaration in the economy is strictly positive: $D > 0$.
2. The average declaration in the economy is zero and the agency is "tough": $D = 0$ and $\lambda > \tilde{\lambda}$ (where $\tilde{\lambda}$ is defined as in proposition 2).

Consider first case 1: From equation 53, this scenario occurs when $\lambda > \hat{s} - \varepsilon$.

Case 2, on the other hand, requires both $\lambda \leq \hat{s} - \varepsilon$ (from equation 53) and $\lambda > \tilde{\lambda}$ (and therefore, implicitly, that $\tilde{\lambda} < \hat{s} - \varepsilon$). Combining the two, case 2 occurs when $\tilde{\lambda} < \lambda \leq \hat{s} - \varepsilon$.

Therefore, the agency audits a taxpayer who declares low income only if $\lambda > \hat{\lambda}$, where $\hat{\lambda} := \min\left\{\hat{s} - \varepsilon, \tilde{\lambda}\right\}$.

The agency's optimal strategy can then be reduced to the following expression

$$a_i(d_i, \lambda) = \begin{cases} 0 & \text{if} \quad d_i = 0 \\ \in [0,1] & \text{if} \quad d_i = 1 \quad \text{and} \quad \lambda \leq \hat{\lambda} \\ 1 & \text{if} \quad d_i = 1 \quad \text{and} \quad \lambda > \hat{\lambda} \end{cases} \tag{54}$$

The expected utility of a taxpayer i who evades is given by expression

$$E_i(u(0, a_i, 1)) = E[u(0, a_i(0, \lambda), 1) \mid s] \tag{55}$$

Taxpayer i's posterior distribution of λ conditional on her private signal s is uniformly distributed around s, so there are three cases to consider:[12]

[12] f is the fine paid if caught evading, as defined in equation 5 .

1. If $\hat{\lambda} \leq s - \varepsilon$, then $a_i = 1$ and $E_i\left(u\left(0, a_i, 1\right)\right) = \int_{s-\varepsilon}^{s+\varepsilon} \frac{u(0,1,1)}{2\varepsilon} d\lambda = \int_{s-\varepsilon}^{s+\varepsilon} \frac{1-f}{2\varepsilon} d\lambda = 1 - f$;
2. If $s - \varepsilon < \hat{\lambda} < s + \varepsilon$, then $E_i\left(u\left(0, a_i, 1\right)\right) = \int_{s-\varepsilon}^{\hat{\lambda}} \frac{u(0,0,1)}{2\varepsilon} d\lambda + \int_{\hat{\lambda}}^{s+\varepsilon} \frac{u(0,1,1)}{2\varepsilon} d\lambda = \int_{s-\varepsilon}^{\hat{\lambda}} \frac{1}{2\varepsilon} d\lambda + \int_{\hat{\lambda}}^{s+\varepsilon} \frac{1-f}{2\varepsilon} d\lambda = 1 - \frac{s+\varepsilon-\hat{\lambda}}{2\varepsilon} f$; and
3. If $s + \varepsilon \leq \hat{\lambda}$, then $a_i = 0$ and $E_i\left(u\left(0, a_i, 1\right)\right) = \int_{s-\varepsilon}^{s+\varepsilon} \frac{u(0,0,1)}{2\varepsilon} d\lambda = \int_{s-\varepsilon}^{s+\varepsilon} \frac{1}{2\varepsilon} d\lambda = 1$.

Intuitively, if the signal is so high (case 1) that even her lowest estimate of λ, $s - \varepsilon$, is high enough as to trigger an audit (from equation 54), then her payoff from evasion is $1 - f$ with certainty. On the other hand, if the signal is so low (case 3) that even her highest estimate of λ, $s + \varepsilon$, is low enough as to avoid triggering an audit, then her payoff from evasion is 1 with certainty. In intermediate cases, the taxpayer's payoff is not certain, and the expected utility takes values in the $(1 - f, 1)$ range.

Thus, the taxpayer's expected utility of evasion is given by the following expression:

$$E_i\left(u\left(0, a_i, 1\right)\right) = E\left[u\left(0, a_i\left(0, \lambda\right), 1\right) \mid s\right] = \begin{cases} 1 & \text{if} \quad s \leq \hat{\lambda} - \varepsilon \\ 1 - \frac{s+\varepsilon-\hat{\lambda}}{2\varepsilon} f & \text{if} \quad \hat{\lambda} - \varepsilon < s < \hat{\lambda} + \varepsilon \\ 1 - f & \text{if} \quad \hat{\lambda} + \varepsilon \leq s \end{cases} \tag{56}$$

Note that this function is a continuous and (weakly) decreasing function of the taxpayer's signal and that it can take values in the range $[1 - f, 1]$. From the definition of f (equation 5), it is straightforward to show that

$$1 - f < 1 - t < 1 \tag{57}$$

This means that the utility of compliance (the term in the centre, from equation 35) is higher than the utility if caught evading (the left-hand side term) but lower than the utility if evasion goes undetected (the right-hand side term). This implies that exists a signal $\hat{s}$ such that the expected utility of evasion (equation 56) equals the utility of compliance $1 - t$. Formally

$$E\left[u\left(0, a_i\left(0, \lambda\right), 1\right) \mid \hat{s}\right] := 1 - t \tag{58}$$

or, equivalently, using equation 56,

$$\hat{s} := \hat{\lambda} - \varepsilon + 2\varepsilon P \tag{59}$$

where P is defined as in equation 36.

When $\hat{\lambda} = \hat{s} - \varepsilon$, then equation 59 becomes

$$\hat{s} = \left(\hat{s} - \varepsilon\right) - \varepsilon + 2\varepsilon P \tag{60}$$

which simplifies to

$$2\varepsilon\left(1-P\right)=0 \tag{61}$$

and is only satisfied in extreme and rather uninteresting cases: when $\varepsilon = 0$ (no fundamental uncertainty regarding the type of agency λ) and/or $P := \frac{1}{1+\varsigma} = 1$ (when no fine is paid if caught evading). Thus, this case will be ignored.

When $\hat{\lambda} = \tilde{\lambda}$, on the other hand, the switching point becomes

$$\hat{s} := \tilde{\lambda} - \varepsilon + 2\varepsilon P \tag{62}$$

and, since the expected utility of evasion (equation 56) is a weakly decreasing function of the private signal received by the taxpayer and $1 - f < 1 - t < 1$, this means that $\hat{s}$ is unique. This proves the first part of the proposition: every taxpayer follows the same threshold strategy.

Furthermore, it is now straightforward to show that the expected utility of evasion is higher (respectively, lower) than the utility of compliance when the private signal s is lower (respectively, higher) than the threshold $\hat{s}$, thus proving that the threshold strategy of equation 40.1 is indeed optimal (i.e., satisfies the condition in equation 52) and hence, trivially, that the optimal declaration strategy is a weakly increasing function of the private signal received.This proves the second part of the proposition. ■

Proof. Proposition 8

The average declaration is defined as

$$D := \int_s d_i\left(1, s\right) dG\left(s \mid \lambda\right) \tag{63}$$

where $G\left(s \mid \lambda\right)$ is the probability distribution of signals, conditional on the type of agency being λ. From equation 13, $s \mid \lambda$ is uniformly distributed on the $[\lambda - \varepsilon, \lambda + \varepsilon]$ segment. Note that, because of the taxpayer's optimal strategy in good years (proposition 7), the average declaration can be interpreted as the fraction of the population that gets a signal above the threshold $\hat{s}$.

Depending on the value of λ, three cases can occur:

1. Full evasion ($\lambda < \hat{s} - \varepsilon$): Even the person with highest signal (i.e., $s_i = \lambda + \varepsilon$) would evade. Formally, $D := \int_{\lambda-\varepsilon}^{\lambda+\varepsilon} (0) \frac{1}{2\varepsilon} d\mathbf{s} = 0$.
2. Partial evasion ($\hat{s} - \varepsilon < \lambda < \hat{s} + \varepsilon$): Those with signals between $\lambda - \varepsilon$ and $\hat{s}$ evade, those with signals between $\hat{s}$ and $\lambda + \varepsilon$ comply. Formally, $D := \int_{\lambda-\varepsilon}^{\hat{s}} (0) \cdot \frac{1}{2\varepsilon} d\mathbf{s} + \int_{\hat{s}}^{\lambda+\varepsilon} (1) \frac{1}{2\varepsilon} d\mathbf{s} = \frac{\lambda+\varepsilon-\hat{s}}{2\varepsilon}$.

3. Full compliance ($\hat{s} + \varepsilon < \lambda$): Even the person with the lowest signal (i.e., $s_i = \lambda - \varepsilon$) would comply. Formally, $D := \int_{\lambda-\varepsilon}^{\lambda+\varepsilon} (1) \frac{1}{2\varepsilon} d\mathbf{s} = 1$.

The level of evasion in good years is simply the fraction of the population that gets a signal below the threshold $\hat{s}$. That is, $\kappa = 1 - D$. ■

Proof. Proposition 11

Consider first the full evasion case ($\lambda < \hat{s} - \varepsilon$). Since the threshold $\hat{s}$ is defined as in equation 62, the condition $\lambda < \hat{s} - \varepsilon$ becomes

$$\varepsilon < \frac{\tilde{\lambda} - \lambda}{2(1-P)} \tag{64}$$

where P is the auditing intensity that eliminates evasion.

Since $P \in \left(\frac{1}{2}, 1\right)$, $1 - P$ can only take values in the interval $\left(0, \frac{1}{2}\right)$. Also, since the noise of the signals cannot be negative, it must be the case that

$$0 < \varepsilon \tag{65}$$

Combining equations 64 and 65, the full evasion case requires $0 < \varepsilon < \frac{\tilde{\lambda}-\lambda}{2(1-P)}$, which is only feasible if $\lambda < \tilde{\lambda}$ (i.e., full evasion is only feasible if the government is soft).

In the full compliance case ($\hat{s} + \varepsilon < \lambda$), the condition $\hat{s} + \varepsilon < \lambda$ becomes

$$\varepsilon < \frac{\lambda - \tilde{\lambda}}{2P} \tag{66}$$

Combining equations 66 and 65, the full compliance case requires $0 < \varepsilon < \frac{\lambda-\tilde{\lambda}}{2P}$, which is feasible only if $\tilde{\lambda} < \lambda$ (i.e., full compliance is only feasible if the government is tough).

Finally, the condition needed for the existence of the partial evasion case ($\hat{s} - \varepsilon < \lambda < \hat{s} + \varepsilon$) becomes $\varepsilon > \max\left\{\frac{\tilde{\lambda}-\lambda}{2(1-P)}, \frac{\lambda-\tilde{\lambda}}{2P}\right\}$. If $\lambda < \tilde{\lambda}$ it becomes $\varepsilon > \frac{\tilde{\lambda}-\lambda}{2(1-P)}$. If $\tilde{\lambda} < \lambda$, it is $\varepsilon > \frac{\lambda-\tilde{\lambda}}{2P}$.

Summarising the results so far, there are two cases to consider: (1) if the government is soft ($\lambda < \tilde{\lambda}$) the full evasion case arises when the noise is low ($\varepsilon < \frac{\tilde{\lambda}-\lambda}{2(1-P)}$) and the partial evasion one when it is high; and (2) if the government is tough ($\tilde{\lambda} < \lambda$) the full compliance case occurs when the noise is low ($\varepsilon < \frac{\lambda-\tilde{\lambda}}{2P}$) and the partial evasion when it is high.

Hence, using proposition 8, the average declaration in each of the two cases is given by

$$\begin{array}{ll} (1) & D^{*}=\left\{\begin{array}{lll} 0 & \text{if} & 0<\varepsilon<\frac{\tilde{\lambda}-\lambda}{2(1-P)} \\ 1-P+\frac{\tilde{\lambda}-\lambda}{2\varepsilon} & \text{if} & \frac{\tilde{\lambda}-\lambda}{2(1-P)}<\varepsilon \end{array}\right. \\ (2) & D^{*}=\left\{\begin{array}{lll} 1 & \text{if} & 0<\varepsilon<\frac{\lambda-\tilde{\lambda}}{2P} \\ 1-P+\frac{\tilde{\lambda}-\lambda}{2\varepsilon} & \text{if} & \frac{\lambda-\tilde{\lambda}}{2P}<\varepsilon \end{array}\right. \end{array} \tag{67}$$

It is straightforward from here to prove the first part of the proposition by simply computing the derivative of D with respect to ε.

For the second part, using the two cases considered above and proposition 10, the expected loss of the agency is as follows

$$\begin{array}{ll} (1) & EL^{*}=\left\{\begin{array}{lll} \gamma\lambda & \text{if} & 0<\varepsilon<\frac{\tilde{\lambda}-\lambda}{2(1-P)} \\ 0 & \text{if} & \frac{\tilde{\lambda}-\lambda}{2(1-P)}<\varepsilon \end{array}\right. \\ (2) & EL^{*}=(1-\gamma)(1-\lambda) \end{array} \tag{68}$$

Deriving with respect to ε yields the result stated in the proposition. ■