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Rodrigo Alfaro

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HIGHER ORDER PROPERTIES OF THE SYMMETRICALLY NORMALIZED INSTRUMENTAL VARIABLE ESTIMATOR

Rodrigo Alfaro
Gerencia de Estabilidad Financiera
Banco Central de Chile

Resumen

Este artículo computa el sesgo de segundo orden para el estimador de variables instrumentales simétricamente distribuido (SNIV) utilizando expansiones asintóticas de Edgeworth tanto para el estimador como para el valor propio mínimo. El estimador fue propuesto por Alonson-Borrego y Arellano (1999) como un estimador alternativo para el estimador de máxima verosimilitud con información limitada (LIML). Sin embargo, sus resultados se basan en simulaciones de Monte Carlo. El artículo rechaza dicha conclusión mostrando que el sesgo de segundo orden de SNIV es similar al de 2SLS, mientras que LIML es insesgado.

Abstract

This paper provides the second order bias for the Symmetrically Normalized Instrumental Variable Estimator (SNIV), using Edgeworth expansions for both the estimator and the minimum eigenvalue. SNIV was proposed by Alonso-Borrego and Arellano (1999) as an alternative for the Limited Information Maximum Likelihood Estimator (LIML), based solely on simulations. The paper shows that second order biases of SNIV and 2SLS are similar meanwhile LIML is second order unbiased. Previous results can be obtained in a specific design: small number of strong instruments, where biases of 2SLS, SNIV, and LIML are zero.

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1 Introduction

There is an extensive literature in econometrics analyzing the properties of IV estimators in large samples. These properties include, for example, the higher-order bias (e.g., see Nagar (1959), Sawa (1969), Fuller (1977), Phillips (1985) and Ullah (2004)), and Mean Squared Error (MSE) (Donald and Newey (2001) and Hahn, Hausman and Kuersteiner (2004)). The analysis can also be conducted under different asymptotic approximations (Bekker (1994), Newey (2004), Hansen, Hausman and Newey (2005) and Chao and Swanson (2005)).

When the endogenous variables are jointly normally distributed, and only one equation is specified, the efficient estimator is the Limited Information Maximum Likelihood (LIML). LIML is not affected by the normalization on the coefficients of the endogenous variables. Alonso-Borrego and Arellano (1999) propose an alternative estimator to LIML, which is also invariant to this normalization, in the context of dynamic panel data models. They name it as Symmetrically Normalized Instrumental Variable (SNIV) estimator. Brown (1960) introduces this estimator for the simultaneous equations problem, and Hillier (1990) studies its properties.

The difference between LIML and SNIV estimators is that LIML uses the variance-covariance matrix of the error terms as a weight for the endogenous variables, whereas SNIV uses the identity matrix. This difference in weights is irrelevant in large samples. LIML and SNIV estimators are both consistent and asymptotically normally distributed.

Using a second order approximation, I find that the SNIV estimator has higher order bias and therefore it is likely to perform worse than LIML in finite samples. Monte Carlo experiments agree with this theoretical results. The size of bias for SNIV is similar to the bias obtained by Nagar (1959) for the 2SLS estimator.

2 Model and Estimators

For simplicity I consider a problem with one explanatory variable which is also endogenous.

$$y_i = \beta x_i + e_i, \tag{1}$$

$i = 1, \ldots, n$. A $K \times 1$ set of valid instruments (Z) for the endogenous variable x is available. Z is not correlated with the error term (e), $E(Z'x) \neq 0$, and $E(ZZ')$ has a rank of K.

The parameter β can be estimated by the 2SLS estimator

$$\hat{\beta}_{2SLS} = \frac{\left(\sum_{i=1}^n z_i x_i\right)' \left(\sum_{i=1}^n z_i z_i'\right)^{-1} \left(\sum_{i=1}^n z_i y_i\right)}{\left(\sum_{i=1}^n z_i x_i\right)' \left(\sum_{i=1}^n z_i z_i'\right)^{-1} \left(\sum_{i=1}^n z_i x_i\right)}.$$

Following the current literature (see, e.g., Staiger and Stock (1997)) I will say that the instruments are weak if the correlation between the instruments and the endogenous variables is small. A measure for weakness is the concentration parameter, that for this particular case has the same information than the first-stage population R^2.

The model (1) can be alternatively written as follows

$$x_i = \gamma y_i + w_i, \tag{2}$$

where $\gamma = 1/\beta$, and $w_i = -e_i/\gamma$. 2SLS in (2) gives

$$\hat{\gamma}_{2SLS} = \frac{\left(\sum_{i=1}^n z_i y_i\right)' \left(\sum_{i=1}^n z_i z_i'\right)^{-1} \left(\sum_{i=1}^n z_i x_i\right)}{\left(\sum_{i=1}^n z_i y_i\right)' \left(\sum_{i=1}^n z_i z_i'\right)^{-1} \left(\sum_{i=1}^n z_i y_i\right)}.$$

Note that in general $\hat{\beta}_{2SLS} \neq 1/\hat{\gamma}_{2SLS}$, so 2SLS is not invariant to normalization. If the specification is correct, however, the differences between the $\hat{\beta}_{2SLS}$ (forward) and the $\hat{\gamma}_{2SLS}$ (reverse) estimators are due to the sampling error. This fact is exploited by Hahn and Hausman (2002). They use $(\hat{\beta}_{2SLS} - 1/\hat{\gamma}_{2SLS})$ to derive a new specification test.

For comparison, I consider the Least Squares Estimator (LS) of the model

$$\hat{\beta}_{LS} = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}.$$

This estimator is valid only when the degree of endogeneity of the system is zero.

The k-class estimator introduced by Theil (1958) is defined as follows for a fixed value of k.[1]

$$\hat{\beta}(k) = \frac{k\left(\sum_{i=1}^{n} z_i x_i\right)' \left(\sum_{i=1}^{n} z_i z_i'\right)^{-1} \left(\sum_{i=1}^{n} z_i y_i\right) + (1-k)\sum_{i=1}^{n} x_i y_i}{k\left(\sum_{i=1}^{n} z_i x_i\right)' \left(\sum_{i=1}^{n} z_i z_i'\right)^{-1} \left(\sum_{i=1}^{n} z_i x_i\right) + (1-k)\sum_{i=1}^{n} x_i^2}.$$

Nagar (1959) suggests to use $k = 1 + (K-2)/n$, which eliminates the second order bias obtained from an Edgeworth expansion of this estimator. A problem with this choice is that for $k > 1$ the estimator does not have finite-sample moments under normal errors in the reduced form system (see, e.g., Mariano (1982)).

The lack of moments implies that for finiste samples the estimator can give estimates very far from the true parameter values. From the point of view of simulations, the results from estimator without moments must be analyzed in terms of the empirical distribution only.

2.1 Normalization

The estimators $\hat{\beta}_{LS}$, $\hat{\beta}_{2SLS}$, $\hat{\gamma}_{2SLS}$ and $\hat{\beta}(k)$ are computed using only one equation. However, it is possible to express the model (1) as

$$\begin{aligned} y_i &= z_i'\theta + u_i, \\ x_i &= z_i'\pi + v_i, \end{aligned} \tag{3}$$

where $\theta = \beta\pi$.

Because x and y are both endogenous variables, it is unclear which variable should be on the left hand side in (1). However, in the reduced form system (3) all endogenous variables are explained by a set of exogenous instruments Z.

In order to estimate β using this reduced form, each equation of (3) is solved using LS estimators

[1]It is easy to see that $\hat{\beta}(0) = \hat{\beta}_{LS}$ and $\hat{\beta}(1) = \hat{\beta}_{2SLS}$.

$$\hat{\theta} = \left(\sum_{i=1}^{n} z_i z_i'\right)^{-1} \left(\sum_{i=1}^{n} z_i y_i\right) = \theta + \left(\sum_{i=1}^{n} z_i z_i'\right)^{-1} \left(\sum_{i=1}^{n} z_i u_i\right),$$

$$\hat{\pi} = \left(\sum_{i=1}^{n} z_i z_i'\right)^{-1} \left(\sum_{i=1}^{n} z_i x_i\right) = \pi + \left(\sum_{i=1}^{n} z_i z_i'\right)^{-1} \left(\sum_{i=1}^{n} z_i v_i\right),$$

then applying a Minimum Distance Estimator, β is estimated as follows

$$(\hat{\beta}, \hat{\pi}) = \arg\min \left\{ \begin{pmatrix} \hat{\theta} - \beta\pi \\ \hat{\pi} - \pi \end{pmatrix}' W^{-1} \begin{pmatrix} \hat{\theta} - \beta\pi \\ \hat{\pi} - \pi \end{pmatrix} \right\},$$

where constraints $\theta = \beta\pi$ and $e_i = u_i - \beta v_i$ are imposed and W is a symmetric and positive definite matrix.

Concentrating out π by LS and using $W = V \otimes (\sum_{i=1}^{n} z_i z_i')^{-1}$, which was suggested by Alonso-Borrego and Arellano (1999), the estimator for β becomes

$$\hat{\beta} = \arg\min_{\beta} q(\beta), \text{ with } q(\beta) \equiv \frac{(\sum_{i=1}^{n} z_i e_i)' (\sum_{i=1}^{n} z_i z_i')^{-1} (\sum_{i=1}^{n} z_i e_i)}{(1, -\beta) V (1, -\beta)'},$$

where $e_i = y_i - \beta x_i$. This estimator is the Limited Information Maximum Likelihood (LIML) if V is the variance-covariance matrix of the error terms of (3). Solving this problem (see Appendix 6.2), I find

$$\hat{\beta}_{LIML} = \frac{(\sum_{i=1}^{n} z_i x_i)' (\sum_{i=1}^{n} z_i z_i')^{-1} (\sum_{i=1}^{n} z_i y_i) - \hat{q}_{LIML} \sum_{i=1}^{n} x_i y_i}{(\sum_{i=1}^{n} z_i x_i)' (\sum_{i=1}^{n} z_i z_i')^{-1} (\sum_{i=1}^{n} z_i x_i) - \hat{q}_{LIML} \sum_{i=1}^{n} x_i^2}.$$

Moreover, this estimator becomes the Symmetrically Normalized Instrumental Variables (SNIV) estimate if V is an identity matrix.[2]

$$\hat{\beta}_{SNIV} = \frac{(\sum_{i=1}^{n} z_i x_i)' (\sum_{i=1}^{n} z_i z_i')^{-1} (\sum_{i=1}^{n} z_i y_i)}{(\sum_{i=1}^{n} z_i x_i)' (\sum_{i=1}^{n} z_i z_i')^{-1} (\sum_{i=1}^{n} z_i x_i) - \hat{q}_{SNIV}}.$$

The SNIV estimator was suggested by Alonso-Borrego and Arellano (1999) as an alternative to

[2]The LS, 2SLS, LIML and SNIV estimator can be defined as solution of a general problem (see Appendix 6.1 for details).

LIML for a Dynamic Panel Data model (DPD).

In the just-identified case ($K = 1$) we have $\hat{q} = 0$, and therefore LIML and SNIV are equivalent to 2SLS. Moreover, LIML can be written as a k-class estimator, with $k = 1/(1 - q_{LIML})$. Note that q_{LIML} is always less than 1, and therefore, LIML is a k-class estimator with $k > 1$ so that the estimator does not have finite-sample moments. A similar argument applies to the SNIV estimator.[3]

3 Properties of SNIV

The exact distributions for some IV estimators have been computed under the assumption of normal errors (see Sawa (1969) for 2SLS, and Phillips (1985) for LIML). The expressions for these distributions usually include infinite series and the computation of moments based on them is cumbersome. For that reason the asymptotic distribution is widely used to approximate the exact finite-sample distribution. In the case of IV estimators, Nagar (1959) and Ullah (2004) have shown that a better approximation to the finite-sample distribution can be obtained by adding higher order terms to the standard first-order normal approximation.

I consider in this analysis the approximation called large-n Edgeworth expansion. This approximation adds terms of higher order to the standard asymptotic distribution. In particular, the first moment presented in this section considers terms up to order $1/n$.

It should be noted that the terms used in this expansion are obtained from the asymptotic Taylor series approximation to the SNIV estimator, which is available in Appendix 6.4. These terms can be used to obtain higher order approximations to the finite-sample moments of the SNIV estimator.[4] Finally, the Berry-Esseen theorem implies that the distribution obtained by the Edgeworth expansion has a maximum error proportional to the third moment of the true distribution (see Serfling (1980), Field and Ronchetti (1990) or Ullah (2004)).

Condition 3.1. *Suppose that (u_i, v_i) are independent and identically distributed (i.i.d.), following*

[3] Alonso-Borrego and Arellano (1999) report that (in simulations) the SNIV estimator does not have well-defined sample moments.

[4] See Ullah (2004) for details of Edgeworth expansion, alternatives procedures and examples.

a bivariate normal distribution with zero-means, variances σ_u^2, σ_v^2, and covariance σ_{uv}.

For the composite error $e_i = u_i - \beta v_i$, Condition 3.1 implies $\sigma_{ev} = \sigma_{uv} - \beta\sigma_v^2$ and $\sigma_e^2 = \sigma_u^2 - 2\beta\sigma_{uv} + \beta^2\sigma_v^2$. It should be noted that the normality assumption can be replaced without further changes by existence of moments up to fourth order, homoskedasticity, and conditional symmetry.

Condition 3.2. *The set of instruments is non-stochastic and the quadratic variation converge to a positive define matrix*

$$\frac{1}{n}\sum_{i=1}^{n} z_i z_i' \quad = \quad \Delta + o\,(1)\,.$$

Under Condition 3.2 the expression $\pi'\Delta\pi$ represents a measure of the goodness of fit of the model. In particular, the first-step population R^2 can be written as $\pi'\Delta\pi/(\pi'\Delta\pi + \sigma_v^2)$. Extending the results for stochastic instruments implies to consider assumptions on the joint distribution of error terms and instruments.

Under Conditions 3.1 and 3.2, the second order bias of the k-class estimator obtained by Nagar (1959) can be written as

$$E\left[\hat{\beta}(k) - \beta\right] \quad = \quad \frac{\sigma_{ev}}{\pi'\Delta\pi}\left[\frac{(K-2)-(k-1)n}{n}\right] + O\left(\frac{1}{n^2}\right).$$

As it was discussed in Section 2, $\hat{\beta}(0) = \hat{\beta}_{LS}$ and $\hat{\beta}(1) = \hat{\beta}_{2SLS}$. It is clear that 2SLS is less biased than LS as long as $K < n$, but the bias of 2SLS converges to the bias of LS as the number of instruments (K) grows. Nagar (1959) proposes to use $k = 1 + (K-2)/n$ for which the second order bias is zero.

LIML does not have a closed form solution, and therefore the procedure proposed by Nagar (1959) is not directly applicable. However, Rothenberg (1984) proposes to approximate $\hat{q}_{LIML}$ and compute the bias for LIML as a k-class estimator with $k = 1 + (K-1)/n$, then the second order bias for LIML is $-\sigma_{ev}/(n\pi'\Delta\pi)$. This result suggests that the bias of LIML is not affected by the number of instruments.[5]

[5] Donald and Newey (2001) refine the second order bias presented in Nagar (1959) and propose a slightly different correction.

As it was shown in section 2.1, the SNIV estimator cannot be computed as a closed form solution, but q_{SNIV} can be approximated using a similar procedure as for LIML.

3.1 Edgeworth Expansion for SNIV estimator

From Condition 3.2, we have

$$\frac{1}{n}\sum_{i=1}^{n} z_i x_i = \frac{1}{n}\sum_{i=1}^{n} z_i(z_i'\pi + v_i) = \Delta\pi + \frac{1}{n}\sum_{i=1}^{n} z_i v_i + o\left(1\right),$$

which is useful to compute the 2SLS asymptotic approximation:

$$\begin{aligned}\hat{\beta}_{2SLS} - \beta &= \frac{\left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)'\left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)}{\left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)'\left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)} \\ &= \frac{\left(\Delta\pi + \frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)'\Delta^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right) + o_p\left(\frac{1}{n}\right)}{\left(\Delta\pi + \frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)'\Delta^{-1}\left(\Delta\pi + \frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) + o_p\left(\frac{1}{n}\right)}.\end{aligned}$$

The standard first order asymptotic approximation ignores the covariance between the error terms e and v, because the order of this covariance is $1/n$, then we have

$$\begin{aligned}\hat{\beta}_{2SLS} - \beta &= \frac{\left(\Delta\pi\right)'\Delta^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right) + o_p\left(1\right)}{\left(\Delta\pi\right)'\Delta^{-1}\left(\Delta\pi\right) + o_p\left(1\right)} \\ &= \frac{1}{\pi'\Delta\pi}\pi'\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right) + o_p\left(1\right).\end{aligned}$$

I use the same argument to approximate the numerator (N) and denominator (D) of q_{SNIV}/n (minimized objective function of SNIV). Note that the numerator (N) can be decomposed into 3 terms, as follows

$$N = \frac{(y - \hat{\beta}_{SNIV}x)'P(y - \hat{\beta}_{SNIV}x)}{n} = T_1 + T_2 + T_3 + o_p\left(\frac{1}{n}\right),$$

with the following definitions

$$
\begin{aligned}
T_1 &= \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right), \\
T_2 &= -2\left(\hat{\beta}_{SNIV} - \beta\right)\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right), \\
T_3 &= (\hat{\beta}_{SNIV} - \beta)^2 \left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right).
\end{aligned}
$$

Taking expectation, I have

$$
E(N) = \frac{(K-1)\sigma_e^2}{n} + o\left(\frac{1}{n}\right).
$$

For the denominator the expected value is

$$
E(D) = 1 + \beta^2 + \frac{\sigma_e^2}{n\pi'\Delta\pi} + O\left(\frac{1}{n}\right).
$$

Lemma 3.1. *Under Conditions 3.1 and 3.2*

$$
\frac{q_{SNIV}}{n} = \frac{(K-1)}{n\kappa} + O\left(\frac{1}{n}\right) \quad \textit{with} \ \ \kappa = \left[\left(\frac{1+\beta^2}{\sigma_e^2}\right) + \left(\frac{1}{n\pi'\Delta\pi}\right)\right].
$$

Proof. See Appendix 6.3. □

Using this result the Edgeworth expansion for SNIV estimator can be obtained. The following theorem states the second order bias for SNIV and it is the main contribution of this paper.

Theorem 3.1. *Under Conditions 3.1 and 3.2 the second order bias for SNIV estimator is*

$$
E(\hat{\beta}_{SNIV} - \beta) = \frac{(K-2)\sigma_{ev}}{n\pi'\Delta\pi} + \frac{\beta(K-1)}{n\kappa\pi'\Delta\pi} + O\left(\frac{1}{n^2}\right),
$$

where κ is defined as in Lemma 3.1.

Proof. See Appendix 6.4. □

We can see here that the first component in the second order bias of SNIV is the same as for the bias of 2SLS. Indeed, under $\beta = 0$ the biases coincide. When the number of instruments is small, relative to the sample size, the bias of SNIV will be as low as for 2SLS. But under many instruments the bias can be large. Moreover, under the Bekker (1994) Alternative Asymptotic (BAA), which consider sequences where the number of instruments grows at the same rate as the sample size with a fixed ratio ($K/n \to \alpha \geq 0$), SNIV is inconsistent

$$p \lim_{n,K\to\infty} (\hat{\beta}_{SNIV} - \beta) = \alpha \left(\frac{\sigma_{ev} + \beta\sigma_e^2}{\pi'\Delta\pi} \right) \neq 0,$$

unless $\alpha = 0$ or $\sigma_{ev} + \beta\sigma_e^2 = 0$.

4 Monte Carlo Experiment

In this section I conduct a Monte Carlo experiment to check the accuracy of the asymptotic approximation of the previous section. There are three issues to discuss in the case of IV estimators that are relevant for the design of the experiment: (1) the level of endogeneity, which is measured as the correlation between the error terms of the reduced form (ρ), (2) the quality of the instrument, which is captured by the correlation between the instrumented variables and the instruments, this can be summarized in the concentration parameter (δ^2), and (3) the number of instruments (K). I analyze these issues using the same design as in Newey and Windmeijer (2007).

Consider $u_i \sim N(0,1)$, $v_i \sim N(0,1)$, $z_i \sim N(0, I_K)$, 1_K a K vector of ones and the following data generating process

$$\begin{aligned} y_i &= x_i\beta + e_i \\ x_i &= z_i'\pi + v_i \\ e_i &= \rho v_i + \sqrt{1-\rho^2}u_i \\ \pi &= 1_K\sqrt{\frac{\delta^2}{Kn}} \end{aligned}$$

Note that the behavior of the structural parameter π is defined as local to zero as long as the concentration parameter is low and/or the number of instruments is large.[6]

I consider a sample size of 200 and set $\delta^2 = \{5, 10, 20, 200, 450, 800\}$, $K = \{3, 10\}$, $\rho = \{0.1, 0.3, 0.5, 0.7, 0.9\}$[7], and $\beta = 0$. The latter implies that bias of 2SLS should be the same as the bias for SNIV. Also, note that the population R^2 for the first step can be computed as $\pi'\pi/(\pi'\pi + 1) = \delta^2/(\delta^2 + n) \equiv R^2(\delta^2)$.

The estimators considered are LS, 2SLS, bias-corrected 2SLS (B2SLS), LIML, a finite sample correction of LIML (LIMLF) and SNIV.

The results for strong instruments (large concentration parameters) are presented in Tables 1 and 2. Note that $R^2(200) = 0.5$, $R^2(450) \approx 0.69$, and $R^2(800) = 0.8$. Here LS is biased and the bias increases with the degree of endogeneity and decreases with the concentration parameter.

For $\delta^2 = 200$ and $K = 3$ the biases of 2SLS, B2SLS, LIML, LIMLF and SNIV are very small, with the figures for SNIV close to the 2SLS estimator, whereas the biases for B2SLS, LIML and LIMLF are similar. When the concentration parameter increases to 450 o above, the bias is not a problem for any of these estimators. Increasing the number of instruments to 10, the bias for SNIV is the same order than the 2SLS, and B2SLS, LIML and LIMLF remain almost unbiased.

It is interesting to note that under the design analyzed $\beta = 0$, Theorem 3.1 predicts that the bias for SNIV (TSOB) should be the same as 2SLS, which was found in the simulations. The possible differences can be attributed to a higher dispersion of SNIV relative to 2SLS, as it is noted from the higher Inter-Quartile Range (IQR) of SNIV estimator. In terms of IQR 2SLS is preferred over SNIV.

It should be noted that IQRs for B2LS, LIML and LIMLF are higher than 2SLS and increase with the number of instruments. This fact is also reported in Hahn, Hausman and Kuersteiner (2004) and Newey and Windmeijer (2007). Finally, the Median Absolute Error (MAE) is similar for all IV estimators, and it decreases with the concentration parameter (Table 3).

[6] Staiger and Stock (1997) compute the asymptotic distribution of the estimator β under near local to zero identification. Chao and Swanson (2005) modify the distribution allowing for the possibility that the number of instruments grows as the sample size does, following Bekker (1994).

[7] Other combinations were computed, $\delta^2 = \{20, 35, 50, 800\}$ and $K = \{2, 5\}$, getting mostly the same conclusions.

Table 1: Median and IQR (in parentheses) for $K = 3$

δ^2	ρ	LS	2SLS	B2SLS	LIML	LIMLF	SNIV	TSOB
200	0.1	0.048	0.003	0.003	0.002	0.003	0.004	0.001
		(0.067)	(0.092)	(0.093)	(0.094)	(0.093)	(0.095)	
	0.3	0.150	0.003	0.002	0.000	0.001	0.003	0.002
		(0.065)	(0.091)	(0.091)	(0.092)	(0.091)	(0.093)	
	0.5	0.250	0.008	0.005	0.001	0.004	0.008	0.003
		(0.063)	(0.090)	(0.092)	(0.092)	(0.091)	(0.092)	
	0.7	0.350	0.004	0.000	-0.004	0.000	0.004	0.004
		(0.061)	(0.099)	(0.099)	(0.102)	(0.101)	(0.101)	
	0.9	0.450	0.010	0.006	0.003	0.007	0.010	0.005
		(0.047)	(0.094)	(0.095)	(0.094)	(0.093)	(0.095)	
450	0.1	0.030	0.000	0.000	0.000	0.000	0.000	0.000
		(0.053)	(0.061)	(0.061)	(0.062)	(0.062)	(0.062)	
	0.3	0.094	0.003	0.002	0.001	0.002	0.003	0.001
		(0.053)	(0.062)	(0.062)	(0.062)	(0.061)	(0.063)	
	0.5	0.153	0.001	0.000	0.000	0.001	0.001	0.001
		(0.051)	(0.061)	(0.061)	(0.062)	(0.062)	(0.062)	
	0.7	0.213	0.003	0.001	-0.001	0.001	0.003	0.002
		(0.049)	(0.060)	(0.060)	(0.061)	(0.061)	(0.061)	
	0.9	0.278	0.007	0.005	0.003	0.005	0.007	0.002
		(0.048)	(0.063)	(0.063)	(0.064)	(0.064)	(0.064)	
800	0.1	0.018	-0.001	-0.001	-0.001	-0.001	-0.001	0.000
		(0.044)	(0.052)	(0.052)	(0.052)	(0.052)	(0.053)	
	0.3	0.060	0.001	0.001	0.000	0.001	0.001	0.000
		(0.043)	(0.047)	(0.047)	(0.047)	(0.047)	(0.048)	
	0.5	0.098	0.000	0.000	-0.001	0.000	0.000	0.001
		(0.043)	(0.047)	(0.047)	(0.048)	(0.048)	(0.048)	
	0.7	0.141	0.001	0.001	-0.001	0.000	0.001	0.001
		(0.042)	(0.050)	(0.050)	(0.051)	(0.051)	(0.051)	
	0.9	0.180	0.002	0.001	0.000	0.001	0.002	0.001
		(0.040)	(0.052)	(0.052)	(0.052)	(0.052)	(0.052)	

Based on 5000 replications. TSOB is the theoretical bias (Theorem 3.1).

Table 2: Median and IQR (in parentheses) for $K = 10$

δ^2	ρ	LS	2SLS	B2SLS	LIML	LIMLF	SNIV	TSOB
200	0.1	0.048	0.006	0.003	0.003	0.003	0.007	0.004
		(0.066)	(0.089)	(0.092)	(0.093)	(0.092)	(0.098)	
	0.3	0.152	0.019	0.008	0.004	0.006	0.021	0.012
		(0.066)	(0.095)	(0.099)	(0.099)	(0.098)	(0.103)	
	0.5	0.248	0.018	0.000	-0.001	0.002	0.020	0.020
		(0.059)	(0.091)	(0.095)	(0.096)	(0.096)	(0.098)	
	0.7	0.347	0.032	0.005	-0.002	0.002	0.034	0.028
		(0.057)	(0.094)	(0.101)	(0.103)	(0.102)	(0.100)	
	0.9	0.450	0.040	0.007	0.002	0.006	0.044	0.036
		(0.052)	(0.087)	(0.096)	(0.098)	(0.096)	(0.095)	
450	0.1	0.031	-0.001	-0.002	-0.003	-0.002	-0.001	0.002
		(0.053)	(0.062)	(0.062)	(0.062)	(0.062)	(0.066)	
	0.3	0.095	0.008	0.003	0.002	0.003	0.009	0.005
		(0.050)	(0.062)	(0.063)	(0.062)	(0.062)	(0.066)	
	0.5	0.153	0.008	0.000	-0.001	0.000	0.008	0.009
		(0.054)	(0.063)	(0.066)	(0.065)	(0.065)	(0.069)	
	0.7	0.215	0.012	-0.001	-0.003	-0.002	0.012	0.012
		(0.052)	(0.065)	(0.066)	(0.068)	(0.068)	(0.068)	
	0.9	0.277	0.020	0.004	0.002	0.004	0.021	0.016
		(0.047)	(0.066)	(0.069)	(0.065)	(0.065)	(0.070)	
800	0.1	0.020	0.000	0.000	-0.001	-0.001	0.000	0.001
		(0.043)	(0.052)	(0.052)	(0.052)	(0.052)	(0.054)	
	0.3	0.061	0.005	0.003	0.001	0.002	0.006	0.003
		(0.044)	(0.048)	(0.049)	(0.049)	(0.049)	(0.051)	
	0.5	0.100	0.006	0.002	0.001	0.001	0.007	0.005
		(0.042)	(0.045)	(0.046)	(0.046)	(0.046)	(0.048)	
	0.7	0.141	0.007	0.000	-0.001	0.000	0.007	0.007
		(0.040)	(0.048)	(0.050)	(0.048)	(0.048)	(0.051)	
	0.9	0.181	0.009	0.000	-0.001	0.000	0.009	0.009
		(0.038)	(0.049)	(0.050)	(0.050)	(0.049)	(0.052)	

Based on 5000 replications. TSOB is the theoretical bias (Theorem 3.1).

Table 3: Median Absolute Error

K	δ^2	ρ	LS	2SLS	B2SLS	LIML	LIMLF	SNIV
3	200	0.1	0.051	0.046	0.046	0.047	0.047	0.048
		0.3	0.150	0.045	0.045	0.045	0.045	0.046
		0.5	0.250	0.048	0.047	0.045	0.046	0.048
		0.7	0.350	0.049	0.050	0.051	0.050	0.050
		0.9	0.450	0.048	0.048	0.047	0.047	0.049
	450	0.1	0.036	0.031	0.031	0.031	0.031	0.031
		0.3	0.094	0.030	0.030	0.030	0.030	0.030
		0.5	0.153	0.030	0.031	0.031	0.031	0.031
		0.7	0.213	0.030	0.031	0.030	0.030	0.031
		0.9	0.278	0.033	0.032	0.032	0.033	0.033
	800	0.1	0.026	0.026	0.026	0.026	0.026	0.027
		0.3	0.060	0.024	0.024	0.024	0.024	0.024
		0.5	0.098	0.024	0.023	0.023	0.024	0.024
		0.7	0.141	0.026	0.025	0.026	0.026	0.026
		0.9	0.180	0.026	0.026	0.026	0.026	0.026
10	200	0.1	0.052	0.045	0.046	0.046	0.046	0.050
		0.3	0.152	0.047	0.048	0.050	0.049	0.052
		0.5	0.248	0.049	0.048	0.047	0.047	0.053
		0.7	0.347	0.053	0.051	0.051	0.051	0.058
		0.9	0.450	0.055	0.050	0.048	0.049	0.060
	450	0.1	0.035	0.031	0.031	0.031	0.031	0.033
		0.3	0.095	0.032	0.031	0.031	0.031	0.034
		0.5	0.153	0.032	0.033	0.033	0.032	0.034
		0.7	0.215	0.033	0.034	0.034	0.033	0.035
		0.9	0.277	0.034	0.034	0.033	0.033	0.037
	800	0.1	0.026	0.026	0.026	0.026	0.026	0.027
		0.3	0.061	0.024	0.025	0.025	0.025	0.026
		0.5	0.100	0.023	0.023	0.023	0.023	0.024
		0.7	0.141	0.024	0.025	0.024	0.024	0.025
		0.9	0.181	0.025	0.025	0.024	0.025	0.026

Based on 5000 replications.

When instruments are weak the concentration parameter is small and the Edgeworth expansion becomes imprecise. Note that $R^2(5) = 0.024$ and $R^2(10) = 0.05$. It is clear from Tables 4 that SNIV behaves like 2SLS having even more bias. Even thought the design of the experiment includes $\beta = 0$, it is not possible to validate the result proposed in Theorem 3.1 because the model is weakly identified.[8] However, the results presented here show: (1) the bias of SNIV is the same order as that of 2SLS, but the IQR is higher, (2) B2SLS improves 2SLS in terms of bias, but not in terms of IQR, and (3) LIML is almost unbiased, but it has the highest IQR.

Increasing the number of instruments (Table 5), the results show: (1) bias and IQR for SNIV are bigger than 2SLS estimator, (2) under low degree of endogeneity ($\rho = 0.1$) LS seems to be reasonable competitor for 2SLS, (3) LIML is still unbiased under high degree of endogeneity, small concentration parameter and many number of instruments, (4) B2SLS is a reasonable competitor for LIML under small degree of endogeneity, and (5) LIMLF improves B2SLS in terms of bias and IQR under high degree of endogeneity.

These results confirm the suggestions proposed by Hahn, Hausman and Kuersteiner (2004), that in terms of Mean Squared Error (MSE), LIMF has the minimum MSE, offering the optimal trade-off between bias and dispersion.

Finally, the MAE under weak instruments shows that SNIV is inferior relative to other IV estimators (Table 6).

[8] A more accurate approach can be done computing the weak-instrument asymptotic proposed by Staiger and Stock (1997).

Table 4: Median and IQR (in parentheses) for $K = 3$

δ^2	ρ	LS	2SLS	B2SLS	LIML	LIMLF	SNIV	TSOB
5	0.1	0.094	0.033	0.014	0.008	0.023	0.045	0.020
		(0.094)	(0.518)	(0.567)	(0.741)	(0.551)	(0.750)	
	0.3	0.295	0.088	0.052	0.014	0.066	0.122	0.060
		(0.094)	(0.487)	(0.571)	(0.697)	(0.532)	(0.666)	
	0.5	0.487	0.164	0.095	0.008	0.111	0.209	0.100
		(0.086)	(0.494)	(0.548)	(0.693)	(0.508)	(0.673)	
	0.7	0.682	0.225	0.134	0.043	0.172	0.306	0.140
		(0.065)	(0.409)	(0.508)	(0.660)	(0.441)	(0.530)	
	0.9	0.880	0.302	0.200	0.031	0.200	0.368	0.180
		(0.041)	(0.366)	(0.467)	(0.628)	(0.331)	(0.449)	
10	0.1	0.100	0.051	0.043	0.044	0.051	0.062	0.010
		(0.095)	(0.381)	(0.416)	(0.480)	(0.427)	(0.479)	
	0.3	0.283	0.043	0.020	0.000	0.028	0.060	0.030
		(0.086)	(0.380)	(0.417)	(0.463)	(0.406)	(0.463)	
	0.5	0.477	0.099	0.048	-0.012	0.049	0.125	0.050
		(0.084)	(0.369)	(0.427)	(0.490)	(0.401)	(0.470)	
	0.7	0.664	0.125	0.075	0.012	0.078	0.146	0.070
		(0.069)	(0.373)	(0.432)	(0.514)	(0.389)	(0.454)	
	0.9	0.856	0.175	0.110	0.027	0.106	0.204	0.090
		(0.045)	(0.305)	(0.379)	(0.418)	(0.308)	(0.353)	
20	0.1	0.088	0.008	0.001	-0.002	0.003	0.009	0.005
		(0.089)	(0.292)	(0.313)	(0.337)	(0.318)	(0.338)	
	0.3	0.271	0.017	0.006	-0.012	0.007	0.021	0.015
		(0.091)	(0.276)	(0.298)	(0.302)	(0.288)	(0.315)	
	0.5	0.456	0.055	0.032	0.003	0.029	0.060	0.025
		(0.083)	(0.292)	(0.313)	(0.316)	(0.292)	(0.328)	
	0.7	0.635	0.062	0.032	-0.011	0.025	0.069	0.035
		(0.069)	(0.261)	(0.287)	(0.305)	(0.269)	(0.294)	
	0.9	0.817	0.087	0.045	0.004	0.051	0.093	0.045
		(0.046)	(0.280)	(0.310)	(0.329)	(0.282)	(0.308)	

Based on 5000 replications. TSOB is the theoretical bias (Theorem 3.1).

Table 5: Median and IQR (in parentheses) for $K = 10$

δ^2	ρ	LS	2SLS	B2SLS	LIML	LIMLF	SNIV	TSOB
5	0.1	0.095	0.066	0.000	0.039	0.048	0.166	0.160
		(0.093)	(0.336)	(0.577)	(0.901)	(0.749)	(0.904)	
	0.3	0.295	0.197	0.016	0.056	0.089	0.414	0.480
		(0.093)	(0.345)	(0.633)	(0.854)	(0.694)	(0.729)	
	0.5	0.488	0.333	0.068	0.052	0.135	0.607	0.800
		(0.084)	(0.333)	(0.621)	(0.897)	(0.674)	(0.644)	
	0.7	0.683	0.461	0.165	0.113	0.213	0.681	1.120
		(0.064)	(0.265)	(0.546)	(0.762)	(0.526)	(0.421)	
	0.9	0.879	0.579	0.233	0.042	0.212	0.742	1.440
		(0.042)	(0.205)	(0.491)	(0.736)	(0.399)	(0.285)	
10	0.1	0.098	0.060	0.004	0.005	0.012	0.124	0.080
		(0.093)	(0.316)	(0.529)	(0.646)	(0.555)	(0.631)	
	0.3	0.283	0.148	0.037	0.021	0.048	0.254	0.240
		(0.088)	(0.281)	(0.486)	(0.578)	(0.500)	(0.523)	
	0.5	0.476	0.252	0.072	0.004	0.064	0.428	0.400
		(0.082)	(0.296)	(0.517)	(0.629)	(0.523)	(0.495)	
	0.7	0.662	0.341	0.092	0.018	0.083	0.502	0.560
		(0.070)	(0.245)	(0.516)	(0.540)	(0.421)	(0.362)	
	0.9	0.855	0.437	0.135	0.012	0.099	0.566	0.720
		(0.046)	(0.201)	(0.503)	(0.497)	(0.355)	(0.274)	
20	0.1	0.087	0.021	-0.004	-0.017	-0.013	0.033	0.040
		(0.089)	(0.274)	(0.372)	(0.415)	(0.387)	(0.430)	
	0.3	0.274	0.088	0.016	-0.007	0.009	0.132	0.120
		(0.091)	(0.240)	(0.349)	(0.374)	(0.345)	(0.363)	
	0.5	0.457	0.155	0.030	0.004	0.027	0.229	0.200
		(0.081)	(0.225)	(0.342)	(0.379)	(0.351)	(0.332)	
	0.7	0.634	0.226	0.065	0.008	0.041	0.313	0.280
		(0.073)	(0.218)	(0.340)	(0.348)	(0.311)	(0.277)	
	0.9	0.819	0.289	0.063	-0.018	0.030	0.381	0.360
		(0.047)	(0.189)	(0.366)	(0.334)	(0.287)	(0.222)	

Based on 5000 replications. TSOB is the theoretical bias (Theorem 3.1).

Table 6: Median Absolute Error

K	δ^2	ρ	LS	2SLS	B2SLS	LIML	LIMLF	SNIV
3	5	0.1	0.095	0.258	0.283	0.368	0.279	0.371
		0.3	0.295	0.256	0.289	0.349	0.280	0.355
		0.5	0.487	0.283	0.299	0.354	0.268	0.395
		0.7	0.682	0.286	0.292	0.328	0.246	0.407
		0.9	0.880	0.339	0.324	0.292	0.242	0.428
	10	0.1	0.100	0.192	0.208	0.245	0.218	0.252
		0.3	0.283	0.199	0.209	0.230	0.205	0.242
		0.5	0.477	0.209	0.225	0.239	0.203	0.258
		0.7	0.664	0.224	0.230	0.242	0.212	0.278
		0.9	0.856	0.220	0.210	0.201	0.185	0.265
	20	0.1	0.089	0.146	0.156	0.168	0.159	0.169
		0.3	0.271	0.140	0.147	0.148	0.143	0.155
		0.5	0.456	0.153	0.158	0.155	0.149	0.171
		0.7	0.635	0.147	0.151	0.146	0.139	0.169
		0.9	0.817	0.158	0.160	0.156	0.151	0.173
10	5	0.1	0.095	0.179	0.290	0.456	0.367	0.473
		0.3	0.295	0.226	0.324	0.433	0.350	0.522
		0.5	0.488	0.341	0.355	0.458	0.350	0.637
		0.7	0.683	0.462	0.357	0.405	0.304	0.686
		0.9	0.879	0.579	0.390	0.348	0.257	0.742
	10	0.1	0.098	0.163	0.265	0.330	0.282	0.343
		0.3	0.283	0.182	0.246	0.286	0.254	0.360
		0.5	0.476	0.260	0.280	0.310	0.274	0.468
		0.7	0.662	0.341	0.272	0.259	0.228	0.505
		0.9	0.855	0.437	0.297	0.232	0.201	0.566
	20	0.1	0.087	0.134	0.185	0.206	0.194	0.219
		0.3	0.274	0.140	0.172	0.177	0.167	0.217
		0.5	0.457	0.181	0.179	0.182	0.181	0.266
		0.7	0.634	0.231	0.187	0.172	0.161	0.324
		0.9	0.819	0.290	0.196	0.165	0.152	0.385

Based on 5000 replications.

5 Conclusion

In this paper I study the finite sample properties of the SNIV estimator proposed by Alonso-Borrego and Arellano (1999) as an alternative to LIML. I developed a second order approximation for the bias of SNIV using an Edgeworth expansion. The result, presented in Theorem 3.1 is new in the literature and it is the main contribution of this paper.

The expression of the bias can be decomposed into 2 elements: the first is the same as the second order bias of 2SLS, and the second is an additional term of the same order that depends on the true parameter of the model (β). The bias of SNIV should vanish in the cases where the number of instrument is small relative to the sample size as in 2SLS.

The theoretical result is confirmed with Monte Carlo experiments. In particular, I show that for strong instruments (when the instruments are highly correlated with the instrumented variable), the second order approximation is valid, and the bias of SNIV is close to the bias of 2SLS. For small concentration parameter the expansion is less accurate and numerical results show that alternative estimators, such as B2SLS, LIML or LIMLF should be preferred over SNIV. Indeed, LIML is almost unbiased in all the scenarios, whereas LIMLF has an optimal trade-off between lower bias and lower IQR.

In conclusion, I would not recommend SNIV as alternative estimator to LIML. Moreover, robust estimators that use SNIV estimator as initial estimator could be biased.

References

Alonso-Borrego, C., and Arellano, M. (1999) "Symmetrically Normalized Instrumental-Variable Estimation using Panel Data," *Journal of Business & Economics Statistics*, 17, 36-49.

Bekker, P. (1994) "Alternative Approximations to the Distribution of Instrumental Variable Estimators," *Econometrica*, 62, 657-681.

Brown, T. (1960) "Simultaneous Least Squares: a distribution free method of equation system structure estimation," *International Economic Revies*, 1, 173-191.

Chao, J., and Swanson, N. (2005) "Alternative Approximations of the Bias and MSE of the IV Estimator Under Weak Identification With an Application to Bias Correction" *Working Paper*, University of Maryland.

Davidson, R., and MacKinnon, J. (1993) *Estimation and Inference in Econometrics*, New York: Oxford University Press.

Donald, S., and Newey, W. (2001) "Choosing the number of instruments," *Econometrica*, 69, 1161-1191.

Field, C., and Ronchetti, E. (1990) *Small Sample Asymptotics* Hayward, CA: Institute of Mathematical Statistics.

Fuller, W. (1977) "Some Properties of a Modification of the Limited Information Estimator," *Econometrica*, 45, 939-953.

Hahn, J., Hausman, J., and Kuersteiner, G. (2004) "Estimation with weak instruments: Accuracy of higher-order bias and MSE approximations," *Econometrics Journal*, 7, 272-306.

Hansen, C., Hausman, J., and Newey, W. (2005) "Estimation with Many Instrumental Variables" *Working Paper*, Massachusetts Institute of Technology.

Hillier, G. (1990) "On the normalization of Structural Equations: Properties of Direction Estimators," *Econometrica*, 50, 1181-1194.

Mariano, R. (1982) "Analytical Small-Sample Distribution Theory in Econometrics: The Simultaneous Equations Case," *International Economic Review*, 23, 503-533.

Nagar, A. (1959) "The Bias and Moment Matrix of the General k-class estimators of the Parameters in Simultaneous Equations," *Econometrica*, 27, 575-595.

Newey, W. (2004) "Many Instruments Asymptotics," *Working Paper*, Massachusetts Institute of Technology.

Newey, W., and Windmeijer, F. (2007) "GMM with Many Weak Moment Conditions" *Working Paper*, Massachusetts Institute of Technology.

Phillips, P. (1985) "The Exact Distribution of LIML: II" *International Economic Review*, 26, 21-36.

Rothenberg, T. (1984) "Approximating the Distribution of Econometrics Estimators of Test Statistics," in *Handbook of Econometrics, Vol II,* ed. by Z. Griliches and M. Intriligator. Amsterdam: North-Holland, 881-936.

Sawa, T. (1969) "The Exact Sampling Distribution of Ordinary Least Squares and the Two Stage Least Squares Estimates," it Journal of American Statistical Association, 64, 928-980.

Serfling, R. (1980) *Appoximation Theorems of Mathematical Statistics* New York: Wiley & Sons.

Staiger, D., and Stock, J. (1997) "Instrumental Variables Regression with Weak Instruments," *Econometrica*, 65, 557-586.

Theil, H. (1958) *Economic Forecast and Policy.* Amsterdam: North-Holland Publishing Company.

Ullah, A. (2004) *Finite Sample Econometrics* New York: Oxford University Press.

6 Appendix

6.1 Generalization of IV estimators

Consider the matrix of the endogenous variables $W = (y, x)$ and $\gamma = (1, -\beta)'$, then for given matrices A and B the estimator that minimizes the ratio argument is:

$$\hat{\gamma} \quad = \quad \arg\min_{\gamma} \left(\frac{\gamma' A \gamma}{\gamma' B \gamma} \right) = \arg\min_{\gamma} q(\gamma | A, B).$$

In addition consider $P = Z(Z'Z)^{-1}Z'$ and

$$N = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{4}$$

- LS is obtained by setting $A = W'W$ and $B = N$. Note that $\gamma' B \gamma = 1$ and the numerator becomes

$$\begin{aligned} \gamma' A \gamma &= \begin{pmatrix} 1 & -\beta \end{pmatrix} \begin{pmatrix} y'y & x'y \\ x'y & x'x \end{pmatrix} \begin{pmatrix} 1 \\ -\beta \end{pmatrix} \\ &= \begin{pmatrix} y'y - \beta x'y & x'y - \beta y'y \end{pmatrix} \begin{pmatrix} 1 \\ -\beta \end{pmatrix} \\ &= (y'y - \beta x'y) - \beta(x'y - \beta y'y) = \sum_{i=1}^{n} (y_i - \beta x_i)^2. \end{aligned}$$

- 2SLS by $A = W'PW$ and $B = N$. Again, $\gamma' B \gamma = 1$ and

$$
\begin{aligned}
\gamma' A \gamma &= \begin{pmatrix} 1 & -\beta \end{pmatrix} \begin{pmatrix} y'Py & x'Py \\ x'Py & x'Px \end{pmatrix} \begin{pmatrix} 1 \\ -\beta \end{pmatrix} \\
&= \begin{pmatrix} y'Py - \beta x'Py & x'Py - \beta y'Py \end{pmatrix} \begin{pmatrix} 1 \\ -\beta \end{pmatrix} \\
&= (y'Py - \beta x'Py) - \beta(x'Py - \beta y'Py) \\
&= (y'Py - 2\beta x'Py + \beta y'Py) = (y - \beta x)'P(y - \beta x) \\
&= \left(\sum_{i=1}^{n} z_i(y_i - \beta x_i)\right)' \left(\sum_{i=1}^{n} z_i z_i'\right)^{-1} \left(\sum_{i=1}^{n} z_i(y_i - \beta x_i)\right).
\end{aligned}
$$

- R2SLS by $A = W'PW$ and $B = I - N$. Similar to 2SLS.
- LIML by $A = W'PW$ and $B = W'W$. Using the previous results we have

$$
\begin{aligned}
\gamma' A \gamma &= (y - \beta x)'P(y - \beta x) \\
&= \left(\sum_{i=1}^{n} z_i(y_i - \beta x_i)\right)' \left(\sum_{i=1}^{n} z_i z_i'\right)^{-1} \left(\sum_{i=1}^{n} z_i(y_i - \beta x_i)\right) \\
\gamma' B \gamma &= (y - \beta x)'(y - \beta x) \\
&= \sum_{i=1}^{n} (y_i - \beta x_i)^2.
\end{aligned}
$$

- SNIV by $A = Y'PY$ and $B = I$. In this case $\gamma' B \gamma = 1 + \beta^2$ and

$$
\begin{aligned}
\gamma' A \gamma &= (y - \beta x)'P(y - \beta x) \\
&= \left(\sum_{i=1}^{n} z_i(y_i - \beta x_i)\right)' \left(\sum_{i=1}^{n} z_i z_i'\right)^{-1} \left(\sum_{i=1}^{n} z_i(y_i - \beta x_i)\right).
\end{aligned}
$$

6.2 LIML: Maximum Likelihood and Least Variance Ratio

Let $w_i = (u_i, v_i)' \sim N(0, B)$ then the joint density of w_i is

$$f(w_i) = \frac{1}{2\pi}|B|^{-1/2}\exp\left(-\frac{1}{2}w_i' B^{-1} w_i\right).$$

The loglikelihood associated with a sample size of n is $l = -n\log(2\pi) - (n/2)\log(|B|) - (1/2)\sum_{i=1}^n w_i' B^{-1} w_i$. The concentrated loglikelihood function can be computed using

$$\frac{\partial l}{\partial B^{-1}} = \frac{n}{2}B - \frac{1}{2}\sum_{i=1}^n w_i' B^{-1} w_i \text{ , then } B = \frac{1}{n}\sum_{i=1}^n w_i w_i'.$$

Also $w_i' B^{-1} w_i = tr(w_i' B^{-1} w_i) = tr(B^{-1} w_i w_i')$ then $\sum_{i=1}^n w_i' B^{-1} w_i = \sum_{i=1}^n tr(B^{-1} w_i w_i') = tr(B^{-1}\sum_{i=1}^n w_i w_i') = 2n$. Finally the concentrated loglikelihood is

$$l^c = -n[\log(2\pi) + 1] - \frac{n}{2}\log(|B|).$$

The maximization of the concentrated loglikelihood is similar to the minimization of the determinant of B, which for the case of IV estimators can be expressed as follows

$$\begin{aligned} |B| &= \frac{1}{n}\left|\begin{pmatrix} \sum_{i=1}^n (y_i - x_i\beta)^2 & \sum_{i=1}^n (y_i - x_i\beta)(x_i - z_i'\pi) \\ \sum_{i=1}^n (y_i - x_i\beta)(x_i - z_i'\pi) & \sum_{i=1}^n (x_i - z_i'\pi)^2 \end{pmatrix}\right| \\ &= \frac{1}{n}\left|\begin{pmatrix} \gamma' W' W\gamma & \gamma' W'(x - z\pi) \\ (x - z\pi)' W\gamma & (x - z\pi)'(x - z\pi) \end{pmatrix}\right|. \end{aligned}$$

Following Davidson and MacKinnon (1993), I define $\upsilon = W\gamma$ and $M_A \equiv I - A(A'A)^{-1}A'$ then

$$\begin{aligned} |B| &= \frac{1}{n}\left|\begin{pmatrix} \upsilon'\upsilon & \upsilon'(x - z\pi) \\ (x - z\pi)'\upsilon & (x - z\pi)'(x - z\pi) \end{pmatrix}\right| \\ &= \frac{1}{n}\upsilon'\upsilon\left|(x - z\pi)' M_\upsilon (x - z\pi)\right|. \end{aligned}$$

The last step also involves the computation of the determinant for partitioned matrices. Finally, π can be estimated by LS, then[9]

[9] See Davidson and MacKinnon (1993) for details.

$$|B| = \frac{1}{n}\kappa|W'M_ZW|,$$

where $\kappa \equiv \gamma'W'W\gamma/\gamma'W'M_ZW\gamma$. Minimizing κ, the concentrated loglikelihood is maximized. Note that $\kappa = 1/(1-q)$ where q is the objective function presented in 6.1.

6.3 Eigenvalue Approximation

Standard arguments can be used to solve T_1 as follows

$$\begin{aligned} E\left(T_1\right) &= E\left(\frac{1}{n^2}\sum_{i=1}^{n} e_i z_i'\Delta^{-1}z_i e_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} E(e_i z_i'\Delta^{-1}z_i e_i) \\ &= \frac{1}{n}E(tr(e_i z_i'\Delta^{-1}z_i e_i)) = \frac{\sigma_e^2}{n}E(tr(z_i z_i'\Delta^{-1})) = \sigma_e^2\frac{K}{n}. \end{aligned}$$

For T_2, I can use the first order asymptotics of this estimator to replace the argument $\hat{\beta}_{SNIV} - \beta$. Note that first order asymptotics of SNIV and 2SLS coincide, then

$$E(T_2) = E\left[\left(\frac{-2\pi'}{\pi'\Delta\pi}\sum_{i=1}^{n} z_i e_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)'\Delta^{-1}\left(\Delta\pi + \frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)\right],$$

under the normality assumption the third moments are zero, then

$$\begin{aligned} E(T_2) &= E\left[\left(\frac{-2\pi'}{n\pi'\Delta\pi}\sum_{i=1}^{n} z_i e_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)'\pi\right] \\ &= \frac{-2\pi'}{\pi'\Delta\pi}E\left[\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)'\right]\pi \\ &= \frac{-2\pi'}{\pi'\Delta\pi}E\left(\frac{1}{n^2}\sum_{i=1}^{n} e_i^2 z_i z_i'\right)\pi \\ &= \frac{-2\sigma_e^2\pi'}{n\pi'\Delta\pi}\left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)\pi = -2\frac{\sigma_e^2}{n}. \end{aligned}$$

As for the last term, I use the same 2SLS asymptotic than before and the definition for $(1/n)\sum_{i=1}^{n} z_i x_i$, then

$$
\begin{aligned}
T_3 &= \left(\frac{\pi'}{n\pi'\Delta\pi}\sum_{i=1}^{n} z_i e_i\right)^2 \left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right) + o_p\left(\frac{1}{n}\right) \\
&= \left(\frac{\pi'}{n\pi'\Delta\pi}\sum_{i=1}^{n} z_i e_i\right) \left(\frac{\pi'}{n\pi'\Delta\pi}\sum_{i=1}^{n} z_i e_i\right)' \\
&\quad \times \left(\Delta\pi + \frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\Delta\pi + \frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) + o_p\left(\frac{1}{n}\right) \\
&= T_{31}\left(\pi'\Delta\pi + T_{32} + T_{33}\right) + o_p\left(\frac{1}{n}\right),
\end{aligned}
$$

where

$$
\begin{aligned}
T_{31} &= \frac{\pi'}{\pi'\Delta\pi}\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)' \frac{\pi}{\pi'\Delta\pi} \\
T_{32} &= 2\left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \pi \\
T_{33} &= \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right).
\end{aligned}
$$

Under the normality assumption $E(T_{31}T_{32}) = 0$ because the third moments are zero. Also $T_{31} \times T_{33}$ is of smaller order than $1/n$, therefore I discard it. As for T_2, it is clear that

$$
\begin{aligned}
E(T_3) &= E(T_{31})\pi'\Delta\pi = \frac{\pi'}{\pi'\Delta\pi} E\left(\frac{1}{n^2}\sum_{i=1}^{n} e_i^2 z_i z_i'\right)\pi \\
&= \frac{\sigma_e^2 \pi'}{n\pi'\Delta\pi} E\left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)\pi = \frac{\sigma_e^2}{n}.
\end{aligned}
$$

Finally, collecting terms

$$
E(N) = (K-1)\sigma_e^2/n.
$$

Using the results for 2SLS, the denominator ($D = 1 + \hat{\beta}_{SNIV}^2$) can be expressed as follow

$$
\begin{aligned}
1+\hat{\beta}_{SNIV}^2 &= 1+\left(\beta+\frac{1}{\pi'\Delta\pi}\pi'\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)\right)^2 + o_p(1) \\
&= 1+\beta^2+\frac{2\beta}{\pi'\Delta\pi}\pi'\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right) \\
&\quad +\left(\frac{1}{\pi'\Delta\pi}\right)^2 \pi'\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)'\pi + o_p(1).
\end{aligned}
$$

Taking expected value as I did with T_2, we have

$$
E(1+\hat{\beta}_{SNIV}^2) = 1+\beta^2+\sigma_e^2\left(\frac{1}{\pi'\Delta\pi}\right)^2\pi'\left(\frac{1}{n^2}\sum_{i=1}^{n} z_i z_i'\right)\pi = 1+\beta^2+\frac{\sigma_e^2}{n\pi'\Delta\pi}.
$$

6.4 Estimator Approximation

Consider the following decomposition of the SNIV estimator

$$
\begin{aligned}
\hat{\beta}_{SNIV}-\beta &= \frac{\left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)'\left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i y_i\right)}{\left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)'\left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)-\hat{q}/n} - \beta \\
&= \frac{\beta\hat{q}/n+\left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)'\left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)}{\left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)'\left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)-\hat{q}/n}.
\end{aligned}
$$

Define the numerator of the last expression as R, and the denominator as S, then the following approximation applies for the numerator

$$
\begin{aligned}
R &\equiv \left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)'\left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)+\frac{\beta\hat{q}}{n} \\
&= \left(\Delta\pi+\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)'\Delta^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)+\frac{\beta\hat{q}}{n}+o_p\left(\frac{1}{n}\right) \\
&= \pi'\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)+\left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)'\Delta^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)+\frac{\beta\hat{q}}{n}+o_p\left(\frac{1}{n}\right),
\end{aligned}
$$

and the following for the denominator

$$
\begin{aligned}
S &\equiv \left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right)' \left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i x_i\right) - \frac{\hat{q}}{n} \\
&= \left(\Delta\pi + \frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\Delta\pi + \frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) - \frac{\hat{q}}{n} + o_p\left(\frac{1}{n}\right) \\
&= \pi'\Delta\pi + \left(\frac{2\pi'}{n}\sum_{i=1}^{n} z_i v_i\right) + \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) - \frac{\hat{q}}{n} + o_p\left(\frac{1}{n}\right).
\end{aligned}
$$

By assumption $\pi'\Delta\pi = O(1)$, the second term in S is $O_p\left(1/\sqrt{n}\right)$, and the third and fourth are $O_p\left(1/n\right)$. Following the footnote 8 in Nagar (1959), it is possible to compute the inverse of S as follows

$$
\begin{aligned}
S^{-1} &= \frac{1}{\pi'\Delta\pi} - 2\left(\frac{1}{\pi'\Delta\pi}\right)^2 \pi' \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) \\
&\quad - \left(\frac{1}{\pi'\Delta\pi}\right)^2 \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) \\
&\quad + \frac{\hat{q}}{n}\left(\frac{1}{\pi'\Delta\pi}\right)^2 + O_p\left(\frac{1}{n}\right)
\end{aligned}
$$

Finally the SNIV estimator can be approximated as $\hat{\beta}_{SNIV} - \beta = \sum_{j=1}^{12} H_j + O_p\left(\frac{1}{n}\right)$, with

$$
\begin{aligned}
H_1 &= \frac{\beta \hat{q}}{n\pi' \Delta \pi} \\
H_2 &= \frac{\pi' \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)}{\pi' \Delta \pi} \\
H_3 &= \frac{\left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)}{\pi' \Delta \pi} \\
H_4 &= \frac{-2\beta \hat{q} \pi' \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)}{n(\pi' \Delta \pi)^2} \\
H_5 &= -\frac{2\pi' \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)' \pi}{(\pi' \Delta \pi)^2} \\
H_6 &= -\frac{2\pi' \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)}{(\pi' \Delta \pi)^2} \\
H_7 &= \frac{\beta \hat{q} \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)}{n(\pi' \Delta \pi)^2} \\
H_8 &= -\frac{\left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) \pi' \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)}{(\pi' \Delta \pi)^2} \\
H_9 &= -\frac{\left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)}{(\pi' \Delta \pi)^2} \\
H_{10} &= -\frac{\beta \hat{q}^2}{(n\pi' \Delta \pi)^2} \\
H_{11} &= -\frac{\hat{q} \pi' \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)}{n(\pi' \Delta \pi)^2} \\
H_{12} &= -\frac{\hat{q} \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)}{n(\pi' \Delta \pi)^2}.
\end{aligned}
$$

It is clear that $E(H_2) = E(H_4) = E(H_{11}) = 0$ because the errors have zero mean. Also by normality H_6 and H_8 have zero expectation because skewness is zero. Note that H_7, H_9, H_{10} and H_{12} are higher order, then I discard them. For the others terms (H_1, H_3 and H_5), I have to compute their expected value. Note that H_1 is the only new term relative to 2SLS estimator, therefore the second order bias is expected to be similar to bias for 2SLS estimator.

$$
\begin{aligned}
E(H_1) &= \frac{\tilde{\beta}(K-1)}{n\pi'\Delta\pi} \\
E(H_3) &= E\left[\frac{\left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)}{\pi'\Delta\pi}\right] \\
&= \frac{1}{\pi'\Delta\pi} E\left[\left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right)' \Delta^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)\right] \\
&= \frac{1}{n\pi'\Delta\pi} E\left(\frac{1}{n}\sum_{i=1}^{n} v_i z_i' \Delta^{-1} z_i e_i\right) \\
&= \frac{E(v_i z_i' \Delta^{-1} z_i e_i)}{n\pi'\Delta\pi} = \frac{\sigma_{ev} K}{n\pi'\Delta\pi} \\
E(H_5) &= -E\left[\frac{2\pi' \left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)' \pi}{(\pi'\Delta\pi)^2}\right] \\
&= -\frac{2\pi'}{(\pi'\Delta\pi)^2} E\left[\left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i\right) \left(\frac{1}{n}\sum_{i=1}^{n} z_i e_i\right)'\right] \pi \\
&= -\frac{2\pi'}{n(\pi'\Delta\pi)^2} E\left[\left(\frac{1}{n}\sum_{i=1}^{n} z_i v_i e_i z_i'\right)\right] \pi = -\frac{2\sigma_{ev}}{n\pi'\Delta\pi}.
\end{aligned}
$$

Collecting the terms the second order bias can be computed as follow

$$
E(\hat{\beta}_{SNIV} - \beta) = \frac{(K-2)\sigma_{ev}}{n\pi'\Delta\pi} + \frac{\beta(K-1)}{n\kappa\pi'\Delta\pi} + O\left(\frac{1}{n^2}\right).
$$

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