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ESTIMATION OF A DYNAMIC PANEL DATA: THE CASE OF CORPORATE INVESTMENT IN CHILE

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Resumen

En este artículo se discute la estimación de modelos de panel dinámicos y se muestra que el sesgo de muestras finitas del estimador Arellano-Bond puede ser reducido cuando se restringe el número de rezagos incluidos en la estimación. A través de una aplicación empírica al caso de la inversión de empresas FECUS se corroboran los resultados teóricos.

Abstract

In this paper I discuss about the estimation of Dynamic Panel Data model, showing that we can reduce the finite-sample bias of the Arellano-Bond estimator by truncation of the number of lags used in this estimator. We check our theoretical result in an empirical application using a panel of Chilean firms.

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1 Introduction

After the seminal work by Hansen (1982) the Generalized Method of Moment (GMM) procedure is widely used to test theoretical economic models.[1] The advantage of GMM over other procedures is that the estimator only needs a set of moment conditions, without imposing restrictions on the distribution of the variables. This framework is particularly suitable for estimating forward-looking rational expectations models. In particular, the so called Euler equation can be interpreted as a moment condition equation, where the possible instruments are all the variables that belong to the information set at each time period. A challenge in the estimation of these models is that the number of moment conditions used may affect the properties of the IV estimators.

In this paper, I use GMM to estimate a corporate investment model with financial frictions using Chilean firm-level panel data. The setup follows Gilchrist and Himmelberg (1998)[2], where the firms are forward-looking optimizers and their investment decisions are affected by an external finance premium that depends on the state variables of the model. Under additional assumptions on the adjustment costs and the structure of the depreciation of the capital, the model can be expressed as a linear Dynamic Panel Data (DPD), which can be estimated directly by GMM or Maximum Likelihood (ML).

In empirical applications, GMM has been preferred over ML due to the structure of the problem and the availability of data. However, new results in the literature, such as Hahn and Kuersteiner (2002) and Alvarez and Arellano (2003), have shown that the performance of GMM is poor in comparison with ML when the number of time periods (T) is large. The intuitive reason is that increasing T in the DPD model increases the number of instruments affecting the finite-sample properties of GMM estimator, whereas the ML estimator becomes consistent as T grows.

In Section 2, I review some available procedures to estimate a DPD model. Section 3

[1] For applications in macroeconomics see Hansen (1990).

[2] I would like to thank Simon Gilchrist for his help in this chapter and to Carmen G. Silva for providing me with the data.

discusses what are common practices used by the empirical researchers. Section 4 describes the theoretical model of investment. In Section 5 the results obtained using actual data from Chilean firms are presented. Section 6 concludes.

2 Estimation of DPD: Theory

The discussion of Dynamic Panel Data (DPD) was opened by Balestra and Nerlove (1966). In that paper, the authors proposed to estimate the model with unobserved component using the Generalized Least Squares (GLS) estimator. However, GLS or ML-Random Effects (RE) estimators are not consistent if the unobserved individual effects are correlated with the exogenous variables. In the latter case the Fixed Effects (FE) specification is preferred. Nerlove (1967, 1971) showed that FE is biased using numerical simulations. That evidence is formalized by Nickell (1981). He shows that the within groups estimator of a dynamic panel data models is biased even in large samples, under asymptotic sequences where the number of time periods (T) is fixed and the number of cross sectional units (n) is large. Accordingly, IV estimators were proposed by Anderson and Hsiao (1981), and Arellano and Bond (1991), among others. These estimators are consistent under fixed T asymptotic, but show finite-sample biases when T is moderate relative to n. An alternative in this case is the Within Group (WG) estimator. It should be noted that the bias computed by Nickell is negligible when the number of time periods is large relative to the number of cross sectional units.

Hahn and Kuersteiner (2002) and Alvarez and Arellano (2003) consider the following DPD model without exogenous variables

$$\begin{aligned} y_{it} &= \theta y_{i,t-1} + w_{it}, \qquad (1) \\ w_{it} &= \eta_i + e_{it}, \end{aligned}$$

where η_i is the unobserved individual effect, the parameter $|\theta| < 1$ meaning that the model is stationary, and e_{it} is an iid zero mean time varying error component with variance σ^2

not correlated with η_i. In addition, suppose that the initial condition y_{i0} is observed by the researcher, given an actual number of periods of $T+1$.

For the posterior asymptotic analysis under sequences where n and T grow large, it is convenient to define the large-T asymptotics version of (1) as follows

$$y_{it}^{+} = \frac{\eta_i}{1-\theta} + u_{it} \text{ , with } u_{it} = \sum_{k=0}^{\infty} \theta^k e_{i,t-k}.$$

The large-T asymptotics is expected to work in the cases where T is large, for that y_{it} will behave similarly to y_{it}^{+}. Moreover, using the definition above it is clear that $E(u_{it}) = 0$, $Var(u_{it}) = \sigma^2/(1-\theta^2)$ and $Cov(u_{it}, u_{i,t-k}) = \theta^k \sigma^2/(1-\theta^2)$.

2.1 IV estimators

The individual effects in (1) can be removed by first differences. However, the estimator obtained by simple LS in the transformed model (called here FD) is not consistent. This result follows from $p\lim(\Delta y_{i,t-1}\Delta e_{it}) = \lim E(\Delta u_{i,t-1}\Delta e_{it}) = -\sigma^2$, using the large-$T$ asymptotics approximation and the fact that e_{it} is not correlated over time. Also, $p\lim[(\Delta y_{i,t-1})^2] = \lim E[(\Delta u_{i,t-1})^2] = 2\sigma^2/(1+\theta)$. Taking these results $p\lim(\hat{\theta}_{FD}) = (\theta-1)/2$.

Anderson and Hsiao (1981) propose two IV estimators to address this problem: (1) IV on the model in first differences and using instruments in levels (AHL) and (2) IV on the model in levels and instruments in first difference (AHD). The corresponding estimators are:

$$\hat{\theta}_{AHD} = \frac{\sum_{i=1}^{n}\sum_{t=3}^{T}\Delta y_{it}\Delta y_{i,t-2}}{\sum_{i=1}^{n}\sum_{t=3}^{T}\Delta y_{i,t-1}\Delta y_{i,t-2}} \text{ , and } \hat{\theta}_{AHL} = \frac{\sum_{i=1}^{n}\sum_{t=2}^{T}\Delta y_{it} y_{i,t-2}}{\sum_{i=1}^{n}\sum_{t=2}^{T}\Delta y_{i,t-1} y_{i,t-2}}.$$

These estimators are both consistent given that $E(\Delta y_{i,t-2}\Delta e_{it}) = 0$ and $E(y_{i,t-2}\Delta e_{it}) = 0$. It should be noted that the consistency of both estimators (AHL and AHD) rely on the absence of serial of e_{it} and the no-correlation between e_{is} and η_i for any period s.

Arellano (1989) and Arellano and Bond (1991) show that asymptotic variance of AHD estimator is not well-defined for some combinations of the model parameters.

2.2 GMM estimator

Arellano and Bond (1991) propose to use a different set of instruments for each observation. For example, for Δe_{i2} the valid instrument is y_{i0}, same as for AHL estimator, but for Δe_{i3} the valid instruments are y_{i0} and y_{i1}. The latter is the only instrument used in AHL at $t = 3$. The number of valid instruments sum up to $T(T-1)/2$ moment conditions that can be used in the estimation of θ through the GMM estimator developed by Hansen (1982). Under the validity of the additional instruments the proposed estimator (AB/GMM) is more efficient than AHL.

It is important to note that AB/GMM estimator was designed for cases where n is large relative to T, therefore the number of moment conditions is usually small. Intuitively, we can think that the number of instruments required for the estimation is the same as the number of moment conditions ($\approx T^2/2$) whereas the total sample size is nT. Then, the ratio number of instruments to total sample is $T/(2n)$.

Bekker (1994) proposes an Alternative Approximation to the behavior of IV estimators (BAA) in the cross-sectional context where the number of instruments (K) increases along with the sample size (n), but K/n converges to a fixed number $\alpha < 1$. Under BAA, the standard IV is inconsistent but the Limited Information Maximum Likelihood estimator (LIML) is consistent.

In the case of DPD, Alvarez and Arellano (2003) analyzes consistency and asymptotic distribution under the double asymptotics on n and T, for the following IV estimators: AB/GMM and AB/LIML (the Limited Information Maximum Likelihood estimator version of Arellano-Bond procedure). For AB/LIML, the necessary condition for consistency is $T/n \to \alpha \leq 2$, this condition is similar to the intuitive condition presented above. A weaker condition is required for the consistency of AB/GMM, which is $(\log T)^2/n \to 0$. It is interesting to note that AB/GMM is consistent even when the number of instruments is growing to infinity. Alvarez and Arellano (2003) explain that the intuition behind the consistency is based on the fact that by increasing the number of time periods, the endogeneity bias tends to zero. Finally the asymptotic distribution for AB/GMM is, as $n, T \to \infty$,

$$\sqrt{nT}\left[\hat{\theta}_{AB/GMM} - \left(\theta - \frac{1}{n}(1+\theta)\right)\right] \xrightarrow{d} N(0, 1-\theta^2).$$

2.3 Without Groups estimator

Under double asymptotic sequences on n and T, Hahn and Kuersteiner (2002) and Alvarez and Arellano (2003) show that the Within Group estimator (WG) is consistent. The intuition behind this result is based on the fact that as T grows the estimators of the individual effects η_i become consistent, see Hahn and Kuersteiner (2002). The asymptotic distribution for WG is, as $n, T \to \infty$,

$$\sqrt{nT}\left[\hat{\theta}_{WG} - \left(\theta - \frac{1}{T}(1+\theta)\right)\right] \xrightarrow{d} N(0, 1-\theta^2).$$

The asymptotic bias for WG is $(1+\theta)/T$, which Hahn and Kuersteiner (2002) use to construct a bias-corrected version of WG (BWG). This estimator can be computed in the second stage as $\widetilde{\theta}_{BWG} = (T+1)\hat{\theta}_{WG}/T + 1/T$. Similar results are presented in Alvarez and Arellano (2003) where they also note that the number of periods (T) is usually lower than the number of cross sectional units (n) then the asymptotic bias for WG is typically higher than the asymptotic bias for AB/GMM which is $(1+\theta)/n$.[3]

Kiviet (1995) proposes a correction for WG, based on Edgeworth expansion, that requires an initial value of θ. This initial parameter is desired to be consistent, for that he uses AHL and AHD as possible candidates for this purpose. The Monte Carlo experiments presented in that paper do not show a dominant initial estimator. The key argument in Hahn and Kuersteiner (2002) to get a bias-correction that does not require an initial value of θ is the observation that $\hat{\theta}_{WG}$ is consistent under large-T asymptotics. Under similar setting, Bun and Kiviet (2003) use higher order terms of the bias of WG obtaining a refinement of Hahn-Kuersteiner's correction. Bun and Carree (2005) explore the performance

[3]For completeness, it is worth noting that the asymptotic bias for the AB/LIML is $(1+\theta)/(2n-T)$. In most cases $n > T$, then AB/LIML has the smallest asymptotic bias relative to WG and AB/GMM.

of this correction under small number of time periods, finding that the correction is poor. They also propose an alternative correction that helps to fix the bias for small T, but it requires to impose additional assumptions in the model.

Finally, Bruno (2005) extends the bias corrections proposed by Bun and Kiviet (2003) to the cases where the panel is not balanced.

3 Estimation of DPD: Practice

In this section, I discuss the main empirical issues in the estimation of DPD models and how these could affect the empirical application.

3.1 Unit Root

In the presence of unit roots AB/GMM and WG have different asymptotic distributions than the ones discussed in Alvarez and Arellano (2003). In particular, Hahn and Kuersteiner (2002) show that the asymptotic distribution for WG under unit root ($\theta = 1$) depends on the distribution of the individual effects, which are by definition unknown to the researcher. Moreover, the asymptotic distributions (with and without unobserved components) are very different under this scenario in comparison with the case of a stationary process.

In the case of AB/GMM estimator, the presence of unit root reduces the correlation between the instruments and the instrumented variables leading to finite-sample biases.[4]

For the purpose of this chapter, the empirical literature related with the dynamic of investment tends to find small autoregressive first order coefficients, which implies that the model is stationary and the lags level of the variable should be correlated with the current changes. The latter also implies (in theory) that instruments are not weak.[5]

[4]Hahn, Hausman and Kuersteiner (2001) explore the behavior of AB/GMM estimator under unit root. They propose the use of Long Difference estimator for this case.

[5]The model has weak instruments if θ is close to one.

3.2 Truncated AB/GMM

Another issue with the AB/GMM is the large number of instruments used in the computation of the estimator. From a practical point of view the computation of AB/GMM estimator becomes highly demanding in computer-time when the number of periods is large.[6] This motivates the use of some kind of truncated AB/GMM estimator (TAB) with specific number of lags. This convenient practical solution has some theoretical foundation as well. Thus, it is possible that very long lags are less correlated with current changes in the dependent variable than the most recent levels, and that adding lags the estimator becomes imprecise in the sense that valid but weak instruments are added.

It is possible to show that the TAB estimator is also consistent. Following Alvarez and Arellano (2003), I define the AB/GMM estimator using the orthogonal deviations as follows

$$\hat{\theta}_{AB/GMM} = \frac{\sum_{t=1}^{T-1} x_t^{*\prime} P_t y_t^*}{\sum_{t=1}^{T-1} x_t^{*\prime} P_t x_t^*},$$

where $x_{it} \equiv y_{i,t-1}$, Z_t is a $n \times t$ vector of instruments, $P_t = Z_t(Z_t' Z_t)^{-1} Z_t'$, and x_t^* denotes a $n \times 1$ vector of orthogonal deviations, defined as $x_{it}^* = (x_{it} - \bar{x}_{tT})/c_t$, where $c_t^2 \equiv (T - t)/(T - t + 1)$ and $\bar{x}_{tT} \equiv (x_{it} + \cdots + x_{iT})/(T - t + 1)$.

For the computation of the asymptotic distribution of $\hat{\theta}_{AB/GMM}$, I define l as the maximum lag allowed, to be included in the computation of the TAB estimator, which collapses to AB/GMM when $l = T - 1$.

Theorem 3.1. *Under the assumptions established in Alvarez and Arellano (2003) for the consistency of AB/GMM estimator, the numerator of the truncated AB/GMM estimator (TAB) has the following expected value*

$$E\left(\sum_{t=1}^{T-1} x_t^{*\prime} P_t e_t^*\right) = -\frac{T\sigma^2}{(1-\theta)}\left[\frac{l}{T} - \frac{1}{T(1-\theta)} \sum_{t=T-l+1}^{T} \left(\frac{1-\theta^t}{t}\right)\right].$$

Proof. See section A.2. □

[6] Note that the number of moment conditions increases at the rate T^2.

This is the generalization of Lemma 2 in Alvarez and Arellano (2003) and it collapses to the same conclusion under $l = T-1$

$$
\begin{aligned}
E\left(\sum_{t=1}^{T-1} x_t^{*'} P_t e_t^*\right) &= -\frac{T\sigma^2}{(1-\theta)}\left[\frac{T-1}{T} - \frac{1}{T(1-\theta)}\sum_{t=2}^{T}\left(\frac{1-\theta^t}{t}\right)\right] \\
&= -\frac{T\sigma^2}{(1-\theta)}\left[1 - \frac{1}{T(1-\theta)}\sum_{t=1}^{T}\left(\frac{1-\theta^t}{t}\right)\right].
\end{aligned}
$$

Clearly the second term inside the brackets in Theorem 3.1 converges to zero as T goes to infinity, regardless if l is fixed or not. It is easy to see that under double asymptotic (large n and large T)

$$
p\lim\left(\frac{1}{\sqrt{nT}}\sum_{t=1}^{T-1} x_t^{*'} P_t e_t^*\right) = -\sqrt{\frac{T}{n}}\left(\frac{\sigma^2}{1-\theta}\right)\frac{l}{T}.
$$

Again when $l = T-1$, the ratio $l/T \to 1$ and there is an asymptotic bias for the AB/GMM estimator that is decreasing in n. But it is interesting to note that for fixed l the estimator is asymptotically unbiased.

It should be noted that the first statement is proved in Theorem 2 of Alvarez and Arellano (2003), but the second is based on the assumption that the denominator of AB/GMM is well-defined under truncation, in other words $p\lim \sum_{t=1}^{T-1} x_t^{*'} P_t x_t^* \neq 0$.

When $l = 1$ TAB becomes the AHL estimator. TAB is less efficient than AB/GMM because the latter uses more instruments, reducing the standard errors of the estimation.

In practical terms, I recommend researchers to use TAB estimator with a lower l, but adding lags of the predetermined variables.

3.3 Unbalancedness

Bruno (2005) computes bias corrections for WG under the presence of missing at random observations in the data. The formulae for the bias are slightly different than the case of balanced panel data. Consider the arithmetic and the harmonic averages of number of time periods as follows

$$T_A \equiv \frac{\sum_{i=1}^{n} T_i}{n}, \text{ and } T_H \equiv n\left(\sum_{i=1}^{n} \frac{1}{T_i}\right)^{-1}.$$

It is clear that for balanced panels $T_i = T$ then $T_H = T_A = T$.

Following Bruno (2005), let $\omega = T_H/T_A$ be the Ahrens and Pincus index. His Monte Carlo experiments show that the bias of WG is similar for mild ($\omega = 0.96$) and severe ($\omega = 0.36$) unbalanced panels, if θ is small. Following Hahn and Kuersteiner (2002), the bias for WG is generated by the sample correlation between $u^{+}_{i,t-1}$ and $\bar{e}_i$. For the case of unbalanced panels

$$\begin{aligned} \sum_{i=1}^{n}\sum_{t=1}^{T_i} E(u^{+}_{i,t-1}\bar{e}_i) &= \sum_{i=1}^{n}\frac{1}{T_i}\sum_{t=1}^{T_i}\sum_{s=1}^{T_i} E(u^{+}_{i,t-1}e_{is}) \\ &= \frac{\sigma^2}{(1-\theta)}\left[n - \frac{1}{1-\theta}\sum_{i=1}^{n}\left(\frac{1-\theta^{T_i}}{T_i}\right)\right]. \end{aligned} \tag{2}$$

Note that for small positive θ, θ^{T_i} is small, then $(1-\theta^{T_i})/T_i \approx 1/T_i$. With that the expression can be approximated as follows

$$\begin{aligned} \sum_{i=1}^{n}\sum_{t=1}^{T_i} E(u^{+}_{i,t-1}\bar{e}_i) &\approx n\frac{\sigma^2}{(1-\theta)}\left[1 - \frac{1}{1-\theta}\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{T_i}\right)\right] \\ &= \frac{n\sigma^2}{(1-\theta)}\left[1 - \frac{1}{T_H(1-\theta)}\right] \leq \frac{n\sigma^2}{(1-\theta)}\left[1 - \frac{1}{T_A(1-\theta)}\right]. \end{aligned}$$

The last expression is the approximated sample correlation for balanced panels of T_A periods. Then the bias for unbalanced panels with small θ should be slightly lower than the one obtained for a balanced panel with T_A periods.

A straightforward bias correction is $(T_A+1)\hat{\theta}_{WG}/T_A + 1/T_A$. It is expected that the bias correction will be accurate enough for cases where degree of unbalancedness, $\omega \approx 1$, is small (measured by the Ahrens-Pincus index).

The bias of AB/GMM and TAB for unbalanced panel can be computed taking the limit of the following expectation

$$E\left(\sum_{t=1}^{T-1} x_t^{*'} P_t e_t^*\right) = \frac{\sigma^2}{(1-\theta)^2}\sum_{i=1}^{n}\sum_{t=1}^{T_i-1} p_{ii}^t \left[\frac{1-\theta^{T_i-t+1}}{T_i-t+1} - \frac{1-\theta^{T_i-t}}{T_i-t}\right].$$

where p_{ij}^t is the position (i,j) of the matrix P_t. Note that the expression in bracket is not constant, then it is not possible to compute a closed form solution for this expectation.

4 Investment Neoclassical Approach

In this section, I describe the standard neoclassical model of Investment with Financial Frictions proposed by Gilchrist and Himmelberg (1998). In this model, firms are forward-looking optimizers that have full access to the credit market, but with a specific premium that depends on the state variables: the level of capital (k_t), the level of debt (b_t) and a stochastic shock (ζ_t).

Let R_t be the gross risk-free rate of return and $S_t \geq 1$ be the finance (gross) premium described above, then the actual gross interest rate associated on debt is $R_t S_t$. For a given technology, the firm maximizes the following dynamic problem

$$v_t(k_t, b_t, \zeta_t) = \max_{k_{t+1}, b_{t+1}} \left\{d_t + R_t^{-1} E_t\left[v_{t+1}(k_{t+1}, b_{t+1}, \zeta_{t+1})\right]\right\},$$

subject to $d_t = \pi(k_t, \zeta_t) - c(k_{t+1}, k_t) - p_t i_t + b_{t+1} - R_t S_t b_t$ (dividends are equal to profit minus adjustment cost of investment, investment, and net debt), $k_{t+1} = k_t(1-\delta) + i_t$ (capital accumulation equation) and $d_t \geq 0$ (liquidity constraint).

Let λ_t be the multiplier associated with the non-negative constraint on dividends, then the Langrangian is $\mathcal{L} = (1+\lambda_t)d_t + E_t\left[R_t^{-1} v_{t+1}(k_{t+1}, b_{t+1}, \zeta_{t+1})\right]$. Under constant interest rate, the following relation is obtained[7]

$$p_t + \frac{\partial c_t}{\partial i_t} = E_t\left[\sum_{s=0}^{\infty}\left(\frac{1-\delta}{R}\right)^s H_{t+1,s}\left(\frac{\partial \pi_{t+1+s}}{\partial k_{t+1+s}}\right)\right], \tag{3}$$

[7] See Appendix A.3 for details.

where $H_{t+1,s} = \prod_{j=0}^{s}(1+\lambda_{t+1+j})/(1+\lambda_{t+j})$ represent the changes in financial constraints of the firm. In particular, without restriction on the dividends ($\lambda_t = 0$) the expression $H_{t+1,s}$ collapses to one and the adjusted price of capital (price of capital plus marginal adjustment cost) is the present value of the future sequence of Marginal Profitability of Capital (MPK). A similar argument applies when the firm faces no changes in its financial situation (λ_t is constant over time). For that reason it is expected that $H = 1$ in steady state.

Gilchrist and Himmelberg (1998) note that for a firm with market-power: $\partial\pi_t/\partial k_t = (\partial p_t/\partial y_t)(\partial y_t/\partial k_t)y_t + (\partial y_t/\partial k_t)p_t = [(\partial p_t/\partial y_t)(y_t/p_t) + 1](\partial y_t/\partial k_t)p_t$. If ϵ is the elasticity of demand and the production function is Cobb-Douglas ($y_t = Ak_t^{\gamma}$) then $\partial\pi_t/\partial k_t = \gamma(1 + 1/\epsilon)p_t y_t/k_t$ or $(\partial\pi_t/\partial k_t)$ is proportional to the sales over capital (S_t), implying

$$p_t + \frac{\partial c_t}{\partial i_t} = \gamma\left(1 + \frac{1}{\epsilon}\right)E_t\left[\sum_{j=0}^{\infty}\left(\frac{1-\delta}{R}\right)^j H_{t+1,j}S_{t+1+j}\right].$$

Consider $H_{t+1,j} = \sum_{i=0}^{j}\varphi X_{t+1+i}$, where X represents the financial situation of the firm (net financial assets) over capital. Then $H_{t+1,j}S_{t+1+j}$ can be approximated by $\gamma_0 + \gamma_0 H_{t+1,j} + S_{t+1+j}$, where $\gamma_0 \equiv \gamma(1 + 1/\epsilon)\overline{S}$ and $\overline{S}$ is steady state value for the ratio sales to capital.

Assuming a quadratic adjustment cost function as $c(k_{t+1}, k_t) = (\phi/2)(i_t/k_t - \eta)^2 k_t$, where η is an unobserved idiosyncratic component, then (3) can be written as

$$p_t + \phi\left(\frac{i_t}{k_t} - \eta\right) = \frac{\gamma_0}{1-\beta} + \gamma_0\varphi\sum_{s=0}^{\infty}\sum_{j=0}^{s}\beta^s E_t(x_{t+1+j}) + \sum_{s=0}^{\infty}\beta^s E_t(S_{t+1+s}), \qquad (4)$$

where $\beta \equiv (1-\delta)/R$. The second term on the right hand side of the equation represents the effect of the non-negative dividends constraint in the model on the choice of investment. It is important to note that the expression is a discounted forward-looking sequence of financial situations of the firm in the future. The conditional expectation in that expression implies that the information is taken in period t to get a forecast of the following periods. With an appropriate set of instruments, GMM can be used to estimate the parameters of this model.

The last term on the right hand side of 4 is known as Tobin's Q. This represents the value of the firm as the present value of the futures MPK that the firm could obtain. The usual practice in the empirical research of investment is to compute that expression using the market-value of the firm, which is the price of the share times the number of shares issued plus the market-value of the debt. From a theoretical point of view, that value will reflect the expectations of agents about the future performance of the firm. In particular, the price of the share measures the future discounted dividends of the firm. However, this theory is valid only if the capital markets are competitive. We might expect that Chilean capital markets are not perfectly competitive since the information set of each agent in the market is probably different and there is not sufficient number of financial instruments to allow for a complete distribution of the uncertainty.

In order to compute the Tobin's Q (also known as Fundamental Q) and to construct the forward-looking financial situation (also called Financial Q), Gilchrist and Himmelberg (1998) propose to generate the endogenous variables using a multivariate autoregressive process. This implies that the expectations of agents in the economy are generated through this model, using all the available information at each time period. Taking this structure the model is

$$\frac{i_{it}}{k_{i,t-1}} = \eta_i + \tau_t + \theta \frac{i_{i,t-1}}{k_{i,t-2}} + \gamma X_{i,t-1} + \psi S_{i,t-1} + e_{it},$$

where η_i is the unobserved component that is constant for each firm, τ_t is the time effect that captures the change in the aggregate variables such as the market-discount factor (interest rate) or technology shocks, θ is the autoregressive parameter for the dependent variable (the ratio of investment over capital)[8], γ captures the effect of the measure of financial stress such as cash equivalent (CE) or net working capital (NWK), ψ captures the effect of the MPK measure,[9] and e_{it} is the error term of the model.

[8]In the empirical section 5, I report the results for AR(2) processes. Also higher order autoregressive processes were computed, leading to similar conclusions.

[9]The model implies Sales, but other measures are considered as well.

5 Empirical Results

In this section I estimate the parameters of the Neoclassical Investment model using data from Chile. These data are taken from balance sheets reported to the Chilean Securities and Insurance Supervisor (*Superintendencia de Valores y Seguros* or SVS) by private firms that are registered in the SVS. The companies are medium or large size, measured in terms of workers, and may or may not trade shares in the stock market. However, for corporations that trade public share it is mandatory to be registered in the SVS, reporting their balance sheets.[10]

For firms that trade public shares, it is possible to compute the market value of the equity using the information provided by the Santiago Stock Market (*Bolsa de Comercio de Santiago*). However, the Chilean stock market is very small and therefore prices may reflect speculative movements rather than the fundamental value of the firms.

5.1 Description of the Data

The data is an unbalanced panel of firms with quarterly information from the first quarter of 1986 to the last quarter of 2005, including 80 periods. The number of firms is about 240 and each firm is observed on average around 40 quarters. Previous studies have used a subset of this sample. For example Medina and Valdés (1998) and Gallego and Loayza (1999) use 78 and 79 firms over the period 1985-1995 (balanced panel). The main purpose of these studies was to analyze the effect of the ownership on the investment dynamics of firms. In particular, Chile has a private Social Security system which is managed by broker firms that are specialized in the administration of retirement funds.[11] The broker-firms can invest only in some small number of the private firms, that are called AFP-able. In the sample of the previous studies some firms were AFP-able and some not.

For the definition of capital, I follow Blundell, Bond, Devereux, and Schiantarelli, (1995), and Gallego and Loayza (1999) taking the difference between the total-assets and the

[10] This report is known as FECUS (*Ficha Estadistica Codificada Uniforme*).
[11] These companies are called *Aseguradoras de Fondos de Pensiones* or AFP.

current-assets of a firm plus the change in cumulative-depreciation. The results are slightly different when fixed-assets are used instead of total-assets minus current-assets. Keeping the first definition implies that other-assets are considered as investment as well as the change in the fixed-assets.

The profitability measures used in this analysis are sales, operational-result (marginal) and total-profit. The first measure is standard. The main difference between the second and third one is that the latter includes additional profits obtained from investment in other firms.

The liquidity measures considered are: (1) Net Working Capital (NWK) which is defined as the difference between the current-assets and the current-liabilities, (2) Adjusted Working Capital (AWK) which is NWK minus inventories[12] and (3) Cash Equivalent (CE) which are the most liquid components of the current-assets. All the measures are considered over the total capital of the firm. Alternative measures haven been proposed in Finance as a part of the Analysis of Financial Ratio. These measures, including Current-Ratio (ratio between current-assets and current-liabilities), Quick-Ratio (ratio between current-assets minus inventories over current-liabilities) and Cash-Ratio (ratio between the most liquid current-assets over current-liabilities), tend to capture the same effects as the measures used here, but they are not adjusted properly by firm size. For this reason the expected effect tend to be seriously downward biased and usually non-significant.

Table 1 shows the main descriptive statistics of the variables used in the analysis (T_A and T_H are the arithmetic and the harmonic averages of time periods).[13] In order to avoid the effect of outliers on the estimations, I drop the tails of the variables: 5 and 95 percentiles. This practice is common for this kind of data, because extreme values in these variables are usually generated by mergers, bankruptcy or even accounting typos (see for example Baum and Caglayan (2006)).

[12]Balance Reports in Chile include inventories as current-assets. The definition of NWK here corresponds to FWK in Gilchirst and Himmelberg (1998). AWK is an additional measure that controls for the fact that inventories are not as liquid as other terms considered in the current-assets.

[13]The sample reported is the joint sample required to estimate the AR(1) process by WG. The actual sample used in each model includes slightly more observations.

Table 1: Descriptive Statistics

Variable (over capital)	Mean	Std. Dev. (overall)	(between)*	(within)*	Min	Max
Investment	0.0195	0.0375	0.0928	0.9072	-0.0813	0.1573
Sales	0.3240	0.3996	0.5914	0.4086	0.0000	2.1708
Margin	0.0480	0.0621	0.4706	0.5294	-0.0377	0.2902
Profits	0.0640	0.0700	0.3660	0.6340	-0.0759	0.3402
Net Working K (NWK)	0.1094	0.1854	0.5958	0.4042	-0.1961	1.2332
Adjusted NWK	0.0412	0.1505	0.4700	0.5300	-0.3663	0.7620
Cash Equivalent	0.0513	0.0789	0.4628	0.5372	0.0002	0.5246

*In percent (over total variance). Also $n = 239$, $T_A = 37$ and $T_H = 17$.

It is interesting to note that the sample has a very low level of investment compared to the descriptive statistics available for the U.S. (see for example Gilchrist, Natalucci and Zakrajsek (2007)). In particular, the level of investment in Chile is low and the dispersion of this variable is mostly within firm variation. This observation suggests that aggregate investment must be relatively stable during the sample period (1986-2005). The variables Net and Adjusted Working Capital have similar variation, while the difference in their means are around 0.1. These variables show that the average firm in the sample tends to have 14% of the value of the capital as excess on the current-assets and almost 10% of that excess is kept as inventories. Comparing the first variable with the U.S., the mean is similar but the overall variance is much lower than in the case of the U.S. (see Gilchrist and Himmelberg (1998)). The descriptive statistics for the Cash Equivalent variable shows that the average firm in the sample tends to have around 6% of the value of the capital in highly liquid assets, with a very small dispersion. These figures are very different to the U.S. where the average firm keeps around 27% of the value of the capital in highly liquid assets and the dispersion is 0.4.

5.2 Main Estimation Results

The Within Groups (WG) estimates of the model parameters results for the model are presented in Table 2. The AR(1) has an autoregressive coefficient is approximated 0.11 (first three columns). For those models, the R^2 (within) is approximated 12%.

These estimates tend to be downward biased and the bias can be corrected applying the $(T_A + 1)/T_A$ factor to the actual estimator and adding the intercept correction $1/T_A$. Approximating these factors for the case of unbalanced panel by the average sample period for each firm ($T_A = 37.44$), then the bias-corrected coefficient is about 0.14. But the bias-corrected estimator is about 0.19 if T_H is used instead of T_A.

The MPK measures have effects of 0.003 when Sales is used, 0.03 in the case of Margins and 0.05 for the case of Profits. Margins and Profits are statistically significant (at 1% level), having the expected sign. However, Sales has a small effect (not significant when NWK is used). The financial variables have the expected sign and all are significant (at 1% level). Both NWK and AWK have an estimated coefficient of 0.02 and CE a coefficient of 0.04. The results are robust to the inclusion of an additional lag.[14]

In Table 3 the results for the AB/GMM are presented. These estimations were computed using the first-step GMM.[15] Each estimator includes all the available valid instruments of the dependent variable, the exogenous variables at time $t - 1$ as instruments and time-dummies.

The AR(1) model has a average coefficient of 0.14, higher that the one obtained using WG, but similar to the bias-corrected WG. This results is not surprising in the light of Alvarez and Arellano (2003). They show that for a balanced dynamic panel data model the biases are $(1+\theta)/T$ for WG and $(1+\theta)/n$ for AB/GMM, then it is expected that AB/GMM bias should be smaller in magnitude than WG. Simulations presented in that paper show that for all cases where $T/n \leq 2$ the bias of WG is always higher or equal to the bias of

[14]Due to the information is in quarterly basis other models were computed: AR(4) and a seasonal AR(1). The conclusions are similar.

[15]The algorithm implemented by Doornik, Arellano and Bond (2006) for the software Ox was used. For reference on the software see Doornik (2006).

AB/GMM. The empirical results presented here with an unbalanced panel confirm that conclusion. The standard errors of AB/GMM are higher for the autoregressive coefficient relative to WG estimator.

The coefficients for the exogenous variables are similar to the ones obtained by WG. In addition, the standard errors are also similar for these variables. Adding one lag the results do not change substantially.

5.3 Alternative Estimation Results

In empirical applications, it is a common practice to use a restricted AB/GMM procedure, using less number of lags as instruments. In the previous sections, it was shown that the moment conditions increase with the number of periods, therefore for panels that cover many periods, it is possible to truncate the lags included in the estimation.

It is important to note that the main motivation for Alvarez and Arellano (2003) was to show that including all the possible lags the AB/GMM is consistent. But, it is well-known that GMM estimators tend to be biased even when strong instruments are used. In Theorem 3.1, a slightly modification of Alvarez and Arellano (2003) states that the truncated AB/GMM (called TAB) is asymptotically unbiased when the truncation limit (l) does not depend on T, therefore $l/T \to 0$. However, it is expected that TAB is less efficient than AB/GMM, therefore the computation of asymptotic MSE could help in this matter, giving an optimal number of lags included in the computation of TAB.

Table 4 shows the results of TAB with $l = 8$. Given that the data is available on quarterly basis.[16] The AR(1) model has a coefficient of 0.2, which is higher than the one obtained by AB/GMM. This was expected because AB/GMM estimator is asymptotically biased, but not TAB. Also, the standard errors of the autoregressive coefficient for TAB is higher than the one obtained under AB/GMM. The coefficients and standard errors of the exogenous variables is similar to AB/GMM estimation. A similar conclusion is obtained from AR(2) model.

[16]The theoretical support for taking that number is based on the fact that firms could have contracts and others duties to do within 2 years

Table 2: Investment Equation: WG estimation

	NWK	AWK	CE	NWK	AWK	CE
Sales						
i_{t-1}/k_{t-2}	0.1134	0.1177	0.1232	0.0997	0.1019	0.1074
	(0.0161)	(0.0158)	(0.0167)	(0.0151)	(0.0148)	(0.0150)
i_{t-2}/k_{t-3}				0.0785	0.0778	0.0796
				(0.0143)	(0.0143)	(0.0151)
p^k_{t-1}/k_{t-2}	0.0020	0.0043	0.0036	0.0018	0.0043	0.0039
	(0.0021)	(0.0020)	(0.0018)	(0.0021)	(0.0020)	(0.0019)
x_{t-1}/k_{t-2}	0.0243	0.0214	0.0365	0.0248	0.0228	0.0391
	(0.0044)	(0.0046)	(0.0073)	(0.0043)	(0.0045)	(0.0074)
Margins						
i_{t-1}/k_{t-2}	0.1120	0.1163	0.1149	0.0966	0.1000	0.1008
	(0.0167)	(0.0163)	(0.0160)	(0.0155)	(0.0153)	(0.0150)
i_{t-2}/k_{t-3}				0.0727	0.0739	0.0721
				(0.0143)	(0.0144)	(0.0148)
p^k_{t-1}/k_{t-2}	0.0285	0.0338	0.0318	0.0251	0.0319	0.0302
	(0.0117)	(0.0115)	(0.0111)	(0.0117)	(0.0116)	(0.0112)
x_{t-1}/k_{t-2}	0.0229	0.0205	0.0383	0.0239	0.0227	0.0394
	(0.0044)	(0.0049)	(0.0077)	(0.0043)	(0.0049)	(0.0079)
Profits						
i_{t-1}/k_{t-2}	0.1033	0.1069	0.1064	0.0900	0.0915	0.0926
	(0.0173)	(0.0170)	(0.0167)	(0.0164)	(0.0162)	(0.0157)
i_{t-2}/k_{t-3}				0.0706	0.0706	0.0707
				(0.0142)	(0.0144)	(0.0147)
p^k_{t-1}/k_{t-2}	0.0468	0.0480	0.0513	0.0434	0.0447	0.0471
	(0.0101)	(0.0101)	(0.0095)	(0.0098)	(0.0098)	(0.0094)
x_{t-1}/k_{t-2}	0.0184	0.0165	0.0359	0.0192	0.0182	0.0371
	(0.0043)	(0.0048)	(0.0079)	(0.0043)	(0.0048)	(0.0080)

Robust Standard errors in parentheses.

Table 3: Investment Equation: AB estimation

	NWK	AWK	CE	NWK	AWK	CE
Sales						
i_{t-1}/k_{t-2}	0.1347	0.1506	0.1604	0.1114	0.1220	0.1179
	(0.0241)	(0.0239)	(0.0240)	(0.0222)	(0.0213)	(0.0223)
i_{t-2}/k_{t-3}				0.1045	0.1022	0.0910
				(0.0192)	(0.0194)	(0.0218)
p^k_{t-1}/k_{t-2}	0.0018	0.0040	0.0034	0.0016	0.0040	0.0038
	(0.0020)	(0.0019)	(0.0018)	(0.0021)	(0.0019)	(0.0019)
x_{t-1}/k_{t-2}	0.0244	0.0216	0.0359	0.0251	0.0232	0.0389
	(0.0043)	(0.0045)	(0.0071)	(0.0042)	(0.0043)	(0.0073)
Margins						
i_{t-1}/k_{t-2}	0.1305	0.1461	0.1349	0.1020	0.1112	0.0931
	(0.0237)	(0.0242)	(0.0253)	(0.0205)	(0.0206)	(0.0217)
i_{t-2}/k_{t-3}				0.0808	0.0882	0.0757
				(0.0208)	(0.0206)	(0.0236)
p^k_{t-1}/k_{t-2}	0.0271	0.0317	0.0305	0.0244	0.0306	0.0306
	(0.0114)	(0.0111)	(0.0109)	(0.0118)	(0.0115)	(0.0113)
x_{t-1}/k_{t-2}	0.0230	0.0208	0.0380	0.0240	0.0230	0.0395
	(0.0043)	(0.0048)	(0.0076)	(0.0043)	(0.0048)	(0.0079)
Profits						
i_{t-1}/k_{t-2}	0.1307	0.1445	0.1367	0.1122	0.1146	0.1230
	(0.0238)	(0.0229)	(0.0236)	(0.0235)	(0.0269)	(0.0238)
i_{t-2}/k_{t-3}				0.0716	0.0844	0.0865
				(0.0226)	(0.0227)	(0.0215)
p^k_{t-1}/k_{t-2}	0.0436	0.0434	0.0480	0.0408	0.0409	0.0429
	(0.0103)	(0.0100)	(0.0097)	(0.0101)	(0.0101)	(0.0094)
x_{t-1}/k_{t-2}	0.0187	0.0171	0.0359	0.0195	0.0188	0.0373
	(0.0043)	(0.0047)	(0.0077)	(0.0042)	(0.0047)	(0.0078)

Robust Standard errors in parentheses.

Table 5 has the results for TAB when the truncated number of lags is 12. The AR(1) model has a coefficient of 0.19, whereas AR(2) model has coefficients of 0.18 and 0.11. The coefficients and standard errors of exogenous variables are not affected by the changing of the truncated lag. However, the standard error for the autoregressive coefficients are higher than AB/GMM but lower than the case when TAB uses 8 lags only.

It is interesting to note that the results for AR(1) are similar to the ones obtained with bias-corrected WG estimator when T_H is used in the correction instead of T_A.

6 Conclusions

In this paper an empirical application of DPD model is developed to compare different estimation methods. I show how a Truncated AB/GMM estimator (TAB) could lead to bias reduction to the original AB/GMM. This implies that the common practice in the empirical research of truncating the number of lags has a theoretical support to reduce the finite-sample bias.

The estimation of a dynamic investment equation for Chilean firms confirms the neoclassic modeling of investment function. The coefficients obtained are in the right direction, indicating that the investment is positively correlated with measures of Marginal Profitability of Capital, such as Sales, Margin or Profits and with the financial position of the firm. The first argument is usually tested using the average Tobin's Q, which is the ratio between the market-value of the firm (external value) over the value of current capital (internal value). However, in the application presented here, I used only balance sheet information, following the proposal of Gilchrist and Himmelberg (1998). This is completely applicable for the case of Chile where the capital markets are not well developed, therefore the market-value of the firm could be affected by measurement error. The second argument is defined as Financial Q, by the same authors, and it could be used with the appropriate factor as a measure of financial-stress of the corporate sector in Chile.

Table 4: Investment Equation: TAB estimation (8 lags)

	NWK	AWK	CE	NWK	AWK	CE
Sales						
i_{t-1}/k_{t-2}	0.1845	0.2039	0.2110	0.1922	0.1852	0.1583
	(0.0328)	(0.0339)	(0.0328)	(0.0421)	(0.0427)	(0.0442)
i_{t-2}/k_{t-3}				0.1314	0.1299	0.1069
				(0.0316)	(0.0310)	(0.0315)
p^k_{t-1}/k_{t-2}	0.0013	0.0036	0.0030	0.0008	0.0034	0.0035
	(0.0020)	(0.0019)	(0.0017)	(0.0020)	(0.0019)	(0.0019)
x_{t-1}/k_{t-2}	0.0247	0.0221	0.0349	0.0260	0.0241	0.0383
	(0.0042)	(0.0043)	(0.0068)	(0.0040)	(0.0040)	(0.0070)
Margins						
i_{t-1}/k_{t-2}	0.2022	0.2037	0.1616	0.1576	0.1598	0.1287
	(0.0350)	(0.0342)	(0.0358)	(0.0410)	(0.0426)	(0.0434)
i_{t-2}/k_{t-3}				0.1264	0.1294	0.1005
				(0.0317)	(0.0350)	(0.0344)
p^k_{t-1}/k_{t-2}	0.0220	0.0276	0.0289	0.0187	0.0259	0.0278
	(0.0109)	(0.0108)	(0.0108)	(0.0118)	(0.0117)	(0.0112)
x_{t-1}/k_{t-2}	0.0234	0.0214	0.0377	0.0248	0.0241	0.0392
	(0.0041)	(0.0045)	(0.0075)	(0.0040)	(0.0045)	(0.0077)
Profits						
i_{t-1}/k_{t-2}	0.1862	0.1844	0.1588	0.1716	0.1432	0.1644
	(0.0368)	(0.0368)	(0.0350)	(0.0489)	(0.0525)	(0.0515)
i_{t-2}/k_{t-3}				0.1039	0.1346	0.0973
				(0.0358)	(0.0382)	(0.0355)
p^k_{t-1}/k_{t-2}	0.0370	0.0385	0.0457	0.0314	0.0337	0.0379
	(0.0108)	(0.0107)	(0.0099)	(0.0110)	(0.0112)	(0.0100)
x_{t-1}/k_{t-2}	0.0194	0.0178	0.0359	0.0207	0.0201	0.0376
	(0.0041)	(0.0045)	(0.0076)	(0.0041)	(0.0045)	(0.0076)

Robust Standard errors in parentheses.

Table 5: Investment Equation: TAB estimation (12 lags)

	NWK	AWK	CE	NWK	AWK	CE
Sales						
i_{t-1}/k_{t-2}	0.1747	0.2006	0.2049	0.1852	0.1659	0.1779
	(0.0307)	(0.0324)	(0.0302)	(0.0352)	(0.0350)	(0.0388)
i_{t-2}/k_{t-3}				0.1141	0.1218	0.1024
				(0.0281)	(0.0282)	(0.0286)
p^k_{t-1}/k_{t-2}	0.0014	0.0036	0.0030	0.0009	0.0036	0.0033
	(0.0020)	(0.0019)	(0.0017)	(0.0020)	(0.0019)	(0.0018)
x_{t-1}/k_{t-2}	0.0247	0.0220	0.0350	0.0257	0.0238	0.0380
	(0.0042)	(0.0043)	(0.0068)	(0.0040)	(0.0041)	(0.0070)
Margins						
i_{t-1}/k_{t-2}	0.1835	0.1841	0.1540	0.1465	0.1449	0.1102
	(0.0312)	(0.0309)	(0.0332)	(0.0345)	(0.0329)	(0.0364)
i_{t-2}/k_{t-3}				0.1069	0.0970	0.0989
				(0.0275)	(0.0292)	(0.0305)
p^k_{t-1}/k_{t-2}	0.0233	0.0290	0.0294	0.0202	0.0279	0.0290
	(0.0111)	(0.0109)	(0.0109)	(0.0117)	(0.0116)	(0.0112)
x_{t-1}/k_{t-2}	0.0233	0.0212	0.0378	0.0246	0.0235	0.0394
	(0.0041)	(0.0046)	(0.0075)	(0.0041)	(0.0047)	(0.0078)
Profits						
i_{t-1}/k_{t-2}	0.1758	0.1707	0.1623	0.1435	0.1249	0.1586
	(0.0333)	(0.0329)	(0.0311)	(0.0410)	(0.0415)	(0.0399)
i_{t-2}/k_{t-3}				0.0880	0.1309	0.1069
				(0.0305)	(0.0312)	(0.0292)
p^k_{t-1}/k_{t-2}	0.0382	0.0401	0.0453	0.0359	0.0362	0.0379
	(0.0105)	(0.0103)	(0.0099)	(0.0108)	(0.0107)	(0.0097)
x_{t-1}/k_{t-2}	0.0193	0.0175	0.0359	0.0201	0.0197	0.0377
	(0.0042)	(0.0046)	(0.0076)	(0.0042)	(0.0046)	(0.0076)

Robust Standard errors in parentheses.

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A Appendix

A.1 Formulae for unbalanced DPD

Alvarez and Arellano (2003) consider the Helmert transformation, which is

$$x^*_{it} = \left(\frac{T_i - t}{T_i - t + 1}\right)^{1/2} \left(x_{it} - \frac{x_{i,t+1} + \ldots + x_{iT_i}}{T_i - t}\right).$$

Taking $c^2_{it} \equiv (T_i - t)/(T_i - t + 1)$ I can define $\bar{x}_{tT_i} \equiv (x_{it} + \cdots + x_{iT_i})/(T_i - t + 1)$ then $x_{i,t+1} + \cdots + x_{iT_i} = (T_i - t + 1)\bar{x}_{tT_i} - x_{it}$ and

$$x^*_{it} = c_{it}\left(\frac{x_{it} - \bar{x}_{tT_i}}{c^2_{it}}\right) = \left(\frac{x_{it} - \bar{x}_{tT_i}}{c_{it}}\right).$$

Now I consider $x_{it} \equiv y_{i,t-1}$ and $u_{it} \equiv \sum_{k=0}^{\infty} \theta^k e_{i,t-k}$, then

$$u_{it} = \sum_{k=0}^{t-1} \theta^k e_{i,t-k} + \theta^t u_{i0}.$$

Define $\phi_i = y_{i0} - u_{i0} - \eta_i/(1-\theta)$, then the process can written as follows

$$\begin{aligned} y_{it} &= \eta_i + \theta y_{i,t-1} + e_{it} \\ &= \frac{\eta_i}{1-\theta} + \theta^t \phi_i + u_{it}. \end{aligned}$$

Then $x_{it} = \eta_i/(1-\theta) + \theta^{t-1}\phi_i + u_{i,t-1}$. It is possible to compute

$$\begin{aligned} \bar{y}_{tT_i} &= \frac{y_{it} + y_{i,t+1} + \ldots + y_{iT_i}}{T_i - t + 1} \\ &= \frac{\eta_i}{1-\theta} + \frac{\phi_i}{T_i - t + 1}\sum_{k=t}^{T_i} \theta^k + \frac{1}{T_i - t + 1}\sum_{k=t}^{T_i} u_{ik} \\ &= \frac{\eta_i}{1-\theta} + \frac{\phi_i \theta^t}{T_i - t + 1}\left(\frac{1 - \theta^{T_i - t + 1}}{1-\theta}\right) + \bar{u}_{tT_i}. \end{aligned}$$

And $\bar{x}_{tT_i} = \eta_i/(1-\theta) + \theta^{t-1}\phi_i(1 - \theta^{T_i - t + 1})/((1-\theta)(T_i - t + 1)) + \bar{u}_{t-1,T_i - 1}$, then

$$x_{it}^{*} = \frac{x_{it} - \bar{x}_{tT_i}}{c_{it}} = \frac{1}{c_{it}}\left(\theta^{t-1}\phi_i\left(1 - \frac{1-\theta^{T_i-t+1}}{(1-\theta)(T_i-t+1)}\right) + u_{i,t-1} - \bar{u}_{t-1,T_i-1}\right).$$

Consider the matrix $P_t = Z_t(Z_t'Z_t)^{-1}Z_t'$ and p_{ij}^t the (i,j) of P_t. In addition, the following expected values are zero: $E(u_{i,t-1}e_{jt})$ and $E(\bar{u}_{t-1,T_i-1}e_{jt})$, then

$$\begin{aligned}
E(x_t^{*'}P_t e_t^*) &= \sum_{i=1}^{n}\sum_{j=1}^{n} E(x_{it}^* p_{ij}^t e_{jt}^*) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} E\left[\left(\frac{u_{i,t-1} - \bar{u}_{t-1,T_i-1}}{c_{it}}\right) p_{ij}^t \left(\frac{e_{jt} - \bar{e}_{tT_j}}{c_{jt}}\right)\right] \\
&= \sum_{i=1}^{n} \frac{p_{ii}^t}{c_{it}^2}\left[E(\bar{u}_{t-1,T_i-1}\bar{e}_{tT_i}) - E(\bar{u}_{t-1,T_i-1}e_{it})\right].
\end{aligned}$$

Taking the fact that $u_{it} = e_{it} + \theta e_{i,t-1} + \theta^2 e_{i,t-2} + \ldots$:

$$\begin{aligned}
E(\bar{u}_{t-1,T_i-1}e_{it}) &= \frac{1}{T_i-t+1}E\left[\left(u_{i,t-1} + u_{it} + \ldots + u_{T_i-1}\right)e_{it}\right] \\
&= \frac{\sigma^2}{T_i-t+1}\left(0 + 1 + \theta + \ldots + \theta^{T_i-t-1}\right) \\
&= \frac{\sigma^2}{T_i-t+1}\sum_{k=0}^{T_i-t-1}\theta^k = \frac{\sigma^2}{T_i-t+1}\left(\frac{1-\theta^{T_i-t}}{1-\theta}\right),
\end{aligned}$$

$$\begin{aligned}
E(\bar{u}_{t-1,T_i-1}e_{i,t+1}) &= \frac{1}{T_i-t+1}E\left[\left(u_{i,t-1} + u_{it} + \ldots + u_{T_i-1}\right)e_{i,t+1}\right] \\
&= \frac{\sigma^2}{T_i-t+1}\left(0 + 0 + 1 + \theta + \ldots + \theta^{T_i-t-2}\right) \\
&= \frac{\sigma^2}{T_i-t+1}\sum_{k=0}^{T_i-t-2}\theta^k = \frac{\sigma^2}{T_i-t+1}\left(\frac{1-\theta^{T_i-(t+1)}}{1-\theta}\right),
\end{aligned}$$

It is easy to see that

$$E(\bar{u}_{t-1,T_i-1}e_{i,k}) = \frac{\sigma^2}{T_i-t+1}\left(\frac{1-\theta^{T_i-k}}{1-\theta}\right).$$

The latter can be used in the following equation

$$
\begin{aligned}
E(\bar{u}_{t-1,T_i-1}\bar{e}_{tT_i}) &= \frac{1}{T_i-t+1}E[\bar{u}_{t-1,T_i-1}(e_{it}+e_{i,t+1}+\ldots+e_{T_i})] \\
&= \frac{\sigma^2}{(T_i-t+1)^2}\sum_{k=t}^{T_i}\left(\frac{1-\theta^{T_i-k}}{1-\theta}\right) \\
&= \frac{\sigma^2}{(T_i-t+1)^2}\left(\left(\frac{T_i-t+1}{1-\theta}\right)-\frac{1}{1-\theta}\sum_{j=0}^{T_i-t}\theta^j\right) \\
&= \frac{\sigma^2}{(T_i-t+1)}\left(\left(\frac{1}{1-\theta}\right)-\frac{1}{(1-\theta)(T_i-t+1)}\left(\frac{1-\theta^{T_i-t+1}}{1-\theta}\right)\right).
\end{aligned}
$$

Replacing into $E(x_t^{*'}P_te_t^*)$

$$
\begin{aligned}
E(x_t^{*'}P_te_t^*) &= \sum_{i=1}^{n}\frac{p_{ii}^t}{c_{it}^2}\left[E(\bar{u}_{t-1,T_i-1}\bar{e}_{tT_i})-E(\bar{u}_{t-1,T_i-1}e_{it})\right] \\
&= \sum_{i=1}^{n}\frac{\sigma^2p_{ii}^t}{c_{it}^2(T_i-t+1)}\left[\left(\frac{1}{1-\theta}\right)-\frac{1-\theta^{T_i-t+1}}{(1-\theta)^2(T_i-t+1)}-\left(\frac{1-\theta^{T_i-t}}{1-\theta}\right)\right] \\
&= \sum_{i=1}^{n}\frac{\sigma^2p_{ii}^t}{c_{it}^2(1-\theta)^2}\left[\frac{1-\theta}{T_i-t+1}-\frac{1-\theta^{T_i-t+1}}{(T_i-t+1)^2}-\frac{(1-\theta)(1-\theta^{T_i-t})}{(T_i-t+1)^2}\right] \\
&= \sum_{i=1}^{n}\frac{\sigma^2p_{ii}^t}{(1-\theta)^2}\left(\frac{1-\theta^{T_i-t+1}}{T_i-t+1}-\frac{1-\theta^{T_i-t}}{T_i-t}\right).
\end{aligned}
$$

The expected value of the numerator of AB/GMM estimator is therefore

$$
E\left(\sum_{t=1}^{T-1}x_t^{*'}P_te_t^*\right) = \frac{\sigma^2}{(1-\theta)^2}\sum_{i=1}^{n}\sum_{t=1}^{T_i-1}p_{ii}^t\left(\frac{1-\theta^{T_i-t+1}}{T_i-t+1}-\frac{1-\theta^{T_i-t}}{T_i-t}\right).
$$

A.2 Proof of Theorem 3.1

Now, consider a balanced panel ($T_i=T$), and a truncated number of lags used in AB/GMM estimator. In particular let l be the maximum lags allowed, then for AB/GMM estimator $l=T-1$ meanwhile for AHL $l=1$. With this generalization $\sum_{i=1}^{n}p_{ii}^t=\min\{t,l\}$, then

$$
E\left(x_t^{*'}P_te_t^*\right) = \frac{\sigma^2\min\{t,l\}}{(1-\theta)^2}\left(\frac{1-\theta^{T-t+1}}{T-t+1}-\frac{1-\theta^{T-t}}{T-t}\right).
$$

With that

$$
\begin{aligned}
E\left(\sum_{t=1}^{T-1} x_t^{*'} P_t e_t^*\right) &= \frac{\sigma^2}{(1-\theta)^2}\sum_{t=1}^{T-1}\min\{t,l\}\left(\frac{1-\theta^{T-t+1}}{T-t+1}-\frac{1-\theta^{T-t}}{T-t}\right)\\
&= \frac{\sigma^2}{(1-\theta)^2}\sum_{t=1}^{l-1} t\left(\frac{1-\theta^{T-t+1}}{T-t+1}-\frac{1-\theta^{T-t}}{T-t}\right)\\
&\quad +\frac{\sigma^2 l}{(1-\theta)^2}\sum_{t=l}^{T-1}\left(\frac{1-\theta^{T-t+1}}{T-t+1}-\frac{1-\theta^{T-t}}{T-t}\right)\\
&= \frac{\sigma^2}{(1-\theta)^2}\sum_{s=T-l+2}^{T}(T-s+1)\left(\frac{1-\theta^{s}}{s}-\frac{1-\theta^{s-1}}{s-1}\right)\\
&\quad +\frac{\sigma^2 l}{(1-\theta)^2}\sum_{s=2}^{T-l+1}\left(\frac{1-\theta^{s}}{s}-\frac{1-\theta^{s-1}}{s-1}\right)\\
&= \frac{\sigma^2}{(1-\theta)^2}\sum_{s=T-l+2}^{T}(T-s+1)\left(\frac{1-\theta^{s}}{s}\right)\\
&\quad -\frac{\sigma^2}{(1-\theta)^2}\sum_{s=T-l+1}^{T-1}(T-s)\left(\frac{1-\theta^{s}}{s}\right)\\
&\quad +\frac{\sigma^2 l}{(1-\theta)^2}\left[\sum_{s=2}^{T-l+1}\left(\frac{1-\theta^{s}}{s}\right)-\sum_{s=1}^{T-l}\left(\frac{1-\theta^{s}}{s}\right)\right]\\
&= -\frac{T\sigma^2}{(1-\theta)}\left[\frac{l}{T}-\frac{1}{T(1-\theta)}\sum_{t=T-l+1}^{T}\left(\frac{1-\theta^{t}}{t}\right)\right].
\end{aligned}
$$

Therefore $p\lim(nT)^{-1/2}\sum_{t=1}^{T-1} x_t^{*'} P_t e_t^* = -\sqrt{T/n}[\sigma^2/(1-\theta)](l/T)$. For the case of AB/GMM $l = T-1$ then that limit is $-\sqrt{T/n}\sigma^2/(1-\theta)$. However, for a fixed l the probability limit is zero.

A.3 Euler Equations for Neoclassical Investment

Let $d_t = \pi_t(k_t,\zeta_t) - c_t(k_{t+1},k_t) - p_t[k_{t+1}-(1-\delta)k_t] + b_{t+1} - R_t S_t b_t$ be the dividend obtained at period t, then the Lagrangian of the problem can be written as

$$\mathcal{L} = (1+\lambda_t)d_t + E_t[R_{t+1}^{-1} v_{t+1}(k_{t+1}, b_{t+1}, \zeta_{t+1})].$$

The First Order Condition (FOC) with respect to the control variables (k_{t+1} and b_{t+1}) are

$$0 = (1+\lambda_t)\frac{\partial d_t}{\partial k_{t+1}} + E_t\left[R_{t+1}^{-1}\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \text{ , and } 0 = (1+\lambda_t)\frac{\partial d_t}{\partial b_{t+1}} + E_t\left[R_{t+1}^{-1}\frac{\partial v_{t+1}}{\partial b_{t+1}}\right].$$

The Envelope Theorem can be used to obtain the derivatives of the value function with respect to k and b.

$$\frac{\partial v_t}{\partial k_t} = (1+\lambda_t)\frac{\partial d_t}{\partial k_t} \quad \text{, and} \quad \frac{\partial v_t}{\partial b_t} = (1+\lambda_t)\frac{\partial d_t}{\partial b_t}.$$

Updating the previous equations it is possible to rewrite the FOC as follows

$$\begin{aligned} 0 &= (1+\lambda_t)\frac{\partial d_t}{\partial k_{t+1}} + E_t\left[R_{t+1}^{-1}(1+\lambda_{t+1})\frac{\partial d_{t+1}}{\partial k_{t+1}}\right], \\ 0 &= (1+\lambda_t)\frac{\partial d_t}{\partial b_{t+1}} + E_t\left[R_{t+1}^{-1}(1+\lambda_{t+1})\frac{\partial d_{t+1}}{\partial b_{t+1}}\right]. \end{aligned}$$

Finally, $\partial d_t/\partial k_{t+1} = -(\partial c_t/\partial k_{t+1} + p_t)$, $\partial d_t/\partial b_{t+1} = 1$, $\partial d_t/\partial k_t = \partial \pi_t/\partial k_t - \partial c_t/\partial k_t + (1-\delta)p_t$ and $\partial d_t/\partial b_t = -R_t(b_t \partial S_t/\partial b_t + S_t)$. Replacing these into the FOC

$$\begin{aligned} (1+\lambda_t)\left(p_t + \frac{\partial c_t}{\partial k_{t+1}}\right) &= E_t\left\{R_{t+1}^{-1}(1+\lambda_{t+1})\left[\frac{\partial \pi_{t+1}}{\partial k_{t+1}} - \frac{\partial c_{t+1}}{\partial k_{t+1}} + (1-\delta)p_{t+1}\right]\right\} \\ (1+\lambda_t) &= E_t\left[(1+\lambda_{t+1})\left(b_{t+1}\frac{\partial S_{t+1}}{\partial b_{t+1}} + S_{t+1}\right)\right]. \end{aligned}$$

Define $\Lambda_t \equiv (1+\lambda_t)/(1+\lambda_{t-1})$, $G_t \equiv R_t^{-1}\Lambda_t$, $p_t^i \equiv p_t + \partial c_t/\partial i_t$ and $p_t^k \equiv \partial \pi_t/\partial k_t$. Moreover, by chain rule: $\partial c_t/\partial k_t = (\partial c_t/\partial i_t)(\partial i_t/\partial k_t) = -(1-\delta)(\partial c_t/\partial i_t)$ and $\partial c_t/\partial k_{t+1} = (\partial c_t/\partial i_t)(\partial i_t/\partial k_{t+1}) = (\partial c_t/\partial i_t)$, then

$$\begin{aligned} p_t^i &= E_t\left\{G_{t+1}\left[p_{t+1}^k + (1-\delta)p_{t+1}^i\right]\right\} = E_t[G_{t+1}p_{t+1}^k] + (1-\delta)E_t[G_{t+1}p_{t+1}^i] \\ &= E_t[G_{t+1}p_{t+1}^k] + (1-\delta)E_t\left[G_{t+1}\left(E_{t+1}[G_{t+2}p_{t+2}^k] + (1-\delta)E_{t+1}[G_{t+2}p_{t+2}^i]\right)\right] \\ &= E_t[G_{t+1}p_{t+1}^k] + (1-\delta)E_t[G_{t+1}G_{t+2}p_{t+2}^k] + (1-\delta)^2E_t[G_{t+1}G_{t+2}p_{t+2}^i] \\ &= E_t\left[\sum_{s=0}^{\infty}(1-\delta)^s\left(\prod_{j=0}^{s}G_{t+1+j}\right)p_{t+1+s}^k\right]. \end{aligned}$$

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