AN OVERVIEW OF MIP MODELING ALGORITHMS WITH SETUP TIMES & CAPACITY MANAGEMENT ROUTINES

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ABSTRACT

This paper surveys lot-sizing and scheduling models emphasizing single and multiple stage production processes. It includes capacity – constrained and uncapacitated applications at different manufacturing stages. Instances of the single and multi-item lot-sizing problems with setup times often appear in practice, either in standard form or with additional constraints, but they have generally been difficult to solve to optimality. A system has been developed for the manufacturing environment, dealing with multiple products and having multiple assembly and sub-assembly lines capable of assembling more than one model and a manufacturing facility that produces the required parts is a multi-stage, multi-machine assembly line such as parallel machines or even parallel multistage machines. In this paper, formulations and solution procedures for different stages of manufacturing processes have been discussed and evaluated. Assumptions and variables are varied with respect to their algorithms.

I. BACKGROUND AND METHODOLOGIES

Capacity has several definitions such as design capacity, effective capacity, excess capacity, short-term capacity, long-term capacity, and constrained capacity. Most researchers (Krajewski & Ritzman, 2001); (Meredith & Shafer, 2001); (Heizer & Render, 2003) agree that capacity is the ability to produce products or services, and effective capacity results, after considering the available factors of production and limitations imposed by product design and the process design. Process constraints limit the actual effective capacity of a production process. Included in these process constraints are raw material availability, plant location, plant layout, design and method of processing machinery, skill levels and training of the process operators relative to the learning curve or manufacturing progress function (Meredith & Shafer, 2001), system or preventive maintenance strategy (Nicholas, 1998) and replacement plans for obsolete and inefficient processes. Constrained capacity is effective capacity which is less than or equal to demand, and which limits the production output of the system. The limits on constrained capacity are caused by disruptions in product or service design including the product design itself, product quality requirements, required volume, location, price, or any possible combination of these. Models and applications of the Continuous Time Lot-sizing and Scheduling Problem (CTLSP), including the Batching and Scheduling problem (BSP), it can be found in (Belvaux & Wolsey, 2000), (Drexl & Kimms, 1997). The proportional lot-sizing and scheduling problem occurs when the CSLP model does not use the full capacity of a period. In order to produce a certain item in a certain period of time, it is necessary to have the machine already configured either at the beginning or at the end of that period. Since the work of Trigiero, Thomas, and McClain, many other researchers have tried to find near-optimal solutions for MCL using heuristic methods. A multi-stage lot sizing problem involving multiple capacitated resources which are shared by different products is presented. The complexity of the problem is increased by various soft and hard constraints which model real-life planning situations.

Multiple Products with Multiple Capacitated Batches

Among the first to try to solve MCL were (Trigiero, Thomas, and McClain, 1989), who used a heuristic that employs Lagrangean relaxation to obtain near optimal solutions to MCL. Since the Lagrangean solutions they obtained were not always feasible, they used a production smoothing heuristic that sought to shift production from the Lagrangean solution in order to obtain a feasible production plan. They were able to solve many of their test problems this way; however, with problems that had a tight capacity restriction, they were not always able to find a feasible solution. (Pochet & Wolsey,1991) and (Belvaux & Wolsey, 2000) have solved instances of MCL and related problems to optimality by strengthening the LP formulation with valid inequalities and then invoking a branch-and-bound algorithm. There are two obvious advantages of using such an approach. The first is that the algorithm, if it has time to terminate, finds a provably optimal solution. The second is that a feasible solution is found if one exists; this characteristic is not shared by the many heuristic methods (such as that proposed by Trigiero, Thomas, and McClain, 1989). A disadvantage of such an optimization approach is that it can require much time and memory, possibly an indefinite amount of both; however, this disadvantage has been mitigated somewhat in recent years by advances in computer technology and mathematical programming theory.

$$\text{Minimize } \sum\sum h_{it} \,.I^{+}_{it} + g_{it}\, I_{it}^{-} \qquad i = 1,\ldots,P; t = 1,\ldots,T \qquad (1)$$

$$I^{+}_{i,\,t-1} - I^{-}_{i,\,t-1} + X_{it} - I^{+}_{it} + I^{-}_{it} = D \qquad (2)$$

I^{+}_{it} and I_{it}^{-} represent respectively the inventory and the backlog of product i at the end of period t and X_{it} represents the quantity of product i produced in period t. The data $\mathbf{D_{it}}$ are the demand of product i at the end of period t and h_{it} the unit invent any holding cost of product i at the end of period t. Constraint (3) represents the capacity version of the problem u_i is the capacity need for one unit of product i and CAP_t is the total capacity of machine at period t. $\sum u_i .\, X_{it} \leq CAP_t$ (3)

Capacitated Format of Multiple Products

The Multiple-Family ELSP with Safety Stocks: The ELSP with normally distributed, time-stationary demand is considered in a manufacturing setting where the relevant costs include family setup costs, item setup costs, and inventory holding costs for both cycle and safety stocks. A family is a subset of the items that share a common family setup with its associated setup cost and setup time. Each item within the family may have its own setup time and setup cost. The families form a partition of the set of items manufactured on a single facility. The Multiple-Family ELSP with safety stocks differs from multi-level inventory models with family setups in that the former assumes non-instantaneous inventory replenishment and considers the cost of holding safety stocks; the latter assumes instantaneous replenishment and does not directly assess the impact of safety stock levels on the total cost. The solution to the mathematical model is comprised of the basic period length, the family multipliers, and the item multipliers that give the lowest total cost of setups and carrying inventory. The family multipliers and items multipliers are restricted to integer powers of two. An efficient solution procedure is developed for this model. Properties of the non-convex feasible space are identified and used in the solution approach. For a truly optimal solution, the product volume and product mix decisions must be coordinated with the economic run decisions to ensure that profit is maximized overall. To minimize the total of set-up and holding costs, the model in (Drexl & Kimms, 1997) is:

$$\text{Minimize } \sum_{t=1}^{P} \sum_{t=I}^{T} \boldsymbol{s_i . Y_{it} + h_i . I_{it}} \qquad i=1,...,P; t=1,...,T \quad (4)$$

Subject to

$$\boldsymbol{I_{it} = I_{i,\, t-1} + X_{it} - D_{it}} \qquad i=1,...,P; t=1,...,T \quad (5)$$

$$u_i . X_{it} \leq CAP_t\, \mathbf{Y_{it}}; \qquad i=1,...,P; t=1,...,T \quad (6)$$

$$\sum u_i . \boldsymbol{X_{it}} \leq CAP_t \qquad i=1,...,P; t=1,...,T \quad (7)$$

$$\boldsymbol{Y_{it} \in (0,1)} \qquad i=1,...,P; t=1,...,T \quad (8)$$

$$\boldsymbol{I_{it}} , \boldsymbol{X_{it} \geq 0} \qquad i=1,...,P; t=1,..., T \quad (9)$$

CAP_t / u_i is used as an upper bound on $\boldsymbol{X_{it}}$ in expression (6). A unit production cost $\boldsymbol{c_i}$ can be also be inserted in the objective function, as follows:

$$\text{Minimize } \sum_{t=1}^{P} \sum_{t=1}^{T} \boldsymbol{s_i . Y_{it} + h_i . I_{it} + c_i X_{it}} \qquad i=1,...,P; t=1,...,T \quad (10)$$

Modeling of Time Periods: Time and Other Capacity Dimensions

Capacity is a time-related resource, if a unit of capacity for the immediate hour is not used this hour, it becomes a forgone, non-retrievable resource, and much of the attendant costs are incurred whether or not the capacity to produce is used. If this process causes a bottleneck, or capacity constrained resource (CCR) (Chase et al. 2003), then the idle time of the processes before and after the bottleneck is also forgone. The only savings attributable to this unused capacity are the deferred wages of process operators and possible ‘wear and tear’ on equipment. Two terms very much used in Lot-sizing Problems, are *small* and *big (or large) bucket*, as defined in (Belvaux & Wolsey, 2000). The CLSP is called a large bucket (or big bucket) problem because several items may be produced per period. The case where the (macro-periods) are subdivided in several micro-periods leads to the Discrete Lot-sizing and Scheduling Problem (DLSP), called a small bucket problem, in (Drexl & Kimms, 1997; Belvaux & Wolsey, 2000), because at most one item can be produced per period. The DLSP has the same objective function as the CLSP, but constraints (7) and (8) need to be replaced by:

$$u_i . \boldsymbol{X_{it}} = CAP_t\, \boldsymbol{s_{it}} \qquad i=1,...,P; t=1,...,T \quad (11)$$

and following the constraints:

$$\sum \boldsymbol{s_{it}} \leq 1 \qquad i=1,...,P; t=1,...,T \quad (12)$$

s_i is the set-up cost for item i and h_i is a holding costs of product i.

$$\boldsymbol{Y_{it} \geq s_{it} - s_{i,t-1}} \qquad (13)$$

where $\boldsymbol{s_{it}} \in \{0,1\}$ indicates whether the machine is configured for item *i* in period *t* ($\boldsymbol{s_{it}}=1$) or not ($\boldsymbol{s_{it}} = 0$) and $\boldsymbol{s_{it}} \in \{0,1\}$, $Y_{it} \in \{0,1\}$ now indicates the start-up of a lot of item *i* in period *t*. The 'all-or-nothing' assumption of the DLSP comes in via equation (11), where, in contrast to CLSP, the left- and right-hand sides must be equal. Constraint (12) makes sure that at most one item can be produced per period and that with (11), capacity limits are taken into account. The start-up of a new lot is spotted by the inequality (13). In the Continuous Set-up Lot-sizing Problem (CSLP) constraint (11) of the DLSP is replaced by

$$u_i.\, \boldsymbol{X_{it}} \leq CAP_t\, \boldsymbol{s_{it}} \qquad i = 1,...,P; t = 1,...,T \qquad (14)$$

Constraint (11) of the DLSP forces the production system to produce to its full capacity, whereas constraint (14) of the CLSP allows the system to produce under its full capacity. The Proportional Lot-sizing and Scheduling Problem (PLSP) occurs when the CSLP model does not use the full capacity of a period. The basic idea of PLSP is to use remaining capacity for scheduling a second item in the particular period, the PLSP can be observed by:

$$u_i.\, \boldsymbol{X_{it}} \leq CAP_t\, (\boldsymbol{s_{i,t-1}} + \boldsymbol{s_{it}}) \qquad i = 1,...,P;\ t = 1,...,T \qquad (15)$$

it replaces (11) and also includes

$$\sum \boldsymbol{u_{it}}\, \boldsymbol{X_{it}} \leq CAP_t \qquad i=1, ... P,\ t = 1,...,T \qquad (16)$$

In equation (24) we see that in order to produce a certain item in a certain period of time, it is necessary to have the machine already configured either at the beginning or at the end of that period. Once it has been configured the total capacity requirement per period has its limit in equation (16). In (Drexl & Haase, 1995) the PLSP with set-up times (PLSPST) is presented where the objective function is expression (13). The capacity constraint now include is set-up time $\boldsymbol{w_{it}}$

$$\sum(\boldsymbol{u_{it}}\, \boldsymbol{X_{it}} + \boldsymbol{w_{it}}) \leq CAP_t, \qquad i=1, ... P,\ t = 1,...,T \qquad (17)$$

The General Lot-Sizing and Scheduling Problem (GLSP), proposed by (Drexl & Kimms, 1997; features multiple products, single-machine sequence-dependent set-up costs, small bucket time, but with no set-up times nor backlogging. The macro-periods t are each divided into a fixed number of non-overlapping micro-periods with variable length, where $S,$ denotes the set of micro-periods s belonging to the macro-period t and all micro-periods are ordered in the sequence s=1,..., S. Two models are provided in, the first being GLSP-CS (Conservation of Set-up State) and the second GLSP-LS (Loss of Set-up State). The mathematical model for GLSP-CS is as follows:

$$\text{Minimize} \sum_{Ijs} \boldsymbol{s_{ij}}.\boldsymbol{z_{ijs}} + \sum_{i,t} \boldsymbol{h_i}.\ \boldsymbol{I_{it}} \qquad i = 1,...,P; t = 1,...,T \qquad (18)$$

Subject to

$$\boldsymbol{I_{it}} = \boldsymbol{I_{i,\, t-1}} + \sum \boldsymbol{X_{it}} - \mathbf{D_{it}} \qquad i = 1,...,P; t = 1,...,T,\ s€S_t \qquad (19)$$

$$\sum a_i\, \boldsymbol{X_{is}} \leq K_t \qquad i, s€S_t \qquad (20)$$

$$a_i\, \boldsymbol{X_{is}} \leq K_t\, \boldsymbol{Y_{is}} \qquad (21)$$

$$\boldsymbol{X_{is}} \geq \boldsymbol{m_i(Y_{is} - Y_{i,s-1})} \qquad (22)$$

$$\sum \boldsymbol{y_{in}} = 1 \qquad (23)$$

$$\boldsymbol{z_{ijs}} \geq \boldsymbol{Y_{i,s-1}} + \boldsymbol{Y_{j,s-1}} - 1 \qquad (24)$$

The variables are: $I_{it} \geq 0$ is the inventory of product i at the end of macro-period t, $\boldsymbol{X_{is}}$ is the production of quantity of item i in micro period s; $y_{is} = 1$, if the machine is set-up for product i in the micro-period s and 0 otherwise; $z_{ijs.} = 1$ if the changeover of the product i to product j take a place in the beginning of micro-period s and 0 otherwise. The parameters are: k_t is the capacity (time) available in macro-period t, a_i

is a capacity consumption (time) needed to produce one unit of product m_i is the minimum lot size of product i , $\boldsymbol{s_{ij}}$ is the cost of a changeover from product i to product j, $\boldsymbol{I_{i0}}$ is the initial inventory of product i at the beginning of the planning horizon. Y_{i0}= 1 if the machine is already set-up for product i at the beginning of period 1 and 0 otherwise. Constraint (22) enforces minimum lot sizes in order to avoid set-up changes without product changes, avoiding an incorrect calculation of set-up costs/times in an optimal solution if set-up costs/times do not satisfy the triangle inequality ($\boldsymbol{s_{ik}} + \boldsymbol{s_{kj}} \geq \boldsymbol{s_{ij}}$) as can occur in the chemical industry, constraint (20) is modified to include set-up times:

$$\sum a_i \boldsymbol{X_{is}} + \sum \boldsymbol{st_{ij}}\, \boldsymbol{z_{ijs}} \leq K_t \qquad (25)$$

Algorithms for the Two-Stage System

Any optimal policy for the two-stage system is such that the set is bounded. If this property holds true then we need to search the optimal policy among a finite number of distinct fixed policies. The most naive way for searching for the optimal policy over the set is by evaluating the expected cost of all fixed policies within the set. Grosfeld-Nir (2005) considered set policies for the two-stage problem with binomial yields: For any demand level D, a control-limit CD is associated, so that production takes place only if Work in Process≥ CD. Two terms very much used in Lot-sizing Problems, are small and big (or large) bucket, which makes a distinction: "between "Big Bucket" models having long time periods in which several items can be set up and produced and "Small Bucket" models have short time periods in order to be able to model start-ups, switch-offs and/or changeovers. The "Small Bucket" models are then split further into those in which only one item can be set up per period and those with possibly two set-ups per period". The CLSP is called a large bucket (or big bucket) problem because several items may be produced per period. The case where the macro-periods are subdivided in several micro-periods leads to the discrete lot-sizing and scheduling problem, called a small bucket problem.

Single-Bottleneck System with Binomial Yields (SBNS)

A stage with non-zero setup cost as a bottleneck (BN) and to a system with only one BN as a single-bottleneck system (SBNS). Similarly, a system with no BNs (all setups are zero) is referred to as a “zero-bottleneck system” (0-BNS). Typically, a SBNS consists of two 0-BNS and the BN. When a binomial 0-BNS faces a rigid demand *D*, it is optimal to process units one at a time until the demand is satisfied. The resulting expected cost is mD, where *m*, the minimal expected cost to satisfy a demand of one unit. When a binomial SBNS faces a rigid demand *D*, it is optimal to process units one at a time on the first 0-BNS until a certain batch size is ready to be processed on the BN. These units are then processed in one batch on the BN, and, finally, the usable units exiting the BN are processed, one at a time, on the second 0-BNS until the demand is satisfied or all units are exhausted. Therefore, the problem of optimally controlling a SBNS is completely characterized by the optimal lot to be processed by the BN (Chase et al. 2003).

II. SUMMARY

The lot-sizing problem is formulated as a mixed integer liner program and uses the concept of variable redefinition to solve it. Using this technique one can solve large MIP problems to optimality or near optimality. In the job shop scheduling, each job has its own unique route and also, a given job can visit the same machine type, for subsequent operations. A stage functions as a bank of parallel machines; at each stage a job requires only one machine and usually any machine can process any job. This job oriented scheduling procedure constitutes generation on of an initial schedule by backward scheduling, determination of core operation based on initial schedule and adjustment of other operations of the job by rescheduling them close to the core operation. Logically, the backward scheduling is considered first, followed by an adjustment procedure and then does the forward schedule for non-critical operations. This has the effect of achieving the due date and at the same time finishing the job with minimum in process time.

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