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CAPITAL REGULATION AND BANK RISK TAKING: COMPLETING BLUM'S PICTURE

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Resumen

Este trabajo estudia los efectos intertemporales que los requerimientos de adecuación de capital tienen sobre las decisiones de toma de riesgo de la banca, usando el modelo seminal propuesto en Blum (1999). Se calculan valores umbrales en cada periodo, a partir de los cuales la regulación de capital comienza a afectar la toma de riesgo de los bancos. Una importante lección obtenida de este ejercicio es que requerimientos de capital constantes (como se establece en Basilea I) son de hecho capaces de reducir el riesgo por debajo de niveles no regulados, e incluso alcanzar el óptimo social sin costos de bancarrota. No obstante, eso podría ocurrir para valores muy altos del requerimiento, y al costo de reducir la intermediación financiera. Puesto que la dinámica de riesgo de- pende de estos valores umbrales, que a su vez dependen de la capitalización inicial de cada banco, una segunda lección es que el conocimiento del nivel de capitalización de los bancos es esencial para el regulador. Otros instrumentos de mercado y una supervisión efectiva por parte del regulador (como propone Basilea II), podrían ser útiles para alcanzar este objetivo.

Abstract

This paper studies the intertemporal effects that capital regulation has on curbing bank risk taking, using the seminal model proposed in Blum (1999). Threshold values of the requirement in each period, for which capital regulation start affecting bank risk taking decisions, are calculated. One main lesson from this exercise is that constant capital requirements (as considered in Basel I) are indeed capable of reducing risk taking below the unregulated solution, and can even achieve the zero bankruptcy cost, socially efficient level of risk. However, that might happen for very high levels of the requirement, and at the cost of reducing financial intermediation. A second important lesson is that as the dynamic of risk depends on these thresholds, and they in turn depend upon the initial equity of the bank; knowing the latter is essential for the regulator to determine the effectiveness of capital regulation. Additional market instruments and effective monitoring and supervision (as proposed in Basel II) could be helpful on this task.

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1 Introduction

The Basel Committee was established at the end of 1974 by the central bank Governors of the Group of Ten countries, with the aim of gathering central bankers and bank supervisors and regulators to discuss issues related to prudential banking supervision. As a result of these talks, in 1988 emerged the first version of the Basel Capital Accord, introducing a common minimum 8 percent risk-weighted capital to asset ratio (the so called *Cooke ratio*) for internationally active G-10 banks, which in the earliest version only considered credit risk. Although some countries had adopted minimum requirements before the agreement (the USA and the UK in 1981, for example), it was only after the agreement that capital requirements became common ground for the banking industry worldwide.[2]

The objective of this form of regulation is to strengthen the soundness and stability of the international banking system, and to reduce competitive inequalities across markets. However, some theoretical results suggest that banks may have found ways of overcoming the limitations that fixed capital requirements impose on their risk taking relative to their capital, either through asset substitution (Koehn and Santomero (1980), Kim and Santomero (1988), Flannery (1989), Rochet (1992)), the reduction of monitoring incentives (Bensako and Kanatas (1993), Boot and Greenbaum (1993)) or through substantial volumes of securitization[3] (Jones (2000)).

The empirical evidence as to whether capital requirements reduce the probability of default or induce banks to increase risk taking in some periods is not conclusive. In a study for 98 USA banks over the period 1975-1986, Furlong (1988) inverts the Black and Scholes (1973) pricing formula to infer the volatility of the portfolio assets of banks. He concludes that volatility was higher after the introduction of capital requirements in 1981, though it grew both for badly and well capitalized banks. In a different study over 219 G-10 banks in the period 1987-1994, Sheldon (1996) finds that while the volatility on US banks increased in the period, independent of the level of the capital requirement, that of Japanese banks fell as capital ratios rose.

On the theoretical side the picture is blurry too. The work of Kahane (1977), Kareken and Wallace (1978) and Sharpe (1978), justify the use of capital requirements to control the solvency of banks which asset allocation is distorted by the presence of deposit insurance, but both assume complete markets. Under an incomplete market approach, Koehn and Santomero (1980) and Kim and Santomero (1988), using a mean variance portfolio model with fixed liabilities, prove that in the absence of a solvency requirement and abstracting from the limited liability clause, the probability of bank failure is a decreasing function

[2] In 1993 all commercial banks in the European union were subject to a common solvency requirement. By 1999 the Basel capital accord was being implemented in about 100 countries. Indeed, since the introduction of the capital accord, risk weighted capital ratios in developed countries have increased significantly. Nonetheless, it is has to be established whether this respond to regulation itself or to increased market discipline (Jackson (1999)).

[3] These are techniques used by banks to mitigate the credit risk to which they are exposed, as collateral, guarantees and credit derivatives.

of its capital ratio, which is independent of the (non-negative) weights used in the computation of the ratio. However, the introduction of capital requirements changes the asset allocation of the bank, so that while the volume of the risky portfolio decreases (because the bank shifts to those assets within a lower weight category), its composition is distorted in the direction of more risk (inside the chosen weight category), increasing the probability of failure. As a way of correcting this problem, they propose the introduction of risk weights proportional to the systemic risk of the assets.

Since then, the literature has given a lot of attention to market based refinements on risk weights.[4] For example, Thakor (1996) shows how a bad selection of risk weights could have a negative impact on the real sector through credit crunches, given that the asset allocation of a bank can be distorted by the difference between market and regulatory assessments of asset risks. Furfine (2001) uses a panel of large US banks between 1990-1997, and a structural dynamic model of bank behavior to show that the credit crunch in the USA in the 1990s could be explained by increasing non-market based risk weighted capital requirements and excessive regulatory monitoring, instead of pure demand effects.

However, when limited liability is taken into account, Rochet (1992) shows that even with the correct weights, capital requirements are not enough to control for moral hazard and additional regulation, in the form of minimum levels of capital, independent of the size of the assets, may be needed.

Furlong and Keeley (1989) advocate capital requirements, arguing that when limited liability and the option value of (flat) deposit insurance are properly taken into account, a bank that maximizes the value of its stock, and therefore diversifies its portfolio, will always reduce risk with more stringent capital requirements. The same result is obtained by Santos (1999), in a model that considers asymmetric information between the bank and the borrowing firm, and the distortions induced by the presence of deposit insurance on the optimal funding contract. More stringent capital requirements make the bank to ask for a (larger) equity stake in the firm, which in turn induces the firm to lower its risk, reducing the bank's probability of default.

Nonetheless a static framework fails to capture important intertemporal effects that capital requirements might have on the behavior of banks. One of the first theoretical models studying the intertemporal effects of capital constraints is given by Blum (1999). In a discrete time model he studies the incentives for asset substitution coming from the reduction in expected profits imposed by the requirement. In order to raise the amount of equity in the following period, a bank may find it optimal to increase risk today, in which case strengthening the requirement would have the opposite effect for which it was designed, to curb bank risk taking.

Following contributions, most of them in continuous time, study the combined effects of capital regulation and the two additional pillars of Basel II

[4] Compare, for example, the great deal of attention devoted to it in Basel II. Almost 60% of the document goes about the way of calculating appropriate risk weighted capital ratios (Pillar I), both in a standardized and non-standardized fashion (see BIS 2005).

(Battacharya et al. (2002), Decamps et al. (2003), Rochet (2004)). But that will be the topic of a different study. In this paper I will build on Blum's model to obtain some important lessons neglected in his original work. While keeping his basic assumptions, I will explicitly make use of some optimization and calculus techniques that allow for a formal proof of some intuitive results. Although his main results are confirmed, I am able to identify threshold values for the requirement in each period which determine the effectiveness of capital regulation in an intertemporal framework. Indeed, depending on the relationship between these threshold levels it is possible to compute values of the requirement for which the risk chosen by the bank converges to the zero bankruptcy cost, social optimum. Both thresholds and the optimal regulation are shown to depend critically on the initial equity of the bank. The paper is subtitled "completing Blum's picture". It is not a new model, but it brings into the scene some important pieces overlooked in Blum's paper.

The paper is organized as follows. Section 2 sets up the basic three periods model for a regulated bank, and establishes an upper bound for the social efficient level of risk. Section 3 studies the equilibrium when capital requirements are slack, which is equivalent to the risk choice of an unregulated bank. When the bank chooses a higher exposure to risk than socially efficient, capital regulation may be justified by the decreasing behavior of risk with respect to initial equity. Threshold values of capital requirements for which they just start to bind are also calculated. Section 4 studies the equilibrium choice of risk when capital requirements bind, given three scenarios: when capital requirements bind only in the initial period, when they bind only in the intermediate period, and when they bind in both periods. While this is the same scheme followed by Blum (1999), the main innovation here is to show that for known values of initial equity, the regulator would be able to compute threshold values of capital requirements and, as a function of them, the value of the requirement capable of achieving a lower or, whether possible, the social optimum level of risk.

Finally, a discussion of the main results and possible extensions are given in section 5.

2 The Model

Consider a bank operating in an economy over three periods $t \in \{0, 1, 2\}$, with an exogenous initial equity of W_0. The bank manager is risk neutral and acts perfectly in the interest of shareholders (so there is no agency problems), which implies she maximizes the expected value of equity.

A safe asset is available in periods 0 and 1, which gross rate of return is normalized to one. That is, for each unit of consumption invested at t, this technology returns 1 unit at $t+1$. At $t = 0$ there is also a risky portfolio, which risk-return structure can be influenced by the bank, that with probability $p(R)$ returns R units at $t = 1$ per unit invested at $t = 0$, and zero otherwise. The probability function, $p(.)$, satisfies $p(1) = 1$, $p'(R) < 0$ and $p''(R) \leq 0$ for all $R \geq 1$. The safe asset is (weakly) dominated by this technology[5] if, in accordance with finance theory, there is a range of values (though eventually small) where a positive trade-off exist between risk and expected return. With the assumptions above, the expected return of the risky portfolio, $p(R)R$, is strictly concave for $R \geq 1$, and corner solutions with infinite risk are ruled-out. The unique level of risk that maximizes this expected return function is given by

$$p'(R^*)R^* + p(R^*) = 0,$$

where $R^* > 1$ if $p'(1) + p(1) > 0$.

For the sake of tractability, at $t = 1$ only one "risky" project is available, returning $\overline{R} > 1$ with probability 1. Even though it would be desirable to replicate period 0 structure of the risky asset in the intermediate period, the model would become analytically intractable. While this simplification eliminates the incentive for assets substitution in the intermediate period, it also allows us to concentrate on the risk choice of the bank at $t = 1$. Moreover, it is realistic to assume that at $t = 0$ the bank manager does not know the full spectrum of investment possibilities available to her in the following periods, though she might have an idea of their average value, $\overline{R}$, which is what is assumed here.

Finally, assume the bank is able to raise fully insured deposits D_t in period t, that cost $C(D_t) \geq D_t$ at $t + 1$, an strictly increasing and convex function that satisfies $C(0) = 0$ and $C'(0)$ bounded.[6] The assumption of full deposit insurance, in a model of complete information, makes the demand for deposits independent of the level of risk chosen by the bank. In other words, depositors are risk neutral. The assumption of universal risk neutrality is useful here to separate risk effects due to risk choice from those due to risk aversion of agents.

2.1 First best

Given the assumption of universal risk neutrality, without lost of generality assume the utility function of the representative agent is $U(y) = y$. Assume consumption is postponed until $t = 2$, and that in each period there is an endowment of M_t, which society needs to allocate between the risk free and risky technologies. Let me call $x_0 \leq M_0$, the amount invested in the risky asset at $t = 0$. If the project succeed, which happens with probability $p(R)$, society will have a wealth of $(M_0 - x_0) + x_0 R$ at $t = 1$; while if the project failed, with probability $1 - p(R)$, the wealth of society will only be $M_0 - x_0$.

[5] Clearly, all projects with $R < 1$ are dominated by the safe asset.

[6] The cost function $C(.)$ can be thought of as the gross interest rate paid on deposits in each period. Assuming the function $C(.)$ is equal in both periods may be a strong simplification, given that, in this model, the risk borne by depositors is higher in the initial period.

The convexity assumption can be justified by incomplete competition arguments. In Blum's own words *"If banks are horizontally differentiated, they each enjoy a local monopoly. If they want to attract more deposits, they have to raise interest rates to capture a greater market share. Doing so the bank not only incurs the cost of these marginal deposits, but also raises the cost on all infra-marginal deposits. Hence the cost of deposits are rising at an increasing rate"*.

A new endowment of M_1 is realized at $t = 1$, which is fully invested in $\overline{R} > 1$ (because that technology is dominant). Hence with probability $p(R)$, society will have a wealth of $[(M_0 - x_0) + x_0 R + M_1]\,\overline{R}$ at $t = 2$, and with probability $1 - p(R)$, a final wealth of $[M_0 - x_0 + M_1]\,\overline{R}$ for consumption.

A risk neutral social planner then maximizes

$$\max_{x_0,R} p(R)\,[(M_0 - x_0) + x_0 R + M_1]\,\overline{R} + (1 - p(R))\,[M_0 - x_0 + M_1]\,\overline{R}$$
$$\text{st. } x_0 \leq M_0$$

or equivalently

$$\max_{x_0,R} x_0 \overline{R}\,[p(R)R - 1] + [M_0 + M_1]\,\overline{R}$$
$$\text{st. } x_0 \leq M_0$$

For the region where the risky technology is dominant ($p(R)R \geq 1$) this function is increasing in x_0, then at the optimum $x_0 = M_0$.

The first order condition for this problem is : $p'(R)R + p(R) = 0$, and as the objective function is concave, this condition is sufficient for a social optimum.

Of course, the previous exercise has not considered the social cost of bank failure, understood as the forgone value of intermediation, or the cost borne by the deposit insurance agency in case of failure. Therefore, absent bankruptcy costs, social returns should equal private profits and a risk neutral social planner should choose the level of risk that maximizes expected returns, i.e., R^*.

When considering bankruptcy costs[7], the social efficient level of risk would be lower than private optimal levels of risk . However, given than R^* proposes a simpler framework for comparison, in the remaining of this paper when referring to the socially efficient level of risk, I will be talking about R^*, the zero bankruptcy cost, socially efficient level of risk, keeping in mind this is an upper bound for the social optimum.

2.2 Capital Requirements

Capital requirements limit the resources that can be invested in the risky technology -though any remaining funds can be invested in the safe asset without restrictions. For a regulated bank, a capital requirement c_t on its original formulation (the Cooke ratio) imposes that capital over risk weighted loans should be at least of an 8 percent ($0.08 \leq c_t \leq 1$). Denoting by $I_t \leq W_t + D_t$, the investment in the risky portfolio in period t, capital requirements in this model translate to:[8]

[7] A more general way of stating this problem would be to consider bankruptcy costs as a convex and increasing function of R, $\phi(R)$. The conditional expected return of the risky portfolio would then be $p(R)R - (1 - p(R))\phi(R)$.

The FOC would be given by: $p'(R)R + p(R) + p'(R)\phi(R) - (1 - p(R))\phi'(R) = 0$ or $p'(R^*)R^* + p(R^*) = (1 - p(R^*))\phi'(R^*) - p'(R^*)\phi(R^*) \geq 0$, which implies that, because marginal returns are decreasing, by including bankruptcy costs risk would be reduced.

[8] By convention, risk free assets have zero weight. The definition of capital requirements considered here only takes into account credit risk. It also assumes that the risky assets are

$$\frac{W_t}{I_t} \geq c_t.$$

Clearly, for a given level of equity at any period, the more stringent the requirement (the higher the value of c_t) the lower the allowed investment in the risky portfolio. Capital regulation is usually presented as a natural counterpart to deposit insurance, in an attempt to control for moral hazard on the banking industry, that is implicitly receiving a subsidy from the government.

The expected equity of the bank in each period is given by the return of the investment in the risky asset, plus the return of any remaining funds invested in the safe technology, minus the cost of deposits; provided the bank had survived to that period (figure 1). Otherwise, and because of limited liability, all remaining resources are transferred to the deposit insurance agency and the bank closes down (in other words, its equity equals 0).[9] Therefore, the bank's expected equity in each period is given by:

$$\begin{aligned}
&W_0 \\
&\mathbb{E}[W_1] = p(R)\left\{I_0 R + (W_0 + D_0 - I_0) - C(D_0)\right\} \\
&\qquad\quad = p(R)\left\{I_0 (R-1) + W_0 - (C(D_0) - D_0)\right\} \\
&\mathbb{E}[W_2] = p(R)\left\{I_1 \overline{R} + (W_1 + D_1 - I_1) - C(D_1)\right\} \\
&\qquad\quad = p(R)\left\{I_1 \left(\overline{R} - 1\right) + W_1 - (C(D_1) - D_1)\right\}
\end{aligned}$$

The regulated, risk neutral bank manager maximizes the expected value of final equity, subject to capital constraints and standard feasibility conditions for investment in each period:[10]

$$\begin{array}{llll}
(P) & \max\limits_{R,D_0,I_0,W_1,D_1,I_1} & p(R)\left\{I_1\left(\overline{R}-1\right) + W_1 + D_1 - C(D_1)\right\} & \\
& st. & & \\
& (1) & I_0 (R-1) + W_0 + D_0 - C(D_0) - W_1 \geq 0 & (\theta) \\
& (2) & W_0 + D_0 - I_0 \geq 0 & (\lambda_0) \\
& (3) & W_0 - c_0 I_0 \geq 0 & (\mu_0) \\
& (4) & W_1 + D_1 - I_1 \geq 0 & (\lambda_1) \\
& (5) & W_1 - c_1 I_1 \geq 0 & (\mu_1)
\end{array}$$

weighted 100%, although the new proposed amendment of the capital accord includes weights as high as 350% (BIS (2005)). Under Pillar I of Basel II, a justification for the assumption made here would be for the assets in the risky portfolio to be unrated.

[9] If the bank fails at $t = 1$ (which happens with probability $1 - p(R)$), because of limited liability its equity is $\max\left\{(W_0 + D_0 - I_0) - C(D_0), 0\right\}$.

However, it is always the case that $(W_0 - I_0) + (D_0 - C(D_0)) \leq 0$. The first term is negative because the risky technology is weakly dominant, therefore investment in the safe asset is effective only when capital requirements are binding, that is, if $I_0 = \frac{1}{c_0} W_0 > W_0$. The second term is also negative, because as I said before by assumption $C(D_0) \geq D_0$.

[10] Non negativity constraints in all the variables, in order to rule out the possibility for a short sale of assets, should also be considered. In order to simplify the algebra, instead of explicitly including them, I will check they are satisfied at the optimum (see appendix).

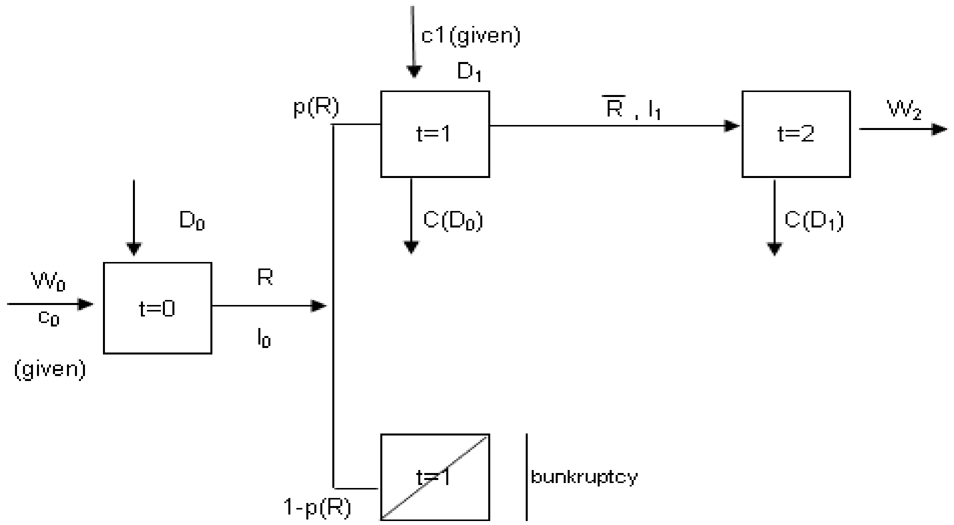

Figure 1: Timeline for the optimal decision of the regulated bank.

where θ is the shadow price of equity in $t = 1$, λ_t is the shadow value of the risky portfolio in period t, and μ_t is the shadow cost of capital requirement in period t.

Sufficient conditions for an optimum are (see appendix):

$$\theta = \begin{cases} p(R^r)\overline{R} & \text{if } c_1 = 0 \\ \dfrac{p(R^r)}{c_1}\left\{\overline{R} + (c_1 - 1)\, C'(D_1^r)\right\} & \text{if } c_1 \neq 0 \end{cases} \tag{1}$$

$$\lambda_0 = \theta\left(C'(D_0^r) - 1\right) \tag{2}$$

$$\mu_0 c_0 = \theta\left(R^r - C'(D_0^r)\right) \tag{3}$$

$$\lambda_1 = p(R^r)\left(C'(D_1^r) - 1\right) \tag{4}$$

$$\mu_1 c_1 = p(R^r)\left(\overline{R} - C'(D_1^r)\right) \tag{5}$$

$$p'(R^r)\left\{I_1^r\left(\overline{R} - 1\right) + W_1^r + D_1^r - C(D_1^r)\right\} + \theta I_0^r = 0 \tag{6}$$

where r stands for the "*regulated*" solution. Notice from equations (3) and (5) of program (P), μ_t are well defined for $c_t = 0$, provided $W_t > 0,\ t = 1, 2$. Second order conditions are checked in the appendix.

First notice that $\theta > 0$ for all R such that $p(R) \neq 0$.[11] As I said before, θ can be interpreted as the shadow price of equity in period one, which is always

[11] $\theta > 0 \Leftrightarrow c_1 > \frac{C'(D_1^r) - \overline{R}}{C'(D_1^r)} \leq 0.$

valuable to the bank that has not gone bankrupt, and is equal to the earnings realized until that period.

The non-negativity of all the multipliers implies that $R^r \geq C'(D_0^r) \geq 1$ and $\overline{R} \geq C'(D_1^r) \geq 1$. Observe that, because $R^r > 1$ and $\overline{R} > 1$, solutions involving $\lambda_0 = \mu_0 = 0$ or $\lambda_1 = \mu_1 = 0$ are not feasible. When money is invested in the safe asset in a determined period, either equation (2) or (4) in program (P) are slack, and the corresponding capital requirement (equation (3) or (5), respectively) should be binding. In such a case, investment in the safe asset takes place because the marginal cost of deposits equals the marginal return of the safe technology in that period. On the other hand, every time a capital requirement constraint is slack, the corresponding investment in the safe asset in that period is nil (because of weak dominance), as in that case the marginal cost of deposits would be strictly higher than the marginal return of the safe technology. Summing up, because the risky technology is weakly dominant, money is invested in the safe asset if and only if capital adequacy requirements in a determined period are binding.

3 The unregulated solution (slack capital adequacy requirements)

When capital requirements do not bind in neither of the two periods, all funds are invested in the risky portfolio ($I_0^u = W_0 + D_0^u$ and $I_1^u = W_1^u + D_1^u$). By complementary slackness $\mu_0 = \mu_1 = 0$, $\lambda_1 > 0$, $\lambda_2 > 0$, and first order conditions become:

$$\theta = p(R^u)\overline{R} \tag{7}$$

$$\lambda_0 = \theta\,(R^u - 1) > 0 \tag{8}$$

$$C'(D_0^u) = R^u \tag{9}$$

$$\lambda_1 = p(R^u)\left(\overline{R} - 1\right) > 0 \tag{10}$$

$$C'(D_1^u) = \overline{R} \tag{11}$$

$$p'(R^u)R^u + p(R^u) = \frac{p'(R^u)}{\overline{R}(W_0 + D_0^u)}\left\{\overline{R}C(D_0^u) - \left(D_1^u\overline{R} - C(D_1^u)\right)\right\} \tag{12}$$

where "u" stands for the non binding case or "*unregulated*" solution. The second order condition relevant to this problem is (see appendix):

$$p''(R^u)W_2^u + 2p'(R^u)\overline{R}\,(W_0 + D_0^u) + \frac{p(R^u)\overline{R}}{C''(D_0^u)} < 0 \tag{13}$$

3.1 Unregulated solution versus first best

Let me compare the risk choice of the unregulated bank with the optimum to society. If $\overline{R}C(D_0^u) < D_1^u\overline{R} - C(D_1^u)$, the RHS of equation 12 would be positive, then $p'(R^u)R^u + p(R^u) > 0 = p'(R^*)R^* + p(R^*)$, and because marginal returns are decreasing, this inequality would imply that the risk chosen by the unregulated bank would be below the efficient level ($R^u < R^*$). This is so because future rents are so high that the banking industry would be rationing credit (reducing risk) in order to increase the probability of getting those rents. In that case other measures, different from minimum capital requirements, would be needed.

Therefore, in the remainder of this paper I will assume (as in Blum (1999)) that future rents are bounded above by period 0 "operational" costs, $\overline{R}C(D_0^u) > D_1^u\overline{R} - C(D_1^u)$, which is equivalent to

$$p'(R^u)R^u + p(R^u) < 0 \text{ or } R^u > R^*.$$

In principle, one would expect the correlation between risk and equity to be negative, because the more capital has the bank, the more is at stake in the event of failure. Consider a quadratic form for the cost function, $C(x) = ax + bx^2$, and a linear probability function $p(R) = \frac{U}{U-1} - \frac{1}{U-1}R$, with support $[1, U]$, satisfying the assumptions of this model.[12] The previous conjecture is confirmed by this numerical example (see figure 2), as the optimal level of risk chosen by the unregulated bank decreases with the level of initial equity. This result can be formally proved as follows.

Proposition 1 *The risk chosen by the unregulated bank is decreasing in the initial level of equity, and converges to the (zero bankruptcy cost) socially efficient level of risk as* $W_0 \to +\infty$.

Proof. For $\dfrac{dR^u}{dW_0} < 0$ see comparative statics in the appendix (section 6.2.1).

Taking limit $W_0 \longrightarrow +\infty$ in equation 12, it is clear that the RHS goes to zero (R^u, D_0^u and D_1^u are bounded), and so the risk chosen by the unregulated bank converges to the social optimum. ■

3.2 Threshold values for capital requirements

Proposition 2 *There exist critical threshold values for capital requirements in each period ($\widetilde{c_0}$ and $\widetilde{c_1}$), depending on the value of initial equity, for which regulatory constraints just start to bind. These thresholds are* $\widetilde{c_0} = \dfrac{W_0}{W_0 + D_o^u}$ *and* $\widetilde{c_1} = \dfrac{R^u\,(W_0 + D_0^u) - C(D_0^u)}{R^u\,(W_0 + D_0^u) - C(D_0^u) + D_1^u}$, *respectively.*

[12] $\int_1^U p(R)dR = 1 \Rightarrow U = 3$, $C(D_t) \geq D_t$ and $C'(D_t) \geq 1\ \forall t \Rightarrow a = 1$, $C''(D_t) > 0 \Rightarrow b > 0$.

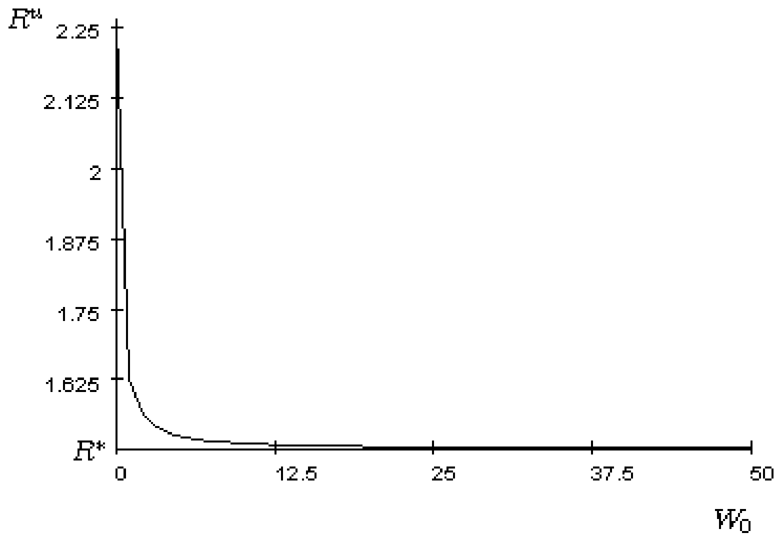

Figure 2: Example of the equilibrium relationship between risk taking and the initial level of equity for the unregulated bank.

Proof. Period 0 requirements do not bind if
$W_0 > c_0 I_0^u = c_0 \left(W_0 + D_0^u\right)$, that is if $c_0 < \widetilde{c}_0 = \dfrac{W_0}{W_0 + D_o^u}$.

Period 1 requirements do not bind if
$W_1^u > c_1 I_1^u = c_1 \left(W_1^u + D_1^u\right)$
$\Rightarrow \left(W_0 + D_0^u\right) R^u - C(D_0^u) = c_1 \left(\left(W_0 + D_0^u\right) R^u - C(D_0^u) + D_1^u\right)$,
that is if $c_1 < \widetilde{c}_1 = \dfrac{R^u \left(W_0 + D_0^u\right) - C(D_0^u)}{R^u \left(W_0 + D_0^u\right) - C(D_0^u) + D_1^u}$.
■

This result is consistent with the intuition that for well capitalized banks the application of small values of capital requirements should be irrelevant; and indeed, with the empirical evidence that capital requirements are slack in the majority of banks in countries that have adopted the Basel principles. Nevertheless, sufficiently tight regulation will eventually force them to modify their capital to asset ratios.

The impact of initial equity (W_0) over the effectiveness of capital requirements in period 0 is clear. Using equations 9 , 11 and 12, total differentiation of $\widetilde{c}_0$ leads to

$$\frac{d\widetilde{c}_0}{dW_0} = \frac{D_0^u}{\left(W_0 + D_0^u\right)^2} \left(1 - \frac{W_0}{D_0^u C''\left(D_0^u\right)} \frac{dR^u}{dW_0}\right) > 0$$

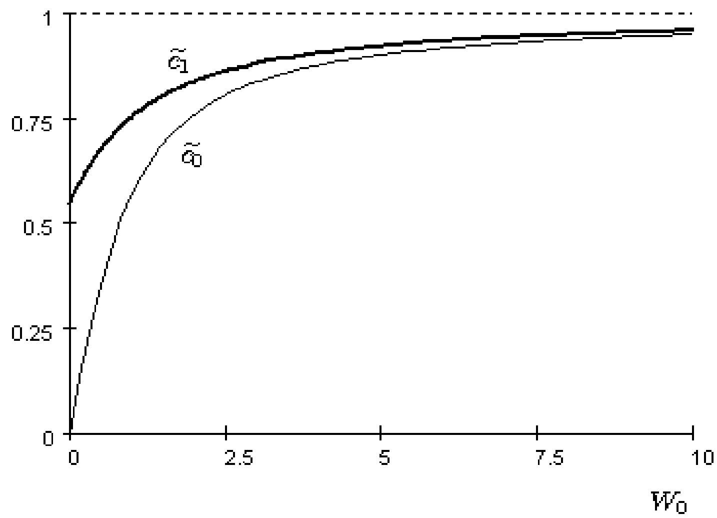

Figure 3: Example of binding capital requirement thresholds as a function of initial equity.

Hence, as risk decreases with initial equity $\left(\frac{dR^u}{dW_0} < 0\right)$, the threshold for which period 0 capital requirements start to bind is increasing, because with a higher equity the bank is at the same time more solvent and less risky. On the other hand, total differentiation of $\widetilde{c}_1$ leads to

$$\frac{d\widetilde{c}_1}{dW_0} = \frac{D_1^u}{\left(W_1^u + D_1^u\right)^2}\left(R^u + (W_0 + D_0^u)\,\frac{dR^u}{dW_0}\right) = \frac{D_1^u R^u}{\widetilde{c}_0\left(W_1^u + D_1^u\right)^2}\left(\widetilde{c}_0 + \frac{W_0}{R^u}\frac{dR^u}{dW_0}\right)$$

Hence,

$$\frac{d\widetilde{c}_1}{dW_0} = \begin{cases} > 0 & \text{if } \left|\dfrac{W_0}{R^u}\dfrac{dR^u}{dW_0}\right| < \widetilde{c}_0 \\ \leq 0 & \text{if } \left|\dfrac{W_0}{R^u}\dfrac{dR^u}{dW_0}\right| \geq \widetilde{c}_0 \end{cases}$$

and the evolution of this threshold will depend upon a sort of “income elasticity” of the demand for risk.

At least for the chosen parameters of the numerical example shown here, both $\widetilde{c}_0$ and $\widetilde{c}_1$ appears to depend increasingly on the level of initial equity (figure 3). Indeed, we can establish that these thresholds goes to 1 as $W_0 \longrightarrow +\infty$, which is consistent with the previous findings.

Finally, given the definitions of $\widetilde{c}_1$ and $\widetilde{c}_0$ and the assumption on R^u,

$$\widetilde{c_1} - \widetilde{c_0} > \frac{1}{I_0^u I_1^u} \left\{ \underbrace{[D_0^u R^u - C(D_0^u)]}_{\geq 0} I_0^u - \tfrac{W_0}{R} C(D_1^u) \right\}.$$

Notice D_1^u is independent of W_0, hence at least for small values of W_0, $\widetilde{c_1} > \widetilde{c_0}$. In particular, for $W_0 = 0$, $\widetilde{c_0} = 0$ and $\widetilde{c_1} = \dfrac{C(D_0^u)}{C(D_0^u) - D_1^u} > 0$, and though capital requirements bind for arbitrarily small values of the Cooke ratio in period 0, a much tighter regulation would be needed in period 1 for it to bind. A general relationship between $\widetilde{c_0}$ and $\widetilde{c_1}$ for general values of W_0, however, cannot be established at this point.

4 Binding Capital Requirements

The main results in Blum's paper establish that tightening the requirement in period 0 leads to less risk, while increasing requirements in the future raises the level of risk above that chosen by the unregulated bank.

Blum (1999) main results: *(i) If a bank faces a binding requirement in the initial period, an increase in the requirement reduces the level of risk.*
(ii) When the capital requirement in the intermediate period first becomes binding, tightening the requirement raises the level of risk. If the requirement is further increased, risk eventually falls again but never below the level of an unregulated bank.

The first part of the result is intuitive, because by rising c_0 the return per unit invested in the risky portfolio is reduced as well (each unit invested at $t = 0$ returns $\dfrac{1}{c_0}R$ units at $t = 1$). This result, however, does not say much about when is the requirements active, how is the regulated equilibrium compared to the unregulated solution, or if the social optimum is attainable at all.

Although these main results will not change in this study, using the information coming from the langrangian multipliers and the thresholds values of capital requirements previously discussed, some interesting conclusions can be drawn. In particular, when only period 0 requirement bind, the socially efficient level of risk can be achieved through a unique value of the capital requirement, which depends increasingly on W_0. This is, the more capitalized the bank, though its risk is smaller, the harder is to bring it to the social optimum through regulation. When constant requirements are active in both periods, the risk chosen by the bank converges to the social optimum, though for very tight values of the requirement, and its dynamic depends on whether $\widetilde{c_1}$ is higher or lower than $\widetilde{c_0}$.

4.1 Initial period binding requirement

A numerical example for the case of a binding requirement in period 0 only, considering the same functions mentioned before, is shown in figure 4.[13] The

[13] Different forms of the probability distribution, that keeps the assumptions of the model, give similar qualitative results.

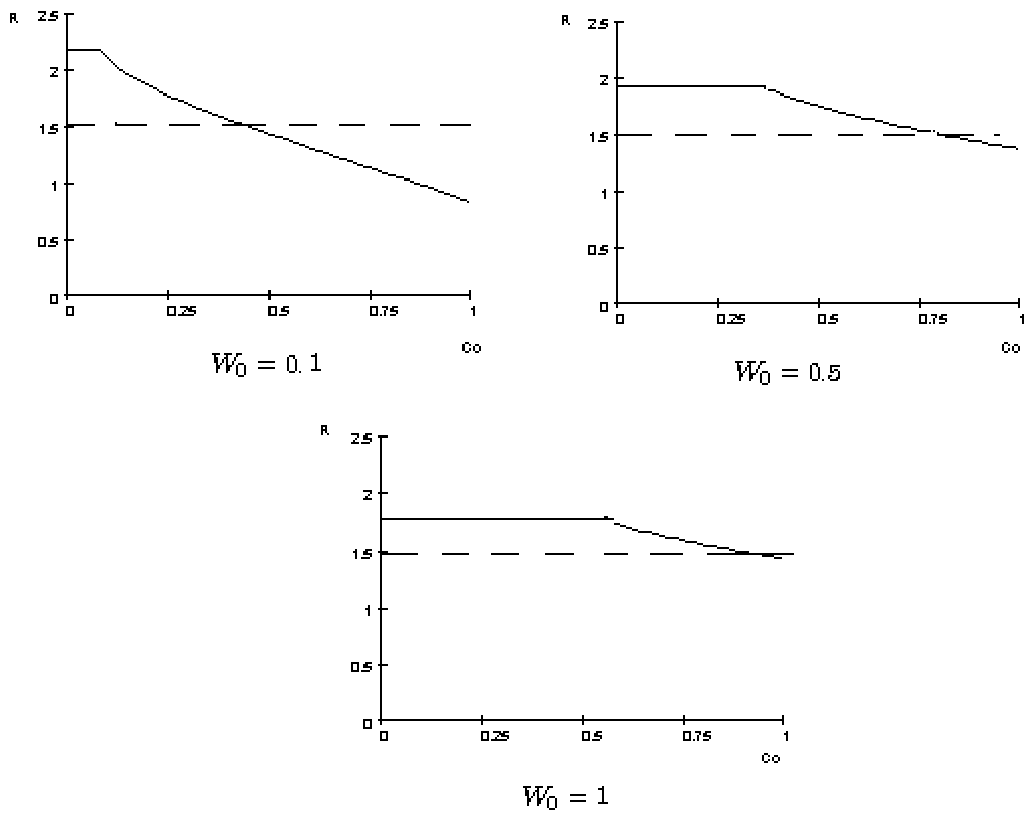

Figure 4: Risk chosen by the bank facing a binding capital requirement in the initial period

discontinuous line shows the social optimum. Up to $\widetilde{c_0}$ (that depends on the value of initial equity), capital requirements are slack and the bank chooses the unregulated level of risk. After that, risk is reduced until at some point the socially efficient level of risk is achieved. The following proposition formally proves this result.

Proposition 3 *If capital requirements bind only in the initial period, tightening the requirement always reduces the level of risk below the unregulated solution and indeed, for all $W_0 > 0$ there exist a unique value of c_0 (c_0^*) for which the bank chooses the (zero bankruptcy cost) socially efficient level of risk.*

Proof. For $\dfrac{dR}{dc_0} < 0$ see comparative statics in the appendix (section 6.2.2).

When the requirement just starts binding ($c_0 = \widetilde{c_0}$), $R^r = R^u > R^*$. Conversely, for the tightest possible regulation ($c_0 = 1$) equation 6 can be

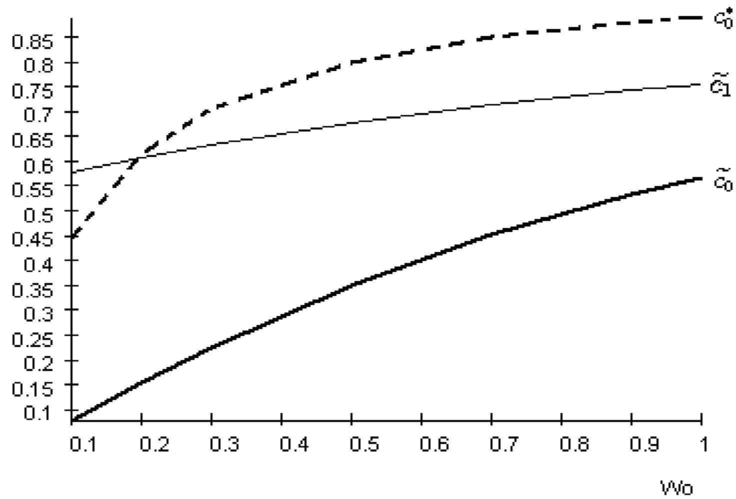

Figure 5: Optimal value of the capital requirement in the initial period.

re-written:[14]
$(p(R) + p'(R)R)\, W_0\overline{R} = -p'(R)\left\{D_1\overline{R} - C(D_1)\right\} > 0.$
Therefore $p(R) + p'(R)R > 0 = p(R^*) + p'(R^*)R^*$ and $R^r < R^*$. Continuity and strict monotonicity implies that there exist a unique value of c_0 for which $R^r = R^*$. ■

Notice c_0^* does not exist for $W_0 = 0$. In fact, as in such a case regulation must always bind ($W_0 = 0 \geq c_0 I_0 \ \forall c_0$) the bank cannot invest in the risky portfolio ($I_0 = 0$, so $R = 1 < R^*$). In this case, any positive value of the requirement at $t = 0$ forces a "chronically undercapitalized" bank to go safe.

Additionally, as the requirement only binds for values above $\widetilde{c_0}$, $c_0^* \geq \widetilde{c_0}$. But we now that $\widetilde{c_0}$ is increasing in W_0, hence $c_0^* \to 1$ as $W_0 \to +\infty$ as well (see figure 5).

Proposition 4 *The higher the initial equity of the bank, the tighter the regulation required to make the bank converge to the (zero bankruptcy cost) social efficient level of risk* $\left(\frac{dc_0^*}{dW_0} \geq 0\right)$.

Proof. When the requirement bind only in the initial period, equation 6 can be re-written as
$(p'(R)R + p(R))\, \frac{W_0}{c_0}\overline{R} = -p'(R)\left\{\left[D_0 - C(D_0) - \frac{1-c_0}{c_0}W_0 + D_1\right]\overline{R} - C(D_1)\right\}$

[14] When $c_0 = 1$, if $\lambda_0 > 0 : D_0 = \left(\frac{1-c_0}{c_0}\right) W_0 = 0$
$\Rightarrow C(D_0) = D_0$. Else, if $\lambda_0 = 0 : C'(D_0) = 1 \Rightarrow C(D_0) = D_0$.

Therefore, $R = R^*$ if and only if $\left[D_0 - C(D_0) - \frac{1-c_0^*}{c_0^*}W_0 + D_1\right]\overline{R} - C(D_1) = 0$. Implicit differentiation of the equation above leads to

$\frac{dD_0}{dW_0}(1 - C'(D_0)) + \frac{W_0}{c_0^2}\frac{dc_o^*}{dW_0} - \frac{1-c_0^*}{c_0^*} = 0$

If $\lambda_0 = 0 \Rightarrow C'(D_0) = 1$ and $\frac{dc_o^*}{dW_0} = \frac{c_0^*(1-c_0^*)}{W_0}$.

If $\lambda_0 > 0 \Rightarrow I_0 = W_0 + D_0 \Rightarrow D_0 = \left(\frac{1-c_0}{c_0}\right)W_0$, therefore $\frac{dD_0}{dW_0} = -\frac{W_0}{c_0^2}\frac{dc_0}{dW_0} + \frac{1-c_0}{c_0}$. Hence the first expression becomes $C'(D_0)\left(\frac{W_0}{c_0^2}\frac{dc_0^*}{dW_0} - \frac{1-c_0^*}{c_0^*}\right) = 0$ and it follows that $\frac{dc_0^*}{dW_0} = \frac{c_0^*(1-c_0^*)}{W_0} \geq 0$. ■

4.2 Intermediate period binding requirement

Figure 6 shows the dynamic of D_1 and R for the chosen parameters of the numerical example presented in this paper, when capital requirements bind only in period 1.

Proposition 5 *If capital requirements bind only in the intermediate period, tightening the requirement will increase the bank risk taking, most likely above the unregulated solution for all values of c_1.*

Proof. See comparative statics in the appendix (section 6.2.3) ■

4.3 Binding requirements in both periods

So far, I have computed critical values for which capital requirements start to bind ($\widetilde{c_0}$ and $\widetilde{c_1}$) and I have shown that when they bind only in period 0, tightening the requirement decreases risk taking, while the opposite is true when the requirement binds only in period 1. Although these results are useful to identify and separate the effects of binding regulation in different periods; given that both $\widetilde{c_0}$ and $\widetilde{c_1}$ are by definition less than 1, when regulators apply constant capital requirements it is no longer feasible for regulation to be binding in only one period throughout the whole spectrum of values of c.

Depending on the relationship between the thresholds one of the following two situations is possible: either $\widetilde{c_0} < \widetilde{c_1}$ or $\widetilde{c_0} \geq \widetilde{c_1}$. Although figure 3 shows $\widetilde{c_0} < \widetilde{c_1}$ for all values of W_0, this might be highly dependent on the functions chosen for that example. In principle, nothing precludes $\widetilde{c_0} > \widetilde{c_1}$ for some (high) values of W_0.

Figure 7 depicts both cases. The blank areas show the regions of values of c_0 and c_1 where capital requirements are slack. The horizontally dashed areas show the regions for which only period 0 capital requirements bind. The vertical dashed areas depict the regions for which only period 1 regulation binds. Finally, the diagonal dashed areas show the values for which capital requirements bind in both periods.

I will concentrate here on constant capital requirements (the solid black line), this is $c_0 = c_1 = c$, which is in fact the framework regulators apply. For values of the requirement above this line, period 1 effects would be stronger, while a decreasing risk effect would be more likely for values below the identity line.

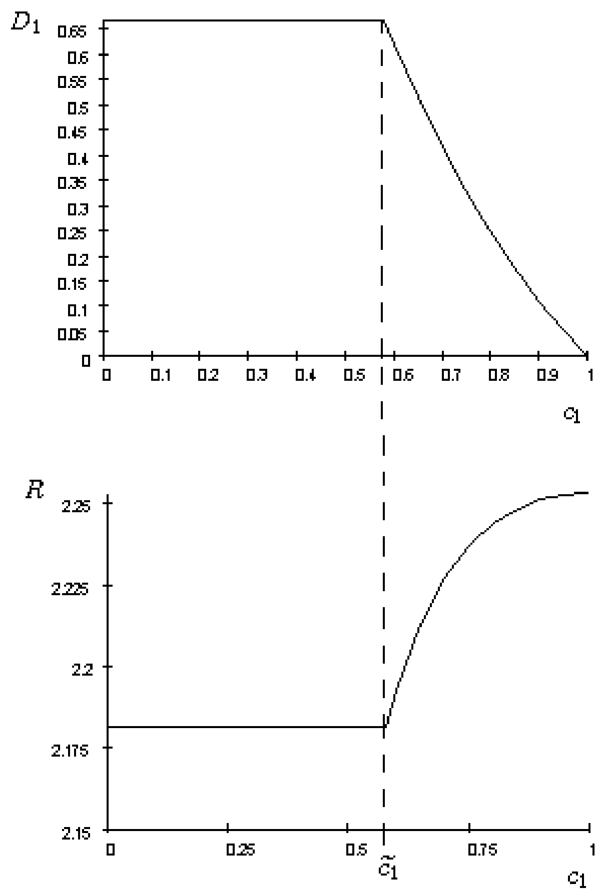

Figure 6: Period 1 binding requirement: Example of the evolution of the equilibrium values of R^r and D_1^r.

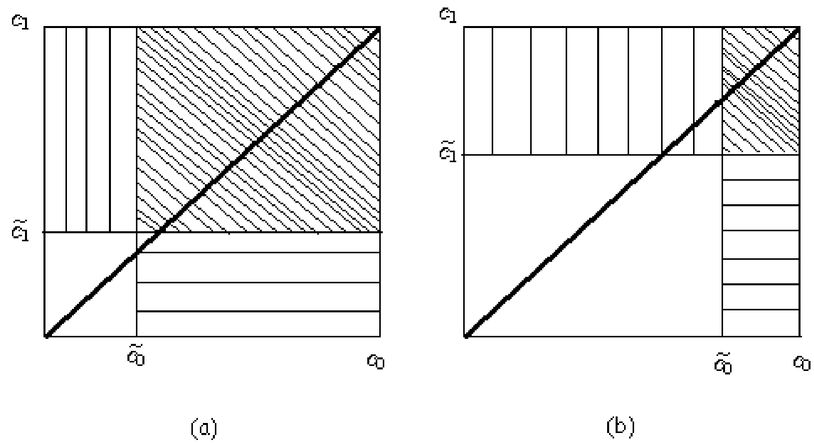

Figure 7: (a) $\widetilde{c_0} < \widetilde{c_1}$, (b) $\widetilde{c_0} \geq \widetilde{c_1}$

Proposition 6 *With constant capital requirements, R^r equals R^* when c equals 1.*

Proof. With $c = 1$, as capital requirements bind in both periods $W_0 = I_0$ and $W_1 = I_1$. Equation 6 becomes:

$$[p'(R)R + p(R)]\, W_0 \overline{R} = p'(R) \left\{ (C(D_0) - D_0)\, \overline{R} + C(D_1) - D_1 \right\} = 0,$$

because

$$\left. \begin{array}{lll} \lambda_t > 0 & \Rightarrow & D_t = \left(\frac{1-c}{c}\right) W_t = 0 \\ \lambda_t = 0 & \Rightarrow & C'(D_t) = 1 \Rightarrow D_t = 0 \end{array} \right\} \Rightarrow C(D_t) = D_t \quad t = 1, 2$$

Therefore $R^r\,(c = 1) = R^*$.

Indeed, the regulated solution approaches the optimum "from above", i.e., $R^r \geq R^*$ in a neighborhood of $c = 1$, because $\left.\dfrac{dR}{dc_1}\right|_{c_1=1} < 0$ (see comparative statics in the appendix, section 6.2.4).

■

Proposition 7 *If $\widetilde{c_0} < \widetilde{c_1}$, $R^r = R^u$ for all $c < \widetilde{c_0}$, R decreases in c for all $\widetilde{c_0} \leq c < \widetilde{c_1}$ and increases right afterwards. Moreover, there exist $c^* \leq 1$ at which $R^r = R^*$.*

Proof. This is situation (a) in figure 7. When $c < \widetilde{c_0}$ none of the requirements bind and so $R^r = R^u$. Afterwards, only period 0 requirements bind and in that case it has already been proved that $\frac{dR}{dc} < 0$.
When c reaches $\widetilde{c_1}$, period 1 requirements start to bind, which in terms of the multipliers means $\mu_1 = 0$ and $\lambda_1 > 0$. So $C'(D_1) = \overline{R}$ and $D_1 = \left(\frac{1-c}{c}\right) W_1$, where

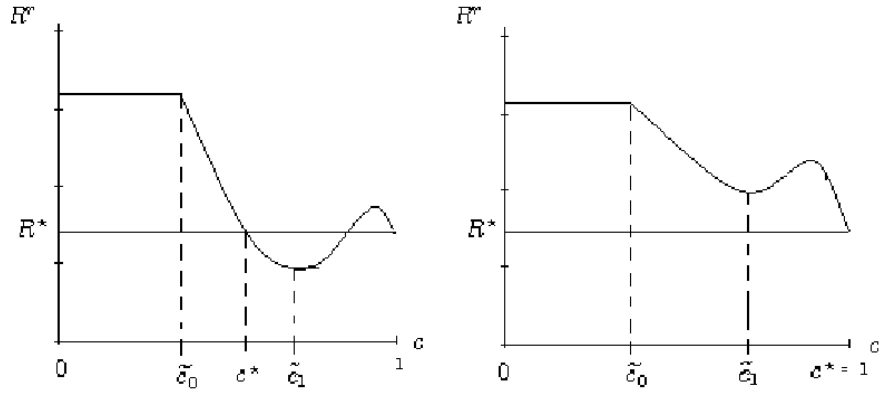

Figure 8: Possible paths of R^r as a function of c, with constant capital requirements and $\widetilde{c_0} < \widetilde{c_1}$.

$$W_1 = W_0 \left(\frac{R+c-1}{c}\right) + D_0 - C(D_0).$$

Implicit differentiation of these expressions gives:

$$\frac{dD_1}{dc} = -\frac{W_1}{c^2} - \left(\frac{1-c}{c}\right)(R - C'(D_0))\frac{W_0}{c^2} + \left(\frac{1-c}{c^2}\right) W_0 \frac{dR}{dc} = 0 \Longrightarrow \frac{dR}{dc} \geq 0.$$

Therefore, at $\widetilde{c_1}$ the risk chosen by the unregulated bank starts to increase. Comparative statics do not give a clear sign for $\frac{dR}{dc}$, (see appendix, section 6.2.4), but from proposition 6 we know that R^r approaches the optimum "from above".

This case presents two possibilities: either $c_0^* < \widetilde{c_1}$ or $\widetilde{c_1} \leq c_0^*$. In the first case, when period 1 requirement starts binding $R^r(\widetilde{c_1}) < R^*$ and afterwards R increases above R^*, and then decreases to hit it once again when $c = 1$. Otherwise, $R^r(\widetilde{c_1}) \geq R^*$ which means R equals R^* only when $c = 1$ (see figure 8). ■

Proposition 7 establishes that for poorly capitalized banks (as in that case $\widetilde{c_0} < \widetilde{c_1}$), constant capital requirements in an intertemporal framework are able first to reduce risk taking below the unregulated solution and second to reach the social optimum with the appropriate (however high) level of the requirement, c^*.

Proposition 8 *If $\widetilde{c_0} > \widetilde{c_1}$, $R^r = R^u$ for all $c < \widetilde{c_1}$, R increases right after $\widetilde{c_1}$ and decreases right after $\widetilde{c_0}$. Moreover, only at $c = 1$, $R = R^*$.*

Proof. This is situation (b) in figure 7. When $c < \widetilde{c_1}$ none of the requirements bind and so $R^r = R^u$. Afterwards, only period 1 requirements bind and in that case it has been already been proved that in a neighborhood of $\widetilde{c_1}$, $\frac{dR}{dc} > 0$. When c reaches $\widetilde{c_0}$, period 0 requirements start to bind, which in terms of the multipliers means $\mu_0 = 0$ and $\lambda_0 > 0$. So $C'(D_0) = R$ and $D_0 = \left(\frac{1-c}{c}\right) W_0$. Implicit differentiation of these expressions gives:

$$\frac{dD_0}{dc} = \frac{1}{C''(D_0)}\frac{dR}{dc} = -\frac{1}{c^2}W_0 = 0 \Longrightarrow \frac{dR}{dc} < 0.$$

Comparative statics do not give a clear sign for $\frac{dR}{dc}$, (see appendix, section 6.2.4),

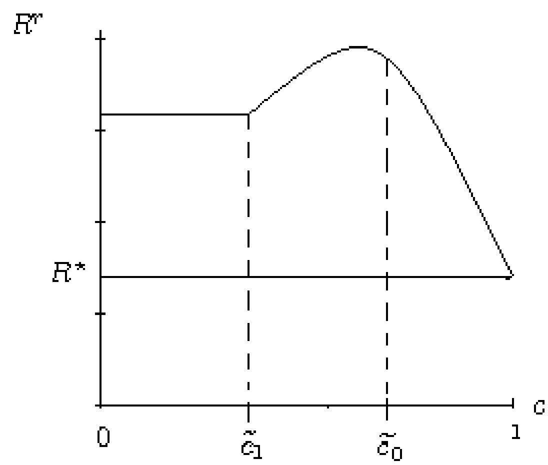

Figure 9: Possible path of R^r as a function of c, when constant capital requirements bind in both periods $\widetilde{c}_0 > \widetilde{c}_1$.

but from proposition 6 we know that R^r approaches the optimum "from above" (see figure 9). ■

5 Concluding Remarks

In this paper I have explored some features of Blum's (1999) model neglected in his original work. I have computed threshold values for capital requirements in each period for which regulation start to bind and, depending on the relationship between these values, I have studied the effectiveness of capital adequacy requirements in an intertemporal framework. In particular, conventional levels of capital requirements (like the 8 percent in Basel I) stop to matter in the decisions of the banks when they have built enough equity, however their risk taking can still be high (compared to social efficient levels).

One lesson from this exercise is that constant capital requirements (as considered in Basel I) are indeed capable of reducing risk taking below the unregulated solution, and can even achieve the zero bankruptcy cost, socially efficient level of risk, but that might happen for very high (and very unpopular) levels of the requirement (eventually $c = 1$), and at the cost of reducing financial intermediation.

A second important lesson is that as $\widetilde{c}_0$ and $\widetilde{c}_1$ depend upon W_0, knowing the value of the initial equity of a bank is essential for the regulator to determine the effectiveness of capital regulation. The fact that $\widetilde{c}_0$ depends increasingly on

W_0 justifies the empirical observation that capital requirements are slack in the vast majority of banks across countries that have adopted the Basel approach.

This model has assumed initial equity is public information. However, if the model were to be replicated infinite times, the initial equity required as an input in each period would be the result of past periods decisions, and so private information to banks. In this sense, effective monitoring and market instruments (like subordinated debt) might be efficient in revealing W_0. However, the introduction of these instruments will again modify the equilibrium choice of risk in the bank.

Of course all of Blum's original disclaimers apply here. This is not a general model, because the results rely strongly on the assumptions made on the probability function and the cost of rising deposits. The assumption of universal risk neutrality is also an issue. While it was useful to separate risk effects due to risk choice from those due to risk aversion of agents, it certainly introduces an atmosphere of "too much risk taking". Also, assuming $\overline{R}$ fixed in the intermediate period may be a restrictive assumption, although one could think of it as the net present value of all future profitable investment decisions, estimated by the bank manager at each time period. While all these simplifications facilitated the development of the main conclusions, they too imply they are incomplete.

In spite of that, the analysis is suggestive, and it does help to understand the effects and limitations of capital requirements in an intertemporal framework.

A remark concerning risk weights in capital requirements applies here. Let me explain briefly how they do modify the solution presented before. Consider a weight factor $\delta > 0$, applied to the risky portfolio I_t. This factor changes the computation of the capital to investment ratio, and so the effective value of the requirement faced by the regulated bank, in the following way:

$$\frac{W_t}{\delta I_t} \geq c_t \Leftrightarrow \frac{W_t}{I_t} \geq \delta c_t$$

If $\delta < 1$ the effective requirement is smaller so less likely to be biding. The opposite is true for $\delta > 1$. However, as I just noticed, a tighter regulation will be more or less desirable depending on the value of W_0. Therefore, δ should not only depend on measures of systemic risk of the investment portfolio, but also on the available information about W_0.

Finally, the commitment of the regulator to monitor the bank is an issue that has not been addressed here. The computation of the threshold levels of capital requirements have assumed the bank is willing to comply with the requirement, which happens only whether there is an effective threat of punishment in case of not doing so. This reasoning reinforces the need for establishing Basel II on three pillars: capital requirements, to curb bank risk taking; market discipline, to infer the values of R and/or W_0; and improved supervision, in order to ensure compliance with -and an appropriate design of- regulation.

6 Appendix

6.1 The regulated bank

$$
\begin{array}{llll}
(P) & \max\limits_{R,D_0,I_0,W_1,D_1,I_1} & p(R)\left\{I_1\left(\overline{R}-1\right)+W_1+D_1-C(D_1)\right\} & \\
& st. & & \\
& (1) & I_0\left(R-1\right)+W_0+D_0-C(D_0)-W_1\geq 0 & (\theta) \\
& (2) & W_0+D_0-I_0\geq 0 & (\lambda_0) \\
& (3) & W_0-c_0I_0\geq 0 & (\mu_0) \\
& (4) & W_1+D_1-I_1\geq 0 & (\lambda_1) \\
& (5) & W_1-c_1I_1\geq 0 & (\mu_1)
\end{array}
$$

The Lagrangian for this problem is:

$\mathcal{L}=p(R)\left\{I_1\left(\overline{R}-1\right)+W_1+D_1-C(D_1)\right\}$

$+\theta\left\{I_0\left(R-1\right)+W_0+D_0-C(D_0)-W_1\right\}$

$+\lambda_0\left(W_0+D_0-I_0\right)+\mu_0\left(W_0-c_0I_0\right)+\lambda_1\left(W_1+D_1-I_1\right)+\mu_1\left(W_1-c_1I_1\right)$

$FOC(R): p'(R)\left\{I_1\left(\overline{R}-1\right)+W_1+D_1-C(D_1)\right\}+\theta I_0=0$

$FOC(D_0): \theta\left(1-C'(D_0)\right)+\lambda_0=0$

$FOC(D_1): p(R)\left(1-C'(D_1)\right)+\lambda_1=0$

$FOC(I_0): \theta\left(R-1\right)-\lambda_0-c_0\mu_0=0$

$FOC(I_1): p(R)\left(\overline{R}-1\right)-\lambda_1-c_1\mu_1=0$

$FOC(W_1): p(R)-\theta+\lambda_1+\mu_1=0$

Solving simultaneously

$\lambda_0=\theta\left(C'(D_0)-1\right)$

$\lambda_1=p(R)\left(C'(D_1)-1\right)$

$\theta\left(R-1\right)-\theta\left(C'(D_0)-1\right)-c_0\mu_0=0\Rightarrow\theta\left(R-C'(D_0)\right)=c_0\mu_0$

If $c_0=0$, equation (3) of program (P) becomes $W_0>0$, so by complementary slackness $\mu_0=0$.

$p(R)\left(\overline{R}-1\right)-p(R)\left(C'(D_1)-1\right)-c_1\mu_1=0\Rightarrow p(R)\left(\overline{R}-C'(D_1)\right)=c_1\mu_1$

If $c_1=0$, equation (3) of program (P) becomes $W_1>0$, so by complementary slackness $\mu_1=0$. Provided $p(R)>0, \overline{R}=C'(D_1)$

$\theta=p(R)+\lambda_1+\mu_1=\frac{p(R)}{c_1}\left\{\overline{R}+\left(c_1-1\right)C'(D_1)\right\}=p(R)\left\{C'(D_1)+\frac{\overline{R}-C'(D_1)}{c_1}\right\}$, that converges to $p(R)\overline{R}$ as $c_1\to 0$.

While I have not ruled out short sale of assets explicitly in the constraints, the non-negativity of the multipliers imply $D_0, D_1\geq 0$. By definition of $p(.)$, $R\geq 1$, and $\{I_t\}_{t=1,2}$ equal either $\frac{W_t}{c_t}$ or W_t+D_t, both being non-negative numbers. Given that solutions where $\mu_t=\lambda_t=0$ are ruled out, there are only nine possible combinations for the sign of the multipliers:

	μ_0	λ_0	μ_1	λ_1
1)	$=0$	>0	$=0$	>0
2)	>0	>0	$=0$	>0
3)	>0	$=0$	$=0$	>0
4)	$=0$	>0	>0	>0
5)	$=0$	>0	>0	$=0$
6)	>0	>0	>0	>0
7)	>0	>0	>0	$=0$
8)	>0	$=0$	>0	>0
9)	>0	$=0$	>0	$=0$

a) Unregulated case (slack capital requirements): $\mu_0 = \mu_1 = 0$, and $\lambda_0 > 0, \lambda_1 > 0$ (case 1).

b) Binding requirement at $t = 0$ only: $\mu_1 = 0, \lambda_1 > 0$ (cases 2 and 3).

c) Binding requirement at $t = 1$ only: $\mu_0 = 0, \lambda_0 > 0$ (cases 4 and 5).

d) Both periods binding requirements (cases 6 to 9).

6.1.1 a) Unregulated case:

$W_0 > c_0 I_0 \Rightarrow \mu_0 = 0 \Leftrightarrow \theta\left(R - C'(D_0)\right) \Rightarrow C'(D_0) = R$
$W_1 > c_1 I_1 \Rightarrow \mu_1 = 0 \Rightarrow C'(D_1) = \overline{R} = cnst$
$\lambda_0 > 0 \Rightarrow I_0 = W_0 + D_0$
$\lambda_1 > 0 \Rightarrow I_1 = W_1 + D_1$
$\theta = p(R)\overline{R}$
$W_1 = \left(W_0 + D_0\right) R - C(D_0)$
$p'(R)\left\{\left[\left(W_0 + D_0\right) R - C(D_0) + D_1\right]\overline{R} - C(D_1)\right\} + p(R)\overline{R}\left(W_0 + D_0\right) = 0$

Hence, R and D_0 are determined simultaneously from
$\mathcal{L}_R : p'(R)\left\{\left[\left(W_0 + D_0\right) R - C(D_0) + D_1\right]\overline{R} - C(D_1)\right\} + p(R)\overline{R}\left(W_0 + D_0\right) = 0$
$\mathcal{L}_{D_0} : p(R)\overline{R}\left[R - C'(D_0)\right] = 0$

The second order conditions reduces to prove that the following matrix of second derivatives for R and D_0 is negative semi-definite, this is, that its leading principal minors alternate in sign:

$$L_a^2 = \begin{bmatrix} p''(R)W_2^a + 2p'(R)(W_0 + D_0)\overline{R} & p(R)\overline{R} \\ p(R)\overline{R} & -p(R)\overline{R}C''(D_0) \end{bmatrix}$$

where $W_2^a = \left[\left(W_0 + D_0\right) R - C(D_0) + D_1\right]\overline{R} - C(D_1) \geq 0$.

$\det\left[L^2\right]_1 = p''(R)W_2^a + 2p'(R)(W_0 + D_0)\overline{R} < 0$
$\det\left[L^2\right]_2 = -p(R)\overline{R}C''(D_0)\left(p''(R)W_2^a + 2p'(R)(W_0 + D_0)\overline{R}\right) - \left(p(R)\overline{R}\right)^2$

$= -p(R)\overline{R}\left\{C''(D_0)\left[p''(R)W_2^a{+}2p'(R)(W_0{+}D_0)\overline{R}\right] + p(R)\overline{R}\right\} \geq 0$
$\Leftrightarrow C''(D_0)\left[p''(R)W_2^a{+}2p'(R)(W_0{+}D_0)\overline{R}\right] {+}p(R)\overline{R}{\leq}\ 0$

6.1.2 b) Binding requirement at $t = 0$ only

$\mu_0 > 0 \Rightarrow W_0 = c_0 I_0 \Rightarrow I_0 = \frac{W_0}{c_0}$
$W_1 > c_1 I_1 \Rightarrow \mu_1 = 0 \Rightarrow C'(D_1) = \overline{R} = cnst$
$\lambda_1 > 0 \Rightarrow I_1 = W_1 + D_1$
$\theta = p(R)\overline{R}$
$W_1 = \frac{W_0}{c_0}(R - 1 + c_0) + D_0 - C(D_0)$
If $\lambda_0 > 0 : I_0 = W_0 + D_0 \Rightarrow D_0 = \left(\frac{1-c_0}{c_0}\right) W_0 = cnst$
If $\lambda_0 = 0 : C'(D_0) = 1 \Rightarrow D_0 = cnst$

Hence, R is determined alone from
$\mathcal{L}_R$: $p'(R)\left\{\left(\frac{W_0}{c_0}(R - 1 + c_0) + D_0 - C(D_0) + D_1\right)\overline{R} - C(D_1)\right\} {+}p(R)\overline{R}\frac{W_0}{c_0}{=}\ 0$
and the relevant second order condition is
$\mathcal{L}_{RR}$: $p''(R)\left\{\left(\frac{W_0}{c_0}(R - 1 + c_0) {+}D_0{-}C(D_0) + D_1\right)\overline{R}{-}C(D_1)\right\} {+}2p'(R)\overline{R}\frac{W_0}{c_0}{<}\ 0$

6.1.3 c) Binding requirement at $t = 1$ only

$\mu_1 > 0 \Rightarrow W_1 = c_1 I_1 \Rightarrow I_1 = \frac{W_1}{c_1}$
$W_0 > c_0 I_0 \Rightarrow \mu_0 = 0 \Leftrightarrow \theta\left[R - C'(D_0)\right] = 0 \Rightarrow C'(D_0) = R$
$\lambda_0 > 0 \Rightarrow I_0 = W_0 + D_0$
$\theta = \dfrac{p(R)}{c_1}\left\{\overline{R} + (c_1 - 1)\, C'(D_1)\right\}$
$W_1 = (W_0 + D_0)\, R - C(D_0)$

If $\lambda_1 > 0 : I_1 = W_1 + D_1 \Rightarrow D_1 = \left(\frac{1-c_1}{c_1}\right)\left[(W_0 + D_0)\, R - C(D_0)\right]$
If $\lambda_1 = 0 : C'(D_1) = 1 \Rightarrow D_1 = cnst$

In both cases, R and D_0 are determined simultaneously from the system:

$$\mathcal{L}_R : p'(R)\left\{\frac{\left[(W_0{+}D_0)\, R - C(D_0)\right]}{c_1}\left(\overline{R}{+}c_1{-}1\right) + D_1{-}C(D_1)\right\} + \frac{p(R)}{c_1}\left(\overline{R}{+}\,(c_1{-}1)\, C'(D_1)\right)(W_0{+}D_0) = 0$$

$$\mathcal{L}_{D_0} : \frac{p(R)}{c_1}\left(\overline{R} + (c_1 - 1)\, C'(D_1)\right)\left[R - C'(D_0)\right] = 0.$$

The second order conditions reduces to prove that the following matrix of second derivatives for R and D_0 is negative semi-definite, this is, that its leading principal minors alternate in sign:

$$L_c^2 = \begin{bmatrix} \mathcal{L}_{RR} & \frac{p(R)}{c_1}\left(\overline{R}+(c_1-1)\,C'(D_1)\right) \\ \frac{p(R)}{c_1}\left(\overline{R}+(c_1-1)\,C'(D_1)\right) & -\frac{p(R)}{c_1}\left(\overline{R}+(c_1-1)\,C'(D_1)\right)C''(D_0) \end{bmatrix}$$

where

$$\mathcal{L}_{RR} = p''(R)W_2^c + 2p'(R)\left(\tfrac{W_0+D_0}{c_1}\right)\left(\overline{R}+(c_1-1)\,C'(D_1)\right) - p(R)\,(W_0+D_0)^2\,\Phi(\lambda_1,R,W_0,c_1),$$

$$\Phi(\lambda_1,R,W_0,c_1) = \begin{cases} C''(D_1)\left(\frac{1-c_1}{c_1}\right)^2 & \lambda_1>0 \\ 0 & \lambda_1=0 \end{cases} \geq 0,$$

$$W_2^c = \left[(W_0+D_0)\,R - C(D_0)\right]\left(\tfrac{\overline{R}+c_1-1}{c_1}\right) + D_1 - C(D_1) \geq 0.$$

$$\det\left[L^2\right]_1 = \mathcal{L}_{RR} \leq 0$$

$$\det\left[L^2\right]_2 = -\tfrac{p(R)}{c_1}\left(\overline{R}+(c_1-1)\,C'(D_1)\right)\left\{\mathcal{L}_{RR}C''(D_0)+\tfrac{p(R)}{c_1}\left(\overline{R}+(c_1-1)\,C'(D_1)\right)\right\} \geq 0$$

$$\Leftrightarrow \left\{p''(R)W_2^c+2p'(R)\left(\tfrac{W_0+D_0}{c_1}\right)\left(\overline{R}+(c_1-1)\,C'(D_1)\right)\right\}C''(D_0) + p(R)\left[\frac{\left(\overline{R}+(c_1-1)C'(D_1)\right)}{c_1} - (W_0+D_0)^2\,C''(D_0)\Phi(\lambda_1,R,W_0,c_1)\right] \leq 0$$

6.1.4 d) Both periods binding requirements ($c_0 = c_1 = c$)

$\mu_0 > 0 \Rightarrow W_0 = cI_0 \Rightarrow I_0 = \frac{W_0}{c}$

$\mu_1 > 0 \Rightarrow W_1 = cI_1 \Rightarrow I_1 = \frac{W_1}{c}$

$W_1 = \frac{W_0}{c}\,(R+c-1) + D_0 - C\,(D_0)$

$\theta = \dfrac{p(R)}{c}\left\{\overline{R}+(c-1)\,C'(D_1)\right\}$

If $\lambda_0 > 0 \Rightarrow I_0 = W_0 + D_0 \Rightarrow D_0 = \left(\frac{1-c}{c}\right)W_0 = cnst$

  If $\lambda_1 > 0 \Rightarrow I_1 = W_1 + D_1 \Rightarrow D_1 = \left(\frac{1-c}{c}\right)\left[\frac{1}{c}W_0R - C(\left(\frac{1-c}{c}\right)W_0)\right]$

  If $\lambda_1 = 0 \Rightarrow C'(D_1) = 1 \Rightarrow D_1 = cnst$

If $\lambda_0 = 0 \Rightarrow \theta\,(1 - C'\,(D_0)) = 0 \Rightarrow C'\,(D_0) = 1 \Rightarrow D_0 = cnst$

  If $\lambda_1 > 0 \Rightarrow I_1 = W_1 + D_1 \Rightarrow D_1 = \left(\frac{1-c}{c}\right)\left[\frac{W_0}{c}\,(R+c-1)+D_0-C(D_0)\right]$

  If $\lambda_1 = 0 \Rightarrow C'(D_1) = 1 \Rightarrow D_1 = cnst$

In all cases, R is determined alone from

$$\mathcal{L}_R : p'(R)\left\{\left[\tfrac{W_0}{c}\,(R+c-1)+D_0-C(D_0)\right]\left(\tfrac{\overline{R}+c-1}{c}\right) + D_1-C(D_1)\right\} + \frac{p(R)}{c^2}\left(\overline{R}+(c-1)\,C'\,(D_1)\right)W_0 = 0,$$

and the relevant second order condition is:

$$p''(R)W_2^d+2p'(R)\tfrac{W_0}{c^2}\left(\overline{R}+(c-1)\,C'(D_1)\right) - p(R)\tfrac{W_0^3}{c^2}\Phi(\lambda_1,R,W_0,c) \leq 0,$$

which always holds true, for

$$W_2^d = \left[\tfrac{W_0}{c}\,(R+c-1)+D_0-C(D_0)\right]\left(\tfrac{\overline{R}+c-1}{c}\right) + D_1-C(D_1) \geq 0.$$

6.2 Comparative statics

So far, for given values of the parameters W_0, c_0 and c_1, I have derived a set of first order conditions determining the optimal solution for R, D_0, D_1, I_0, I_1, and W_1; and each possible combination of the sign of the multipliers. I have also shown that D_1, I_0, I_1 and W_1 can be written in terms of W_0, c_0, c_1, R and D_0, where either D_0 is constant, or R and D_0 are determined simultaneously from the system:

$$\mathcal{L}_R = \frac{\partial \mathcal{L}}{\partial R} = 0$$

$$\mathcal{L}_{D_0} = \frac{\partial \mathcal{L}}{\partial D_0} = 0$$

From the implicit function theorem it is known that

$$\begin{bmatrix} \dfrac{dR}{d\varphi} \\ \\ \dfrac{dD_0}{d\varphi} \end{bmatrix} = \Bigg[\underbrace{\begin{matrix} \mathcal{L}_{RR} & \mathcal{L}_{RD_0} \\ \mathcal{L}_{D_0R} & \mathcal{L}_{D_0D_0} \end{matrix}}_{L^2}\Bigg]^{-1} \begin{bmatrix} \mathcal{L}_{R\varphi} \\ \\ \mathcal{L}_{D_0\varphi} \end{bmatrix},$$

or simply

$$\begin{aligned} \frac{dR}{d\varphi} &= \frac{\mathcal{L}_{D_0\varphi}\mathcal{L}_{RD_0} - \mathcal{L}_{R\varphi}\mathcal{L}_{D_0D_0}}{\mathcal{L}_{RR}\mathcal{L}_{D_0D_0} - \mathcal{L}_{RD_0}\mathcal{L}_{D_0R}}, \\ \frac{dD_0}{d\varphi} &= \frac{\mathcal{L}_{R\varphi}\mathcal{L}_{D_0R} - \mathcal{L}_{D_0\varphi}\mathcal{L}_{RR}}{\mathcal{L}_{RR}\mathcal{L}_{D_0D_0} - \mathcal{L}_{RD_0}\mathcal{L}_{D_0R}}, \end{aligned}$$

where $\varphi \in \{W_0, c_0, c_1\}$.

$$\begin{aligned} sign\left[\frac{dR}{d\varphi}\right] &= sign\left[\frac{1}{\det L^2}\right] \times sign\left[\mathcal{L}_{D_0\varphi}\mathcal{L}_{RD_0} - \mathcal{L}_{R\varphi}\mathcal{L}_{D_0D_0}\right], \\ sign\left[\frac{dD_0}{d\varphi}\right] &= sign\left[\frac{1}{\det L^2}\right] \times sign\left[\mathcal{L}_{R\varphi}\mathcal{L}_{D_0R} - \mathcal{L}_{D_0\varphi}\mathcal{L}_{RR}\right], \end{aligned}$$

But since SOCs imply $\det L^2 \geq 0$,

$$\begin{aligned} sign\left[\frac{dR}{d\varphi}\right] &= sign\left[\mathcal{L}_{D_0\varphi}\mathcal{L}_{RD_0} - \mathcal{L}_{R\varphi}\mathcal{L}_{D_0D_0}\right], \\ sign\left[\frac{dD_0}{d\varphi}\right] &= sign\left[\mathcal{L}_{R\varphi}\mathcal{L}_{D_0R} - \mathcal{L}_{D_0\varphi}\mathcal{L}_{RR}\right], \end{aligned}$$

6.2.1 a) Unregulated case:

$\mathcal{L}_R : p'(R)\left\{[(W_0 + D_0)\,R - C(D_0) + D_1]\,\overline{R} - C(D_1)\right\} + p(R)\overline{R}\,(W_0 + D_0) = 0$
$\mathcal{L}_{D_0} : p(R)\overline{R}\,[R - C'(D_0)] = 0$
$\varphi = W_0$

$$\begin{bmatrix} \mathcal{L}_{RR} & \mathcal{L}_{RD_0} \\ \mathcal{L}_{D_0R} & \mathcal{L}_{D_0D_0} \end{bmatrix} = L_a^2 = \begin{bmatrix} p''(R)W_2{+}2p'(R)(W_0{+}D_0)\overline{R} & p(R)\overline{R} \\ p(R)\overline{R} & -p(R)\overline{R}C''(D_0) \end{bmatrix}$$

$\mathcal{L}_{RW_0} = (p'(R)R + p(R))\,\overline{R}$
$\mathcal{L}_{D_0W_0} = 0$

$$sign\left[\frac{dR}{dW_0}\right] = sign\left[(p'(R)R + p(R))\,p(R)\overline{R}^2 C''(D_0)\right]$$
$$sign\left[\frac{dD_0}{dW_0}\right] = sign\left[(p'(R)R + p(R))\,\overline{R}^2 p(R)\right]$$

As I have assumed that for the unregulated bank $p'(R)R + p(R) < 0$, then

$$\frac{dR}{dW_0} < 0 \text{ and } \frac{dD_0}{dW_0} < 0.$$

6.2.2 b) Binding requirement at $t = 0$ only

$\mathcal{L}_R$: $p'(R)\left\{\left(\frac{W_0}{c_0}\,(R - 1 + c_0) + D_0 - C(D_0) + D_1\right)\overline{R} - C(D_1)\right\} + p(R)\overline{R}\frac{W_0}{c_0} = 0$
$\mathcal{L}_{D_0} : D_0 - fnct\,(W_0, c_0) = 0$

In this case D_0 is independent of R, therefore comparative statics reduce to

$$\frac{dR}{d\varphi} = -\left[\mathcal{L}_{RR}\right]^{-1}\mathcal{L}_{R\varphi}$$

But since $\mathcal{L}_{RR} < 0$,

$$sign\left[\frac{dR}{d\varphi}\right] = sign\left[\mathcal{L}_{R\varphi}\right]$$

Notice that for $c_0 \geq \widetilde{c_0}$:

$\frac{\partial D_0}{\partial c_0} = \begin{cases} -\frac{W_0}{c_0^2} & \lambda_0 > 0 \\ 0 & \lambda_0 = 0 \end{cases} \leq 0$

$\frac{\partial D_0}{\partial W_0} = \begin{cases} \frac{1-c_0}{c_0} & \lambda_0 > 0 \\ 0 & \lambda_0 = 0 \end{cases} \geq 0$

Then

$\mathcal{L}_{RW_0} = \frac{\overline{R}}{c_0}\left[p'(R)\,(R - (1 - c_0)\,C'(D_0)) + p(R)\right] \geq 0.$
$\mathcal{L}_{Rc_0} = -\overline{R}\frac{W_0}{c_0^2}\left[p'(R)\,(R - C'(D_0)) + p(R)\right] \leq 0.$
$\mathcal{L}_{Rc_1} = 0.$

This comes from the first order condition on R ($\mathcal{L}_R = 0$):
$p'(R)\left(R - C'(D_0)\right) + p(R)$
$= -\frac{c_0}{\overline{R}W_0}p'(R)\left\{C'(D_0)W_0\overline{R} + \left[C'(D_0)D_0 - C(D_0)\right]\overline{R} + \left[D_1\overline{R} - C(D_1)\right]\right\} \geq 0$

This is because $C'(0)$ bounded and $C(.)$ convex, imply $C'(D_t)D_t \geq D_t$ for all D_t.[15]

Also

$$p'(R)\left(R - (1-c_0)\, C'(D_0)\right) + p(R)$$
$$= -\frac{c_0}{\overline{R}W_0} p'(R) \left\{ \left[C'(D_0)D_0 - C(D_0)\right]\overline{R} + \left[D_1\overline{R} - C(D_1)\right] \right\} \geq 0$$

Hence, for $c_0 \geq \widetilde{c_0}$:

$$\frac{dR}{dW_0} \geq 0$$
$$\frac{dR}{dc_0} \leq 0$$

6.2.3 c) Binding requirement at $t = 1$ only

$$\mathcal{L}_R\!: p'(R)\left\{ \frac{\left[(W_0+D_0)\,R - C(D_0)\right]}{c_1}\left(\overline{R}+c_1-1\right) + D_1 - C(D_1) \right\}$$
$$+\frac{p(R)}{c_1}\left(\overline{R} + (c_1-1)\, C'(D_1)\right)(W_0+D_0) = 0$$

$$\mathcal{L}_{D_0} : \frac{p(R)}{c_1}\left(\overline{R} + (c_1-1)\, C'(D_1)\right)\left[R - C'(D_0)\right] = 0$$

$$\begin{bmatrix} \mathcal{L}_{RR} & \mathcal{L}_{RD_0} \\ \mathcal{L}_{D_0R} & \mathcal{L}_{D_0D_0} \end{bmatrix} = L_c^2$$

$$= \begin{bmatrix} \mathcal{L}_{RR} & \frac{p(R)}{c_1}\left(\overline{R} + (c_1-1)\, C'(D_1)\right) \\ \frac{p(R)}{c_1}\left(\overline{R} + (c_1-1)\, C'(D_1)\right) & -\frac{p(R)}{c_1}\left(\overline{R} + (c_1-1)\, C'(D_1)\right) C''(D_0) \end{bmatrix}$$

where

$$\mathcal{L}_{RR} = p''(R)W_2^c + 2p'(R)\left(\frac{W_0+D_0}{c_1}\right)\left(\overline{R} + (c_1-1)\, C'(D_1)\right)$$
$$-p(R)\,(W_0+D_0)^2\,\Phi(\lambda_1, R, W_0, c_1).$$

Notice that for $c_1 \geq \widetilde{c_1}$:

$$\frac{\partial D_1}{\partial c_1} = \begin{cases} \left(-\frac{1}{c_1^2}\right)\left[(W_0+D_0)\,R - C(D_0)\right] & \lambda_1 > 0 \\ 0 & \lambda_1 = 0 \end{cases} \leq 0,$$

$$\mathcal{L}_{RW_0} = \left[p'(R)R + p(R)\right]\left(\frac{\overline{R}+(c_1-1)C'(D_1)}{c_1}\right) - p(R)R\,(W_0+D_0)\,\Phi(\lambda_1, R, W_0, c_1),$$

$$\mathcal{L}_{Rc_0} = 0,$$

$$\mathcal{L}_{Rc_1} = p'(R)\left[\frac{(W_0+D_0)R - C(D_0)}{c_1^2}\right]\left(C'(D_1) - \overline{R}\right) + p(R)\left(\frac{W_0+D_0}{c_1^2}\right)\left(C'(D_1) - \overline{R}\right)$$
$$+p(R)\,(W_0+D_0)\left[\frac{(W_0+D_0)R - C(D_0)}{(1-c_1)c_1}\right]\Phi(\lambda_1, R, W_0, c_1),$$

$$\mathcal{L}_{D_0W_0} = -p(R)R\left(R - C'(D_0)\right)\Phi(\lambda_1, R, W_0, c_1) = 0,$$

$$\mathcal{L}_{D_0c_0} = 0,$$

$$\mathcal{L}_{D_0c_1} = -\frac{p(R)}{c_1^2}\left(\overline{R} + (c_1-1)C'(D_1)\right)\left(R - C'(D_0)\right)$$
$$+\frac{p(R)}{c_1}\left[C'(D_1) + (c_1-1)\, C''(D_1)\frac{\partial D_1}{\partial c_1}\right]\left(R - C'(D_0)\right) = 0,$$

[15] Define $F(D) = C'(D)D - C(D)$

As $C'(0)$ is bounded, $F(0) = 0, F'(D) = C''(D)D \geq 0$. Hence $F(D) \geq 0\ \forall D$, i.e. $C'(D)D \geq C(D)$.

because a non binding requirement at $t=0$ implies $C'(D_0)=R$.

Hence,

$$sign\left[\frac{dR}{d\varphi}\right] = sign\left[\mathcal{L}_{D_0\varphi}\mathcal{L}_{RD_0} - \mathcal{L}_{R\varphi}\mathcal{L}_{D_0D_0}\right]$$
$$= -sign\left[\mathcal{L}_{D_0D_0}\right] sign\left[\mathcal{L}_{R\varphi}\right] = sign\left[\mathcal{L}_{R\varphi}\right]$$

From $\mathcal{L}_R = 0$,

$$[p'(R)R+p(R)](W_0{+}D_0)$$
$$= p'(R)C(D_0){+}p'(R)\left(\tfrac{C(D_1)-D_1}{\overline{R}+c_1-1}\right)c_1{+}p(R)\frac{(1-c_1)}{\overline{R}{+}c_1{-}1}(W_0{+}D_0)\left(C'(D_1)-1\right)$$

Then,

$$\mathcal{L}_{Rc_1} = \left\{[p'(R)R+p(R)](W_0{+}D_0) - p'(R)C(D_0)\right\}\left(\tfrac{C'(D_1)-\overline{R}}{c_1^2}\right)$$
$$+p(R)(W_0+D_0)\left[\tfrac{(W_0+D_0)R-C(D_0)}{(1-c_1)c_1}\right]\Phi(\lambda_1, R, W_0, c_1)$$
$$= \left\{p'(R)\left(\tfrac{C(D_1)-D_1}{\overline{R}+c_1-1}\right)c_1{+}p(R)\frac{(1-c_1)}{\overline{R}{+}c_1{-}1}(W_0{+}D_0)\left(C'(D_1)-1\right)\right\}\left(\tfrac{C'(D_1)-\overline{R}}{c_1^2}\right)$$
$$+p(R)(W_0+D_0)\left[\tfrac{(W_0+D_0)R-C(D_0)}{(1-c_1)c_1}\right]\Phi(\lambda_1, R, W_0, c_1)$$

For all values of c_1 where $\lambda_1 > 0$:

$$\mathcal{L}_{Rc_1} = -p'(R)\tfrac{(C(D_1)-D_1)\left(\overline{R}-C'(D_1)\right)}{\left(\overline{R}+c_1-1\right)c_1}$$
$$+p(R)(W_0{+}D_0)\left(\tfrac{1-c_1}{c_1^2}\right)\left\{[(W_0+D_0)R-C(D_0)]\tfrac{C''(D_1)}{c_1} - \frac{(C'(D_1)-1)}{\overline{R}{+}c_1{-}1}\left(\overline{R}{-}C'(D_1)\right)\right\}$$

The first term is positive, hence

$$\frac{dR}{dc_1} \geq 0 \text{ if } A(c_1) = [(W_0{+}D_0)R - C(D_0)]\tfrac{C''(D_1)}{c_1} - \frac{(C'(D_1)-1)}{\overline{R}{+}c_1{-}1}\left(\overline{R}-C'(D_1)\right) \geq 0$$

When the requirement just starts binding (at $c_1 = \widetilde{c_1}$), by continuity $\mu_1 = 0$ and $\lambda_1 > 0$, which implies that $C'(D_1) = \overline{R}$.
Hence,

$$A(\widetilde{c_1}) = [(W_0{+}D_0)R - C(D_0)]\tfrac{C''(D_1)}{c_1} \geq 0,$$

so $\dfrac{dR}{dc_1} \geq 0$, and by continuity this is also true for values of c_1 in a neighborhood of $\widetilde{c_1}$.

When $c_1 = 1$ (or if $\lambda_1 = 0$)
$A(1) = [(W_0{+}D_0)R - C(D_0)]C''(D_1) \geq 0$, and indeed
$\mathcal{L}_{Rc_1}|_{c_1=1} = p'(R)\left(\tfrac{C(D_1)-D_1}{\overline{R}}\right)\left(C'(D_1){-}\overline{R}\right) = 0$, because $C(D_1) = D_1$.

Therefore, $\left.\dfrac{dR}{dc_1}\right|_{c_1=1} = 0.$

Also notice that $\lim\limits_{c_1\to 1} A(c_1) = [(W_0{+}D_0)R - C(D_0)]C''(D_1) \geq 0$, which implies that in a neighborhood of $c_1 = 1$, $\dfrac{dR}{dc_1} \geq 0$.

$\mathcal{L}_{RW_0} = [p'(R)R + p(R)] \left(\frac{\overline{R}+(c_1-1)C'(D_1)}{c_1}\right) - p(R)R\,(W_0 + D_0)\,\Phi(\lambda_1, R, W_0, c_1)$

Until $c_1 = \widetilde{c_1}$ no requirement bind, therefore by assumption $p'(R)R + p(R) < 0$. Hence, when the requirement just starts binding $\left(C'(D_1) = \overline{R}\right)$:

$\mathcal{L}_{RW_0} = [p'(R)R + p(R)]\,\overline{R} - p(R)R\,(W_0 + D_0)\,\Phi(\lambda_1, R, W_0, c_1) < 0$

Therefore, $\left.\dfrac{dR}{dW_0}\right|_{c_1=\widetilde{c_1}} < 0.$

Also, replacing $p'(R)R + p(R)$ from $\mathcal{L}_R = 0$:

$\mathcal{L}_{RW_0} = p'(R)\left[C(D_0) + \left(\frac{C(D_1)-D_1}{\overline{R}+c_1-1}\right)c_1\right]\left(\frac{\overline{R}+(c_1-1)C'(D_1)}{c_1(W_0+D_0)}\right)$
$+p(R)\left\{\left(\frac{1-c_1}{c_1}\right)(C'(D_1)-1)\left(\frac{\overline{R}+(c_1-1)C'(D_1)}{\overline{R}+c_1-1}\right) - R\,(W_0 + D_0)\,\Phi(\lambda_1, R, W_0, c_1)\right\}$

If $c_1 = 1$:

$\mathcal{L}_{RW_0} = p'(R)C(D_0)\left(\frac{\overline{R}}{W_0+D_0}\right) < 0.$

Therefore, $\left.\dfrac{dR}{dW_0}\right|_{c_1=1} < 0.$

Summing up, provided $R > R^u$ (as it is likely to happen, given the sign of $\dfrac{dR}{dc_1}$ around the corner values of c_1), $\dfrac{dR}{dW_0} \leq 0.$

6.2.4 d) Both periods binding requirements ($c_0 = c_1 = c$)

$\mathcal{L}_R$: $p'(R)\left\{\left[\frac{W_0}{c}\,(R + c - 1) + D_0 - C(D_0)\right]\left(\frac{\overline{R}+c-1}{c}\right) + D_1 - C(D_1)\right\}$
$+p(R)\left(\overline{R} + (c-1)\,C'(D_1)\right)\frac{W_0}{c^2} = 0$

$\mathcal{L}_{D_0}$: $D_0 - fnct\,(W_0, c_0) = 0$

In this case D_0 is independent of R, therefore comparative statics reduce to

$$\frac{dR}{d\varphi} = -\left[\mathcal{L}_{RR}\right]^{-1}\mathcal{L}_{R\varphi}$$

But since $\mathcal{L}_{RR} < 0$,

$$sign\left[\frac{dR}{d\varphi}\right] = sign\left[\mathcal{L}_{R\varphi}\right]$$

For all $c \geq \max\{\widetilde{c_0}, \widetilde{c_1}\}$:

$\frac{\partial D_0}{\partial c} = \begin{cases} -\frac{W_0}{c^2} & \lambda_0 > 0 \\ 0 & \lambda_0 = 0 \end{cases}$

$\frac{\partial D_1}{\partial c} = \begin{cases} -\frac{1}{c^2}\left\{W_1 + W_0\left(\frac{1-c}{c}\right)[R - C'(D_0)]\right\} & \lambda_1 > 0 \\ 0 & \lambda_1 = 0 \end{cases}$

$\mathcal{L}_{Rc}$: $p'(R)\left\{\left[-\frac{W_0}{c^2}\,(R-1) + \left(1 - C'(D_0)\right)\frac{\partial D_0}{\partial c}\right]\left(\frac{\overline{R}+c-1}{c}\right) + \left(1 - C'(D_1)\right)\frac{\partial D_1}{\partial c}\right\}$

$$+p'(R)\left\{-\left[\tfrac{W_0}{c}\left(R+c-1\right)+D_0-C(D_0)\right]\left(\tfrac{\overline{R}-1}{c^2}\right)\right\}$$
$$-2p(R)\frac{W_0}{c^3}\left(\overline{R}+\left(c-1\right)C'\left(D_1\right)\right)+\frac{p(R)}{c^2}W_0\left\{C'\left(D_1\right)+\left(c-1\right)C''\left(D_1\right)\tfrac{\partial D_1}{\partial c}\right\}$$
$$=p'(R)\left\{-\tfrac{W_0}{c^3}\left(R-C'(D_0)\right)\left(\overline{R}+\left(c-1\right)C'(D_1)\right)-\tfrac{W_1}{c^2}\left(\overline{R}-C'(D_1)\right)\right\}$$
$$-\frac{p(R)}{c^3}W_0\left\{2\overline{R}-\left(2-c\right)C'\left(D_1\right)\right\}$$
$$+\frac{p(R)}{c^5}W_0\left[W_1+W_0\left(1-c\right)\left(R-C'(D_0)\right)\right]\left(1-c\right)C''\left(D_1\right)$$

From the FOC ($\mathcal{L}_R=0$):

$$[p'(R)R+p(R)]\tfrac{W_0}{c^2}\left(\overline{R}+\left(c-1\right)C'\left(D_1\right)\right)$$
$$=-p'(R)\left\{\left[D_0-C(D_0)\right]\left(\tfrac{\overline{R}+c-1}{c}\right)+D_1-C(D_1)+\ldots\right.$$
$$\left.\ldots+\tfrac{W_0}{c^2}\left(c-1\right)\left[\overline{R}+c-1+R\left(1-C'\left(D_1\right)\right)\right]\right\}$$

or equivalently

$$[p'(R)R+p(R)]\left(\overline{R}+c-1\right)\tfrac{W_0}{c^2}=-p(R)\left(c-1\right)\left(C'\left(D_1\right)-1\right)\tfrac{W_0}{c^2}$$
$$-p'(R)\left\{\left[D_0-C(D_0)\right]\left(\tfrac{\overline{R}+c-1}{c}\right)+D_1-C(D_1)+\tfrac{W_0}{c^2}\left(c-1\right)\left(\overline{R}+c-1\right)\right\}$$

In particular, when $c=1$

$[p'(R)R+p(R)]\,\overline{R}W_0=0$, so $R\left(c=1\right)=R^*$

and

$$\mathcal{L}_{Rc}\text{: }p'(R)W_0\left\{-\left(R-C'(0)\right)R-R\left(\overline{R}-C'(0)\right)\right\}-p(R)W_0\left\{2\overline{R}-C'\left(0\right)\right\}$$
$$=-\left[p'(R)R+p(R)\right]W_0\left(2\overline{R}-C'\left(0\right)\right)+p'(R)W_0\overline{R}C'(0)=p'(R)\overline{R}W_0C'(0)<0$$

Therefore, $\left.\dfrac{dR}{dc_1}\right|_{c_1=1}<0.$

If $\widetilde{c_0}<\widetilde{c_1}$, when c reaches $\widetilde{c_1}$, period 0 requirement is binding and period 1 requirement starts to bind, which in terms of the multipliers means $\lambda_1>0$ and $\mu_1=0$, so $C'(D_1)=\overline{R}$ and $C''\left(D_1\right)=0$.

$\mathcal{L}_{Rc}$: $-\tfrac{W_0}{c^2}\overline{R}\left[p'(R)\left(R-C'(D_0)\right)+p(R)\right]$

Also, $\lambda_1>0$ implies $D_1=\left(\tfrac{1-c}{c}\right)W_1$, therefore $\mathcal{L}_R=0$ reduces to its form in case (b) and

$$p'(R)\left\{\left(\tfrac{W_0}{c}\left(R-C'(D_0)+C'(D_0)+c-1\right)+D_0-C(D_0)\right)\tfrac{\overline{R}}{c}-C(D_1)\right\}$$
$$+p(R)\overline{R}\tfrac{W_0}{c}=0$$
$$p'(R)\tfrac{W_0}{c}\left(R-C'(D_0)\right)\tfrac{\overline{R}}{c}+p(R)\overline{R}c\tfrac{W_0}{c^2}$$
$$=-p'(R)\left\{\left(\tfrac{W_0}{c}\left(C'(D_0)+c-1\right)+D_0-C(D_0)\right)\tfrac{\overline{R}}{c}-C(D_1)\right\}$$
$$\left[p'(R)\left(R-C'(D_0)\right)+p(R)\right]\overline{R}\tfrac{W_0}{c^2}$$
$$=-p'(R)\left\{\left(\tfrac{W_0}{c}\left(C'(D_0)+c-1\right)+D_0-C(D_0)\right)\tfrac{\overline{R}}{c}-C(D_1)\right\}+p(R)\left(1-c\right)\overline{R}\tfrac{W_0}{c^2}$$

If $\widetilde{c_0} \geq \widetilde{c_1}$, when c reaches $\widetilde{c_0}$, period 1 requirement is binding and period 0 requirement starts to bind, which in terms of the multipliers means $\lambda_0 > 0$ and $\mu_0 = 0$, so $C'(D_0) = R$ and $D_0 = \left(\frac{1-c}{c}\right) W_0$.

$$
\begin{aligned}
&\mathcal{L}_{Rc}\colon \left\{-p'(R)\left(\overline{R} - C^{'}(D_1)\right) + p(R)W_0\left(\tfrac{1-c}{c^3}\right)C''(D_1)\right\}\tfrac{1}{c^2}\left(\tfrac{1}{c}W_0R - C(D_0)\right) \\
&\quad -\frac{p(R)}{c^3}W_0\left\{2\overline{R} - (2-c)\,C'(D_1)\right\} \\
&\quad = \left\{-p'(R)\left(\overline{R} - C^{'}(D_1)\right) + p(R)W_0\left(\tfrac{1-c}{c^3}\right)C''(D_1)\right\}\tfrac{1}{c^2}\left(\tfrac{1}{c}W_0R - C(D_0)\right) \\
&\quad -\frac{p(R)}{c^3}W_0\left(\overline{R} - C'(D_1)\right) - \frac{p(R)}{c^3}W_0\left(\overline{R} + (c-1)\,C'(D_1)\right) \\
&\quad = -\left[p'(R)R + p(R)\right]\left(\overline{R} - C^{'}(D_1)\right)\tfrac{W_0}{c^3} + p'(R)\left(\overline{R} - C^{'}(D_1)\right)\tfrac{1}{c^2}C(D_0) \\
&\quad + p(R)W_0\left(\tfrac{1-c}{c^5}\right)C''(D_1)\left(\tfrac{1}{c}W_0R - C(D_0)\right) - \frac{p(R)}{c^3}W_0\left(\overline{R} + (c-1)\,C'(D_1)\right)
\end{aligned}
$$

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