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A New Proof that Every Weakly Compact Operator with Domain $L_1(\mu)$ is Representable

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In 1940, Dunford and Pettis [3] proved if (Ω,Σ,μ) is a finite measure space, X a Banach space and $T\colon L_1(\mu) \longrightarrow X$ is a weakly compact linear operator then T is representable. Indeed, there is $g\colon \Omega \longrightarrow X$, $g \in L_\infty(\mu,X)$ such that

$$T(f) = \int_\Omega f \cdot g \, \mathrm{d}\mu .$$

Here we get the same result by using the well known fact that every weakly compact subset of a Banach space X is dentable.

Furthermore, we get

THEOREM. *The following statements are equivalent*:

1. If $T\colon L_1(\mu) \longrightarrow X$ *is a weakly compact linear operator then* T *is representable.*

2. If $\nu\colon \Sigma \longrightarrow X$ *is a countable additive* μ*—continuous vector measure and the set*

$$\{\nu(E)/\mu(E) : E \in \Sigma, \mu(E) > 0\}$$

is weakly compact, then ν *has a Radon—Nikodym derivative.*

3. Every relatively weakly compact subset of a Banach space X *is dentable.*

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