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NEAR LOOP RINGS OF MOUFANG LOOPS

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[1] calls a non-empty set L to be a loop if L is endowed with the following properties. (i) For all $a,b \in L$, $a.b \in L$ where . is a binary operation from $L \times L \rightarrow L$. (ii) For every ordered pair $(a,b) \in L \times L$ there is one and only one x such that $ax = b$ in L and only one y such that $ya = b$ in L. (iii) There exists an element $e \in L$ such that $ae = ea = a$ for every $a \in L$ called the identity element of L. L is said to be a Moufang loop if it satisfies any one of the following identities. $(xy)(zx) = [x(yz)]x$, $[(xy)z]y = x[y(zy)]$, $x[y(xz)] = [(xy)x]z$. For more properties of loops please refer [1].

__Definition 1__. Let (L, .,1) be a loop. N a near-ring with multiplicative identity. The loop over the near-ring, that is a near-loop ring denoted by NL with identity is the non-associative near-ring of all formal sums $\alpha = \Sigma\ \alpha(m)m$, $m \in L$ and $\alpha(m) \in N$ such that supp $\alpha = \{m/\alpha(m) \neq 0\}$ the support of α is finite; with the following operational rules

(i) $\Sigma\ \alpha(m)m = \Sigma\ \mu(m)m \iff \alpha(m) = \mu(m)$ for all $m \in L$.

(ii) $\Sigma\ \alpha(m)m + \Sigma\ \mu(m)m = \Sigma\ (\alpha(m) + \mu(m))m$, $m \in L$.

(iii) $(\Sigma\ \alpha(m)m)\ (\Sigma\ \mu(m)m) = \Sigma\ \gamma(m)m$, $m \in L$ where

$$\gamma(m) = \Sigma\ \alpha(x)\ \mu(y), \quad xy = m \in L.$$

(iv) $n\ (\Sigma\ \alpha(m)m) = \Sigma\ n(\alpha(m)m$ for all $n \in N$ and $m \in L$.

Dropping the zero components of the formal sum we may write $\alpha = \Sigma\ \alpha_i m_i$, $i = 1,2, \ldots, n$. Thus $n \rightarrow n.1$ is an embedding of N in NL. After the identification of of N with N.1. We shall assume that N is contained in NL. Clearly nm = mn for all $n \in N$ and $m \in L$. NL is a non-associative near ring, unlike the group near-ring which are associative structures. Throughout this paper N denotes a right near-ring with identity. For more properties about near-rings please refer [4]. By L we mean only a Moufang loop unless otherwise stated.

Lemma 2. Let L be a Moufang loop, N a near-ring with identity. Then the near loop ring NL is a N-group.

Proof. NL is a non-associative near-ring. But NL is a group under addition and contains 0. We have $\mu : N \times NL \rightarrow NL$ by (iv) of definition 1. Hence (NL, μ) is a N-group. We can say something about ideals in KL.

Proposition 3. Let L be a Moufang loop N a near ring G a subgroup generated by x,y in L. Then for $KG \subseteq KL$, we have (i) The sets of all ideals (right ideals, left ideals, N-subgroup invariant sub near-rings) form inductive Moore systems on NG. (ii) The sets of all ideals (N-subgroups) of an N-group, NL form inductive Moore systems on NL.

Proof. Since every Moufang loop L is di-associative, every pair of elements in L generate a subgroup. So NG is a near-ring, and $NG \subseteq NL$. (i) and (ii) are grue from [4].

Note. Nothing can be said about NL in (i).

Theorem 4. Let N be a finitely generated near-ring L a Moufang loop. $G = \langle x,y \rangle$ be the subgroup generated by $x,y \in L$. Then $NG \subseteq NL$ is such that (a) each ideal different from NG is contained in a maximal one.

Proof. $NG \subseteq NL$. Since N is finitely generated and $G = \langle x,y \rangle$ is finitely generated NG is a finitely generated near-ring. So each ideal different from NG is contained in a maximal one [4].

Proposition 5. Let N be a near-ring L a Moufang loop. Then the near loop ring NL contains NG such that G is a (subgroup) subsemigroup of (NG.). NG has a near ring of left quotients with respect to G if and only if (a) $G \neq \emptyset$. (b) for all $s \in G$: s is (left and right) cancellative (c) NG satisfies the left ore condition with respect to G.

Proof. Since L is a Moufang loop, L is diassociative, so every pair of elements generate a subgroup G in L. Hence $G \subseteq L$ and NG is a near ring. G is clearly a semigroup. If in the proof given in [4] take G = S and N to be NG the result is true.

Theorem 6. Let L be a Moufang loop which has no elements of finite order and N a near-ring without divisors of zero. Then the near loop ring NL contains nontrivial commutative domains.

Proof. L is Moufang. So L is diassociative and power associative. Hence every element in L generates a subgroup in L, $G = \langle x \rangle$ is torsion free, since L has no elements of finite order. N has no divisors of zero. Hence NG is a domain by ([2] and [3]). We pose the following problem.

Problem. For what loops L will the near loop ring NL be a domain?

References

[1] Bruck, R.H., A Survey of Binary Systems, Springer-Verlag (1958).

[2] Passman, D.S., Infinite group rings, Marcel Dekker (1971).

[3] Passman, D.S., The Algebraic Structure of Group Rings, Interscience Wiley (1977).

[4] Pilz, G., Near-Rings, North-Holland Mathematics Studies (1977).