EXTRACTA MATHEMATICAE 4, n.1, 45-47 (1989)

ON THE SURJECTIVE DUNFORD-PETTIS PROPERTY

Fernado Bombal, Pilar Cembranos and José Mendoza [1]

Departamento de Análisis Matemático
Facultad de Ciencias Matemáticas
Universidad Complutense de Madrid
28040 Madrid (España)

1.-Introduction and Notations.

A Banach space E has the Dunford-Pettis Property (DPP) if every weakly compact operator on E is Dunford-Pettis (or completely continuous), i.e., maps weakly Cauchy sequences into norm convergent ones. By the Davis-Figiel-Johnson-Pelczynski's factorization theorem , this is equivalent to saying that every operator from E into a *reflexive* Banach space is Dunford-Pettis. In [L] a formally weaker property is introduced, the so called *surjective Dunford-Pettis Property* (SDPP), by imposing that every operator from E onto a reflexive Banach space is Dunford-Pettis. This property is used in [L] to obtain sustantive extensions of previous results of Lotz on ergodic operators and strongly continuous semigroups of operators. Also it is proved in [L] that the SDPP is, in fact, genuinely weaker than the DPP, building a Banach space L with the SDPP that does not have the DPP.

In this note, we give several results on the SDPP.

2.- The surjective Dunford-Pettis Property.

Definition 1. ([L]) A Banach space E is said to have the *surjective Dunford-Pettis property* (SDPP in short) if every operator from E onto a reflexive Banach space is a Dunford-Pettis operator.

---

1 Supported in part by CAICYT grant 0338/84.

It is clear that every Banach space with the DPP has the SDPP. Also Banach spaces without reflexive infinite dimensional quotients have the SDPP. Examples of Banach spaces with the SDPP that fails the DPP are the followig: the Banach space L built by Leung in [L] and its bidual L";and the preduals of the Banach spaces defined by Azimi and Hagler in [A-H].

Obviously, complemented subspaces of Banach spaces enjoying the SDPP have this property too. It is not the same for quotients: many Banach spaces having even the DPP (f.i., ℓ_1 or ℓ_∞) have infinite dimensional reflexive quotients, but it is clear that among reflexive spaces only finite dimensional ones have the SDPP. However we have the following result.

Proposition 2. *If E has the SDPP and F is a subspace of E which does not contain a copy of ℓ_1, then E/F has the SDPP too.*

The SDPP and the DPP are different properties. However, the following theorem shows that for certain classes of Banach spaces they coincide.

Theorem 3. *If E is a Banach space containing a complemented copy of ℓ_1, then E has the DPP if and only if it has the SDPP.*

Theorem 3 shows that the SDPP, on the contrary than the DPP, is not preserved by finite products in general. In fact, the Leung's space L has the SDPP but not the DPP; then, according to theorem 3, $L\times\ell_1$ does not have the SDPP.

3.- $L_1(\mu, E)$ and $C(K, E)$ spaces enjoying SDPP.

As an immediate consequence of Theorem 3 one has that if E is a Banach space and (Ω,Σ,μ) is a non trivial measure space (i.e., $L_1(\mu)$ is not finite dimensional) then $L_1(\mu,E)$, the usual Banach space of E-valued Bochner μ-integrable functions, has the SDPP if and only if it has the DPP. This shows that the problem of determining in which conditions $L_1(\mu,E)$ has the SDPP is actually a problem already studied and for which there are some answers. For

C(K,E), the Banach space of E-valued continuous functions defined on a compact Haussdorff space K, the problem seems to be new. We have the following result.

Theorem 4. *For a Banach space E, the following assertions are equivalent:*

(a) $E^n = E\times E\times\ldots\times E$ (n) *has the SDPP for every* $n \in \mathbb{N}$.

(b) The space $c_0(E)$ *has the SDPP.*

(c) The space C(K,E) has the SDPP, for every dispersed, compact Hausdorff space K.

We do not know if condition (a) in the preceding theorem is satisfied for all Banach spaces enjoying the SDPP. Therefore the following question arises:

Question 5: Let E be a Banach space with the SDPP. Does $E\times E$ have the SDPP ?

4. The space L of Leung.

Finally we obtain the following properties of the Banach space constructed by Leung

Theorem 4. *Let L be the Leung's space, then one has:*

(a) $L\times L$ *is isomorphic to L.*

(b) Even duals of L are Grothendieck spaces which have the SDPP and fail the DPP.

(c) Odd duals of L fail the SDPP.

(d) If $1\leq p < \infty$ *then every operator* $T \in \mathcal{L}(L,\ell_p)$ *is compact.*

(e) If R is a reflexive quotient of ℓ_∞ *then each operator* $T \in \mathcal{L}(L'',R)$ *is a Dunford-Pettis operator.*

REFERENCES

[A-H] Azimi, P., Hagler, J.N.: Examples of hereditarily ℓ_1 Banach spaces failing the Schur property. Pacific J. Math. 122, 287-97 (1986).

[L] Leung, D.: Uniform convergence of Operators and Grothendieck spaces with the Dunford-Pettis property. Math. Z. 197, 21-32 (1988).