

PRODUCT THEOREMS FOR THE D-DIMENSION IN NON-METRIZABLE SPACES

by

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We establish some product theorems for the D-dimension, introduced by D.W. Henderson in [He] for metrizable spaces and studied for non-metrizable spaces in [CT1], [CT2] and [CT3]. We formulate the product theorem for the D-dimension as  $D(X \times Y) \leq D(X) \oplus D(Y)$  where the sum " $\oplus$ " is the upper sum of ordinals defined by G.H. Toulmin in [T].

For finite-dimensional spaces, the product theorem for the large inductive dimension ( $\text{Ind}$ ) —  $\text{Ind}(X \times Y) \leq \text{Ind}(X) + \text{Ind}(Y)$  — requires some restrictions over the spaces  $X$ ,  $Y$  and  $X \times Y$ : In [F] one shows compact Hausdorff spaces  $X$  and  $Y$  such that  $\text{Ind}(X)=1$ ,  $\text{Ind}(Y)=2$  and  $\text{Ind}(X \times Y) \geq 4$ . Also, M.L. Wage in [W] gives a normal space  $Z$  such that  $\text{Ind}(Z)=0$  and  $\text{Ind}(Z \times Z) > 0$ . These examples are valid for the D-dimension because this transfinite dimension coincides with the large inductive dimension in the finite case.

For metrizable spaces, D.W. Henderson shows in [He] that for this class of spaces the product theorem holds. In the general case we can formulate the next results:

**THEOREM 1.** *If  $X$  and  $Y$  are non-empty strongly hereditarily normal spaces and  $X \times Y$  is strongly paracompact and strongly hereditarily normal, then,  $D(X \times Y) \leq D(X) \oplus D(Y)$ .*

**THEOREM 2.** *If  $X$  is a non-empty perfectly normal space and  $Y$  is a non-empty metrizable space,  $D(X \times Y) \leq D(X) \oplus D(Y)$ .*

THEOREM 3. For every non-empty paracompact space  $X$  and every non-empty compact space  $Y$  such that  $X \times Y$  is totally normal,  
 $D(X \times Y) \leq D(X) \oplus D(Y)$ .

THEOREM 4. Let  $X$  and  $Y$  be two non-empty topological spaces such that  $X \times Y$  is completely paracompact and totally normal. Then,  
 $D(X \times Y) \leq D(X) \oplus D(Y)$ .

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