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D-DIMENSION IN NON-METRIZABLE SPACES

by

Regino Criado and Juan Tarrés

Departamento de Geometría y Topología. Facultad de C. Matemáticas
Universidad Complutense. 28040, MADRID

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In this paper we generalize to the non-metrizable spaces the D-dimension introduced by D.W. Henderson in [5] for metrizable spaces. This dimension is a transfinite extension of the large inductive dimension "Ind" for normal (T_4) spaces. We stablish the subspace and the locally finite sum theorems for strongly hereditarily normal spaces; other properties of the D-dimension are stablished for paracompact and perfectly normal (T_{5*}) spaces.

For every ordinal number α, let $\lambda(\alpha$ be the largest limit ordinal which is $\leq\alpha$, and let $n(\alpha)$ be the finie ordinal such that $\alpha=\lambda(\alpha)+n(\alpha)$. We considere the extra symbols Δ and -1 with the conventions that for each ordinal α we have $-1<\alpha<\Delta$ and $\lambda(-1)=0$, $\lambda(\Delta)=\Delta$, $n(-1)=-1$, $n(\Delta)=0$.

Definition 1. *Let X be a T_4-space and β an ordinal or -1 with $\lambda(\beta)=\gamma$. A* ***β-D-representation*** *of X is an expression:*

$$X = \bigcup_{0\leq\alpha\leq\gamma} A_\alpha$$

such that:

a) For $0\leq\alpha\leq\gamma$, A_α is a closed set of X with $Ind(A_\alpha)<\infty$.

b) For every $\delta\leq\gamma$ the set $\bigcup\{A_\alpha / \delta\leq\alpha\leq\gamma\}$ is closed in X.

c) $Ind(A_\gamma)=n(\beta)$.

d) For every $x\in X$ there exists a largest ordinal δ such that $x\in A_\delta$.

Definition 2. *For a T_4-space X, the* ***D-dimension*** *of* ***X*** *- D(X) - is defined as an ordinal, -1 or Δ according the following conditions:*

D1. $D(X)=-1$ if and only if $X=\emptyset$.

D2. If $X\neq\emptyset$, D(X) is the smallest ordinal β (if any exists) such that X has a β-D-representation.

D3. If $X\neq\emptyset$ and no such β exists we set $D(X)=\Delta$.

It is clear that the D-dimension is a transfinite extension of the large inductive dimension in the sense of that if either Ind(X) or D(X) is finite, they are equal.

Theorem 1. (Subspace theorem). *If X is a strongly hereditarily normal space and $M\subset X$, $D(M)\leq D(X)$.*

Theorem 2. (Locally finite sum theorem). *Let X be a strongly hereditarily normal space which is the union of a locally finite family Ω of closed sets such that for each $C\in\Omega$, $D(C)\leq\beta$. Then, we have $D(X)\leq\beta$.*

Corollary. *If a strongly hereditarily normal space X is the union of a finite family Ω od closed sets we have $D(X)=$***máx***$\{D(C)\mid C\in\Omega\}$.*

A relation between the D-dimension and the large transfinite inductive dimension "trInd" is given in the next theorem:

Theorem 3. *Let X be a paracompact, T_{5a}-space. If trInd(X) exists, $trInd(X)\leq D(X)$.*

For an ordinal α let $|\alpha|$ be the cardinal of α. We denote by w(X) the weight of the space X. Now, we have:

Theorem 4. *For a paracompact, T_{5a}-space, if $D(X)<\Delta$, $|D(X)|\leq w(X)$.*

REFERENCES

[1] C.H. DOWKER. *"An imbedding theorem for paracompact metric spaces"*. Duke Math. Journal **14**(1947), 639-645.

[2] R. ENGELKING. *"General Topology"*. North-Holland. Amsterdam, Varszawa. 1977.

[3] ------- *"Dimension Theory"*. North-Holland. Amsterdam, Varszawa. 1978.

[4] ------- *"Transfinite Dimension"*. Surveys in General Topology, ed. G.M. Reed. New York (1980), 131-161.

[5] D.W. HENDERSON. *"D-Dimension I. A New Transfinite Dimension"*. Pacific Journ. Math. **26**(1968), 91-107.

[6] ------- *"D-Dimension II. Separable spaces and Compactifications"*. Pacific Journ. Math. **26**(1968), 109-113

[7] F. HAUSDORFF. *"Set Theory"*. 2^{nd} Ed. Chelsea Publ. Co. New York. 1962.

[8] W. HUREWICZ and H. WALLMAN. *"Dimension Theory"*. Princeton Univ Press. 1941.

[9] L.A. LUXEMBURG. *"On Compactifications of Metric Spaces with Transfinite Dimensions"*. Pacific J. Math. **101**(1982) 399-450

[10] ------- *"On Universal Infinite-Dimensional Spaces"*. Fund Math. **122**(1984), 129-147

[11] NAGATA, J.I. *"Modern Dimension Theory"* North-Holland. Amsterdam. 1965

[12] ------- *"A Survef of Dimension Theory"*. Gen. Top. Appl. **1**(1971), 65-77.

[13] E. POL and R. POL. *"A Hereditarily Normal Strongly Zero-Dimensional Space Containing Subspaces of Arbitrarily large Dimension"*. Fund. Math. **102**(1979), 136-142.

[14] A.R. PEARS. *"Dimension Theory of General Spaces"*. Cambridge Univ. Press. 1975.

[15] L.POLKOWSKI. *"On Transfinite Dimension"*. Coll. Math. **50**(1985) 61-79.

[16] W. SIERPINSKI. *"Cardinal and Ordinal Numbers"*. 2ª Ed. P.W.N. Varszawa. 1965.

[17] G.H. TOULMIN. *"Shuffling Ordinals and Transfinite Dimension"*. Proc. London Math. Soc. **4**(1954), 177-195.