

ON MAXIMAL SUBALGEBRAS OF CENTRAL SIMPLE MALCEV ALGEBRAS*

Alberto Elduque

Departamento de Matemáticas I

E.T.S.I.I.Z., Edificio Interfacultades

Universidad de Zaragoza, 50009 ZARAGOZA

Malcev algebras (see [2] and [3]) are an extension of Lie algebras in the same way as alternative algebras with respect to associative ones. Central simple Malcev algebras are either Lie algebras or seven-dimensional algebras obtained from the Cayley-Dickson algebras. Actually, if C is a Cayley-Dickson algebra, over a field of characteristic different from two and three, with multiplication denoted by juxtaposition, and C^- is the algebra defined over the same vector space structure of C but with new multiplication $[x,y] = xy - yx$, then C^- is a direct sum of its one dimensional centre, $F1$, and a seven dimensional ideal which is a simple non-Lie Malcev algebra. Moreover, any central simple non-Lie Malcev algebra may be obtained in this way.

In this paper, the structure of the maximal elements of the lattice of subalgebras of such algebras is considered. Such maximal subalgebras are studied in two ways: first by using theoretical results concerning Malcev algebras, and second by using the close connection between these simple non-Lie Malcev algebras and the Cayley-Dickson algebras, which have been extensively studied (see [4]). Our main result is:

THEOREM: Let M be a subalgebra of the central simple non-Lie Malcev algebra A . Then M is a maximal subalgebra of A if and only if it is either a three-dimensional non split simple Lie algebra or it verifies $M = V \dot{+} S$ with S a subalgebra isomorphic to the Lie algebra of 2×2 traceless

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matrices and V is an abelian ideal of M which is an S -module of type M_2 (in the notation of [1]).

The associated quadratic form to any Cayley-Dickson algebra plays an important role in the proof of this Theorem.

As a consequence of this result we get that the intersection of the maximal subalgebras of any simple non-Lie Malcev algebra is trivial.

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