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IN A COSTLY EXTERNAL FINANCE MODEL**

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**THE DEFAULT RATE AND PRICE OF CAPITAL
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Resumen

Para enriquecer los mecanismos de transmisión en la macroeconomía, se han incorporado las fricciones financieras. Sin embargo, las predicciones de los modelos de ciclos económicos reales con financiamiento externo costoso implican que la tasa de morosidad, el premio por riesgo y el precio relativo del capital son procíclicos, lo que aparece como contradictorio con los datos. En este trabajo se incluyen las fluctuaciones tecnológicas que afectan la productividad promedio y el riesgo idiosincrásico de los productores de capital en un modelo estándar de financiamiento externo costoso. Estos elementos permiten que en el modelo la tasa de morosidad, el premio por riesgo y el precio relativo del capital sean contracíclicos. Esto es más coherente con los datos y es contrario al resultado que se obtiene cuando sólo priman fluctuaciones de productividad neutrales al sector de la economía. Intuitivamente, si los proyectos de inversión de los empresarios- productores de capital llegan a ser más productivos en promedio, el precio relativo del capital y la tasa de morosidad caen, mientras que la inversión y el producto aumentan. Usando datos sobre el precio relativo del capital, este trabajo realiza una calibración de este tipo de fluctuaciones lo cual resalta su relevancia para los ciclos económicos.

Abstract

Financial frictions have been used to enrich mechanism transmission in macroeconomics. However, the predictions of real business cycle models of costly external finance imply a procyclical default rate, external premium and relative price of capital which seems at odd with the data. In this article, we include technology shocks that affect the average productivity and idiosyncratic risk of capital producers in a standard costly external finance model. These elements enhance the model to deliver a countercyclical default rate, external finance and relative price of capital premium which is more consistent with the data and contrary to the result obtained with a sector neutral productivity shock. Intuitively, if the entrepreneurs' investment projects become more productive in average, the relative price of capital and the default rate fall while the investment and output increase. Using data on the relative price of capital, we perform a calibration of this type of shocks which highlights its business cycle relevance.

1 Introduction

The presence of financial frictions have been used as a key element to improve the quantitative performance of economic models. In macroeconomics, the existence of financial markets imperfections have enhanced the transmission mechanisms in business cycle models helping to replicate the empirical responses of aggregate variable to the shocks of the economy.¹ In finance, incompleteness in the financial markets has resulted in a higher equity premium which is more in line with the level observed in the data.²

In this article, we analyze quantitatively some asset pricing properties of a real business cycle model with costly external finance. There are several reasons to focus on a costly external finance model. First, this type of financial friction in a standard dynamic macroeconomic model offers a rationale for the amplifications and persistence of shocks observed in the macroeconomic aggregate variables. Second, models of this nature are consistent with the corporate finance literature which has justified the imperfect substitution between internal and external funds.³ Moreover, this gap of cost for the external funds over the internal funds has generated empirical research testing if cash flow, leverage and other balance-sheet factors have effects on the investment decisions of firms beyond their implicit information about investment opportunities.⁴ The result of this empirical literature has been to argue that financial frictions are important in the investment decisions. Third, the basic costly external finance can be justified through the “costly state verification” problem first analyzed by Townsend (1979). The advantages of this model are its simplicity and descriptive realism which allow it to be embedded

¹Some prominent examples of this propagation features are Bernanke and Gertler (1989), Bernanke et al. (1996, 1999), Carlstrom and Fuerst (1997) and Kiyotaki and Moore (1997).

²For instance, Mankiw (1986), Constantinides and Duffie (1996) and Lustig (2001) use incompleteness in the financial markets to obtain a high equity premium in line with the postwar data.

³Examples where the asymmetric information has been the main reason for this imperfect substitution are Gale and Hellwig (1985), Jensen and Meckling (1976) and Myers and Majluf (1984).

⁴In the empirical research about investment and financial constraints we can find examples of this idea in Fazzari et al. (1988) and Hoshi et al. (1991).

inside of a dynamic general equilibrium framework.

Hence, if the reason to explain the amplification and propagation mechanism at the aggregate is associated with costly external finance it seems natural to see whether the assets pricing fluctuations in this kind of model are quantitatively appealing. Gomes et al. (2003a) have argued that using a costly external finance model can give a higher mean and volatility of the equity premium than other standard real business cycle models. However, their results show that the size of the equity premium is still very low compared with the data and the propagation mechanism is driven by a procyclical default rate and external premium which is a property that seems at odd with the data. They argue that these findings cast doubt on the presence of financial frictions as a realistic channel for the propagation mechanism in macroeconomics models.

In this article, we show that the inclusion of changes in the average productivity and idiosyncratic risk of capital producers in the basic model of costly external finance (see Carlstrom and Fuerst (1997)) can give a source of countercyclical default rate and external premium which is consistent with the data. The intuition of this result is as follows. For instance, when the capital producers become more productive in average the supply of new capital expands driving down the equilibrium price of capital, the default rate and the external premium. At the same time, investment and consequently output expand. A similar effect has a reduction in the dispersion of the productivity of capital producers.

Using US data on the relative price of capital we calibrate the stochastic process of the aggregate and capital specific productivity change. This calibration confirms the importance of the capital specific technological change in this costly external finance model. On one hand, these types of fluctuations are required to move the supply of investment goods as a way to obtain a countercyclical relative price of capital. On the other hand, if the aggregate productivity fluctuations are eliminated, the capital specific

productivity fluctuations can explain about 30% of the volatility in the US total output.

The rest of the article is organized as follows. In section 2, we describe the model with emphasis on the financial contract between entrepreneurs and financial intermediaries. After a base parameterization of the model in section 3, we analyze the response of the economy to different sources of fluctuations in section 4. In section 5, we make the final calibration of the capital specific technological change using some dynamic properties of the relative price of capital in US. The final section 6 concludes and describes directions for future research. Appendix A derives some functions related with the financial contract while appendix B describes the log-linearized system of equations used to solve and simulate the model.

2 Model

The model presented in this section is based on Carlstrom and Fuerst (1997). The framework is a standard neoclassical model with a costly external finance driven by endogenous agency costs. This element introduces financing constraints that contribute to distort the optimal capital accumulation and thus generate a model with a much richer set of dynamics. The economy consists of continuum of consumers of unit mass. A fraction $(1 - \eta)$ are households and fraction η are entrepreneurs. The latter consumers produce capital goods and use external funds to finance this activity. There are a set of competitive financial intermediaries that provide funds to the entrepreneurial sector. Finally, there are competitive final goods producers that do not face any financing constraint. We can now examine in detail each of these agents.

2.1 Households

The households are infinitely lived with preferences given by:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t, 1 - l_t)\right]$$

where $\mathbb{E}_0[\cdot]$ denotes the expectation operator conditional on the time 0 information, $\beta \in (0, 1)$ is the subjective discount factor, c_t and l_t are the consumption and fraction of the time that households work at t , respectively. In each period the household derives income from renting the labor services and capital holdings at a competitive rates, w_t and r_t , respectively. Also, they can sell the undepreciated capital. This income is used to purchase consumption goods and capital for the next period such that household budget constraint is:

$$c_t + q_t a_{t+1}^h = w_t l_t + (r_t + q_t(1 - \delta))a_t^h$$

where a_{t+1}^h , q_t and δ are the household capital holding for the next period, relative price of capital and the depreciation rate, respectively. The optimal choices of household can be summarized in the following first order conditions:

$$u_1(c_t, 1 - l_t)w_t = u_2(c_t, 1 - l_t)$$

$$q_t u_1(c_t, 1 - l_t) = \beta \mathbb{E}_t[u_1(c_{t+1}, 1 - l_{t+1})(r_{t+1} + q_{t+1}(1 - \delta))]$$

The first equation is the household labor supply. The second one is the Euler equation that governs the household intertemporal substitution using capital holding to move resources across periods.

2.2 Entrepreneurs

The entrepreneurs are able to produce capital goods and live infinitely with preferences characterized by:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta\gamma)^t c_t^e\right]$$

The entrepreneurs are different to the household in two dimensions. They are risk neutral and more impatient than households ($\gamma < 1$). Risk neutrality implies that they only care about expected returns which in turn will ensure that they will bear all risk. That simplifies the financial contract explained below considerably. The extra discount factor prevents that they are wealthy enough to overcome the financing constraint in equilibrium. At the beginning of each period they have a net worth given by renting labor and capital to the final good producers at competitive rates, w_t^e and r_t , respectively.⁵ They also can sell the undepreciated capital such that their net worth can be written as:

$$n_t = w_t^e + (r_t + q_t(1 - \delta))a_t^e$$

where a_t^e is the capital holdings of the entrepreneur at the beginning of the period.

Within the period each entrepreneur can earn additional income by investing i_t units of consumption goods in a technology that produces $\theta_t i_t$ of capital goods at the end of the period. It is assumed that θ_t is a random variable independent and identically distributed across entrepreneurs, but not over time with cumulative distribution and probability density functions $F_t(\theta_t) = F(\theta_t, \lambda_t)$ and $f_t(\theta_t) = f(\theta_t, \lambda_t)$, respectively. λ_t is a vector of parameters that defines the distribution of θ_t . Following the costly state verification literature, it is considered that θ_t is observed only by the entrepreneur at the end of the period. It can be observed by outsiders at a cost of μi_t units of capital goods. In the beginning of the period entrepreneurs and outsiders know the distribution

⁵Since leisure does not enter in the entrepreneur preferences, they work all their available time.

of θ_t but the entrepreneurs learn their types in the end of the period without any cost.

Investment is financed by borrowing funds from financial intermediaries if needed. However, the asymmetric information about the productivity of the entrepreneur makes the external finance costly. Gale and Hellwig (1985) in a static setting show that the optimal financial contract between a lender and an entrepreneurs resembles a risky debt. Here to keep this type of contract is that is eliminated any repeated game aspects of the financial contract assuming that the contract can be only based on the current level of net worth and investment. Also, we assume that the financial markets are incomplete. So, they cannot write contracts that are functions of all public information.⁶

Let r_t^l denote the lending rate of this risky debt in terms of capital goods. Therefore, the contract at t states that if entrepreneurs borrows $(i_t - n_t)$ he commits to repay $(1 + r_t^l)(i_t - n_t)$ in terms of capital goods. However, if the realization of θ_t is too low the entrepreneur will not able to repay and must default. In other words, this kind of contract determines a cutoff $\bar{\theta}_t$ such that:

- if $\theta_t < \bar{\theta}_t$, the entrepreneur defaults, the lender monitors the project outcome. It follows that the entrepreneur sets $c_t^e, a_{t+1}^e = 0$
- if $\theta_t > \bar{\theta}_t$, the entrepreneur repays $(1 + r_t^l)(i_t - n_t) = \bar{\theta}_t i_t$ and his budget constraint is $q_t a_{t+1}^e + c_t^e = q_t(\theta_t i_t - (1 + r_t^l)(i_t - n_t))$

With this financial contract the euler equation for the entrepreneur can be expressed as:

$$q_t = \beta \gamma \mathbb{E}_t[(r_{t+1} + q_{t+1}(1 - \delta))R_{t+1}^n(n_{t+1}, i_{t+1})]$$

⁶In a partial equilibrium framework, Albuquerque and Hopenhayn (2002) and Clementi and Hopenhayn (2002) characterize the full dynamic optimal contract between a financial constrained entrepreneur and a lender when the financial frictions come from enforceability problems and asymmetric information, respectively. The contracts are intertemporal optimal and based on all public information.

where $R_t^n(n_t, i_t)$ is the expected return to the internal funds at the beginning of period t in terms of consumption goods given a net worth of n_t and an investment of i_t . Using the financing contract we can get:

$$R_t^n(n_t, i_t) = \frac{q_t}{n_t} \mathbb{E}_t[(\theta_t i_t - (1 + r_t^l)(i_t - n_t))1\{\theta_t \geq \bar{\theta}_t\}] = \frac{q_t i_t}{n_t} \mathbb{E}_t[(\theta_t - \bar{\theta}_t)1\{\theta_t \geq \bar{\theta}_t\}]$$

where $1\{\theta_t \geq \bar{\theta}_t\}$ is the indicator function that is equal to 1 if $\theta_t \geq \bar{\theta}_t$ and zero otherwise.

2.3 Financial Intermediaries

Financial intermediaries allocate household savings by financing entrepreneur investment project. By funding a large number of entrepreneurs, the intermediaries diversify project specific risk and, thus, guarantee a safe return to the households since there is no aggregate risk during the life of the project.

Now we can get the expected income of intermediaries after financing a project of size i_t with a loan of $(i_t - n_t)$:

$$\begin{aligned} q_t i_t g_{1t}(\bar{\theta}_t) &= q_t \left[\int_0^{\bar{\theta}_t} \theta i_t f_t(\theta) d\theta - \mu F_t(\bar{\theta}_t) i_t + (1 - F_t(\bar{\theta}_t))(1 + r_t^l)(i_t - n_t) \right] \\ q_t i_t g_1(\bar{\theta}_t, \lambda_t) &= q_t i_t \left[\int_0^{\bar{\theta}_t} \theta f_t(\theta) d\theta - \mu F_t(\bar{\theta}_t) + (1 - F_t(\bar{\theta}_t)) \bar{\theta}_t \right] \\ &= q_t i_t \left[\int_0^{\bar{\theta}_t} \theta f(\theta, \lambda_t) d\theta - \mu F(\bar{\theta}_t, \lambda_t) + (1 - F_t(\bar{\theta}_t, \lambda_t)) \bar{\theta}_t \right] \end{aligned}$$

where g_1 denotes the fraction of the expected net production of capital goods received by the financial intermediary.

Similarly, the expected income received by the entrepreneur is:

$$\begin{aligned}
q_t i_t g_{2t}(\bar{\theta}_t) &= q_t \left[\int_{\bar{\theta}_t}^{\infty} \theta i_t f_t(\theta) d\theta - (1 - F_t(\bar{\theta}_t))(1 + r_t^l)(i_t - n_t) \right] \\
q_t i_t g_2(\bar{\theta}_t, \lambda_t) &= q_t i_t \left[\int_{\bar{\theta}_t}^{\infty} \theta f_t(\theta) d\theta - (1 - F_t(\bar{\theta}_t))\bar{\theta}_t \right] \\
&= q_t i_t \left[\int_{\bar{\theta}_t}^{\infty} \theta f(\theta, \lambda_t) d\theta - (1 - F(\bar{\theta}_t, \lambda_t))\bar{\theta}_t \right]
\end{aligned}$$

where g_2 denotes the fraction of the expected net production of capital goods received by the entrepreneur.

We can check that $g_1(\bar{\theta}_t, \lambda_t) + g_2(\bar{\theta}_t, \lambda_t) = E_t[\theta_t] - \mu F(\bar{\theta}_t, \lambda_t)$ so that an amount $\mu F(\bar{\theta}_t, \lambda_t)$ of the capital produced is lost due to monitoring cost.⁷

The optimal contract is determined by solving the following problem:

$$\begin{aligned}
&\max_{\bar{\theta}_t, i_t} q_t i_t g_2(\bar{\theta}_t, \lambda_t) \\
&\text{s.t.} \\
&q_t i_t g_1(\bar{\theta}_t, \lambda_t) \geq (i_t - n_t) \quad (\text{P1}) \\
&q_t i_t g_2(\bar{\theta}_t, \lambda_t) \geq n_t \quad (\text{P2})
\end{aligned}$$

where (P1) and (P2) are the participation constraint for financial intermediaries and entrepreneurs, respectively.⁸ (P1) will be binding while (P2) will not be binding. This conclusion comes from the fact that there are many competitive financial intermediaries and hence they must break even at the optimal contract.

⁷In appendix A is described g_1 , g_2 , F and f for the case that θ_t has a log-normal distribution function which is the case used in the calibration.

⁸Recall that the financial contract is within the period so that opportunity cost for the funds of entrepreneurs and financial intermediaries is zero.

Since only (P1) is binding the maximization above can be written as:

$$\max_{\bar{\theta}_t} \frac{q_t g_2(\bar{\theta}_t, \lambda_t) n_t}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)}$$

where the FOC with respect to $\bar{\theta}_t$ is

$$\left[\frac{\partial g_2}{\partial \bar{\theta}}(\bar{\theta}_t, \lambda_t) \right] (1 - q_t g_1(\bar{\theta}_t, \lambda_t)) + q_t \left[\frac{\partial g_1}{\partial \bar{\theta}}(\bar{\theta}_t, \lambda_t) \right] g_2(\bar{\theta}_t, \lambda_t) = 0$$

Now using $g_1(\bar{\theta}_t, \lambda_t) + g_2(\bar{\theta}_t, \lambda_t) = \mathbb{E}_t[\theta_t] - \mu F(\bar{\theta}_t, \lambda_t)$ we can rewrite the FOC as:

$$1 = q_t \left[\mathbb{E}_t[\theta_t] - \mu F(\bar{\theta}_t, \lambda_t) + \mu f(\bar{\theta}_t, \lambda_t) \frac{g_2(\bar{\theta}_t, \lambda_t)}{\frac{\partial g_2}{\partial \bar{\theta}}(\bar{\theta}_t, \lambda_t)} \right]$$

This equation defines an implicit relationship among q_t , $\bar{\theta}_t$ and λ_t as $\bar{\theta}_t = \bar{\theta}(q_t, \lambda_t)$. From that last equation can be proved that $\frac{\partial \bar{\theta}}{\partial q_t} > 0$. Also, given our parameterization of the distribution of θ_t , we can express $\mathbb{E}_t[\theta_t]$ as function $m(\lambda_t)$.

The external premium in terms of consumption good is $q_t(1 + r_t^l) - 1$ which can be expressed as $\bar{\theta}_t/g_1(\bar{\theta}_t, \lambda_t) - 1$. From here we can see that for a given λ_t the external premium is increasing in $\bar{\theta}_t$.⁹ This is important since a procyclical external premium is equivalent to have a procyclical probability of default if the distribution of θ is invariant to the cyclical position of the economy. This is the case in Carlstrom and Fuerst (1997). However, if the distribution of θ changes with the cyclical position of the economy we do not know how the external premium moves with the business cycles clearly. A critical element in this model is going to be how the set of parameters of the distribution of θ moves along the business cycles.

Other financial statistic that we can derive from this model is the default rate which

⁹From the definition of $g_1(\bar{\theta}_t, \lambda_t)$ it can be derived that $\frac{\partial g_1(\bar{\theta}_t, \lambda_t)}{\partial \bar{\theta}_t}$ is negative which guarantees that $\frac{\partial \bar{\theta}_t/g_1(\bar{\theta}_t, \lambda_t)}{\partial \bar{\theta}_t} > 0$.

is defined by $F(\bar{\theta}_t, \lambda_t)$. Using the definition of $g_2(\cdot)$ we obtain:

$$\begin{aligned} \mathbb{E}_t [(\theta_t - \bar{\theta}_t)1\{\theta_t \geq \bar{\theta}_t\}] &= \int_{\bar{\theta}_t}^{\infty} (\theta - \bar{\theta}_t)f(\theta, \lambda_t)d\theta \\ &= \int_{\bar{\theta}_t}^{\infty} \theta f(\theta, \lambda_t)d\theta - (1 - F(\bar{\theta}_t, \lambda_t))\bar{\theta}_t \\ &= g_2(\bar{\theta}_t, \lambda_t) \end{aligned}$$

which be can used to express the return of internal funds to the entrepreneur as:

$$R_t^n(i_t, n_t) = \frac{q_t i_t g_2(\bar{\theta}_t, \lambda_t)}{n_t} = \frac{q_t g_2(\bar{\theta}_t, \lambda_t)}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)}$$

where the second equality comes from using (P1) with equality.

Finally, using the fact (P1) is binding we can express the investment as a function of the net worth, the cutoff and the vector of parameters of the distribution or as a function of the net worth, the price of the capital and the vector of parameters of the distribution:

$$i_t = \frac{n_t}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)} = \frac{n_t}{1 - q_t g_1(\bar{\theta}(q_t, \lambda_t), \lambda_t)}$$

This equation can be interpreted as the supply of investment goods obtained from this costly external finance model. For fixed values for n_t and λ_t , this investment supply will in general be increasing in q_t .

2.4 Final Goods Producers

The final set of agents are the final good producers. They are competitive firms using constant return to scale technology :

$$Y_t = \exp(z_t)G(K_t, H_t, H_t^e)$$

where K_t is the aggregate level of capital in the economy in period t , H_t is the aggregate supply of household labor, H_t^e is the aggregate supply of entrepreneurial labor and z_t is the aggregate sector neutral productivity factor. The optimality conditions implies the following equations:

$$r_t = \exp(z_t)G_1(K_t, H_t, H_t^e)$$

$$w_t = \exp(z_t)G_2(K_t, H_t, H_t^e)$$

$$w_t^e = \exp(z_t)G_3(K_t, H_t, H_t^e)$$

The first expression defines the rental rate of capital. The second one determines the household labor demand while the last one specifies the entrepreneur labor demand.

2.5 Aggregation

By the law of large numbers, the aggregate investment at t is the expected value of the production of capital goods minus the monitoring cost incurred:

$$I_t = \int_0^\infty \theta i_t f(\theta, \lambda_t) d\theta - \int_0^{\bar{\theta}_t} \mu i_t f(\theta, \lambda_t) d\theta = i_t [m(\lambda_t) - \mu F(\bar{\theta}_t, \lambda_t)]$$

Similarly, aggregating across the entrepreneurs' budget constraint we can write:

$$q_t A_{t+1}^e + \eta c_t^e = [\eta w_t^e + A_t^e (r_t + q_t(1 - \delta))] \frac{q_t g_2(\bar{\theta}_t, \lambda_t)}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)}$$

where A_t^e is the aggregate capital holding of entrepreneurs at the beginning of t and with some abuse of notation c_t^e is now the average entrepreneurial consumption. Finally, given the linearity of the investment as function of the net worth and the mapping between the price of capital and $\bar{\theta}_t$ we have:

$$n_t = w_t^e + \frac{A_t^e}{\eta} [r_t + q_t(1 - \delta)]$$

$$i_t = \frac{n_t}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)}$$

where now n_t and i_t are the average entrepreneurial net worth and investment.

2.6 Equilibrium Conditions

A competitive equilibrium satisfies the following markets clearing conditions:

$$H_t = (1 - \eta)l_t$$

$$H_t^e = \eta$$

$$(1 - \eta)c_t + \eta c_t^e + \eta i_t = Y_t$$

$$K_{t+1} = (1 - \delta)K_t + \eta i_t [m(\lambda_t) - F(\bar{\theta}_t, \lambda_t)\mu]$$

A recursive competitive equilibrium is defined by decision rules for K_{t+1} , A_{t+1}^e , H_t , n_t , i_t , $\bar{\theta}_t$, c_t^e , c_t , and pricing functions q_t , w_t , w_t^e , r_t , where these decision rules and pricing functions are invariant functions of $(K_t, A_t^e, z_t, \lambda_t)$ and satisfy the following equations:

$$u_1(c_t, 1 - l_t)w_t = u_2(c_t, 1 - l_t) \quad (1)$$

$$q_t u_1(c_t, 1 - l_t) = \beta \mathbb{E}_t [u_1(c_{t+1}, 1 - l_{t+1})(r_{t+1} + q_{t+1}(1 - \delta))] \quad (2)$$

$$K_{t+1} = (1 - \delta)K_t + \eta i_t [m(\lambda_t) - F(\bar{\theta}_t, \lambda_t)\mu] \quad (3)$$

$$q_t [m(\lambda_t) - \mu F(\bar{\theta}_t, \lambda_t) + \mu f(\bar{\theta}_t, \lambda_t) \frac{g_2(\bar{\theta}_t, \lambda_t)}{\frac{\partial g_2}{\partial \theta}(\bar{\theta}_t, \lambda_t)}] = 1 \quad (4)$$

$$(1 - \eta)c_t + \eta c_t^e + \eta i_t = Y_t = \exp(z_t)G(K_t, H_t^e, H_t) \quad (5)$$

$$i_t = \frac{n_t}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)} \quad (6)$$

$$n_t = w_t^e + \frac{A_t^e}{\eta} [r_t + q_t(1 - \delta)] \quad (7)$$

$$A_{t+1}^e = \frac{\eta n_t g_2(\bar{\theta}_t, \lambda_t)}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)} - \frac{\eta c_t^e}{q_t} \quad (8)$$

$$q_t = \beta \gamma \mathbb{E}_t [(r_{t+1} + q_{t+1}(1 - \delta)) \frac{q_{t+1} g_2(\bar{\theta}_{t+1}, \lambda_{t+1})}{1 - q_{t+1} g_1(\bar{\theta}_{t+1}, \lambda_{t+1})}] \quad (9)$$

$$H_t = (1 - \eta)l_t \quad (10)$$

$$r_t = \exp(z_t)G_1(K_t, H_t, \eta) \quad (11)$$

$$w_t = \exp(z_t)G_2(K_t, H_t, \eta) \quad (12)$$

$$w_t^e = \exp(z_t)G_3(K_t, H_t, \eta) \quad (13)$$

3 Based Calibration

3.1 Preferences, Technologies and Financial Parameters

The base calibration is designed to make a simple comparison among the effects on the economy of different sources of fluctuations. We follow the lines of Carlstrom and Fuerst (1997) in almost all parameters except for some financial statistics. We begin assuming that the utility function of consumer is :

$$u(c, 1 - l) = \ln(c) + \nu(1 - l)$$

where ν is chosen such that in steady state the households work 30% of their time and $\beta = 0.99$. This is a standard preference used in the real business cycle literature to explain quarterly US data.¹⁰

The final goods production is assumed to be of the Cobb-Douglas form:

$$Y_t = \exp(z_t)K_t^{\alpha_1}H_t^{\alpha_2}(H_t^e)^{(1-\alpha_1-\alpha_2)}$$

where $\alpha_1 = 0.36$, $\alpha_2 = 0.6399$. The share of the entrepreneurial labor is chosen so small such that labor income plays a very irrelevant role both in determining net worth and income distribution in this model. z_t is the final good production technological change. The stochastic process for this exogenous variable will be explained below.

Regarding the parameters that defines the financial contract as in Carlstrom and

¹⁰See Hansen (1985).

Fuerst (1997) we use a monitoring cost $\mu = 0.25$. Also, the distribution of θ_t is assumed to be log-normal. Then the set of parameters that define the distribution of θ in t are mean m_t and variance σ_t^2 . The specific stochastic process for these two variables will be described in the next subsection. We assume as normalization that at the steady state $m = 1$. Hence, to match the default rate and the external premium we just need to pin down $\bar{\theta}$ and σ at the steady state. Using a default premium of 203 basis points and a default rate of 0.974% we can get $\bar{\theta} = 0.1361$, $\sigma = 0.6590$. These values imply a steady state relative price of capital of $q = 1.0861$. Also, to avoid self-financing outcomes for the entrepreneurs, we should set γ such that $\gamma q g_2(\bar{\theta}, m, \sigma^2)/(1 - q g_1(\bar{\theta}, m, \sigma^2)) = 1$. That condition gives a $\gamma = 0.9111$. Finally, the depreciation rate δ is set in 2%.

3.2 Stochastic Process for Shocks

The technological shocks follow a joint autoregressive process:

$$\begin{pmatrix} z_t \\ \ln(m_t) \\ \ln(\sigma_t^2/\sigma^2) \end{pmatrix} = \begin{pmatrix} \rho_z & \rho_{zm} & \rho_{z\sigma} \\ \rho_{mz} & \rho_m & \rho_{m\sigma} \\ \rho_{\sigma z} & \rho_{\sigma m} & \rho_\sigma \end{pmatrix} \begin{pmatrix} z_{t-1} \\ \ln(m_{t-1}) \\ \ln(\sigma_{t-1}^2/\sigma^2) \end{pmatrix} + \begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{m,t} \\ \varepsilon_{\sigma,t} \end{pmatrix}$$

where we consider that $\varepsilon_{z,t}$, $\varepsilon_{m,t}$, and $\varepsilon_{\sigma,t}$ have a joint normal distribution independent and identically distributed over time. Also, the mean of these perturbations are zero and are independent each other with variance denoted by v_z^2 , v_m^2 , and v_σ^2 , respectively.

In the real business cycle literature when the final good technological change is the only source of exogenous fluctuations is typically used $\rho_z = 0.95$ and $v_z = 0.712\%$. Unfortunately, we do not have unambiguous data to obtain values for other parameters governing the joint stochastic process of these exogenous fluctuations. For example, being consistent with this model, data on the fluctuations of the relative price of capital or

the default premium contain information of the three shocks considered here and there are multiple ways to decompose the fluctuations of the endogenous variables as coming from these shocks.

Since the parameters in the stochastic process of the exogenous variables will imply a particular dynamic of the endogenous variables one method to get these parameters is to choose them such that the model matches some moments of the endogenous variables observed in the data. However, there is a large number of potential moments that can be used and an equally large set of parameters to be calibrated (8 in the autoregressive matrix and 2 variances). Hence, it seems important to understand the quantitative dynamics in a simpler context of the stochastic process. On these lines, in the next section we will analyze the impulse response functions in a constrained case which fixes $\rho_{ij} = 0$ for all $i, j = z, m, \sigma$ and $i \neq j$. In other words, there are no spillover effects among exogenous fluctuations.

4 Impulse Responses for a Simple Case

After having the base calibration of the model we can make numerical analysis using the well known method of taking a log-linear expansion of the equations of the model around the deterministic steady state. Then the log-linear decision rules are computed using the method of undetermined coefficients. Having this we are ready to compute the impulse response functions to the three source of fluctuations: (i) aggregate-sector-neutral productivity changes (z); (ii) changes in the average productivity of capital producer (m) and; (iii) changes in the idiosyncratic risk of the capital producer technology (σ^2).

The impulse responses are computed with an initial shock that deviates each one of the exogenous variables 1% from the steady state. Since we did not choose a particular value for ρ_m and ρ_σ we consider two cases: (a) $\rho_m = \rho_\sigma = 0.9$; and (b) $\rho_m = \rho_\sigma = 0$.

The first case describes a highly persistent evolution of these exogenous variables and the second considers a path that is independent over time.

4.1 Shock in the Aggregate Sector Neutral Productivity

The responses of the main economic variables to a aggregate productivity shock of 1% are displayed in figures 1 and 2. The results are equivalent to Carlstrom and Fuerst (1997). The output, investment and consumption exhibit a hump-shape pattern that reflects a delayed response to this shock that is not present in the standard neoclassical model. The agency cost and the persistency of the aggregate productivity shock generate a higher autocorrelation in the output and investment growth. Output increases slowly until to reach a positive deviation from its steady state of almost 1.8% at the third quarter after the shock. Investment also displays the same path profile that output with a peak deviation of more than 5%. The relative price of capital increases a 0.8% instantaneously to die out monotonically afterwards. The external premium and the default rate increase about 25 and 15 basis points at the time of the shock having a path similar to the one of the relative price of capital.

The intuition behind this result is related with the increase in the marginal cost of investment due to the agency problem. An increase in the productivity of the final goods producers shifts out the demand for new capital and entrepreneurs then want to increase the production of capital goods. However, the increase in investment needs external funds which is costly and the net worth of entrepreneurs does not raise too much initially since they cannot adjust their capital holding until the next period. Hence, the supply of investment does not shift out too much compared to the increase in the demand of new capital which delivers a increase in the price of capital. This rise in the price of capital drives up the cutoff $\bar{\theta}_t$ and therefore it pushes up the default rate and the external premium. Although we obtain a richer propagation dynamics of the

aggregate productivity shock in this costly external finance model, the impulse responses characterize a procyclical default rate, external premium and relative price of capital.

4.2 Shock in the Average Entrepreneurial Productivity

Figures 3 and 4 depict the responses of the economy to an increase of 1% in the mean of the entrepreneurial productivity when this shock is persistent ($\rho_m = 0.9$). This can be interpreted as an exogenous force that drives up the average productivity of entrepreneurs in the production of capital goods. This change shifts out the supply curve of investment goods without affecting simultaneously the demand for investment. Hence, the investment and the equilibrium price of capital goes down which lowers the default rate and external premium. Hence, we get a countercyclical relative price of capital, default rate and external premium which is more consistent with the data.

It is worth noting that the biggest deviation in the premium and default rate is observed around the second or third quarter after the initial shock in the average productivity. In those periods the premium is 30 basis points below its steady state value while the default rate displays almost 20 basis points of reduction with respect its steady state value. This hump-shaped path of these two variables are not present under aggregate sector productivity shock (see Figure 2). Although the magnitude in output response is little lower after the shock in the average entrepreneurial productivity than after the aggregate sector productivity shock, the response in the investment has the same quantitative reach which coincides with a decrease in the external finance premium.

The responses of the economy to this same shock but for the *iid* case ($\rho_m = 0$) are displayed in figures 5 and 6. Although, the responses of the economic variables die out very fast in this case, the result still shares the basic property of having a countercyclical relative price of capital. That pattern is driven by a short period shift out of the supply of investment which in turn implies a contraction of the the default rate and external

premium.

4.3 Shock in the Variance of Entrepreneurial Productivity

Other source of fluctuation is a change in the variance of θ_t . This can be interpreted as a change in the idiosyncratic risk of entrepreneurs. An increase in σ^2 implies a rise in the idiosyncratic risk of entrepreneurs. The response of the main variables to an increase in the idiosyncratic risk of entrepreneurs for the persistent case ($\rho_\sigma = 0.9$) are depicted in figures 7 and 8.

An increase in the idiosyncratic risk of entrepreneurial productivity shift inward the supply of investment driving up the price of capital and down the investment in the equilibrium. The rise in the price of the capital pushes up the cutoff $\bar{\theta}_t$ such that the default rate and the external premium go up. Thus, we are back to the case of a countercyclical relative price of capital, default rate and external premium. Given the persistence of this shock, the effect stays for a bunch of period.

However, the quantitative results are not very significant under this type of shocks. The output and investment fall only -0.025% and -0.15%, respectively. Moreover, the hump-shaped of these two variables is not observed and only the entrepreneur net worth displayed that feature but also with a low quantitative magnitude (0.045%). These features ensure that this type of shock loses its attractiveness from a quantitative point of view. Even though if we were to scale up the path in the impulse responses, we will need an increase of 40% in the variance of the entrepreneurial productivity to get a reduction of 1% in output which would imply a big increase of 800 and 450 basis points in the premium and default rate, respectively.

When the shock to the idiosyncratic risk is *iid* we have the same pattern but as it is

expected that this effect is short lived. This case is displayed in figures 9 and 10 where the magnitude is even smaller.

5 Calibration of the Stochastic Process

From the last section we noted that only the shocks to the aggregate sector productivity and the average entrepreneurial productivity will have a relevant magnitude. Hence, in this section we will focus on calibrating the stochastic process assuming the only presence of these two shocks.

Using the notation introduced earlier we will suppose that the aggregate sector productivity (z_t) and the average of the entrepreneurial productivity ($\ln(m_t)$) follow a joint autoregressive process:

$$\begin{pmatrix} z_t \\ \ln(m_t) \end{pmatrix} = \begin{pmatrix} \rho_z & \rho_{zm} \\ \rho_{zm} & \rho_m \end{pmatrix} \begin{pmatrix} z_{t-1} \\ \ln(m_{t-1}) \end{pmatrix} + \begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{m,t} \end{pmatrix}$$

where $(\varepsilon_{z,t}, \varepsilon_{m,t})'$ have independent and identical distribution over time with mean zero and variance-covariance matrix given by:

$$\text{Var} \begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{m,t} \end{pmatrix} = \begin{pmatrix} v_z^2 & 0 \\ 0 & v_m^2 \end{pmatrix}$$

In this specification we have assumed that the spillover effects are symmetric: $\rho_{zm} = \rho_{mz}$. For simplicity we will keep the same parameters values known for the aggregate sector productivity fluctuation, i.e., $\rho_z = 0.95$ and $v_z = 0.712\%$. This implies that we need to calibrate three parameters: ρ_m , ρ_{zm} , and v_m . Hence, we will require to match three moments observed in the data to obtain these parameters.

In Table 5 we displayed some moments computed quarterly from the US economy during the period 1954:Q1 - 2004:Q1.¹¹ The variables shown are total real GDP (Y), real fixed investment (I), and the relative price of fixed investment defined as investment deflator divided by the GDP deflator. The statistics are computed using the log deviation of each series with respect to their Hodrick-Prescott filter.

Table 1: Standard Deviation and Autocorrelations with GDP

	st. dev.	x(-1)	x(0)	x(+1)
Y	1.00	0.85	1.00	0.85
I	3.01	0.79	0.90	0.84
q	0.44	-0.20	-0.10	0.03

From the table we can conclude that the relative price of capital is slightly countercyclical and a reduction in the last quarter can predict an increase in total output in the current one. Recalling the impulse responses graphed in the last section, we can see that the aggregate sector productivity shock can not produce this negative correlation with output. However, the presence of fluctuations to the average productivity of capital goods producers can induce a negative correlation between output and relative price of capital.

Also, since in this model the cyclical properties of the default premium resembles that of the relative price of capital, this result will imply a countercyclical default premium. This last property has been well documented in finance (see for example Gomes et al. (2003b)), but since this model contains a highly simplified corporate sector, it is harder to see what premium in the data is actually described by this model. Hence, we prefer to focus on the relative price of capital as the variable containing informa-

¹¹The standard deviation are expressed relative to the GDP standard deviation which is 1.6%.

tion regarding the shift in the supply and demand for investment good which in turn will convey something about the default rate and external finance premium in this model.

Finally, we consider the following moments from the data to be matched with this model: (i) the standard deviation of the relative price of capital with respect to the one of the GDP (0.44) ; (ii) the contemporaneous correlation between GDP and the relative price of capital (-0.10) and; (iii) the correlation of the GDP with one lag of the relative price of capital (-0.20). These values imply $\rho_m = 0.81$, $\rho_{zm} = 0.12$ and $v_m = 0.38\%$.

This final calibration highlights the relevance of the capital specific technological change to explain business cycle and at the same time gets plausible cyclical properties for the default rate and external premium in a model with financing constraints. If we shut down the aggregate sector productivity shocks we can get an estimate of the relative importance of the capital specific technological change in terms of the output volatility. This estimation concludes that a 30% of the fluctuations in GDP could be attributed to a capital specific technological change. Surprisingly, Greenwood et al. (2000) obtain the same result for the contribution of the capital specific technological change using a different model and methodology.

6 Final Thoughts

Financial frictions have been used to explain persistence in macroeconomics and asset pricing anomalies. Also, the empirical research on the determinants of the aggregate demand has given a important role to the credit markets imperfections.

In this article, we analyze simultaneously the quantitative implications of financial frictions on macroeconomics and their implications on the behavior of the default rate, the external finance premium and the relative price of capital. We extend a costly exter-

nal finance model to allow for more plausible cyclical properties of default rate, external premium and relative price of capital.

The basic ingredient is the inclusion of changes in the average entrepreneurial productivity or idiosyncratic entrepreneurial risk. These elements deliver a countercyclical pattern for the default rate, external premium and relative price of capital which is more consistent with the data. This is a very simple case that makes the supply of investment goods shift more than the demand for investment which implies a reduction in the relative price of capital when investment rises. The increasing relationship between the price of capital and the default rate in the model gives a source to reconcile the default rate behavior along the cyclical position of the economy. In other words, in good times when investment and output go up, the default rate, external finance premium and relative price of capital fall.

The result of countercyclical relative price of capital is very important since the US postwar aggregate fluctuations show this feature. Greenwood et al. (2000) show that investment-specific technology shocks can explain that behavior of the relative price of capital and about the 30% of output fluctuations. This article obtains the same quantitative result and offers another dimension for their observations because the shocks that induce a countercyclical price of capital are specific to the capital producers. In other words, the modification to the costly external finance model suggested here shows another way to rationalize the sectoral specific technological changes as an important source of economic fluctuations and be consistent with the countercyclical features of the relative price of capital. It is also worth noting that this cyclical behavior of these variables is obtained without affecting the propagation mechanism emphasized by the costly external finance model.

It is worth stating several directions for future research. First, the modifications to

the basic model of costly external finance were done with the purpose of getting the right directions for the cyclical behavior of the default rate. It is still the case that the technological change in the capital production are not completely understood from a economic point of view. For that reason is interesting to investigate other ways to induce a shift in the supply of investment that comes from the economic environment and not just from new exogenous state variables.

Second, this model does not offer a complete theoretical counterpart for asset pricing. In particular, it is hard how to interpret what can be called equity in this model. Gomes et al. (2003a) use the return of household capital holding as equity, but that definition is not very satisfactory. Hence, other extension would be how to introduce clearly the presence of two sources of financing: risky debt and equity in the model.

Third, having defined clearly equity in the model, it is straightforward ask whether the costly external finance model gives sensitive results in other dimension of the asset pricing. For example, it is interesting to find out what other elements should be added to have a high equity premium.

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7 Appendix A: Distribution of θ and Financial Contract Functions

As described in the article, we assume that θ follows a log-normal distribution function. In this case the set of parameters that determine the distribution can be reduced to the mean (m_t) and variance (σ_t^2) of θ_t . Let $\Phi(\cdot)$ and $\phi(\cdot)$ denote the cumulative and density function of a normal standard distribution. Then the default rate is given by:

$$F(\bar{\theta}_t, m_t, \sigma_t^2) = \Phi \left(\frac{\ln(\bar{\theta}_t) - \ln(m_t) + \frac{1}{2} \ln(\sigma_t^2/m_t^2 + 1)}{(\ln(\sigma_t^2/m_t^2 + 1))^{1/2}} \right) \quad (14)$$

The density function of θ_t can be written as:

$$f(\bar{\theta}_t, m_t, \sigma_t^2) = \Phi \left(\frac{\ln(\bar{\theta}_t) - \ln(m_t) + \frac{1}{2} \ln(\sigma_t^2/m_t^2 + 1)}{(\ln(\sigma_t^2/m_t^2 + 1))^{1/2}} \right) \frac{1}{\bar{\theta}_t (\ln(\sigma_t^2/m_t^2 + 1))^{1/2}} \quad (15)$$

The fraction of the expected net production of capital goods received by the financial intermediaries is:

$$g_1(\bar{\theta}_t, m_t, \sigma_t^2) = m_t \Phi \left(\frac{\ln(\bar{\theta}_t) - \ln(m_t) - \frac{1}{2} \ln(\sigma_t^2/m_t^2 + 1)}{(\ln(\sigma_t^2/m_t^2 + 1))^{1/2}} \right) \quad (16)$$

The fraction received by the entrepreneurs is:

$$g_2(\bar{\theta}_t, m_t, \sigma_t^2) = m_t \left[1 - \Phi \left(\frac{\ln(\bar{\theta}_t) - \ln(m_t) - \frac{1}{2} \ln(\sigma_t^2/m_t^2 + 1)}{(\ln(\sigma_t^2/m_t^2 + 1))^{1/2}} \right) \right] - \bar{\theta}_t \left[1 - \Phi \left(\frac{\ln(\bar{\theta}_t) - \ln(m_t) + \frac{1}{2} \ln(\sigma_t^2/m_t^2 + 1)}{(\ln(\sigma_t^2/m_t^2 + 1))^{1/2}} \right) \right] \quad (17)$$

8 Appendix B: Log-linearized Equations

This appendix lists the set of equations that characterizes the equilibrium and their log-linearized versions used to solve and simulate the model. The variables without a subscript t are the steady state values and the symbol ‘ $\tilde{\cdot}$ ’ denotes the log deviation of the variable with respect its steady state value.

Household-leisure decision

$$w_t/c_t = \nu$$

$$\tilde{w}_t - \sigma \tilde{c}_t = 0 \quad (1)$$

Household-Euler Equation

$$q_t/c_t = \beta \mathbb{E}_t\{(q_{t+1}(1 - \delta) + r_{t+1}^k)/c_{t+1}\}$$

$$\mathbb{E}_t\{\beta q(1 - \delta)\tilde{q}_{t+1} + \beta r^k \tilde{r}_{t+1}^k - q\tilde{c}_{t+1} - q\tilde{q}_{t+1} + q\tilde{c}_t\} = 0 \quad (2)$$

Aggregate Capital Evolution

$$K_{t+1} = (1 - \delta)K_t + \eta i_t [m_t - F(\bar{\theta}_t, m_t, \sigma_t^2)\mu]$$

$$(1 - \delta)K\tilde{K}_t + \eta i [m_t - F(\bar{\theta}, m, \sigma^2)\mu]\tilde{i}_t + \eta i [1 - \frac{\partial F}{\partial m}(\bar{\theta}, m, \sigma^2)\mu]m\tilde{m}_t - \eta i \frac{\partial F}{\partial \sigma^2}(\bar{\theta}, m, \sigma^2)\mu\sigma^2\tilde{\sigma}_t^2 - \eta i \frac{\partial F}{\partial \theta}(\bar{\theta}, m, \sigma^2)\mu\tilde{\theta}_t - K\tilde{K}_{t+1} = 0 \quad (3)$$

Aggregate Expenditure

$$(1 - \eta)c_t + \eta c_t^e + \eta i_t = Y_t$$

$$(1 - \eta)c\tilde{c}_t + \eta c^e \tilde{c}_t^e + \eta i\tilde{i}_t - Y\tilde{Y}_t = 0 \quad (4)$$

Price of capital

$$\begin{aligned}
q_t [m_t - \mu F(\bar{\theta}_t, m_t, \sigma_t^2) + \mu f(\bar{\theta}_t, m_t, \sigma_t^2) g_2(\bar{\theta}_t, m_t, \sigma_t^2) / \frac{\partial g_2}{\partial \bar{\theta}}(\bar{\theta}_t, m_t, \sigma_t^2)] &= 1 \\
q \left[m - \mu F(\bar{\theta}, m, \sigma^2) + \mu f(\bar{\theta}, m, \sigma^2) g_2(\bar{\theta}, m, \sigma^2) / \frac{\partial g_2}{\partial \bar{\theta}}(\bar{\theta}, m, \sigma^2) \right] \tilde{q}_t &+ \\
q \left[1 - \mu \frac{\partial F}{\partial m}(\bar{\theta}, m, \sigma^2) + \mu \frac{\partial}{\partial m} \left(f(\bar{\theta}, m, \sigma^2) g_2(\bar{\theta}, m, \sigma^2) / \frac{\partial g_2}{\partial \bar{\theta}}(\bar{\theta}, m, \sigma^2) \right) \right] m \tilde{m}_t &+ \\
q \left[-\mu \frac{\partial F}{\partial \sigma^2}(\bar{\theta}, m, \sigma^2) + \mu \frac{\partial}{\partial \sigma^2} \left(f(\bar{\theta}, m, \sigma^2) g_2(\bar{\theta}, m, \sigma^2) / \frac{\partial g_2}{\partial \bar{\theta}}(\bar{\theta}, m, \sigma^2) \right) \right] \sigma^2 \tilde{\sigma}_t^2 &+ \\
q \left[-\mu \frac{\partial F}{\partial \theta}(\bar{\theta}, m, \sigma^2) + \mu \frac{\partial}{\partial \theta} \left(f(\bar{\theta}, m, \sigma^2) g_2(\bar{\theta}, m, \sigma^2) / \frac{\partial g_2}{\partial \bar{\theta}}(\bar{\theta}, m, \sigma^2) \right) \right] \tilde{\theta} \tilde{\theta}_t &= 0 \quad (5)
\end{aligned}$$

Investment Supply

$$i_t(1 - q_t g_{1t}(\bar{\theta}_t; m_t, s_t)) = n_t$$

$$\begin{aligned}
i(1 - q g_1(\bar{\theta}, m, \sigma^2)) \tilde{i}_t - n \tilde{n}_t - i q g_1(\bar{\theta}, m, \sigma^2) \tilde{q}_t - i q \frac{\partial g_1}{\partial \bar{\theta}}(\bar{\theta}, m, \sigma^2) \tilde{\theta} \tilde{\theta}_t \\
- i q \frac{\partial g_1}{\partial m}(\bar{\theta}, m, \sigma^2) m \tilde{m}_t - i q \frac{\partial g_1}{\partial \sigma^2}(\bar{\theta}, m, \sigma^2) \sigma^2 \tilde{\sigma}_t^2 = 0 \quad (6)
\end{aligned}$$

Net Worth

$$n_t = w_t^e + A_t^e (q_t(1 - \delta) + r_t^k) / \eta$$

$$w^e \tilde{w}_t^e + A^e q(1 - \delta) / \eta \tilde{q}_t + A^e r^k / \eta \tilde{r}_t^k + A^e (q(1 - \delta) + r^k) / \eta \tilde{A}_t^e - n \tilde{n}_t = 0 \quad (7)$$

Entrepreneurs Capital Holding

$$A_{t+1} = \eta g_{2t}(\bar{\theta}_t; m_t, s_t) \frac{n_t}{1 - q_t g_{1t}(\bar{\theta}_t; m_t, s_t)} - \eta c_t^e / q_t$$

$$\begin{aligned} & \eta g_2(\bar{\theta}, m, \sigma^2) \tilde{i}_t + \eta i \frac{\partial g_2}{\partial \theta}(\bar{\theta}, m, \sigma^2) \tilde{\theta} \tilde{\theta}_t + \eta i \frac{\partial g_2}{\partial m}(\bar{\theta}, m, \sigma^2) m \tilde{m}_t \\ & + \eta i \frac{\partial g_2}{\partial \sigma}(\bar{\theta}, m, \sigma^2) \sigma^2 \tilde{\sigma}_t^2 - \eta c^e / q \tilde{c}_t^e + \eta c^e / q \tilde{q}_t - A^e \tilde{A}_{t+1}^e = 0 \quad (8) \end{aligned}$$

Entrepreneurs Euler Equation

$$\begin{aligned} q_t &= \mathbb{E}_t \left\{ \beta \gamma (q_{t+1} (1 - \delta) + r_{t+1}^k) \frac{q_{t+1} g_2(\bar{\theta}_{t+1}, m_{t+1}, \sigma_{t+1}^2) i_{t+1}}{n_{t+1}} \right\} \\ \mathbb{E}_t \left\{ \beta \gamma [2q(1 - \delta) + r^k] \frac{q g_2(\bar{\theta}, m, \sigma^2) i}{n} q \tilde{q}_{t+1} + \beta \gamma r^k \frac{q g_2(\bar{\theta}, m, \sigma^2) i}{n} r_{t+1}^k \right. \\ & + \beta \gamma (q(1 - \delta) + r^k) q \frac{\partial g_2}{\partial \theta}(\bar{\theta}, m, \sigma^2) \frac{i}{n} \tilde{\theta} \tilde{\theta}_{t+1} + \beta \gamma (q(1 - \delta) + r^k) q \frac{\partial g_2}{\partial m}(\bar{\theta}, m, \sigma^2) \frac{i}{n} m \tilde{m}_{t+1} \\ & \left. + \beta \gamma (q(1 - \delta) + r^k) q \frac{\partial g_2}{\partial \sigma^2}(\bar{\theta}, m, \sigma^2) \frac{i}{n} \sigma^2 \tilde{\sigma}_{t+1}^2 + \right. \\ & \left. \beta \gamma (q(1 - \delta) + r^k) q g_2(\bar{\theta}, m, \sigma^2) \frac{i}{n} (\tilde{i}_{t+1} - \tilde{n}_{t+1} - q \tilde{q}_t) \right\} = 0 \quad (9) \end{aligned}$$

Final Good Production

$$Y_t = \exp(z_t) K_t^{\alpha_1} H_t^{\alpha_2} \eta^{1-\alpha_1-\alpha_2}$$

$$z_t + \alpha_1 \tilde{K}_t + \alpha_2 \tilde{H}_t - \tilde{Y}_t = 0 \quad (10)$$

Rental rate of Capital

$$r_t^k = \alpha_1 \exp(z_t) K_t^{\alpha_1-1} H_t^{\alpha_2} \eta^{1-\alpha_1-\alpha_2}$$

$$z_t + \alpha_2 \tilde{H}_t - (1 - \alpha_1) \tilde{K}_t - \tilde{r}_t^k = 0 \quad (11)$$

Household wage

$$w_t = \alpha_2 \exp(z_t) K_t^{\alpha_1} H_t^{\alpha_2 - 1} \eta^{1 - \alpha_1 - \alpha_2}$$

$$z_t + \alpha_1 \tilde{K}_t - (1 - \alpha_2) \tilde{H}_t - \tilde{w}_t = 0 \quad (12)$$

Entrepreneurs wage

$$z_t + w_t^e = (1 - \alpha_1 - \alpha_2) \exp(z_t) K_t^{\alpha_1} H_t^{\alpha_2} \eta^{-\alpha_1 - \alpha_2}$$

$$z_t + \alpha_1 \tilde{K}_t + \alpha_2 \tilde{H}_t - \tilde{w}_t^e = 0 \quad (13)$$

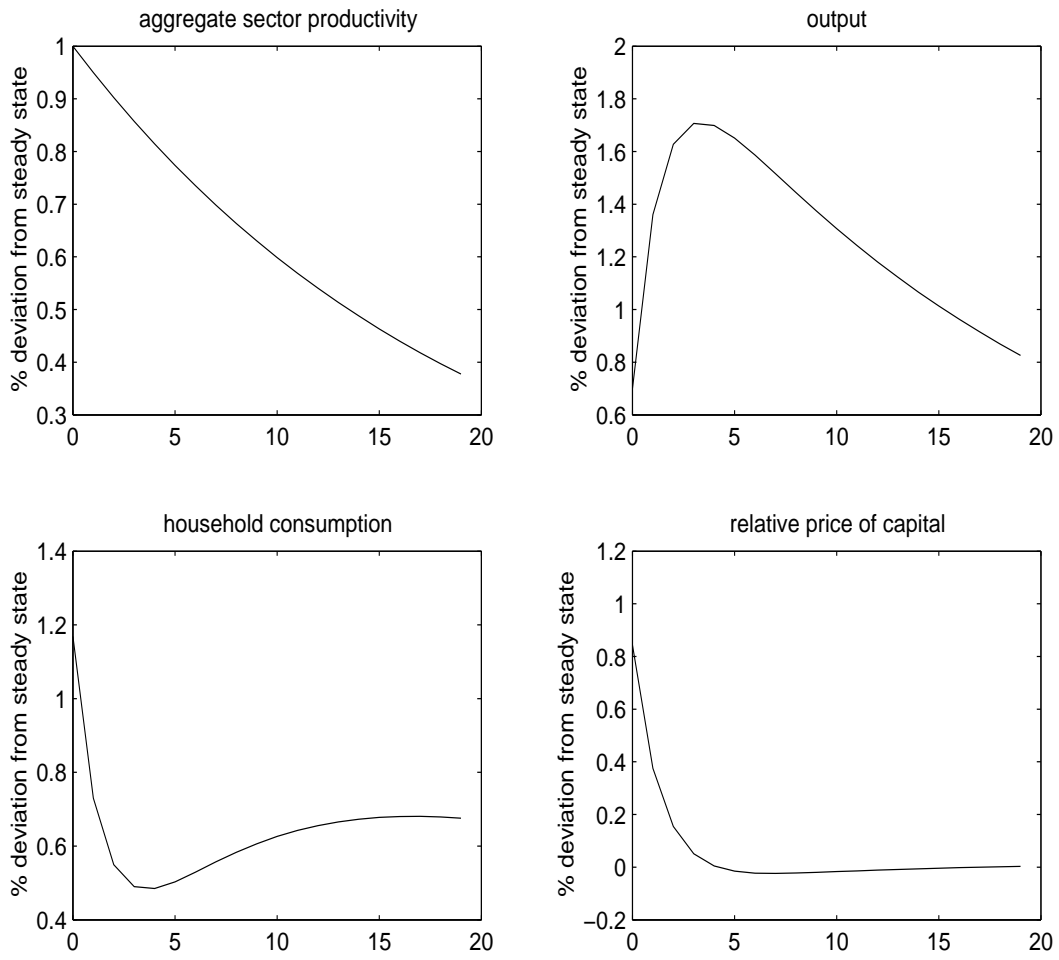


Figure 1: Responses to a shock in the Aggregate Sector Productivity (Part 1)

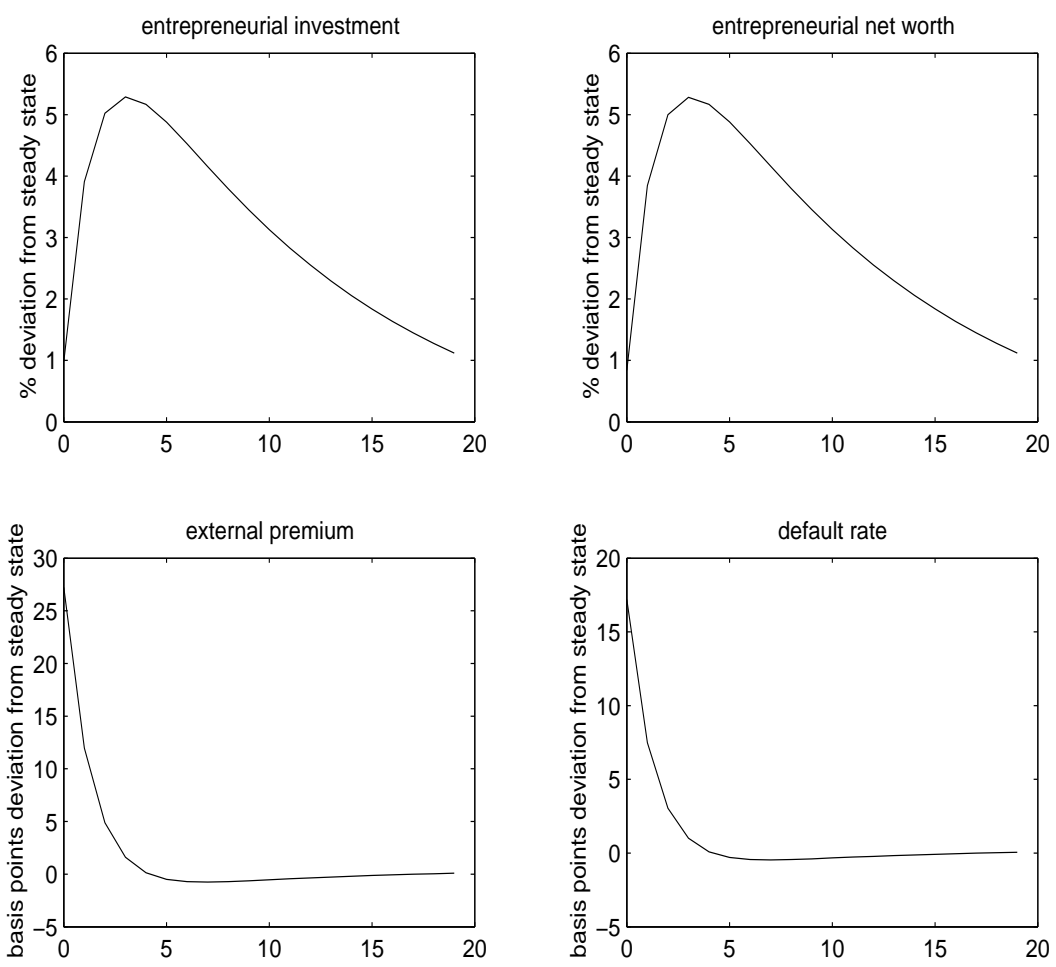


Figure 2: Responses to a shock in the Aggregate Sector Productivity (Part 2)

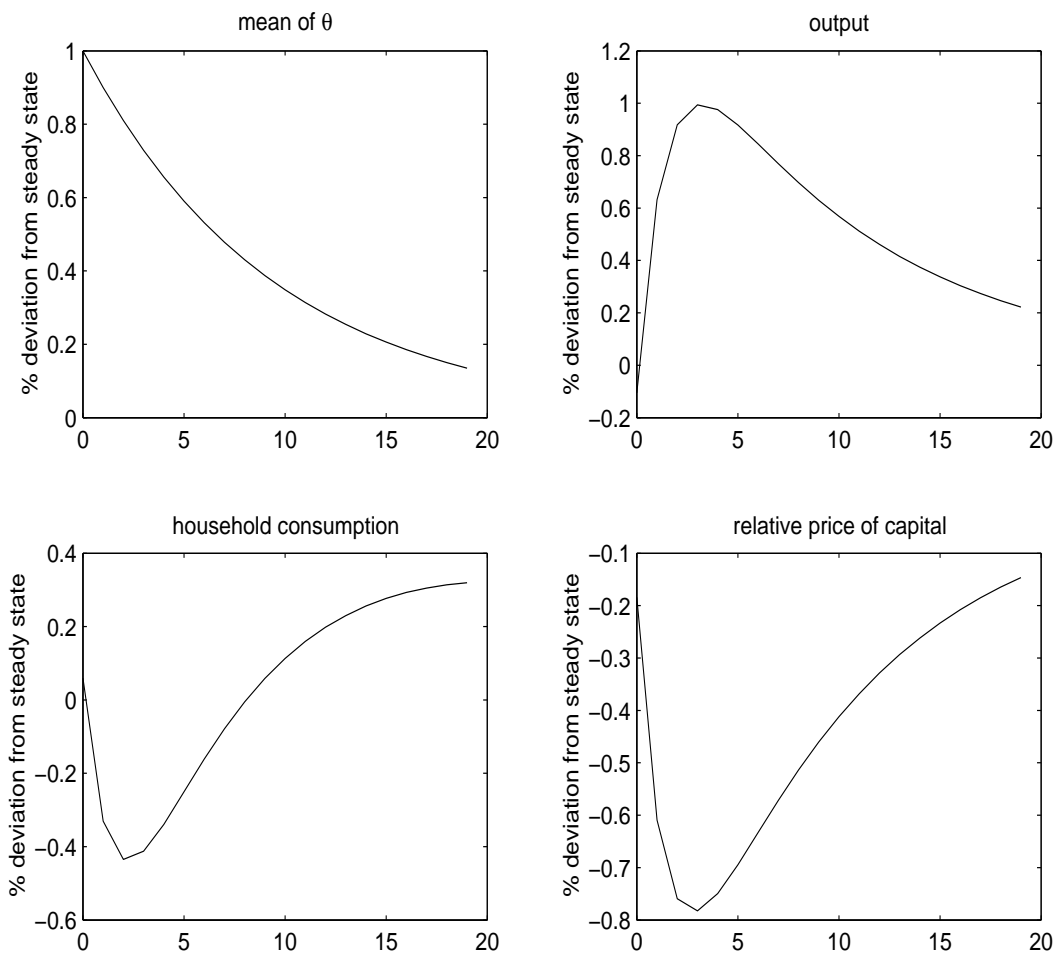


Figure 3: Responses to a shock in the Average Entrepreneurial Productivity ($\rho_m = 0.9$, Part 1)

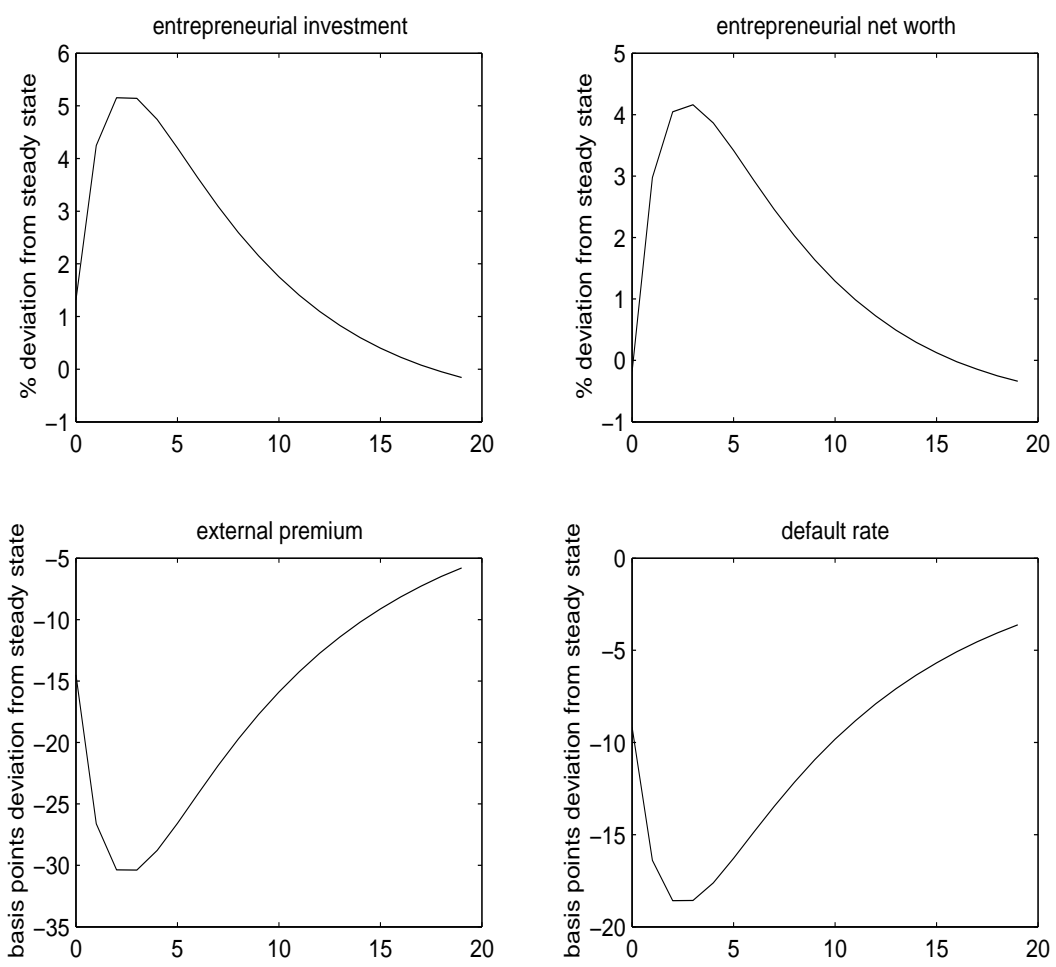


Figure 4: Responses to a shock in the Average Entrepreneurial Productivity ($\rho_m = 0.9$, Part 2)

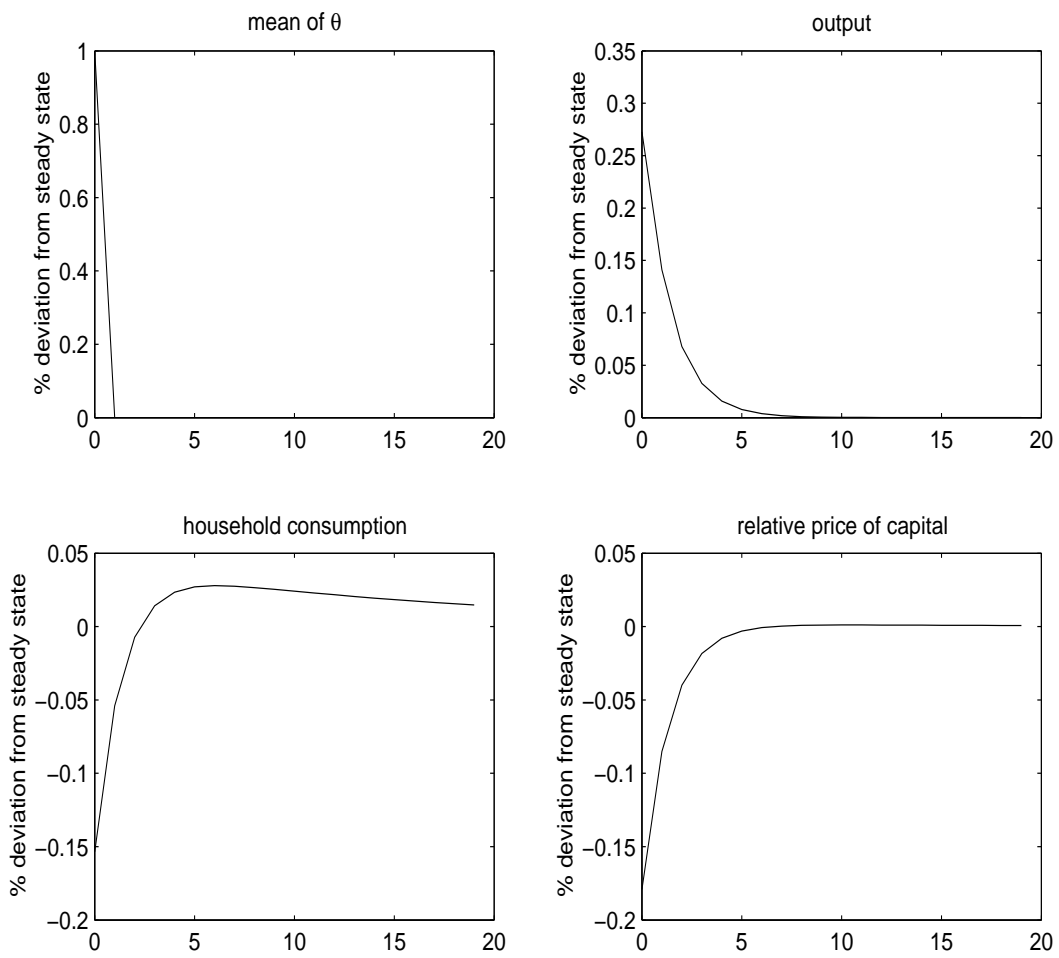


Figure 5: Responses to a shock in the Average Entrepreneurial Productivity ($\rho_m = 0$, Part 1)

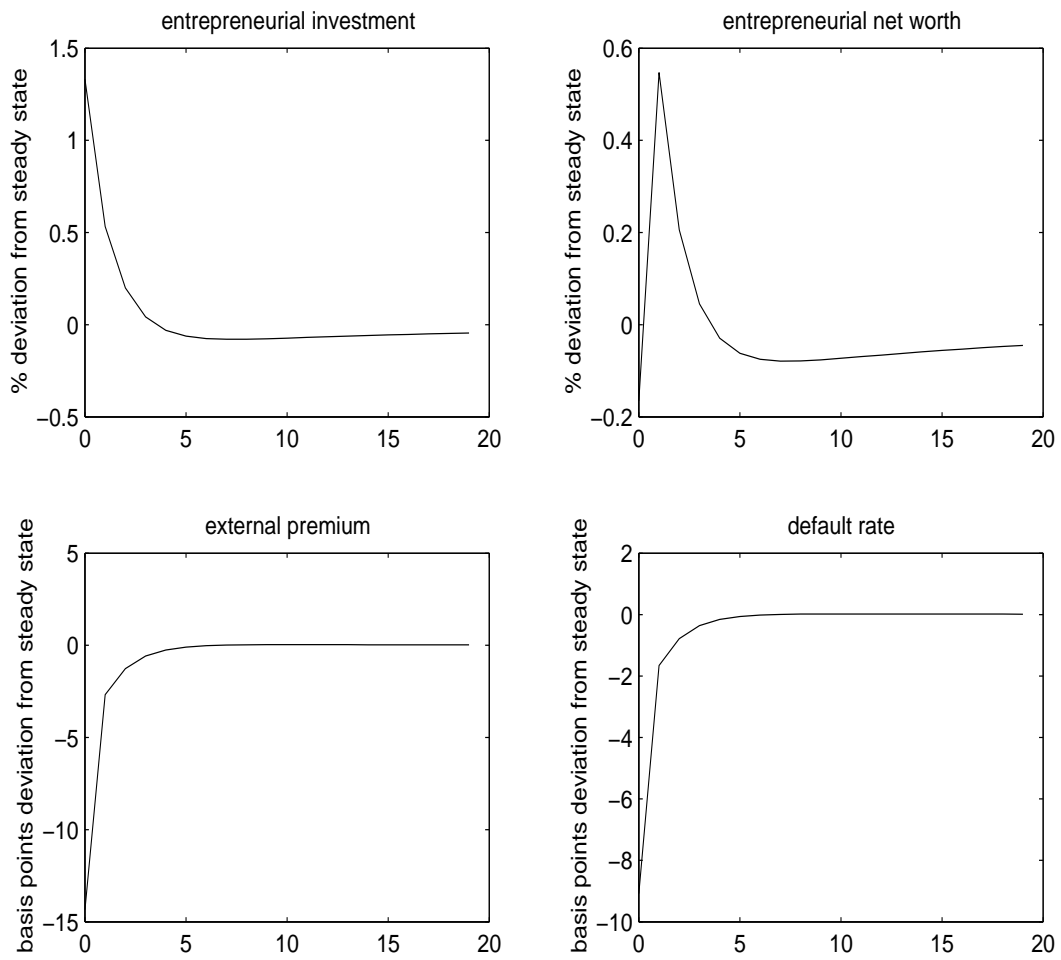


Figure 6: Responses to a shock in the Average Entrepreneurial Productivity ($\rho_m = 0$, Part 2)

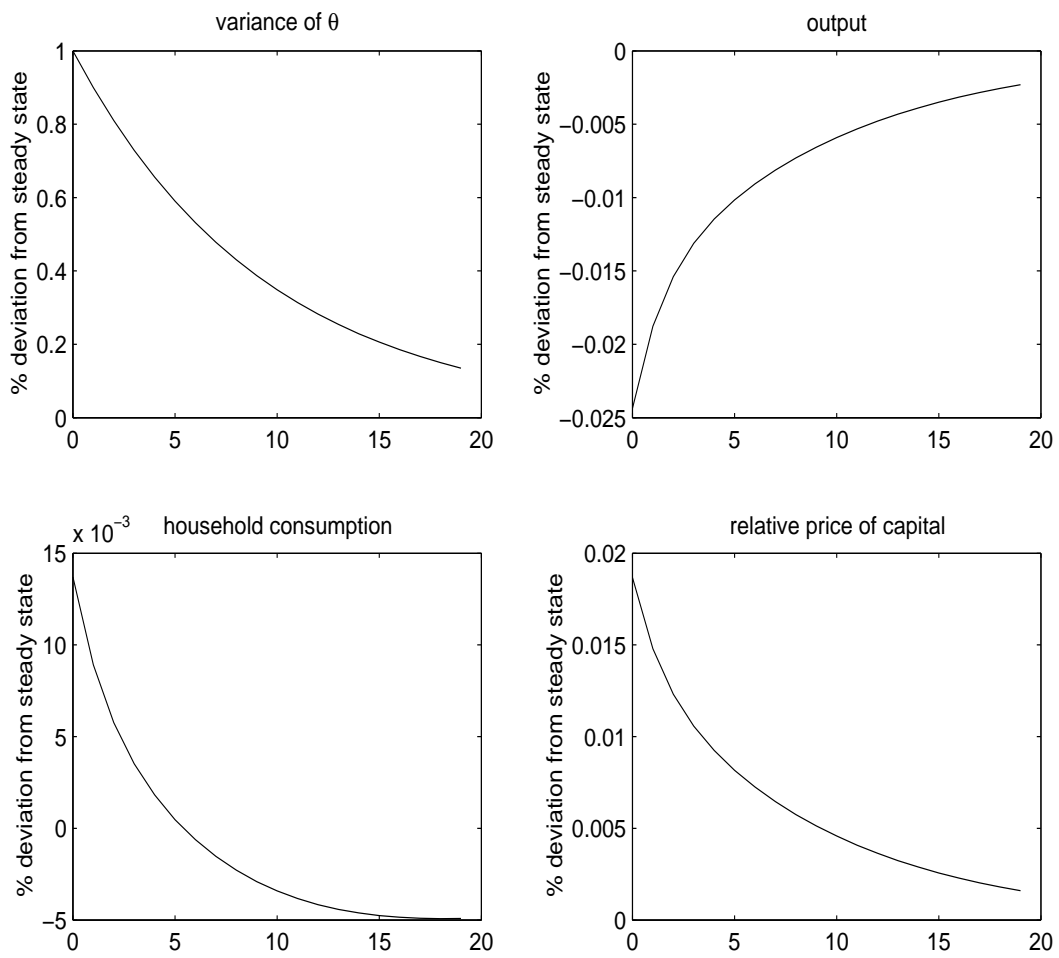


Figure 7: Responses to a shock in the Variance of Entrepreneurial Productivity ($\rho_\sigma = 0.9$, Part 1)

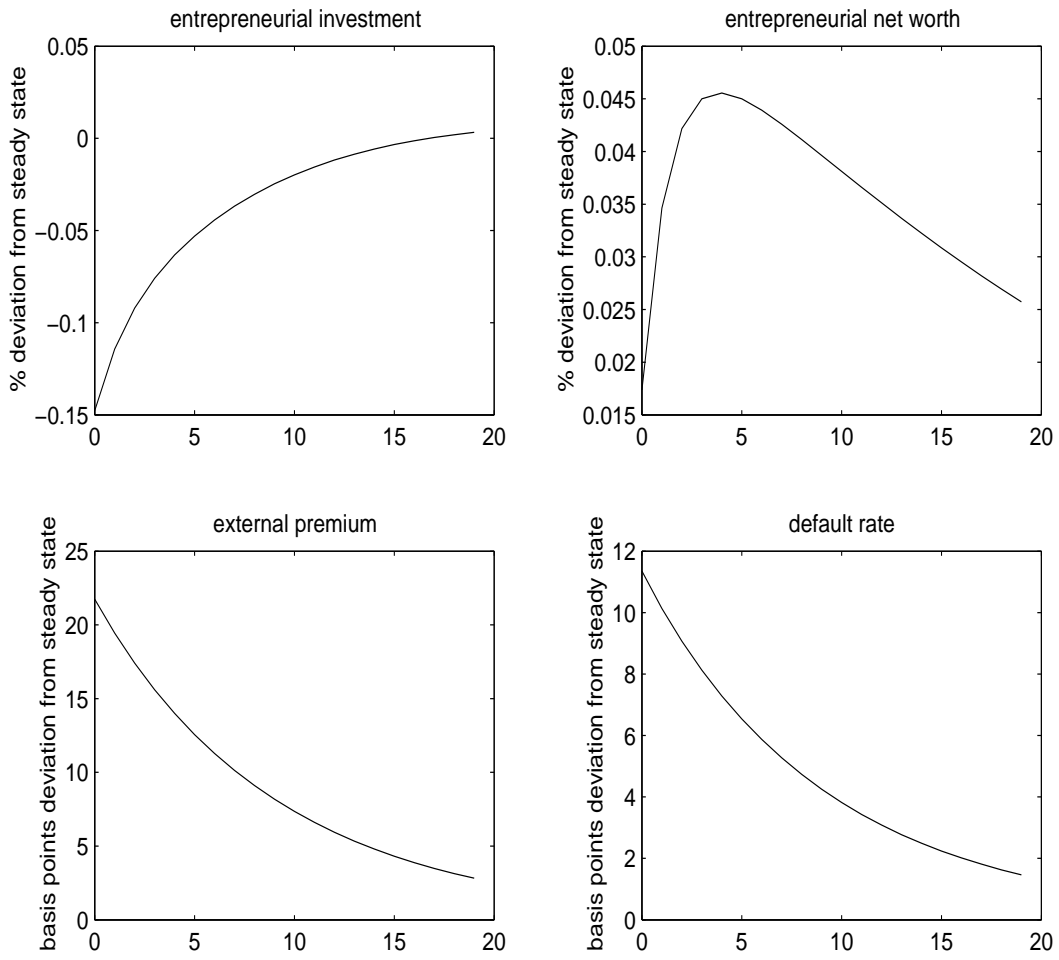


Figure 8: Responses to a shock in the Variance of Entrepreneurial Productivity ($\rho_\sigma = 0.9$, Part 2)

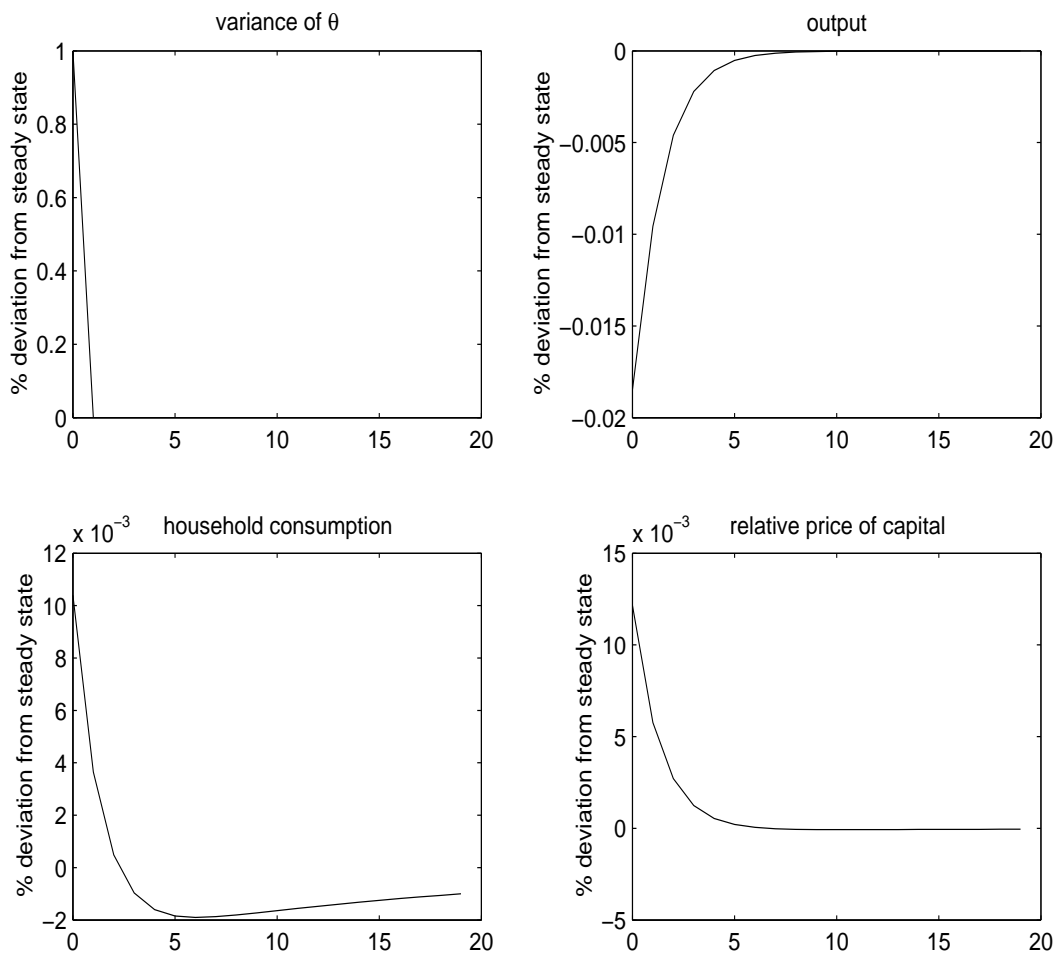


Figure 9: Responses to a shock in the Variance of Entrepreneurial Productivity ($\rho_\sigma = 0$, Part 1)

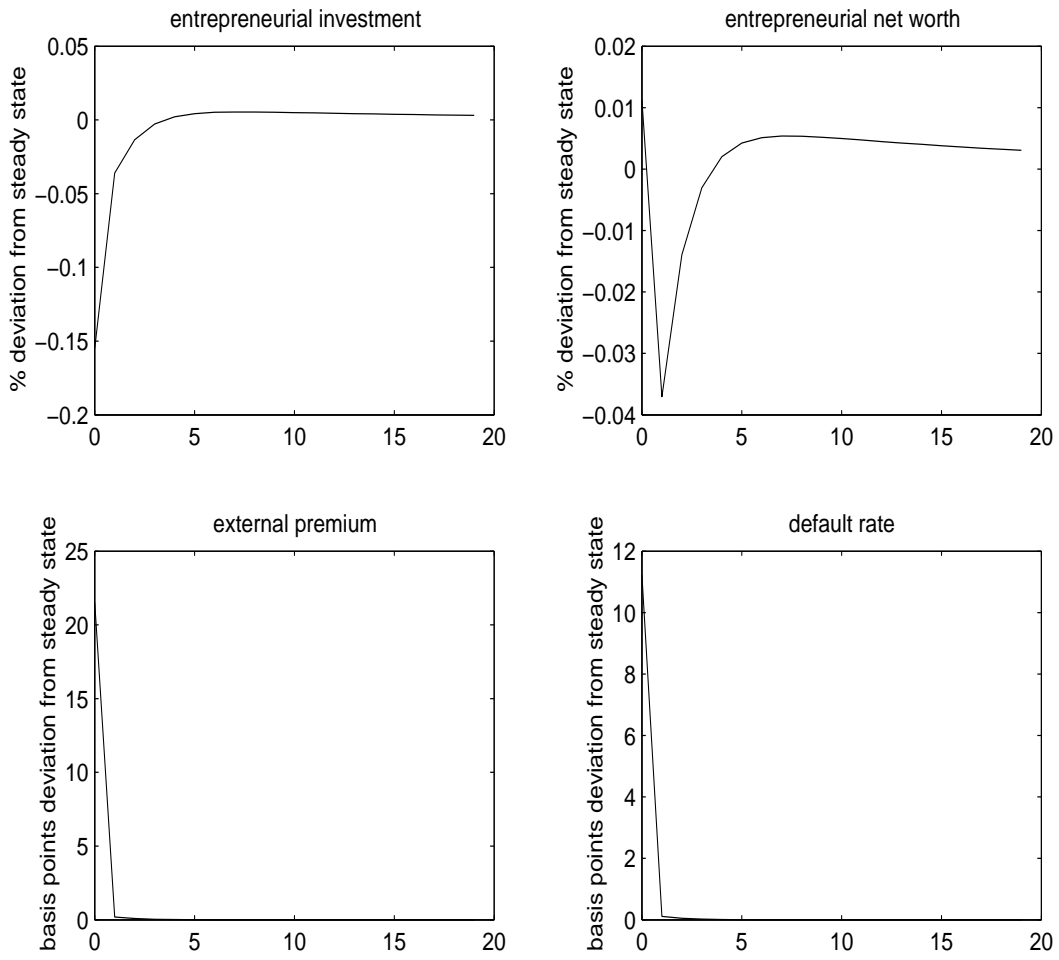


Figure 10: Responses to a shock in the Variance of Entrepreneurial Productivity ($\rho_\sigma = 0$, Part 2)

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