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RESUMEN

Este artículo examina el papel que loa inmigrantes de baja cualificación laboral juegan a la hora de determinar los beneficios de las pensiones públicas en el país de acogida. Con un modelo de generaciones solapadas en tiempo continuo que nos permite identificar que grupos de la población nativa están mejor o peor con la inmigración y un sistema de pensiones completamente redistributivo, nosotros encontramos que las pensiones y por lo tanto los niveles de bienestar de la población local dependen de si coinciden o no con los inmigrantes durante el período de jubilación. En este sentido, la población local más joven puede preferir, frente a los más viejos, una política de fronteras cerradas.

Palabras clave: Inmigración, Bienestar, Pensiones.

ABSTRACT

This paper examines the role that low-skilled immigrant labor force plays in determining the benefits of the public pension of the host population. With an overlapping-generations model in continuous time which allows to identify which groups of native population are better or worse off with immigration and a fully redistributive pension system, we find that the retirement benefits and hence the welfare levels of the host population are affected in a different way whether sharing or not pension benefits with immigrants. In this sense, the youngest local population may prefer, contrary to the oldest ones, a policy of closed borders.

Keywords: Immigration, welfare, pension benefits.
JEL Classification: F22, H55, J61

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1. Introduction

Nowadays the flow of immigrants into developed countries with generous welfare systems constitutes one of the most important economic issues. In most of these countries, pay-as-you-go (PAYG) financed public pensions are suffering from demographic problems because of the ongoing aging of their populations. In such a manner that immigration is frequently being thought as a means to overcome the financial problems of the public pensions.

In parallel, it is normally argued that immigrants are net beneficiaries of the welfare state (see Sinn (2002) and Chand and Paldam (2004) among others). Whether or not it is true that immigrants may be beneficiaries of the social schemes, concerning to PAYG social security system, immigration rises unambiguously the number of contributors.

For these reasons, the public choice approach towards immigration has gained increasing interest within the academic literature. Razin and Sadka (1999) , in an overlapping-generations model with two periods, show that even though immigrants may be low-skilled and net beneficiaries of the pension system, all groups living at the time of the immigrants' arrival would be better off. This result depends crucially on the assumptions that immigrants have the same birth rate as the native population and that the ability index of their offspring is distributed similarly. Krieger (2004) replicates the Razin and Sadka model by relaxing these assumptions. He shows that the Razin and Sadka result is no longer unambiguous.

As they suggest, the equal ability distribution assumption is a subject of open debate.[1] Because of this, the assumption in which the ability index of immigrants' offspring is distributed similarly as the native population has been also relaxed in this paper.

Our theoretical framework considers an overlapping-generations model in continuous time which allows to identify specifically, unlike Razin and Sadka (1999) and Krieger (2004), which groups of native population are better or worst off with immigration. The present article explicitly examines the role which low-skilled immigrant labor force plays in determining the benefits of the public pension and hence the whole host population's welfare.

In a country with open borders and a PAYG, redistributive pension system, we abstract from the effect that immigration may have on very different social and economic issues and concentrate on the effect it has on the benefits of such a domestic pension system. As a time horizon is needed to assess the true impact of immigration, this paper takes into account that at some point in the future immigrants will start collecting retirement benefits. Moreover, in order to avoid individuals ignoring the impact of current decisions on future retirement benefits, we assume fully rational individuals. That is, their future income benefits become quantitatively as important as their present income benefits.

We find that, with the arrival of immigrants, the retirement benefits and hence the welfare levels of the host population are affected in a different way according to *share or not* pension benefits with them. On one hand, the welfare of the oldest individuals (retirees and workers who are close to retirement), since do not coincide with immigrants in the retirement period, unambiguously increases. On the other hand, the welfare of the youngest native individuals, since coincide with immigrants in the retirement period, depends on the total (quota) and the labor productivity of immigrants, and on the own age of the native individual. Concretely, the older a native individual and the higher the quota, the more likely to get benefits from immigration. Moreover, if the average labor productivity of immigrants is high (although always smaller than the native one), then young native generations could improve with immigration. On the contrary, if the immigrant labor productivity is low then the youngest native individuals could be worst off. Consequently, depending on the level of immigrant labor productivity, the youngest local population may prefer, against the oldest ones, a policy of closed borders.

The paper is organized as follows. Section 2 develops the model. Section 3 analyzes how the arrival of immigrants affects the welfare of the host-population. Section 4 summarizes the main results. Proofs are placed in the appendix.

2. The model

We consider an overlapping-generations model in continuous time. At each point of time t a new cohort of individuals is born. We assume a constant birth rate which is normalized to the unity. In this model we have a continuous and uniform distribution of agents on age, with no uncertainty on the length of their lives, going from zero to a fixed age, T. Besides, a continuous distribution of agents on labor productivity between a minimum and a maximum level $[l_-, l_+]$ is assumed .

The government levies an income labor tax, τ, for redistributive social security program. The social security program is a balanced budget ”pay as you go” system

(PAYG), in which current workers are net contributors while the retired are net beneficiaries.

The utility function of individuals over their life-cycle is similar to Crawford and Lilien, (1981). These individuals have a stationary and temporally independent utility function, which is separable and strictly increasing in consumption and leisure. We assume that leisure yields utility to the individual only when this individual is retired. The pension or retirement benefits are received only after they stop working. The instantaneous utility function is then as follows

$$U\left(c_i^t, \theta^t\right) = u\left(c_i^t\right) + v\left(\theta^t\right) \tag{2.1}$$

where c_i^t is the consumption at period t of individual i. The utility of consumption is twice differentiable with $u' > 0$, $u'' < 0$. Let θ^t be the leisure per unit of time t, equal for all individuals, being the utility of leisure $v\left(\theta_w^t\right) = 0$, in their working years and $v\left(\theta_R^t\right) = v$, in their retirement years.

Let δ, r be the subjective rate of time preference and the market rate of interest respectively. Let p be the annual pension benefits that workers get when they are retired. Let R be the current mandatory retirement age at which pension benefits are available.[2] Then the lifetime utility that an individual i has to maximize over his life-cycle can be written as

$$\int_0^T U\left(c_i^t, \theta^t\right) dt = \int_0^R u\left(c_i^t\right) e^{-\delta t} dt + \int_R^T \left[u\left(c_i^t\right) + v\left(\theta^t\right)\right] e^{-\delta t} dt \tag{2.2}$$

subject to

$$\int_0^T c_i e^{-rt} dt = \int_0^R (1-\tau)\, l_i e^{-rt} dt + \int_R^T p e^{-rt} dt \tag{2.3}$$

where l_i is the labor productivity for an individual i.

Concerning to the individual pension benefits, the pension system is assumed to be unfunded and fully redistributive so that the discounted value of the total pensions received by any individual is as follows

$$P = \int_R^T \frac{\tau R L}{T-R} e^{-rt} dt \tag{2.4}$$

where L is the average labor productivity of the working population.

For the sake of simplicity, we assume that there are no returns on savings and that individuals do not discount the future, so both discount rates are zero $(\delta = r = 0)$. This assumption implies that each individual will set a constant consumption per period. That is, individuals are fully rational and hence their future income benefits become quantitatively as important as their present income benefits.[3] Then the indirect remaining lifetime utility function of an individual i and age a (as Breyer (1994)) can be reduced to

$$U(a, c_i, \theta) \equiv (T - a)\, u(c_i) + (T - R)\, v \tag{2.5}$$

and the optimal consumption is given by

$$c_i = \frac{1}{T - a}\left((R - a)(1 - \tau)\, l + (T - R)\, p + \pi\right) \tag{2.6}$$

where R is the retirement age, τ the tax rate on wage, p the annual pension benefits and π the accumulated wealth up to a period t :[4]

$$\pi = a\left((1 - \tau)\, l - \frac{R}{T}\left(l\,(1 - \tau) + \tau L\right)\right) \tag{2.7}$$

At period t, a quota of $m \in (0, 1)$ immigrants are allowed in. As Razin and Sadka (1999), it is assumed that these immigrants are all young $(a = 0)$ and unskilled workers (low l). They have the same preferences and the same fertility rate as the native population. Hence, in the host-country a cohort of $1 + m$ individuals is born at each point of time from period t on.[5]

With the arrival of immigrants at period t, the retirement benefits of each native individual are affected in a different way according to age, $p(a)$. Four benchmark cases can be defined.

The first benchmark, retired people. Native individuals whose age at period t is higher than mandatory *retirement age*, $a \geq R$. These individuals are net beneficiaries of the pension system and pass away before the first immigrants will start collecting benefits. Thus, the discounted value of the total pension benefits received by an retired individual aged a is as follows

$$P(a) = \int_a^T \frac{\tau R L + \tau t m I}{T - R}\, dt \tag{2.8}$$

where I denotes the average labor productivity of immigrants. It is assumed that $I < L$. The component on the right hand side (RHS) of (2.8) represents the total tax contribution of native working population and immigrants to individual pension benefits of currently retired people.[6]

In the second benchmark, we consider native individuals whose age at period t is greater than mandatory *retirement period*, $a \geq T - R$. These native individuals are presently working but pass away before the first immigrants will start collecting benefits. Thus, the discounted value of the total pension benefits received by an individual aged a is as follows

$$P(a) = \tau RL + \int\limits_{R-a}^{T-a} \frac{\tau tmI}{T-R} dt \tag{2.9}$$

The first component on the RHS of (2.9) represents the total tax contribution of native population to the pension benefits of native individual. The second term reflects the additional tax contribution of immigrants.[7]

In the third benchmark, we consider the case in which the age of native individuals at period t is less than mandatory *retirement period*, $0 < a \leq T - R$. These native individuals coincide with immigrants in the retirement period and, thus, during at least some years, *share* pension benefits with them. The discounted value of the total pension benefits received by an individual aged a changes significantly

$$\begin{aligned} P(a) &= \frac{a}{T-R} \tau RL + \int\limits_{R-a}^{R} \frac{\tau tmI}{T-R} dt + \\ &+ \int\limits_{R}^{T-a} \frac{\tau R (L + mI)}{T - R + (t - R) m} dt \end{aligned} \tag{2.10}$$

The first two components on the RHS of (2.10) represent the total tax contribution of native population and immigrants (respectively) to pension benefits of native individual when the first generation of immigrants is not still in the retirement period. However, the third term reflects the tax contribution of native population and immigrants to pension benefits when the first generation of immigrants is already in the retirement period, that is, when at least there exist R generations of immigrants in the host-country.[8]

Finally, we examine the case of native individuals who are born at period t, that is, their age is $a = 0$. These native individuals coincide with immigrants during the whole retirement period and therefore from beginning to end *share* pension benefits. The discounted value of the total pension benefits can be expressed as

$$P(0) = \int_{\hat{R}}^{T} \frac{\tau R (L + mI)}{T - R + (t - R) m} dt \tag{2.11}$$

The component on the RHS of (2.11) represents the total tax contribution of native population and immigrants to the individual pension when the first generation of both immigrants and native individuals aged 0 at period t reaches the mandatory retirement age.[9]

Consequently, with the arrival of immigrants, the retirement benefits of the host population are affected in a different way according to *share or not* pension benefits. In the first two benchmarks, for native individuals who do not coincide (and thus do not share) with immigrants in the retirement period, immigrants are net contributors of the social security system. In the last two ones, for native individuals who coincide (and thus share) with immigrants in the retirement period, immigrants are also beneficiaries of pension benefits.

3. Welfare analysis

Once it has been determined the setting that each native individual faces, the focus will be on how the arrival of immigrants affects the welfare of whole host population. Analytically, we derive how the utility function of each individual evaluated in the optimal consumption changes with immigration

$$\frac{\partial U(.)}{\partial m} = (T - a) \frac{\partial c}{\partial m} u'(c) \tag{3.1}$$

Since we assumed that $u'(c)$ is positive, the sign of (3.1) depends on the sign of $\partial c / \partial m$. That is, the effect of immigration on welfare is always via its effect on consumption levels through the pension benefits. Regarding the immigration's impact, because of the concavity of $u(c)$, notice that different levels of labor productivity among the host population just affect the intensity and not the sign of the impact. Specifically, the larger the labor productivity of a native individual is, the smaller the impact of immigration on their individual consumption levels. In other words, when unskilled immigrants come in, the most skilled native people are the least affected and vice versa.

3.1. Retirees and workers without sharing pension benefits

- **Individuals with** $a \geq R$

At first we analyze retired native individuals. These individuals do not share pension benefits with immigrants. Upon inspection of (2.8), one can easily deduce that retirement benefits and thus consumption levels rise with immigration. Therefore, as expected, these individuals are clearly better off with immigration. See proposition 3.1.

- **Individuals with** $a \geq T - R$

In second place we examine the oldest native workers.[10] These individuals do not share either pension benefits with immigrants. Inspecting now (2.9), one can again easily deduce that the retirement benefits increase with immigration.

Therefore for any native individual without sharing pension benefits with immigrants the following proposition can be stated.

Proposition 3.1. *The sign of* $\partial U\left(.\right)/\partial m$ *is always positive* $\forall m \in (0, 1)$.

Through retirement benefits, an increase in the quota of immigrants rises the consumption level ($\partial c/\partial m > 0$) and thus improves the welfare level of this cohort of host population. These native individuals would have incentives to support a policy of open borders.

3.2. Workers sharing pensions benefits

- **Individuals with** $0 < a \leq T - R$

Next we analyze how the arrival of immigrants affects the welfare of native population whose age is less than the mandatory retirement period. These individuals share, at least some years (according to age), pension benefits with immigrants.

In this case, the expression (3.1), after some simplifications, can be expressed as follows

$$\frac{\partial U\left(.\right)}{\partial m}=\left(\frac{a}{T-R}I\left(R-\frac{1}{2}a\right)+RI\gamma_{1}+RL\gamma_{2}\right)\tau u'\left(c\right) \tag{3.2}$$

with $\gamma_1 = \frac{T-R-a}{T-R+m(T-R-a)}$ and $\gamma_2 = \frac{T-R-a}{(T-R+m(T-R-a))m} - \frac{\ln(T-R+m(T-R-a))-\ln(T-R)}{m^2}$. It can be proved that $R - \frac{1}{2}a > 0$, $\gamma_1 \geq 0$ and $\gamma_2 \leq 0$.[11] Hence, the sign of this expression is uncertain.

The first two components on the RHS of (3.2) represent the positive effect of an increase in the arrival of immigrants on the native individual's welfare: the first term denotes the additional contribution of immigrants to the pension benefits of native population when the first generation of immigrants is not still in the retirement period; and the second term reflects the additional contribution of immigrants to the pension benefits when the first generation of immigrants is already in the retirement period.

In contrast, the remaining third term represents the negative effect of an increase in the arrival of immigrants on native individuals' retirement benefits and thus on their welfare. This third term denotes the additional decrease to pension benefits of native population as consequence immigrants are already in the retirement period and therefore they also receive pension benefits.

- **Individuals with** $a = 0$

Finally it is examined how the entrance of immigrants affects the welfare of individuals who have just incorporated to the labor market. These native individuals live together with immigrants the remaining life. Now the expression (3.1) can be equivalently stated as

$$\frac{\partial U\left(.\right)}{\partial m}=\left(I\delta_{1}+L\delta_{2}\right)\tau Ru'\left(c\right) \tag{3.3}$$

where $\delta_1 = \frac{1}{1+m}$ and $\delta_2 = \frac{1}{(1+m)m} - \frac{\ln(1+m)}{m^2}$. It can be showed that $\delta_1 > 0$, $\delta_2 < 0$ and $|\delta_1| > |\delta_2|$.[12] Hence, the sign of this expression is uncertain.

The first term on the RHS of (3.3) represents the positive impact of an increase in the arrival of immigrants on the native individuals' welfare. This first term denotes the additional contribution of immigrants to pension benefits of native population when at least the first generation of immigrants is already in the retirement period.

In contrast, the second term represents the negative impact of an increase in the arrival of immigrants on native individuals' retirement benefits and thus on their welfare. This second term denotes the additional decrease in pension benefits of the native population as consequence immigrants are already in the retirement period and therefore they also receive a pension benefits from social security system.

For any native individual of age $a \in [0, T-R]$ and a quota of immigrants $m \in (0,1)$, $I(a,m)$ denotes the level of average labor productivity of immigrants in such a manner that $\partial U(.)/\partial m = 0$. We can state the following proposition.

Proposition 3.2. *For any native individual with age $a \in [0, T-R]$ and quota of immigrants $m \in (0,1)$ there exists an average labor productivity of immigrants $I(a,m) \in \left(\underline{I}, \overline{I}\right)$ with $0 < \underline{I} < \overline{I} < L$ such that $\partial U(.)/\partial m > 0$, for any $I > I(a,m)$; and $\partial U(.)/\partial m < 0$, for any $I < I(a,m)$. Moreover, $\partial I(a,m)/\partial a < 0$ and $\partial I(a,m)/\partial m < 0$.*

From proposition 3.2. can be deduced that immigration does not always entail a Pareto improvement for the population in the host-country. In this respect, regarding the level of the immigrant labor productivity and its impact on the host welfare, there exists like an threshold value which depends inversely on the age and the quota of immigrants. Namely, each native individual has (according to age) her own threshold value and the older a native individual and the higher the quota are, the less her threshold value and the more likely to get benefits from immigration.

For any level of immigrant labor productivity greater than the threshold value of a native individual, $I > I(a,m)$, the arrival of immigrants would improve her welfare level. The intuition underlying is the following. Because of the high labor productivity of immigrants, an increase in the quota would increase her consumption levels $(\partial c/\partial m > 0)$ and thus would improve her individual pension benefits. That is, the additional tax contribution of immigrants to her pension benefits would be larger than the additional decrease as consequence immigrants also receive retirement benefits. This young native individual would have incentives to support a policy of open borders.

However, for any level of immigrant labor productivity smaller than the threshold value of a native individual, $I < I(a, m)$, the arrival of immigrants would reduce her welfare level. The intuition beneath is as follows. Due of the low labor productivity of immigrants, an increase in the quota of immigrants would decrease the consumption levels ($\partial c/\partial m < 0$) and thus would lower her individual pension benefits. In this case, the additional decrease in the pension benefits would be larger than the additional tax contribution of immigrants to the retirement benefits. This native individual would have incentives to advocate a policy of closed borders.

Consequently, since each native individual has (according to age) her own threshold value which determines the impact of immigration on her welfare levels, for a determined quota and level of labor productivity of immigrants, while some native individuals (the oldest ones) may gain, others may lose (the youngest ones) from immigration.

4. Conclusions

With an overlapping-generations model in continuous time which allows to identify specifically which groups of native population are better or worst off with immigration and a fully redistributive pension system, we find that, with the arrival of immigrants, the retirement benefits and hence the welfare levels of the host population are affected in a different way according to share or not pension benefits with them.

For native individuals without sharing pension benefits (retirees and old workers), immigration unambiguously increases the number of net contributors to the pension system. In this way, the arrival of immigrants increases the consumption levels (through higher pension benefits) and thus rises the welfare levels of these native individuals.

However, for individuals sharing pension benefits (young workers), the incentives for immigration vary significantly for different labor productivity of immigrants. Furthermore, the native individual's net benefit from immigration also depends on the quota and on her own age. Concretely, the older a native individual and the higher the quota, the more likely to get benefits from immigration.

For each native individual of this latter cohort, we show that if the labor productivity of immigrants is low (high), or at least less (greater) than a determined threshold value, then the welfare of the native individuals sharing pensions would worsen (improve) with immigration. Therefore, according to the level of immigrant labor productivity, the youngest native individuals may not welcome a large number of immigrants and prefer, against the oldest ones, a policy of closed borders.

Summarizing, we find that in spite of low-skilled immigrants and a fully redistributive social security system, immigration might entail a Pareto improvement for the whole native population. However, as Razin and Sadka [1999] suggest, the children of immigrants could appear to have attributes such as relatively low school completion rates that weaken their earnings' potential later in life, if immigrants keep over time a low labor productivity, the youngest native individuals may lose from immigration and opt to halt it.

5. Appendix

Proof of proposition 3.1. (for $a \geq R$) From the first order conditions and after some simplifications, the following expression is obtained:

$$\frac{\partial U(.)}{\partial m} = \tau I a \frac{T+a}{2(T-R)} u'(c) \tag{5.1}$$

Since that $R < T$, it is straightforward to deduce that the sign of this expression is always positive. Q.E.D.

Proof of proposition 3.1. (for $a \geq T - R$) From the first order conditions and after some simplifications, the following expression is obtained:

$$\frac{\partial U(.)}{\partial m} = \tau I \left(\frac{T+R}{2} - a \right) u'(c) \tag{5.2}$$

Since that $a < R < T$, it is easy to verify that the sign of this expression is always positive. Q.E.D.

Proof of proposition 3.2. (for $a = 0$) From first order conditions of utility function we obtain

$$\frac{\partial U(.)}{\partial m} = T u'(c) \frac{\partial c}{\partial m} \tag{5.3}$$

where

$$\frac{\partial c}{\partial m} = \frac{\tau\left(m\left(L+mI\right)-\ln\left(1+m\right)L\left(1+m\right)\right)}{Tm^2\left(1+m\right)} \tag{5.4}$$

By definition $u'(c)$ is strictly positive, therefore the sign of (5.3) depends on the sign of $\partial c/\partial m$. Let us to define $I = p * L$. Then $\partial U(.)/\partial m$, after some simplifications, can be expressed as follows

$$\frac{\partial U(.)}{\partial m} = u'(c)\left[pm^2 + m - (1+m)\ln(1+m)\right]\frac{I\tau R}{p(1+m)m^2} \tag{5.5}$$

In order to clarify the definitive sign of $\partial c/\partial m$ and $\partial U(.)/\partial m$ we proceed:

For $p \in [0,1]$ and $m \in (0,1)$ define a function $S(m,p) :=pm^2 + m - (1+m)\ln(1+m)$. Notice that the sign of $S(m,p)$ is equivalent to the sign of $\partial c/\partial m$ and $\partial U(.)/\partial m$. The function $S(m,p) > 0$ if and only if $pm^2 + m - (1+m)\ln(1+m) > 0$, that is, if and only if $p > \frac{(1+m)\ln(1+m)-m}{m^2}$. For $m \in (0,1)$, define a function $H(m) :=\frac{(1+m)\ln(1+m)-m}{m^2}$. We obtain that $\lim_{m\to 0} H(m) =0.5$ and $H(1) =0.38$. Let $\overline{p} = 0.5$ and $\underline{p} = 0.38$. Then for any $p > \overline{p}$ we have that $p > H(m), \forall m \in (0,1)$. Consequently, for any $p > \overline{p}$, we have that $S(m,p) > 0$, i.e., $\partial c/\partial m$ and $\partial U(.)/\partial m$ are positive, $\forall m \in (0,1)$.

Analogously, for any $p < \underline{p}$ we have that $p < H(m)$ and thus $S(m,p)$, $\partial c/\partial m$ and $\partial U(.)/\partial m$ are negative $\forall m \in (0,1)$.

Moreover, since $H(m)$ is a continuous, monotone and decreasing function. For any $\underline{p} < p < \overline{p}$ there exists a $m^* \in (0,1)$ such that $p = H(m^*)$. Thus $S(m^*,p)$, $\frac{\partial c}{\partial m}|_{m^*}$ and $\frac{\partial U(.)}{\partial m}|_{m^*}$ are equal to zero. Then, for any $m' < m^*$ implies that $H(m') > H(m^*)$ what means that $H(m') > p$. As we showed earlier, this is equivalent to that $S(m,p)$, $\partial c/\partial m$ and $\partial U(.)/\partial m$ are negative $\forall m \in (0,m^*)$. Symmetrically, for any $m' > m^*$ this leads to that $H(m') < H(m^*)$ what yields that $H(m') < p$. That is, $S(m,p)$, $\partial c/\partial m$ and $\partial U(.)/\partial m$ are positive $\forall m \in (m^*,1)$.

Let $\overline{I}$ and $\underline{I}$ be the levels of average labor productivity of immigrants for $\overline{p}$ and $\underline{p}$ respectively. Therefore, we can state that following:

i) For any $I \in (0,\underline{I})$ then $\partial c/\partial m$ and $\partial U(.)/\partial m$ are strictly negative.

ii) For any $I \in \left(\overline{I},L\right)$ then $\partial c/\partial m$ and $\partial U(.)/\partial m$ are strictly positive.

iii) For any $I \in \left(\underline{I},\overline{I}\right)$ the signs of $\partial c/\partial m$ and $\partial U(.)/\partial m$ depend on m. In this case, there exists a m^* such that for any $m \in (0,m^*)$ the signs of $\partial c/\partial m$

and $\partial U(.)/\partial m$ are negative and for any $m \in (m^*, 1)$ the signs of $\partial c/\partial m$ and $\partial U(.)/\partial m$ are positive.

Let us to define $p_0 = H(m_0)$ and $p_1 = H(m_1)$. Let us to suppose that $p_1 < p_0$ and $m_1 < m_0$. Since $H(m)$ is a continuous, monotone and decreasing function, $m_1 < m_0$ implies that $H(m_1) > H(m_0)$ or equivalently $p_1 > p_0$, that is a contradiction. Therefore, if $m_1 < m_0$ then $p_1 > p_0$. That is, the larger the quota m is, a lower immigrant labor productivity I is needed to obtain $\partial c/\partial m$ and $\partial U(.)/\partial m$ positive or negative. Q.E.D.

Proof of proposition 3.2. (for $0 < a \leq T - R$) From first order conditions of utility function we obtain

$$\frac{\partial U(.)}{\partial m} = (T-a)\, u'(c)\, \frac{\partial c}{\partial m} \tag{5.6}$$

where

$$\frac{\partial c}{\partial m} = \frac{\tau \left(\frac{a}{T-R} I \left(R - \frac{1}{2}a\right) + RI\gamma_1 + RL\gamma_2 \right)}{(T-a)} \tag{5.7}$$

where $\gamma_1 = \frac{T-R-a}{T-R+m(T-R-a)}$ and $\gamma_2 = \frac{T-R-a}{(T-R+m(T-R-a))m} - \frac{\ln(T-R+m(T-R-a)) - \ln(T-R)}{m^2}$.

By definition $u'(c)$ is strictly positive, thus the sign of (5.6) depends on the sign of $\partial c/\partial m$. Let us to define $I = p * L$. Then $\partial U(.)/\partial m$, after some simplifications, can be expressed as follows

$$\frac{\partial U(.)}{\partial m} = u'(c) \left(\frac{a}{T-R} \left(R - \frac{1}{2}a \right) + R\gamma_1 + \frac{1}{L} R\gamma_2 \right) \tau I \tag{5.8}$$

In order to clarify the definitive sign of $\partial c/\partial m$ and $\partial U(.)/\partial m$ we proceed:

For $a \in (0, T-R]$, $R \in [T/2, T]$, $p \in [0,1]$ and $m \in (0,1)$ define a function $T_{a,R}(m,p) := \frac{a}{T-R}\left(R - \frac{1}{2}a\right) + R\gamma_1 + \frac{1}{p}R\gamma_2$. Notice that the sign of $T_{a,R}(m,p)$ is equivalent to the sign of $\partial c/\partial m$ and $\partial U(.)/\partial m$. The function $T_{a,R}(m,p) > 0$ if and only if $\frac{a}{T-R}\left(R - \frac{1}{2}a\right) + R\gamma_1 + \frac{1}{p}R\gamma_2 > 0$, that is, if and only if $p > -\frac{2R\gamma_2 T - 2R^2\gamma_2}{2aR - a^2 + 2R\gamma_1 T - 2R^2\gamma_1}$. For $a \in (0, T-R]$, $R \in [T/2, T]$ and $m \in (0,1)$, define a function $Q_{a,R}(m) := -\frac{2R\gamma_2 T - 2R^2\gamma_2}{2aR - a^2 + 2R\gamma_1 T - 2R^2\gamma_1}$. Besides, $Q_{a,R}(m)$ is a continuous, monotone and decreasing function. Let $\overline{p}$ be the maximum value of $Q_{a,R}(m)$ with $a \in (0, T-R]$ and $R \in [T/2, T]$. Then for any $p > \overline{p}$ we obtain that $p > Q_{a,R}(m)$,

$\forall m \in [0,1]$. Consequently, for any $p > \overline{p}$, we have that $T_{a,R}(m,p) > 0$, i.e., $\partial c/\partial m$ and $\partial U(.)/\partial m$ are positive, $\forall m \in (0,1)$.

Analogously, let $\underline{p}$ be the minimum value of $Q_{a,R}(m)$ with $a \in (0, T-R]$ and $R \in [T/2, T]$. Then, for any $p < \underline{p}$ we have that $p < Q_{a,R}(m)$ and thus $T_{a,R}(m,p)$, $\partial c/\partial m$ and $\partial U(.)/\partial m$ are negative $\forall m \in (0,1)$.

Moreover, since $Q_{a,R}(m)$ is a continuous, monotone and decreasing function. For any $\underline{p} < p < \overline{p}$ there exists a $m^* \in (0,1)$ such that $p = Q_{a,R}(m^*)$ with $a \in (0, T-R]$ and $R \in [T/2, T]$. Thus $T_{a,R}(m^*, p)$ $\frac{\partial c}{\partial m}|_{m^*}$ and $\frac{\partial U(.)}{\partial m}|_{m^*}$ are equal to zero. Then, for any $m' < m^*$ implies that $Q_{a,R}(m') > Q_{a,R}(m^*)$ what means that $Q_{a,R}(m') > p$. As we showed earlier, this is equivalent to that $T_{a,R}(m,p)$, $\partial c/\partial m$ and $\partial U(.)/\partial m$ are negative $\forall m \in (0, m^*)$. Symmetrically, for any $m' > m^*$ this leads to that $Q_{a,R}(m') < Q_{a,R}(m^*)$ what yields that $Q_{a,R}(m') < p$. That is, $T_{a,R}(m,p)$, $\partial c/\partial m$ and $\partial U(.)/\partial m$ are positive $\forall m \in (m^*, 1)$.

Let $\overline{I}$ and $\underline{I}$ be the levels of average labor productivity of immigrants with $\overline{p}$ and $\underline{p}$ respectively. Therefore, we can state the following:

i) For any $I \in (0, \underline{I})$ then $\partial c/\partial m$ and $\partial U(.)/\partial m$ are strictly negative.

ii) For any $I \in \left(\overline{I}, L\right)$ then $\partial c/\partial m$ and $\partial U(.)/\partial m$ are strictly positive.

iii) For any $I \in \left(\underline{I}, \overline{I}\right)$ the signs of $\partial c/\partial m$ and $\partial U(.)/\partial m$ depend on m. In this case, there exists a m^* such that for any $m \in [0, m^*)$ the signs of $\partial c/\partial m$ and $\partial U(.)/\partial m$ are negative and for any $m \in (m^*, 1]$ the signs of $\partial c/\partial m$ and $\partial U(.)/\partial m$ are positive.

Llet us to define $p_0 = Q_{a,R}(m_0)$ and $p_1 = Q_{a,R}(m_1)$. Let us to suppose that $p_1 < p_0$ and $m_1 < m_0$. Since $Q_{a,R}(m)$ is a continuous, monotone and decreasing function, $m_1 < m_0$ implies that $Q_{a,R}(m_1) > Q_{a,R}(m_0)$ or equivalently $p_1 > p_0$, what is a contradiction. Therefore, if $m_1 < m_0$ then $p_1 > p_0$. Therefore, the larger m, a lower I is needed to obtain $\partial c/\partial m$ and $\partial U(.)/\partial m$ positive or negative.

At last, there is no analytical solution to verify the inverse relationship between I and a, but numerically one can easily obtain it.[13]. Q.E.D.

Notes

[1] Djajic (2003) argues that the assimilation of immigrants is a multidimentional process of enormous complexity. In each of theses dimensions (earnings, human capital occupational status, consumption, fertility...) they assimilate at rates that may differ from those of their children.

[2] For the sake of simplicity, we suppose that the mandatory retirement age $R \in (T/2, T)$. That is, the number of workers must be larger than the number of retirees. In fact, even in the most pessimistic estimations about aging, the ratio pensioners-workers is always less than one.

[3] We consider that both assumptions fully redistributive pension system and fully rational individuals are the most appropriate scenario to analyze the impact over time of unskilled immigration on the local population's welfare through pension benefits.

[4] The accumulated wealth arises from the difference between total earned income and total consumption until period t.

[5] This framework is similar to assume that the new constant birth rate is normalized to $1+m$.

[6] The individual annual pension of retirees can be rewritten as

$$p(a) = \frac{\tau (T-a)}{(T-R)^2} \left(RL + mI \left(\frac{T+a}{2} \right) \right)$$

[7] The individual annual pension can be rewritten as

$$p(a) = \frac{\tau}{(T-R)} \left(RL + mI \left(\frac{T+R}{2} - a \right) \right)$$

[8] The individual annual pension benefits can be rewritten as

$$p\left(a\right)=\frac{\tau}{\left(T-R\right)}\left[\frac{a}{T-R}\left(RL+mI\left(R-\frac{a}{2}\right)\right)+RL\alpha+RmI\alpha\right]$$

where $\alpha=\frac{\ln(T-R+mT-mR-ma)-\ln(T-R)}{m}$.

[9] The individual annual pension benefits can be rewritten as

$$p\left(0\right)=\frac{\tau R}{T-R}\frac{\ln\left(1+m\right)}{m}\left(L+mI\right)$$

[10] Notice that this cohort of population is not a negligible number. For instance, for a life-cycle of 40 years as worker plus 15 years as retired, this cohort would represent the 62.5% of working population.

[11] Since $R\in(T/2,T)$ and $0<a\leq T-R$ it is straightforward to verify that $R>\frac{1}{2}a$ and $\gamma_1\geq 0$.

Let $G(m)=\gamma_2$ be a function of m. For any $m\in(0,1)$ it is obtained that $\lim\limits_{m\to 0}G\left(0\right)<G\left(1\right)\leq 0$ with $G'\left(m\right)>0$. Thus it can be concluded that $\gamma_2\leq 0$.

[12] Since $m\in(0,1)$ it is always verified that $\delta_1>0$.

Let $J(m)=\delta_2$ be function a of m. For any $m\in(0,1)$ it is obtained that $\lim\limits_{m\to 0}J\left(0\right)<J\left(1\right)<0$ with $J'\left(m\right)>0$. Thus it can be concluded that $\delta_2<0$.

Let $K\left(m\right)=\delta_1+\delta_2$ be function a of m. For any $m\in(0,1)$ it is obtained that $\lim\limits_{m\to 0}K\left(0\right)>K\left(1\right)>0$ with $K'\left(m\right)<0$. Thus it can be concluded that $\left|\delta_1\right|>\left|\delta_2\right|$.

[13] For the following parameter values is verified the inverse relationship between I and a: $T=57$; $R=48$; $l>0$; $\tau\in\left(0,1\right)$; $L>0$; $I\leq L$; $m\in(0,1)$ and $a\leq T-R$.

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