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Dynamic Instabilities Induced by Irrational Behavior in Financial Markets: Causes and Consequences for Risk Assessment

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CONTENT

CAPÍTULO 1

1 RESUMEN Y CONCLUSIONES[1]

1.1 OBJETIVO DE LA INVESTIGACIÓN

El objetivo de esta Tesis Doctoral es mostrar cómo el riesgo puede medirse sobre la base del comportamiento irracional en el movimiento de los precios de las acciones en los mercados bursátiles. El comportamiento irracional se mide en el dominio espectral descomponiendo el movimiento de los precios mediante transformaciones de Fourier discretas, resultando en una media del riesgo alternativa a la volatilidad. La tesis consiste en tres partes.

- La primera parte, que se refleja en el trabajo Schädler y Grabinski (2015), muestra cómo las transacciones financieras especulativas pueden generar inestabilidades dinámicas. La idea consiste en extender el bien conocido modelo NAIRU (Non-Accelerating Interest Rate of Unemployment) para el capital, bajo el supuesto de que el desempleo es proporcional al capital invertido mientras que el capital invertido depende del tipo de interés. El modelo no lineal resultante se escrutan los resultados con el fin de comprobar si el equilibrio es estable o inestable.

  El modelo dinámico en su formulación básica en el que el capital depende del ahorro previo muestra un estado de equilibrio estable. Después, la diferente influencia de los cambios en el tipo de interés sobre el comportamiento de la inversión en la economía real se compara con el comportamiento en el sector financiero. El siguiente paso consiste en añadir renta procedente de transacciones financieras especulativas. Esto conduce a un acoplamiento más

[1] Este capítulo fue traducido del inglés al español por Prof. José Mª Labeaga.

fuerte de ecuaciones diferenciales. Los resultados indican que las soluciones se convierten en inestables si el volumen de renta proveniente de transacciones financieras especulativas llega a ser suficientemente grande. Se muestra que, incluso en modelos tan simples como el NAIRU, el cambio en ciertos parámetros puede conducir a inestabilidades, que en el peor caso desemboca en caos matemático. Ello es debido a la mayor respuesta del tipo de interés en los mercados financieros que en la economía real.

- En la segunda parte, Schädler (2018) analiza el impacto del comportamiento no racional en el movimiento de los precios en los mercados bursátiles a través de la descomposición de frecuencia en el dominio espectral. El espectro de potencia permite que las frecuencias sean sumadas a bandas de frecuencia y de esa manera se conducen los excesos del comportamiento especulativo en relación con los ajustes de precios racionales impulsados por eventos basados en nueva información individual. El concepto de especulación comúnmente asociado a los participantes del mercado que negocian acciones se extiende se extiende al comportamiento especulativo de, por ejemplo, la administración de una sociedad anónima que intenta influir en el precio de la acción en cualquier dirección. Con el fin de distinguir los conceptos y relacionarlos con la volatilidad como una medida de riesgo alternativa, la relación se denomina irracionalidad. La irracionalidad determina el impacto que las fluctuaciones de tres meses a un año tienen en el desarrollo del precio en relación con la influencia de las fluctuaciones de hasta tres meses, lo que representa la incertidumbre en el modelo de negocio. Se elige el período de fluctuaciones de hasta tres meses, ya que representa el intervalo máximo dentro del cual se publica la nueva información de las empresas públicas. Para determinar las influencias, los precios históricos de las acciones se transfieren del dominio del tiempo al dominio de la frecuencia a través de una transformada de Fourier discreta. El análisis basado en el índice bursátil alemán (DAX) durante un período de 20 años muestra que la influencia del rango especulativo es 2,5 a 5,5 veces mayor que la del rango racional. Además, se obtienen indicios de que la irracionalidad se puede usar para medir el riesgo.

- La tercera parte la constituye el trabajo de Schädler y Steurer (2019), que evalúa el comportamiento no racional como una media alternativa al riesgo en comparación con la volatilidad y examina de forma empírica su poder explicativo. Para el análisis ex - post que abarca más de veinte años, los componentes individuales de los índices de mercado alemanes, europeos y estadounidenses se analizan de dos formas: primero, en conjunto con la volatilidad y segundo, en un análisis exclusivamente basado en la irracionalidad. En el ejemplo de la irracionalidad como una medida de riesgo independiente, los índices se clasifican en orden ascendente de acuerdo con su nivel de irracionalidad y se dividen en carteras de cuartiles de riesgo bajo a alto. Entre los resultados, encontramos una relación negativa entre riesgo y rendimiento, en términos de irracionalidad, en los tres índices. El efecto es estadísticamente significativo e importante cuando se comparan las carteras de mercado con las carteras de alto riesgo. Además, el análisis proporciona una indicación de cómo se puede replicar un índice con menos gastos y complejidad, particularmente en relación con el hecho de que la volatilidad de la cartera de inversión sintética es equivalente al portafolio del mercado.

1.2 FUNDAMENTOS TEÓRICOS

La comprensión de los mecanismos y las características de los mercados de capital en términos de riesgo y rentabilidad es importante para el manejo de los riesgos financieros y, actualmente es una línea de investigación en proceso economía financiera. Una clave de la moderna teoría financiera fue el descubrimiento que el riesgo de una cartera diversificada se puede cuantificar a través de la varianza de sus rentabilidades (Markowitz, 1952) y, por tanto, el retorno esperado resulta ser una función lineal de dicho riesgo. Aunque Fama, trabajando bajo la influencia de Mandelbrot[2], rechazó el supuesto de la normalidad en la distribución de la rentabilidad de las acciones a favor de la distribución estable de Pareto[3] (Fama, 1965a), solo a través de la comprensión de la evaluación de riesgos basada en cambios históricos en los precios fue capaz de formular la teoría de los mercados eficientes (Fama, 1970). Bajo el supuesto de que los mercados financieros son un juego justo en el que los precios reflejan fielmente la información disponible, la teoría distingue tres formas diferentes. En su forma débil, la teoría asume que solo se proporciona como información los precios históricos y, por tanto, la tendencia es conocida y no permite exceso de rentabilidad basada en esta información histórica. En la forma semi-fuerte, los precios, además, incluyen y reflejan toda la información pública disponible. Finalmente, la forma fuerte de la teoría establece que toda la información tanto pública como privada, incluida la información privilegiada, se incorpora de forma instantánea en los precios de los activos financieros. En este contexto, no vale la pena incluso en la versión fuerte disponer de las distribuciones históricas de las rentabilidades ya que se asume que son estacionarias, por lo que contienen toda la información valiosa acerca de la distribución de los retornos esperados y constituyen la mejor fuente de información disponible.

Una teoría tan sencilla y plausible como el supuesto de expectativas racionales de movimientos de precios en el corto plazo no puede explicar el excesivo movimiento

[2] Mandelbrot (1963) mostró que los movimientos de precios siguen una distribución de Pareto estable.

[3] Más tarde, Blattberg y Gonedes (1977) mostraron que las distribuciones t-Student explican mejor los rendimientos diarios de las acciones que las distribuciones de Pareto; esto sucede, sin embargo, bajo la hipótesis improbable de que las rentabilidades son independientes, dado que rentabilidades altas suelen preceder a rentabilidades altas en cualquier dirección.

en el largo plazo basado en la especulación ni el comportamiento irracional debido a sesgos cognitivos.

En relación con el comportamiento no racional, Kahneman y Tversky proporcionaron evidencia que los individuos utilizan al menos parcialmente la heurística para convertir problemas complejos en tareas juiciosas. Mientras este comportamiento puede conducir a errores sistemáticos, estos autores encuentran que la heurística es útil (Tversky & Kahneman, 1974). Un ejemplo de error sistemático resultante de esta heurística es la falacia del jugador. En la misma, un jugador que solo apuesta al color o negro observa una ruleta. Después de que la bola termine en negro un par de veces en una secuencia, el jugador tiende a pensar que es más probable que la bola termine en rojo la siguiente tirada. Inconscientemente, el jugador espera una reversión a la media para un giro en particular. Estudios posteriores muestran que las emociones pueden constituir un factor crucial en la toma de decisiones bajo incertidumbre. En la teoría de la perspectiva (Prospect Theory), siendo una alternativa a la teoría de la utilidad esperada, Kahneman y Tversky muestran que, debido a la aversión al riesgo, las elecciones con resultados ciertos se ven favorecidas sobre elecciones con resultados probable, lo que denominan el "efecto de la certidumbre", y muestran también el efecto de la asimetría entre la percepción de valores de pérdidas y de ganancias (Kahneman & Tversky, 1979). Como resultado, prueban que limitadas cantidades de información irrelevante para la solución racional pueden cambiar los resultados de la decisión de los agentes.

Incluso el estado anímico de los participantes en el mercado puede tener influencia sobre el desarrollo del mismo, tal como muestran Hirshleifer y Shunman (2003), quienes relacionan los cambios diarios en los precios en varios índices con el tiempo climatológico en un período de 15 años. Sus resultados muestran que cambios en los precios están correlacionados significativamente con las condiciones soleadas de forma que incluso después de tener en cuenta los costes de transacción, una estrategia de inversión puede beneficiarse de la predicción del tiempo meteorológico. Estos resultados no se pueden explicar mediante la teoría de mercados eficientes.

Los excesivos movimientos de mercado de largo plazo, que son inexplicables mediante cambios en las expectativas de descuento de dividendos futuros, han sido estudiados extensamente (West (1988), Shiller & Beltratti (1992), Campbell & Ammer (1993)). Las explicaciones posibles para el exceso de volatilidad y, por tanto, para la inestabilidad de los precios está explicada en Case y Shiller (2003) y Shiller (2017). Ambos trabajos constituyen un conjunto de creencias y narrativas alrededor de sobre los participantes en el mercado. En el caso del mercado de vivienda, el miedo a perder y no ser capaz de conseguir una vivienda en el futuro debido a subidas en los precios puede afectar a la demanda y, en consecuencia, a los precios. Además, el exceso de confianza en la probabilidad de subidas considerables de precios y, al mismo tiempo, la subestimación del riesgo de que los precios bajen en un futuro próximo puede justificar unos precios demasiado altos basados en los fundamentales. En el caso de precios especulativos de activos como las acciones atraen la atención pública debido a las narrativas acerca del rápido crecimiento de los precios por la vía de la cobertura mediática de una empresa en particular o del mercado en general y pueden modificar la demanda y, en consecuencia, los precios. Cueva et al. (2015) relacionan los altos niveles de cortisona a tomas de decisiones con altos niveles de riesgo. Además, elevados niveles de testosterona se asociaron con un aumento del optimismo acerca de las expectativas de desarrollos de precios futuros. Ambas hormonas tienden a apoyar inversiones en activos más volátiles, de riesgo mayor. Como novedad, información no esperada puede aumentar los riesgos de cortisol y ganancias repetidas en acciones con movimiento ascendente pueden incrementar el optimismo a través del aumento de los niveles de testosterona, de forma que ambas hormonas podrían contribuir a la aparición de burbujas financieras.

Siguiendo la propuesta fundamental de la teoría de los mercados eficientes de que el precio de la deuda refleja la información actual disponible y es, por tanto, el mejor estimador del valor corriente, Shiller (1981) contrastó la volatilidad de los movimientos de precios reales de las acciones frente a la variabilidad de los cambios en los futuros dividendos. Como el valor de una inversión se deriva del flujo descontado de su cash-flow, los movimientos en los precios debieran ser explicados por cambios en las expectativas acerca de los dividendos futuros basados en nueva información.

Tomando como ejemplo el índice de precios de la cartera compuesta de Standard & Poor's para el período 1871-1979 y las medias del índice industrial del Dow Jones para el período 1928-1979, mostró que *"… la volatilidad en el siglo pasado es aparentemente distinta de ser alta – de cinco a trece veces demasiado alta – para ser atribuida a la nueva información acerca de los dividendos futuros si la incertidumbre sobre estos se mide por la desviación estándar muestral de los dividendos reales en torno a su crecimiento exponencial de largo plazo."* (Shiller, 1981, p. 433). Su conclusión es que ni los cambios en los tipos de interés real esperados ni el miedo a movimientos mayores en los precios pueden explicar los movimientos observados en los precios.

Además, Appel y Grabinski (2011) muestran que los valores fundamentales de una empresa solo están sujetos a fluctuaciones menores en relación con los movimientos del precio de la deuda, ajustando el valor presente de su cash-flow futuro para valores no conservados.

En los límites de la volatilidad, Sornette, Cauwels y Smilyanov encuentran que *"… la volatilidad no es un indicador de confianza de la maduración de una burbuja ni su impedimento para que termine en un crash."* (Sornette, Cauwels, & Smilyanov, 2018, p. 95). De las 40 burbujas históricas que analizan, desde el mercado bursátil, pasando por el mercado de futuros o el de monedas, 26 muestran rentabilidades positivas sin un significativo incremento de la volatilidad durante el aumento del pico de la burbuja. Para resolver este inconveniente, se recomienda el uso de los modelos Log-Periodic Power Law Singularities como en Sornette y Cauwels (2014), Zhou y Sornette (2008) o Zhou y Sornette (2009). Un punto clave de estos modelos es que permiten realizar el test de significatividad estadística de las oscilaciones periódicas aceleradas en logaritmos durante la generación de una burbuja sea positiva o negativa con un espectro de poder discreto normalizado a datos que pueden no ser equidistantes.

Aplicaciones adicionales de métodos de Fourier se presentan en Bormetti, Cazzola, Livan, Montagna y Nicrosini (2010), que muestran como el VaR (Value at Risk) y el CVaR (Conditional Value at Risk) o caída esperada se puede calcular de forma eficiente en el dominio de la frecuencia aplicando transformaciones de Fourier generalizadas. Barunik y Krehlik (2018) un método de Fourier para calcular el VaR en series de rentabilidad de las acciones con colas gordas y en carteras con derivados.

Sobre la base de estas consideraciones teóricas, se puede concluir que mientras que los precios de las acciones se pueden modelar de forma óptima a través de un paseo aleatorio, en el corto plazo este método resulta menos eficiente y puede llegar a ser ineficiente a largo plazo debido a comportamientos no racionales. Además, la aplicación de técnicas de Fourier puede constituir una valiosa contribución a la valoración del riesgo de los activos financieros.

1.3 Impacto en la Evaluación del Riesgo

Una cuestión fundamental de las finanzas, desde el punto de vista académico, es la cuantificación del riesgo. El supuesto generalmente aceptado es que los inversores han de soportar riesgos altos para obtener altas rentabilidades. El efecto de este supuesto tanto para inversores como para fondos de inversión e instituciones gubernamentales es habitualmente sobreestimado, dado que el concepto es transversal a todas las áreas involucradas. Esto sucede especialmente en el momento actual de bajos tipos de interés, que se extiende ya durante un período de más de diez años, lo que refuerza la tendencia a la mayor exposición a riesgos.

Aunque la medida del grado de variabilidad de la rentabilidad de los activos financieros ha llegado a ser una medida estadística clásica, la percepción y la medición de los riesgos puede variar de forma considerable a lo largo del tiempo. Por ejemplo, la volatilidad es una parte fundamental del modelo CAPM (Capital Asset Pricing Model) y los modelos VaR y sus refinamientos subsiguientes. Además, la volatilidad es en sí misma volátil. Así, un periodo de baja o alta volatilidad puede cambiar y, por tanto, se puede adaptar la percepción del riesgo a lo largo del tiempo cuando se analizan los activos financieros en términos relativos. Una forma evolucionada de ajustar la volatilidad es a través de la utilización de momentos inferiores parciales. Bajo este prisma, el método de medición del riesgo que se presenta en la tesis es disruptivo y su uso no necesita ningún supuesto subyacente.

A través de la transformación de series temporales históricas al dominio de frecuencias, el concepto de volatilidad se puede extender a través del análisis de la influencia de las diferentes bandas de frecuencia en el movimiento del precio de las acciones. De esta forma, no solo es posible examinar globalmente el grado de variación de diferentes intervalos de tiempo sino también se puede analizar el cambio en la composición a lo largo de la serie temporal. Esta propiedad permite cuantificar las variaciones y extender lo que es posible en los métodos financieros con comportamiento. El método que se aplica en la tesis, que incluye dentro del análisis la posibilidad de comportamiento no racional, resulta en una ratio que asocia riesgos altos a rentabilidades bajas. El uso de ratios con esta característica puede alterar la percepción del riesgo y conducir a una reducción de la exuberancia irracional en los mercados de capital. En el mejor de los casos, las burbujas en los mercados financieros se pueden reducir y las recesiones se pueden mitigar.

1.4 METODOLOGÍA

La Tesis Doctoral enfoca el problema de valoración del riesgo con un enfoque basado en comportamiento no racional bajo dos perspectivas diferentes. Primero, el capítulo 3 analiza la estabilidad de un modelo no lineal muy general extendido para incluir transacciones financieras. Segundo, en los capítulos 4 y 5 se analiza el peso del comportamiento especulativo en relación con el ajuste explicado por el comportamiento racional y su capacidad explicativa para la selección de la cartera de inversión.

En el capítulo 3 se estudia la estabilidad e inestabilidad de los estados estables en modelos dinámicos no lineales, utilizando como ejemplo el modelo NAIRU. Como dicho modelo consiste en una ecuación diferencial con dos variables desconocidas – los cambios en las tasas de inflación y desempleo a lo largo del tiempo – se requiere otra ecuación diferencial para resolver el modelo. Para completar dicho modelo, se realiza el supuesto de que la cantidad de individuos a emplear es proporcional al capital invertido. En el segundo caso, la ecuación diferencial para el capital invertido se extiende para permitir transacciones financieras especulativas. La lógica que está en la diferenciación entre ambos casos es que las empresas invierten, por ejemplo, en propiedades, instalaciones y equipamiento o capital intangible en I+D si la rentabilidad esperada es superior a la media ponderada del coste de uso del capital (WACC). Incluso asegurando la comparabilidad de los estados financieros como una característica cualitativa por sí misma, el WACC es de forma muy limitada expuesto a cambios menores en el tipo de interés de los bancos centrales de, por ejemplo, un cuartillo. Por el contrario, estos cambios en los tipos de interés tienen un impacto directo y significativo en la inversión o desinversión en activos financieros. De cara a obtener información sobre la estabilidad de ambos modelos, se linealizan alrededor de los puntos de equilibrio y se determinan los valores propios. Si todas las partes reales de los valores propios son negativas, la solución es asintóticamente estable. Si uno solo de los valores propios es positivo, la solución es inestable. Si las partes reales son iguales a cero, la solución en los modelos analizados es una oscilación no amortiguada alrededor del punto de equilibrio.

Sobre la base del resultado de que el comportamiento especulativo conduce a la inestabilidad en determinadas circunstancias, el capítulo 4 propone una medida del riesgo que discrimina entre movimientos de precios racionales e irracionales. La aplicación de la transformación discreta de Fourier tiene, en este caso, varias ventajas. Primero, se puede aplicar tanto con procesos estocásticos como determinísticos. Segundo, la composición de los movimientos de precios se puede analizar mediante la frecuencia y la intensidad asociada. Además, en comparación con la volatilidad, permite un análisis más profundo del comportamiento de los precios, ya que la volatilidad mide de forma exclusiva el margen general de las fluctuaciones. De forma adicional, el supuesto de normalidad representa una simplificación que no se adecúa a la evidencia empírica de que la rentabilidad histórica de las acciones sigue una distribución estable de Pareto. Aplicaciones previas del análisis espectral sobre el precio de las acciones están presentes en el trabajo de Granger y Morgenstern (1963). Estudios posteriores como Johansen y Sornette (1999) aplican el análisis del periodograma de Lomb, un tipo de transformada discreta de Fourier para datos espaciados de forma desigual, de cara a investigar el rebasamiento de los mercados bursátiles antes de la llegada de recesiones críticas. Al contrario que en los dos estudios antes mencionados, los movimientos de precios no se examinan a través de frecuencias dominantes o a través de tests de significación estadística de la tendencia del logaritmo de oscilaciones periódicas. En lugar de ello, se relaciona la influencia de ciertas bandas de frecuencia sobre el movimiento de los precios.

Para evaluar el poder explicativo de la nueva medida irracional de riesgo, se realiza un estudio de selección de cartera de inversión en el capítulo 5. La objetividad del estudio se asegura al utilizar un universo de inversión extensivo sin introducir alteraciones ni eliminar valores atípicos. Además, de cara a preservar tanta información como sea posible en relación con el movimiento de los precios, se aplican procedimientos de eliminación de tendencia que filtran los datos lo menos posible. El análisis se lleva a cabo desde dos ópticas diferentes. Por una parte, se analizan los efectos de la combinación de las dos ratios de riesgo, irracionalidad y volatilidad, sobre la rentabilidad de la cartera. Las acciones se ordenan en orden ascendente de riesgo, primero por irracionalidad y después por volatilidad y en cada momento del tiempo se divide la cartera entre alto y bajo riesgo. Por otra parte, la deuda se ordena de forma ascendente por riesgo exclusivamente de acuerdo a los valores de la ratio de irracionalidad y se divide la cartera en cuatro diferentes del mismo tamaño. Se analizan las carteras utilizando las ratios de Sharpe y Calmar y también se contrasta la significación estadística de los resultados, enfrentando con las correspondientes carteras de mercado. Las conclusiones se toman teniendo en cuenta el poder explicativo de la irracionalidad como medida del riesgo.

1.5 Conclusiones e Investigaciones Futuras

En la Tesis Doctoral que se presenta se propone una nueva forma de medir el riesgo de las carteras de inversión en carteras teniendo en cuenta el comportamiento especulativo mediante una nueva medida que tiene en cuenta este comportamiento no racional. Se propone siguiendo el procedimiento tradicional que utiliza la volatilidad como medida del riesgo. El comportamiento no racional se basa en el análisis de las fluctuaciones de las series temporales de precios. La diferencia es que la medida propuesta no considera todo el margen de las fluctuaciones, sino que la ratio representa la importancia relativa de los comportamientos racional e irracional sobre el movimiento de los precios. En Schädler (2018) se muestra que la media de la influencia del comportamiento irracional en los componentes individuales de los precios que componen el índice DAX en un período de 20 años fue del 79.1%. La influencia del comportamiento no racional en el movimiento de los precios fue cuatro veces mayor que el efecto del modelo tradicional.

Dado que la irracionalidad evalúa las fluctuaciones de los precios para caracterizar los activos financieros, parece razonable aplicar las ratios como parte de la evaluación del riesgo de cara a analizar los riesgos de inversión en activos en los mercados financieros. La aproximación de la irracionalidad como un indicador de riesgo tiene la característica distintiva de que la rentabilidad esperada decrece cuando se incrementa el riesgo. En particular, las carteras de mayor riesgo tienen ratios de Sharpe y Calmar estadísticamente significantes bajos en comparación con la cartera de mercado. Este resultado puede tener una gran influencia en los procesos de decisión de inversiones y análisis de riesgos. La clave no es tanto tomar mayores riesgos para incrementar la rentabilidad esperada sino evitar inversiones con un grado relativo elevado de irracionalidad en las fluctuaciones de los precios. Ampliamente aplicado, esto conduce a bajadas de precios en inversiones especulativas y a una reducción de los “espíritus animales” por parte de los inversores. La exuberancia del mercado puede, de estar forma, reducirse y se pueden mitigar las recesiones.

De cara a investigaciones futuras, se puede mencionar que las propuestas de modelar la irracionalidad se pueden aplicar para valorar estrategias de inversión de medio y largo plazos. El análisis que se ha realizado considerando la evolución de los precios de los últimos 36 meses ha resultado bastante prometedora. Dado que el análisis de períodos de tiempo cortos medidos a intervalos constantes conduce a espectros más amplios, se puede considerar el análisis de valores intradía. De la misma forma que con la combinación de irracionalidad y volatilidad, se pueden examinar otros factores en el análisis empírico. Dado que la irracionalidad se basa únicamente en movimientos de precios, la ratio precio-valor contable u otros análisis fundamentales representan extensiones naturales.

CHAPTER 2

2 SUMMARY AND CONCLUSIONS

2.1 RESEARCH OBJECTIVES

The objective of this doctoral thesis is to show how risk can be measured based on irrational behavior in the price movements of shares listed on stock exchanges. Irrational behavior is measured in the spectral domain by decomposing price movements via the discrete Fourier transform, resulting in an alternative risk measure to volatility. The thesis consists of three parts.

- The first part, which is reflected in Schädler and Grabinski (2015), shows how speculative financial transactions can induce dynamic instabilities. The idea was to extend the model of the non-accelerating interest rate of unemployment (NAIRU) for capital under the assumption that employment is proportional to the capital invested whereby the capital invested is dependent on the interest rate. The resulting nonlinear models are scrutinized to determine whether their equilibrium solutions are stable or unstable.

  The dynamic model in its basic form, in which the capital depends on previous savings, shows a stable equilibrium state. Subsequently, the different influence of interest rate changes on the investment behavior of the real economy compared to the financial sector is outlined. In a next step, income from speculative financial transactions is added. This leads to a stronger coupling of the differential equations. The results indicate that the solutions become unstable if income from speculative financial transactions becomes sufficiently large. It is shown that even in models as simple as NAIRU the change of certain parameters can lead to instabilities, in the worst case to mathematical chaos. In the presented case, this is due to financial operations reacting more strongly to interest rate changes than the real economy.

- The second part, Schädler (2018) analyzes the impact of irrational behavior on stock price movements via frequency decomposition in the spectral domain. The power spectrum enables individual frequencies to be summed up in frequency bands and thereby the measurement of excesses of speculative behavior in relation to event-driven rational price adjustments based on new information. The concept of speculation commonly associated with market participants trading stocks is extended for speculative behavior of, for example, the management of a stock corporation trying to influence the stock price in either direction. To distinguish the concepts and relate to volatility as an alternative risk measure, the ratio is named irrationality. Irrationality determines the impact longer fluctuations of three months to one year have on the price development in relation to the influence of fluctuations of up to three months, which represent the uncertainty of the respective business model. The period of fluctuations of up to three months was chosen, as it represents the maximum interval within which new information from publicly held companies is published. To determine the influences, historical stock prices are transferred from the time domain to the frequency domain via the discrete Fourier transform. The analysis based on the German stock index (DAX) over a period of 20 years shows that the influence of the speculative range is 2.5 to 5.5 times higher than that of the rational range. In addition, there are indications that irrationality can be used to measure risk.

- The third part, Schädler and Steurer (2019), evaluates irrationality as an alternative risk measure in comparison to volatility and empirically examines its explanatory power. For the ex-post analysis ranging over twenty years, the individual constituents of the German, European and American market indices are analyzed in two ways: first, in conjunction with volatility, and second, exclusively based on irrationality. Exemplified by irrationality as a standalone risk measure, the indices are sorted in ascending order according to their amount of irrationality and split into portfolios of quartiles from low to high risk. The findings indicate a negative relationship between risk and return in terms of irrationality across the three indices. The effect is statistically significant and remarkably evident when comparing the market portfolios to the high-risk portfolios. Further, the analysis provides an indication of how an index can be replicated with fewer expenses and complexity, particularly regarding the fact that the volatility of the synthetic portfolio is equivalent to the market portfolio.

2.2 THEORETICAL FOUNDATIONS

Understanding the mechanisms and characteristics of capital markets both in terms of risk and return is essential for financial risk management and represents an ongoing research field in financial economics. A key point for modern financial theory was the finding that the risk of a diversified portfolio can be quantified by the variance of its returns (Markowitz, 1952) and therefore the expected return is a linear function of that risk. Although Fama, under Mandelbrot's influence[4], rejected the assumption of stock returns being normally distributed in favor of a stable Paretian distribution[5] (Fama, 1965a), it was only through the understanding of risk assessment based on historical price changes that he was able to formulate the theory of efficient markets (Fama, 1970). Under the assumption of financial markets being a fair game in that prices fully reflect available information, the theory distinguishes three different forms: In its weak form, the theory presumes that only historical price data is provided as information; therefore, the trend is known and does not allow for excess returns based on historical data. In the semi-strong form, prices further include and directly reflect all publicly available information. Lastly, the strong form states that all publicly available as well as private information, including insider information, is incorporated instantaneously into the prices of financial assets. In this context, it is worth noting that even in the strong form historical distributions of returns, due to being assumed to be stationary over time, contain valuable information about the distribution of future expected returns and might be the best source of information available.

As simple and plausible as the assumption of rational expectations of the short-term price changes is, it can neither explain long-term excessive market movements based on speculation nor irrational behavior due to cognitive biases.

Regarding irrational behavior, Kahneman and Tversky provided evidence that people use a small set of heuristics to convert complex problems into simple

[4] Mandelbrot (1963) showed stock price movements to follow a stable Paretian distribution.
[5] Blattberg and Gonedes (1977) later showed student distributions to better fit daily stock returns than stable Paretian distributions. However, under the empirically disproved assumption that returns are independent, as high returns tend to be followed by high returns in either direction.

judgmental tasks. While sometimes this may lead to systematic errors, they found these heuristics to be quite useful (Tversky & Kahneman, 1974). One example of a systematic error resulting from these heuristics is the gambler's fallacy: Here, a gambler only betting either on red or black observes a roulette table. After the ball has ended up in black pockets a couple of times in a row, the player tends to believe that the ball is more likely to end up in a red pocket on the next spin. Subconsciously the gambler expects a reversion to the mean for a particular spin. Further studies showed that emotions can be a crucial factor in decision making under uncertainty. In Prospect Theory, being an alternative to the expected utility theory, Kahneman and Tversky showed that, due to risk aversion, choices with certain outcomes are favored over choices with probable outcomes, which they called the "certainty effect" as well as the asymmetry between the perceived values of gains and losses (Kahneman & Tversky, 1979). As a result, they showed that small pieces of information irrelevant to the rational solution of the task can change the outcome of the decision.

How even the mood of the market participants can influence market development was shown by Hirshleifer and Shunman (2003), who linked the daily price changes of various indices to the prevailing weather over a period of 15 years. They found changes in price to be significantly correlated with sunny conditions in a way that, even after accounting for transaction costs, an investment strategy could benefit from the inclusion of the weather forecast. These findings cannot be explained with the efficient market theory.

Long-term excessive market movements inexplicable by changes in expectation about discounted future dividends have been studied thoroughly (West (1988), Shiller & Beltratti (1992), Campbell & Ammer (1993)). Possible explanations for the excess volatility and therefore unstable prices can be found in Case and Shiller (2003) and Shiller (2017). Both are a set of beliefs and narratives surrounding market participants. For the housing market the fear of missing out as well as not being able to afford a house in the future due to rising prices can drive demand and therewith prices. Furthermore, overconfidence in the probability of prices rising considerably and, at the same time, underestimation of the risk of prices falling any time soon can justify otherwise too-high prices based on fundamentals. In the case of speculative asset prices like stocks, heightened public attention due to narratives about rapidly rising prices via high media coverage of a particular company or the market as a whole can drive demand and therefore prices. Cueva et al. (2015) associate high levels of cortisone to higher risk-taking behavior. Further, elevated levels of testosterone were associated with an increase in optimism about future price developments. Both hormones tend to support investments in riskier, more volatile assets. As new, unexpected information can ramp up cortisol levels, and repeated gains in an upward moving stock market can increase optimism through heightened testosterone levels, the two hormones may contribute to the build-up of financial bubbles.

Following the fundamental proposition of the theory of efficient markets that equity prices reflect current available information and are therefore the best estimate of the present value, Shiller (1981) tested the volatility of real stock price movements against the variability of changes in future dividends. Since the value of an investment is derived from future discounted cash flows, price movements should be explained by changes in expectations about future dividends based on new information.

Using the example of the Standard and Poor's Composite Stock Price Index for the period from 1871 to 1979 and the Dow Jones Industrial Average from 1928 to 1979, he showed that the "… volatility over the past century appears to be far too high — five to thirteen times too high — to be attributed to new information about future real dividends if uncertainty about future dividends is measured by the sample standard deviations of real dividends around their long-run exponential growth path" (Shiller, 1981, p. 433), concluding that neither changes in expected real interest rates nor fear of much bigger price movements could explain the price movements.

Furthermore, Appel and Grabinski (2011) showed that fundamental company values are only subjected to minor fluctuations in relation to the respective equity price movements by adjusting the present value of future cash flows for non-conserved values.

On the limits of volatility, Sornette, Cauwels and Smilyanov found that „… volatility is neither a reliable indicator of the maturation of a bubble nor of its impeding ending in a crash“ (Sornette et al., 2018, p. 95). Out of the 40 historical bubbles analyzed ranging from stock and commodity futures to currencies, 26 showed high positive returns not accompanied by a significant rise in volatility during the ramp-up to the bubble peak. To resolve this shortcoming, the use of Log-Periodic Power Law Singularities models as in Sornette and Cauwels (2014), Zhou and Sornette (2008) or Zhou and Sornette (2009) is recommended. A key point of the models is the test for statistical significance of the accelerating log-periodic oscillations during the build-up of positive or negative bubbles via a normalized discrete power spectrum applicable to non-equidistant data.

Further applications of Fourier methods can be found in Bormetti, Cazzola, Livan, Montagna and Nicrosini (2010) who showed how the Value at Risk (VaR) and and Conditional Value at Risk (expected shortfall) can be efficiently calculated in the frequency domain by applying generalized Fourier transforms. Barunik and Krehlik (2018) showed a Fourier method for calculating the Value at Risk for fat-tailed stock returns as well as for portfolios with derivatives.

Based on these theoretical foundations, it can be concluded that while stock prices are best modeled through a near-random walk, they are efficient in the short run and can be inefficient due to irrational behavior in the longer run. Further, the application of Fourier techniques can make a valuable contribution to the risk assessment of financial assets.

2.3 Impact on Risk Assessment

A central question of academic finance is the quantification of risk. The generally accepted assumption is that for higher returns more risk must be borne by the investor. The impact of this assumption, from retail investors to mutual funds and government institutions, can hardly be overestimated, as this concept encompasses all areas. Especially in the current era of low interest rates, which has lasted for more than ten years, this reinforces the trend toward higher risk exposure.

Even though the measurement of the degree of variation in the returns of financial assets has become the classical statistical measure, the perception and measurement of risks can vary considerably and change over time. For instance, volatility is a fundamental part of the Capital Asset Pricing Model (CAPM) and VaR as well as their subsequent refinements. Furthermore, volatility itself is volatile. Thus, a period of low or high volatility can change and therefore adapt risk perception over time when financial assets are analyzed in relative terms. An evolutionary way to adjust volatility is to use the lower partial moments. Under this aspect, the method of risk measurement presented here is disruptive without the need for assuming an underlying distribution.

Through the transformation of historical time series into the frequency domain, the concept of volatility can be extended by the analysis of the influence of different frequency bands on the stock price movements. Hereby it is not only possible to examine the overall degree of variation of different time intervals or time periods but also to analyze the change in the composition of the time series. This property enables the quantitative measurement of variations to be extended by behavioral finance approaches. The approach applied in this thesis, which includes irrational behavior into the analysis, results in a ratio that associates higher risks with lower returns. The use of ratios with this characteristic may change the perception of risk and lead to a reduction of irrational exuberance in capital markets. At best, bubbles on the financial markets can be reduced and subsequent downturns mitigated.

2.4 METHODOLOGY

This doctoral thesis approaches the topic of risk assessment with a focus on irrational behavior from two perspectives. First, the stability of a very general nonlinear model extended for financial transactions is analyzed in chapter 3. Second, in chapter 4 and 5, the proportion of speculative behavior in relation to rational explainable adjustments in the price movements of financial assets is analyzed and its explanatory power for the portfolio selection is examined.

In chapter 3, the stability and instability of steady states of nonlinear dynamic models are analyzed using the example of the non-accelerating inflation rate of unemployment (NAIRU). As NAIRU is an incomplete model consisting of a differential equation with two unknown variables – the change in the inflation rate and the unemployment rate over time – a second differential equation is required to solve it. To complete the model, in the first case the assumption is made that the amount of people employed is proportional to the capital invested. In the second case, the differential equation for the capital invested is further extended for speculative financial transactions. The logic behind the distinction between the two cases is that companies invest, for example, in property, plant and equipment, intangible assets or research and development if the expected rate of return is above the weighted average cost of capital (WACC). Even on account of ensuring the comparability of financial statements as a qualitative characteristic alone, the WACC is to a very limited extent exposed to minor changes in central bank interest rates of, for example, one quarter of a percent. In contrast, such changes in interest rates have a direct and significant impact on the investment or divestment decisions of financial assets. In order to obtain information about the stability of the two models, they are linearized around their equilibrium points and the eigenvalues are determined. If all real parts of the eigenvalues are negative, the solution is asymptotically stable. As soon as one eigenvalue is positive, the solution is unstable. If the real parts equal zero, the solution is an undamped oscillation around the equilibrium point in the models studied.

Based on the finding that speculative behavior leads to instability under certain circumstances, chapter 4 proposes a risk-measure that distinguishes between rational and irrational price movements. The application of the discrete Fourier transformation in this case has several advantages: First, it is applicable to both stochastic and deterministic processes. Second, the composition of price movements can be investigated by frequency and the associated intensity. Furthermore, in comparison to volatility, it allows for a deeper analysis of price movements, as volatility exclusively measures the overall margin of fluctuations. Additionally, the assumption of a normal distribution represents a simplification which falls short of the empirical evidence that historical stock returns follow a stable Paretian distribution. Early applications of spectral analysis on stock prices can be found in Granger and Morgenstern (1963). Further studies like Johansen and Sornette (1999) applied the Lomb periodogram analysis, a type of discrete Fourier transform for unequally spaced data, to research the overshooting of stock markets before critical crashes. In contrast to the two studies mentioned above, the price movements are not examined toward dominant frequencies or a test for statistical significance of the trend of log-periodic oscillations. Instead, the influence of certain frequency bands on the price movements are put in relation to one another.

To assess the explanatory power of the new risk measure irrationality, an empirical study on portfolio selection follows in chapter 5. The objectivity of the study was ensured by the fact that an extensive investment universe was analyzed, and there were no alterations by removing outliers. Further, in order to preserve as much information as possible concerning price movements, detrending procedures are applied, which filter out as little of the relevant details as possible. The analysis was carried out from two angles: On the one hand, the effects of the combination of the two risk ratios irrationality and volatility on the total returns of the portfolios were studied. The shares were sorted by ascending risk, first by irrationality, then by volatility and each time divided into portfolios with high and low risk. On the other hand, equities were sorted in ascending order by risk exclusively according to their irrationality values and divided into four portfolios of equal size. By analyzing the portfolios based on their Sharpe Ratios and Calmar Ratios as well as testing the statistical significance of the results against the corresponding market portfolio, conclusions were drawn regarding the explanatory power of irrationality as a risk measure.

2.5 Conclusions and Future Research

To determine the extent to which the price movements of equities result from speculative behavior, a new risk measure – irrationality – was developed. In the same way as historical volatility, irrationality is based on the analysis of the fluctuations of time series. It differs, however, in that it does not consider the overall margin of fluctuations, but rather represents a ratio of the relative impact of rational and irrational fluctuations on the price movements. In Schädler (2018), it was shown that the mean of the irrational influences on the individual stock prices of members of the DAX over a period of twenty years was 79.1%. The influence of irrational behavior on the price movement was correspondingly four times higher than the fluctuations caused by the underlying business model.

Since irrationality evaluates price fluctuations to characterize financial assets, it is reasonable to apply the ratio as part of the risk assessment of investments in financial markets. The approach of irrationality as a risk indicator has the distinctive property that the expected return decreases with increasing risk. In particular, higher-risk portfolios have statistically significant lower Sharpe ratios and Calmar ratios compared to the market portfolio. This insight can have a fundamental influence on the investment process and risk analysis. The focus is no longer on taking higher risks to increase expected returns, but on avoiding investments with a high relative degree of irrational price fluctuations. Broadly applied, this leads to falling prices for highly speculative investments and a reduction in animal spirits of investors. Market exuberances may thereby be reduced, and subsequent downturns mitigated.

For future research, it is worth mentioning that research is currently being conducted into the implementation of irrationality as an investment strategy for medium to long-term investments. A rolling analysis of the last 36 months seems promising. As the analysis of shorter time periods at constant measurement intervals leads to a broader spectrum, intra-day values may be considered. In the same way as the combination of irrationality and volatility, other factors may also be examined and added to the analysis. Since irrationality is based solely on price movements, the book/market ratio or other fundamental analyses represent natural extensions.

CHAPTER 3

3 INCOME FROM SPECULATIVE FINANCIAL TRANSACTIONS WILL ALWAYS LEAD TO MACRO-ECONOMIC INSTABILITY

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3.1 ABSTRACT

Starting with a macro-economic model based upon the NAIRU (the nonaccelerating inflation rate of unemployment), we show that, in a world with no (speculative) financial transactions, the macro- economy shows a stable equilibrium state. Including income from (speculative) financial transactions will lead to instability if the amount is sufficiently large. Considering the present amount of financial transactions, stability is impossible. Therefore, further financial crashes are not only likely but inevitable.

Keywords:

System Dynamics, Instability, Speculative Financial Transactions, Conserved Value, Chaos.

JEL Codes:

C62, E37, E44

3.2 INTRODUCTION

Economic models are in some sense the experiments of economists. Real experiments, such as in physics, are impossible in economics, so scrutinizing the economy is either done through executing fiscal policy or by "playing" with economic models. While access to the first method is limited to very few people (and may be immoral), the second method is the way of choice for most economists in science. Needless to say, there are very many different economic models. It is virtually impossible to address even a small part of them.

Our focus is not to add yet another new macro-economic model to the long list already existing. The main focus in many economic models is on finding the equilibrium state when some parameter is changed. One may, for instance, want to know by how much inflation would rise if the central bank lowered the interest rate by a certain percentage. Furthermore, one might like to know within what time span this new equilibrium would be reached. These are important questions and answering them is essential for fiscal policy makers.

Besides creating and solving an economic model, one should always prove its stability. What happens when all input parameters (e.g., tax rates) stay the same and the model is put off equilibrium for a short period of time? Will it come back to equilibrium (stability) or shift to a new state (instability). In a stable economic system, conducting fiscal policy makes sense. If an economic system is unstable, however, fiscal policy will never steer toward a proper equilibrium. In some sense capitalism should be declared a failure. Since real economic systems are extremely complex, instability will lead to chaotic fluctuations (in a mathematical sense, cf., e.g., Schuster (1984)). Neither predictions nor governance are possible then.

Surprisingly little can be found about stability analysis in economic systems. If done, the focus is typically on a very particular model, and the goal is to prove or forecast a particular scenario like a financial crisis. Chiarella, Flaschel, Köper, Proaño and Semmler (2012) examined the "financial meltdown" in that vein in 2008. Our focus is slightly different, though. We want to give a more general answer.

To see the point, we have to take a step back. An economy is nothing other than an arrangement of individual human beings and companies. Trying to forecast a particular person's income for the next year or a company's cash flow is surprisingly simple, and the forecasts are highly accurate, see, e.g., SAP, a manufacturer of ERP systems, in Appel and Grabinski (2011). To predict its cash flow is quite simple. It can be represented as a slightly rising line over many years with hardly any fluctuations. Its stock price should be proportional to its (future) cash flow. Surprisingly, it fluctuates by ± 20 % within months as a typical result. As explained by Appel (2012), this is due to the often-ignored difference between price and value. Unfortunately, a rising stock price may create real cash and therefore value to be invested in the real economy. Based on the work of Grabinski (2004, 2007), Appel (2012) defined the term conserved value in contrast to non-conserved value. Both values can be measured in monetary units, and they may be physically existent. However, conserved value can only change if something else changes accordingly. This is like the conserved quantity energy in physics; it is highly predictable. In contrast, there is such a thing as the "value" of a stock. It is not conserved and may change without notice at any time. A non-conserved quantity is by no means suitable to describe a system. If the system is sufficiently complex, there will be chaotic fluctuations which are not predictable.

Within this concept, the momentum effect could be explained (Appel, Dziergwa, & Grabinski, 2012). Furthermore, one can show that a Tobin tax would always be positive and could even be introduced nationally (Dziergwa, Klinkokva, & Grabinski, 2013). As far as stability is concerned, one can also show that dealing in financial products is in most cases identical to gambling, (Klinkokva & Grabinski, 2013). Dziergwa (2015) applied the concept of conserved value to a new accounting principle: Conserved value-based accounting principles (CVBAP). Our goal is to apply it to the macro-economic world.

The conjecture that something has gone wrong in economic modeling is not far-fetched. Consider, for instance, a situation where a central bank raises or lowers the interest rate by half a percentage point. Reactions won't be long in coming. With lower interest, for example, borrowing is cheaper, and investment should increase, eventually leading to higher GDP. The reasoning behind this seems almost trivial. The magnitude of its impact, however, is surprising: Lowering interest rates by half a percentage point will have a measurable effect. It will not affect the investment decisions of (real-world) companies, though. Typically, one may demand a gross return on investment of around 20 %, with an assumed capital borrowing rate of, perhaps, 7 %. Not a single decision would be changed if the borrowing rate were 6 % or 8 %. (The latter of the authors of this paper has advised many companies on investment decisions in the past. As a rule, an interest rate varying by ± 1 percentage point wouldn't even lead to the calculation being redone) There must be another reason for this effect, and the only candidate is trade in financial products. There, non-conserved value is created by (regularly) borrowing money, investing it in the stock market, and paying it back after rapidly selling the stocks (or derivatives). Depending very strongly on the interest rate, such deals can be profitable or unprofitable (in the short run). Hence the turnover and, with it, the profits (and losses) on the stock market depend heavily on interest rates.

Therefore, a suitable economic model should distinguish between investments from the real economy (in general savings from work) and the proceeds from financial transactions being invested. It is hard to imagine that the former will lead to instability. Based on the work of Klinkova and Grabinski (2013), it is almost likely that the latter may imply instability. Ryzhenkov had a conjecture that instability may occur in his own models, such as Ryzhenkov (1999, 2001, 2002). That is the starting point of this work.

In chapter 2, we will construct a model. It has to be a model that is very general and assumed to be valid in all cases. On the other hand, it is not necessary for this model to lead to accurate economic forecasts. In other words, it should be a model that will be accepted by (almost) everybody. Arguably, there are two things agreed upon within the otherwise much divided economic community: comparative advantage and NAIRU (short for "non-accelerating inflation rate of unemployment").

A famous supporter and architect of the NAIRU concept is Tobin (1980). NAIRU isn't actually an economic model in its pure form. It links the change in inflation to the rate of unemployment, so one gets two variables and one equation, which makes it insoluble from a mathematical point of view. Therefore, in chapter 3.3, we will create two independent (non-linear) differential equations based upon NAIRU. There, we will strictly stick to investments from the real economy into the real economy and will avoid income from (speculative) financial transactions. Our model is very general and therefore always valid. (Please note that it is not very suitable for making economic forecasts, as it contains (unknown) constants. But this is of no consequence for our purpose here.) The equations are soluble or at least integrable even in their non-linear version. Their solutions are always stable.

In chapter 3.4, we will introduce "speculation" to our model, allowing investment from (speculative) financial transactions. In other words, we will allow non-conserved value to be transferred into the real economy. As a result, the differential equations become more coupled. A rigorous stability analysis shows that the solutions are unstable as soon as the percentage of investment from speculations increases too much. Assuming realistic values for the constants, the solution will always be unstable and this, in turn, results in the sorry fact that (over a longer period of time) financial crises are inevitable. Setting the "correct" interest rates can at best soften the effect or prolong the period of time between two crises. In order to avoid a future financial crisis, income from financial transactions should be sufficiently low. A Tobin tax would be a good way of accomplishing this (Dziergwa et al., 2013), but, most likely, it would not suffice. New accounting principles such as the ones suggested by Dziergwa (2015) and additional tax legislation are the only possible way to achieve this, but a discussion of this is essentially left to further research, as stated in chapter 3.6.

In chapter 3.5, we will discuss our model critically. As a result, we will see that, despite all possible shortcomings, our postulate that speculation always lead to instability will remain valid. In chapter 3.4, we will show that financial crises are (almost) inevitable. Describing the dynamics of a financial crisis itself is impossible not only within our model but within all models based upon differential equations.

In Appendix A, we will derive our models from a very general mathematical point of view. This will prove that they are correct in the lowest non-trivial order. While it is impossible to tell whether this lowest order is sufficiently accurate to describe real economic dynamics, it has no influence on the stability analysis. In other words, our results about instability due to speculation hold true even for the most general model. In Appendix B, we will comment on the connection between our model and neo-classical and Keynesian approaches.

3.3 THE EXTENDED NAIRU MODEL

NAIRU is arguably the most fundamental approach in macro-economics. It states that there is a certain equilibrium rate n of unemployment u(t) so that inflation I(t) stays constant (in equilibrium). If unemployment u(t) > n, inflation will decrease. This is logical because many unemployed people are typically willing to work for less money, which will result in a deflation in labor costs. While labor is cheap, employers tend to hire, which brings down unemployment until equilibrium has been reached.

A similar mechanism works for too low unemployment u(t) < n. Workers are scarce, so labor costs will rise, resulting in an inflation in labor costs. Because of the inflation, more money is needed to build such things as factories, for example, which will lead to less jobs being created and, therefore, to an increase in unemployment until eventually u(t) = n is reached. Please note that we do not add effects such as the ones of minimum wages or job security, as we want to have the "pure" model and prove its stability or instability. Doing the same in a more advanced model would always lead to the question whether the original model or the add-ons produced the stability or instability.

Classic textbooks will normally give a formula such as this

$$\partial_t I(t) = -a \cdot (u(t) - n) \tag{1}$$

Here, the derivative with respect to time t is proportional to the negative deviation of unemployment u(t) from equilibrium unemployment n. The constant "a" must be positive (a > 0), else the argumentation above would not hold. Equation (1) contains two variables (I(t) and u(t)). Therefore, a second differential equation is necessary to solve it. The employment rate 1 – u is proportional to the number of jobs and therefore to the capital c(t) invested in jobs:

$$1 - u(t) = b \cdot c(t) \tag{2a}$$

The constant "b" is obviously positive because the capital c(t) is positive, and the unemployment rate u(t) ≤ 1 (u(t) = 1 means nobody is employed. Equation (2a) implies

$$\partial_t u(t) = -b \cdot \partial_t c(t) \tag{2b}$$

In order to find capital c(t) to be invested in jobs, said capital must be created first. In our case, people have to work for it and save or invest what they do not consume. ("The creation" of money through financial transactions will be addressed in the next chapter.) This means the change in capital is proportional to the employment rate 1-u (the number of people who are working) and the incentive they get for saving, the interest rate z. Of course, interest alone is no incentive; only the difference between interest and inflation can be an incentive. This leads to

$$\partial_t c(t) = \frac{\kappa}{b \cdot a} \cdot \big(z - I(t)\big) \cdot \big(1 - u(t)\big) \tag{2c}$$

The proportional constant κ/(b a) has been chosen in this way to keep the final result simpler. Of course, this proportional constant must be positive and so is κ. Eliminating ċ(t) from Equation (2c) by using Equation (2b) yields

$$\partial_t u(t) = \frac{\kappa}{a} \cdot (I(t) - z) \cdot \big(1 - u(t)\big) \tag{2}$$

Equations (1, 2) are a set of coupled ordinary first order non-linear differential equations which can be solved. The interest z from Equation (2) and normal rate of unemployment n from Equation (1) are the equilibrium rates of I(t) and u(t), respectively. Please note that the interest rate z is generally not equal to the interest rate set by a central bank. However, it is a monotonous function of it. (For a more general approach, please see Appendix A.) The interest rate z is a rate which makes people save money. A number of psychological factors may be involved in that. The same is true for the strength of the investment (or divestment) κ. With the following substitution

$$I(t) = z + \varepsilon(t) \quad and \quad u(t) = n + \eta(t) \tag{3}$$

Equations (1, 2) become

$$\partial_t \varepsilon(t) = -a \cdot \eta(t) \tag{4}$$

$$\partial_t \eta(t) = \frac{\kappa}{a} \cdot \varepsilon(t) \cdot \big(1 - n - \eta(t)\big) \tag{5}$$

Equations (4, 5) are differential equations for inflation ε(t) (= deviation from equilibrium inflation) and unemployment η(t) (= deviation from equilibrium unemployment). They yield no more information than Equations (1, 2), but they are more convenient for our purpose. Of course, Equations (4, 5) are easily transformed into two decoupled second order differential equations:

$$\ddot{\varepsilon} = -\kappa \cdot \varepsilon \cdot \left(1 - n + \frac{1}{a} \cdot \dot{\varepsilon}\right) \tag{6}$$

$$\ddot{\eta} = -\kappa \cdot \eta \cdot (1 - n + \eta) - \frac{\dot{\eta}}{1 - n - \eta} \tag{7}$$

Just by taking the linear parts of Equations (6, 7), it is easy to see that they display a harmonic oscillator with a frequency of

$$\sqrt{\kappa \cdot (1 - n)}$$

Even in their non-linear version Equations (6, 7) can be integrated. Their solutions are almost identical to their linear versions. Only for extremely high inflation (say, 70 %) will the sinusoidal variation of inflation turn into a saw-tooth like shape with a lower frequency. Unemployment hardly changes due to the non-linear terms. The details of this will be published elsewhere. As has been stated several times already, we are not focusing on solving an economic model; we want to prove or disprove its stability. This is done by linearizing Equations (4, 5) to

$$\partial_t \begin{pmatrix} \varepsilon(t) \\ \eta(t) \end{pmatrix} = \begin{pmatrix} 0 & -a \\ \frac{\kappa}{a} \cdot (1-n) & 0 \end{pmatrix} \begin{pmatrix} \varepsilon(t) \\ \eta(t) \end{pmatrix} \tag{8}$$

The eigenvalues λ_i of the matrix of Equation (8) are

$$\lambda_{1,2} = \pm\sqrt{\kappa \cdot (1-n)} \tag{9}$$

With both λ_i being purely imaginary, we have an undamped harmonic oscillation, just as stated above. Just for the sake of completeness, we will also give the corresponding eigenvectors:

$$\vec{e}_{1,2} = \frac{1}{\sqrt{1 + \frac{a^2}{\kappa \cdot (1-n)}}} \begin{pmatrix} \frac{\mp a}{\sqrt{\kappa \cdot (1-n)}} \\ 1 \end{pmatrix}$$

This gives the formal solution of Equation (8), which can also be obtained by using a linear combination of sin and cos functions as an ansatz:

$$\varepsilon(t) = A \cdot sin(t \cdot \lambda) + B \cdot cos(t \cdot \lambda) \tag{10}$$

$$\eta(t) = B\frac{\lambda}{a} \cdot sin(t \cdot \lambda) - A\frac{\lambda}{a} \cdot cos(t \cdot \lambda) \tag{11}$$

where $\lambda = |\lambda_1| = |\lambda_2|$ from Equation (9) and A and B are arbitrary constants determined by the initial conditions. Figure 1 shows a typical plot of the Equations (10, 11). The parameter κ essentially determines the period, here chosen so that the "economic cycle" is seven years. Of course, any other length would also be possible. The parameter "a" (defined in Equation (1)) determines the shift between inflation and unemployment. It also determines the strength of the non-linearity, cf. Equation (6). As stated, it does not matter here.

The solution does not show any instability. Please note that the solution might never look as smooth as shown in Figure 1. This has essentially to do with the fact that κ is influenced by the willingness to save, which may change. The same is true for the perceived interest z (cf. Equation (2c)). And, of course, the interest rate set by the central bank may change too.

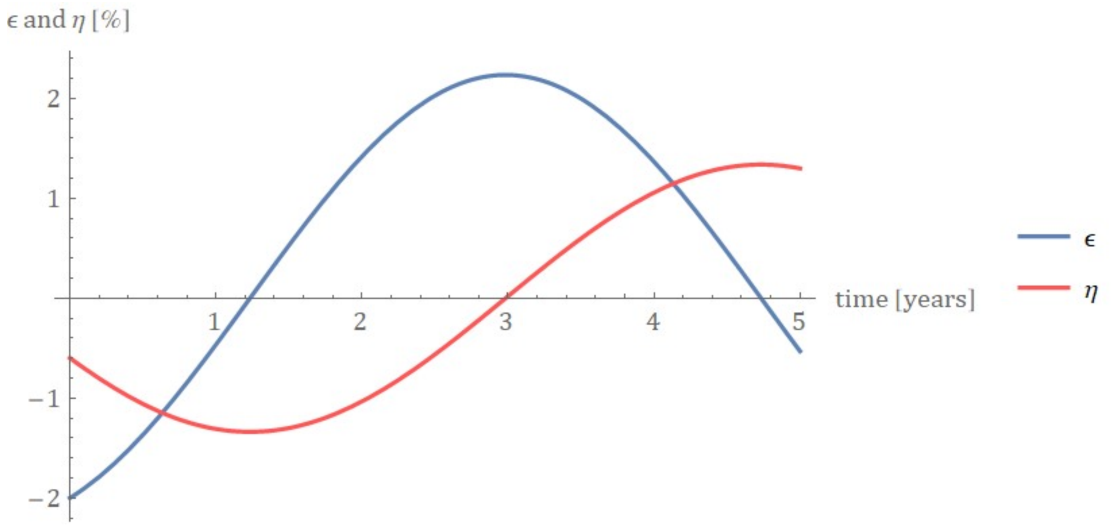

Figure 1. Plot of off-equilibrium inflation ε and unemployment η, n = 5%, κ ≈ 0.85/year² and a = 1.5/year

Please note that the interest rate z does not appear in the general solution of Equations (10, 11). This is surprising at first glance only. As long as the difference between inflation and interest rate remains constant, nothing will change. But, of course, changing the interest rate within a certain system will change its behavior. In order to see how it works, take a look at Figure 2.

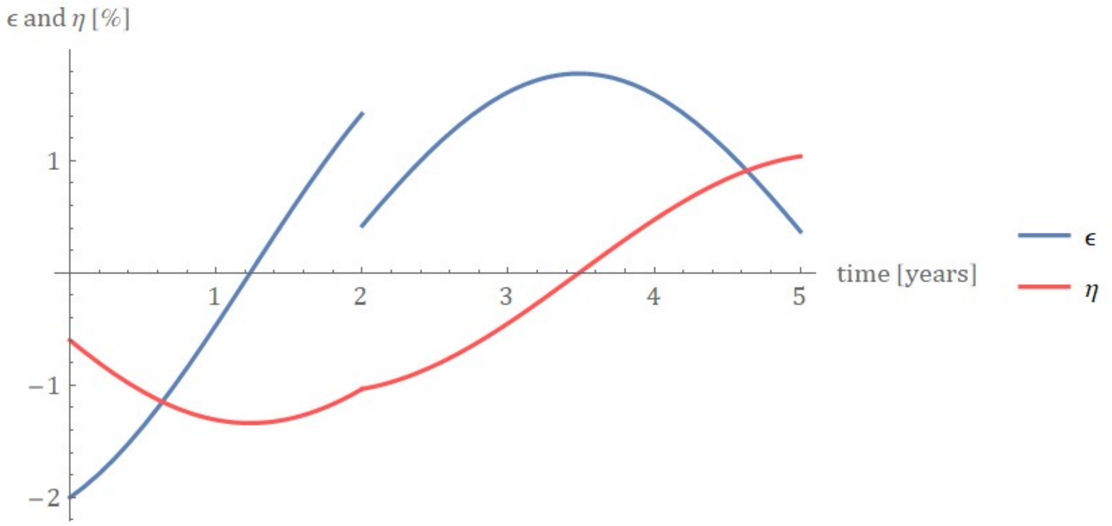

Figure 2. Plot of off-equilibrium inflation ε and unemployment η, n = 5%, κ ≈ 0.85/year² and a = 1.5/year with one percentage raise in interest after two years

After two years, inflation has grown by over three percentage points, which is why the central bank might raise the interest rate by one percentage point, and this has the same effect on z. As can be seen in Equation (2), $\dot{u}(t)$ will decrease immediately. Such sudden change will certainly have a big effect on the non-linear terms. A detailed discussion will be published elsewhere. Here, we will stick to the linear equations. Of course, these will not give the correct result in close proximity of t = 2 years, but otherwise the result should be fine. It is displayed in Figure 2. After the rise in interest, inflation and unemployment are growing less rapidly, but, as stated earlier, in this chapter, we assume a world without speculation, which is unrealistic anyway. The next chapter will eliminate this shortcoming.

3.4 THE NAIRU MODEL WITH SPECULATION

Besides "creating" money through an increase in conserved value, the financial industry also provides money by changing it into non-conserved value, Appel (2011, 2012) and Dziergwa (2015). This process is commonly referred to as speculation. This does not change Equations (2a, 2b), but it will change the mechanism of how capital is created. Therefore, Equation (2c) will get an extension:

$$\partial_t c(t) = \frac{\kappa}{b \cdot a} \cdot \big(z - I(t)\big) \cdot \big(1 - u(t)\big) + \frac{\kappa_S}{b \cdot a} \cdot \big(I(t) - z_S\big) \qquad (2c_S)$$

If inflation I(t) is sufficiently high compared to an effective interest z_S, the amount of capital created through speculative financial transactions will grow. Please note that the effective interest rate z_S will typically change with the interest rate set by the central bank without being identical to it. For a more formal consideration, please see Appendix A. The constant κ_S is assumed to be positive. However, the *willingness* to invest in stocks and especially in the more advanced financial products, such as derivatives and the like, can change quite suddenly. As shown by Appel and Grabinski (2011) and Dziergwa, Klinkova and Grabinski (2013), no capital is created in the long run. Mathematically speaking, we have

$$\int_{-\infty}^{+\infty} dt \, \frac{\kappa_S}{b \cdot a} \cdot \big(I(t) - z_S\big) = 0 \qquad (11)$$

Typically, κ_S will be positive for a long time. For very short periods of time, it will turn into a large negative number, though. Any such period of time is commonly referred to as a financial crisis. Changing Equation (2c) into Equation ($2c_S$) leads to an extended Equation (2):

$$\partial_t u(t) = \frac{\kappa}{a} \cdot \big(I(t) - z\big) \cdot \big(1 - u(t)\big) - \frac{\kappa_S}{a} \cdot \big(I(t) - z_S\big) \qquad (2_S)$$

Equations (1, 2_S) are the new set of differential equations to be solved, and the procedure is identical to the one in the previous chapter. Note that a discussion of the non-linear terms can be found elsewhere. Here, we will stick to the linear version around the equilibrium. The ansatz like Equation (3) transforms into

$$I(t) = \bar{z} + \varepsilon(t) \quad and \quad u(t) = n + \eta(t) \tag{12}$$

Because of the new couplings in Equations (1, 2_S), the equilibrium inflation $\bar{z}$ is some combination of z and z_S:

$$\bar{z} = \frac{\kappa\,(1-n)}{\kappa\,(1-n) - \kappa_S} \cdot z + \frac{\kappa_S}{\kappa_S - \kappa\,(1-n)} \cdot z_S \tag{13}$$

Besides the slightly more complicated form of the equilibrium inflation, it can be positive or negative:

$\boldsymbol{case}$ **1**: $\kappa_S > \kappa\,(1-n) \;\; and \;\; z_S < z \cdot \dfrac{\kappa\,(1-n)}{\kappa_S} \;\Rightarrow\; \bar{z} < 0$

$\boldsymbol{case}$ **2**: $\kappa_S < \kappa\,(1-n) \;\; and \;\; z_S > z \cdot \dfrac{\kappa\,(1-n)}{\kappa_S} \;\Rightarrow\; \bar{z} < 0$

$\boldsymbol{case}$ **3**: $else \;\; \bar{z} > 0$

Case 1 holds for a sufficiently large amount of speculation. Unfortunately, this is quite likely because proceeds from speculative financial transactions are much higher than the ones from the real economy, see, e.g., Dziergwa et al. (2013). Case 2 also leads to a negative equilibrium, but this may not occur very often in reality. It is only case 3 that leads to a positive equilibrium value. In summary, a sufficiently high level of speculation implies a negative equilibrium inflation, which is, of course, never attainable. This does not come as a surprise. “Profits” from speculative financial transactions are nothing other than inflation (within a certain area), cf. Dziergwa et al. (2013). This is identical to “printing money” in order to invest in jobs and will always lead to too high inflation.

But the problem of no equilibrium inflation is a minor one compared to the problem of instability. To see the point, one has to derive an equation analogous to Equation (8) from Equations (1, 2_S). A straightforward calculation yields

$$\begin{pmatrix} \dot{\varepsilon}(t) \\ \dot{\eta}(t) \end{pmatrix} = \begin{pmatrix} 0 & -a \\ \frac{\kappa}{a}\,(1-n) - \frac{\kappa_S}{a} & \frac{\kappa}{a}\,\Delta z \end{pmatrix} \begin{pmatrix} \varepsilon(t) \\ \eta(t) \end{pmatrix} \tag{14}$$

with

$$\Delta z \equiv \bar{z} - z = \frac{\kappa_S}{\kappa_S - \kappa\,(1-n)} \cdot (z_S - z) \tag{15}$$

The matrix in Equation (14) has the eigenvalues

$$\lambda_{1,2} = \frac{\kappa\,\Delta z}{2\,a} \pm \sqrt{\kappa_S - \kappa\,(1-n) + \left(\frac{\kappa\,\Delta z}{2\,a}\right)^2} \tag{16}$$

Depending on κ_S, one can distinguish between five cases:

case* 1**: $\kappa_S > \kappa\,(1-n) \quad \Rightarrow \quad \boldsymbol{\lambda_1 > 0}$ ***and $\boldsymbol{\lambda_2 > 0}$ ***if*** $\boldsymbol{\Delta z > 0}$

***case* 2**: $\kappa\,(1-n) \geq \kappa_S > \kappa\,(1-n) - \left(\frac{\kappa\,\Delta z}{2\,a}\right)^2$ *and* $z_S < z$ *implies at least* $\boldsymbol{\lambda_1 > 0}$

***case* 3**: $\kappa\,(1-n) \geq \kappa_S > \kappa\,(1-n) - \left(\frac{\kappa\,\Delta z}{2\,a}\right)^2$ *and* $z_S > z$ *implies* $\boldsymbol{\lambda_{1,2} < 0}$

***case* 4**: $\kappa_S < \kappa\,(1-n) - \left(\frac{\kappa\,\Delta z}{2\,a}\right)^2$ *and* $z_S < z$ *implies* $\boldsymbol{Re\{\lambda_1\} > 0}$

***case* 5**: $\kappa_S < \kappa\,(1-n) - \left(\frac{\kappa\,\Delta z}{2\,a}\right)^2$ *and* $z_S > z$ *implies* $\boldsymbol{Re\{\lambda_{1,2}\} < 0}$

Please note that the inequalities above are implicit because Δz is a function of κ_S, cf. Equation (15), and that it is not straightforward to make them explicit. Because it is impossible to stick to one eigenvector in real-life situations, instability will occur in the above cases 1, 2, and 4. Stability, on the other hand, will only occur in the above cases 3 or 5. In other words, a sufficiently large amount of speculation will always imply instability. This is the major result of this publication.

In order to make the result more transparent, we also give the explicit results of Equation (14):

$$\varepsilon(t) = A \cdot e^{\lambda_1 \cdot t} + B \cdot e^{\lambda_2 \cdot t} \tag{17}$$

$$\eta(t) = -\frac{\lambda_1}{a} A \cdot e^{\lambda_1 \cdot t} - \frac{\lambda_2}{a} B \cdot e^{\lambda_2 \cdot t} \tag{18}$$

A and B are arbitrary real constants. Of course, only the real parts of Equations (17, 18) are solutions in the real world. It is now possible to discuss the five cases above in detail, which can be found elsewhere. To illustrate the general line of our argument, we will stick to case 4 here. Cases 1 and 2 are trivially instable. Case 3 is an untypical stable solution, and case 5 is stable because it is the limit toward no speculation. As a typical result, case 4 gives a plot such as in Figure 3.

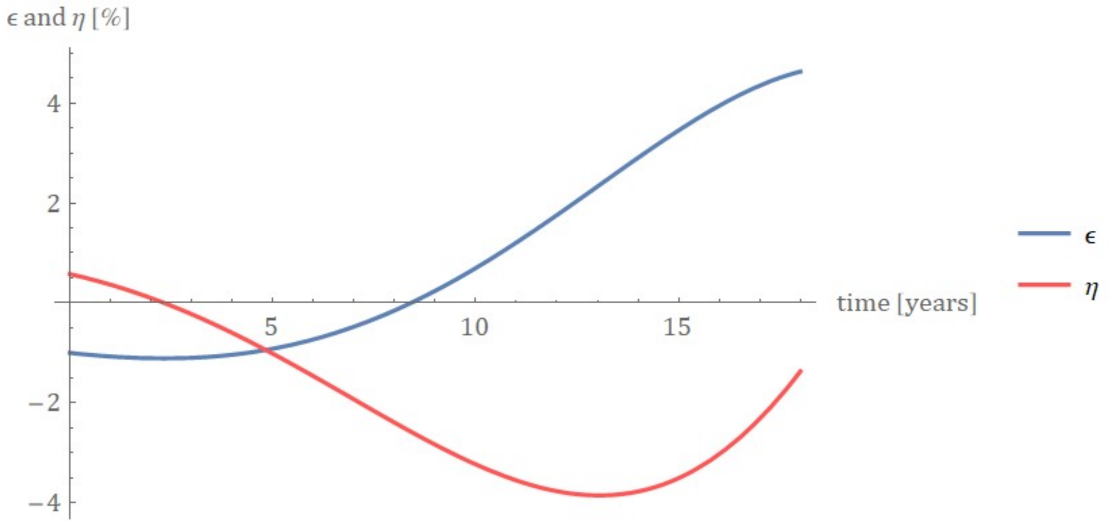

Figure 3. Plot of off-equilibrium inflation ε and unemployment η, n = 5%, κ ≈ 0.85/year², a = 1.5/year, z = 2%, z_S = 0.3% and κ_S = 0.9 κ

We stayed close to the values of Figure 2 and included some small amount of speculation. Both inflation and unemployment show an oscillation with an increasing amplitude. The length of the business has roughly tripled compared to Figure 2. But this is not very important here because it is not our goal to insert the economic data of any one particular real country.

After twelve years, Figure 3 shows a rise in inflation of about 4 percentage points. Maybe the central bank will decide to raise interest rates, which would typically have a big effect on z_S and a smaller one on z. (This is because the financial world reacts strongly to changes in interest, while the Rea real economy is usually left fairly unimpressed, as already stated in the introduction.)

As one can see in Figure 4, an increase in interest slows down inflation but will also raise unemployment (in this case, from a very low base).

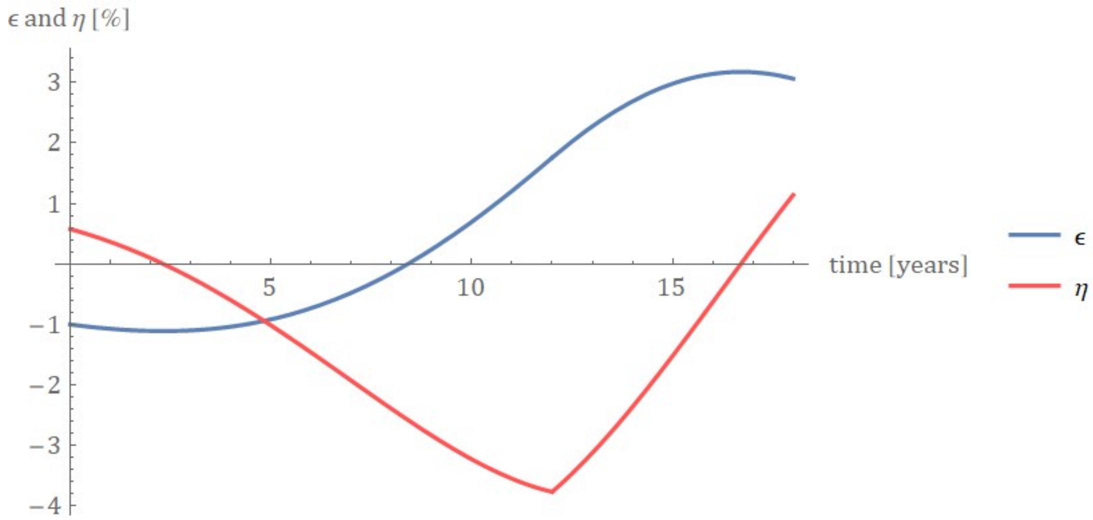

Figure 4. Plot of off-equilibrium inflation ε and unemployment η, n = 5%, κ ≈ 0.85/year², a = 1.5/year, z = 2%, z_S = 0.3% and $κ_S$ = 0.9 κ; z = 2.1% and z_S = 0.5% after twelve years

Please note that the non-linear terms will also have a big effect on the curves of Figure 4 at t ≈ 12 years, but the general line of argumentation will stay the same. Within our model (speculation included), non-linear terms may have an effect for reasons discussed in Appendix A. Although these may or may not have an effect on areas of stability or instability, the general line of argumentation should not change.

3.5 Critical Evaluation

In the previous chapter, we have shown that speculative financial transaction will, in almost all cases, lead to non-stable solutions in the dynamics of unemployment and inflation. Most of the other economic variables show some more or less close relationship to unemployment and inflation so that their stability is affected in the same way. And it is the important quantity GDP, in particular, that is strongly tied to unemployment (and inflation).

Although our model is very general and contains the perhaps most basic economic variables, unemployment and inflation, these variables are not without flaws. In contradiction to some of the basic textbooks, one has to say that inflation is defined precisely yet hard to measure accurately. The opposite is true for unemployment: It is generally ill-defined but very easy to measure within its particular definition.

Inflation occurs when the same product or service will cost more at a later point in time than they do now. No one, of course, can take account of all products and services. Therefore, a basket of goods is defined as representative, which results in the emergence of at least two separate inflation rates: “consumer price inflation” and “industrial price inflation.” As a matter of fact, there should be different baskets depending on the specific industry or the individual’s style of living. (This is the same problem as with the definition of purchasing power parity) With it, inflation becomes an almost arbitrary quantity. Furthermore, any basket is normally dominated by energy and housing. Speculation or a bursting bubble can cause a huge inflation spike or our current problem of deflation. Hence the strangely low inflation or outright deflation in Japan has most likely to do with the bursting property bubble of the 1990s, cf. The Economist (2015) and is no counterexample to the theory of inflation and demography.

It is also hard to decide what is meant by the terms “same product or service.” Consider a laptop, for example. If we take the word same at face value, we have a huge deflation where laptops are concerned. If we however assume that "the same product" only ever means the premium laptop model, then inflation is highly overstated.

Unemployment has an exact definition, which differs from country to country, and it is impossible to give the one most reasonable definition. Of course, one could take all non-working people in a country and divide this figure by the total population. But what does non-working mean? How many hours a week does a person need to work to be considered working? Furthermore, dual education, for instance, counts as work, while a university student is considered non-working. And when a rich single parent hires a nanny, a job is created and unemployment decreases. If, however, he or she marries the nanny, the job is destroyed, and unemployment rises. Similar arguments apply to the elderly, the disabled, or people wealthy enough to stay at home.

These remarks on inflation and unemployment apply to all economic models. Therefore, it is almost impossible to decide whether an economic model reflecting, say, 90 % of reality is better than a model that shows 80 % accuracy.

In addition, there is another problem with almost all economic models (and with the ones in management science). Its formulation goes back to Grabinski (2004): Any economic outcome is the sum of all actions of all participating human beings. Humans have free will, which means that, strictly speaking, even equations for NAIRU such as Equation (1) are always wrong and can only be understood as a statistical result. In order to use statistics, one has to consider many actions, without a single one of them being dominant. This, by the way, does not have its origin in man's free will. A gas consists of a large number of molecules (with no free will whatsoever). A macroscopic description by differential equations is only possible if one considers time scales which are long compared to the time of the individual interactions between the molecules. The same is true for the length scale.

At first glance, this does not seem to be an important limitation. There are, however, particular situations in physics where the internal length scales become very long. This is, for instance, the case when water freezes into ice or ice melts into water.

At this very point, none of the differential equations that otherwise describe the behavior of water or ice perfectly at almost all other temperatures can be used.

For some strange reason, physicists sometimes speak of the "catastrophe theory" when, for instance, describing the phase transition of water to ice. Similar things may occur in economic models. Consider Equation (11) of our model, for example. Typically, it implies a leap from $\kappa_S > 0$ to $\kappa_S < 0$ at a certain point in time (e.g., because suddenly almost everybody is selling his or her stocks). From a purely mathematical point of view, one can solve the model as long as non-linear terms are taken into account. This would be a waste of time and effort, though. Here, we have individual actions triggering an avalanche, which makes any statistical approach, the prerequisite for using differential equations, impossible. This means we are faced with the sorry fact that none of the models based upon differential equations and the like is useful for describing the dynamics of a financial crisis. In a typical financial crisis, any economic model will leave the range of validity.

It is hard to imagine that a proper description, such as "catastrophe theory" in physics, will ever be found in order to simulate the dynamics of a financial crisis although (unlike in physics) the word catastrophe theory seems very appropriate here.

3.6 Conclusions

We have shown that speculation will (almost) always lead to non-stability. Furthermore, the equilibrium rate of inflation can be negative due to speculative financial transactions. Therefore, central banks and financial policy makers can at best mitigate the severity of financial crises. The only way out would be to make speculative financial transactions become extinct, with one way being to prohibit them altogether. But this approach would be hard to manage (What exactly is meant by “speculative financial transaction”?). Furthermore, prohibiting them would not fit into our liberal world. Another way would be to implement a proper tax policy.

There are two ways of achieving this that appear to be easy and very effective: one would be the introduction of a Tobin tax, as suggested by Dziergwa et al. (2013). The effect on reasonable financial transactions would be minimal because they occur less frequently than speculative financial transactions by a factor of one thousand or even one million. The other possible measure would be to tax derivatives differently. As stated by Klinkova et al. (2013), their market is much more instable than the “ordinary” stock market, and the potential crashes there are much more severe. Supporters of derivatives claim that they are a reasonable way of providing insurance against such risks as fluctuating oil prices. But if it is an insurance, it should be treated as such. For one, it is not allowed, for instance, to insure your neighbor’s house against fire (e.g., to receive money in case it burns down without having suffered a financial loss). Again, it may be difficult to judge whether a person or company is really exposed to damage due to changing oil prices or whether that is just speculation. However, when there is a real risk, people are willing to pay some sort of tax on this insurance. To see how it works, consider insurance in Germany, for example: An insurance tax of 19 % is imposed on the premium you pay. And unlike the value added tax, it is not refundable. Individuals and companies experience substantial losses in case their house or factory burns down. Therefore, almost all homeowners and companies in Germany have fire insurance, and the 19 % insurance tax does not seem to hurt anybody. So why not introduce a similar tax on “oil price insurance?”

The next steps in our area of research will be to:

- Scrutinize the present model, and especially the five cases in chapter 3.4, in more detail.
- Take non-linear terms into account. This should prove that our general line of argumentation is not affected by non-linearities. Furthermore, effects of changes in interest can be displayed more realistically. There may be a chance to find interesting effects, such as mathematical chaos.
- Check other models (unrelated to this one) for instability.

3.7 APPENDIX A

Equations (2c, 2cS) give the relation between change in capital and interest. Although our argumentation should be very plausible, one could not say that other terms are forbidden or of less importance. Because Equation (2cS) is a generalization of Equation (2c), it will do to stick to the first one.

A change in capital c(t) may come from the real economy. In that case, it must be proportional to the employment rate 1 – u(t). It may also come from speculative financial transactions. Both parts will also depend on the effective interest rate. Hence the most general formulation of Equation (2cS) will take the following form:

$$\partial_t c(t) = f\big(i - I(t)\big) \cdot \big(1 - u(t)\big) + g\big(i - I(t)\big) \tag{19}$$

Here, "i" is the interest set by the central bank; f and g are arbitrary functions. It is hard to imagine having a more general approach. As long as f and g are analytical functions, they have a Taylor expansion. (If they were non-analytic, there would always be arbitrarily accurate approximations to them which would be analytic) The Taylor expansions of f and g are as follows:

$$f(i - I) = a_0 + a_1 \cdot (i - I) + O((i - I)^2) \tag{20}$$

$$g(i - I) = b_0 + b_1 \cdot (i - I) + O((i - I)^2) \tag{21}$$

Of course, the a_n and b_n are easily given by

$$a_n = \frac{1}{n!}\frac{\partial^n f(x)}{\partial x^n}\bigg|_{x=0}, \; b_n = \frac{1}{n!}\frac{\partial^n g(x)}{\partial x^n}\bigg|_{x=0}$$

Making the following substitutions in Equation (20,21)

$$a_0 \equiv \frac{\kappa}{b \cdot a} \cdot (z - i) \quad and \quad a_1 \equiv \frac{\kappa}{b \cdot a}$$

$$b_0 \equiv \frac{\kappa_S}{b \cdot a} \cdot (i - z_S) \quad and \quad b_1 \equiv -\frac{\kappa_S}{b \cdot a}$$

and inserting them into Equation (19) will transform Equation (19) into Equation ($2c_S$) if higher order terms are neglected. So, we have a proof that Equation ($2c_S$) is correct in the lowest order.

It is an interesting question whether this lowest order expansion makes sense. If z and z_S were the equilibrium values of the inflation I(t), it would be absolutely correct within our stability analysis. However, the $\bar{z}$ from Equation (13) is the true equilibrium value of I(t). Linearization and a stability analysis are, of course, still possible. However, the values of κ and κ_S will change with the deviation of z and z_S from the central bank interest rate i. This might change the regimes of stability.

3.8 Appendix B

Our models are constructed from a very general approach, especially when considering Appendix A. Quite often macro-economic models are classified into two categories: the neo- classical (or neo-liberal) models on the one hand and the Keynesian approach on the other. We did deliberately not follow this classification. To have two schools and not to know which one of them is the correct one resembles a religious approach to macro-economics. Our model should be scientific rather than creedal.

Nevertheless, some readers might ask whether our model is Keynesian or neo-classical. In short, it is both (or maybe neither). To see the point, consider Equation ($2c_S$) again:

$$\partial_t c(t) = \frac{\kappa}{b \cdot a} \cdot \big(z - I(t)\big) \cdot \big(1 - u(t)\big) + \frac{\kappa_S}{b \cdot a} \cdot \big(I(t) - z_S\big) \qquad (2c_S)$$

The first part (with κ) connects labor and its proceeds with change in capital. The main point is that one has to work and save in order to invest in the economy. This is the typical neo-classical approach. The Keynesian critique of it would be that, if everybody (or at least a lot of people) saves money (instead of spending it), employment will shrink, and the economy will enter a downward spiral. As Keynes put it, the otherwise reasonable micro-economic approach is not valid in the macro-economic world. (We will comment on this "conundrum" further below)

The second part of Equation (2c_S) (with κ_S) implies an increase in capital (for the creation of jobs) as long as borrowing money is sufficiently cheap. It does not question the origin of said money. This is exactly the kind of financial stimulus Keynes would have suggested. So, our model encounters both worlds, the neo-classical and the Keynesian one. Again, this confirms that this is the most general of models. Sadly, although the Keynesian stimulus may create jobs, it will never lead to stability.

Now we will come back to the "conundrum" mentioned above. If micro-economic mechanisms were not valid in the macro-economic world, *all* macro-economic models would be invalid. This is because integrating (solving) differential equations is nothing but the summing up of particular (micro-economic) happenings.

To solve this puzzle, consider a heavily indebted country, for example. All economists will agree that this country obviously consumed more than it earned by working, whereas people in countries abroad earned more than they consumed, or else they couldn't have lent any money to this country. The neo-classical remedy would be to save money. In other words, the indebted country ought to consume less and work more in order to repay its debts. The Keynesian critique would be that, if the people in the indebted country consume less, fewer goods will be needed. And producing fewer goods will imply less work and, thus, fewer jobs. However, this outcome is not the only possible one: The people in the indebted country could still produce more goods and consume less. Using the excess, they could repay their debt. In the real world, Keynesians, in particular, might argue that the goods these people produce might not be greatly sought after in foreign countries. This might even be the reason why their country got into debt in the first place. But generally, this is not true; people do not like or dislike certain products. What they do like or dislike is the product-to-price-ratio. In other words, you can flood the world market with almost any product as long as it is sufficiently cheap. So, the neo-classical answer to an indebted people would be as follows: Work more, without receiving more pay. Consume only part of these products.

Because of their low production costs, the rest of these goods can be exported. And since you consume less, you will be able to repay your debts.

This means that fiscal policy should only soften the hardship the indebted people most likely experience. One way of achieving this would be to encourage investments, either to produce new products or to improve the efficiency in producing the old ones. But even if the invested money is borrowed, that does not mean that debt isn't sometimes a good way of helping countries to get out of it. It is a common misunderstanding to assume that borrowing money in order to invest it means getting into debt. This is only the case if one only considers cash flow. However, as any accountant knows, one has to consider both sides of the balance sheet. Sadly, countries do not do so in their "accounting." So, if there is a reason why one cannot add up all the micro-economic entries to describe the macro- economy, it lies in the cameralistics of governmental accounting. It may not be easy to include assets and liabilities in governmental accounting but ignoring them and drawing the wrong conclusions is just plain stupid. As shown by Atkinson, Agarwala and Muñoz (2012), even a rough estimate can lead to completely new and interesting results.

Chapter 4

4 Measuring Irrationality in Financial Markets

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4.1 ABSTRACT

This paper presents the measurement of irrationality contained in the continuous pricing of individual stocks. Irrationality is used to extend the concept of historical volatility by decomposing historical stock quotes into frequencies via Fourier transformation. The analysis in the frequency domain enables clustering of the contributions of short and long-term fluctuations to the overall price changes. With the resulting ratio it is possible to rank stocks within an index according to their specific fluctuation profile. The analysis is performed on daily stock quotes over a period of 20 years (1997-01-02 until 2016-12-30). Although the analysis presented here focuses on the stock market, the concept of irrationality is transferable to other financial markets as for bonds, housing prices or derivatives as well as to different time periods.

Keywords:

Irrationality, Volatility, Risk, Speculative Behavior, Fourier Transform.

JEL Codes:

G12, G14, G40, E44.

4.2 INTRODUCTION

Mandelbrot proved that price movements of stock markets do not always follow a Gaussian distribution, what he describes as mild randomness. Instead historical stock quotes should be better described by a stable Paretian process to account for wild randomness (Mandelbrot, 1963). The wild behavior of stock markets can be investigated with the use of Fourier techniques. The spectral density being proportional to $1/f^{\alpha}$ and α = 2 in the frequency domain proves for mild randomness known as red noise or random walk. In this case the Gaussian distribution would hold, and the market is seen as efficient. Mild behavior of the market with thin tailed distributions therefore relates to the Efficient Market Hypothesis (EMH) as described by Fama (1965b, 1970). $\alpha < 2$ proves for wilder randomness and may be described with the concept of "irrational exuberance" (Shiller, 2016) which is better suited to explain fat tails. Furthermore, irrational behavior and therefore unexplained price movements by the EMH may be induced by the divergence of value and price as illustrated by Summers (1986) as well as Shiller (1981) and in a more recent study by Appel and Grabinski (2011).

The purpose of this publication is to develop an applicable method to quantify the extent of irrationality in the valuation of the share prices of corporates. The resulting ratio is consciously named irrationality to express that it does not only include speculative behavior of market participants but also the behavior of the corporate management including its decisions. Other influences such as political decisions or interest rate decisions by central banks influence the overall market and are not company specific. Therefore, stocks within an index should be comparable. The general idea is to transfer the time series into the spectral domain, where it is possible to measure the influence of specified frequency ranges with longer frequencies from three months to one year to short ones ranging from ten days to three months. The reason to exclude the frequencies below ten days is that with only one data point per day for the calculation, this range equals white noise and is further based on unreliable data in this range. This could lead to signals which are nothing else than data errors.

Even though there is irrational behavior in the timespan from the tick basis to three months, it is not possible to separate adjustments to new information from components of irrationality like speculation. Long-term changes of over one year reflect actual changes in the macroeconomic environment as well as strategic decisions made by the board. So, the timespan between three month and twelve months was taken as the closest representation for irrationality.

Irrationality in this sense is neither good nor bad per se, it is the part not explained by rational behavior as proposed by the EMH. It is also not indicating under- or overvaluation of the asset as neither fundamental values nor discounted cash flows are considered. Therefore, stock prices even though they adjust to new information on the short-term do not need to reflect the actual future economic benefits of the stock.

For the spectral estimation of functions or time series a broad literature body exists which is still extending. The mathematical details leading to the spectral estimation method used in this paper are given in the next chapter 4.3 where the presented method to stock prices will be applied as well.

In chapter 4.4 the results for stocks within an index as well as between indices are discussed. Therefore, a ratio of low frequencies to high frequencies closely linked to the concept of empirical probability is build. It determines the amount of irrationality in contrast to the inherent variance of the specific stock for adjusting to new information. Please note that this is a relative measure. It can be transformed to fit into analyses based on volatility or beta if needed. To get the real proportions of the power frequency bands, one may divide the sum of the irrational band by the short-term basis, which will stretch the results but keep the ranking.

The last chapter summarizes the results and gives hints for further research. The main result of this analysis is that by applying Fourier techniques, the strength of different periodic components within financial time series can be used to measure and classify risk within an index.

4.3 THE FOURIER TRANSFORMATION

The Fourier transformation is an over 200-year-old tool mostly applied to analyze frequencies in a signal (spectral analysis) or to solve an arbitrary set of linear partial differential equations. First, the Fourier series is introduced.

Any periodic function $f(t)$ can be written as a series of harmonic functions:

$$f(t) = \sum_{k=0}^{\infty} a_k cos(k \cdot \omega t) + b_k sin(k \cdot \omega t) \tag{1}$$

Here a period of T was assumed so that $\omega = 2\pi/T\ k \in \mathbb{N}_0$. The coefficients a_k and b_k are determined by

$$a_k = \frac{2}{T}\int_0^T dt\, f(t) \cdot cos(k \cdot \omega t)\ \ and\ \ b_k = \frac{2}{T}\int_0^T dt\, f(t) \cdot sin(k \cdot \omega t) \tag{2}$$

A proof of Equation (2) is performed by inserting $f(t)$ of Equation (1) into Equation (2) and performing the integration. A Fourier series exists only if the integrals of Equation (2) exist.

The interpretation of a Fourier series evaluates a_k and b_k which represent the strength of the frequency $k\omega$. Equation (1) can also be used as an approximation for a periodic function e.g. in forecasting. Then a_k and b_k are considered fitting parameters. This is analogous to a Taylor series up to a certain power.

A series of stock prices is most likely not periodic. Analyzing the frequencies of stock prices for e.g. 20 years, one may assume that stock prices have a period of 20 years.

Instead of having discrete frequencies, also continuous frequencies can be applied. a_k or b_k are then becoming a function rather than a set of discrete parameters. This leads to the so-called Fourier transformation. The Fourier transformed $\tilde{f}(\omega)$ of a not necessarily periodic function $f(t)$ is defined by

$$\tilde{f}(\omega) \equiv \int_{-\infty}^{\infty} dt\, f(t) \cdot e^{-i\omega t} \tag{3}$$

As usual $i^2 \equiv -1$. There is also a backward transformation given by

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt\, \tilde{f}(\omega) \cdot e^{i\omega t} \tag{4}$$

The proof of Equation (3) or (4) is again performed by inserting Equation (3) into (4) or vice versa. The Fourier transformed exists if the integral in Equation (3) exists. Equation (4) is the continuous analogue to Equation (1). The sum in Equation (1) is transformed into an integral and the discrete coefficients a_k and b_k are now a function $\tilde{f}(\omega)$. Please do not be confused that $\tilde{f}(\omega)$ has complex values (even if $f(t)$ is real). The identity

$$e^{i\omega t} = cos(\omega t) + i \cdot sin(\omega t) \tag{5}$$

shows that the real part of $\tilde{f}(\omega)$ corresponds to a_k and the imaginary part to b_k. In this sense one sometimes speaks of the cosine or sine transformed function. In the same token one may use Equation (5) to rewrite Equation (1) into

$$f(t) = \sum_{k=-\infty}^{\infty} c_k \cdot e^{k \cdot i\omega t} \tag{6}$$

with $c_k \in \mathbb{C}$ given by

$$c_k = \frac{1}{T} \int_0^T dt\, f(t) \cdot e^{-k \cdot i\omega t} \tag{7}$$

Applying it to share prices is now possible. The variation of stock prices can be described by a function $f(t)$. Equation (2), (3), or (7) can be used to determine the frequency spectrum and build a ratio like:

$$\frac{\sum_{k=0}^{K} |c_k|}{\sum_{k=0}^{\infty} |c_k|} \tag{8}$$

With $\omega = 2\pi/T$ the sum in the numerator gives the sum of the (angular) frequencies starting from zero until $2\pi K/T$. Of course, low frequencies belong to a longer time span. In the case of stock quotes, one may find it reasonable that price fluctuations adjust for new information within one day to three months, as being the frequency of publishing quarterly financial statements. Longer timespans until one year indicate irrationality as dividends and cash flows rather change as a smooth polynomial function.

The main mathematical problem with stock prices is that they are (strictly speaking) not a function of time in the sense required here. They are always mapping a finite number of time points on a finite number of prices. However, any integral over such functions is zero. This contrasts with discrete time series in natural science. There one may observe or calculate data at certain time points via digital or numerical approaches. However, the data does exist at any time. A function from $\mathbb{R} \to \mathbb{R}$ or even $\mathbb{C} \to \mathbb{C}$ can be obtained by interpolation.

Stock prices are men-made due to trading. They do not exist per se. Therefore, one may gather stock prices every second. Connecting them by straight lines appears to be a reasonable interpolation. However, it must be kept in mind that stock exchanges are predominantly closed for more than 10 hours in between trading days and around 30% of calendar days are no trading days. Additionally, having a stock price every second will never cover every price change on a tick basis. Furthermore, it is hardly possible to scrutinize reliable data covering every second especially from several stock exchanges. It is even challenging to get these data for a longer period of say the last 20 years. In this paper each stock price per day was generated by taking the mean of the daily open and close price. Beyond filtering for unreliable data even with cleaned data there is still an ambiguity at weekends and holidays. Therefore, the most reasonable approach with these data would be to connect the consecutive points by straight lines or even better exponential functions. Because of dealing with financial markets an interpretation of (positive or negative) compound interest seems to be appropriate. From that results a continuous function of time and Equation (3) can be used to get the Fourier transformed. Of course, the backward transformation via Equation (4) will lead to the exact stock price. The integral in Equation (3) can be calculated analytically, because it is just an integral over a piecewise exponential function. However, this approach consumes vast amounts of computing power. Furthermore, it is doubtful whether it will lead to better results than given by the method described below. Though one should perform these calculations in the future, when faster data processing power will reach the threshold to reduce calculation time into a matter of days, in order to prove it.

A second approach would be to assume that the price of a stock is always constant over one trading day. This will lead to a rectangular curve. Because integrals can be taken over discontinuous functions Equation (3) or (7) can be used to calculate the Fourier transformed or the amplitudes c_k respectively. Please note that this will consume the same computing power as the exponential interpolation of the last paragraph. However, there is another problem. A rectangular curve changes within zero time at its discontinuous points. So, it will lead to infinite high frequencies, which are not connected to the stock price. The so found short frequencies will be high, even if there are none. For instance, consider a stock which grows constantly by five percent annually. Having just one price per day, the approach would lead to high frequencies, though a Fourier transformed of an exponential curve will decay with $1/\omega$ for $\omega \to \infty$.

Therefore, the best approach to realize a fast and practicable process is via the discrete Fourier transform, even though the historical stock quotes are obviously neither weakly stationary nor equidistant. These shortcomings must be accounted for via detrending and the assumption that there is no difference if the market was opened or closed the day before. The mathematics behind the discrete Fourier transform is as follows. Instead of starting with $\omega = 2\pi/T$ (with T in this paper being 20 years) which would lead to arbitrary high frequencies and an infinite number of c_k, one may choose a $T_d \approx T$ with $k_{max}/T_d \triangleq 1\,\text{day}$. This will reduce calculation time tremendously. Furthermore, it will be consistent to have a finite amount of amplitudes for a finite amount of data points. Squaring the time series as well as its frequency spectrum equals the same power on both sides known as the Parseval's theorem which leads to the power spectrum and shows how the variances are distributed over the frequencies.

In the next chapter the transformation of the historical stock quotes of SAP SE from the time domain to the frequency domain will be demonstrated and afterwards the power spectra of various stock price series will be evaluated.

4.4 RESULTS FOR STOCK MARKETS

As described above, the discrete Fourier transformation has been applied to all stocks in various indices such as the Deutsche Boerse AG German Stock Index (DAX). A period of 20 years from 1997-01-02 until 2016-12-30 was considered, containing slightly over 5,000 trading days or data points for each stock. All raw data were derived from Thomson Reuters Eikon. Figure 5 shows the historical stock quotes of SAP SE in the time domain as well as its spectral counterpart with absolute values of c_k in the frequency domain containing the equivalent information:

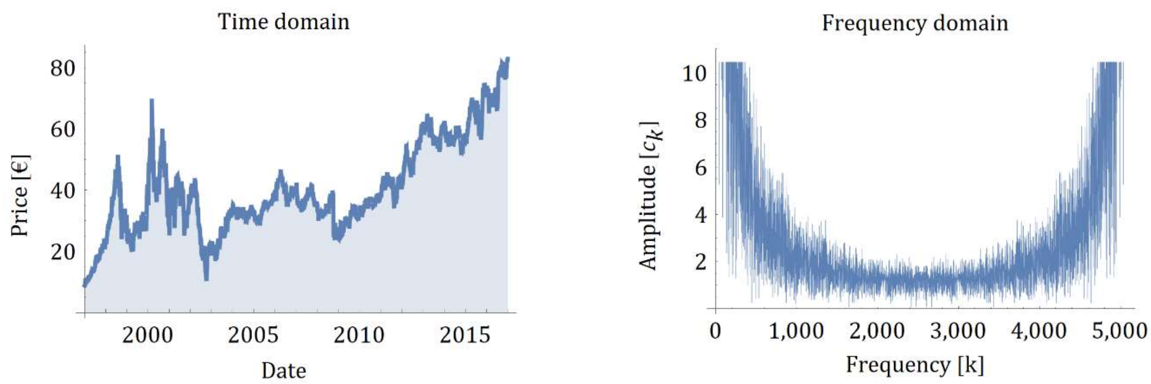

Figure 5. Time domain and Frequency domain of SAP SE - historical stock quotes

The absolute values of c_k were taken. This also implies symmetry because the discrete Fourier transformation gives a complex amplitude and its conjugate. Furthermore, a_k and b_k of Equation (2) may have positive or negative values while $|c_k|$ is always positive. Also, the real and imaginary part of c_k can be used, which correspond to a_k and b_k of Equation (2), respectively. The representation of the frequency domain in this form is for demonstration purposes only.

Several procedures have been applied to avoid undue disturbances. One important step was to extract the (exponential) growth or decay of the stock price. Essentially, the trend has been subtracted from the time series. It leads to a stock price fluctuating around zero. That leaves all relevant frequencies untouched but removes the trivial ones due to growth or decay.

The lowest frequency is given by 1/20 years. This is artificial due to the period of 20 years under consideration. Because of the detrending, very low frequencies are reduced to a minimum. The high frequencies above 1/10 days are not reliable because of having just one price per day, so only frequencies from 1/10 days until 1/1 year were considered. The accumulated signal power in the range of three months to one year, defined as the sum squared amplitudes, in the numerator is divided by the power of the range from 10 days until 1 year which results the irrationality of the stock. The power of the range from 10 days until three months is therefore seen as the inherent variance of the specific stock.

The irrationality of the DAX shares with uninterrupted data for the period under consideration is shown in Table 1:

Table 1. Irrationality of DAX members

Company	Irrationality	Volatility
Muenchener Rueck AG	73.7%	26.7%
Beiersdorf AG	76.6%	22.8%
Bayer AG	76.9%	25.2%
E.ON SE	77.2%	25.0%
Siemens AG	77.4%	28.1%
Henkel AG & Co KGaA	77.5%	21.9%
Adidas AG	77.7%	25.5%
BASF SE	77.8%	23.7%
RWE AG	77.8%	25.3%
Allianz SE	77.9%	29.2%
Linde AG	78.2%	22.5%
Fresenius Medical Care AG & Co KGaA	78.7%	23.7%
Merck KGaA	79.0%	23.6%
Bayerische Motoren Werke AG	79.1%	28.7%
SAP SE	79.2%	32.5%
Volkswagen AG	79.8%	30.9%
Daimler AG	80.1%	28.4%
Continental AG	80.3%	30.8%
HeidelbergCement AG	80.5%	29.1%
Deutsche Lufthansa AG	80.9%	28.3%
Commerzbank AG	82.2%	36.8%
Deutsche Bank AG	82.8%	33.1%
Thyssenkrupp AG	82.8%	30.1%
Deutsche Telekom AG	84.6%	27.0%
Mean	79.1%	27.4%
Median	78.9%	27.6%

On average the irrationality in the index was 79.1% for the stocks included with the median being slightly below the mean. A cluster of conservative titles like insurance and basic utilities forms the lower band of irrationality. The automotive industry begins above the mean and is very concentrated in the range from 79.1 % to 80.3 %.

The volatile and structurally risky companies Commerzbank AG, Deutsche Bank AG and Thyssenkrupp AG form the upper end of the ranking together with the Deutsche Telekom AG. The high irrationality of Deutsche Telekom AG is no surprise. After being privatized in 1996 right before the rise and fall of the Dotcom bubble it became one of the most notorious examples of irrational exuberance.

The difference between irrationality and volatility becomes apparent when comparing Muenchener Rueck AG and Deutsche Telekom AG, which have the lowest and highest irrationality within the evaluation. Muenchener Rueck AG has a volatility of 26.7% and thus a value slightly below the mean and median, although it has the lowest irrationality. The low value of irrationality results from the fact that the contribution of the irrational frequency range to the historical stock quotation is just 2.8 times higher than that of the rational range. Deutsche Telekom AG on the other side, while having a similar volatility, has a factor of 5.5. The higher factor results both from a lower contribution of the short frequencies to the overall share price development and from a higher contribution of the longer frequencies compared to the Muenchener Rueck AG.

In addition to the results above, an overview of irrationality ranges measured for selected indices is presented in Table 2. The values refer to individual shares contained in the index, not to the index itself.

Table 2. Irrationality range of selected indices

Index	**Min**	**Max**	**Mean**	**Median**
S&P 500	66.7%	89.0%	78.1%	78.3%
FTSE	68.3%	84.3%	77.1%	77.7%
Nasdaq	69.0%	84.1%	78.5%	78.4%
Dow Jones	72.7%	80.4%	77.0%	77.0%
DAX	73.4%	84.6%	79.1%	78.9%
IBEX	73.8%	82.4%	79.1%	79.5%
HSI	74.0%	83.8%	79.9%	80.4%

Starting with the mean and median, both give a clear picture of the Dow Jones being the index with the lowest irrationality. The minimum (Min) and maximum (Max) of the range of irrationality behaves logically. The more companies included in the index, the wider the range of expected possible outcomes. Therefore, the similar ranges of irrationality for DAX, Spanish Exchange Index (IBEX) and Hang Seng Index (HIS) are plausible.

Please note, that as described above only qualifying stocks regarding data length and consistency are included. Further studies with shorter time periods and other data sources like from the Center for Research in Security Prices (CRSP) may include more companies.

Possible shortcomings and sources of errors lie in the fact that historical stock quotes are neither weakly stationary nor equidistant nor periodic. Different detrending methods will lead to slightly different results due to possible influence on the first frequency bins under one year. In this case, the values of c_k in the low frequencies are reduced and thus the proportion of irrationality is lower than it would be. The ranking should nevertheless be stable, as the impact on each stock has a similar effect. Possible detrending methods are linear regression models e.g. via the ordinary least square method or one can apply a discrete wavelet-analysis as it can handle nonstationary time series, also low-pass filters are reasonable. Differencing of logarithmized time series is a very specific method of detrending which - even though being commonly applied in economics - cannot be applied here.

The point that weekends and holidays were ignored is probably a minor point for the time span under consideration. It will not add extra frequencies or eliminate others. Further studies with intraday data where non-equidistant data may impact the evaluation should consider the use of the Lomb–Scargle periodogram (Lomb, 1976, Scargle, 1982).

Moreover, it should be noted that a reduction in measurement values necessarily leads to a broader frequency spectrum comparable to Heisenberg's "Uncertainty Principle" and vice versa (Hill, 2013). Because of dealing with nonperiodic time series it is therefore important that the time period for the analysis is well above the longest frequency considered (in this analysis one year).

In conjunction with the above-mentioned methods the concept of irrationality is transferable to other financial markets like bonds, housing prices or derivatives as well as to different time periods.

A more technical aspect are dividends. After a dividend has been paid, the stock price drops correspondingly. This rapid drop has an impact on the high frequencies in the frequency domain but only occurs at less than 1.6 percent of the trading days for quarterly dividend-paying companies.

4.5 CONCLUSIONS

The analysis shows that irrationality can classify stocks within an index according to risk and can be directly reconciled with historical volatility or beta analyses with minor transformations in representation. Therefore, the same restrictions as for volatility and beta apply. A single ratio cannot capture all risks at every point in time and must be accompanied by stress tests or scenario analyses including further information like liquidity or funding.

Additionally, the results show that there exists valuable information unobservable in the time domain with promising results for analyses within an index. Further studies should therefore compare irrationality to volatility and beta with respect to the compound annual growth rate (CAGR) to test the applicability and informative power of this risk measure. At least, the results are expected to show irrationality being somehow proportional to the other two risk measures. Furthermore, it should be investigated whether conclusions can be drawn about the instability of volatility by comparing long frequency ranges with short ones.

The concept of irrationality is transferable to other financial markets as for bonds, housing prices or derivatives as well as to different time periods.

CHAPTER 5

5 PORTFOLIO SELECTION BASED ON A VOLATILITY MEASURE ADJUSTED FOR IRRATIONALITY

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5.1 ABSTRACT

This paper investigates the combination of the two risk measures irrationality and volatility for portfolio selection as well as irrationality as a standalone risk measure. The study is conducted for a period of 20 years, ranging from 1999-01-04 to 2018-12-31, using daily closing prices from 1.295 stocks. The companies are derived from the indices S&P 500, STOXX 600 and a representative index of the German market out of the indices DAX, MDAX, SDAX and TecDAX. The findings indicate a negative relationship between risk and return in terms of irrationality across the three indices. The effect is particularly evident when comparing the portfolios with the lowest and highest values of irrationality. Further, the analysis provides an indication of how an index can be replicated with fewer expenses and complexity, particularly regarding the fact that the volatility of the synthetic portfolio is equivalent.

Keywords:

Irrationality, Volatility, Portfolio Selection, Portfolio Management.

JEL Codes:

G11, G40.

5.2 INTRODUCTION

The linear relationship between the expected risk and return is one of the fundamental assumptions of modern portfolio theory. The basic idea behind this relationship goes back to Markowitz (Markowitz, 1952) and lies in the fact that investors must accept a corresponding "variance of profits" for their expected returns. If an investment with a better risk-return ratio exists, this will increase the demand for the investment opportunity and thus place the ratio back in the line of all other investment opportunities. Based on the resulting portfolio selection theory, which includes the liquidity preference, the Capital Asset Pricing Model (CAPM) was established by Sharpe (1964) and Lintner (1969). The CAPM assumes that the specific risks of the individual investments are offset by diversification and that only the market risk remains, which cannot be further reduced. This risk is therefore borne by the shareholders who invest via a diversified portfolio along the capital market line, which represents the linear relationship between expected risk and return. Under the assumption of unrestricted borrowing and lending, an investor with maximum risk aversion would theoretically receive exactly the risk-free return.

Assuming that the market portfolio should take into consideration all risk-bearing investment opportunities, the hypothesis of a linear relationship is not empirically verifiable or falsifiable (Roll, 1977). Empirical studies therefore analyze samples of the market portfolio for example through broad stock indices that cover a large proportion of the total market capitalization of a country or region.

Early empirical studies such as Black, Jensen and Scholes (1972) therefore used portfolios instead of individual stocks. The portfolios covering all shares of the New York Stock Exchange (NYSE) for the period from 1926 to 1966 were sorted in ascending order from low to high beta values. The study confirms the linear relationship between risk and return, but with a flatter curve than assumed by the CAPM. The returns for portfolios with lower risk are higher and vice versa lower with higher risk. In a more recent study, Fama and French (2004) confirmed the results of Black, Jensen and Scholes for the shares of the NYSE over a period from 1928 to 2003 and the National Association of Securities Dealers Automated Quotations (NASDAQ) from 1972 to 2003. The authors conclude as well that the relationship between risk and return is flatter than assumed by the single factor model and thus the CAPM cannot be verified for the portfolios under consideration. Hence low-risk equities are underpriced by the market and high-risk equities are attributed too high future returns.

All these studies are conducted based on the assumption that risk is fully captured in terms of volatility - the annualized standard deviation of returns - which is a measure of the overall margin of fluctuations. In our analysis we examine the inclusion of irrationality as a measure of risk in addition to volatility as well as irrationality as a standalone risk measure. Irrationality as defined by Schädler (2018) measures the ratio of long to short frequency components of historical stock quotes in relation to one another. Therefore, it is a relative measure which in contrast to volatility does not consider the overall fluctuation margin. A possible advantage in the combination of the two risk measures may be that irrationality, unlike volatility, does not assume a standard normal distribution of returns.

The difference to volatility becomes apparent when one considers an ideal company which grows linearly without uncertainties about the future course of business and no exogenous cycles. In this case, under the premise of efficient markets, the price of the shares would move within a narrow corridor around the present value of the company, which results purely from the usual trading of the shares. The irrationality would then tend towards zero. In the next step, market participants are introduced, who, driven by irrational behavior, cause the share price to fluctuate around the actual company value. In the extreme case of very high fluctuations around the actual value, irrationality would tend towards one, as the additional fluctuations would have a much stronger impact on the development of the share price than the usual trading in the above example. Empirical values are somewhere between these two extremes of the idealized company.

Both Shiller (1981) and Appel (2012, p. 153) conclude that price fluctuations are around two to fourteen times higher than the change in the discounted future cash-flows of the companies considered. To take this into account, it is assumed that equity markets are efficient, meaning that all available information is priced directly into the market. Therefore, it is not possible to generate a constant excess return based on new information. "Efficient" however, does not mean that valuations on the stock exchange should by no means deviate from the actual company value. In order to reflect this in the risk indicator irrationality, it is assumed that the maximum time interval between the release of new information is no more than three months apart and possible price fluctuations accounting for the information are offset within the same period.

This rational part of the fluctuations is compared to fluctuations that are in a range of more than three months to one year, in which no new information is available. Therefore, this irrational part is adjusted for the overall trend of the company's development, which affects fluctuations over a period longer than one year. The historical development of a share price can thus be described by the following model

$$\begin{aligned} Stock\ Price = {} & Trend + Irrational\ Fluctuations + Rational\ Fluctuations \\ & + Random\ Trade \end{aligned}$$

whereby solely the rational and irrational fluctuations are considered in the calculation of irrationality.

The next chapter outlines the selection and the way the individual stocks used for the analysis are preprocessed. The calculation of the two risk ratios is then presented in detail. While volatility is calculated using logarithmic returns, irrationality is calculated over three steps. First, the logarithmized time series are detrended with a linear regression model whose parameters are estimated via the ordinary least squares. The resulting weakly stationary time series can then be used to calculate the power spectrum via the discrete Fourier transform. The last step is to accumulate the corresponding frequency bands and to put them in relation to one another.

5.3 RESEARCH DESIGN

We analyzed daily historical stock quotes ex-post on a total return basis for a period of 20 years, ranging from 1999-01-04 to 2018-12-31. The data used were taken from Datastream. Since it is assumed that shares with high market capitalization within developed countries are efficient, the following indices were selected.

For the German market, the performance indices DAX, MDAX, SDAX and TecDAX were combined to form the German Major Indices Index (GMII), adjusted for double listings of individual stocks as provided by Deutsche Boerse AG. The STOXX 600 was used as a proxy for the European market as provided by STOXX AG. The American market is represented by the S&P 500 as provided by the Standard and Poor's Corporation (S&P). All data for the analysis were accessed on 2019-05-29.

The daily values of the time series had to be checked for missing values first. Shares with a history of less than 20 years were excluded from the analysis. Furthermore, price recordings on public holidays were eliminated. In order to exclude systematic errors in the data series, these were cross-checked with the stock price data of Thomson Reuters Eikon. The existing survivorship bias is accepted due to data availability and required history of 20 years.

Table 3 shows the number of shares contained in the respective index compared to the number of shares available for our analysis.

Table 3. Stock count after cleaning per index

Indices	Total individual stocks	Individual stocks count after cleaning
GMII	190	73
STOXX 600	600	368
S&P 500	505	380

The next step is to calculate the historical volatility of the time series. Historical stock volatility as a measure for dispersion of returns defined as

$$Volatility = \sqrt{\frac{1}{q-1}\sum\nolimits_{p=1}^{q}(r_p - \bar{r})^2}\ \sqrt{q/a}, \qquad r_p = ln\frac{S_p}{S_{p-1}} \tag{1}$$

is the annualized square root of the variance, with r_p being the logarithmic returns of the respective historical stock quotes S_p and $\bar{r}$ the expected value. Due to the handling of daily data with the length q and a the number of years under consideration, the raw variance is annualized by the square root of q/a. The annualization is done for consistency and has no influence on the relative risk in sense of a risk ranking as all stocks are handled equally.

As described above, irrationality is calculated over three steps. First, the logarithmized time series are detrended with a linear regression model as the data must be weakly stationary for being applicable to the discrete Fourier transform. In this paper, the adjustment was made using a fourth-degree polynomial regression model, which was found to be a good fit for the analyzed data series.

In the general case, the parameters of a k-th degree polynomial regression model

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \cdots + \beta_k x_t^k + \varepsilon_t \tag{2}$$

are estimated via the ordinary least squares in its well-known matrix form

$$\hat{\beta} = (X^T X)^{-1} X^T y_t \tag{3}$$

by minimizing the sum of squares residuals where y_t represents the logarithmized time series. $\hat{\beta}$ being the coefficient vector with the parameters $\beta_0, \dots, \beta_k$ and X the $n \times m$ design matrix. To detrend the stock quotes, the values of the regression model will be deducted from the original logarithmized time series. In the next step, the Power spectrum is estimated.

For a finite, stationary time series the power spectrum $P(n)$ is defined as the squared convolution of the magnitude of the frequency components and its complex conjugate.

$$P(n) = |F(n)|^2, \qquad F(n) = \sum_{t=0}^{N-1} f(t)\, e^{\frac{-2\pi i n}{N}} \tag{4}$$

The frequency components are derived via the discrete Fourier Transform. $f(t)$ denoted as the logarithmized, detrended historical stock quotes, N the number of daily observations of the time series, n the respective frequency bins and $i^2 \equiv -1$. Although stock prices do not hold energy in a physical sense, the use of this analogy is useful to compare the strength or influence of frequency bands relative to each other. It is important to note, that a comparison of the summed absolute magnitudes of frequency bands would be biased towards shorter frequencies. In the final step the values of the corresponding frequency bands are accumulated and put in relation to one another.

The limits of the frequency bands depend on the number of years under observation and the number of trading days per year. While the one-year boundary h matches the number of years under observation, the three-month m and the ten days threshold j are selected as the closest to the corresponding frequency bin. The sum of the power of the frequency components relative to each other returns the value of irrationality.

$$Irrationality = \frac{\sum_{n=h}^{n=m} P(n)}{\sum_{n=h}^{n=j} P(n)} \tag{5}$$

Irrationality can thus rise in two ways. On the one hand, it increases if the influence of long frequencies on the price development increases. On the other hand, it increases when the influence of short-term frequencies declines if all else is equal. We would like to point out anew that short-term fluctuations reflect the risk inherent in the respective business model and thus form the basis of the calculation which the long-term fluctuations are compared to.

From the calculation logic it can further be concluded that irrationality is less volatile than volatility itself. An increase in very short fluctuations under 10 days will not have a direct effect on the results. Significant fluctuations over a period of more than 10 days increase the signal strength of the short frequencies and only influence long-term frequency ranges over a longer period. In addition, it should be noted that a single data error, even when more than 5.000 measurement points are considered, can have such a strong influence on volatility that a value which is within the market average in terms of volatility may be classified as very risky. By excluding all frequencies below 10 days, irrationality is resistant to such data errors.

5.4 RESULTS

As described above, the individual shares of the indices analyzed were split according to their respective values of irrationality and volatility. The division of the portfolios results from the process of first sorting the individual stocks in ascending order from low to high irrationality. This enables the subdivision into two halves, while in the case of an odd number of shares, the portfolio with a comparatively higher risk receives one more share. Subsequently, each of the two portfolios were separated according to low and high volatility and divided into lower and higher risk portfolios.[6] For each quadrant, the compound annual growth rate (CAGR) of the respective portfolio was calculated. As the market capitalization changes significantly over the period of 20 years, we preferred to measure the CAGR on an equal weighted basis. The total return CAGRs of the equal weighted portfolios are presented in Figure 6. The colors in the plot are scaled from red to green whereby red indicates the lowest, green the highest value in terms of CAGR.

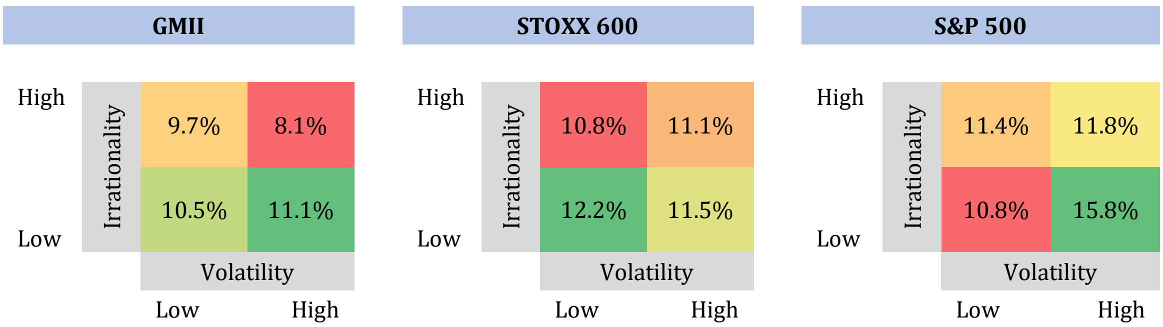

Figure 6. Total return CAGRs per index and equal weighted portfolios

Across all three indices a consistent pattern for the CAGRs emerges. High returns can be found in portfolios in the lower half of irrationality regardless of volatility. This is evident for all indexes analyzed. In terms of volatility, the American

6

Thresholds	GMII	STOXX 600	S&P 500
Irrationality Threshold	77.8%	77.6%	77.2%
Volatility Threshold low Irrationality	32.1%	30.8%	30.9%
Volatility Threshold high Irrationality	37.8%	34.4%	37.4%

market differs from the European market. While the European markets have a higher CAGR for the lower half, the American market shows a higher CAGR for the higher half of volatility.

At an individual portfolio level, no definite conclusion can be drawn for the lowest CAGR value. It is noticeable that all the portfolios with the highest CAGR are in the range of low irrationality. Especially the portfolios with low irrationality and high volatility show higher returns than the market portfolio.

From the results it appears that a strategy with low irrationality is advantageous for the European market as well as the American market, while an investment decision favoring high volatility irrespective of irrationality is most advantageous for the US market. It is interesting to note that low irrationality is associated with higher CAGRs. In other words, the assumption of the CAPM that higher returns are associated with higher risk - defined as volatility - cannot be confirmed for irrationality. Therefore, an analysis of irrationality as a standalone risk measure by dividing it into quarters should confirm the results.

The analysis with irrationality as a standalone risk ratio is presented in the Table 4 based on equal-weighted portfolios. All Shares portfolios contain all evaluated equities within the respective index. The other portfolio names are in ascending order by irrationality whereby the First Quarter contains the shares with the lowest and the Fourth Quarter the stocks with the highest irrationality values. The thresholds[7] result from this classification and represent the upper limit for the respective portfolio.

Sharpe ratios are calculated with a risk-free rate of zero. The maximum Drawdown (MDD) measures the highest loss throughout the whole time series in percentage points from the highest point before the drawdown and the subsequent trough. Calmar Ratios are defined as the CAGR of the portfolio divided by the corresponding MDD. Statistical significance[8] of Sharpe ratios and Calmar ratios of the

7

Thresholds	GMII	STOXX 600	S&P 500
First Quarter	75.6%	75.3%	74.3%
Second Quarter	77.8%	77.6%	77.2%
Third Quarter	80.6%	80.1%	79.6%

8 *, ** and *** denote statistical significance at the 90%, 95% and 99% levels.

four portfolios based on irrationality was tested against die All Shares portfolios on a rolling annual basis using the Mann-Whitney U test.

Table 4. Descriptive statistics of the total return portfolios

Index	Portfolio	CAGR	Volatility	Sharpe Ratio	MDD	Calmar Ratio
GMII	All Shares	10.0%	18.9%	0.50	59.4%	0.17
	First Quarter	11.8%	19.6%	0.57*	59.9%	0.20***
	Second Quarter	9.8%	21.0%	0.44***	59.9%	0.16
	Third Quarter	9.9%	20.7%	0.46***	63.9%	0.16***
	Fourth Quarter	7.9%	24.7%	0.31***	65.6%	0.12***
STOXX 600	All Shares	11.4%	17.6%	0.62	54.5%	0.21
	First Quarter	12.0%	17.1%	0.66	46.3%	0.26***
	Second Quarter	11.7%	16.4%	0.68	50.7%	0.23
	Third Quarter	11.3%	18.1%	0.60**	55.8%	0.20*
	Fourth Quarter	10.5%	20.7%	0.48***	66.7%	0.16***
S&P 500	All Shares	12.8%	19.9%	0.61	50.6%	0.25
	First Quarter	15.0%	20.0%	0.70	47.0%	0.32***
	Second Quarter	12.4%	18.9%	0.62**	46.2%	0.27***
	Third Quarter	11.1%	21.3%	0.49***	54.6%	0.20***
	Fourth Quarter	12.1%	22.6%	0.51***	58.1%	0.21***

The Sharpe Ratios of the First Quarter portfolios are consistently higher compared to the All Shares portfolios. Both the First Quarter and the All Shares portfolios have higher Sharpe Ratios than the Fourth Quarter portfolios. Besides the lower CAGR, the main reason for the lower Sharpe Ratios is that the highest volatility is measured for the portfolios with the highest risk. Further, the high risk of the Fourth Quarter portfolios is reflected in the highest MDD values over the 20-year period. Rising MDD in accordance with risk from the first to the fourth Quarter portfolios and the CAGR falling simultaneously lead to decreasing Calmar ratios. This effect is evident across all three indices. In summary, we conclude, that higher risk in terms of irrationality is not compensated by higher returns for the analyzed equity universe.

The statistical significance of the Sharpe ratios from the 95% level upwards indicates that the All Shares portfolio can be replicated by the First Quarter portfolio. In this case, an equivalent volatility with statistically significant higher Calmar ratios can be assumed.

5.5 CONCLUSIONS

Due to the exclusion of frequencies below 10 days, irrationality is per design less volatile than volatility itself. Furthermore, irrationality is more robust against erroneous data series. While even a single measurement error may have a significant effect on volatility, the irrationality in this case would hardly change. Irrationality can therefore be a valuable addition to volatility when measuring risk.

Irrationality differs markedly from volatility. Whereas the literature assumes a linear relationship of higher CAGRs and increasing volatility, the expected returns decrease with increasing irrationality. Since irrationality is associated with speculative behavior, it can be concluded that equities with a high speculative component in relation to rationally explainable fluctuations are associated with lower CAGRs.

For the management of equity portfolios, the analysis provides an indication of how an index can be replicated with fewer expenses and complexity, particularly regarding the fact that the volatility of the synthetic portfolio is equivalent.

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