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"UTILIZACIÓN DE MODELOS FACTORIALES DINÁMICOS EN LA PREDICCIÓN INTEGRADA A CORTO PLAZO DE AGREGADOS MACROECONÓMICOS Y SUS COMPONENTES"

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ÍNDICE GENERAL

CAPÍTULO 1: Resumen y conclusiones

1. Objetivo de la investigación

El objeto de la presente investigación se centra en la utilización de los denominados modelos factoriales dinámicos para la previsión o estimación en el corto plazo de determinadas magnitudes macroeconómicas y sus subagregados. Concretamente, el trabajo de investigación tiene tres vertientes:

- La primera vertiente, que se plasma en el primer artículo (Cuevas y Quilis (2011)), es la de plantear la utilización de un modelo factorial dinámico para la predicción en tiempo real del Producto Interior Bruto (PIB). Concretamente se presenta un modelo factorial dinámico de mediana escala para prever la tasa de crecimiento de la economía española en el muy corto plazo.

  El tamaño intermedio del modelo supera los graves problemas de especificación asociados a los modelos a gran escala y la posible pérdida implícita de información de pudiera achacarse los modelos pequeños. El factor común estimado se usa para pronosticar el PIB por medio de un modelo de función de transferencia. Asimismo, el modelo resuelve los límites operacionales y de información planteados por la presencia de un panel incompleto de indicadores e implícitamente genera pronósticos en un entorno multivariante de los indicadores implicados.

- En un segundo artículo (Cuevas et al. (2015a)), se extiende su uso para la predicción simultánea de un conjunto de subagregados que cumplan algún tipo de restricción con un determinado agregado. Concretamente se implementa un sistema de estimación y predicción en tiempo real de los PIB trimestrales por Comunidades Autónomas, que necesariamente han de cumplir la restricción transversal impuesta por el PIB trimestral nacional que proporciona la Contabilidad Nacional. Más específicamente, se propone una metodología para estimar en forma trimestral el PIB de las diferentes regiones de España, proporcionando asimismo perfiles trimestrales para el dato oficial anual observado de las mismas estimado por la Contabilidad Regional. De esta manera, se ofrece un nuevo instrumento para el seguimiento a corto plazo que permite a los analistas el poder cuantificar el grado de sincronía entre los ciclos económicos regionales.

Técnicamente, se combinan los modelos de series de tiempo con los métodos de benchmarking, para procesar indicadores mensuales y trimestrales a corto plazo y estimar los PIB regionales trimestrales asegurando su consistencia temporal y transversal con los datos de las cuentas nacionales. Asimismo, la metodología considera adecuadamente el problema ligado a la no-aditividad, teniendo en cuenta explícitamente las limitaciones transversales impuestas por los índices de volumen encadenados utilizados por la Contabilidad Nacional.

- En último lugar, y como tercer artículo (Cuevas et al. (2015b)), se planteará el uso de los modelos factoriales dinámicos para la predicción en tiempo real de un cuadro macroeconómico completo desde el punto de vista de la demanda y en términos reales, donde también se establecen restricciones contables entre sus componentes.

  La principal característica distintiva de este enfoque es que, como ya se ha mencionado, se pronostica en una base de tiempo real no sólo el PIB, sino también su desglose completo desde el punto de vista de la demanda. De esta forma se tienen modelos específicos para pronosticar el consumo privado, el consumo público, la inversión en bienes de capital, la inversión en construcción, las exportaciones y las importaciones. Se integran todos ellos en un conjunto coherente de previsiones mediante el uso de la técnica de equilibrado desarrollada en van der Ploeg (1982, 1985), que permite tener en cuenta la incertidumbre con que han sido generadas cada una de las predicciones individuales.

2. Importancia del tema de investigación

El análisis del ciclo económico siempre ha suscitado un elevado interés entre investigadores, analistas y decisores en el ámbito de la política económica. Ello es así porque un conocimiento certero de la realidad económica que se vive en un momento concreto condiciona la toma decisiones tanto en el ámbito público, a la hora de instrumentar políticas o estrategias, como en el privado, relacionándolo por ejemplo con las decisiones de inversión de los agentes.

Adicionalmente, este interés se ha visto estimulado por la gravedad y amplitud de la relativamente reciente recesión de la economía española y mundial. La evaluación de las políticas económicas requiere información oportuna y precisa sobre las condiciones macroeconómicas generales. En este sentido, el uso de medidas estándar de la actividad económica general, basadas en las medidas tradicionales encuadradas en el ámbito de la Contabilidad Nacional, pueden imponer un retraso en el proceso de toma de decisiones que pueden dificultar su eficacia.

Con el fin de mitigar las limitaciones de información impuestas por estas medidas estándar, se va a recurrir a la utilización de los mencionados modelos factoriales dinámicos, cuyo desarrollo en los últimos años ha sido exponencial, a la vez que han demostrado ser muy eficaces a la hora de diagnosticar el estado del ciclo económico en el muy corto plazo sobre una base en tiempo real.

Por otra parte su versatilidad y flexibilidad de aplicación en diversos contextos van a permitir, combinándolos con técnicas de equilibrado que permitan tener en cuenta las restricciones contables subyacentes, formar un sistema completo de estimación para un conjunto de agregados económicos.

3. Fundamentos teóricos en que se inscribe la investigación

Una manera de sintetizar la información que proporcionan los distintos indicadores coyunturales y ponerla en un marco común, va a ser la de incluirlos en un modelo factorial dinámico. El núcleo principal del modelo es la estimación de un factor común dinámico subyacente al conjunto de indicadores mensuales o trimestrales, de forma que este factor recoja de forma parsimoniosa las interacciones dinámicas de los mismos.

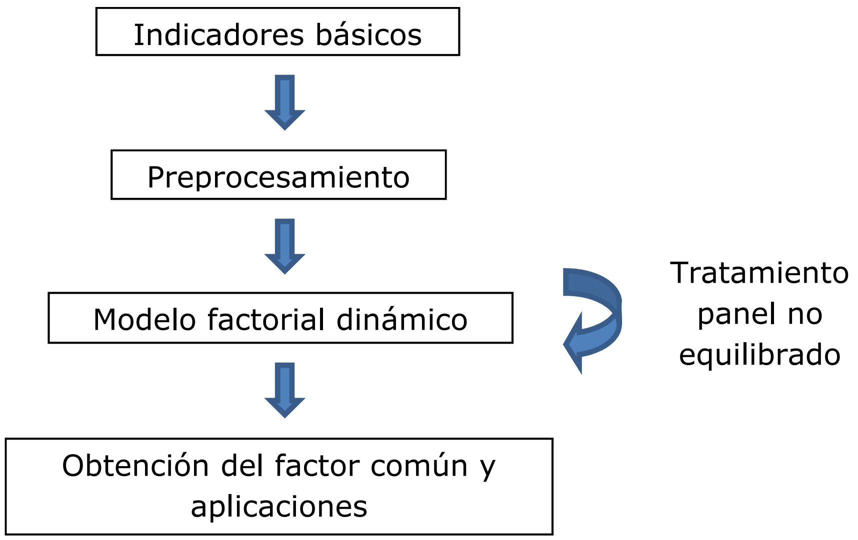
El número de indicadores de coyuntura que informan de la evolución de la actividad económica se ha ido incrementando de forma constante, abarcando desde las variables cuantitativas o *hard indicators*, es decir, aquellas que son expresión de unidades físicas, precios o monetarias (ejemplos son el Índice de Producción Industrial, el Índice de Cifra de Negocios en la Industria, etc.) a las cualitativas o *soft indicators* que son generalmente relativas a opiniones (tales como el Indicador de Clima Industrial, el Indicador de Confianza de los Consumidores, etc.). La elaboración de estos indicadores suele implicar cierto *trade-off* entre la prontitud en su publicación y el ruido estadístico: los datos disponibles más rápidamente se suelen basar en muestras más pequeñas, lo que puede dar lugar a una señal de más difícil interpretación.

Cabe destacar que este último punto ha sido una de las ventajas que presentan los modelos factoriales, ya que han permitido traducir las señales que proporcionan los *soft indicators*, que a menudo son puramente direccionales, en una cuantificación directa de la variación de una macro magnitud.

La idea básica del análisis factorial es que las relaciones que se puedan dar entre el conjunto de indicadores son el resultado de una estructura latente más simple, en la que un reducido número de variables inobservables afectan a las series observadas. Estas variables se llaman factores comunes o, simplemente, factores y se suele asumir que cada uno de ellos es independiente de los demás. No obstante, esta representación es una aproximación, ya que los factores no pueden explicar toda la variabilidad de las series observadas. El elemento residual se denomina factor específico o factor idiosincrásico. Estos elementos se presumen independientes tanto respecto a los factores comunes como entre sí.

Una exposición detallada del proceso completo de estimación de este tipo de modelos se encuentra precisamente en el primer artículo de la tesis (Cuevas y Quilis (2011)), la Figura 1 muestra de forma esquemática dicho proceso.

Figura 1: Proceso completo de estimación en tiempo real



Fuente: Cuevas y Quilis (2011)

Se parte de un conjunto de indicadores susceptibles de reflejar la evolución de la actividad. Posteriormente se preprocesan, esto es, se corrigen de variaciones estacionales y calendario, y a continuación son transformados logarítmicamente[1] y diferenciados de forma regular (esto equivale a calcular las correspondientes tasas de crecimiento intermensual).

Seguidamente se plantea el modelo factorial dinámico, donde es preciso destacar que el conjunto de información sobre el que se implementa el procedimiento es de tipo «no equilibrado», es decir, indicadores cuya muestra no se solapa de forma necesaria. Para solventarlo y obtener el factor común a la totalidad de la muestra, incluyendo la especificación dinámica, se escribe el modelo en forma de espacio de estados pudiendo así aplicar en su estimación el filtro de Kalman. Esto va a permitir a su vez obtener predicciones implícitas de cada uno de los indicadores, así como la posibilidad de proyectar hacia el futuro el factor.

[1] Excepto las series de encuestas de opinión, expresadas como saldo de respuestas extremas, a las que no se aplica la transformación logarítmica.

4. Antecedentes de la investigación

La literatura que relata la utilización de modelos factoriales para la previsión en el corto plazo de agregados macroeconómicos tiene importantes precedentes, comenzando con los trabajos de Stock y Watson (1991, 2002), que pueden considerarse descendientes modernos del trabajo seminal de Burns y Mitchell (1946) sobre indicadores cíclicos.

Más recientemente, los modelos de factoriales se han convertido en una de las técnicas más utilizadas en el análisis macro econométrico aplicado debido a que proporcionan una forma parsimoniosa para parametrizar los modelos dinámicos para vectores de series de tiempo, ver Bai y Ng (2008) y Stock y Watson (2010) para una revisión exhaustiva.

Recientemente, diversos bancos centrales e instituciones académicas han creado todo tipo de indicadores en tiempo real y que, en algunos casos, los difunden a través de sus sitios en Internet. Ejemplos notables de este tipo de trabajos son el indicador diseñado por Aruoba et al. (2009), publicado en tiempo real por el Banco de la Reserva Federal de Dallas, Giannone et al. (2008) o Higgings (2014) para la economía de los EE.UU.; Angelini et al. (2008) y Camacho y Pérez-Quirós (2009) para la zona euro; Liu et al. (2010) para América Latina, Barhoumi et al. (2008) para Francia, Nunes (2005) para Portugal, etc.

Para el caso de España, ya se han publicado tres modelos que merece la pena resaltar. Camacho y Pérez Quirós (2010) construyeron un modelo factorial de pequeña escala para el Banco de España (España-Sting). Cuevas y Quilis (2011) propusieron un modelo de escala media para el Ministerio de Economía (FASE) (*artículo que se presenta como parte de esta tesis doctoral*) y Camacho y Domenech (2011) construyeron otro modelo de pequeña escala para el BBVA (MICA), en el que prestan especial atención a diversas variables financieras a disposición del BBVA.

5. Metodología

El enfoque econométrico utilizado en los diferentes artículos comprende diversos elementos, dentro de los que destacan dos:

En primer lugar, se utilizan los mencionados modelos factoriales dinámicos que representan de una manera compacta y parsimoniosa la dinámica conjunta de cada agregado macroeconómico y los correspondientes indicadores de corto plazo.

Un segundo elemento presente son los procedimientos de equilibrado, que garantizan de una manera objetiva y razonable la consistencia de las previsiones del PIB con las previsiones de sus componentes, o bien la del agregado con sus correspondientes contrapartidas regionales.

Modelos factoriales dinámicos

Tal como se ha comentado anteriormente, la noción básica del análisis factorial dinámico es que un reducido número de variables inobservables generan las series objeto de estudio a través de estructuras lineales estocásticamente perturbadas. De esta manera, la pauta de co-movimientos del vector de series se descompone en dos partes: comunalidad (variación debida a un pequeño número de factores comunes) y variabilidad idiosincrásica (elementos específicos de cada serie, no susceptibles de una interpretación sistémica).

Siguiendo la exposición de Cuevas y Quilis (2011), y para el caso en el que sólo se consideren indicadores mensuales, se asume que las señales de crecimiento de los *k* indicadores son generadas mediante un modelo factorial de la forma:

$$z_{i,t} = \lambda_i f_t + u_{i,t}$$

Siendo:

- $z_{i,t}$: la señal de crecimiento del indicador i-ésimo, en la observación t.
- λ_i: carga de la señal de crecimiento en el factor común.
- f_t: valor del factor común en la observación t.
- $u_{i,t}$: elemento específico o idiosincrásico que recoge la variabilidad de la señal de crecimiento del indicador i-ésimo que no ha sido explicada por el factor común del modelo.

La variable f_t representa el factor común e inobservable que afecta a la evolución conjunta de las k series consideradas. Los parámetros λ_i (llamados "cargas") cuantifican la sensibilidad de la señal de crecimiento de cada indicador respecto a cambios en el factor.

El término $u_{i,t}$ denota el componente idiosincrásico de la señal del indicador i, esto es, la porción de variabilidad que no obedece a elementos compartidos con las demás series y que representa, por tanto, factores específicos de cada una.

El modelo examinado hasta este momento sólo tiene en cuenta las interacciones estáticas o contemporáneas entre las señales de crecimiento de los indicadores y sus factores subyacentes. Con el fin de obtener predicciones y completar la representación de los datos, es menester incluir en el modelo de forma explícita el comportamiento dinámico tanto de los factores comunes como de los idiosincrásicos.

Una representación suficientemente general para el factor común se basa en un modelo autorregresivo estacionario de cuarto orden:

$$\begin{aligned} &\varphi_4(B)f_t = (1-\varphi_1 B - \varphi_2 B^2 - \varphi_3 B^3 - \varphi_4 B^4) f_t = a_t \\ &a_t \sim iid\ N(0,1) \end{aligned}$$

En la expresión anterior B es el operador de desfase $Bf_t = f_{t-1}$ y la varianza de la innovación ha sido normalizada. Este tipo de modelos permite una reversión a la media tanto monótona como oscilatoria, dependiendo de los valores numéricos de sus raíces características.

La dinámica de los elementos específicos se puede considerar de menor complejidad que la del factor común, pero se admite cierto grado de persistencia:

$$\begin{aligned} &(1-\psi_i B) u_{i,t} = b_{i,t} \quad |\psi_i| < 1 \\ &b_{i,t} \sim iid\ N(0, v_i) \end{aligned}$$

Para completar la representación de los factores, se considera que sus innovaciones son ortogonales:

$$\begin{aligned} &E(a_t b_{i,s}) = 0 \quad \forall i,t,s \\ &E(b_{i,t} b_{j,s}) = 0 \quad \forall i,j,t,s \end{aligned}$$

Representación en el espacio de los estados y filtrado de Kalman

De esta manera, el modelo anterior recoge tanto los aspectos estáticos como los dinámicos. Su estimación completa puede hacerse definiéndolo en el espacio de los estados y aplicando el filtro de Kalman.

La representación de modelos econométricos dinámicos utilizando la formulación propia del espacio de los estados es una forma sencilla de expresarlos separando nítidamente los aspectos de medida (¿qué variables generan las series temporales observadas y de qué forma?) de los dinámicos (¿cómo evolucionan en el tiempo los factores determinantes del sistema?). Para realizar esta tarea es necesario introducir un vector de estado, esto es, un conjunto de variables de dimensión mínima que permite proyectar el estado futuro del sistema a partir de su pasado. El vector de estado correspondiente es:

$$X_t = \begin{pmatrix} f_t & f_{t-1} & f_{t-2} & f_{t-3} & u_{1,t} & \cdots & u_{k,t} \end{pmatrix}'$$

La ecuación de medida asociada es:

$$Z_t = \begin{bmatrix} L & 0_{kx1} & I_k \end{bmatrix}' X_t = H\, X_t$$

Siendo $L=\{\lambda_i\ i=1..k\}$ la matriz de cargas. Esta ecuación permite derivar las series observadas a partir del vector de estado (inobservable) que incluye tanto los factores comunes como los idiosincrásicos.

La ecuación de transición completa el sistema y caracteriza su dinámica:

$$X_t = GX_{t-1} + V_t$$

Siendo G una matriz cuadrada de dimensión $k+4$:

$$G = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 & 0 & \cdots & 0 \\ 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \psi_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \psi_k \end{bmatrix}$$

El vector de innovaciones V_t queda definido como:

$$V_t = \begin{pmatrix} a_t & 0 & 0 & 0 & b_{1,t} & \cdots & b_{k,t} \end{pmatrix}'$$

Este vector se distribuye de forma gaussiana con vector de medias cero y matriz de varianzas-covarianzas:

$$Q = E\left[V_t V_t^{'}\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & v_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & v_k \end{bmatrix}$$

Se asume que la muestra recorre el índice temporal *t* desde 1 hasta *T.* La aplicación del filtro de Kalman para estimar el factor y su desviación típica asume que los parámetros involucrados *Θ=[H, G, Q]* son conocidos.

Procedimientos de equilibrado

La aplicación de los modelos factoriales dinámicos llega a poder proporcionar pronósticos independientes de diferentes agregados macroeconómicos (PIB, el consumo de los hogares, etc.). Como se ha mencionado, estas previsiones combinan la información disponible de los indicadores pertinentes a corto plazo con la dinámica de la variable macroeconómica de una manera eficiente, pero no tienen en cuenta las limitaciones transversales que enlazan las variables macroeconómicas. Estas limitaciones se derivan del proceso de compilación de las cuentas nacionales, ya sea por ejemplo a partir de la descomposición del PIB desde el lado de la demanda, o la desagregación regional del mismo.

Con el fin de incorporar estas restricciones en el proceso de previsión, se puede recurrir a un procedimiento de equilibrado que garantice su coherencia interna. En particular, se puede citar el propuesto por van der Ploeg (1982, 1985), que se expone a continuación.

Sea *Y* un vector que representa las estimaciones de *M* variables, cuya distribución es:

$$Y \sim N(\mu, \Sigma)$$

Se asume que las estimaciones conciliadas *Z* han de satisfacer *h* restricciones lineales de la forma:

$$AZ = a$$

donde *A*: hxM y *a: hx1* representan, respectivamente, la estructura general y los valores numéricos finales de dichas restricciones. Así, por ejemplo, *A* puede recoger que determinados componentes de *Z* sean iguales entre sí o que la suma de un subconjunto de variables iguale a la de otro subconjunto.

En el método de van der Ploeg se propone la determinación de *Z* como solución del siguiente programa de optimización condicionada:

$$\underset{Z}{MIN} \quad \varphi = (Z-Y)'\Sigma^{-1}(Z-Y) \qquad s.a. \quad AZ = a$$

La función objetivo pondera las desviaciones cuadráticas de cada estimación no conciliada respecto a su versión conciliada de forma inversa al error con que se estiman. Estos pesos tienen también en cuenta la estructura de covariación de dichos errores.

La solución final del programa de optimización es:

$$Z = Y - \Sigma A'\left[A\Sigma A'\right]^{-1}(AY - a)$$

La interpretación de esta ecuación es inmediata: el vector de variables conciliadas es el resultado de ajustar las estimaciones preliminares (*Y*) en función de la discrepancia observada (*AY-a*), teniendo en cuenta la estructura de varianzas y covarianzas de las estimaciones preliminares. De esta manera, las estimaciones iniciales son modificadas teniendo en cuenta sus discrepancias al incorporar las restricciones lineales. Las discrepancias son ponderadas según su fiabilidad o, si se prefiere, de forma inversa a la incertidumbre que se asocia a las estimaciones iniciales.

Este procedimiento posee algunas propiedades interesantes, del tipo *ceteris paribus*:

1. La magnitud de las revisiones, en valor absoluto, es tanto mayor cuanto mayor es la varianza de la estimación inicial (σ_{ii}), esto es,

cuanto mayor es la incertidumbre que rodea a la estimación inicial mayor es la cuantía de la modificación a que puede verse sujeta.

2. Si se considera que una determinada estimación preliminar se conoce con exactitud absoluta ($\sigma_{ii}=0$), entonces no se realiza ajuste alguno: $z_i=y_i$.

3. Si la incertidumbre en la estimación de dos variables evoluciona en el mismo sentido ($\sigma_{ij}>0$), sus revisiones también se registrarán en dicho sentido: las dos al alza o las dos a la baja. Si, por el contrario, su covariación es negativa los ajustes se realizarán en sentidos opuestos: una al alza y la otra a la baja o viceversa.

Obsérvese que, dada la forma de la solución, el conocimiento de la matriz de varianzas y covarianzas de las estimaciones preliminares (Σ) es un elemento crucial. Por el contrario, su valor esperado (μ) no juega papel alguno. Habitualmente, Σ no es conocida por lo que ha de ser estimada, usualmente en dos etapas: (a) estimación de las varianzas y (b) estimación de las covarianzas.

La estimación de las varianzas puede realizarse al mismo tiempo que la preliminar, por ejemplo, acompañando la estimación puntual preliminar de un intervalo de confianza o a partir de la varianza de las revisiones históricas. Concretamente, en el contexto del tercer artículo (Cuevas et al. (2015b)) estas varianzas va a provenir de la aplicación de los modelos factoriales a las series que posteriormente han de ser conciliadas.

Las covarianzas son, por su naturaleza, más difíciles de estimar. Usualmente se recurre a algún procedimiento indirecto basado en las correlaciones históricas entre las variables. En ese caso, una vez estimadas las varianzas, se derivan las covarianzas mediante la expresión siguiente:

$$\sigma_{ij} = \rho_{ij}\sqrt{\sigma_{ii}\sigma_{jj}}$$

6. Conclusiones y futuros desarrollos

Queda establecido en la literatura que los modelos de factores dinámicos que explotan la información contenida en la dinámica conjunta de las variables macro y sus indicadores puntuales relacionados, son las mejores herramientas para la predicción a corto plazo. Banbura et al. (2013) o Camacho et al. (2013).

En este trabajo se ha plasmado su uso más común y directo para la previsión del PIB. Primero bajo la óptica de un modelo de media escala y con posterioridad con un modelo de pequeña escala que potencialmente mejora el rendimiento predictivo.

Asimismo se ha extendido su uso en varias direcciones:

- Por un lado para la estimación y predicción en tiempo real de un conjunto completo de PIB trimestrales a nivel regional. Con ello se ha solventado la falta de dicho PIB trimestral desglosado por CCAA, proporcionando estimaciones consistentes con los datos oficiales disponibles publicados por la Contabilidad Nacional (Contabilidad Regional de España (CRE) y Contabilidad Nacional Trimestral (CNTR)). Estas estimaciones son un producto independiente que se puede utilizar como input en modelos econométricos regionales.

- Por otro lado para la predicción en tiempo real de un cuadro macroeconómico completo desde el punto de vista de la demanda. La principal característica distintiva de la metodología es que se pronostica, sobre una base de tiempo real, no sólo el PIB sino también su completa descomposición del lado del gasto. Hay modelos específicos para pronosticar el consumo privado, el consumo público, inversión en equipo, la inversión en construcción, exportaciones e importaciones. Se integran todos ellos en un conjunto coherente de predicciones coherentes con la predicción del PIB.

Por su parte, cabe mencionar que hay numerosas líneas futuras de investigación:

- Adaptar de este tipo de modelos para para analizar temas relacionados con la sincronía ciclos económicos regionales así como su patrón de co-movimientos.

- No limitarse sólo a la desagregación del PIB desde el punto de vista de la demanda, sino también ampliar el objetivo a las otras dos vertientes de la obtención del PIB: oferta y rentas.

- Plantear un modelo que integre el procedimiento de equilibrado dentro del modelo de espacio de estados, en la línea de Proietti (2011).

- Integrar este tipo de modelos con otros de más medio plazo, i.e. tipo BVAR, que simultáneamente den cabida a datos de frecuencias mixtas, con idea de aportar información estructural que permita extender el horizonte de predicción.

CAPÍTULO 2: A Factor Analysis for the Spanish Economy (FASE)[2]

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Abstract

We present a medium-scale dynamic factor model to estimate and forecast the rate of growth of the Spanish economy in the very short term. The intermediate size of the model overcomes the serious specification problems associated with large-scale models and the implicit loss of information of small-scale models.

The estimated common factor is used to forecast the Gross Domestic Product (GDP) by means of a transfer function model. Likewise, the model solves the operational and informational limits posed by the presence of an unbalanced panel of indicators and generates multivariate forecasts of the basic indicators.

Keywords

Dynamic factor model, short-term economic analysis, Spanish economy, Kalman filter, transfer function, temporal disaggregation, forecasting, nowcasting.

JEL Codes

C22, C53, C82, E27, E32.

1. Introduction

Business cycle analysis has been spurred by the severity of the recent downturn of the world economy. The assessment of economic policies does require timely and precise information about general macroeconomic conditions. In this vein, the use of standard measures of aggregate economic activity based on the Quarterly National Accounts (QNA) imposes a delay in the decision-making process that may hamper its effectiveness.

In order to alleviate the information constraints imposed by these standard measures, we design a coincident indicator to estimate the state of the business cycle in the very short term on a real-time basis using dynamic factor analysis.

This attempt has some precedents, starting with Stock and Watson (1991, 2002) which may be considered modern descendants of the seminal work on cyclical indicators of Burns and Mitchell (1946). Factor models have become one of the most widely used techniques in applied econometric analysis because they provide a parsimonious way to parameterize dynamic models for vector time series, see Bai and Ng (2008) and Stock and Watson (2010) for a comprehensive review. The relationship of factor models with other multivariate techniques is analyzed in Galeano and Peña (2000), Peña and Poncela (2006b) and Stock and Watson (2005). Since the pioneering work of Sargent and Sims (1977) and Geweke (1977), these models have been used for macroeconomic analysis and forecasting. More recently, both central banks and academic institutions have created all sorts of real time indicators and disseminated them through their websites. These estimates and forecasts influence policy-makers and shape up public opinion. Notable examples are the indicator designed by Aruoba et al. (2009), published in real time by the Federal Reserve Bank of Dallas; Chauvet (1998), both for the United States economy (U.S.) and for Brazil; Giannone et al. (2008) and Evans et al. (2002) also for the U.S. economy; Angelini et al. (2008) for the Eurozone and Camacho and Pérez-Quirós (2009a, 2009b) for both the Eurozone and the Spanish economy.

Applications related to finance are also numerous, many of them linked to risk management and term structure modeling, see Chamberlain (1983), Chamberlain and Rothschild (1983), Litterman and Scheinkman (1988), Knez et al. (1994), Bechikh (1998), Reimers and

Zerbs (1999), Ang and Piazzesi (2003) and Christensen et al. (2009), among others. Factor models have been used to asses economic policies, see Bernanke et al. (2005), Boivin and Giannoni (2006), and Forni and Gambetti (2010); estimation and inferential issues are analyzed in Box and Peña (1987), Watson and Engle (1983), Watson and Kraft (1984), Stock and Watson (1988), Escribano and Peña (1994), Peña and Poncela (2004, 2006a). Factor models in the frequency-domain are described in Priestley et al. (1974), Geweke (1977), Sargent and Sims (1977), Geweke and Singleton (1981) and Forni et al. (2000, 2005), among others.

All the preceding models are designed either as small-scale or large-scale. Both methodologies present important shortcomings. On the one hand, small-scale models are relatively exposed to idiosyncratic shocks and suffer an implicit loss of information. On the other hand, the estimation of large-scale models by quasi-maximum likelihood methods, akin to those used in our model, can violate the weak cross-correlation assumption needed to ensure the consistency of their estimators. By contrast, our model has an intermediate size that provides a natural hedge against the pitfalls of both small-scale and large-scale models.

The debate concerning the forecasting performance of small-scale models versus large-scale models is still an open issue. Our main contribution to the literature is twofold. First, we increase the number of indicators in a controlled way, fulfilling the assumption of weak cross-correlation among the idiosyncratic components which ensures the consistency of the estimators. Second, our model combines dynamic factor analysis with transfer function modeling, instead of ad hoc bridge equations.

The common factor underlying the observed indicators is estimated by means of the Kalman filter, after a suitable reparameterization of the model in state space form. In this way, we solve simultaneously the problem posed by the presence of an unbalanced panel (i.e., indicators with non-overlapping samples) and the generation of forecasts for individual indicators using a multivariate approach.

It must be emphasized that these predictions of the individual indicators are made in an explicit multivariate setting, avoiding the overparameterization and overfitting risks posed by other approaches (e.g. VAR models). Therefore, when making individual forecasts, the model makes an efficient use of the information contained in related indicators.

Moreover, transfer function models provide a simple and quantitatively consistent relationship between the common factor and the macroeconomic aggregates, GDP in particular. This linkage allows us to compile a contemporaneous estimate of GDP on a real-time basis. These models also provide confidence intervals for the GDP estimates, quantifying the uncertainty that surrounds them. It is important to note that the specification of the transfer function is checked with the results of a bivariate vector autoregressive and moving average (VARMA) model. The VARMA model provides an additional and rigorous foundation for the transfer function and prevents data mining.

This two-step approach (common factor estimation and transfer function) effectively disentangles the uncertainty due to the real-time estimation of actual business cycle conditions using monthly indicators from the uncertainty due to the relationship between GDP and monthly short-term indicators. This separation hedges us from idiosyncratic GDP changes that may distort the historical relationship between monthly indicators and quarterly macroeconomic aggregates measured by the QNA.

Additionally, the fact that the GDP compilation features (chain-linking, benchmarking, seasonal adjustment and balancing) are so different from the usual short-term indicators compilation practices, suggests the use of a two-step approach such as the one used in this work. These features, individually considered, are absent in the compilation of most short-term economic indicators and their concurrent use is completely missing. Hence, from the compilation perspective, GDP is a very special type of economic statistics, see INE (2002) and Abad et al (2009).

Additionally, GDP is a synthetic statistic, the result of combining short-term indicators (monthly and quarterly data) with structural sources (annual data provided by the National Accounts and the Input-Output tables, see INE (1993)). Thus, GDP is functionally equivalent to a common factor although not compiled using factor models. As a result, a one-step approach that considers GDP and short-term indicators in a unique framework may overweight the former due to its synthetic (or "artificial") nature, rather than on a genuine communality derived from common economic fundamentals.

This methodology is applied to a broad set of monthly indicators of the Spanish economy, whose selection took into account their economic significance, their temporal and statistical coverage, and an appropriate degree of sources diversification. The size of the model (31 indicators) allows a feasible computerized processing and reduces the risks implied by idiosyncratic shocks affecting the estimation and forecasting of the common factor as well as its link to the quarterly GDP.

It should be also mentioned that a natural extension of the model would be its integration with a Markov switching model in the line of Cancelo (2005) and Camacho et al. (2010). These nonlinear integrated models allow simultaneous calculation of probabilities of recession while dealing with some specificity of common factor models (mixing frequencies, data revisions and ragged edges). However, in the context of the size of this model is computationally more complex, while the integrated models may be more sensitive in their results to changes in information, especially in the delimitation of the states.

The document is organized as follows. The second section outlines the econometric methodology, detailing the nature of the dynamic factor model, its estimation by means of the Kalman filter and its relationship with macroeconomic variables using transfer function models. The third section presents the basic short-term indicators and their preliminary statistical treatment. The empirical results appear in section four. Finally, a set of appendices describes the technical details of the model, in order to ensure the self-contained nature of the text.

2. Econometric approach

The starting point of our modeling approach is a dynamic one-factor model that captures in a parsimonious way the dynamic interactions of a set of monthly economic indicators. The common factor of the system is estimated by means of the Kalman filter, after casting the factor model in state space form. On the basis of this factor we design a synthetic index that is related to quarterly aggregate output through a transfer function model. The entire procedure has been adapted to operate with unbalanced data panels, in order to forecast both indicators as well as macroeconomic aggregates in real time (nowcasting).

2.1 Dynamic factor model

Dynamic factor analysis is based on the assumption that a small number of latent variables generate the observed time series trough a stochastically perturbed linear structure. Thus, the pattern of observed co-movements is decomposed into two parts: communality (variation due to a small number of common factors) and idiosyncratic effects (specific elements of each series, uncorrelated along the cross-section dimension).

In this paper we assume that the observed, stationary growth signals of *k* monthly indicators are generated by a factor model:

[2.1] $$z_{i,t} = \lambda_i f_t + u_{i,t}$$

Being:

- $z_{i,t}$: i-th indicator growth signal at time *t*.
- λ_i: i-th indicator loading on common factor.
- f_t: common factor at time *t*.
- $u_{i,t}$: specific or idiosyncratic component of i-th indicator at time *t*.

The loadings λ_i measure the sensitivity of the growth signal of each indicator for changes in the factor.

Equation [2.1] considers only static (i.e., contemporaneous) interactions among the observed indicators trough its common dependence on a latent factor. The model must be expanded in order to adapt it to a time series framework, thereby adding a dynamic specification for the common factor and the idiosyncratic elements. A finite autoregression of order *p*, AR(p), provides a sufficiently general representation for dynamics of the common factor:

[2.2] $$\begin{aligned} &(1-\phi_1 B-\phi_2 B^2-\phi_3 B^3-\cdots-\phi_p B^p) f_t = a_t \\ &a_t \sim iid\ N(0,1) \end{aligned}$$

In [2.2] *B* is the backward operator $Bf_t = f_{t-1}$ and the variance of the innovation has been normalized. Depending on the characteristic roots of $\phi_p(B)$ the model may exhibit a wide variety of dynamic behaviors. Determining the order *p* of the model is made taking into account the empirical dynamics of the static factor, according to standard order selection criteria. As will be seen below in the section on empirical results, the most appropriate order is *p=4*.

We consider an AR(1) specification for the dynamics of the specific elements, allowing for some degree of persistence:

[2.3] $$\begin{aligned} &(1-\psi_i B)u_{i,t} = b_{i,t} \quad |\psi_i| < 1 \\ &b_{i,t} \sim iid\ N(0,v_i) \end{aligned}$$

Finally, we assume that all the innovations of the system are orthogonal:

[2.4] $$\begin{aligned} &E(a_t b_{i,s}) = 0 \quad \forall i,t,s \\ &E(b_{i,t} b_{j,s}) = 0 \quad \forall i,j,t,s \end{aligned}$$

2.2 State-space representation and Kalman filter

Model [2.1]-[2.4] attempts to represent the static as well as the dynamic features of the data. We estimate the common and idiosyncratic factors using the Kalman filter, after a suitable reparameterization of the model in state-space form. This reparameterization requires the introduction of a state vector that encompasses all the required information needed to project future paths of the observed variables from their past realizations. In our case, this vector is:

[2.5] $$X_t = \begin{pmatrix} f_t & f_{t-1} & f_{t-2} & f_{t-3} & u_{1,t} & \cdots & u_{k,t} \end{pmatrix}'$$

The corresponding measurement equation is:

[2.6] $$Z_t = (L \quad 0_{kx3} \quad I_k)\ X_t = H X_t$$

Where $L=\{\lambda_i\ i=1..k\}$ represents the loading matrix. This equation allows us to derive the observed indicators from the (unobservable) state vector.

The transition equation completes the system and characterizes its dynamics:

[2.7] $$X_t = GX_{t-1} + V_t$$

Where G is a square matrix with dimension $k+4$:

$$G = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & \phi_4 & 0 & \cdots & 0 \\ 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \psi_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \psi_k \end{bmatrix} \quad [2.8]$$

The innovations vector V_t is:

$$V_t = \left(a_t \quad 0 \quad 0 \quad 0 \quad b_{1,t} \quad \cdots \quad b_{k,t}\right)' \quad [2.9]$$

V_t evolves as a Gaussian white noise with diagonal variance-covariance matrix as follows:

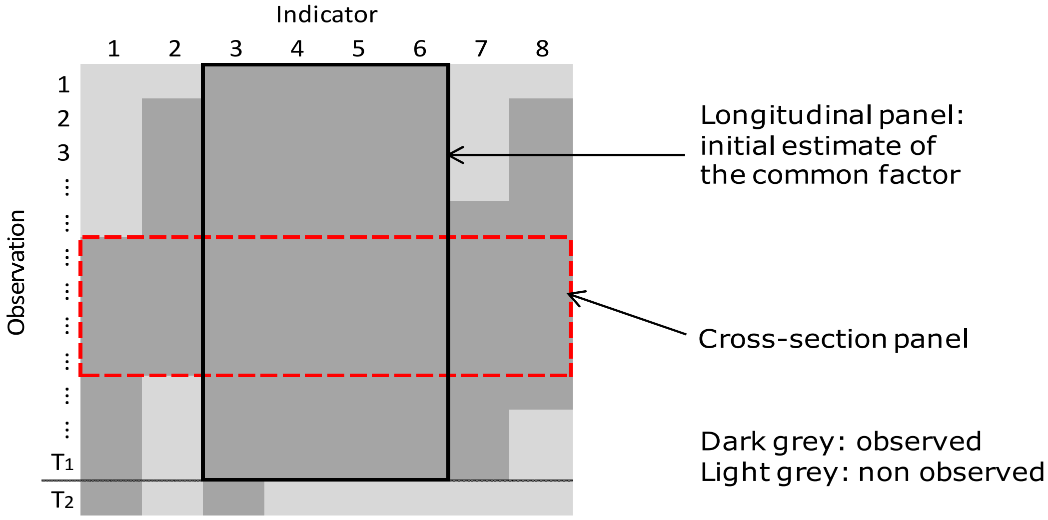
$$Q = E\left[V_t V_t^{'}\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & v_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & v_k \end{bmatrix} \quad [2.10]$$

We assume that the time index t goes from 1 to T. The application of the Kalman filter requires $\Theta = [H, G, Q]$ to be known. Since the model is not small-scale, full-system maximum likelihood estimates for Θ are not feasible. Our solution was to derive them from the static version of the model estimated using bootstrap methods, see Appendix A for details. The Kalman filter is explained in O'Connell (1984) and Kim and Nelson (1999), among others.

2.3 Dealing with an unbalanced data panel

One of the major operational problems faced while analyzing multiple time series is the incomplete nature of the available information. In general, the availability of different indicators is not homogeneous, which leads to the generation of a non-overlapping sample.

Figure 2.1: Unbalanced panel data



One way to deal with unbalanced panels consists in working only with complete panels, in the time dimension or in the cross-section dimension. As shown in Figure 2.1, in the first case we may discard a large number of the relevant indicators, with likely adverse effects on factor estimation accuracy and forecasting performance. In the second case, the number of observations may be too small when some series have a short span, making the forecasting or backasting horizon too long.

Given these drawbacks we propose a way to utilize all available information, both on the cross-section dimension and on the time dimension. The method, which is partially based on Stock and Watson (2002) and Giannone et al. (2006), relies on an iterative process with the following steps:

I. Estimation of a static factor model by principal components using the longitudinal panel data. Obviously, the use of this panel involves a loss of information that will be compensated in the following stages.

II. The indicators that have been excluded from the longitudinal panel are individually regressed (by ordinary least squares, OLS) on the common factor. The estimated parameters are then used to calculate the missing data in these series from $t=1$ to $t=T_1$.

III. A new factor is calculated from the statically balanced panel, from $t=1$ to $t=T_1$, using the same procedure as in step 1. Hence, new parameters $\Theta = [H, G, Q]$ are available.

IV. Using the new parameters Θ we apply the Kalman filter from $t=1$ to $t=T_1$ to estimate the common factor. This factor is in turn projected to $t=T_2$.

V. With the estimated common factor derived from step 4 as a regressor, we rebalance again the panel using the same procedure used in step 2. Steps 2 to 5 are iterated until convergence is achieved. The convergence criterion states that the change of the likelihood function should not trespass a given threshold.

The initial longitudinal panel should be wide enough to be representative, easing the usual trade-off between temporal coverage and cross-section coverage. After several tests, we selected January 1990 as the starting point of the panel data, providing a sensible balance in the above mentioned trade-off.

2.4 Linkage with macroeconomic variables via transfer function modeling

One of the main goals of the model consists in designing a connection between high-frequency indicators and the key variables that shape the macro scenario. In order to do it in a simple and efficient way, a transfer function model emerges as the ideal solution, providing real-time estimates of quarterly GDP using monthly indicators.

Once we have completed the estimation process of the dynamic factor model, taken into account the basic nature of the indicators as (standardized) period on period rates of growth, we can follow Mariano and Murasawa (2003) and represent the factor at the quarterly frequency combining the monthly observations according to:

[2.11] $$f_T = (\frac{1}{3} + \frac{2}{3}B + B^2 + \frac{2}{3}B^3 + \frac{1}{3}B^4)\ f_t$$

where f_t represents the monthly dynamic common factor and f_T is its temporally aggregated (quarterly) counterpart with time indexes related by $T=3t$. Hence, quarter T comprises months t, $t-1$ and $t-2$.

We consider that the dynamic relationship at the quarterly frequency between the common factor and the GDP may be articulated using a linear transfer function:

[2.12] $$y_T = c + V(B)f_T + n_T$$

where:

- y_T is the GDP, quarter on quarter rate of growth.
- f_T is the dynamic common factor, temporally aggregated according to Mariano-Murasawa.
- n_T is a stochastic disturbance that obeys a stationary and invertible ARMA(p,q) model.

The intercept c represents the non-stochastic component of y_T and $V(B)$ is the filter that passes on the information contained in f_T to contemporaneous and future values of y_T.

In order to specify the impulse-response $V(B)$ in a parsimonious way we follow Box and Jenkins (1976) and represent it in a rational form. Hence, the model [2.12] becomes:

$$y_T = c + \frac{\omega_s(B) B^b}{\delta_r(B)} f_T + \frac{\theta_q(B)}{\phi_p(B)} u_T$$

[2.13]

Where $u_T \sim iid\ N(0,v_u)$ and $\delta_r(B)$, $\omega_s(B)$, $\phi_p(B)$ and $\theta_q(B)$ are polynomials on the backward operator B with orders r, s, p and q, respectively. We assume that all of them have their roots outside of the unit circle. The term $b \geq 0$ is the pure delay of the transfer function.

We arrive at the final form for [2.13] following the adaptive methodology of Box-Jenkins, refined and tailored to the transfer function case by Liu and Hanssens (1982), Hanssens and Liu (1983) and Tsay and Wu (2003), among others. In particular, tentative identification of the orders *b*, *r* and *s* of the (rational) impulse response is performed using the corner method (Beguin et al., 1980) as implemented by Liu (2005). The orders *p* and *q* of the model for the perturbation are determined using the the so-called *Smallest Canonical Analysis* (SCAN), see Tsay and Tiao (1985).

This methodology provides a statistically well-rooted method to determine the dynamic form of the relationship between y_T and f_T, avoiding *ad hoc* data mining and other pitfalls of the standard bridge equation approach.

3. Data

This section details the indicators that have been selected for model estimation and the preliminary processing that they have gone through.

3.1 Selection of indicators

Given the objective of the model and the econometric methodology at hand, we have made a relatively wide selection of monthly indicators. The selection process was carried out under the premise that indicators should be available timely and should provide a synthetic measure of the growth rate of the Spanish economy, being selected at their more aggregated level[3]. Additionally, they should have a correlation with GDP growth greater than 0.4 in absolute value. The 31 selected economic indicators, listed in Table 3.1, can be divided into five large blocks.

The first set includes information related to the domestic production. Among them we include the traditional series that are used to capture the evolution of economic activity, such as apparent consumption of cement, energy consumption or the industrial production index.

In the second block we have considered those economic variables related to the external sector, such as exports and imports of goods and services suitably deflated.

The third block consists of "soft" or qualitative indicators, where the economic sentiment indicator plays an important role due to their prompt availability. The financial variables are represented by (deflated) credit to firms and households.

Finally, the number of social security contributors, the number of registered contracts and the number of employed provided by the Labor Force Survey (LFS)[4], stands for the aggregate evolution of the Spanish labor market.

[3] The initial set on which the selection has been made is available in the *Synthesis of Economic Indicators* published by the Ministry of Economy and Finance.

[4] The data provided by the LFS are the only ones compiled on a quarterly basis. In order to preserve the monthly nature of the data set, we have used temporal disaggregation techniques to derive consistent monthly figures, see Boot et al. (1967). The transformation has been applied to the seasonally adjusted levels.

Table 3.1: List of indicators[5]

	Code	Source	Start Date	Unit	Release Date
Domestic production					
TOTAL AIR TRAFFIC	AER	State Agency for Air Navigation	1990 01	passengers	t + 12 days
APPARENT CONSUMPTION OF CEMENT	CEMN	Cement Partnership	1990 01	thousand tons	t + 22 days
CONSUMER GOODS AVAILABILITIES	DISPOCONS	GDMA*	2000 01	volume index	t + 50 days
CAPITAL GOODS AVAILABILITIES	DISPOEQ	GDMA	2000 01	volume index	t + 50 days
ELECTRIC POWER CONSUMPTION	ELE	Spanish Electricity Network	1990 01	million Kw/h	t + 1 day
ENTRY OF TOURISTS	ENT	Institute of Tourism Studies	1995 01	thousand people	t + 22 days
CONSUMPTION OF GASOLINE AND DIESEL	GASOL	Petroleum Products Corporation	1990 01	thousand of metric tons	t + 30 days
INDUSTRIAL TURNOVER INDEX DEFLATED BY IPRI	ICNI	National Statistical Institute	2002 01	deflated value index	t + 47 days
SERVICES TURNOVER INDEX DEFLATED BY CPI OF SERVICES	ICNSS	National Statistical Institute	2002 01	deflated value index	t + 47 days
INDUSTRIAL ORDER BOOKS INDEX DEFLATED BY IPRI	IEPI	National Statistical Institute	2002 01	deflated value index	t + 47 days
INDUSTRIAL PRODUCTION INDEX	IPI	National Statistical Institute	1990 01	volume index	t + 35 days
CONSTRUCTION INDUSTRY PRODUCTION INDEX	IPIC	Eurostat	1990 01	deflated value index	t + 47 days
RETAIL TRADE INDEX	IVCM	National Statistical Institute	1995 01	deflated value index	t + 27 days
SEA GOODS TRANSPORT	MARM	Ministry of Public Works	1990 01	thousand Mt	t + 40 days
CAR REGISTRATIONS	MATT	General Directorate of Traffic	1990 01	units	t + 1 day
TRUCK AND CARGO VAN REGISTRATIONS	MATVC	General Directorate of Traffic	1990 01	units	t + 1 day
OVERNIGHT STAYS IN HOTELS	PERNO	National Statistical Institute	1999 01	units	t + 23 days
TOTAL GROSS SALARIES	RBT	Tax State Agency	1995 01	deflated value index	t + 35 days
RAILWAY GOODS TRANSPORT	REM	Spanish Railways	1990 01	thousand passsengers/km	t + 29 days
LARGE COMPANIES SALES. TOTAL	VGE	Tax State Agency	1995 01	deflated value index	t + 35 days
ROAD PASSENGER TRANSPORT	VICAR	National Statistical Institute	1996 01	units	t + 39 days
PERIODIFIED NUMBER OF HOUSING STARTS	VIVPER	Ministry of Public Works	1990 01	units	t + 1 day
External sector					
EXPORTS OF GOODS DEFLATED BY UVI	EXBQ	Tax State Agency/GDMA	1990 01	deflated value index	t + 50 days
EXPORTS OF SERVICES DEFLATED BY CPI OF SERVICES	EXBS	Bank of Spain	1990 01	deflated value index	t + 60 days
IMPORTS OF GOODS DEFLATED BY UVI	IMPB	Tax State Agency/GDMA	1990 01	deflated value index	t + 50 days
IMPORTS OF SERVICES DEFLATED BY EURO ZONE CPI	IMPS	Tax State Agency/Eurostat	1996 01	deflated value index	t + 60 days
Opinion					
ECONOMIC SENTIMENT INDICATOR. SPAIN	ISE	European Commission	1990 01	index 1990-2008=100	t - 1 day
Financial variables					
CREDIT TO COMPANIES AND FAMILIES DEFLATED BY CPI	FIN	Bank of Spain/National Statistical Institute	1995 01	deflated value index	t + 35 days
Labour market					
SOCIAL SECURITY CONTRIBUTORS	AFI	Ministry of Labour	1990 01	thousand people	t + 2 days
REGISTERED CONTRACTS	CONTRA	Ministry of Labour	1990 01	units	t + 2 days
EMPLOYED LFS	OCU	National Statistical Institute	1990 01	units	t + 30 days

*GDMA: General Diretorate for Macroeconomic Analysis

Another reason for the choice of these variables is to consider all the indicators used in the compilation of the QNA and its main output, GDP. See Álvarez (1990), Martínez and Melis Maynar (1990), INE (1993,1994) and Álvarez (2005). To achieve this goal we attempt to cover in a sensible manner all the operations involved in the GDP compilation, both from the point of view of supply and demand:

[5] All indicators are freely available at: http://serviciosweb.meh.es/apps/dgpe/default.aspx

Table 3.2: Allocation of indicators to macroeconomic aggregates and sectors

Indicator	Consumption	Investment	Exports	Imports	Industry	Construction	Services	Labor market
AER	x						x	
AFI								x
CEMN						x		
CONTRA								x
DISPOCONS	x							
DISPOEQ		x						
ELE	x				x	x	x	
ENT			x				x	
EXBQ			x					
EXSQ			x					
FIN	x	x						
GASOL	x						x	
ICNI					x			
ISE	x							
ICNSS							x	
IEPI					x			
IMPB				x				
IMPS				x				
IPI					x			
IPIC						x		
IVCM							x	
MARM							x	
MATT	x							
MATVC		x						
RBT	x							x
REM							x	
OCU								x
PERNO			x				x	
VICAR							x	
VIVPER		x				x		
VGE					x	x	x	

As shown in the table above, we want that the main macroeconomic aggregates and sectors are adequately represented in the factor model. Such representation is strengthened diversifying the information sources, to the extent feasible by available economic short-term statistics.

3.2 Preliminary processing

As already mentioned, the objective of the model is to provide a synthetic measure of the rate of growth of the economy. This goal requires identifying a reliable signal of growth to be fitted by the factor model. In practice, the identification of this signal requires applying a filter to the series that removes their secular trend from the observed data. A detailed analysis of the different measures of economic growth can be found in Melis (1991) and Espasa and Cancelo (1993).

In order to emphasize the short-term information contained in the indicators, we have chosen the regular first difference of the log time series to perform such decomposition. The high-pass nature of this filter ensures an adequate estimation of the short-term variation, ruling out at the same time the trend component.

For this filtering not to be distorted by the presence of seasonal and calendar factors, they have been removed by means of seasonal adjustment and time series techniques (Maravall and Gómez, 1996; Caporello and Maravall, 2004). These transformations are also necessary in order to set a linkage, via equation [2.11], with the GDP growth, as both are corrected by the same factors (seasonal and calendar factors).

In the specific case of "soft" series, typically measured as balances of qualitative responses, the log transformation is not applied. Naturally, in all cases, the process of seasonal and calendar adjustment applies only if such effects are significant[6]. To facilitate the process of estimation and interpretation of the factor model, the filtered series are standardized:

[3.1] $$z_{i,t}^{st} = \frac{z_{i,t} - \mu_i}{\sigma_i}$$

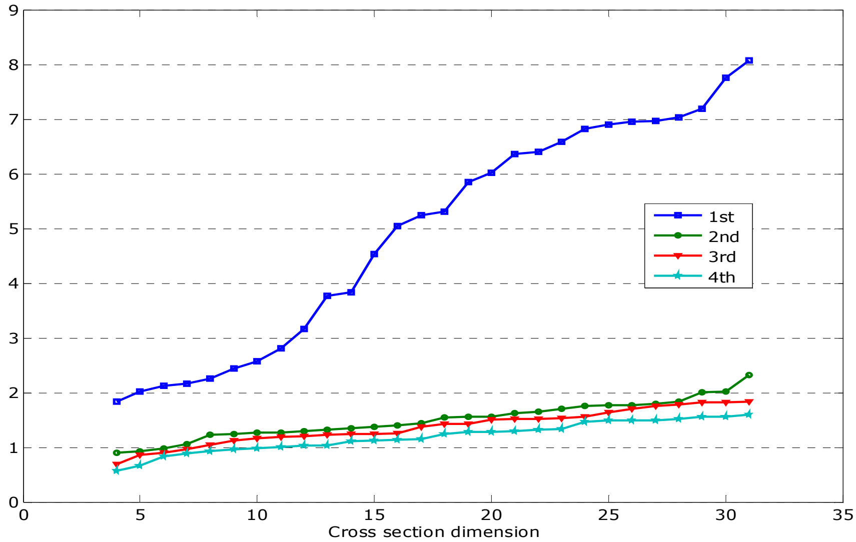
Being μ_i and σ_i the mean and standard deviation of the indicators in the selected sample. Thus, all series contained in the system are expressed in the same units of measurement.

4. Empirical results

The eigenvalues of the indicators correlation matrix across its cross-section dimension indicates the dominance of its maximum over the remaining eigenvalues:

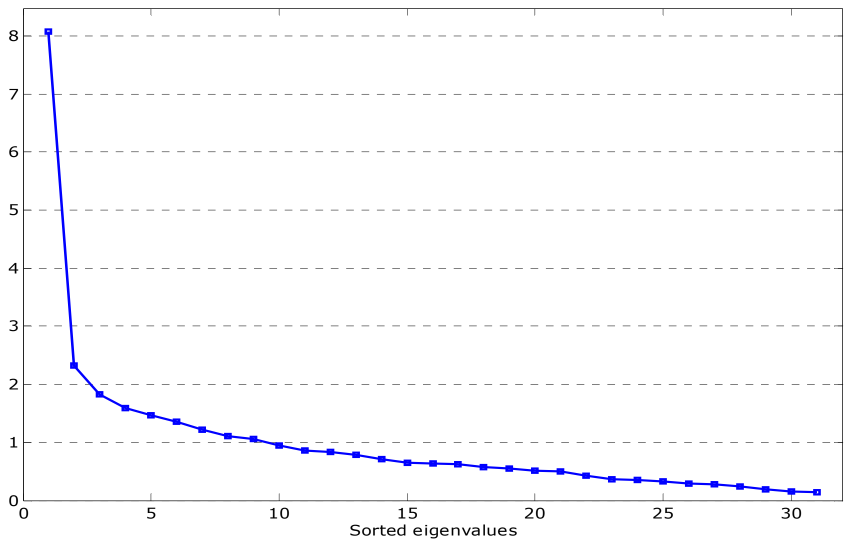
[6] Consumer Goods Availabilities (DISPOCONS), Capital Goods Availabilities (DISPOEQ) and the Economic Sentiment Indicator (ISE) are not adjusted from seasonal and calendar effects because they have already been processed by our data provider, http://serviciosweb.meh.es/apps/dgpe/default.aspx.

Figure 4.1: Correlation matrix eigenvalues across its cross-section



A similar picture emerges from the scree plot of the same eigenvalues computed using the 31 indicators at the same time:

Figure 4.2: Correlation matrix eigenvalues: scree plot



Both results suggest that a one-factor model may be a sensible model for the joint behavior of the 31 indicators.

The loading vector is estimated by means of principal components factor analysis combined with resampling techniques, suitably adapted to the time series context by Politis and Romano (1994). Estimation is based on 10,000 bootstrap replicates. The resampling procedure uses the stationary bootstrap with an expected size block of 41 months. This method provides a measure of the precision of point estimates and does not require any assumption concerning the distributional features of the data. See Appendix A for details. The following table shows the results of estimating equation [2.1], sorted from highest to lowest loading. This table also includes the mean[7] communalities (defined as the ratio between the observed variance of each indicator and the variance explained by the factor model).

Table 4.1: Loading vector: bootstrap estimates and communalities

	Loadings		Communalities
	Estimate	Standard Error	
ICNSS	0.88	0.07	0.78
ICNI	0.85	0.06	0.71
VGE	0.76	0.07	0.59
IPI	0.74	0.06	0.55
IEPI	0.74	0.08	0.53
AFI	0.73	0.11	0.58
CEMN	0.64	0.06	0.41
IVCM	0.60	0.07	0.36
GASOL	0.54	0.05	0.28
OCU	0.54	0.15	0.35
MATVC	0.51	0.05	0.26
DISPOCONS	0.50	0.09	0.24
MATT	0.47	0.06	0.22
CONTRA	0.47	0.06	0.22
FIN	0.46	0.10	0.22
IPIC	0.44	0.09	0.23
EXBQ	0.44	0.05	0.18
IMPB	0.43	0.06	0.18
ENT	0.40	0.06	0.15
VIVPER	0.40	0.16	0.21
DISPOEQ	0.36	0.08	0.12
AER	0.35	0.07	0.13
ISE	0.31	0.06	0.10
EXSQ	0.30	0.06	0.09
RBT	0.30	0.11	0.10
VICAR	0.26	0.04	0.06
IMPS	0.25	0.06	0.01
REM	0.22	0.13	0.06
ELE	0.20	0.06	0.04
MARM	0.19	0.05	0.03
PERNO	0.15	0.12	0.02

To set the lag order of the factor AR model we have used the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC)

[7] The mean is computed averaging over all the resamples.

and the Partial Autocorrelation Function (PACF) applied to the static common factor. The results of these statistics suggest *p=3* or *p=4* as the appropriate order of the model. We have chosen *p=4* in order to fully represent the systematic dynamics of the factor[8].

Using the previous results and the corresponding static factor we estimate the parameters of equation [2.2] by ordinary least squares, obtaining:

Table 4.2: Common factor: AR(4) estimates

	ϕ_1	ϕ_2	ϕ_3	ϕ_4
Estimate	0.03	0.28	0.31	0.14
Standard Error	0.06	0.06	0.06	0.07

Using the same estimation procedure applied to equation [2.3] we get:

[8] The detailed results are available upon request.

Table 4.3: Idyosincratic factors: AR(1) estimates

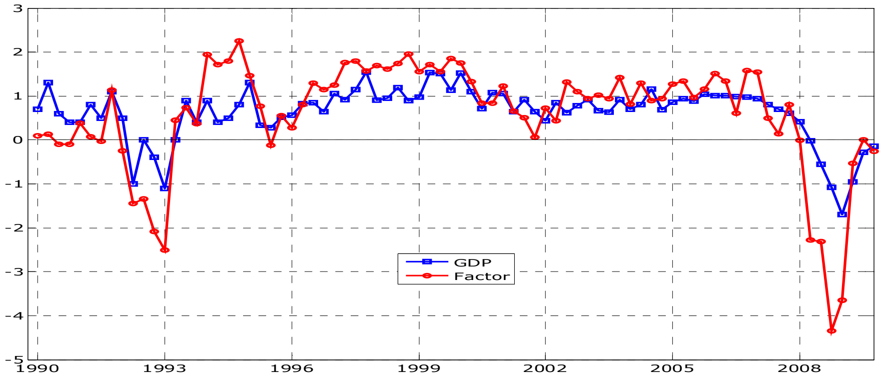
	ψ_1		
	Estimate	Standard Error	σ
ICNSS	0.01	0.07	0.17
ICNI	-0.21	0.06	0.23
VGE	-0.33	0.06	0.36
IPI	-0.15	0.06	0.41
IEPI	-0.42	0.06	0.37
AFI	0.14	0.07	0.42
CEMN	-0.31	0.06	0.52
IVCM	0.06	0.07	0.67
GASOL	-0.34	0.06	0.65
OCU	0.59	0.05	0.42
MATVC	-0.37	0.06	0.66
DISPOCONS	-0.26	0.06	0.72
MATT	0.01	0.07	0.79
CONTRA	-0.22	0.06	0.74
FIN	0.48	0.06	0.62
IPIC	0.46	0.06	0.62
EXBQ	-0.38	0.06	0.70
IMPB	-0.55	0.05	0.58
ENT	-0.40	0.06	0.73
VIVPER	0.64	0.05	0.48
DISPOEQ	-0.51	0.06	0.66
AER	-0.27	0.06	0.82
ISE	-0.15	0.06	0.88
EXSQ	-0.37	0.06	0.79
RBT	-0.30	0.06	0.83
VICAR	-0.22	0.06	0.90
IMPS	-0.46	0.06	0.74
REM	-0.54	0.05	0.67
ELE	-0.50	0.06	0.72
MARM	-0.52	0.06	0.69
PERNO	-0.35	0.06	0.86

Note: the ordering is the same as in table 4.1

The dynamic common factor is estimated using the Kalman filter and its quarterly counterpart, temporally aggregated using the Mariano-Murasawa formula. It shows a remarkable conformity with GDP growth, as may be appreciated in the next graph[9]:

[9] The dynamic common factor has been scaled according to the affine transformation $\alpha+\beta f_T$, being α and β the mean and standard deviation of GDP growth, respectively. This transformation enhances the comparability of both time series and preserves the directional pattern of the factor.

Figure 4.3: Dynamic common factor (scaled) and GDP growth



The figure shows their notable similitude, quantified by a high correlation (0.8) especially if one takes into account the presence of an important irregular component in both series.

The cross correlation function also shows a high degree of conformity between the common factor of the system and the GDP. The function has a maximum at lag zero, confirming the coincident nature of the factor with respect to GDP. Moreover, its asymmetric shape points to a tendency of the factor to lead GDP. This feature is very convenient for nowcasting and short-term forecasting.

Table 4.4: Common factor and GDP: cross correlation function

Lag										
-5	-4	-3	-2	-1	0	1	2	3	4	5
0.24	0.40	0.60	0.68	0.83	0.84	0.62	0.46	0.34	0.12	0.02

Note: Negative (positive) lags indicate that the factor is leading (lagging) GDP.

Following the methodology described in Liu (2005), the orders finally selected for the transfer function are: *b=0*, *s=r=1* and *p=q=0*. The formal expression is:

[4.1] $$y_T = c + \frac{(\omega_0 + \omega_1 B)}{(1 - \delta B)} f_T + u_T$$

Moreover, a separate multivariate analysis, based on the estimation of an autoregressive and vector moving average (VARMA) model, clearly ascertains a unidirectional Granger-causality that goes from factor to GDP and not vice versa. This lack of feedback justifies the use of a transfer function. Furthermore, this analysis suggests a tentative similar model: *b=0*, *r=s=1* and *p=q=1*. It was found that the modeling of the disturbance may ultimately be simplified, obtaining *p=q=0*. See Appendix B for additional details on the VARMA analysis.

The next table displays the estimation of the transfer function model by exact maximum likelihood:

Table 4.5: Transfer function estimates

	c	ω_0	ω_1	δ	σ
Estimate	0.56	0.21	-0.14	0.84	0.25
Standard Error	0.03	0.02	0.03	0.05	

The dynamics implied by the estimated transfer function reveals the high degree of persistence of the GDP (δ=0.84). The $\omega(B)$ operator plays also an important role since, due to its low long-run gain ($\omega(1)$=0.07), compensates the inertia of GDP and links its forecasts more closely to those of the factor.

Following Tsay and Tiao (1985) we have performed a canonical analysis of the residuals (the so-called *Smallest Canonical Analysis*, SCAN). The results do not show any major inadequacy, in line with the autocorrelation function.

In order to check the robustness of the transfer function, we estimate an expanded version of [4.1]. The augmented model is:

[4.2] $$y_T = c + \frac{(\omega_0 + \omega_1 B + \omega_2 B^2)}{(1 - \delta_1 B - \delta_2 B^2)} f_T + \frac{(1-\theta B)}{(1-\phi B)} u_T$$

Following Box and Jenkins (1976), the additional parameters are included one by one, in order to isolate as best as possible its individual contribution. The next table presents the results:

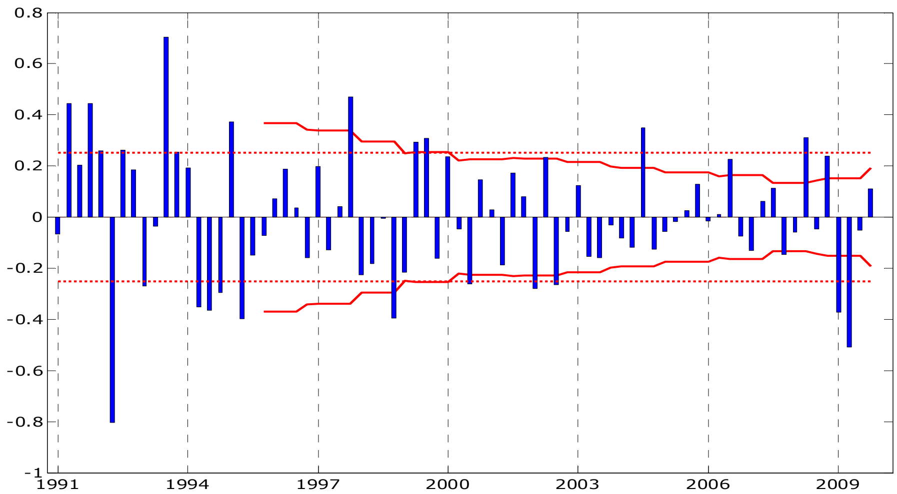
Table 4.6: Extended transfer function estimates

	ω_2	δ_2	ϕ	θ
Estimate	-0.06	0.63	0.08	-0.03
Standard Error	0.03	0.05	0.11	0.11
σ	0.24	0.27	0.27	0.26

In all the cases the additional parameter is not significative and/or it does not improve the fit of the model. As an additional check of the robustness of the model, we estimate the parameters of the transfer function [4.1] recursively from 2006:Q1 onwards. In general, the recursive estimates remain confined in the intervals centered around the last estimate[10].

There is some evidence of changing volatility reflected in the kurtosis (3.62), in the autocorrelation of the squared residuals (systematically positive) and in the variability of the variance of the residuals, as shown in the following graph:

Figure 4.4: Transfer function: residuals



Note: Red lines represent $\pm\sigma$ interval. The dotted line estimates σ using the full sample and the solid line estimates σ using a 5-year rolling window.

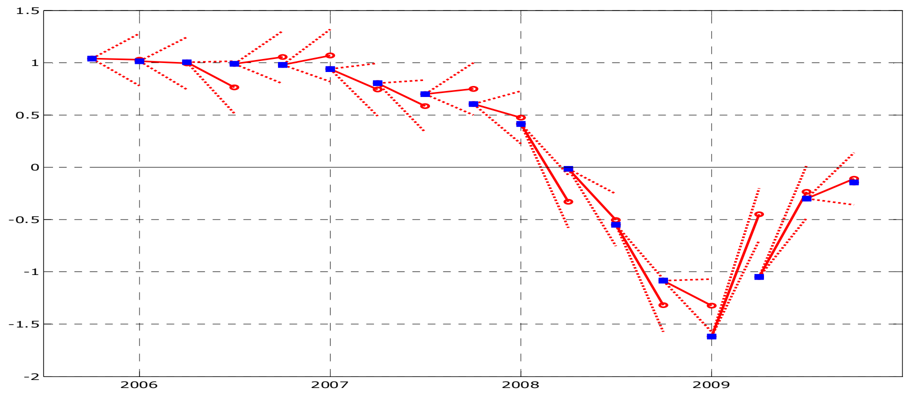
However, this evidence is not strong enough to reject the gaussianity assumption using the Jarque-Bera test[11] but deserves additional analysis using more sophisticated methods in future research (e.g., stochastic volatility models). This issue is important because, as shown in the previous figure, it may reflect changes in the size of the shocks affecting GDP.

In order to evaluate the forecasting performance of the model we have done several backtesting exercises. In all cases, the model has proved its usefulness as a tool for short-term economic analysis and the assessment of the growth pattern. As an example, the following graph shows the good tracking properties of the model during the previous four years.

[10] The graph of the recursive estimates are available upon request.

[11] The test value, 1.38, generates a p-value of 0.41.

Figure 4.5: Backtesting 2006-2009. One-step ahead forecasts (±s.e.)



Note: Blue squares are actual GDP and red circles are recursive one-step ahead forecasts. Dotted lines are $\pm\sigma$ *confidence intervals.*

We have compared the predictions made by the transfer function with those that have been generated by three standard univariate models used in forecasting GDP growth: a random walk, I(1), a first-order autoregressive and moving average, ARMA(1,1), and a fourth-order autoregression, AR(4). The first one represents a "no change" assumption, the second one a univariate transposition of the VARMA(1,1) model and the third one considers only pure AR representations[12].

The table below shows alternative measures of the forecasting performance of the models during the span 2006:Q1-2009:Q4: root of mean squared errors (RMSE) and mean of absolute errors (MAE), both considering one-step ahead forecasts. This time span has been chosen to take into account both a period of high growth and a period of sharp and deep contraction of aggregate output. The table also includes the Diebold-Mariano (1995) test to check the equivalence of the forecasting performance of the alternative models.

[12] The order of the AR model, $p=4$, has been determined using the Bayesian Information Criterion (BIC). The Akaike's Information Criterion (AIC) suggested a much less parsimonious model ($p=6$). Anyway, its forecasting performance is much similar to the AR(4) model.

Table 4.7: Forecasting performance, 2006:Q1 – 2009:Q4

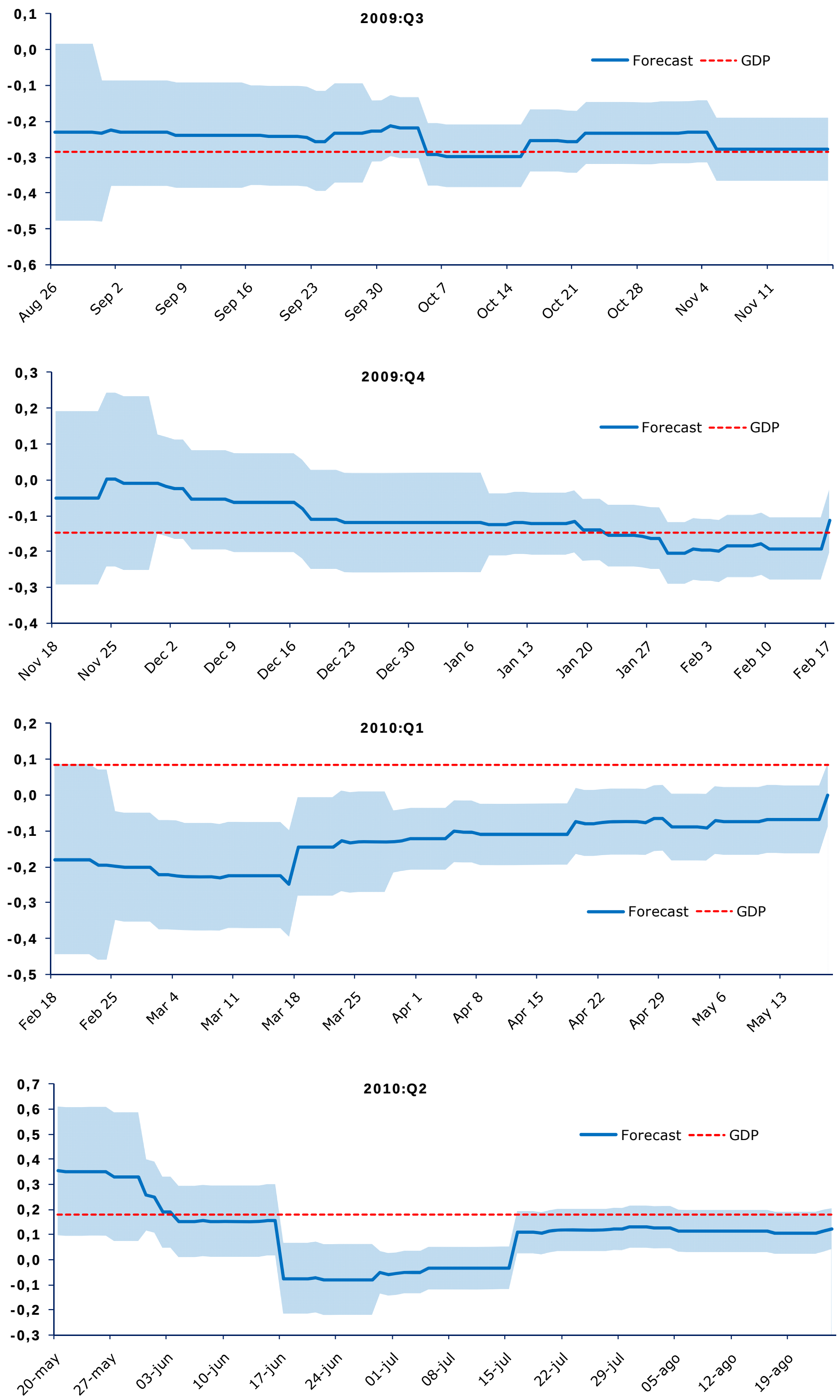
	I(1)	ARMA(1,1)	AR(4)	Transfer function
RMSE	0.36	0.40	0.37	0.21
MAE	0.26	0.29	0.25	0.15
RMSE: DM	0.01	0.02	0.07	
MAE: DM	0.02	0.02	0.10	

Note: DM test is reported using the p-value of the null hypothesis of forecasting performance equivalence.

The competitive edge of the transfer function model relies on its efficient use of monthly information combined with a proper dynamic specification, leading to better outcomes than its peers. The DM test presents the AR(4) model as the most close competitor of the transfer function, although the significance level is still quite small.

In order to complete the evaluation of the forecasting performance, it has been carried out a real time estimation exercise for the last four quarters (2009:Q3 – 2010:Q2). The following graphs plot the evolution of the real-time forecast of GDP in such quarters on a daily basis, including its $\pm\sigma$ confidence interval:

Figure 4.6: GDP growth rate. Real-time forecasts



Observing the graphs, we can see how the model reacts to the coming out of data updates. This process reduces the amplitude of the confidence interval, as the cross-sectional estimates are replaced by actual data. Initially, when only “soft” indicators are available, the uncertainty associated with the estimate is greater. When “hard” information arrives (industrial production index, large companies sales, etc), the estimate becomes more accurate.

The four graphs show that these forecasts were in close agreement with the GDP flash release disseminated by the National Statistical Institute and the subsequent final figure (second estimate)[13]. May be noted that, in most cases, the final data published has fallen within the confidence intervals associated with the estimation.

Finally, it is worth noting the work carried out in the same line for Camacho and Pérez-Quirós (2009) and Camacho and Doménech (2010) who also estimate dynamic factor models for the Spanish economy[14], providing GDP forecasts and synthetic measures of the state of the economy[15]. The following table compares the different characteristics of the models.

Table 4.8: Factor models for the Spanish economy

Model	Number of indicators	Preprocessing	First observation	Factor estimation	GDP Forecasting
S-STING	11	Seasonal and calendar adjustment, seasonal differences and levels	1990:01	Maximum likelihood + Kalman filtering	Internal
MICA	12	Seasonal and calendar adjustment, seasonal differences and levels	1980:01	Maximum likelihood + Kalman filtering	Internal
FASE	31	Seasonal and calendar adjustment, first differences	1990:01	Static factor analysis + Kalman filtering	External

As can be seen, the first of the differential characteristics of the models is their size. Both the S-Sting and MICA took place in a small-scale size, easing their maximum likelihood estimation. The second feature is the different preliminary treatment of the indicators, sharing all of them the seasonal and calendar adjustment. Finally, apart from the greater sample period covered by the MICA model, our model does not include the GDP as an indicator to estimate the factor, since, as mentioned earlier in the article, the GDP is already synthetic statistic, the result of combining short-term indicators.

[13] The GDP flash estimate is released about six weeks after the end of the quarter. The second estimate, incorporating the complete GDP breakdown, is released just one week after the flash (except in August that left nearly two weeks to incorporate the structural information of Annual National Accounts).

[14] They are also known by their acronyms: S-Sting (Spain-Short Term Indicator for Growth) and MICA (*Modelo de Indicadores Coincidentes y Adelantados*, Model of Coincident and Leading Indicators).

[15] Using an affine methodology, Camacho and Pérez-Quirós (2009) estimate and analyze a dynamic factor model for the Eurozone. Camacho et al. (2010) expand the model to incorporate non-linearities (via Markov-switching) in the behavior of the common factor. In the same vein, Cancelo (2005) estimates a dynamic factor model with Markov-switching features to analyze the GDPs of the Eurozone countries.

5. Conclusions

In this paper we have designed a real-time, coincident indicator of the Spanish business cycle. It has a straightforward interpretation as the dynamic common factor of a set of representative short-term monthly economic indicators. This synthetic indicator also plays a critical role in GDP forecasting, by means of a suitable dynamic projection based on transfer function modeling.

The model differs from others proposed in the literature due to its medium-scale. This feature provides a certain advantage over small-scale models due to its higher information content and, at the same time, avoids the technical problems concerning the consistency of the estimators that hamper large-scale models. Moreover, its two-step approach strengthens the operative characteristics of the model, providing a hedge from changes in the relationship between indicators and macroeconomic aggregates.

This work could be extended in several directions. The incorporation of leading indicators would enrich the dynamic structure of the model. Another possibility is to apply this methodology to other macroeconomic aggregates, being the demand-side components of GDP prime candidates. Anyway, since the model is eminently empirical, its use in a production mode will determine the way forward, including changes in the list of indicators and refinements of the estimation procedures.

Appendix A: preliminary static estimation

By rewriting [2.1] in matrix form, we obtain:

[A.1] $$Z_t = Lf_t + U_t$$

Being Z_t: kx1, L: kx1, f_t: 1x1 and U_t: kx1.

The normalized eigenvector associated with the largest eigenvalue of the correlation matrix of *Z*, provides an estimate of the loading matrix *L*:

[A.2] $$\hat{L} = \sqrt{\lambda_1}\; e(\lambda_1)$$

The variance-covariance matrix of the specific factors is then estimated as a residual:

[A.3] $$\hat{\Psi} = diag(R - \hat{L}\hat{L}')$$

In order to obtain estimates of *L* and Ψ with appropriate standard errors, we apply [A.2] and [A.3] to the resampled time series. Resampling is performed using the bootstrap technique suggested by Politis and Romano (1994), in which the resampling is applied with reposition to blocks of varying size. The block size is selected each time according to a predefined probability distribution. In our application we have used the geometric distribution with an expected block size of 41 months[16]. The results are robust with respect to alternative mean block size. The estimation is repeated 10,000 times and the corresponding averages and standard deviations provide the estimates for *L* and Ψ.

The stationary bootstrap provides more robust results than other resampling methods, notably those procedures based on the use of fixed size blocks, e.g. Künsch (1989). In fact, the former may be considered as a weighted average over block size of the latter, generating a smoothed version of it.

With the resulting point estimates of *L* and Ψ we transform the original factor model into one akin to a multivariate regression model. Hence, an initial estimate can be obtained using generalized least squares (GLS):

[A.4] $$\hat{F} = \left[\hat{L}'\hat{\Psi}^{-1}\hat{L}\right]^{-1}\left[\hat{L}'\hat{\Psi}^{-1}Z\right] = \Theta(\hat{L},\hat{\Psi})Z = \Theta Z$$

[16] Following Camacho et al. (2005) in their implementation of stationary bootstrap for business cycle analysis.

A complete analysis of these issues can be found in Mardia et al. (1979).

Appendix B: VARMA analysis

In this section we estimate a vector autoregressive moving-average (VARMA) model to summarize the econometric relationship between the dynamic common factor (f_t) and the GDP quarter on quarter rate of growth (y_t), see Tiao and Box (1981), Lütkepohl (1991), Reinsel (1993) and Tiao (2001), for an in-depth analysis of such models.

Consider a k-dimensional vector, Z_t, which evolves following a VARMA(p,q) model, which can be expressed by the following equation:

[B.1] $$\Phi_p(B)Z_t = c + \Theta_q(B)U_t$$

Being $\Phi_p(B)$ and $\Theta_q(B)$ polynomial matrix operators of degree *p* and *q*, respectively. Furthermore, the vector U_t can be characterized by the following distributional properties:

[B.2] $$U_t : kx1 \sim N(0,\Sigma)$$

Being Σ, in general, a non-diagonal matrix. Additionally, it is assumed that all the roots of the determinantal polynomials $|\Phi_p(B)|$ and $|\Theta_q(B)|$ lie either on or outside the unit circle.

The canonical correlation analysis of Tsay-Tiao suggests that a low-order VARMA(1,1) provide a reasonable fit to the data. This model serves as a benchmark to check the adequacy of several specifications concerning the direction of (Granger) causality. The results are summarized in the following table:

Table B.1: VARMA(1,1): Granger-causality analysis

Hypothesis	Log Likelihood	Δ
Feedback	22.75	--
Factor causes GDP	22.98	0.23
GDP causes factor	-2.24	-25.00
Decoupling	-7.22	-29.98

The results strongly support the hypothesis that the dynamic common factor is an input in the determination of GDP and that the use of transfer function is well grounded.

The estimation of the constrained[17] VARMA(1,1) model by exact maximum likelihood yields the following results:

Table B.2: VARMA Model: $(I-\Phi B)Z_t = c + (I-\Theta B)U_t$

Constrained maximum likelihood estimation

	Estimate		Standard Error		Eigenvalues
C	0.08		0.02		
	0		--		
Φ	0.85	-0.02	0.03	0.03	0.85
	0	0.66	--	0.10	0.66
Θ	1.00	-0.32	0.05	0.04	1.00
	0	-0.37	--	0.12	-0.37
$\Sigma \mid \Gamma$	0.09	0.65			
	0.23	1.43			
Log Likelihood	23.02				

Note: 0 and -- mean restricted parameters. Γ is the correlation matrix linked to Σ.

The residuals obtained from the VARMA model do not show any major inadequacy, as may be seen from their corresponding SCAN table:

Table B.3: VARMA Model. SCAN analysis of the residuals

	q =>	0	1	2	3	4	5	6	7	8
	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0
p	4	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0

To further analyze the underlying structure of the VARMA model we perform a canonical analysis, following Box and Tiao (1977). The results are summarized in the following table:

Table B.4: VARMA Model: $(I-\Phi B)Z_t = c + (I-\Theta B)U_t$
Box-Tiao canonical analysis

Eigenvalues	0.78	0.27
Eigenvectors	1.00	0.94
	0.01	-0.34

[17] The constraint $c_2=0$ is considered in addition to the ones defined in the second row of table C.1.

The main results may be summarized as follows:

- There is a remarkable degree of persistence in the bivariate system, as denoted by the maximum eigenvalues of both Φ (0.85) and the Box-Tiao canonical analysis (0.78). The behavior of the GDP explains most of this feature.

- Adding up to the dynamic (unidirectional) interactions, there is a significative degree of contemporaneous association between both series (0.65). This fact justifies the use of the common factor to nowcast GDP on a real-time basis.

- The system is non-invertible, due to the GDP intrinsic dynamics ($\Theta_{1,1}=1$). This fact may be the result of the seasonal adjustment procedure, see Maravall (1993).

- The Box-Tiao canonical analysis indentifies a stable contemporaneous, positive relationship between GDP and the common factor. Deviations from this "equilibrium" feature revert to the mean at a high speed.

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CAPÍTULO 3: Quarterly Regional GDP Flash Estimates by means of Benchmarking and Chain-Linking[18]

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Abstract

In this article we propose a methodology for estimating the GDP of a country's different regions, providing quarterly profiles for the annual official observed data. Thus the article offers a new instrument for short-term monitoring that allows the analysts to quantify the degree of synchronicity among regional business cycles.

Technically, we combine time-series models with benchmarking methods to process short-term quarterly indicators and to estimate quarterly regional GDPs ensuring their temporal and transversal consistency with the National Accounts data. The methodology addresses the issue of nonadditivity, explicitly taking into account the transversal constraints imposed by the chain-linked volume indexes used by the National Accounts, and provides an efficient combination of structural as well as short-term information.

The methodology is illustrated by an application to the Spanish economy, providing real-time quarterly GDP estimates, that is, with a minimum compilation delay with respect to the national quarterly GDP. The estimated quarterly data are used to assess the existence of cycles shared among the Spanish regions.

Keywords

Benchmarking, Chain-Linking, National Accounts, Regional Accounts, GDP Flash Estimates.

JEL Codes

C53, C43, C82, R11.

1. Introduction

Business cycle analysis and the short-term monitoring of a national economy can be substantially improved if an explicit regional dimension is taken into consideration. In this way, the diffusion of the aggregate (or national) cycle can be analyzed in detail: identifying leading/coincident/lagged regions, detecting common and specific shocks and so on. The relevance of this added geographical dimension is especially important both for large or medium-sized countries as well as for countries with decentralized systems that allow specific economic policies to be applied. Of course, the quarterly regional estimates that we present below are also very useful for regional governments.

The Regional Accounts (RA) are annual data and in this context we here propose a methodology for estimating quarterly Gross Domestic Product (GDP) time series at the regional level, providing a new instrument for short-term monitoring that allows us to gauge the degree of synchronicity and the identification of shared and idiosyncratic shocks to different regions.

Our methodology ensures the consistency of these quarterly regional GDPs with the national quarterly GDP, taking into account the chain-linking procedures that underlie its compilation. Note that the same principles of volume estimation using chain-linked indices have been used in our analysis and we have applied the same procedures of seasonal and calendar adjustment used by the QNA.

Structural consistency is also ensured, since the quarterly regional GDPs are consistent with their annual Regional Accounts counterparts. The fact that both QNA and RA share the same National Accounts (NA) framework, provides the base for the consistency obtained in our analysis. In this way, we can use the quarterly regional estimates to derive structural measures at the regional level.

The modeling approach is highly reliant on a set of regional high-frequency indicators. These indicators provide the ultimate basis used by the model to generate GDP according to time-series techniques ranging from univariate ARIMA models to multivariate dynamic-factor models. The set of indicators and models are homogeneous across regions, ensuring the comparability of the results.

The methodology has three main stages:

1. Processing of the high-frequency indicators available at the regional level and estimation, for each region, of a synthetic index that combines the available short-term information.

2. Temporal disaggregation and interpolation of annual regional GDP using the indicators processed in Step 1.

3. Balancing of these initial quarterly estimates in order to ensure transversal consistency with national quarterly GDP, at the same time preserving the temporal consistency achieved in the previous stage.

It is worth emphasizing that, from an operational perspective, early estimates of quarterly regional GDPs may be available with a minimum delay with respect to the national quarterly GDP release, the so-called "GDP flash estimate". Thus the national figure may have timely regional counterparties, enhancing the informational content of analysis carried out at the aggregate level.

The main contributions of our article are:

- A methodology for obtaining quarterly estimates of GDP for all the regions in a country, derived in a consistent way with the official available data provided by the National Accounts, both RA and QNA.

- Early (or flash) estimates of quarterly GDP at the regional level that may be released at the same time as the national GDP.

- Transversal consistency is compliant with the chain-linking methodology, circumventing its nonadditive features in the balancing step.

The article is organized as follows. The second section outlines the modeling approach, going into detail on its main steps. A complete and in-depth application of the methodology using Spanish data appears in section three. Finally, in the fourth section, we present our conclusions and future lines of research.

2. Modeling Approach

In this section we present the main steps of the proposed methodology. The modeling approach consists of three basic steps: (i) seasonal adjustment of regional short-term raw indicators and construction of synthetic indicators for each region by means of factor analysis, (ii) initial quarterly estimates of regional GDP provided by benchmarking and (iii) enforcement of the transversal constraint that links the regional quarterly GDPs with their national counterpart.

This aggregation constraint must be consistent with the chain-linking procedure used to compile quarterly GDP at the national level, dealing with the nonadditivity issue in an appropriate way. We now turn to examine the three stages in more detail; however, to simplify the exposition, we first present the required information set.

2.1. Information set

The model requires as input three elements that vary according to their sampling frequency (annual or quarterly), their spatial coverage (regional or national) and their method of compilation (National Accounts or short-term indicators).

The variables of the system are: regional GDPs (y), national GDP (z) and regional short-term indicators in original or raw[19] form (xr). Upper-case letters refer to annual variables while lower-case letters refer to quarterly variables. Let $T=1..N$ be the annual (low-frequency) index, $t=1..4$ the quarterly index within a natural year and $j=1..M$ the regional (cross-section) index.

Hence, $Y=\{Y_{T,j}: T=1..N; j=1..M\}$ is a NxM matrix comprising the annual regional GDPs that play the role of temporal benchmarks of the system. Aggregation of the regional GDPs generates the GDP at the national level[20].

Variable z is a $nx1$ vector comprising the observed quarterly GDP provided by the QNA, being $n \geq 4N$. This figure is available more timely than the regional data and shares with them the corresponding annual GDP volume index[21]:

[19] That is, incorporating seasonal and calendar effects.
[20] Again, aggregation is performed according to the chain-linking methodology.
[21] For example, taking 2011 as a reference, the QNA released its first estimate of 2010:Q4 on February, 11 while the RA released its first estimate of 2010 on March, 24. Both estimates share the annual figure for 2010 implicitly provided by the QNA by means of temporal aggregation of the four quarters of 2010.

[1] $$Z_T = \frac{1}{4}\sum_{t \in T} z_{t,T}$$

For example, taking 2011 as a reference, the QNA released its first estimate of 2010:Q4 on February, 11 while the RA released its first estimate of 2010 on March, 24. Both estimates share the annual figure for 2010 implicitly provided by the QNA by means of temporal aggregation of the four quarters of 2010.

Finally, *xr* is a *nxM* matrix comprising the observed raw quarterly indicators that operate as high-frequency proxies for the regional aggregates *Y*. As will be explained later, we work with the seasonally and calendar-adjusted indicators (*x*) instead of the raw indicators (*xr*).

Only the indicators *x* are observed at the three dimensions of the system: *T* (annual index), *t* (quarterly index) and *j* (regional index). Therefore, they provide the interpolation basis for *Y* (across the quarterly dimension *t*) and *z* (across the regional dimension *j*). In other words, our objective is to estimate *y* using *x* as interpolators and consistently with both *Y* and *z*.

Table 1 sets out the relationship among the inputs (*Y*, *z* and *x*) and the output (*y*) of the system for a simplified case with two regions (*M=2*) and two years (*T=2*). The first year is complete while the second year is incomplete (i.e., the last two quarters are not available for *x* and *z* and the annual figure for *Y* is not available either).

Table 1. Information set of the model: Quarterly GDP tracker (x), Annual Regional GDP (Y), Quarterly National GDP (z) and Quarterly Regional GDP (y), which is the variable to be estimated.

		Region 1			Region 2			Nation
Year	Quarter	x_1	y_1	Y_1	x_2	y_2	Y_2	z
1	1	$x_{1,1,1}$	$y_{1,1,1}$	$\mathbf{Y_{1,1}}$	$x_{2,1,1}$	$y_{2,1,1}$	$\mathbf{Y_{2,1}}$	$\mathbf{z_{2,1,1}}$
	2	$x_{1,2,1}$	$y_{1,2,1}$		$x_{2,2,1}$	$y_{2,2,1}$		$\mathbf{z_{2,2,1}}$
	3	$x_{1,3,1}$	$y_{1,3,1}$		$x_{2,3,1}$	$y_{2,3,1}$		$\mathbf{z_{2,3,1}}$
	4	$x_{1,4,1}$	$y_{1,4,1}$		$x_{2.4,1}$	$y_{2,4,1}$		$\mathbf{z_{2,4,1}}$
2	1	$x_{1,1,2}$	$y_{1,1,2}$		$x_{2,1,2}$	$y_{2,1,2}$		$\mathbf{z_{2,1,2}}$
	2	$x_{1,2,2}$	$y_{1,2,2}$		$x_{2,2,2}$	$y_{2,2,2}$		$\mathbf{z_{2,2,2}}$
	3							
	4							

Note: bold variables are temporal constraints (Y) or transversal constraints (z).

In this simplified example, we want to estimate the first year's quarterly regional GDPs $y_{j,t,1}$ consistently with their annual counterparts $Y_{j,1}$ and satisfying the transversal constraint that links the regional GDPs with the national GDP $z_{t,1}$ each quarter. The annual constraints do not apply during the second year since $Y_{j,2}$ are not available. Thus the only binding constraint is the transversal constraint.

2.2. Processing Short-Term Indicators

Typically, short-term regional economic indicators are compiled in raw form by the statistical agencies. However, the volume GDP used for short-term monitoring at the national level is calculated in two ways: using raw indicators or using seasonal and calendar-adjusted indicators. Since seasonal and calendar effects could be quite different between indicators and the macroeconomic aggregates, the second procedure for the calculation of the GDP seems more reliable. Usually these GDP figures are referred as seasonal and calendar adjusted.

In order to ensure the homogeneity between both sources of information, regional raw indicators and seasonally adjusted quarterly national GDP, we apply an ARIMA model-based correction that filters out the raw data from seasonal and calendar effects, if they are present. The procedure has been implemented using the TRAMO-SEATS program, see Gómez and Maravall (1996) and Caporello and Maravall (2004). Formally:

[2] $$x_{j,t,T} = V(B,F;\psi_j)\, xr_{j,t,T}$$

where $xr_{j,t,T}$ is the raw short-term indicator[22]; $V()$ is the Wiener-Kolmogorov filter symmetrically defined on the backward and forward operators B and F and ψ_j are the parameters of the filter derived consistently with those of the ARIMA model for $xr_{j,t,T}$, see Gómez and Maravall (1998a, 1998b) for a detailed exposition of the model-based approach used by TRAMO-SEATS.

If the indicators are available at the monthly frequency, seasonal adjustment is performed on the monthly series. The resulting series are temporally aggregated to the quarterly frequency.

We have used TRAMO-SEATS because it is the method used by the Spanish National Statistical Institute (NSI) to adjust GDP from seasonal effects. Of course, the choice of the seasonal adjustment procedure depends on the official method used by the NSI to produce the GDP figures. In countries where X12-ARIMA is the official procedure this should be also the choice to seasonally adjust the short-term indicators.

In practice, several short-term economic indicators are used to monitor and estimate regional GDPs. These indicators are individually processed according to [2] and then linearly combined, producing a composite indicator that will be used as the high-frequency proxy for regional GDPs. As we shall explain in the third section, we use factor analysis to estimate a synthetic indicator for each region because it provides an objective and simple way to combine the available indicators.

2.3. Initial quarterly regional GDP estimation

Preliminary estimates of quarterly GDP at the regional level are compiled using benchmarking techniques, see Di Fonzo (1987, 2002) and Proietti (2006) for an in-depth exposition. These techniques play an important role in the compilation practices of Quarterly National Accounts around the world, see Eurostat (1998) and Bloem et al. (2001).

We have considered several benchmarking procedures to derive the preliminary GDP estimates: Chow-Lin (1971), Fernández (1981), Santos Silva-Cardoso (2001) and Proietti (2006). All of them hinge

[22] If calendar effects (e.g., Easter effect and trading day effect) are present, a preliminary correction is also performed. Without loss of generality, we will continue to call the possibly calendar-corrected data as raw data.

around a dynamic linear model that links the (observable) high-frequency indicator with the (unobservable) regional GDP[23]:

[3] $$y_t = \phi y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + u_t$$

The innovation u follows an AR(1) process:

[4] $$u_t = \rho u_{t-1} + a_t$$

Finally, the random shock that drives the innovation u is Gaussian white noise process:

[5] $$a_t \sim iid\ N(0, v_a)$$

The model includes a temporal constraint that makes y quantitatively consistent with its annual counterpart Y:

[6] $$Y = Cy$$

C is the temporal aggregation-extrapolation matrix defined as:

[7] $$C = (I_N \otimes c \mid O_{N,n-sN})$$

Where N is the number of low-frequency observations, $\otimes$ stands for the Kronecker product, c is a row vector of size s which defines the type of temporal aggregation and s is the number of high frequency data points for each low frequency data point. If $c=[1,1,\dots,1]$ we would be in the case of the temporal aggregation of a flow, if $c= [1/s,1/s,\dots,1/s]$ in the case of the average of an index and, if $c=[0,0,\dots,1]$, an interpolation would be obtained. In our case, $s=4$.

Extrapolation arises when $n>sN$. In this case, the problem can easily be solved by simply extending the temporal aggregation matrix by considering new columns of zeroes which do not distort the temporal aggregation relationship and that do not pose any difficulty to the inclusion of the last $n-sN$ data points of the indicators in the estimation process of y.

The different benchmarking methods depend on the values of the parameters in [3] and [4] according to table 2:

[23] To keep the notation simple we have omitted the regional index j.

Table 2: Benchmarking methods

Method	Parameter ϕ	β_1	ρ
Chow-Lin	0	0	(0,1)
Fernández	0	0	1
Santos Silva-Cardos	(0,1)	0	0
Proietti	(0,1)	≠0	0

The methods of Chow-Lin and Fernández places the dynamics in the innovation, that may follows a stationary AR(1) process (Chow-Lin) or a non-stationary I(1), random walk process (Fernández)[24]. On the other hand, the methods of Santos Silva-Cardoso and Proietti places the dynamics in the variables *y* and *x*, treating the innovation as a purely random shock[25].

The estimation of the parameters and the unobserved time series *y* is performed by maximizing the implied log-likelihood profile of the low-frequency model[26]. This optimization is performed by means of a grid search on the stationary domain of ϕ or ρ, and pinning down the values of β and σ that maximizes the log-likelihood function conditioned on the selected value for ϕ or ρ, see Bournay and Laroque (1979) for an in-depth exposition. The computations have been carried on using the functions written in Matlab by Abad and Quilis (2005).

2.4. Balancing in a chain-linking setting

The estimates derived in the previous step do not verify the transversal constraint that should relate them to the national quarterly GDP, satisfying the same type of relationship that links annual regional GDPs and annual national GDP. We solve the problem applying a multivariate balancing procedure, in particular a multivariate extension of the Denton (1971) method. This extension can be expressed in matrix form, as in Di Fonzo (1990) and Di Fonzo and Marini (2003), as well as in state space form, see Proietti (2011). In this paper we have adopted

[24] Litterman (1983) proposes a methodology affine to those of Chow-Lin and Fernández. However, the empirical and Monte Carlo evidence show that its performance is sometimes disappointing. This fact is due to the flatness of the implied likelihood profile and, therefore, the corresponding observational equivalence in a wide range of values for its dynamical parameter, see Proietti (2006).
The low-frequency model incorporates the temporal aggregation constraints [2.6] and [2.7].

[25] Gregoir (1994) and Salazar et al. (1994) also propose methods in which the dynamics of *y* and *x* play an explicit role.

[26] The low-frequency model incorporates the temporal aggregation constraints [2.6] and [2.7].

the former approach, using the functions written in Matlab by Abad and Quilis (2005).

This balancing method depends on the formulation of additive constraints. However, volume indexes compiled according to the chain-linking methodology are non-additive, see Bloem et al. (2001) and Abad et al. (2007). Fortunately, we can transform the chain-linked measures in order to write them in an additive form and then use the powerful machinery of balancing procedures to ensure transversal and temporal consistency. Finally, we can express the results in the initial chain-linked format by reversing the transformation.

The constraint that links regional and national quarterly volume GDP is:

[8] $$z_{t,T} = \left(\sum_j W_{j,T-1} \frac{y_{j,t,T}}{Y_{j,T-1}} \right) Z_{T-1}$$

Where $z_{t,T}$ is the national quarterly volume GDP, $W_{j,T-1}$ is the weight of region j in year $T-1$ and $y_{j,t,T}$ is the quarterly volume GDP of the j-th region[27]. Finally, Z_T and $Y_{j,T}$ are the annual counterparts $z_{t,T}$ of and $y_{j,t,T}$.

After some algebraic manipulations, we can express the constraint in additive form:

[9] $$\underbrace{\frac{z_{t,T}}{Z_{T-1}}}_{r_{t,T}} = \sum_j \underbrace{W_{j,T-1} \frac{y_{j,t,T}}{Y_{j,T-1}}}_{wr_{j,t,T}} = \sum_j wr_{j,t,T}$$

In [9] the relationship between the national ratio $r_{t,T}$ and the weighted regional ratios $wr_{j,t,T}$ is additive.

Plugging the initial estimates derived according to [3]-[7] into [9] we obtain the preliminary, unbalanced estimates:

[10] $$wr^*_{j,t,T} = W_{j,T-1} \frac{\hat{y}_{j,t,T}}{Y_{j,T-1}}$$

The balanced and temporally consistent time series $wr^{**}_{j,t,T}$ are the output from the following constrained quadratic optimization program:

[11] $$\underset{wr^*}{MIN} \quad (wr^{**} - wr^*)' D' D (wr^{**} - wr^*) \qquad s.t. \quad H\, wr^{**} = R_e$$

[27] Weights are computed using GDPs valued at current prices, see Abad et al. (2007) for a complete derivation.

being:

$$H = \begin{bmatrix} 1_M' \otimes I_n \\ I_M \otimes C \end{bmatrix} \quad \text{and} \quad R_e = \begin{bmatrix} z \\ WR \end{bmatrix}$$

Where 1_M is a column vector of ones and WR is the annual counterpart of the weighted regional ratios written in matrix form.

In the program [11] the objective function reflects the volatility of the discrepancies between the quarter-to-quarter growth rates of the balanced series and those of the unbalanced ones. After some mathematical manipulation, an explicit expression can be derived:

[12] $$wr^{**} = wr^{*} + (D'D)^{-1}H'\left[H(D'D)^{-1}H'\right]^{-1}(R_e - Hwr^{*})$$

The interpretation of equation [12] is straightforward: the quarterly balanced series are the result of adding up a correction factor to the unbalanced series. This correction factor originates from the distribution of the discrepancy between the preliminary unbalanced estimates and the constraint series R_e.

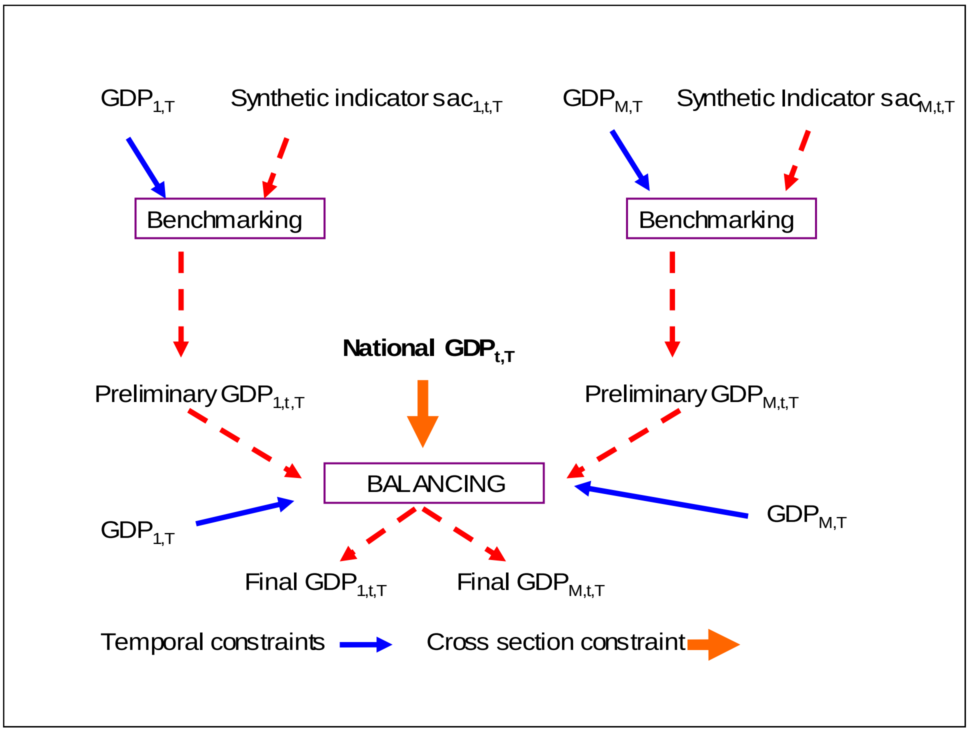
Once we have obtained the consistent weighted ratios, we can reverse the transformation [9] to derive the final estimates of the quarterly regional GDP in volume terms:

[13] $$y^{**}_{j,t,T} = wr^{**}_{j,t,T}\frac{Y_{j,T-1}}{W_{j,T-1}}$$

In this way, the estimates of quarterly GDP derived in the previous equation are quantitatively consistent in their time dimension (taking as benchmark their annual regional counterparts) and in their cross-section dimension (generating the GDP provided by the QNA by regional aggregation). We should also emphasize that the consistency extends to the methodological dimension too, since the chain-linking procedures in current use by the National Accounts have been properly taken into account. Finally, using time series methods to project the basic short-term indicators we can derive nowcasts (or flash estimates) of regional quarterly GDP in a timely way.

As a summary, the figure 1 presents a picture of the complete procedure. The diagram emphasizes the binding constraints and the homogeneous processing of information at the regional level. Note that the box labeled "balancing" embeds the de-chaining and re-chaining steps required to circumvent the non-additive features of the chain-linked volume indexes.

Figure 1: Schedule of steps 2 (Benchmarking) and 3 (Balancing)



Note: In bold national variables. Quarterly index t goes from 1 to 4; annual index T goes from 1 to N and regional index j goes from 1 to M.

2.5. Comparison with other approaches

Table 3 compares our methodology with related approaches along six dimensions: high-frequency model, role of constraints (temporal and transversal), explicit consideration of chain-linking, mixing data frequencies (e.g., annual and quarterly data) and computational approach.

Table 3: Comparison with other methodological approaches

	Di Fonzo (1990)	Di Fonzo & Marini (2003)	Proietti (2011)	Ours
High-frequency model	Static model + I(1) innovations	Unspecified	Static model + I(1) or I(2) innovations	Static or dynamic model + AR(1)/I(1) innovations
Temporal constraints	Yes	Yes	Yes	Yes
Transversal constraints	Yes	Yes	Yes	Yes
Chain-linking constraints	No	No	No	Yes
Mixing frequencies	Yes	Yes	Yes	Yes
Computational approach	Matrix oriented	Matrix oriented	State space	Matrix oriented

Di Fonzo (1990) presents a methodology closely related to ours. We have expanded his approach to cope with the issue of chain-linking and focus the results to flash estimation and benchmarking. Di Fonzo and Marini (2005) may be considered as a variant of Di Fonzo (1990) in which balancing plays also a critical role.

In addition, Proietti (2011) is also a close reference. He generalizes the Di Fonzo (1990) model to take into account integrated random walk innovations and deals with the issue of non-additivity posed by the chain-linking volume indexes implicitly, arranging the measurement equations to consider a statistical discrepancy. His computational approach relies on Kalman filtering of the state space representation of the model. In contrast, our approach is matrix-oriented, following Di Fonzo (1990).

Spatial correlation plays an important role due to the fact that short-term regional indicators are closely related and the estimation of regional GDPs at the quarterly frequency depends also on the national quarterly GDP (step 3: balancing).

However, our procedure is oriented towards the temporal disaggregation of regional aggregates, preserving at the same time the cross-section consistency with the national quarterly GDP rather than the spatial disaggregation of national totals taking as interpolands the information contained in the regional indicators. The last approach is used by the so-called spatial Chow-Lin procedure that adapts the Chow-Lin method to the spatial nature of the data and may use to distribute a grand total into its spatial components at a give point in time, see Vidoli and Mazziotta (2012) and Polasek and Sellner (2010) among others. This procedure is very flexible an can be used to disaggregate national, regional or provincial totals into its spatial components (regions, provinces or areas) but does not consider explicitly the temporal constraints that are the hallmark of the National Accounts, both Regional and Quarterly, and of our procedure.

Finally, we want to emphasize that our approach is focused on the estimation of (unobservable) quarterly regional GDPs rather than on the forecasting of the (observable) annual regional GDPs. To ensure the comparability and homogeneity of those estimates our procedure hinges around the temporal and cross-section consistency in the same way as they are implemented in the National Accounts. The reliance on mimicking the National Accounts limits the selection of indicators as well as the modeling approach. Lehmann and Wolhrabe (2012) present a detailed forecasting exercise at the regional level using a variety of models and a large set of indicators with different spatial coverage.

3. Case study: a system of flash regional quarterly gdp estimates for Spain

In this section we present the main results of a system of regional quarterly GDP flash estimates for the Spanish economy, following the modeling approach previously outlined.

3.1. Selection of monthly regional indicators

This subsection details the indicators that have been selected for model estimation. The selection process was carried out under the premise that indicators should be available timely and should provide a synthetic measure of each of the regional economies.

The criteria for the choice of these variables is to consider the regional counterpart of all the indicators used in the compilation of the Quarterly National Accounts, see Álvarez (1989), Martínez and Melis (1989), INE (1993) and Álvarez (2005). To fulfill this goal, we have prepared a set of monthly regional indicators that provides a fairly comprehensive basis for analyzing and monitoring GDP at the regional level. This set offers a high-frequency approximation to the behavior of the main macroeconomic aggregates: gross added value (industry, construction and services), consumption, external trade and employment. The selected indicators, with a brief description of them, are:

- <u>IPI: Index of Industrial Production.</u>
    - Units: Index number.
    - Source: National Statistical Institute (*Instituto Nacional de Estadística, INE*).
    - Starting date: 1995.01.
    - Retropolation: combining data from 1990 base (1995.01-2002.01) and base 2005 base (2002.01-2011.12), using the oldest period-on-period rates of growth to retropolate the newest base.

- <u>LIC: Municipal construction licenses. Total area to build.</u>
    - Units: squared meters.
    - Source: Ministry of Public Works (*Ministerio de Fomento*).
    - Starting date: 1995.01.
    - Back-calculation: Data for Basque Country (País Vasco) during the period 1995.01-1997.12 have been back-calculated using the average of the remaining regions as indicator. Some specific missing data (Basque Country -2008.08- and Navarra -2009.12-) have been interpolated using the program TRAMO.

- <u>PER: Overnight stays in hotel establishments.</u>
    - Units: Number of overnight stays.
    - Source: National Statistical Institute (*Instituto Nacional de Estadística, INE*).
    - Starting date: 1995.01.
    - Back-calculation: The series have been homogeneized since 1998.12 by means of univariate intervention analysis in order to correct from the methodological change introduced in 1999.01.

- <u>IAS: Services sector activity indicator.</u>
    - Units: Index number. Valuation at current prices.
    - Source: National Statistical Institute (*Instituto Nacional de Estadística, INE*).
    - Starting date: 2005.01.
    - Deflated using the Consumer Price Index (CPI) for services (house renting excluded).
    - Missing data since 1995.01 have been estimated using the static factor derived from the indicators that start in 1995.01 as regressor.

- <u>ICM: Retail sales index.</u>
    - Units: Index number. Valuation at current prices, gas stations excluded.
    - Source: National Statistical Institute (*Instituto Nacional de Estadística, INE*).
    - Starting date: 2001.01.
    - Deflated using the Consumer Price Index (CPI) for services (house renting excluded).
    - Missing data since 1995.01 have been estimated using the static factor derived from the indicators that start in 1995.01 as regressor.

- <u>MAT: Car registrations.</u>
    - Units: Registrations.
    - Source: Traffic Department (*Dirección General de Tráfico, Ministerio del Interior)*.
    - Starting date: 1995.01.

- <u>EXP: Exports of goods.</u>
    - Units: Euros, valuation at current prices.
    - Source: External trade statistics, Ministry of Economy and Competitiveness.
    - Starting date: 1995.01.
    - Deflated using the national exports unit value index.

- <u>IMP: Imports of goods.</u>
    - Units: Euros, valuation at current prices.
    - Source: External trade statistics, Ministry of Economy and Competitiveness.
    - Starting date: 1995.01.
    - Deflated using the national imports unit value index.

- <u>AFI: Social security system: registered workers.</u>
    - Units: persons.
    - Source: Labor department (*Ministerio de Empleo y Seguridad Social*).
    - Starting date: 1995.01.

The short-term indicators, in order to be consistent with the QNA data, as mentioned in section 2, have been seasonally and calendar adjusted.

3.2. Regional Synthetic Indexes

To combine the information contained in the individual monthly indicators in an efficient and operative way, we have calculated a synthetic indicator for each region. In order to convey an idea of the correlation between the individual indicators and the estimated synthetic indicator (common factor), Table 4 shows the loading vectors, estimated by means of principal components factor analysis.

Table 4. Regional synthetic indexes: loading structure

	AFI	EXP	IMP	IPI	LIC	MAT	PER	ICM	IAS
Andalucía (AND)	0.54	0.28	0.05	0.45	0.01	0.77	0.21	0.73	0.90
Aragón (ARA)	0.31	0.63	0.29	0.79	0.04	0.63	0.01	0.51	0.65
Asturias (AST)	0.42	0.41	0.25	0.31	0.17	0.63	0.25	0.74	0.87
Baleares (BAL)	0.29	0.24	0.19	0.33	0.09	0.74	0.07	0.37	0.78
Canarias (CAN)	0.63	0.01	0.01	0.50	0.10	0.54	0.23	0.78	0.84
Cantabria (CANT)	0.35	0.56	0.36	0.57	0.07	0.56	0.06	0.06	0.74
Castilla La Mancha (CLM)	0.50	0.32	0.31	0.57	0.39	0.48	0.03	0.69	0.88
Castilla León (CYL)	0.31	0.55	0.50	0.61	0.01	0.68	0.01	0.08	0.82
Cataluña (CAT)	0.38	0.62	0.45	0.77	0.12	0.69	0.01	0.68	0.90
Extremadura (EXT)	0.41	0.30	0.14	0.14	0.30	0.75	0.01	0.42	0.76
Galicia (GAL)	0.32	0.62	0.24	0.45	0.01	0.70	0.23	0.66	0.89
Madrid (MAD)	0.41	0.43	0.32	0.62	0.01	0.48	0.28	0.75	0.69
Murcia (MUR)	0.45	0.24	0.01	0.37	0.04	0.76	0.19	0.79	0.87
Navarra (NAV)	0.35	0.61	0.54	0.72	0.01	0.32	0.21	0.09	0.64
País Vasco (PV)	0.14	0.58	0.49	0.76	0.08	0.57	0.01	0.62	0.86
La Rioja (RIO)	0.18	0.66	0.44	0.54	0.25	0.43	0.19	0.67	0.88
Valencia (VAL)	0.43	0.53	0.25	0.75	0.06	0.64	0.01	0.72	0.91

We have to note how loadings vary depending on the predominant activities in which each region specializes. Since two of the indicators (IAS and ICM) have been completed using the common factor estimated from the remaining indicators, their correlations with the common factor estimated with the balanced panel are overestimated to a certain extent. This fact complicates the exact quantification of their role. However, their economic relevance (IAS for the whole services sector and ICM for private consumption) recommends their inclusion in the estimation of the regional GDP trackers.

The corresponding monthly regional synthetic indicators are temporally aggregated to the quarterly frequency.

3.3. National Accounts Data: Regional Accounts (RA) and Quarterly National Accounts (QNA)

Apart from the monthly regional indicators mentioned above, regional annual GDPs in chained-volume indices are provided by the Regional Accounts (RA) according to ESA-95 conventions and they are available for the time span 1995-2011. The cross-section dimension includes 17 regions (Comunidades Autónomas) plus two autonomous cities that will be jointly considered, giving M=18, a NUTS-2 regional breakdown according to Eurostat's classification.

Finally, the quarterly transversal constraint is the Spanish quarterly volume GDP provided by the QNA. This variable is compiled seasonally and calendar adjusted.

3.4. Empirical Results

Using the abovementioned data for the period 1995.01 – 2012.12 we can compare now the final results obtained using the different benchmarking techniques mentioned in section two (Fernandez, Chow-Lin, Santos Silva-Cardoso (SSC for brevity), Proportional Denton and Proietti) in order to select the most appropriate in terms of correlation and volatility.

Table 5 shows the summary results obtained with the different methods. Starting with the composite indicators derived by factor analysis for each region in the first stage, we apply different benchmarking methods and compare the different results obtained after final balancing. In order to summarize the results, we present the average correlation of the quarterly growth rate of GDP finally estimated by region with the initial composite indicator and the average standard deviation of the quarterly growth rate of GDP finally estimated by region.

Table 5. Comparison of methods (quarterly rates of growth)

	Fernandez	Chow-Lin	SSC	Denton Prop.	Proietti
Average Standard Deviation	0.821	0.858	0.731	0.843	0.744
Average Correlation	0.767	0.776	0.683	0.670	0.736

This table shows that there seems to be a trade-off relationship between correlation and volatility (except in proportional Denton, which shows high volatility and low correlation). The Fernández and Chow-Lin methods are closest to the evolution of the indicator, without assuming a more complex structure in the errors, as is the case with SSC and Proietti.

Based on these results, we have decided to choose either the Fernández or the Chow-Lin method, because we think it is more important to be as faithful as possible to the information contained in the indicators, despite having higher volatility. Additionally, this is the method currently suggested for the compilation of the Spanish QNA (see Quilis 2005).

Regarding the distinction between the Fernández or Chow-Lin method, the results of the exercise show an innovational parameter with Chow-Lin close to 1 (approximately 0.98-0.99 in most cases), so under this situation both methods are practically equivalent.

With the aim of analyzing both the duration and the date of entry and exit of the recession in each region, Table 6 presents the evolution of the estimated year-on-year rates of growth in the quarterly frequency; for the exercise performed with the Chow-Lin method, for example:

Table 6. Dating recession in quarterly GDP (year-on-year rates of growth)

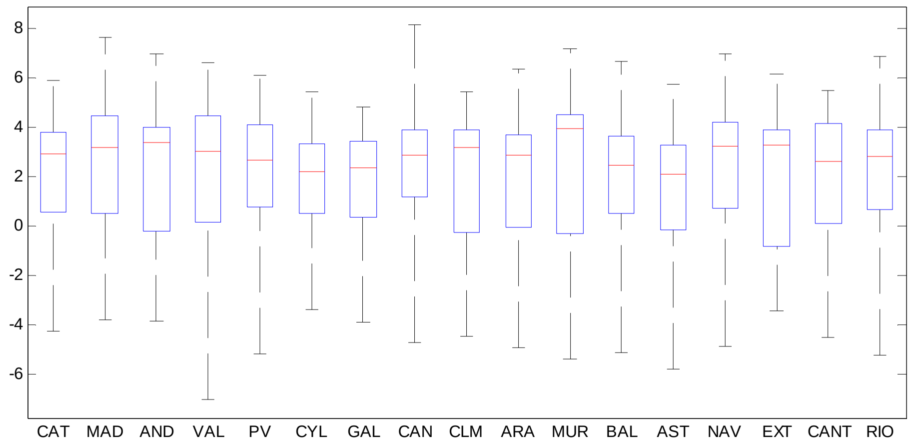
	2008				2009				2010				2011				2012			
	T I	T II	T III	T IV	T I	T II	T III	T IV	T I	T II	T III	T IV	T I	T II	T III	T IV	T I	T II	T III	T IV
Spain	2,7	1,9	0,3	-1,4	-3,4	-4,4	-4,0	-3,1	-1,5	-0,2	0,0	0,4	0,5	0,5	0,6	0,0	-0,7	-1,4	-1,6	-1,5
Andalucía	2,9	1,7	-0,4	-1,8	-2,8	-3,8	-3,8	-3,4	-2,3	-1,0	-0,5	-0,1	-0,2	-0,4	-0,1	0,2	-0,2	-0,7	-1,4	-1,7
Aragón	3,5	2,2	0,7	-2,9	-4,4	-4,9	-4,8	-1,8	-1,3	-1,5	-0,5	0,1	-0,1	1,0	0,8	-1,5	-0,7	-2,1	-2,0	-0,2
Asturias	2,9	2,3	0,2	-1,0	-3,9	-5,6	-5,8	-4,6	-2,0	-0,7	0,0	0,3	0,1	0,4	0,3	-0,6	-1,3	-1,9	-2,0	-2,1
Baleares	2,6	2,3	0,7	-0,5	-2,1	-5,1	-4,3	-3,8	-2,5	-0,9	-0,6	-0,8	-0,6	2,2	2,5	2,0	1,3	-1,1	-0,9	0,1
Canarias	1,6	1,4	-0,3	-1,5	-3,0	-4,5	-4,7	-4,5	-3,1	-2,0	1,1	1,4	2,3	2,8	1,2	1,1	-0,3	-1,0	-2,0	-0,6
Cantabria	2,1	1,9	0,7	-0,5	-2,1	-3,7	-4,5	-4,1	-2,4	-1,3	-1,2	-0,6	-0,1	0,2	1,2	0,7	0,0	-0,6	-0,8	-0,1
Castilla La Mancha	3,8	2,5	0,6	-0,8	-2,8	-3,5	-4,4	-4,1	-3,1	-2,4	-0,4	-0,2	-0,2	0,4	-0,7	-0,5	-1,3	-1,7	-1,3	-1,2
Castilla León	3,2	2,1	0,5	-2,3	-3,2	-3,4	-3,3	-1,4	0,2	1,6	0,2	0,5	0,9	0,3	2,0	0,8	-0,3	-1,6	-1,9	-2,1
Cataluña	1,9	0,9	-0,5	-1,5	-3,7	-4,2	-3,9	-3,1	-1,0	0,2	0,7	1,0	0,6	0,5	0,9	0,0	-0,1	-0,6	-0,9	-0,6
Extremadura	4,4	3,6	0,5	-1,1	-3,1	-3,4	-3,0	-2,0	-0,8	0,0	-2,3	-1,3	-1,5	-1,1	1,4	-0,7	-1,0	-2,7	-2,9	-2,7
Galicia	3,6	2,1	1,1	-0,1	-2,2	-3,8	-3,8	-3,9	-1,9	0,5	0,3	0,7	0,7	0,3	-0,3	-0,8	-0,9	-1,5	-1,5	-1,9
Madrid	2,4	2,0	0,6	-1,1	-2,7	-3,8	-2,4	-1,8	-0,5	0,4	-0,1	0,0	0,8	0,6	0,8	0,0	-1,7	-2,5	-2,8	-2,9
Murcia	3,7	2,8	1,1	-1,2	-3,6	-5,4	-4,6	-4,7	-2,7	-0,6	-0,3	0,0	-0,5	-0,3	-0,3	-0,1	-0,3	-0,7	-1,7	-2,4
Navarra	2,8	3,7	1,0	0,0	-3,8	-4,9	-3,3	-2,4	0,1	0,3	0,4	0,8	1,5	2,0	1,0	0,5	-1,3	-2,4	-2,0	-1,5
País Vasco	2,6	2,5	1,1	-0,8	-3,4	-5,1	-4,7	-3,2	-0,8	0,9	1,2	1,4	1,7	1,4	0,8	0,2	-1,0	-1,6	-1,3	-1,0
La Rioja	3,8	2,3	0,7	-1,0	-3,8	-4,5	-5,2	-5,2	-2,8	-2,7	-1,8	-0,5	-0,7	0,8	1,5	1,3	0,6	0,0	-0,4	-1,0
Valencia	3,4	1,9	0,5	-2,7	-6,1	-7,0	-6,0	-4,4	-2,1	-0,2	-0,6	0,1	0,7	0,2	0,2	-0,8	-1,1	-1,3	-1,0	-1,1

Negative rates
Minimum rate
Positive rates

The table shows how the crisis has affected regions unevenly. For example, we can place the bulk of the recession between the fourth quarter of 2008 and the first quarter of 2010. Most of the regions fell into recession at the same time but not all of them left it simultaneously; this is the case of regions such as Andalucía, where the contractionary period is particularly long. We can see that many regions fall back into recession after the first quarter of 2012.

In relation to the variance of these results, the following figure shows the different box plot of the year-on-year rates of growth in the quarterly frequency for the different regions:

Figure 1. Box plot: annual growth rates by region in quarterly frequency, sorted according to weight on Spanish GDP

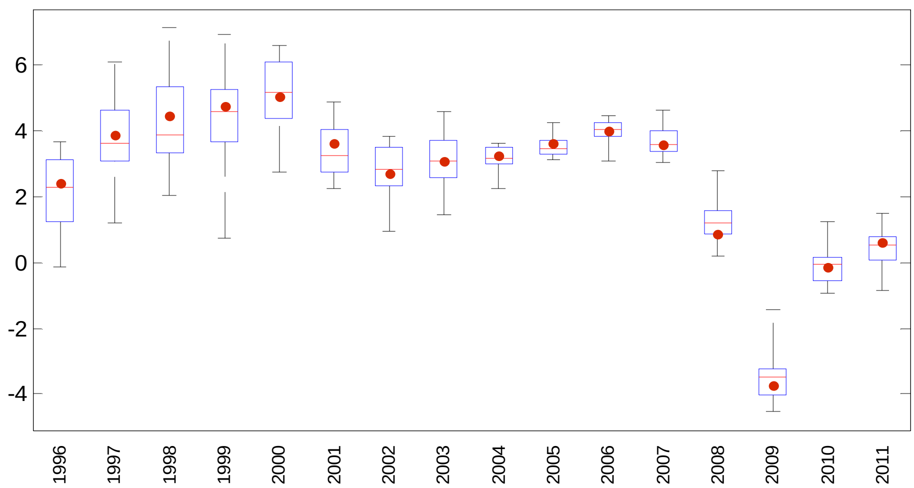


Note: Central line stands for median values, the box represents 50% of the central part of the data and the whiskers are the minimum and maximum of the data.

We observe a greater presence of outliers in periods of recession than in periods of expansion. This is partly due to the longer duration of the latter, rendering the median less representative for recessionary quarters. At the same time, the highest rate of variability is not linked to the larger size (GDP weight) of the region (see Appendix 1).

The temporal dimension of the data allows us to appreciate a reduction in volatility after 2003, although this is a property inherited from the annual data published by the RA (see Figure 2):

Figure 2. Box plot: year-on-year rates of growth (annual data)



Note: Red dot is the aggregate data for Spain

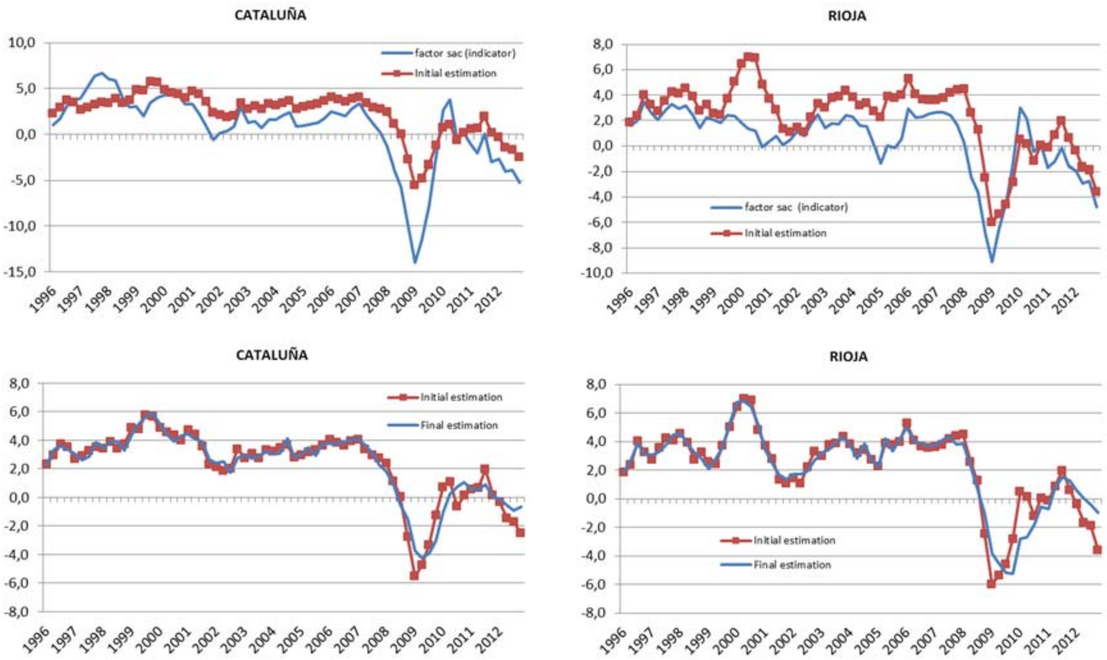
Finally, in order to clarify the importance of the balancing procedure on the final estimate, an exercise on two regions has been carried out: one with a large size (Cataluña) and other with a small size (La Rioja). This exercise is trying to reveal whether a small region can seriously change its initial estimate of quarterly GDP with the final balancing.

Initial or preliminary estimates do not take into account the information contained in the national quarterly GDP. Those initial estimates are modified to be consistent each quarter with the quarterly national GDP, reflecting the fact that the national data is the transversal aggregation of the regions.

The difference between the initial and the final estimates reflects the balancing procedure that ensures the transversal constraint and preserves, for each region, the temporal consistency with the Regional Accounts.

Figure 3 shows, firstly, the initial quarterly regional GDP estimation (distribution of annual regional GDP according to the indicator) against the evolution of the indicator and, secondly, the initial quarterly estimation against the final quarterly GDP.

Figure 3. Initial quarterly estimation vs. final balanced estimation. Small vs. large regions, year-on-year rates of growth



It is easy to see how the first step of estimating quarterly GDP depending on the evolution of the indicator is even more crucial to the subsequent balancing procedure. Furthermore, the small region does not have its initial estimate changed substantially compared with that of the large region. This fact shows the robustness of the balancing procedure, revealing that the variability in the final estimate is driven by the variability of the selected indicator.

4. Conclusions

In this article we have presented a feasible way to add a regional dimension to the short-term macroeconomic analysis, satisfying the temporal and cross-section constraints imposed by the National Accounts. Our procedure generates results that are comparable across regions, are based on meaningful short-term information, and may be updated at the same time as the GDP flash national estimates, providing a solid basis for specific regional estimates.

In summary, the major outcomes of the model are:

- It solves the lack of quarterly GDP at the regional level, providing estimates consistent with the official available data published by the National Accounts (RA and QNA). These estimates are a stand-alone product that may be used as input in regional econometric models.

- It provides a regional breakdown of the early estimates of the quarterly national volume GDP that may be released simultaneously, providing flash estimates at the regional level.

There are several promising lines of research that may broaden the scope of the article. The use of dynamic-factor models to estimate the regional high-frequency synthetic indexes may provide a more complete description of the economic conditions at the regional level.

The modeling approach can be extended easily to accommodate several types of extrapolations. For example, the transversal benchmark of the model (the national quarterly GDP) may be an official release made by the National Statistical Institute or a forecast made by an analyst (e.g. the research department of an investment bank). In the latter case, we can combine these forecasts with the projected path for the underlying short-term quarterly regional indicators to generate the corresponding regional quarterly GDPs. The resulting conditional extrapolations can be used to assess the expected cyclical position of each region with respect to the nation.

Finally, the estimated regional quarterly GDPs can be used to analyze issues related to the synchronicity of the regional business cycles as well as their pattern of comovements.

Appendix 1: Main features of the Spanish regions (2011)

	Population (thousand)	Population weight	GDP weight	Employment weight
Andalucía	8,270.5	17.9%	13.5%	14.7%
Aragón	1,315.5	2.9%	3.2%	3.1%
Asturias	1,054.5	2.3%	2.1%	2.1%
Baleares	1,092.5	2.4%	2.5%	2.6%
Canarias	2,107.0	4.6%	3.9%	4.1%
Cantabria	578.3	1.3%	1.2%	1.2%
Castilla La Mancha	2,045.4	4.4%	3.5%	3.9%
Castilla León	2,483.8	5.4%	5.3%	5.3%
Cataluña	7,303.1	15.8%	18.6%	17.8%
Extremadura	1,083.1	2.3%	1.6%	1.9%
Galicia	2,732.0	5.9%	5.3%	5.7%
Madrid	6,371.6	13.8%	18.0%	16.8%
Murcia	1,471.4	3.2%	2.6%	3.0%
Navarra	622.8	1.4%	1.7%	1.6%
País Vasco	2,127.9	4.6%	6.2%	5.3%
La Rioja	312.7	0.7%	0.8%	0.7%
Valencia	5,001.2	10.8%	9.5%	9.8%
Ceuta y Melilla	151.7	0.3%	0.3%	0.3%
Spain	46,125.0	100.0%	100.0%	100.0%

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CAPÍTULO 4: Integrated model of short-term forecasting of the Spanish economy (MIPred model)

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Abstract

This paper presents a methodology for predicting in real-time GDP and its demand components simultaneously. The model consists of a set of dynamic factor models for both GDP and its demand components, plus a balancing procedure to ensure the transversal consistency of these forecasts, thus providing a consistent set of estimates based on the statistically most useful indicators about current economic activity and demand developments. The methodology is applied to the Spanish economy, presenting real-time quarterly estimates of GDP and its demand components.

Keywords:

Dynamic Factor Models, Short Term Economic Analysis, Spanish Economy, Kalman Filter, Forecasting, Nowcasting, National Accounts, Balancing.

JEL Codes:

C22, C53, C82, E27

1. Introduction

Real time forecasts of GDP are very much discussed in the recent literature. Advances in information technology have made available to the researchers a great amount of information with unprecedented update frequency. Therefore, most central banks or international institutions which are in charge of monitoring and analysing business cycle developments, have estimated models in order to update at high frequency the assessment of business cycle conditions. Recent examples include Angelini et al. (2008) or Camacho and Perez Quirós (2010) for the Euro area, Aruoba et al. (2009), Giannone et al. (2008) or Higgings (2014) for the US, Liu et al. (2010) for Latin America, Barhoumi et al. (2008) for France, Nunes (2005) for Portugal, etc.

For the case of Spain, three models have already been published. Camacho and Pérez Quirós (2008) constructed a small scale factor model for the Bank of Spain (Spain-Sting). Cuevas and Quilis (2011) proposed a large scale model for the Ministry of Economy (FASE) and Camacho and Domenech (2011) constructed another small scale model for BBVA (MICA), where they pay special attention to several financial variables available to BBVA.

The Spanish Independent Fiscal Authority (AIReF) in the exercise of its mandate, is in charge of analysing the reliability of the government macroeconomic and fiscal projections. The key variables that the government has to forecast when preparing macroeconomic and fiscal projections are GDP and its components. The government projects the main macro variables with a time horizon of one to four years ahead, depending on the exercise that has to undertake. Obviously, all the projections are based on short term forecasts. If the current and following quarters are accurately forecasted, the one-year ahead forecast will be reliable and the forecasts for further years ahead will be more precise.

It is well established in the literature that dynamic factor models that exploit the information content in the join dynamics of the macro variable and related timely indicators are the best tools for short term forecasting, as shown in the recent surveys of Banbura et al. (2013) or Camacho et al. (2014). Therefore, the AIReF, in line with has been done by other institutions, relies on its own model for analysing the implications of current conditions of the economy for budgetary stability and financial sustainability.

Obviously, our proposed model cannot ignore previous attempts made to model the Spanish economy data. There is some overlap with previous models, although there are some definitely distinct characteristics, which make our model different with respect to the previous specifications.

The main distinctive feature of our approach is that we forecast on a real time basis not only GDP, but also its complete breakdown from the expenditure side. We have specific models to forecast private consumption, public consumption, investment in capital goods, investment in construction, exports and imports. We integrate all of them in one consistent set of forecasts for all the demand components of GDP by using the balancing technique developed in van der Ploeg (1982, 1985). The name of the model, MIPred makes reference to that integration, (Modelo Integrado de Predicción in Spanish, Integrated Prediction Model in English)

To our knowledge, this is the first integrated methodology to forecast in real time all the variables that define the core of the macroeconomic scenario (GDP and its demand-side components), not only for the case of Spain but for any other country. All the automatized methods developed in the literature forecast only GDP or, additionally, the variables included as indicators in the model.

A second distinctive feature is that, for most of the variables forecasted in the model, and, specially for GDP, we only use information freely available to the general public. We do not rely on any confidential series or any other series whose information is restricted to those who pay a fee. Therefore, the results of the model are fully replicable by any researcher and the forecasts are completely transparent and easy to interpret.

Finally, a third distinctive feature is that the selection of indicators has been made using the proposed methodology of Camacho and Perez Quirós (2010). We start from a very parsimonious specification, in line with Stock and Watson (1991), and we only extend the model if the variance of GDP explained by the common factor increases. The variables included in the model are selected following the order of putting in first the one contributing most to increase the variance of the factor. We stop the process of extending the model when any additional variable biases the factor toward sectors whose indicators are correlated among themselves, following idiosyncratic components, but which do not have any additional explanatory power over GDP movements. Details of the bias-induced problem can be found in Alvarez et al. (2012).

The paper is structured as follows. Section 2 reviews the indicators that have been selected for each macro aggregate and the preliminary processing they have gone through. The econometric methodology is explained in section 3, where we discuss the detailed structure of the dynamic factor model, how we have dealt with missing observations and the balancing procedure used to ensure the transversal consistency of GDP forecasts with the independent forecasts of its demand components. Section 4 presents the output of the model and section 5 concludes.

2. Data

2.1 Selection of indicators

The selection process was carried out under the premise that the indicators should be available timely and should provide a meaningful economic signal of the demand components of the national economy. The estimation sample covers from 1990.Q1 until the last observation available.

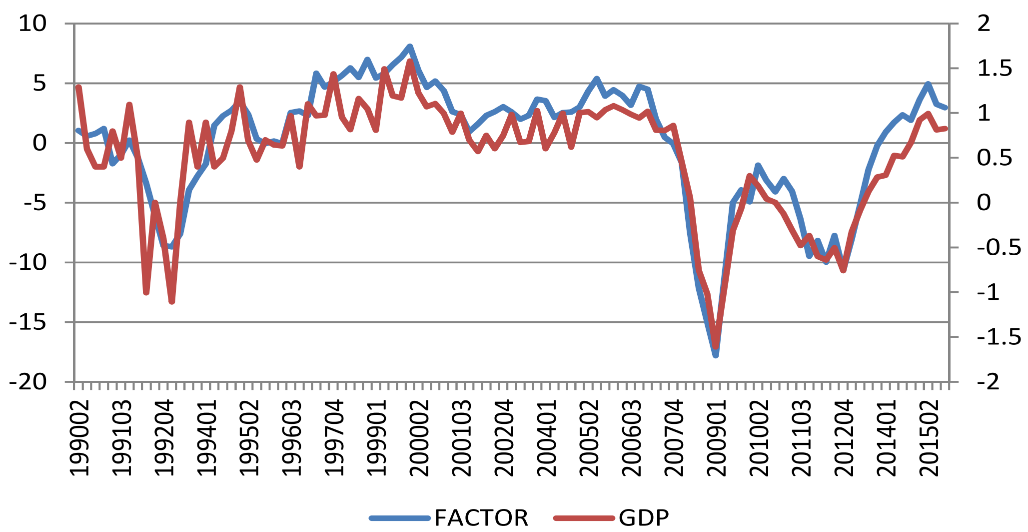
The criteria for the choice of these variables is to consider all the main indicators used in the compilation of the Quarterly National Accounts, see Álvarez (1989), Martínez and Melis (1989), INE (1993) and Álvarez (2005). To fulfill this goal, we have prepared a set of monthly and quarterly indicators, both real and financial, which facilitates a fairly comprehensive basis for analyzing and monitoring GDP and its demand components. In this way, this set offers a high-frequency approximation to the behavior of these main macroeconomic aggregates.

The selection of the final set of indicators has followed a stepwise procedure, as suggested in Camacho and Perez Quirós (2010). The starting point is a minimal set of indicators for each aggregate that represents unequivocally its behavior. For instance, in the GDP model, the "core" group is formed by key economic variables: index of industrial production (supply side indicator of GDP), total deflated sales of large firms (demand side of GDP), large firms´ compensation of employees deflated (income side of GDP) and employment measured by the labor force survey. This initial selection follows Stock and

Watson (1991) and try to mimic the three dimensions of GDP (demand, supply and income) and its direct projection on the labor market (employment). In addition, given the knowledge we have about the determinants of the last recession, we include an indicator of financial conditions (total credit to private resident non-financial sectors) and, as a leading soft indicator the PMI of services, which is freely available. Just with these indicators, we obtain a factor (also named tracker) that it is strongly correlated with GDP growth (the factor is calculated in monthly frequency but can be transformed into quarterly). In particular, the correlation is as high as 0.81 for the 1990.Q1-2015.Q1 sample and 0.83 when the sample starts in 1995.

The selection procedure adds at each step the indicator which is most correlated with the dynamic factor model in order to estimate a new aggregate tracker. If the correlation of the new aggregate tracker increases, the indicator is added to the model. Otherwise, the indicator is dropped from the list. The step is repeated until the full list of possible indicators is exhausted. The final selection produces a correlation of 0.91 for the full sample and 0.96 for the sample starting in 1995. The selected variables are displayed in the first panel of Table 1. Figure 1 represents the factor in quarterly growth rates and the evolution of GDP for the whole sample. As can be seen in Figure 1, the model that we select, which does not include GDP growth itself, shows an extremely close relation with GDP growth. All the turning points are perfectly captured, and it is noticeable that, even with this small set of variables, there is not much room for improvement in the fitting of GDP growth.

Figure 1: GDP growth rate and coincident factor



Regarding the GDP demand components, we repeat the same procedure to select the indicators finally chosen to obtain accurate estimation of each GDP component. Table 1 displays the list of the indicators selected for each variable of interest and its publication lag.

Table 1: List of Indicators

	Starting date	Unit	Source	Release delay
GROSS DOMESTIC PRODUCT (GDP)				
Social security system: registered workers	2001m1	Thousand people	Ministry of Labour	t+1
Employed Labor Force Survey	1990 q1	Thousand people	National Statistical Institute	t+30
Index of Industrial Production	1990 m1	Volume index	National Statistical Institute	t+35
Apparent consumption of cement	1990 m1	Thousand tons	Cement Producers Association	t+22
Electric power consumption	1990 m1	Million Kw/h	Spanish Electricity Network	t+1
Imports of goods deflated by the unit value index	1990 m1	Deflated value index	Tax State Agency/GDMA	t+50
PMI services index for Spain	1999m8	Index between 0 and 100	Markit economics	t+1
Credit to companies and households deflated by consumer price index	1995 m1	Deflated value index	Bank of Spain	t+35
Large companies sales. Deflated total sales	1995 m1	Deflated value index	Tax State Agency	t+35
Large companies sales. Deflated compensation of employees	1995 m1	Deflated value index	Tax State Agency	t+35
HOUSEHOLDS CONSUMPTION				
Index of Industrial Production: consumption goods	1990 m1	Volume index	National Statistical Institute	t+35
Real wage income indicator	1990 m1	Deflated value index	General Directorate Macro. Analysis	t+35
Retail trade index, deflated	1995 m1	Deflated value index	National Statistical Institute	t+27
Consumer confidence index	1990 m1	Index between -100 and 100	European Commission	t-1
Imports of consumption goods deflated by the unit value index	1990 m1	Deflated value index	Tax State Agency/GDMA	t+50
Credit to households for consumption deflated by consumer price index	2003 m1	Deflated value index	Bank of Spain	t+35
Large companies sales. Consumption sales deflated	1995 m1	Deflated value index	Tax State Agency	t+35
Large companies sales. Number of recipients	1995 m1	Deflated value index	Tax State Agency	t+35
GOVERNMENT CONSUMPTION				
Social security system: registered workers in public administration	1995 m1	Thousand people	Ministry of Labour	t+1
State nominal final consumption deflated	1995 m1	Deflated value index	General Audit Office	t+35
Withholding employment income of workers in the public administration deflated	1996 m1	Deflated value index	Tax State Agency	t+35
FIXED CAPITAL INVESTMENT: EQUIPMENT				
Index of Industrial Production: equipment	1990 m1	Volume index	National Statistical Institute	t+35
Cargo and bus registrations	1990 m1	Units	General Directorate of Traffic	t+1
Industrial Confidence Indicator: equipment	1993 m1	Percentage balances	Ministry of Industry, Energy and Tourism	t-1
Imports of capital goods at constant prices	1990 m1	Deflated value index	Tax State Agency/GDMA	t+50
Credit to resident companies deflated	1995 m1	Deflated value index	Bank of Spain	t+35
IBEX-35 Share price index	1990 m1	Index, Jan. 1994=100	Madrid Stock Exchange	t+1
Utilization of productive capacity	1990 q1	Percentage of utilization	Ministry of Industry, Energy and Tourism	t+27
FIXED CAPITAL INVESTMENT: CONSTRUCION				
Social security system: registered workers in construction	2001 m1	Thousand people	Ministry of Labour	t+1
New building visas: total area to build	1991 m11	Buildable floorage (m2)	Ministry of Public Works	t+35
Number of housing transaction: new housing	2007 m1	Units	Ministry of Public Works	t+35
Apparent consumption of cement	1990 m1	Thousand tons	Cement Producers Association	t+22
Confidence index in construction sector	1993 m1	Percentage balances	Ministry of Industry, Energy and Tourism	t-1
Credit to households for housing acquisition and rehabilitation	2003 m1	Deflated value index	Bank of Spain	t+35
Large companies sales. Construction sales deflated	1995 m1	Deflated value index	Tax State Agency	t+35
EXPORTS OF GOODS AND SERVICES				
Total entry of tourist	1995 m1	Thousand people	Ministry of Industry, Energy and Tourism	t+23
Foreign orders. Total industry	1993 m1	Percentage balances	Ministry of Industry, Energy and Tourism	t+23
Total exports of goods at constant prices	1990 m1	Deflated value index	Tax State Agency/GDMA	t+35
Tourism revenues	1990 m1	Deflated value index	Bank of Spain	t+23
World trade in goods	1991 m1	Volume index	Central Planning Bureau (Netherlands)	t+30
PMI index. Industry	1998 m2	Index between 0 and 100	Markit economics	t-1
Large companies sales. Exports sales deflated	1995 m1	Deflated value index	Tax State Agency	t+35
IMPORTS OF GOODS AND SERVICES				
Index of Industrial Production	1990 m1	Volume index	National Statistical Institute	t+35
Total imports of goods at constant prices	1990 m1	Deflated value index	Tax State Agency/GDMA	t+35
Balance of payments. Tourism payments	1996 m1	Deflated value	Bank of Spain	t+60
World trade in goods	1991 m1	Volume index	Central Planning Bureau (Netherlands)	t+20
PMI index. Industry	1998 m2	Index between 0 and 100	Markit economics	t-1
Large companies sales. Imports sales deflated	1995 m1	Deflated value index	Tax State Agency	t+35

2.2 Preliminary processing

The main objective of the model is to provide a synthetic measure of the rate of growth of each macroeconomic variable. This goal requires identifying a reliable signal of growth to be fitted by the factor model. In order to emphasize the short-term information contained in the indicators, we have chosen as signals, for "hard" indicators, the regular first difference of the log time series, and for "soft" indicators, the levels of the series, as in Camacho and Perez Quiros (2010). We consider these indicators in levels for two reasons. On the one side, according to the statistical offices, soft indicators are designed to achieve as high correlation as possible with the year-on-year growth of the coincident series, see European Comission (2006). On the other side, it is in levels how these indicators are interpreted in the industry, as can be seen when they are reported in the press.

For this filtering not to be distorted by the presence of seasonal and calendar factors, they have been removed by means of seasonal adjustment and time series techniques (Maravall and Gómez, 1996; Caporello and Maravall, 2004). We could have estimated the model directly with non-seasonally adjusted data, but following Camacho et al. (2015), we understand that the noise induced by estimating the model with raw data distorts the results and produce worse forecasts than those produced by using seasonally adjusted data. Obviously, out of consistency, all the variables have to be corrected by the same type of factors (seasonal and calendar factors).

3. Econometric approach

The econometric approach used in this paper integrates three main elements. In the first place, a set of dynamic factor models that represent in a compact and parsimonious way the joint dynamics of each macro aggregate and the corresponding short-term indicators. The second element is the treatment of missing observations that can arise as a result of differences in the timing of data publication or as a result of the combination of time series sampled at different frequencies (e.g. monthly and quarterly). Finally, the third element of the methodology is a balancing procedure that ensures in an objective and sensible way the consistency of the GDP forecasts with the forecasts of its components.

3.1 Design of trackers using dynamic factor analysis

For each macro aggregate listed in the previous section (Y_t) a tracker ($f_{j,t}$) is estimated by means of a dynamic one-factor model which captures in a parsimonious way the dynamic interactions of a set of monthly economic indicators ($Z_{i,j,t}$). Given that we are combining quarterly and monthly information for N series, it is important to to clarify the notation from the beginning. The subindex "t" refers to quarterly time, ie, 1990.Q1, 1990,Q2, etc…the subindex "j" refers to monthly time in a given quarter, and it takes the values 1,2,3 referring to the first, second or third month of quarter "t". Finally, the subindex "i" refers to the corresponding ith series when we have more than one series. Therefore, (Y_t) is a quarterly series, ($f_{j,t}$) is a monthly series and ($Z_{i,j,t}$) is the ith monthly series.

The common factor of the system ($f_{j,t}$) is estimated by means of the Kalman filter, after formulating the factor model in state space form. The entire procedure has been adapted to operate with unbalanced data panels, following the procedure of Mariano and Murasawa (2003).

Dynamic factor analysis is based on the assumption that a small number of latent variables generate the observed time series trough a stochastically perturbed linear structure. Thus, the pattern of observed co-movements is decomposed into two parts: commonality (variation due to a small number of common factors) and idiosyncratic effects (specific elements of each series, uncorrelated along the cross-section dimension).

In this paper we assume that the observed, stationary growth signals of k_1 monthly indicators are generated by a factor model:

[1] $$z_{i,j,t} = \lambda_i f_{j,t} + u_{i,j,t}$$

Being:

- $t=1..T$, quarterly time index.
- I=1…k_1
- $z_{i,,j,t}$= i-th indicator growth signal at time j,t.
- λ_i: i-th indicator loading on common factor.
- $f_{i,t}$: common factor at time j,t.
- $u_{i,j,t}$: specific or idiosyncratic component of i-th indicator at time j,t.

The loadings λ_i measure the sensitivity of the growth signal of each indicator with respect to changes in the factor.

When k quarterly indicators –including the variable to track (Y_t)- are considered, we have to take into account that the quarterly indicators are related to monthly activity through time aggregation:

[2] $$Y_t = \frac{1}{3}x_{3,t} + \frac{2}{3}x_{2,t} + x_{1,t} + \frac{2}{3}x_{3,t-1} + \frac{1}{3}x_{2,t-1}$$

Where Y_t is the quarterly macroeconomic aggregate (or a quarterly tracker), and $x_{j,t}$ is the unobserved monthly macroeconomic aggregate (or unobserved monthly tracker).

The unobserved monthly macro aggregate has the same structure than [1]:

[3] $$x_{j,t} = \lambda_Y f_{j,t} + u_{Y,j,t}$$

The subindex Y is just to indicate that we are talking about the decomposition of the Y variable (i.e. GDP, household consumption, etc).

Therefore:

[4]

$$Y_t = \frac{1}{3}\lambda_Y f_{3,t} + \frac{2}{3}\lambda_Y f_{2,t} + \lambda_Y f_{1,t} + \frac{2}{3}\lambda_Y f_{3,t-1} + \frac{1}{3}\lambda_Y f_{2,t-1} + \frac{1}{3}u_{Y,3,t} + \frac{2}{3}u_{Y,2,t} + u_{Y,1,t} + \frac{2}{3}u_{Y,3,t-1} + \frac{1}{3}u_{Y,2,t-1}$$

The case displayed in equation [4] refers to the variable we want to track. If we have some additional quarterly indicators, the structure will be the same (i.e. employment measured by the labor force survey).

Finally, in the special case of the k_2 soft indicators, which are considered in levels, given that they are related to the year on year growth of hard indicators, need a long structure of the factor that covers 12 months. In addition, according to the literature (Camacho and Domenech, 2011) they usually present a leading behavior. Therefore, they are related to the annual growth rate of the series of interest, but with a few periods leading behavior. After trying for different leading periods, we conclude that three quarters is the preferred lead time. Therefore, our specification for the soft indicator variables is:

[5] $$S_{i,j,t,} = \lambda_i\left(f_{3,t+1} + f_{2,t+1} + ... + f_{2,t-3}\right) + u_{i,j,t}$$

Being:

- $S_{i,j,t}$= i-th soft indicator in levels at time j,*t*.
- $I=k_1+1 \; .... k_1+k_2$

- λ_i: i-th indicator loading on common factor.
- $f_{j,t}$: common factor at time j,t.
- $u_{j,t}$: specific or idiosyncratic component of i-th soft indicator at time t.

Equation [1] to [5] do not consider the dynamics in the idiosyncratic part or in the factor structure. Therefore, inference about future activity cannot be made. The model should be expanded in order to adapt it to a time series framework, thereby adding a dynamic specification for the common factor and the idiosyncratic elements, in addition to the dynamics of the series sampled quarterly and the soft indicators.

A second-order autoregression, AR(2), provides a sufficiently general representation for the common factor:

[6] $$\begin{gathered}(1-\phi_1 B-\phi_2 B^2)\, f_{j,t}=e_{f,j,t} \\ e_{f,j,t}\sim iid\ N(0,1)\end{gathered}$$

In [6] B is the backward operator and the variance of the innovation has been normalized. Depending on the characteristic roots of $\phi_2(B)$ the model may exhibit a wide variety of dynamic behaviors.

We also consider an AR(2) specification for the dynamics of the specific elements, allowing for some degree of persistence:

[7] $$\begin{gathered}(1-\psi_{i,1}B-\psi_{i,2}B^2)\,u_{i,j,t}=e_{i,j,t} \\ e_{i,j,t}\sim iid\ N(0,v_i)\ for\, i=1....(k_1+k_2)\end{gathered}$$

[8] $$\begin{gathered}(1-\psi_{Y,1}B-\psi_{Y,2}B^2)\,u_{Y,j,t}=e_{Y,j,t} \\ e_{Y,j,t}\sim iid\ N(0,v_Y)\end{gathered}$$

Finally, we assume that all innovations of the system are orthogonal.

Model [1]-[8] attempts to represent the static as well as the dynamic features of the data. We estimate the common and idiosyncratic factors using the Kalman filter, after a suitable reparameterization of the model in state-space form. The reparameterization requires the introduction of a state vector that encompasses all the required information needed to project future paths of the observed variables from their past realizations. In our case, this vector is:

[9] $$\eta_t = [\, f_{3,t+1} \ldots f_{2-t-2}\,, u_{Y,3,t} u_{Y2,t,} u_{Y1,t,} u_{Y3,t-1,} u_{Y2,t-2,} u_{1,3,t} u_{1,2,t}\,, \ldots . u_{k1+k2,3,t} u_{k1+k2,2,t}\,]'$$

The corresponding measurement equation is:

[10] $$Z_t = H\,\eta_t$$

With $Z_t = (Y_t, Z_{i,t}, S_{it})'$

And H is a vector of coefficients that match the dynamics stated in [1], [4] and [5].

This equation allows us to derive the observed indicators from the (unobservable) state vector.

The transition equation completes the system and characterizes its dynamics:

[11] $$\eta_t = G\eta_{t-1} + V_t$$

Where G is the matrix that capture the dynamic behavior in equations [6] to [8].

The innovations vector V_t is:

[12] $$V_t = [e_{f,3,t+1} \ldots e_{f,2,t-2} \quad e_{Y,3,t} \ldots e_{Y,2,t-1} \quad e_{1,3,t} \ldots . e_{k1+k2,2,t} \qquad]'$$

V_t evolves as a Gaussian white noise with diagonal variance-covariance matrix as follows:

[13] $$Q = E[V_t V_t'] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_Y & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & V_{k1+k2} \end{bmatrix}$$

We assume that the time index *t* goes from 1 to *T*. The application of the Kalman filter requires *Θ = [H, G, Q]* to be known. This requirement is fulfilled using the maximum likelihood estimates of *Θ*, derived by means of numerical maximization of the likelihood function. Note that this optimization is feasible thanks to the iterative computations performed by the Kalman filter.

3.2 Dealing with missing observations

The fact that we have to combine monthly and quarterly frequencies imply that we have necessarily to deal with missing observations, because quarterly data are available only every three months. In addition, our monthly variables are not released simultaneously, and most of them are not available for the whole sample. Therefore, we have to confront daily with an unbalanced dataset, where we have missing observations both at the end and at the beginning of the sample.

In order to deal with this problem we follow Mariano and Murasawa (2003). The idea of this method is to substitute the missing observations with extractions from a random normal distribution. We then estimate a Kalman filter with time varying coefficients where the row that corresponds to the missing observations is multiplied by 0 and we add a noise.

The model is then estimated with this specification. After we estimate the model, the forecast and the filling in of the missing observations is done by substituting the missing value by the number obtained in the Kalman filter with the full matrix H not multiplied by 0 in any of its rows.

3.3 Balancing method

The application of dynamic factor models provides us with independent forecasts of the macro aggregates of MIPred (GDP, Households consumption, etc.). As we have seen, these forecasts combine the available information of the relevant short-term indicators with the dynamics of the macroeconomic variable in an efficient way, but do not take into account the transversal (static) constraints that link the macroeconomic variables. These constraints derive from the compilation process of the National Accounts and, in particular, from the decomposition of GDP from the expenditure side.

In order to incorporate these constraints in the forecasting process, we have relied on a balancing procedure that ensures their internal consistency. In particular, we use the one proposed by van der Ploeg (1982, 1985) for the compilation of the National Accounts[28].

[28] See Abad et al. (2006) for a large-scale application to the Spanish Quarterly National Accounts.

The van der Ploeg method starts with an initial (unbalanced) set of forecasts for each macro aggregate ($Y_{m,t}$) where $m=1..M$, and a measure of their uncertainty embedded in the variance-covariance matrix Σ_t. The final (balanced) forecasts (W_t) must satisfy h linear constraints of the form[29]:

[14] $$A\,W = a$$

Where *A:hxM* and *a:hx1* represent, respectively, the general structure and the final numerical values of such restrictions written in matrix form. For example, *A* may require that certain components of *W* are equal to each other and that the sum of a subset of variables is equal to the sum of another subset. Many other constraints can be envisaged.

The van der Ploeg procedure determines *W* as the solution of the following constrained quadratic optimization program:

[15] $$\underset{W}{\mathrm{MIN}} \quad \phi = (W - Y)'\,\Sigma^{-1}(W - Y) \qquad \text{s.t.} \quad AW = a$$

The objective function weights the squared deviations of each unbalanced forecast with respect to its balanced version, using as weights their precisions (the inverse of their corresponding standard error). Note that in the formulation of the objective function [15] the full covariance of the precisions can be considered (Σ). Solving the quadratic optimization program [15] yield to the following solution:

[16] $$W = Y - \Sigma A' \left[A \Sigma A'\right]^{-1}(AW - a)$$

The interpretation of this equation is straightforward: the balanced vector (*W*) is the result of adjusting the preliminary forecasts (*Y*) on the basis of the observed discrepancy (*AW-a*). These discrepancies are weighted according to their precision, i.e. inversely to the uncertainty associated with the initial forecasts. The van der Ploeg method has some interesting features:

- The (absolute) magnitude of revision increases with the variance of the initial estimate ($\sigma_{m,m}$), where $m=1....M$. That is, the greater the uncertainty surrounding the initial forecast, the greater is the corresponding change.

- Assuming that a given preliminary estimate is known with absolute certainty ($\sigma_{m,m}=0$), then no adjustment is made: $w_m=y_m$. In this way, we can easily perform what-if

[29] In the following, we will drop the time index due to the static nature of the van der Ploeg method.

scenarios or to impose a hierarchy in the forecasting process.

- If the uncertainty in the estimation of two variables evolve in the same direction ($\sigma_{m,n}>0$), their revisions will also adjust them in the same direction, both upward and downward. If, on the other hand, the covariance is negative, adjustments will be made in opposite directions: one upward and one downward.

Note that, given the form of the solution, knowledge of the covariance matrix of the preliminary estimates (Σ) is a crucial element. Usually Σ it is not known, so it must be estimated, usually in two stages: (a) estimation of variances and (b) estimation of the covariances. The estimation of the variances is linked to the standard errors of the forecasts provided by the set of dynamic factor models for each macro aggregate, while covariances can be derived from the historical correlations of the series that must be balanced. In that case, covariances are derived according to:

[17] $$\sigma_{\mathrm{m,n}} = \rho_{\mathrm{m,n}} \sqrt{\sigma_{\mathrm{m,m}} \sigma_{\mathrm{n,n}}}$$

The balancing procedure proposed by van der Ploeg avoids some limitations of competing mehods, like the biproportional RAS method (Bacharach, 1965). In particular, it can manage very general linear constraints, taking into account at the same time different degrees of uncertainty of the forecasts, a quite interesting feature from the point of view of the forecasting practice. In this way, as can be seen in equation [16], the balanced solution avoids the pro-rata adjustment that discredits the RAS method.

The implementation of the van der Ploeg procedure in MIPred considers as inputs the quarter-on-quarter (qoq) rates of GDP and the qoq growth contributions of the remaining macroeconomic variables. The constraint represents the GDP decomposition from the expenditure side:

[18] $$A = \begin{bmatrix} 1 & -1 & \cdots & -1 & 1 \end{bmatrix} \qquad a = 0$$

The final (balanced) forecasts impose a hierarchy among them, conferring priority to the initial GDP forecast, setting $\sigma_{GDP}=0$. This hierarchy reflects the compiling practice of the Spanish QNA, which gives temporal precedence to the estimation of the GDP figure[30] over the estimation of its breakdown. This precedence is not merely a timing issue. When the GDP breakdown is published, the subsequent revisions of the initial GDP estimate are very small. This fact indicates that the information provided by the breakdown has a limited impact on the aggregate GDP estimate, suggesting a top-bottom modelling approach.

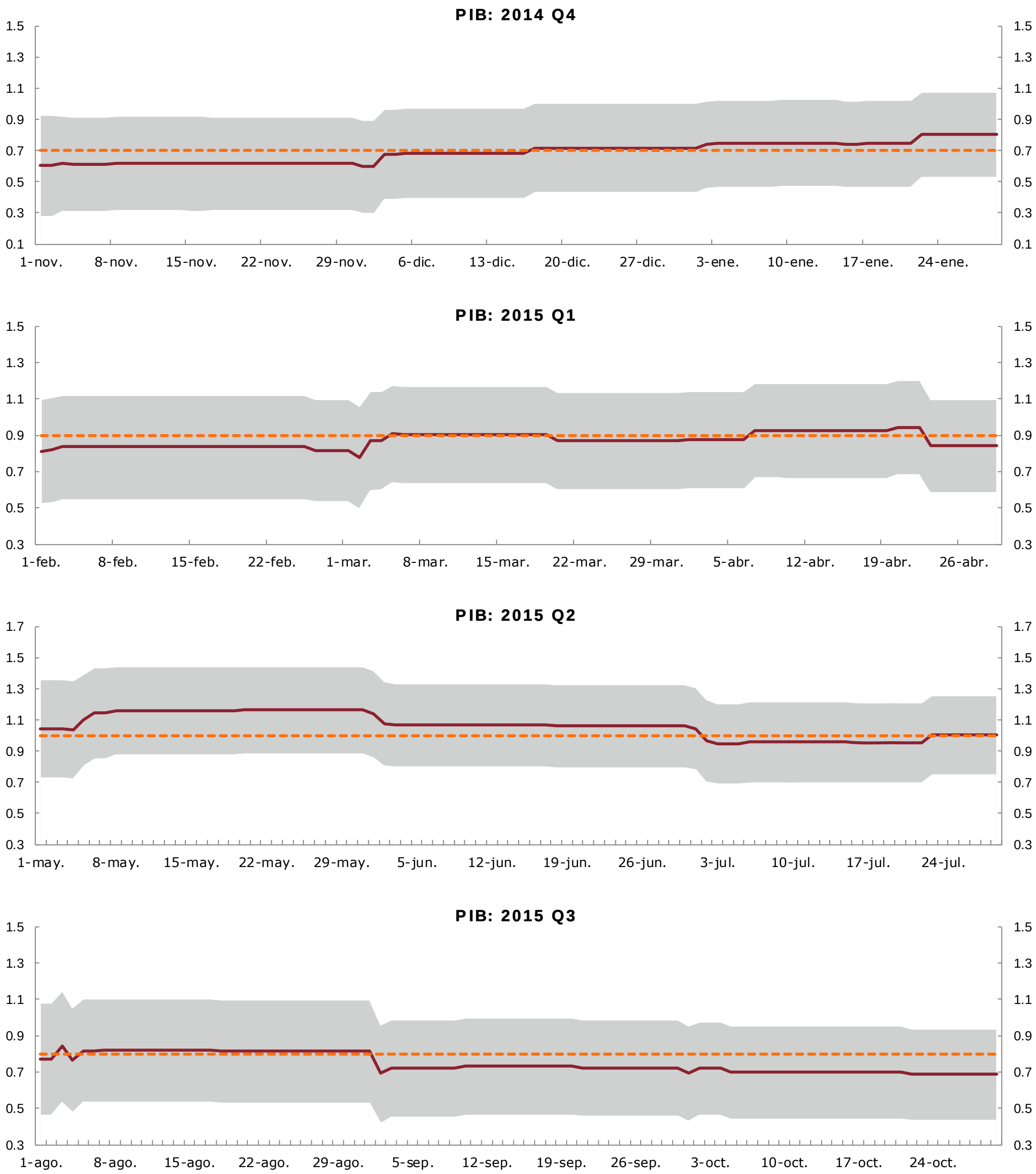
4. Output of the model

In order to show the forecasting performance of the model, it has been carried out a real time estimation exercise for the GDP model in the last four quarters (2014:Q4 – 2015:Q3). The graphs in figure 2 show the evolution of the real-time forecast of GDP in these quarters on a daily basis, including a one standard deviation confidence interval for the forecast value. The time interval during which real time forecasts for each variable are shown in the graphs is defined by the period between two consecutive releases of the corresponding flash estimates published by the National Institute of Statistics (these flash estimates are represented by the dotted line).

Those graphs show how the model reacts to the arrival of the information provided by the indicators. Obviously, this process reduces somewhat the amplitude of the confidence interval, as the cross-sectional estimates are replaced by the observed data. Intuitively, when only "soft" indicators are available, the uncertainty associated with the estimate is greater. Later, when "hard" information arrives (social security contributors, industrial production index, large companies sales, etc.), the estimate becomes less uncertain.

[30] The GDP flash estimate is released about four weeks after the end of the quarter. The second estimate, incorporating the complete GDP breakdown, is released just four weeks after the flash.

Figure 2: GDP growth rate real-time forecasts



Additionally, the four graphs show that these forecasts were close to the GDP flash release disseminated by the National Statistical Institute and the subsequent final figure (second estimate). It can be seen clearly that, in all cases, the flash data published has fallen within the confidence intervals associated with the estimation, and very close to the central estimation.

On the other hand, and summarizing figures for simplicity, Table 2 shows the final forecast for the different macroeconomic variables in those quarters and their corresponding confidence intervals, comparing them with the final data released in the second estimate of the Quarterly National Accounts.

It can be seen that the forecasts of the components, in most cases, fall within the confidence intervals and the ratio error / standard deviation falls within 1 in absolute value (in order to have a measure that weighs the prediction error in relation with the volatility of the series).

It has to be noticed that some sub-aggregates, as in the cases of the series of investment or external trade, have a higher intrinsic volatility that involves wider confidence intervals, making them more difficult to predict.

Table 2: GDP growth rate real-time forecasts

Q-O-Q Rates. Volume SAC data	Lower limit	Central forecast	Upper limit	Observed data	Error	Error / Std. Dev.
	2014 Q4					
Private Consumption	0.3	0.8	1.2	0.9	0.2	0.4
Public Consumption	-1.6	-0.3	1.0	-1.0	-0.7	-0.5
Investment in equipment	0.3	1.7	3.1	1.4	-0.4	-0.3
Investment in construction	-0.5	0.7	1.9	1.4	0.8	0.6
Exports	-0.9	0.7	2.3	0.0	-0.8	-0.5
Imports	-1.8	0.2	2.1	-0.6	-0.8	-0.4
	2015 Q1					
Private Consumption	0.3	0.7	1.2	0.7	0.0	0.0
Public Consumption	0.0	1.3	2.6	1.6	0.3	0.2
Investment in equipment	2.9	4.2	5.6	1.4	-2.8	-2.1
Investment in construction	0.4	1.6	2.8	1.5	-0.1	-0.1
Exports	-0.5	1.1	2.7	1.0	-0.1	-0.1
Imports	0.6	2.3	4.0	0.8	-1.5	-0.9
	2015 Q2					
Private Consumption	0.7	1.0	1.3	1.0	0.0	0.0
Public Consumption	-0.7	0.5	1.7	0.4	-0.1	-0.1
Investment in equipment	2.9	4.3	5.7	3.2	-1.1	-0.8
Investment in construction	1.0	2.1	3.1	1.4	-0.7	-0.6
Exports	2.0	3.3	4.6	1.6	-1.7	-1.3
Imports	2.8	4.3	5.8	2.3	-2.0	-1.3
	2015 Q3					
Private Consumption	0.2	0.8	1.4	1.0	0.2	0.3
Public Consumption	-1.2	0.4	2.1	0.9	0.5	0.3
Investment in equipment	1.7	3.2	4.8	2.3	-0.9	-0.6
Investment in construction	0.1	0.9	1.8	0.6	-0.3	-0.3
Exports	0.0	1.4	2.7	2.8	1.4	1.0
Imports	0.3	1.7	3.2	4.0	2.2	1.5

5. Conclusions

A wide range of public and private institutions (Bank of Spain, AIReF, BBVA, etc.) are interested in monitoring and forecast the main macro variables of the Spanish economy. The key variables that the government has to forecast when preparing macroeconomic and fiscal projections are GDP and its components.

The main distinctive feature of the methodology we use is that we forecast, on a real time basis, not only GDP but also its complete breakdown from the expenditure side. We have specific models to forecast private consumption, public consumption, investment in equipment, investment in construction, exports and imports. We integrate all of them in a consistent set of forecasts for all the variables that compose GDP.

The model provides a judgement-free measure of current economic conditions, thus offering a timely and easy to interpret output which summarizes these conditions through the GDP growth profile, including its demand-side decomposition.

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