Ph.D. Thesis:

Essays in Monetary and Fiscal Policy

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Черепки в огне червонца.

Дед — как в жамковой слюде,

И играет зайчик солнца

В рыжеватой бороде.

an excerpt of Sergei Yesenin's poem "Grandfather"

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Introduction

This thesis contributes to the literature on the joint analysis of monetary and fiscal policy. The contemporary consensus on these two policy branches has become largely dichotomous: fiscal policy is charged with controlling the sustainability of government debt, while monetary policy is charged with stabilizing demand and inflation. This thesis consists of a number of contributions that focus on the interaction of monetary and fiscal policy when the consensus assignment is questioned or does not apply. Addressed questions are particularly relevant in light of the economic developments during the last decade. Since the onset of the global economic downturn in 2007-2008, many advanced economies experienced large economic fluctuations. Stabilizing policy responses in those countries often included large fiscal stimulus packages that in turn triggered discussions of the policy measures—including monetary policy—that would ensure debt sustainability or perform debt adjustment if required. In my work I study policy design in the framework of dynamic general equilibrium models that capture such pressing policy issues. In what follows I provide a brief summary of each chapter.

A liquidity trap due to a binding zero lower bound is a known case to consider otherwise controversial use of fiscal policy for the stabilization of the economy. Many recent studies focused separately on the implications of the zero lower bound either for the use of government spending (see, *e.g.*, Eggertsson (2006), Werning (2011)) or distortionary tax instruments (see, *e.g.*, Eggertsson and Woodford (2004), Correia et al. (2013)). After observing the fiscal developments during the recent crisis in the United States and elsewhere, it is clear that policymakers are apt to exploit both revenue and expenditure fiscal margins. It is therefore instructive to understand if there are reasons to rely on one in favor of the other. The first chapter entitled *"Time-Consistent Management of a Liquidity Trap: Monetary and Fiscal Policy with Debt"* considers the case of using discretionary government spending and labor income taxation jointly as stabilizing instruments at the liquidity trap in a standard New Keynesian model. To mitigate the fall of demand at the liquidity trap, fiscal policy has to lower real interest rates by inducing inflation during the liquidity trap. Increases of government spending and labor income taxes generate demand-pull and cost-push inflation correspondingly. Using quantitative methods I show that when the government is free to vary its debt over time it places more weight on the use of the labor tax. It is then also optimal to avoid large variation in government spending because it produces deviations of the public good consumption from the constant efficient level. Absence of commitment on its own does not overturn the optimal prescription of raising taxes and reducing debt, as first described by Eggertsson and Woodford (2004). The tax instrument's efficacy, however, significantly declines when considering the case of a government budget balanced period by period, since it worsens the time-consistency problem of the dynamic tax response. As a result the change of the labor income tax is three times smaller and the response of government spending is one order of magnitude larger compared to the case with debt dynamics. My results for the balanced budget case suggest that, inability of households to smooth their consumption by running down their assets, could be one of the reasons explaining the active use of the spending instrument in the economies that are currently at the liquidity trap.

Another policy concern that is high on the agenda in many countries is fiscal sustainability in the aftermath of the global economic downturn. Related to these issue is the debate on whether monetary policy and inflation are to play an important role in maintaining fiscal solvency or it is primarily a concern of fiscal policy. In the second chapter entitled *"Debt Maturity, Monetary Policy and Fiscal Sustainability without Commitment"* I focus on how the speed of optimal government debt adjustment and the policy mix that implements it depend on the maturity structure of debt when policy is chosen discretionary. Optimal time-consistent monetary and fiscal policy in a standard New-Keynesian model with one-period bonds does not support the consensus of policy separation when public debt level is moderate or large. Leith and Wren-Lewis (2013) showed that, in the absence of commitment, a high debt burden requires prompt adjustment of debt and monetary policy bears the greatest burden of implementing the adjustment. Using quantitative methods I show that accounting for a plausible average maturity, makes the optimal debt adjustment much more gradual, which is in line with the existing empirical evidence on the persistence of government debt. In the case of bond portfolios with the average maturity ranging from several years and higher, it is no longer optimal for monetary policy to accommodate debt adjustment. I derive intuition for these results using analytical solution of the model's linear-quadratic representation. Higher average maturity reduces sensitivity of the government budget to changes in the market price of bonds. This explains a corresponding reduction in both the incentive of the government to alter current policy and the incentive to strategically affect future self so as to improve the price of borrowing. In equilibrium, this leads to a significant slowdown of the optimal speed of debt adjustment and a shift of the role of monetary policy in debt adjustment.

There are cases when keeping public debt on a sustainable path requires the government to perform debt adjustment via outright default. In the third chapter entitled *"Monetary Policy and Sovereign Default in a Monetary Union: a Risk-Sharing Perspective"* I study sovereign default policy in a monetary union with imperfect risk-sharing. I start off with the monetary union where the government debt in one of the countries is not always supported by budget surpluses and the union-wide monetary authority follows the Taylor principle in setting the nominal interest rate. As in the closed economy studied by Uribe (2006), such a policy arrangement would produce explosive price dynamics unless debt is adjusted through the sovereign default in response to fiscal shocks. At the same time, default policy in the monetary union can take on the role of insuring households across the union against country-specific fiscal risk because international markets are incomplete. I analyze conditions for default policy to efficiently perform both debt stabilization and risk-sharing assignments. I characterize analytically a solution to the model's first-order dynamics and compare equilibrium consumption allocation against a benchmark of the perfect risk-sharing. For these two to coincide one necessary condition has to be satisfied, namely default policy has to be imperfectly discriminatory. The companion result is that, under imperfectly discriminatory default, changes in the monetary policy rule affect real economic activity during the periods of debt adjustment despite the absence of nominal rigidities. Finally, I discuss design of a simple default rule that attains perfect risk-sharing in equilibrium.

1. Time-Consistent Management of a Liquidity Trap: Monetary and Fiscal policy with Debt

1.1. Introduction

A shortfall of demand in an economy calls for a firm policy response. The conventional wisdom until the Great Recession was for monetary policy to bear the burden of stabilizing the economy.[1] In December 2008, in the midst of the Great Recession, the Federal Reserve lowered the federal funds rate almost down to zero. Situations in which the zero lower bound prevents the monetary authority from providing enough stimuli are often referred to as liquidity traps. The European Central Bank, the Bank of Japan, and the central banks of other smaller advanced countries are in liquidity traps today. The constraints imposed on the conventional monetary policy have led academics to consider a number of alternative policy tools to stabilize business cycles, such as fiscal policy and unconventional monetary policy.

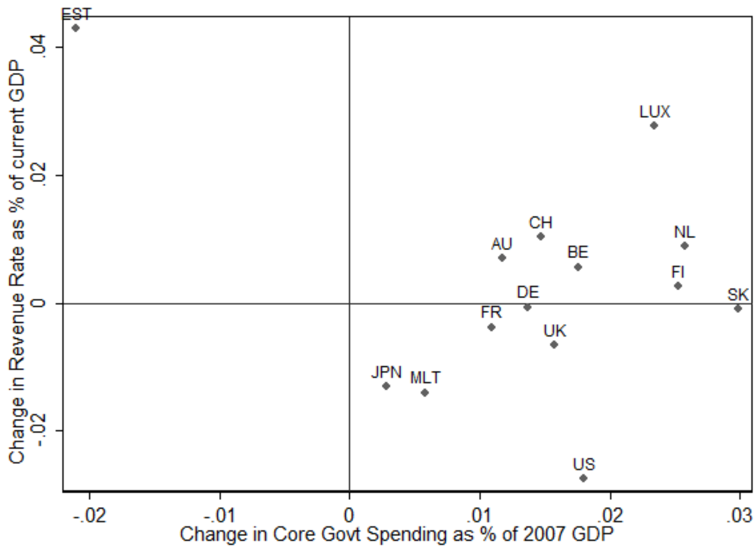
Renewed academic interest in fiscal policy was coupled with the implementation of policy during the Great Recession. In February 2009, an unprecedented stimulus package was enacted in the United States, the "American Recovery and Reinvestment Act." Based on Keynesian ideas, the majority of the package and the subsequent fiscal adjustment came in the form of government spending increases and payroll tax changes. Following the onset of the Great Recession, other countries with policy rates trapped near the minimum feasible level turned to fiscal adjustments of government spending and taxation in various magnitudes and directions. Figure 1.1 illustrates this fact by depicting the changes in government spending and taxation since 2007 for a subset of the OECD countries.

The binding zero lower bound is known to increase the time-inconsistency of the optimal monetary policy and to severely impair stabilization under discretion, as demonstrated by Adam and Billi (2007).[2] Moreover, existing institutional arrangements typically support only a limited horizon of accountability for the treasuries. This motivates me to focus on the challenging case of monetary discretion and to examine the scope of solutions that fiscal policy can offer to improve stabilization while remaining discretionary.

This chapter contributes to the line of research that studies optimal time-consistent monetary and fiscal policy with an occasionally binding zero lower bound constraint on the nominal interest rates. The discretionary use of government spending for stabiliza-

[1] See Kirsanova et al. (2009) for a discussion of the consensus assignment of monetary and fiscal policy.
[2] See also Levine et al. (2008) and Nakov (2008) for assessment of welfare gain from monetary commitment.

Figure 1.1. – Cross-country changes of core government spending and taxation



Notes: Taxation is measured using the revenue rate, which equals total government revenue divided by GDP. Core government spending is total government expenditure net of interest and transfer payments. Time period is 2007-2010. Countries: Eurozone excluding countries affected by the EU debt crisis, Japan, Switzerland, the United Kingdom, and the United States. Data source: Alesina et al. (2014).

tion purposes has been shown to be optimal in a number of studies with New Keynesian models by Nakata (2013), Schmidt (2013), and Werning (2011).[3] The assumptions of lump-sum taxes and Ricardian equivalence are frequently made in such models. The modeling choice of using lump-sum taxes restricts the study of the various ways that taxes can influence the margins of private decisions and how they may be used to partially offset the effect of the zero lower bound. In fact, a diverse set of tax instruments can be used to circumvent the relevance of the zero lower bound along the lines of unconventional fiscal policy, as discussed in Correia et al. (2013). Although their work set an important benchmark in the study of the optimal fiscal policy at the liquidity trap, the application of unconventional fiscal policy requires the kind of consumption tax which is applied after prices are set by firms; in the United States, it is imposed at the state or local level, and it is absent in many other countries. It is therefore instructive to consider the case of the joint use of government spending and a restricted set of tax instruments to determine whether there are reasons to rely on one over the other.

I frame the analysis in a standard New Keynesian model with monopolistic competition and costly price adjustment augmented with a fiscal sector. The government is benevolent and discretionary chooses a short-term nominal interest rate, a level of government spending that has an intrinsic value, and the government debt supply. I assume

[3] Government spending is neutral under joint monetary and fiscal discretion and no zero lower bound; see, *e.g.*, Gnocchi (2013).

that the only tax available to the government is a distortionary flat-rate labor income tax.[4] In doing so, I follow the tradition of the literature on optimal fiscal policy built upon Lucas and Stokey (1983). The model economy is subject to uncertain demand stemming from *ad hoc* variation in the time preference of households. A sufficiently strong preference shock may bring demand for output low enough to call for a negative real interest rate and, at moderate inflation rates, for full stabilization to require a negative nominal interest rate, which would effectively make the zero lower bound binding and bring the economy into the liquidity trap.[5] Failure to offset falling demand leads to downward pressure on prices and output. In order to focus on the dynamic demand stabilization problem, I abstract from the static distortions that would otherwise lead to the usual average inflation bias due to policy discretion. The model is set up in such a way that the deterministic steady state features zero government debt. By establishing zero debt as the *status quo* in the setting without uncertainty, I ensure that any long run level of debt that appears in the stochastic version of the model is due to the risk of binding zero lower bound and any debt dynamics attached to this risk are for stabilization purposes. To solve a stochastic version of the model, I use a global nonlinear numerical method.

With two fiscal instruments the government can improve stabilization of the model economy subject to demand fluctuations that make the zero lower bound effectively binding. For small demand fluctuations that call for positive real rates, it is optimal for the government to rely on nominal interest rate variation in order to stabilize the economy. However, variations in both government spending and labor taxes have a stabilizing role upon entering the liquidity trap. When the nominal interest rate hits the zero lower bound, it is optimal to respond by increasing government spending and raising the labor tax rate. Initial government spending and the labor tax responses monotonically decline over time, and both instruments revert to pre-crisis levels by the time the liquidity trap is over. The magnitude of both responses depends crucially on whether the government budget is balanced or flexible.

The government with a balanced budget relies heavily on the use of government spending. Increasing government spending is helpful in cushioning the fall of aggregate demand, but its use requires deviation from the constant efficient level. The efficacy of the labor tax as a stabilization instrument is improved when it is used in conjunction with debt so that the government shifts reliance away from the spending instrument. Relaxing the government budget makes the increase of spending one order of magnitude smaller and the labor tax change roughly three times larger than in the case with the balanced budget. Raising the labor tax rate, when the nominal interest rate is at the zero lower bound, is inflationary because of the positive cost push effect. Higher taxes can offset deflationary pressure at the liquidity trap and reduce real interest rates. The discretionary government in a given period of time at the liquidity trap, however, does not internalize the effect of the tax increase on demand stabilization in preceding peri-

[4]Assuming, instead, a VAT tax does not affect my considerations.

[5]To build richer quantitative models, one has to incorporate risk premium shocks, as discussed by Amano and Shukayev (2012) and Coibion et al. (2012).

ods. Presence of debt is necessary to create a discretionary incentive for the government to reduce its debt burden via the inflationary effect of higher taxes. Households, anticipating this incentive, are less willing to hold government bonds with a lower return and run down their bond holdings. Households substitute savings with current consumption and improve aggregate demand. Over the course of the liquidity trap the government with a flexible budget runs a surplus and every period rolls over only a fraction of the inherited debt to accommodate smoothing of consumption by households.

In normal times, when the zero lower bound is not binding, optimal discretionary policy dictates that the government accumulate or reduce debt until it reaches a positive optimal stock exceeding the efficient deterministic level of zero. Such an outcome is an example of how responses around the deterministic steady state are sub-optimal and how short run dynamics drive the long run of the economy away from its counterpart without uncertainty. Keeping moderate positive debt in the long run requires a higher labor tax, which pushes marginal costs up. To counteract the pass-through to prices, the nominal interest rate policy has to be tighter, which in the long run increases the magnitude of the shock necessary to make the zero lower bound binding. The optimal long run level of debt balances the cost associated with extra taxes against the benefit of reaching the zero lower bound at a lower frequency.

My work is closely related to Eggertsson (2006), who studied optimal discretionary policy in an economy with a binding zero lower bound and taxes of the costly lump-sum type *à la* Barro (1979).[6] In his analysis, Eggertsson shows that, when taxes do not affect relative prices in the economy, it is optimal to cut taxes and accumulate debt during the liquidity trap. Such a response is effective only because it provides a way to support credibility of the forward guidance on nominal interest rates. When the liquidity trap is over, policy discretion does not allow to sustain the accumulated debt. As a result, a great deal of the debt adjustment is done fiscally irresponsible not just by running budget surpluses but rather by keeping the nominal interest policy loose to reduce real interest rates. My analysis suggests that, when raising taxes pushes marginal costs of the firms up, gains from abandoning fiscal responsibility in order to support the credibility of the monetary forward guidance can be outweighed by the gains from maintaining fiscal responsibility and using taxation as a macroeconomic stabilization instrument during the liquidity trap.

The set of my results for the case of a balanced budget suggests that, inability of households to smooth their consumption by running down their holdings of government bonds, could be one of the reasons explaining the active use of government spending in the economies that are currently in the liquidity trap.[7] The results in this chapter also show that the absence of commitment on its own does not overturn the optimal prescription of raising taxes and reducing debt at the liquidity trap—first described by Eggertsson and Woodford (2004)—which is at odds with conventional wisdom. It is

[6] Burgert and Schmidt (2014) study optimal discretionary monetary and fiscal policy with distortionary taxes, but keep taxes constant in their baseline model.

[7] In a recent study Mckay et al. (2014) show that private borrowing constraints may significantly affect the efficacy of forward guidance at the liquidity trap.

important to note that the fiscal part of the optimal commitment policy mix features a forward guidance component, which is not sustained under discretion. In particular, Nakata (2011) showed that for a government currently in the liquidity trap, it is optimal to commit to expansionary lower taxes and the reversal of government spending expansion following the liquidity trap.[8] Such policy is not part of the optimal time-consistent policy considered in this chapter. Therefore, while the debt level as a natural state variable can largely insulate fiscal policy from the time-consistency problem during the liquidity trap, it may not be capable of making fiscal forward guidance credible.

The chapter is organized as follows. Section 1.2 contains the description of the model. Section 1.3 defines equilibria under two policy regimes: (1) Ramsey equilibrium with commitment policy and (2) Markov-perfect equilibrium with time-consistent policy. I then discuss the design of a labor subsidy that eliminates static distortions and allows to focus on the role of policy for stabilizing demand. Section 1.4 presents numerical results of the optimal time-consistent policy in the Markov-perfect equilibrium with occasionally binding zero lower bound. Section 1.5 concludes.

1.2. The Model

I consider a standard dynamic stochastic general equilibrium New Keynesian model of a closed economy with monopolistically competitive intermediate goods market and costly price adjustment along the lines of Galí (2008) and Woodford (2003). This section describes the economy, defines the competitive equilibrium and derives the first-best allocation as a reference point for the policy problem defined in the next section.

1.2.1. Households

The representative household consumes a composite good, which is produced from a continuum of differentiated products, indexed by $i \in [0, 1]$, using constant-elasticity-of-substitution production technology. Total supply of the aggregate good Y_t is given by

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where $Y_{i,t}$ is the input of differentiated good i, and $\theta > 1$ is the intratemporal elasticity of substitution across different varieties of differentiated goods. This composite good is used either for private and public consumption. Private consumption of the aggregated good is denoted by C_t, and G_t denotes spending on public good provision by the government.

The representative household dislikes labor and values private consumption as well as the public good provided by the government. Preferences of the representative household are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=-1}^{t-1} \xi_s \right) \left[u(C_t) + g(G_t) - \int_0^1 v(h_{i,t}) di \right], \tag{1.1}$$

[8] Belgibayeva and Horvath (2014) showed that this result is robust to wages being sticky.

where $\beta \in (0,1)$ is the time discount factor. Labor of type i is used to produce differentiated good i, and $h_{i,t}$ denotes the quantity (hours) of labor supplied to the firm producing intermediate good of type i. I assume that function v is increasing and convex in hours, function g is increasing and concave in consumption of the public good, and function u is increasing and concave in consumption of the private good. Exogenous stochastic disturbance ξ_t is a preference shock that affects the marginal rate of substitution between consumption at time t and consumption at time $t+1$.[9] I normalize $\xi_{-1} = 1$ and assume that ξ_t for $t \geqslant 0$ evolves according to the following AR(1) process

$$\ln(\xi_t) = \rho \ln(\xi_{t-1}) + \varepsilon_t,$$

where $0 \leqslant \rho < 1$, and $\varepsilon \sim N(0, \sigma^2)$. The preference shock can be interpreted as a shock to a natural real interest rate. The natural rate of interest is the real interest rate associated with the optimal allocation in the flexible price economy. Given described specification of the preference shock, the natural real interest rate in the model economy is equal to $(\beta\xi_t)^{-1}$.

Given prices of the intermediate goods, $P_{i,t}$, the price index P_t that corresponds to the minimum cost of a unit of the aggregate good is computed as follows:

$$P_t = \left(\int_0^1 P_{i,t}^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$

The household enters period t holding assets in the form of nominal non-contingent (risk free, zero coupon) one-period bonds B_{t-1}. Bonds issued in period t can be purchased at a price R_t^{-1}, where R_t is the nominal interest rate. The supply of bonds in equilibrium is determined by government policies, which will be discussed later. Under described market arrangement, a household's flow budget constraint is of the following form:

$$P_t C_t + R_t^{-1} B_t = (1-\tau_t) \int_0^1 W_{i,t} h_{i,t} di + B_{t-1} + \int_0^1 \Pi_{i,t} di - T_t, \tag{1.2}$$

where $W_{i,t}$ is the nominal wage of labor of type i, $\Pi_{i,t}$ is the nominal profit from sales of differentiated good of type i distributed in a lump-sum way, and T_t is the lump-sum government transfer.[10] Labor income of the household is taxed at a linear tax rate τ_t. To avoid complicating the notation I do not explicitly describe the market for private claims. Regardless, this setup is isomorphic to the model with a complete set of private state-contingent securities, as these would not be traded in equilibrium under the assumption of representative household. Also, note that I consider a "cashless" limit of the monetary economy in the spirit of Woodford (2003) and therefore I abstract from money holdings.

To have a well-defined intertemporal budget constraint and rule out "Ponzi schemes" I impose an additional constraint on the behavior of the household that has to hold at

[9] Preference shock enters preferences analogously to Braun et al. (2013), Nakata (2013) and Ngo (2013).

[10] The lump-sum transfer is used for the sole purpose of financing labor (employment) subsidy at the steady state. See discussion below.

each contingency:

$$\lim_{T\to\infty} \mathbb{E}_t \left[\left(\prod_{k=t}^{T} R_k^{-1} \right) B_T \right] \geqslant 0. \tag{1.3}$$

The household maximizes (1.1) by choosing consumption, industry-specific labor and bond purchases $\{C_t, h_t(i), B_t\}_{t=0}^{\infty}$ subject to the budget constraint (1.2) and no-Ponzi condition (1.3), taking as given prices, policies and firms' profits $\{P_t, W_t, R_t, \tau_t, G_t, T_t, \Pi_t(i)\}_{t=0}^{\infty}$, the exogenous stochastic process of preference shocks $\{\xi_t\}_{t=0}^{\infty}$, and initial bond holdings B_{-1}.

The optimal plan of the household has to satisfy (1.2) and (1.3) with equality (the latter is then referred to as the transversality condition), and the following remaining first-order conditions:

$$w_{i,t} = \frac{1}{(1-\tau_t)} \left(\frac{v'(h_{i,t})}{u'(C_t)} \right), \tag{1.4}$$

$$R_t^{-1} = \beta \xi_t \mathbb{E}_t \left(\frac{u'(C_{t+1})}{u'(C_t)\pi_{t+1}} \right), \tag{1.5}$$

where $\pi_{t+1} \equiv {}^{P_{t+1}}/_{P_t}$ is the gross one period inflation rate, and $w_{i,t} \equiv {}^{W_{i,t}}/_{P_t}$ is the real wage of labor of type i. Equation (1.4) describes intratemporal trade-off between consumption and leisure. Equation (1.5) is the Euler equation describing intertemporal allocation of consumption and savings.

1.2.2. Firms

There is a continuum of firms of unit mass producing imperfectly substitutable differentiated goods with a technology that is linear in labor $Y_{i,t} = h_{i,t}$. The firm producing good i sets the price $P_{i,t}$ and hires, in a perfectly competitive labor market, the quantity of labor of type i that is necessary to satisfy realized demand.

I assume that the government allocates its spending on the good varieties identically to the household. The resulting demand schedule for good i is then

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} Y_t. \tag{1.6}$$

Imperfect price-elasticity of demand endows firms with market power. A firm i chooses price $P_{i,t}$ so as to maximize its present discounted real value of profits:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\frac{P_{i,t}}{P_t} Y_{i,t} - (1-s) w_{i,t} Y_{i,t} - \kappa_{i,t} Y_t \right],$$

subject to demand function (1.6), where λ_t is the marginal utility of real income t periods ahead for the representative household, and s is the time-invariant rate of a labor (employment) subsidy.[11] Following Rotemberg (1982), I assume a quadratic cost of price adjustment:

$$\kappa_{i,t} \equiv \frac{\varphi}{2}\left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right)^2.$$

The choice of this specification for sticky prices over Calvo (1983) is commonly used in the literature when solving optimal policy problems without using the linear-quadratic approach, which would require an enlargement of the state-space due to the need to keep track of the price dispersion.[12]

In a symmetric equilibrium where all the firms charge identical prices the first order condition of the firm's problem is

$$\theta\left((1-s)w_t - \frac{(\theta-1)}{\theta}\right) = \varphi\left((\pi_t - 1)\,\pi_t - \beta\xi_t\mathbb{E}_t\frac{u'(C_{t+1})}{u'(C_t)}\frac{Y_{t+1}}{Y_t}(\pi_{t+1}-1)\,\pi_{t+1}\right). \quad (1.7)$$

Condition (1.7) is a New Keynesian Phillips curve, in the nonlinear form, stating that current inflation depends on the marginal cost of production and expected inflation.

Goods market clearing implies the following aggregate resource constraint:

$$(1-\kappa_t)Y_t = C_t + G_t, \quad (1.8)$$

As in Braun et al. (2013), I define GDP as the gross output net of the resource cost of price adjustment incurred by producers of the differentiated goods, $(1-\kappa_t)Y_t$. This definition assumes that the resource costs of the price adjustment is the input of producers of the differentiated goods.

1.2.3. The Government

The government consists of a central bank and a treasury. The treasury decides on the amount of public good G_t in the form of the aggregate consumption good to provide to the household. To cover spending on the public good, it levies a labor income tax, τ_t, and participates in the bond market. The central bank controls the short-term interest rate by means of open-market operations that vary the level of the real money balances held by the household. In equilibrium, the policy rate controlled by the central bank by no arbitrage condition has to be equal to the interest rate on government bonds, R_t.

In the "cashless" economy that I consider, the government supplies nominal claims (also known as "money") that do not provide nonpecuniary return and thus only impose a zero lower bound on gross nominal interest rates:

$$R_t \geqslant 1. \quad (1.9)$$

[11] This subsidy can be used by the government to eliminate the deterministic steady state distortions associated with monopolistic competition and a distortionary labor income tax. See discussion below. The specification follows Leith and Wren-Lewis (2013). One can alternatively design a labor income subsidy, which works equivalently when the labor market is competitive.

[12] See exceptions Anderson et al. (2010), Ngo (2013) for fully nonlinear solutions with Calvo (1983) pricing.

Substituting firms' profit, I can map household's flow budget constraint (1.2) and resource constraint (1.8) into the nominal flow budget constraint of the government:

$$R_t^{-1} B_t = P_t \left(G_t - (\tau_t - s) w_t Y_t \right) + B_{t-1} - T_t .$$

As I leave money out of consideration I automatically forgo seigniorage revenues obtained by the government. I assume that lump sum transfers T_t are used for the sole purpose of transferring resources corresponding to the labor subsidy. Furthermore, since the purpose of the subsidy is to address only permanent (long-run) distortions in the economy, I set the real value of the lump-sum transfers over time equal to the steady-state value of the subsidy level. The flow budget constraint of the government in real terms is then given by

$$R_t^{-1} b_t = \frac{b_{t-1}}{\pi_t} + \left(G_t + \varsigma_t - \tau_t w_t Y_t \right), \tag{1.10}$$

where $b_t \equiv {}^{B_t}/_{P_t}$ is the real value of government bonds, and $\varsigma_t \equiv s w_t Y_t - s w^{ss} Y^{ss}$ is the real deviation of the subsidy from its steady state level.

Following Chari and Kehoe (1993) and Lustig et al. (2008), I assume that households participate at the bond market anonymously, so that bonds issued by households are unenforceable and the government would not buy them. This assumption restricts the range of the government portfolio positions. I capture this assumption with the following inequality constraint that has to hold at all times and in all states:

$$b_t \geqslant 0. \tag{1.11}$$

Under this constraint the government can borrow from households but does not provide loans to households.[13] Note that under (1.11), the no-Ponzi condition (1.3) is automatically satisfied.

Therefore the government, subject to the zero lower bound constraint (1.9), chooses $\{R_t, G_t, \tau_t\}$ that, at the equilibrium prices, uniquely pin down $\{b_t\}$ as satisfying (1.10) and (1.11). The government's problem will be introduced and discussed in the next section.

1.2.4. The Competitive Equilibrium

I focus on the symmetric equilibria where producers of intermediate goods charge equivalent prices, $P_{i,t} = P_t$ for all $i \in [0, 1]$, and therefore face the same demand $Y_{i,t} = Y_t$, hire the same amount of labor $h_{i,t} = h_t$, and pay the same competitive real wage $w_{i,t} = w_t$. The production function for intermediate goods then implies that $Y_t = h_t$. Taking these conditions into account, I now define a competitive equilibrium

Definition 1.1 (Competitive equilibrium)**.** *Given exogenous process for household's preference shocks* $\{\xi_t\}_{t=0}^{\infty}$ *and initial outstanding government debt* $b_{-1} \geqslant 0$*, a rational ex-*

[13] No government lending constraint is also imposed in Faraglia et al. (2013).

pectations symmetric equilibrium is a sequence of stochastic processes $\{C_t, Y_t, \pi_t, w_t, b_t, R_t, G_t, \tau_t\}_{t=0}^{\infty}$ *satisfying*

$$u'(C_t) = \beta\xi_t R_t \mathbb{E}_t \left\{ \frac{u'(C_{t+1})}{\pi_{t+1}} \right\}, \tag{1.12}$$

$$v'(Y_t) = (1-\tau_t) w_t u'(C_t), \tag{1.13}$$

$$\theta(1-s)w_t = (\theta-1) + \varphi\left((\pi_t - 1)\,\pi_t - \beta\xi_t \mathbb{E}_t \left\{ \frac{u'(C_{t+1})}{u'(C_t)} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1)\,\pi_{t+1} \right\} \right), \tag{1.14}$$

$$0 = C_t + G_t - \left(1 - \frac{\varphi}{2}(\pi_t - 1)^2\right) Y_t, \tag{1.15}$$

$$R_t^{-1} b_t = \frac{b_{t-1}}{\pi_t} + (G_t + \varsigma_t - \tau_t w_t Y_t), \tag{1.16}$$

$$R_t \geqslant 1, \tag{1.17}$$

$$b_t \geqslant 0, \tag{1.18}$$

and the transversality condition, for all $t \geqslant 0$*; where the Euler equation (1.12) and equation (1.13) are first-order conditions of the household problem, the New-Keynesian Phillips curve equation (1.14) is the first-order condition of the intermediate goods producer, equation (1.15) is the aggregate resource constraint, equation (1.16) is the government budget constraint, (1.17) represents the zero lower bound constraint on nominal interest rates, and (1.18) is the government no-lending constraint.*

1.2.5. The First-Best Allocation

The first-best allocation is the one maximizing the household's utility (1.1) subject to the technology constraints while abstracting from economic distortions in the private markets. Such an allocation is efficient and coincides with the solution of the fictitious Social Planner's problem described in appendix A.1. Efficiency dictates, for all $t \geqslant 0$, that the marginal utilities of private and public consumption be set equal to the marginal disutility of labor

$$u'(C_t) = v'(Y_t),$$
$$g'(G_t) = v'(Y_t).$$

I use these efficiency conditions as a benchmark when discussing the problem of the government in the next section.

1.3. The Policy Problem

In this section I formulate the optimal policy problem. Throughout the chapter I assume full cooperation between fiscal and monetary authorities. The government is assumed to be benevolent and, hence, have the maximization of the household's utility as an objective. The first-best allocation is in general not attainable even in the absence of any shocks due to a number of economic distortions in the model. First, the intermediate goods producers charge a mark-up over the marginal cost. Second, government spending has to be financed with the distortionary labor tax that introduces a wedge into a private leisure decision. Further distortions include the price-adjustment cost as well as the bounds on bond holdings and nominal interest rate.

I start by establishing a second-best policy problem, as a apart of the Ramsey equilibrium, that takes the described economic distortions into account. The Ramsey policymaker is assumed to fully commit to the policy it announces. Then I proceed by dropping the commitment assumption and describing the optimal Markov-perfect allocation under discretionary policymaking. The last part of this section is devoted to the analysis of the deterministic steady states of the two policy regimes. It discusses the design of an employment subsidy capable of eliminating static differences between the two equilibria that are not related to the demand stabilization problem, which is the focus of this chapter. The stochastic version of the Markov equilibrium is then numerically analyzed in the next section.

1.3.1. The Ramsey Equilibrium

Assume that the government can commit to follow through the policies it announces at the beginning of time. The government then announces policy plan for all future contingencies in order to implement the best competitive equilibrium which I refer to as the Ramsey equilibrium. I formally capture this idea in the following definition.

Definition 1.2 (Ramsey equilibrium)**.** *Ramsey equilibrium consists of a state-contingent plan* $\{C(\xi^t), Y(\xi^t), \pi(\xi^t), w(\xi^t), b(\xi^t), R(\xi^t), G(\xi^t), \tau(\xi^t)\}_{t=0}^{\infty}$ *chosen at the initial time period* $t = 0$ *and for all possible histories* $\xi^t \equiv (\xi_0, \ldots, \xi_t)$ *of preference shocks that, given initial outstanding government debt* $b_{-1} \geqslant 0$*, maximizes expected discounted sum of future utilities*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=-1}^{t-1} \xi_s \right) [u(C_t) + g(G_t) - v(Y_t)]$$

subject to equations (1.12) - (1.18) characterizing competitive equilibrium.

The Lagrangian of the Ramsey problem and the first-order conditions characterizing the Ramsey allocation are described in Appendix A.2.

1.3.2. The Markov-Perfect Equilibrium

Assume that the government is benevolent and credibly repays its debt in the future, but the use of the remaining fiscal and monetary instruments is credible only within a period of announcment.[14] The government has incentives to renege on its past promises of taxes, government spending, and the nominal interest rate. This gives rise to a time-inconsistency problem of the Ramsey policy. Policy without commitment is discretionary. I model optimal discretionary policy as an outcome of a game between the private sector and the government represented by the treasury and the central bank playing cooperatively to maximize expected discounted utility (1.1) of the representative household. More precisely, I look for policy that is a part of the stationary Markov-perfect equilibrium of this game.

The timing of the game is as follows. Each period, the government acts first as a Stackelberg leader and announces current period policies. When doing so, it internalizes the effect of its decision on the actions of the private sector, acting as a Stackelberg follower and taking future policy as given. The government chooses the optimal policy sequentially based on the minimal payoff-relevant state of the economy. Such an aggregate state of the economy at the time t is described by realizations of the preference shock ξ and inherited government liabilities from the previous period b_{-1}. Although the government at a given period can not credibly commit to its future policy and, hence, does not directly influence future actions of the private sector, it can indirectly influence both through the current debt issuance policy that determines the future inherited state of the economy. In the Markov-perfect equilibrium, sequentially optimal choices are time-consistent and recursively determined by stationary rules.

I proceed by using a recursive formulation to formally describe the concept of the optimal discretionary policy. I drop time indices and use b_{-1} and b to denote outstanding and newly issued government bonds in a given perios. For all the remaining variables I denote the next period value of a given variable x by x'. The government, at a given period in time correctly anticipates the future policy as well as the corresponding equilibrium allocation and prices that I denote as governed by the stationary functions $\mathcal{C}, \mathcal{Y}, \mathit{\Pi}, \mathcal{W}, \mathcal{R}, \mathcal{B}, \mathcal{T}, \mathcal{G}$. The future value function is denoted by the stationary function $\mathcal{V}$. When announcing policy for an ongoing period the government is free to deviate from the anticipated rules by implementing the best possible competitive equilibrium. As it is common in the optimal taxation literature, one can think of the government as choosing simultaneously its policy, prices and allocation, provided that they satisfy equilibrium conditions from definition 1.1.[15] I capture this with the following Markov optimization problem of the discretionary government:

$$\max_{C,Y,\pi,w,R,b,\tau,G} u(C) + g(G) - v(Y) + \beta\xi\mathbb{E}\left\{\mathcal{V}\left(b;\xi'\right)\right\}$$

subject to the constraints

[14] Credible debt repayment can be supported by a high implicit cost of the outright debt default.

[15] Strictly speaking, I also assume that the primitives of the model permit correspondence between sequential formulation in definition 1.1 and recursive formulation employed here to be valid.

$$
\begin{aligned}
0 =& \left[\theta(1-s)w - (\theta-1) - \varphi(\pi-1)\pi\right] Y u_c \\
& + \varphi\beta\xi\mathbb{E}\left\{\mathcal{Y}(\cdot)\left(\mathit{\Pi}(\cdot)-1\right)\mathit{\Pi}(\cdot)\, u_c\left(\mathcal{C}(\cdot)\right)\right\}, \\
0 =& \left(1-\frac{\varphi}{2}(\pi-1)^2\right) Y - C - G, \\
0 =& R^{-1}b - \pi^{-1}b_{-1} - \left(G + \varsigma - \tau w Y\right), \\
0 =& u_c - R\beta\xi\mathbb{E}\left\{\frac{u_c\left(\mathcal{C}(\cdot)\right)}{\mathit{\Pi}(\cdot)}\right\}, \\
0 =& w(1-\tau)u_c - v_y, \\
b \geqslant& 0, \\
R \geqslant& 1.
\end{aligned}
$$

For optimal policy to be time-consistent, the government should find no incentives to deviate from the anticipated rules. This idea is captured in the formal definition of the Markov-perfect equilibrium.

Definition 1.3 (Markov-perfect equilibrium)**.** *A Markov-perfect equilibrium is a function $\mathcal{V}(b_{-1};\xi)$ and a tuple of rules $\{\mathcal{C},\mathcal{Y},\mathit{\Pi},\mathcal{W},\mathcal{R},\mathcal{B},\mathcal{T},\mathcal{G}\}$ each being a function of b_{-1} and ξ, such that for all $b_{-1} \geqslant 0$:*

1. *Given $\mathcal{V}(\cdot)$, tuple of rules solves Markov problem of the government,*
2. *$\mathcal{V}(\cdot)$ is the value function of the government*

$$
\mathcal{V}(b_{-1};\xi) = u\left(\mathcal{C}(\cdot)\right) + g\left(\mathcal{G}(\cdot)\right) - v\left(\mathcal{Y}(\cdot)\right) + \beta\xi\mathbb{E}\left\{\mathcal{V}\left(\mathcal{B}(\cdot);\xi'\right)\right\}.
$$

I restrict attention to equilibria with differentiable value and policy functions. Under this assumption, equilibria are characterized by the first-order conditions of the government problem.[16] The derivation of these conditions is delegated to Appendix A.3.

1.3.3. Steady States

Standard in this class of models, there is in fact a continuum of the Ramsey steady states indexed with an initial level of government debt. The Markov steady state in general is different from the Ramsey steady states. Both the Ramsey and the Markov steady states are in general different from the first-best allocation. These differences are discussed in detail in Appendix A.4. The focus of this study is on the discretionary policy that stabilizes demand and I want to study it in isolation from the long run effects caused by the absence of commitment. To do so I assume that a labor subsidy is used

[16] I refrain from a formal general proof of equilibrium existence or uniqueness. For a parametrized model numerical results demonstrate existence. Multiplicity is known to exist in a standard neoclassical economy with capital, see Ortigueira et al. (2012).

by the government to offset permanent distortions in the economy as described in the following proposition.

Proposition 1.1 (Employment subsidy)**.** *For a given debt level b, there exists a unique employment subsidy rate $s^e(b)$ such that*

- *corresponding Ramsey steady state is efficient,*
- *there is a Markov steady state with debt level b that is also efficient.*

Proof. See Appendix A.4.1. □

According to this proposition, in the deterministic setting without uncertainty one can design the subsidy to support any debt-to-GDP ratio with efficient allocation under both policy regimes. In what follows I pick a particular subsidy rate under which efficient steady states of both policy regimes feature zero government debt.

1.4. Optimal Time-Consistent Policy

This section presents results describing stochastic Markov-perfect equilibrium of the model with an occasionally binding zero lower bound. I apply numerical methods because the equilibrium cannot be solved for in the closed form. First, I lay out the parameterization strategy. Then I discuss the optimal long run debt policy and, after that, proceed with discussion of the optimal policy responses to variations in the preference shock when the government budget constraint is flexible. The final part of this section considers the case when the government budget is balanced.

1.4.1. Parameterization and Solution Method

Parameter values are summarized in Table 1.1. Each period in the model represents one quarter of a year. The steady state time discount factor, β, is set to 0.995, corresponding to the annual real interest rate of 2 per cent.

Preferences of the households for consumption of private and public goods are described by $u(c_t) \equiv \frac{c_t^{1-\gamma_c}}{1-\gamma_c}$, $g(G_t) \equiv \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g}$, and the disutility from work is described by $v(h_t) \equiv \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h}$. Curvature parameters of the utility functions, γ_c, γ_g and γ_h, are set to 2, 1, and 1 correspondingly. I choose values of the utility weights ν_h and ν_g equal to 100 and 1.25 correspondingly so that in the deterministic steady state households spend one quarter of their unitary time endowment working and government spending amounts to 20 per cent of the value added.

The monopoly power of the firms is described by the elasticity of substitution between intermediate goods, θ, which I set equal to 11 in order to match markup of the price over the marginal cost of 10 per cent. The parameter of price adjustment cost, φ, is set equal to 116.505, which is consistent with a Calvo (1983) price-setting specification where one quarter of firms reoptimize their prices every period.

The parameter governing persistence of the demand shock, ρ, is set equal to 0.8. This value is a common choice in the literature studying zero lower bound and, as Fernández-Villaverde et al. (2012) discuss, it implies that the demand shock process has a half life of about three quarters. The standard deviation of the preference shock innovations is used to target unconditional probability of the model economy being at the zero lower bound. Coibion et al. (2012) calibrate the duration of the binding zero lower bound at roughly five per cent of the time, which is consistent with the post-WWII to 2011 period in the United States. Reifschneider and Williams (2000) estimate this probability to be in the range of five to six per cent based on the data for the Great Moderation period in the United States. Taking into account that the zero lower bound in the United States is expected to be binding well into the 2015 implies that the zero lower bound will have been binding for almost a quarter of time since the beginning of the Great Moderation period, which by far exceeds the five per cent probability estimate. Even if one considers current events to be the extreme tail event, Chung et al. (2012) included first two years of the Great Recession into the sample and employed a variety of estimation techniques to conclude that five per cent frequency is an underestimation. Also, recently revived hypothesis of the secular stagnation (see Summers (2013)) and IMF (2014) forecast of downward trend in the natural real interest rate would suggest that the probability of hitting the zero lower bound could be higher in the medium run. I set the standard deviation of the preference shock innovations equal to 0.002. This value results in having the unconditional probability of being at the zero lower bound around eight per cent of the time.

Parameter	Description	Value
β	Discount rate	0.995
γ_c	Interpemoral elasticity for C	2
γ_g	Intertemporal elasticity for G	1
γ_h	Inverse Frisch elasticity	1
ν_g	Utility weight on G	1.25
ν_h	Utility weight on labor	100
φ	Price adjustment cost	116.505
θ	Elasticity of substitution among goods	11
ρ	Demand shock persistence AR(1)	0.8
σ_ε	Demand shock innovation s.d.	0.002

Table 1.1. – Baseline parameterization

I solve the model numerically by means of a global nonlinear approximation method. Appendix A.5 contains the details.

1.4.2. Optimal Long Run Debt

In this section I show that the risk of falling into the liquidity trap permanently drives the economy away from its efficient deterministic steady state. Compared to the zero debt in the deterministic steady state, long run debt level is positive in the stochastic equilibrium with preference shocks. The optimal long run level of government debt, b^*, amounts to ~5 per cent of the annual GDP. Figure 1.2 shows the optimal government debt change policy when the preference shock is equal to its average value ($\xi = 1$). It plots the absolute change of the outstanding debt across two consecutive periods as a function of the outstanding debt in the first out of these two periods, scaled by the constant annual efficient GDP. Let the economy start without outstanding government debt, $b_{-1} = 0$. Conditional on $\xi = 1$, the optimal policy prescription is to accumulate government debt until the optimal level b^* is reached. Conversely, an economy starting with a debt level above the optimal level b^*, conditional on $\xi = 1$, should experience debt stock decline until the optimal level b^* is reached.

The allocation corresponding to the optimal long run level of debt, b^*, and the average preference shock, $\xi = 1$, is a *risky* steady state in the terminology of Coeurdacier et al. (2011). Keeping positive debt in the long run is an instrument to permanently change the point of the state space around which the economy fluctuates. The optimal positive long run debt level b^* creates an incentive to reduce the real interest rate that balances out the increase of the real interest rate due to the discretionary response to the binding zero lower bound. Both of these effects on the real interest rate stem from the failure of the discretionary government to provide optimal (history-dependent) dynamic policy control.[17] This general failure of the discretionary policy is often referred to as the stabilization bias.[18] The remainder of this section explains in more detail the two counteracting forms of the stabilization bias in my model that lead to the accumulation of the positive government debt buffer in the long run.

The first form of the stabilization bias present in this model is the debt stabilization bias due to labor income taxation.[19] The government in an economy with an outstanding debt exceeding the efficient level of zero has to collect additional revenue via distortionary taxation to support its liabilities. Under discretion the government can not enjoy the benefits of spreading costs of the distortionary taxation over time and has to stabilize debt stock around its efficient level.[20] At the beginning of each period discretionary government will be tempted to manipulate the real interest rate in order to increase the price of its bonds.[21] Without the second form of the stabilization bias introduced by the binding zero lower bound and discussed below, current discretionary government would have to discipline future self by adjusting the debt level toward the efficient level where

[17]See Currie and Levine (1987) for a discussion of the time-inconsistency of the optimal dynamic control of an economy under commitment.

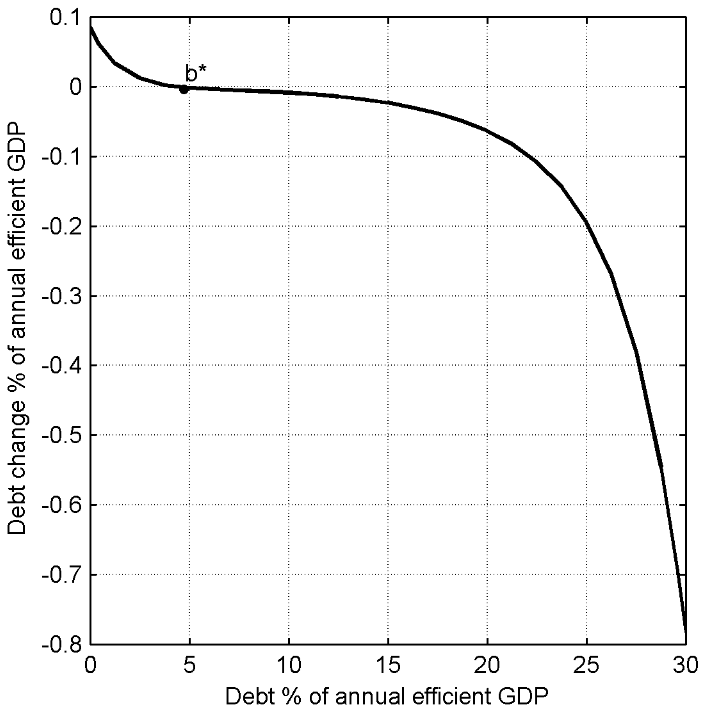
[18]See Clarida et al. (1999) for a discussion of the stabilization bias in application to dynamic inflation control in a New Keynesian model.

[19]See Leith and Wren-Lewis (2013) and Vines and Stehn (2007) for a discussion of the debt stabilization bias in New Keynesian models without the risk of binding zero lower bound.

[20]Given the incomplete markets structure, it would be optimal for the government to commit and spread associated costs of distortionary taxation over time. Under commitment, debt then would follow a random walk type of behavior described in Aiyagari et al. (2002) and Schmitt-Grohé and Uribe (2004).

[21]See Debortoli and Nunes (2012) for a discussion of the discretionary fiscal policy with debt in a real economy.

Figure 1.2. – Conditional optimal debt change policy



Notes: Depicted optimal debt change policy is computed as $(\mathcal{B}(b_{-1}, \xi) - b_{-1})$, conditional on $\xi = 1$. The optimal long run debt level at the risky steady state is marked with b^*.

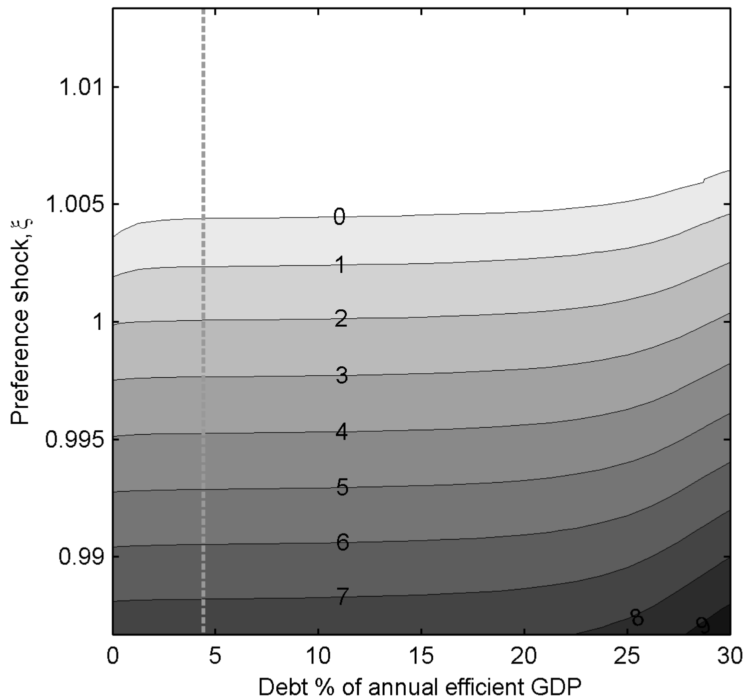
the efficient allocation is implemented and the discretionary incentive is eliminated. Chapter 2 of this thesis discusses that this is true more generally when efficient level of debt is different from zero and average maturity of government debt exceeds one period of time.

The second form of the stabilization bias present in this model is the deflation bias due to an occasionally binding zero lower bound.[22] Following high enough preference shock, the binding zero lower bound prevents the nominal interest rate policy from offsetting the fall of private demand. Abstracting from fiscal policy, falling demand pulls down prices and output contracts. Deflationary pressure is larger under the discretionary monetary policy because it can not credibly compromise stabilization upon exiting the liquidity trap and generate output boom, which would effectively reduce desire of households to save into the future and improve stabilization at the liquidity trap. Matters get worse when the zero lower bound is recurring. The deflationary pressure and output contraction associated with the liquidity trap spill into the rational expectations of households

[22]See Adam and Billi (2007) and Eggertsson (2006) for a discussion of the deflation bias in New Keynesian models without labor income taxation.

even when the nominal interest rate is positive and the economy is away from the liquidity trap. As a result, contemporaneous inflation and output in the times when the zero lower bound is not binding are adjusted downward. The adjustment of the former makes the real interest rate higher (compared to commitment).

Figure 1.3. – Optimal nominal interest rate policy



Notes: The graph shows contour lines of the optimal nominal interest rate policy $\mathcal{R}(b_{-1}, \xi)$. Numbers on the contour lines report corresponding net annualized nominal interest rates. White area corresponds to the state space area where the zero lower bound is binding. Dotted line marks optimal long run debt level b^*.

The two forms of the stabilization bias influence the conduct of nominal interest rate policy in the normal times when the zero lower bound is not binding. The deflation bias makes the central bank to conduct more accommodative interest rate policy, which leads to the lower bound being reached at a higher frequency. The debt stabilization bias at the low debt levels, comparable to the optimal long run level b^*, makes the central bank to conduct tighter interest rate policy. Figure 1.3 shows how the optimal nominal interest rate response to preference shocks on impact changes with the outstanding debt in the beginning of period. An outstanding debt level above b^* makes the debt stabilization bias prevail and requires larger preference shock for the zero lower bound to be reached. Such a debt level in the long run is, however, unsustainable because it would require higher tax rates that can not be smoothed over time under discretion. On the other hand, reducing level of debt below b^* makes the deflation bias prevail, which leads to more frequent liquidity trap episodes. Optimal long run level of debt, b^*, balances the benefit from less frequently binding zero lower bound against the cost of distortionary taxation induced by this level.

1.4.3. Optimal Responses

I proceed with examination of the optimal policy and allocation in the Markov-perfect equilibrium by considering the impulse responses to varying demand of households. I consider the case when the government budget constraint is flexible and the level of outstanding government debt prior to the shock is equal to the optimal long run level b^*. The variation of demand is a result of the exogenous changes in time preference. Temporary increase of the preference shock ξ makes the household more patient and reduces its contemporaneous demand.

Figure 1.4 shows the impulse responses under the optimal discretionary policy following a positive preference shock equal to the one unconditional standard deviation. I report consumption, government spending, labor supply (hours), the GDP, and the real wage in percentage deviation from the deterministic steady state. Inflation and the (net) nominal interest rate are reported in annualized percentage points. The labor tax rate is reported in percentage points. Primary balance is the difference between tax revenue and government spending, reported as a fraction of the quarterly efficient GDP. Real value of debt is reported as a fraction of the annual efficient GDP.

The conventional short-term nominal interest rate policy is fully capable on its own to entirely insulate the economy from moderate negative demand shocks. Lowering the policy rate reduces excess savings desire of the household and keeps the allocation of aggregate economic activity intact. Labor supply and both private and public consumption are stabilized at the values that are slightly below their deterministic steady state counterparts because the risky steady state level of government debt is positive compared to the debt level of zero in the deterministic steady state. Temporary reduction of the government debt stock reflects the reduction of private savings in response to lower interest rates. Keeping allocation constant also requires keeping government spending, the tax rate and, hence, the primary surplus unaltered. Despite adjusting debt, the government can keep the primary balance constant because lower nominal interest rate raises government bond prices and adjustment is self-financed.

If the preference shock deviation from its average generates large enough fall in demand, the zero lower bound starts binding. Figure 1.5 shows the impulse responses under the optimal discretionary policy following a positive preference shock equal to the three unconditional standard deviations; the magnitude of the shock is the only difference compared to Figure 1.4. When the zero lower bound is binding, the nominal interest rate fails to offset the fall of private demand. Falling private demand manifests itself into downward pressure on both prices and hours worked. Resulting deflationary pressure makes households to adjust their expectations of future inflation downward because of the shock persistence and the nominal rigidity in the price setting decision of the firms. Reduction in the expectations of inflation increases the real interest rate, which reinforces savings desire and endogenously aggravates the fall of private demand. The larger and more persistent is the preference shock the more pronounced are the hours worked decline and deflationary pressure. The nominal interest policy is ineffective at the liquidity trap, yet the government has two fiscal instruments remaining at its disposal. The available set of fiscal instruments does not allow to entirely stabilize the economy but is still used to improve stabilization of the economy at the liquidity trap. The remainder of this section discusses the response of fiscal instruments at the liquidity trap.

In models without the fiscal sector where the private sector is the only source of demand, changing private demand is the only force driving changes in aggregate demand. Differently, in the class of models with the endogenous government spending, the government may actively use the spending instrument, which endogenously responds to shocks, so as to stabilize aggregate demand. The optimal policy mix at the liquidity trap features increase of government spending in the impact period. The initial government spending expansion dies out until reaching the pre-crisis level when the zero lower bound stops binding. The response of government spending cushions decline of aggregate demand due to the fall of private demand. The higher aggregate demand improves hiring incentives of the firms. As a result, the fall of the hours worked is mitigated and improving labor demand supports wages and therefore prices from falling. Thus, government spending is optimally used for stabilization purposes when the zero lower bound is binding, which is not the case when demand fluctuations are not strong enough to put the economy into the liquidity trap.[23]

Using government spending for cushioning the fall of aggregate demand is, however, costly because it opens a gap by driving public consumption away from its efficient level. Like government spending that takes on a stabilizing role during the liquidity trap, so does the labor tax respond to the binding zero lower bound when its variation is otherwise avoided. As a part of the optimal fiscal policy response to the shock that puts economy into the liquidity trap, it is optimal to temporarily raise taxes returning them back to the pre-crisis level once the zero lower bound stops being binding. The higher labor tax is optimal due to its supply-side effect on prices. Increasing taxes in a given period makes agents less willing to work. Firms are then forced to increase wages to produce a given amount of output and pass some degree of the marginal cost increase on prices. Keeping taxes elevated in the future remaining periods of the liquidity trap creates inflationary expectations that offset expectations of deflation due to the binding zero lower bound. It reduces the real interest rate in the current period and breaks the feedback loop between high real interest rates and the falling private demand.

[23] Nakata (2013) and Schmidt (2013) make the case for using government spending as stabilization instrument at the liquidity trap when taxes are lump-sum and the Ricardian equivalence holds.

The discretionary government does not internalize the benefit of the contemporaneous tax rate increase in a given period for stabilizing the fall of private demand in the preceding periods. Under discretionary policy, tax hikes can only be sustained to the extent they are time-consistent. The contemporaneous tax increase comes at a cost of driving labor supply down. The benefit of the contemporaneous tax increase comes from its inflationary effect on the real value of the outstanding government bonds. If the government inherits a positive level of debt, it can benefit from reducing the debt burden via inflation. Importantly, this discretionary incentive to manipulate the price of government debt is not time-inconsistent at the liquidity trap because higher inflation reduces the real interest rate in the previous period and, therefore, improves stabilization.[24] Amount of debt issued by the current government regulates the trade-off of tax setting by the government in the next period. Bequeathing a certain amount of debt, the current government influences the dynamic tax response to reduce the real interest rate for stabilization purposes.

Reduction of the real interest rate, resulting from the anticipation by households of the discretionary incentive to manipulate the price of government bonds, offsets the fall of private demand so that households are willing to run down their assets. For households to smooth their consumption by running down their assets, the government has to reduce the amount of the outstanding debt. The price of government bonds is, however, relatively too low because the real interest rate falls short of the level that would entirely offset the fall of private demand. Therefore, debt reduction is not self-financed as in response to moderate demand shocks that do not lead to the binding zero lower bound. The government runs extra surplus to implement debt reduction.

In sum, flexibility of the government budget creates a discretionary incentive that restrains the severity of the time-inconsistency problem of the dynamic tax response. As a result, the government can credibly exploit the intertemporal effect of tax hikes on the real interest rate to improve consumption smoothing of households. Flexibility of the government budget is crucial for the efficacy of taxes in stabilizing the fall of private demand. Next section demonstrates that composition of the fiscal response is reversed when the government budget is balanced period by period.

1.4.4. Comparison with the Balanced Budget Case

I contrast previous analysis to the case where the government is restrained to keep flow budget balanced period by period. When doing so, I do not change the nature of the tax instrument. Government spending has to be financed with revenue collected via the distortionary tax on labor income. The balanced budget assumption eliminates the discretionary incentive to manipulate the real interest rate and prevents households from using variation in government debt holdings to smooth their consumption because bonds are in zero net supply.

[24] This result stands in contrast to the time-inconsistency problem that arises in the models of discretionary policy where the zero lower bound is not binding, see e.g., Debortoli and Nunes (2012) and Leith and Wren-Lewis (2013).

Figure 1.6 shows the impulse responses under the optimal discretionary policy to a positive preference shock equal to the three unconditional standard deviations when the government budget is balanced in every period (solid black lines) compared to the responses from the previous section when the government budget is flexible (dotted red lines, primary balance and debt dynamics are not reported). As in the flexible budget case, following a large negative demand shock the short-term nominal interest rate is brought down to the zero lower bound in an attempt to offset increased savings desire of the household. The fiscal responses are qualitatively similar to the case with government debt dynamics. Both government spending and the labor income tax rate temporarily increase. When the government budget is balanced, however, tax increase becomes much more costly instrument from the perspective of the government in a given period of time at the liquidity trap. Therefore, using taxes to push inflation expectations is not a credible stabilization strategy as it is in the case with the flexible government budget. When the government budget is balanced, the real rate stays high and the economy experiences stronger fall of private demand. In response to falling demand wages plummet and the economy experiences larger private consumption gap and falling prices. As a result, the government is pushed toward considerably more pronounced response of government spending in order to cushion the fall of aggregate demand.

When increasing government spending in order to cushion the fall of aggregate demand, the government goes much further to the point where its response is one order of magnitude larger than in the case with government debt dynamics. Not surprisingly, there is less reliance on the tax instrument: the rate set on impact in response to the binding zero lower bound is around one percentage point lower than in the case with government debt dynamics. This change of the roles leads to a different response of hours worked. Stronger use of the government spending and milder use of the labor tax do not depress labor supply as much as when taxes is the prime stabilizing instrument. Under baseline parameterization I find positive hours worked gap at the liquidity trap with a balanced government budget. Really important, however, is the qualitative fact that the this gap is smaller in absolute value than in the case with government debt dynamics. Smaller gap of hours worked comes at a cost of having larger gaps in public and private consumption accompanied by deflation.

Absence of government debt not only affects fiscal policy and the allocation during the liquidity trap, but it also affects policy and the allocation when the zero lower bound is not binding. Without government debt the economy fluctuates around its efficient deterministic steady state, but moderate fluctuations in demand are not precisely offset with variations in the nominal interest rate. Figure 1.7 shows the impulse responses under the optimal discretionary policy following a positive preference shock equal to the one unconditional standard deviation magnitude.

Under a balanced budget assumption, the government decreases nominal rate more aggressively than it does in the economy without balanced budget in response to the preference shock of the same magnitude. Different dynamics during liquidity trap episodes without government debt spill over into dynamics when the zero lower bound is not binding through expectations. Forward-looking agents and firms take into account higher real rates and lower wages corresponding to the states with the binding zero lower bound and adjust correspondingly their consumption and prices downward even when zero lower bound is not binding. The higher is conditional probability of reaching the liquidity trap, the stronger is such pass-through. Monetary policy lowering nominal interest rate more aggressively counteracts this effect. In the equilibrium, more accommodative monetary policy leads to the GDP boom and positive private consumption gap. More accommodative monetary policy also leads to the zero lower bound being hit more often, which increases the overall time the economy spends at the liquidity trap.

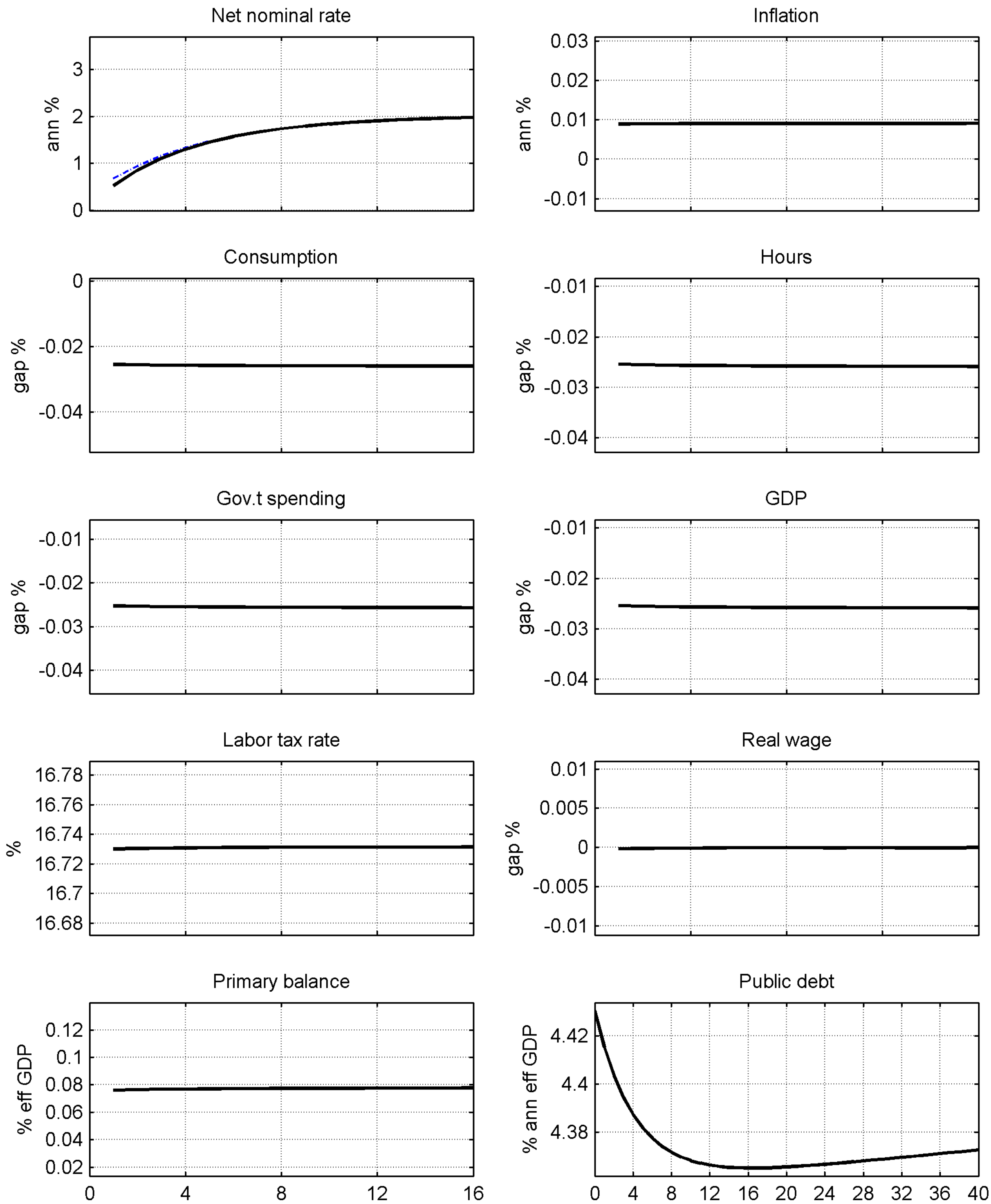
1.5. Conclusion

The global economic downturn put into a liquidity trap many advanced economies. Large fiscal adjustments were implemented in an attempt to stabilize these economies. Interest rates in the advanced countries are projected to stay low in the medium run so that there is a risk of occasionally binding zero lower bound. Hence, it is of policy relevance to assess efficacy of various fiscal instruments in stabilizing the economy during liquidity trap episodes.

In this chapter I characterize optimal monetary and fiscal policy under discretion in a New Keynesian model with recurring episodes of a liquidity trap. I focus the analysis on the use of discretionary government spending and labor income taxation jointly as stabilizing instruments at the liquidity trap. A government that must keep a balanced budget relies more on the spending instrument than a government that is allowed to borrow and can temporarily run down its debt during the liquidity trap. On the other hand, a government that is allowed to borrow and can temporarily run down its debt during the liquidity trap places more weight on the use of the labor income tax.

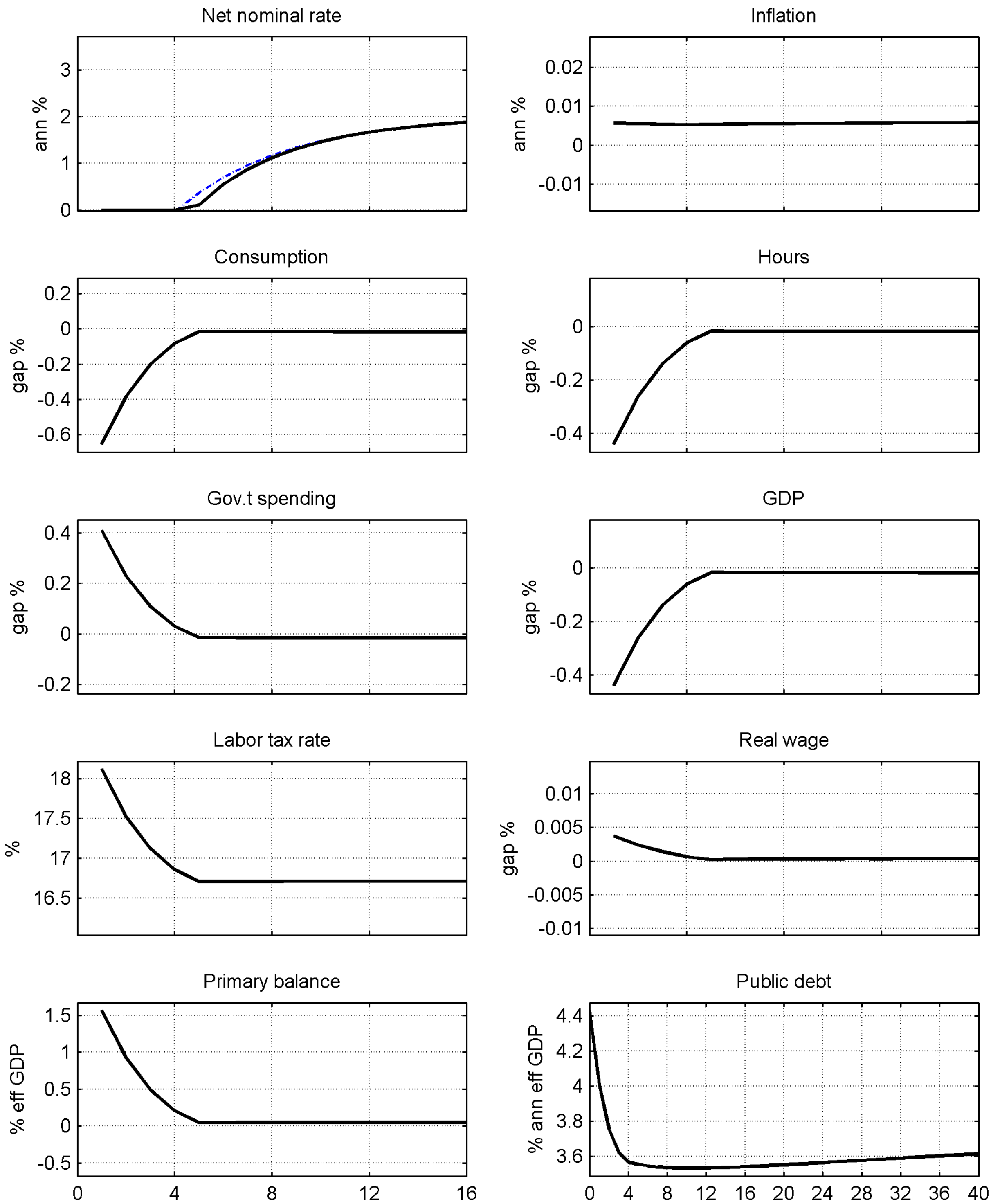
Results of this chapter demonstrate that debt dynamics matter for the policy prescriptions delivered by the model. Along this dimension, the model used in this chapter does not account for important features of government debt in advanced economies such as elevated levels of debt and its market structure with average maturity exceeding several years. In the next chapter I make a step toward overcoming this shortcoming by studying the effect of government debt maturity on the optimal discretionary monetary and fiscal policy in the times when the zero lower bound is not binding.

Figure 1.4. – Dynamics when zero lower bound is not reached



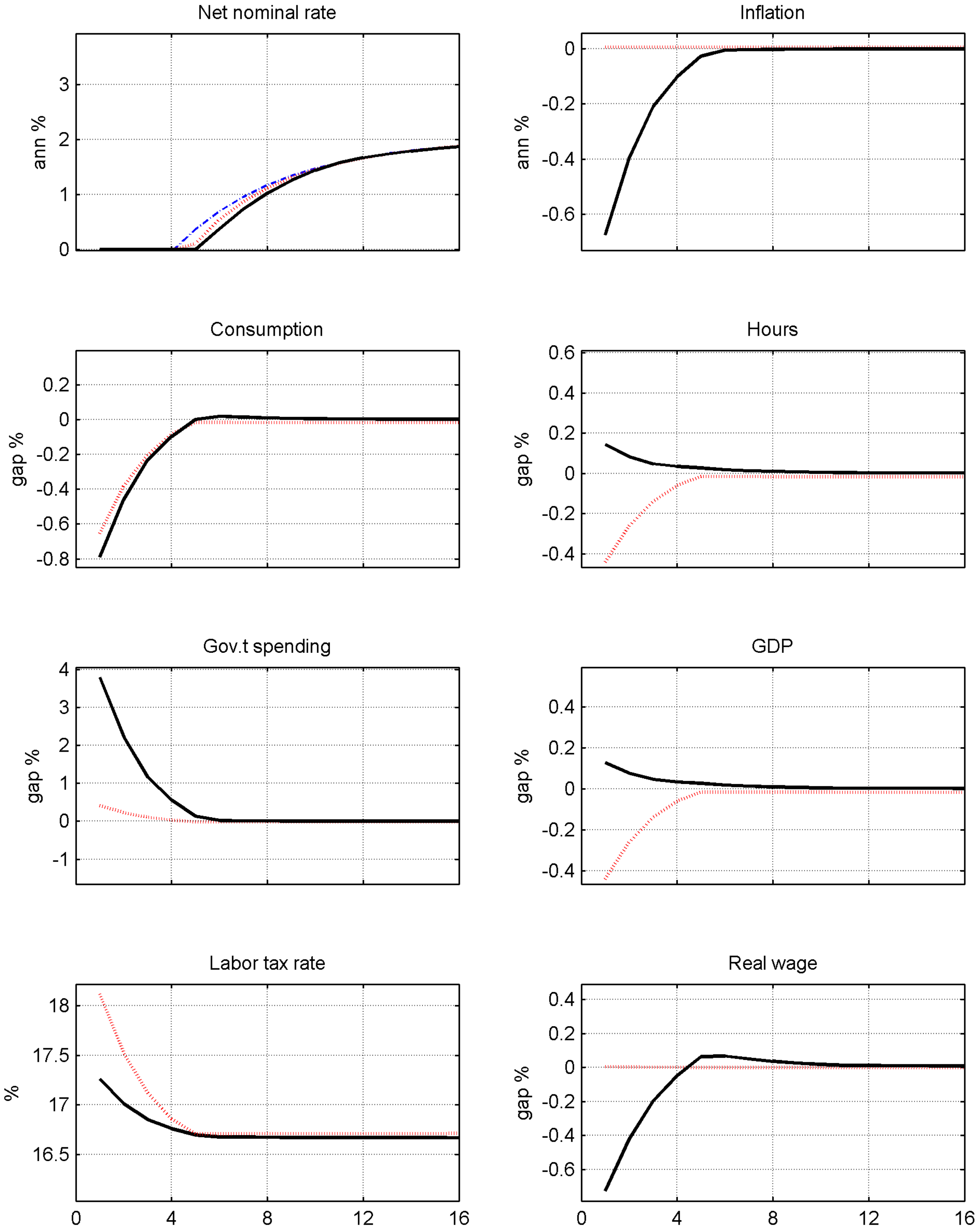
Notes: Impulse responses to a +1 unconditional standard deviation preference shock starting from optimal long run debt level b^*. Dashed blue line in the top left panel shows the (net) natural real interest rate.

Figure 1.5. – Dynamics when zero lower bound is reached



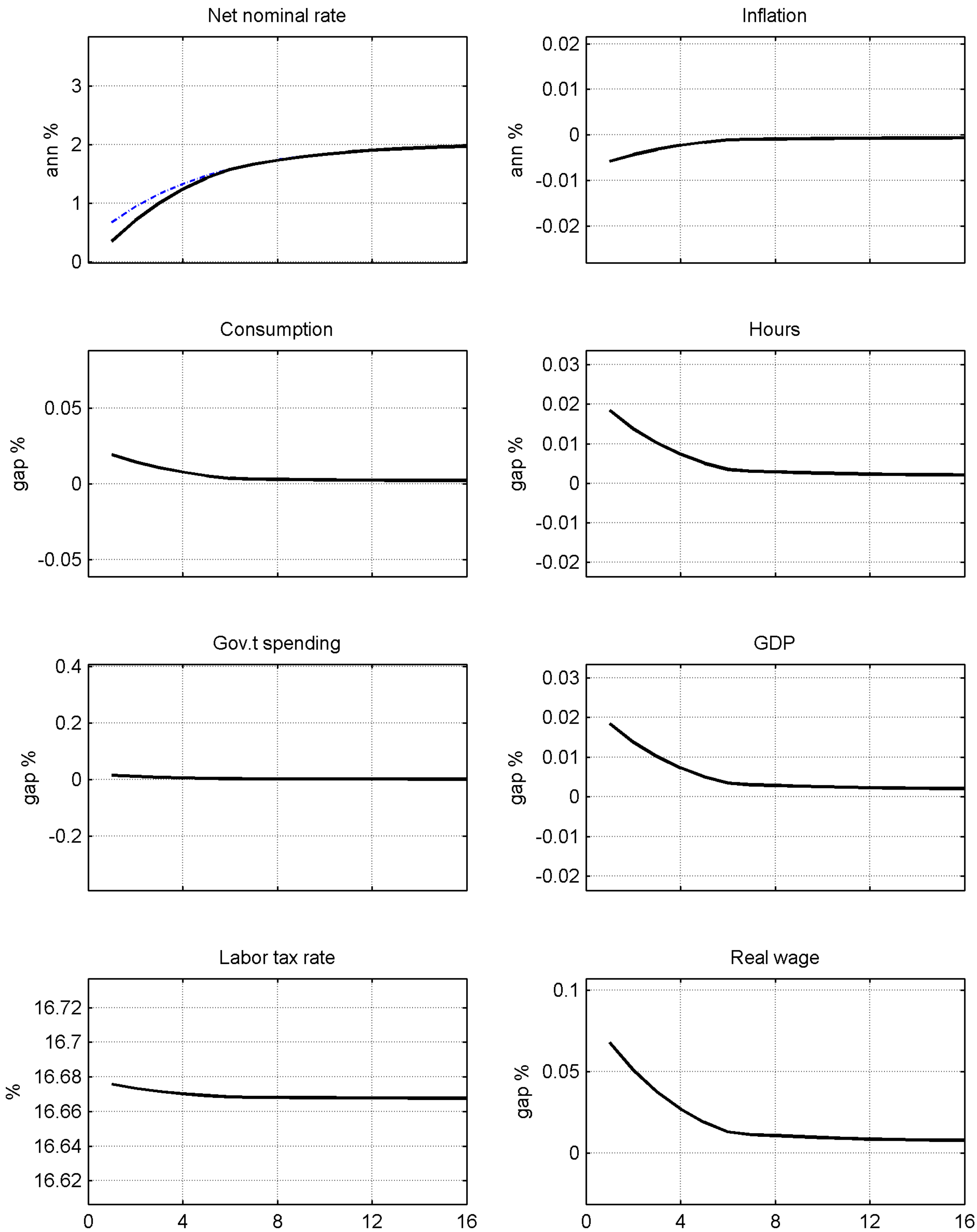
Notes: Impulse responses to a +3 unconditional standard deviations preference shock starting from optimal long run debt level b^*. Dashed blue line in the top left panel shows the natural real interest rate.

Figure 1.6. – Dynamics when zero lower bound is reached: *balanced vs flexible budget*



Notes: Impulse responses to a +3 unconditional standard deviations preference shock. Solid black lines are responses under balanced budget. Dotted red lines are responses under flexible budget. Dashed blue line in the top left panel shows the natural real interest rate.

Figure 1.7. – Dynamics when zero lower bound is not reached under *balanced budget*



Notes: Impulse responses to a +1 unconditional standard deviation preference shock. Dashed blue line in the top left panel shows the natural real interest rate.

2. Debt Maturity, Monetary Policy and Fiscal Sustainability without Commitment

2.1. Introduction

The global financial and economic downturn of 2008-2009 has left a legacy of unprecedented government debt levels in many countries. Not surprisingly, these fiscal developments have resumed and intensified the discussion among policymakers and economists alike regarding issues of fiscal sustainability. Related to these issues is the debate on whether monetary policy and inflation are to play an important role in maintaining fiscal solvency or it is primarily a concern of fiscal policy. The conventional wisdom is that it is optimal for fiscal policy to maintain control over government debt and smooth the associated distortions over time. Renewed concerns of monetary policy taking part in fiscal financing of highly indebted governments are often linked to the maturity structure of debt. Debt of longer maturity can be thought as providing larger scope to achieve fiscal sustainability with the help of monetary policy.

In this chapter, I study normative implications of government debt maturity for the conduct of monetary and fiscal policy. To this end, I use a stylized New Keynesian model where a benevolent government issues nominal bonds and cannot commit to a specific path of policy instruments in the future. The government faces a non-trivial problem of fiscal financing because it does not have access to lump-sum taxes and inflation is not costless. The set of instruments available for the government to maintain fiscal sustainability consists of fiscal policy in the form of government spending on public goods and a flat-rate labor income tax, and monetary policy controlling a short-term nominal interest rate that affects the price at which the government borrows. Adjustment of these policy instruments required to finance government debt is distortionary. The government acts discretionary, each period it determines the amount of distortions it imposes today and the amount it postpones by rolling-over debt into the future. I analyze how the use of policy instruments for fiscal financing depends on the average maturity of government debt. Furthermore, I focus on how debt dynamics depends on its average maturity.

Under lack of commitment, elevated levels of government debt are required to be adjusted over time toward a certain (efficient) long-run level in order to achieve fiscal sustainability. If government debt is assumed to take the form of one-period nominal bonds, for plausible levels of debt, fiscal sustainability requires prompt adjustment of debt, and monetary policy bears a significant burden of implementing the adjustment—see Leith and Wren-Lewis (2013). The novelty of this chapter is to demonstrate how the speed of optimal government debt adjustment and the policy mix that implements it depend on the maturity structure of debt.

I derive linear-quadratic representation of the policy problem and identify two channels that make it optimal for the government to improve policy trade-off in a given period of time by reducing debt in order to strategically affect policy choices in the subsequent period. First, reducing government debt allows to obtain a better inflation-output trade-off, which reduces distortions inflicted by labor taxes required to raise budget revenue. Second, reducing government debt allows to increase the market price of the newly issued bonds. The maturity structure directly affects the second channel that looses its strength as the average maturity of government bonds rises. In particular, lengthening average maturity reduces gains from a *ceteris paribus* rise in the price of the newly issued bonds as those are becoming to a large extent offset by capital losses due to the corresponding increase in the value of the outstanding liabilities. The reduced sensitivity of the government budget to changes in the market price of bonds reduces both the incentive of the government to raise this price and the strategic motive to affect future policy in order to obtain a better trade-off. In equilibrium, this leads to a significant slowdown of the optimal speed of debt adjustment.

Using numerical method to solve nonlinear policy problem, I show that when all government debt is in the form of one-period bonds, it is optimal to cut debt in excess of the long-run level by more than a half within a single quarter under a baseline parameterization. Such a fast speed of adjustment is at odds with an existing empirical evidence on the persistence of the dynamics of government debt. Friedman (2005), for instance, uses the postwar U.S. data and finds a half-life equal to 85 quarters for the debt-to-GDP ratio response to its own shock in a univariate autoregressive setting.[1] Accounting for a plausible average maturity of government debt makes debt adjustment under optimal discretionary policy much more in line with the empirical evidence. I show that in the case of average maturity of government debt equal to 4 years, the half-life of debt adjustment is equal to 92 quarters under the baseline parameterization.

This chapter contributes to the debate on interaction between monetary and fiscal policy in several ways. First, the consensus assignment—discussed in Kirsanova et al. (2009)—dictates to prevent monetary policy from controlling debt dynamics. Optimal discretionary policy, however, does not necessarily support the consensus and may feature monetary policy that accommodates adjustment of government debt towards the long-run level. For plausible levels of government debt in the form of one-period bonds, monetary policy supports fiscal financing by holding real interest rates down. I show that this result is specific to the case of government debt with short average maturity.[2] In the case of government bond portfolios with average maturity in the range from several years and higher real interest rates are held elevated throughout the adjustment of excessive government debt. Such a stance of monetary policy slows down debt adjustment compared to what is implied by the observed fiscal policy alone.

Second, there is a pressing policy concern on whether longer maturity of government debt increases incentives to use inflation so as to maintain fiscal sustainability. Faraglia

[1] Marcet and Scott (2009) provide an extensive evidence of high persistence of the market value of debt.

[2] Bhattarai et al. (2014) show how shortening average can improve stabilization if the nominal interest rate is at the zero lower bound.

et al. (2013) and Leeper and Zhou (2013) provide a rationale for a longer average maturity to lead to a stronger and more persistent inflation under the assumption of the joint monetary and fiscal commitment. In these studies, however, there is no intrinsic incentive to bring the level of debt towards a certain long-run level because government debt optimally follows a near random walk behavior. In this chapter, on the other hand, under discretion debt is required to be reduced until reaching the steady-state.[3] As average maturity of government debt affects the monetary-fiscal policy mix that implements debt adjustment it consequently affects the strength and persistence of inflation. In particular, as the average maturity lengthens monetary policy switches away from accommodating debt adjustment and becomes anti-inflationary. This is consistent with an empirical finding of Rose (2014) who shows that the existence of long-term government bonds markets may help keep inflation low and stable. I show that contribution of inflation to debt adjustment declines by a factor of two in the case of average maturity of government debt equal to 4 years compared to one-period bonds.

More broadly, this chapter is also related to the literature studying optimal maturity structure of government debt. A number of studies incorporate portfolio problem into the optimal problem of the government under the assumption of commitment—see, *e.g.*, Angeletos (2002), Buera and Nicolini (2004), Faraglia et al. (2014), and Lustig et al. (2008). More closely related are the studies that introduce lack of commitment and solve for optimal maturity structure in the real economy, see Debortoli et al. (2015), and in the economy with nominal frictions, see Arellano et al. (2013). Differently from these studies, I treat the maturity structure as determined by a separate debt management authority that is not explicitly modeled. Optimal fiscal and monetary policy are chosen for a given average maturity of government debt that does not change over time, which is not much at odds with the stability of portfolio shares documented in Faraglia et al. (2014).

This chapter is organized as follows. Section 2.2 describes the model economy and the corresponding first-best allocation. Section 2.3 defines the nonlinear policy problem and its linear-quadratic representation. Section 2.4 derives analytical insights in the linear-quadratic setting. Section 2.5 describes numerical solution of the nonlinear policy problem. Section 2.6 concludes.

2.2. The Model

This section describes the economy, defines the competitive equilibrium and characterizes the first-best allocation. I consider a standard dynamic stochastic general equilibrium New Keynesian model of a closed economy with monopolistically competitive intermediate goods markets and costly price adjustment. The model is similar to the one described in Chapter 1. The main differences are: (1) here I abstract from uncertainty and, therefore, from occasionally binding zero low bound; (2) here the structure of the government

[3] Adam (2011) shows that technology shocks produce budget risk that makes it optimal to reduce government debt over time even under commitment. Horvath (2011) shows that assuming unconditional welfare objective also implies gradual adjustment of government debt towards its mean value.

budget is different in that it allows for the average maturity of government debt to go beyond one-period.

2.2.1. Households

The economy is populated by a continuum of ex-ante and ex-post identical households. The representative household consumes a composite good, which is produced from a continuum of differentiated products, indexed by $i \in [0,1]$, using constant-elasticity-of-substitution production technology. Total supply of the aggregate good, Y_t, is given by

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where $Y_{i,t}$ is the input of differentiated good i, and $\theta > 1$ is the intratemporal elasticity of substitution across different varieties of differentiated goods. This composite good can be used for private and public consumption. Private consumption of the aggregated good is denoted by C_t, and G_t denotes spending on public good provision by the government.

The representative household values private consumption, enjoys consumption of the public good, and dislikes labor. Preferences of the representative household are represented by

$$\sum_{t=0}^{\infty} \beta^t \left[u(C_t) + g(G_t) - v(h_t) \right], \tag{2.1}$$

where $\beta \in (0,1)$ is the discount factor. I assume that functions u and g are increasing and concave, and function v is increasing and convex. Total units of labor H_t are composed of the quantities (hours) of labor $H_{i,t}$ supplied to each intermediate goods-producing firm

$$H_t = \int_0^1 H_{i,t} di.$$

Given prices of the intermediate goods, $P_{i,t}$, the price index P_t that corresponds to the minimum cost of a unit of the aggregate good is computed as follows:

$$P_t = \left(\int_0^1 P_{i,t}^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \tag{2.2}$$

The household enters period t holding assets in the form of nominal non-contingent one-period government bonds B_{t-1}^S and a long-term portfolio of government bonds B_{t-1}. As in Woodford (2001), the portfolio of long-term bonds is defined as a set of perpetual bonds with payoffs that decay exponentially at the rate ρ. One-period bonds issued in period t can be purchased at a price R_t^{-1}, where R_t is the nominal interest rate. Non-decayed long-term portfolio, ρB_{t-1}, is exchanged for a new portfolio issued in period t at a market price q_t. The supply of bonds in equilibrium is determined by government policies, which will be discussed later. Under described market arrangement, a household's flow nominal budget constraint is of the following form:

$$P_tC_t + R_t^{-1}B_t^s + q_tB_t = (1-\tau_t)W_tH_t + B_{t-1}^s + (1+\rho q_t)B_{t-1} + \int_0^1 \Pi_{i,t}di - T_t, \quad (2.3)$$

where W_t is the nominal wage, $\Pi_{i,t}$ is the share of profits from sales of differentiated good of type i distributed in a lump-sum way, and T_t is the lump-sum government transfer.[4] Labor income of the household is taxed at a linear tax rate τ_t. To avoid complicating the notation I do not explicitly describe the market for private claims. Regardless, this setup is isomorphic to the model with a complete set of private state-contingent securities, as these would not be traded in equilibrium under the assumption of representative household. Also, note that I consider a "cashless" limit of the monetary economy in the spirit of Woodford (2003) and therefore I abstract from money holdings. To have a well-defined intertemporal budget constraint I implicitly impose an additional constraint that rules out "Ponzi schemes".

The household maximizes (2.1) by choosing consumption, labor and bond purchases $\{C_t, H_t, B_t^S, B_t\}_{t=0}^{\infty}$ subject to the budget constraint (2.3) and no-Ponzi condition, taking as given prices, policies and firms' profits $\{P_t, W_t, R_t, \tau_t, G_t, T_t, \Pi_t(i)\}_{t=0}^{\infty}$, and initial bond holdings B_{-1}^S and B_{-1}.

The optimal plan of the household has to satisfy (2.3) and a standard transversality condition, as well as the following first-order conditions:

$$R_t^{-1} = \frac{\beta}{\pi_{t+1}} \frac{u'(C_{t+1})}{u'(C_t)}, \quad (2.4)$$

$$\frac{q_t}{1+\rho q_{t+1}} = \frac{\beta}{\pi_{t+1}} \frac{u'(C_{t+1})}{u'(C_t)}, \quad (2.5)$$

$$w_t = \frac{1}{(1-\tau_t)} \frac{v'(H_t)}{u'(C_t)}, \quad (2.6)$$

where $\pi_{t+1} \equiv {}^{P_{t+1}}/_{P_t}$ is the gross one period inflation rate, and $w_t \equiv {}^{W_t}/_{P_t}$ is the real wage. Equation (2.6) describes intratemporal trade-off between consumption and leisure. Equations (2.4) and (2.5) are Euler equations describing intertemporal allocation of consumption and savings. Combining Euler equations (2.4) and (2.5) yields the no-arbitrage condition between one-period interest rate and the price of long-term bonds

$$R_t = \frac{1+\rho q_{+1}}{q_t}. \quad (2.7)$$

2.2.2. Firms

There is a continuum of firms of unit mass producing imperfectly substitutable differentiated goods with a technology that is linear in labor $Y_{i,t} = H_{i,t}$. The firm producing

[4] The lump-sum transfer is used for the sole purpose of financing labor (employment) subsidy at the steady state. See discussion below.

good i sets the price $P_{i,t}$ and hires, in a perfectly competitive labor market, the quantity of labor that is necessary to satisfy realized demand. I assume that the government allocates its spending on the good varieties identically to the household. The resulting demand schedule for good i is then

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} Y_t, \tag{2.8}$$

Imperfect price-elasticity of demand endows firms with market power to set prices, which in general distorts the economy. Present discounted real value of a firm's profits is given by

$$\sum_{t=0}^{\infty} \beta^t \lambda_t \left[\frac{P_{i,t}}{P_t} Y_{i,t} - (1-s)\frac{W_t}{P_t} Y_{i,t} - \kappa_{i,t} Y_t\right], \tag{2.9}$$

where λ_t is the marginal utility of real income t periods ahead for the representative household, and s is a time-invariant rate of labor (employment) subsidy. This subsidy can be used by the government to eliminate the deterministic steady state distortions associated with monopolistic competition and a distortionary labor income tax.[5] I also assume that there is a nominal rigidity that makes price adjustment costly for a firm. Following Rotemberg (1982), I model the nominal rigidity by introducing a quadratic cost of adjusting nominal prices:

$$\kappa_{i,t} \equiv \frac{\varphi}{2}\left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right)^2,$$

where $\varphi \geqslant 0$ measures the degree of nominal price rigidity. Higher values of φ indicate greater price stickiness, while $\varphi = 0$ corresponds to the case of perfectly flexible prices.

The pricing problem of a firm is dynamic due to the presence of adjustments costs. A firm i chooses prices $\{P_{i,t}\}_{t=0}^{\infty}$ so as to maximize its profits (2.9) subject to demand function (2.8), taking as given nominal wage, aggregate index of prices, and aggregate demand $\{W_t, P_t, Y_t\}_{t=0}^{\infty}$. In equilibrium firms behave symmetrically and charge identical prices $P_{i,t} = P_t$ for all $i \in [0,1]$.[6] Then, optimizing behavior of firms described by a first-order condition of the firm's problem implies

$$\theta\left((1-s)w_t - \frac{(\theta-1)}{\theta}\right) = \varphi\left((\pi_t - 1)\,\pi_t - \beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Y_{t+1}}{Y_t}(\pi_{t+1}-1)\,\pi_{t+1}\right). \tag{2.10}$$

Condition (2.10) is a New Keynesian Phillips curve stating that current inflation depends on the marginal cost of production and next period inflation.

[5] The specification follows Leith and Wren-Lewis (2013). One can alternatively design a labor income subsidy, which works equivalently when the labor market is competitive.

[6] Symmetric pricing behavior in equilibrium leads to all the firms, for $i \in [0,1]$, produce the same output $Y_{i,t} = Y_t$, and hire the same amount of labor $h_{i,t} = h_t$.

2.2.3. The Government

The government consists of a central bank and a treasury. The treasury decides the amount of spending on public good provision to the household, G_t. To finance spending on public good, the treasury levies a labor income tax, τ_t, and participates in the bond market. The central bank controls the short-term nominal interest rate by means of open-market operations that vary the level of the real money balances held by the household. Using framework of the "cashless" limit allows me to abstract from seigniorage revenues obtained by the government. In equilibrium, the policy rate controlled by the central bank by no arbitrage condition has to be equal to the nominal interest rate on one-period government bonds, R_t. I assume that one-period bonds are in zero net supply, hence consolidated flow budget constraint of the government is given by

$$q_t B_t = P_t \left(G_t - (\tau_t - s) w_t H_t \right) + (1 + \rho q_t) B_{t-1} - T_t.$$

I assume that lump sum transfers T_t are used for the sole purpose of transferring resources corresponding to the labor subsidy. Furthermore, since the purpose of the labor subsidy is to address only permanent distortions in the economy, I set the real value of the lump-sum transfers over time equal to the steady-state value of the subsidy level. The flow budget constraint of the government in real terms is then given by

$$q_t b_t = (1 + \rho q_t) \frac{b_{t-1}}{\pi_t} + \left(G_t + \varsigma_t - \tau_t w_t H_t \right), \tag{2.11}$$

where $b_t \equiv {}^{B_t}/_{P_t}$ is the real value of government bonds, and $\varsigma_t \equiv s w_t Y_t - s w^{ss} Y^{ss}$ is the real deviation of the subsidy from its steady state level. The payoff decay factor ρ parametrizes average maturity of the portfolio of long-term government bonds and is assumed to be time-invariant. Setting $\rho = 0$ makes duration of all the bonds equal to one period. Another extreme is the case of consol bonds when $\rho = 1$. In general, if inflation were to stay at zero the duration of the portfolio of long-term government bonds would be equal to $(1 - \beta\rho)^{-1}$.

No-arbitrage condition (2.7) with recursive substitution can rewritten as

$$q_t = \sum_{k=0}^{\infty} \frac{\rho^k}{R_t R_{t+1} \dots R_{t+k}},$$

which shows that the government sells the portfolio of long-term bonds issued at period t at a market price that in equilibrium depends on the current as well as future monetary policies reflected by choices of the short-term nominal interest rate, R_t. One can then think of $\{R_t, G_t, \tau_t\}_{t=0}^{\infty}$ as of the government policies that, at the equilibrium prices, uniquely pin down $\{b_t\}_{t=0}^{\infty}$ as satisfying (2.11). The government's problem will be introduced and discussed in the next section.

2.2.4. The Competitive Equilibrium

In equilibrium all the firms employ the same amount of labor, $H_{i,t} = H_t$. The aggregate production function is then

$$Y_t = H_t, \tag{2.12}$$

and the aggregate resource constraint resulting from the clearing of the goods market is given by

$$H_t = C_t + G_t + \frac{\varphi}{2}(\pi_t - 1)^2 H_t, \tag{2.13}$$

The aggregate resource constraint shows that nominal rigidity creates a wedge between aggregate demand and aggregate output because a fraction of the output goes into the price adjustment cost.

I define a competitive equilibrium as follows.

Definition 2.1 (Competitive equilibrium)**.** *A rational expectations equilibrium is a sequence* $\{C_t, Y_t, H_t, \pi_t, w_t, q_t, R_t, b_t, G_t, \tau_t\}_{t=0}^{\infty}$ *satisfying equations (2.5)–(2.7), (2.10), (2.11)–(2.13) and the transversality condition, for* $t \geqslant 0$*, given initial outstanding government debt* b_{-1}*.*

2.2.5. The First-Best Allocation

The first-best allocation is the one that maximizes the household's utility (2.1) subject to the technology constraints while abstracting from economic distortions in the private markets. Such an allocation is efficient and coincides with the solution of the fictitious Social Planner's problem. As in the model from Chapter 1, efficiency dictates, for all $t \geqslant 0$, that the marginal utilities of private and public consumption be set equal to the marginal disutility of labor

$$u'(C_t) = v'(H_t),$$
$$g'(G_t) = v'(H_t).$$

These optimality conditions yield the efficient allocation with constant hours worked, consumption, government spending and output.

2.3. The Policy Problem

I assume that there is full cooperation between the central bank and the treasury and that such a government acts benevolently with the objective of maximizing the lifetime utility (2.1) of the representative household. The government controls policy instruments at its disposal in order to achieve optimal allocation as a part of the decentralized competitive equilibrium. I model the government as not being able to commit to its future choices and instead acting discretionary in every period of time. The government, however, can still credibly commit to repay its debt in the future.[7] I abstract from reputation mechanisms and define optimal policy as an outcome of dynamic game between successive selves of the government as if these were separate policymakers in every period of time. I analyze a stationary Markov-perfect equilibrium of this game as defined, for instance, in Klein et al. (2008).

2.3.1. The Markov-Perfect Equilibrium

In a Markov-perfect equilibrium, strategies of the government depend on the minimal payoff-relevant state of the economy, which in every period t is entirely described by the amount of inherited government liabilities from the previous period b_{t-1}. Each period t, the government maximizes utility of the representative household starting from its incumbent period onwards. When making the announcement of policy for the current period t, the government takes into account how the private sector reacts, given anticipated future policy.

Formally, the Markov optimization problem of discretionary government in any period t can be written as choosing $\{C_t, Y_t, H_t, \pi_t, w_t, q_t, R_t, b_t, G_t, \tau_t\}$ that maximize

$$u(C_t) + g(G_t) - v(H_t) + \beta\mathcal{V}(b_t)$$

subject to

$$0 = \mathbf{\Phi}(b_{t-1}; C_t, Y_t, H_t, \pi_t, w_t, q_t, R_t, b_t, G_t, \tau_t; \mathcal{C}(b_t), \mathcal{Y}(b_t), \mathit{\Pi}(b_t), \mathcal{Q}(b_t)),$$

given outstanding amount of debt in the beginning of period, b_{t-1}, and anticipated future policy together with implied allocation and prices in the competitive equilibrium as described by functions $\{\mathcal{C}, \mathcal{Y}, \mathcal{H}, \mathit{\Pi}, \mathcal{W}, \mathcal{Q}, \mathcal{R}, \mathcal{B}, \mathcal{G}, \mathcal{T}\}$ that provide net present utility $\mathcal{V}$. For convenience, the vector-function $\mathbf{\Phi}$ is used to summarize the set of constraints that consists of competitive equilibrium conditions from definition 2.1.

For optimal policy to be time-consistent, the government should find no incentives to deviate from the anticipated rules. This idea is captured in the formal definition of the Markov-perfect equilibrium.

Definition 2.2 (Markov-perfect equilibrium)**.** *A Markov-perfect equilibrium is a function $\mathcal{V}(b_{t-1})$ and a tuple of rules $\{\mathcal{C}, \mathcal{Y}, \mathcal{H}, \mathit{\Pi}, \mathcal{W}, \mathcal{Q}, \mathcal{R}, \mathcal{B}, \mathcal{G}, \mathcal{T}\}$ each being a function of b_{t-1}, such that for all b_{t-1}:*

1. *Given $\mathcal{V}$, tuple of rules solves Markov problem of the government,*

2. *$\mathcal{V}$ is the value function of the government*

$$\mathcal{V}(b_{t-1}) = u(\mathcal{C}(b_{t-1})) + g(\mathcal{G}(b_{t-1})) - v(\mathcal{H}(b_{t-1})) + \beta\mathcal{V}(\mathcal{B}(b_{t-1})).$$

[7]Credible debt repayment can be supported by a high implicit cost of the outright debt default.

I restrict attention to equilibria with differentiable value and policy functions. Assuming that such an equilibrium exists, it can be characterized by the first-order conditions of the government problem. Deriving these conditions is similar to the model in Chapter 1 and I omit this step for brevity.

Following Leith and Wren-Lewis (2013), I assume that a labor subsidy is used by the government to offset permanent distortions in the economy.

Proposition 2.1 (Steady-State)**.** *For a given level b of government debt, there exists a unique employment subsidy rate s such that Markov-perfect equilibrium features a steady-state with debt level b, zero inflation, and an allocation that coincides with the efficient allocation.*

Proof. See the proof of Proposition 1.1. □

The proof of this proposition is instructive in how to design the subsidy to support any debt-to-GDP ratio with efficient allocation. Note, however, that when the economy is away from the steady-state its allocation is not efficient.

For the remainder of the chapter I assume preferences of the households for consumption of private and public goods are described by $u(c_t) \equiv \frac{c_t^{1-\gamma_c}}{1-\gamma_c}$, $g(G_t) \equiv \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g}$, and the disutility from work is described by $v(h_t) \equiv \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h}$.

2.3.2. Linear-Quadratic Representation

Before solving numerically the fully non-linear policy problem defined above, I reformulate the policy problem in the linear-quadratic form. Doing so helps to obtain a number of analytical and clear numerical insights discussed in the following section. In order to obtain linear-quadratic representation of the policy problem, one has to quadratically approximate objective function of the government, whereas equilibrium constraints are approximated (log-)linearly.[8] The approximation is done around an (efficient) deterministic steady-state with zero inflation and non-zero amount of government debt.

In the linear-quadratic representation, the Markov-perfect equilibrium can be associated with a solution to the following Bellman equation

$$\mathcal{W}(\hat{b}_{t-1}) = \max_{\{\hat{C}_t, \hat{Y}_t, \hat{\pi}_t, \hat{w}_t, \hat{q}_t, \hat{i}_t, \hat{b}_t, \hat{\tau}_t, \hat{G}_t\}} \left[\left(-\eta_\pi \hat{\pi}_t^2 - \eta_y \hat{Y}_t^2 - \eta_c \hat{C}_t^2 - \eta_g \hat{G}_t^2 \right) + \beta \mathcal{W}(\hat{b}_t) \right]$$

subject to

[8] See Woodford (2003) for a detailed discussion of the linear-quadratic approach. One could alternatively derive first-order approximation of the first-order optimality conditions of the non-linear Markov-perfect equilibrium.

$$0 = \hat{C}_t - \frac{\bar{Y}}{\bar{c}}\hat{Y}_t + \frac{\bar{G}}{\bar{c}}\hat{G}_t, \tag{2.14}$$

$$0 = \hat{q}_t + \hat{\Pi}(\hat{b}_t) - \beta\rho\hat{\mathcal{Q}}(\hat{b}_t) - \gamma_c\left(\hat{C}_t - \hat{\mathcal{C}}(\hat{b}_t)\right), \tag{2.15}$$

$$0 = \hat{\omega}_t - \frac{\bar{\tau}}{1-\bar{\tau}}\hat{\tau}_t - \gamma_c\hat{C}_t - \gamma_h\hat{Y}_t, \tag{2.16}$$

$$0 = \hat{\omega}_t - \frac{\phi}{\theta-1}\hat{\pi}_t + \beta\frac{\phi}{\theta-1}\hat{\Pi}(\hat{b}_t), \tag{2.17}$$

$$0 = \frac{\hat{b}_{t-1}}{\beta} - (1-\rho)\,\hat{q}_t - \frac{\hat{\pi}_t}{\beta} - \hat{b}_t + \frac{\bar{G}}{\bar{\Gamma}}\hat{G}_t - \frac{\bar{\omega}\bar{Y}}{\bar{\Gamma}}\left((\bar{\tau}-s)\left(\hat{Y}_t + \hat{\omega}_t\right) + \bar{\tau}\hat{\tau}_t\right), \tag{2.18}$$

$$0 = \hat{i}_t + \hat{q}_t - \beta\rho\hat{\mathcal{Q}}(\hat{b}_t) \tag{2.19}$$

where the stabilization weights η_π, η_y, η_c, and η_g depend on the structural parameters. Hat variables denote percentage deviations from the steady-state. Following conventional notation, the deviation of the short-term nominal interest rate is denoted as $\hat{i}_t$. Bar variables denote steady-state values and $\bar{\Gamma} \equiv \bar{b}\bar{q}$ is the market value of government debt in the steady-state.

2.4. Optimal Time-Consistent Policy: LQ Analytics

This section characterizes linear-quadratic representation of the Markov-perfect equilibrium. Such a characterization allows me to provide analytical insights that ease the understanding of fully nonlinear results presented in the next section.

Let $\lambda_{q,t}$ and $\lambda_{b,t}$ denote Lagrange multipliers corresponding to the Euler equation, (2.15), and the government budget constraint, (2.18), respectively. Solving a system of the first-order conditions corresponding to the linear-quadratic representation of the Markov-perfect equilibrium, one can derive the following optimality condition linking these two multipliers:

$$\lambda_{q,t} = (1-\rho)\lambda_{b,t}. \tag{2.20}$$

Condition (2.20) shows the extent to which tightness of the government budget is transmitted on to monetary policy. With increasing average maturity of government debt fiscal financing needs become less of a drag on to monetary policy. As $\rho \to 1$, the term $(1-\rho)\hat{q}_t$ in the government budget constraint, (2.18), goes to zero because the gain from selling bonds at a higher price is exactly offset by an increase of the value of outstanding liabilities.[9] In the case, when the entire debt is in the form of consol bonds, so that $\rho = 1$, monetary policy is completely detached from fiscal financing because government budget becomes effectively immune to variations of the roll-over price, $\hat{q}_t$. This interrelation is key to the effects of government debt maturity for optimal policy and dynamics of the economy described below.

One can also show that there is a direct interrelation between Lagrange multiplier

[9]This statement implictly assumes that the price $\hat{q}_t$ does not change unboundedly as $\rho \to 1$.

corresponding to the government budget constraint, $\lambda_{b,t}$, and the rate of inflation, $\hat{\pi}_t$. Therefore, under an assumption that outstanding amount of debt above the efficient level makes the government budget constraint tighter and *vice versa,* it follows that the inflation rate $\hat{\pi}_t$ is an increasing function of the outstanding government debt, $\hat{b}_{t-1}$.[10] This implicit connection can be used to infer debt dynamics through dynamics of inflation. This connection also means that higher than the steady-state debt levels are associated with positive inflation. The straightforward implication is that part of an excessive debt burden is adjusted by inflating its value away: in the current period and in the future periods if debt dynamics is persistent. Moreover, positive inflation rates also implicitly capture combinations of the use of government policies for fiscal financing through other channels, such as government revenue and the price of borrowing. In what follows I exploit these links to characterize equilibrium in more details.

Using the first-order conditions together with the envelope condition one can eliminate Lagrange multipliers altogether and, employing the resource constraint (2.14), obtain the following subset of the necessary optimality conditions:

$$\hat{C}_t = \Omega_c \hat{\pi}_t, \tag{2.21}$$

$$\hat{Y}_t = \Omega_y \hat{\pi}_t, \tag{2.22}$$

$$\hat{G}_t = \Omega_g \hat{\pi}_t, \tag{2.23}$$

$$0 = \left[\hat{\pi}_{t+1} - \hat{\pi}_t\right] + \frac{\beta}{\bar{\Gamma}}\left[\frac{\phi}{\theta}\bar{Y}\mathit{\Pi}_{b,t+1}\right]\hat{\pi}_t + (1-\rho)\left[\gamma_c \mathcal{C}_{b,t+1} + \mathit{\Pi}_{b,t+1} - \beta\rho\mathcal{Q}_{b,t+1}\right]\hat{\pi}_t, \tag{2.24}$$

where the coefficients Ω_c, Ω_y and Ω_g depend on the structural parameters including parameter ρ governing average maturity of the portfolio of government bonds, as well as on the level of government debt in the steady state, $\bar{b}$. The expressions defining the coefficients Ω_c, Ω_y and Ω_g are delegated to Appendix B.1. The Markov-perfect equilibrium can therefore be characterized by the system of equations (2.15)–(2.18) and (2.21)–(2.24).

2.4.1. Dynamics of Debt Adjustment

Equation (2.24) describes optimal balancing of the intertemporal trade-off faced by the government and is often referred to as a *generalized* Euler equation (GEE) due to the presence of derivatives of equilibrium functions. The GEE is written as a linear combination of one of the welfare-relevant wedges—inflation deviation from the efficient level of zero—in the two consecutive periods. This equation is trivially satisfied in the efficient steady-state where the wedge is closed because inflation is equal to zero. If the economy is away from the steady state with debt level different from the efficient one then the

[10] It is not possible to derive an explicit expression in terms of the structural parameters under which this assumption holds. However, in the numerical simulations this assumption is always satisfied. Moreover, in an otherwise similar model with taxes of the costly lump-sum type *à la* Barro (1979), Bhattarai et al. (2014) prove this property to hold analytically for $\rho < 1$.

optimal policy has to balance direct gains from smoothing inefficient debt burden and corresponding policy distortions, reflected by inflation, over time versus indirect losses coming from anticipated effects of doing so on discretionary policy incentives in the next period. Overall, the GEE is a crucial element that determines whether the steady-state is (locally) stable and if it is stable—the case that I implicitly assume in the discussion that follows—the speed with which the economy converges to the steady-state.

The direct gains from the intertemporal smoothing of distortions are reflected by the first term in the GEE, which introduces a random-walk component into the dynamic behavior of the economy. The indirect losses from postponing adjustment of the inefficient debt burden come from worsening of the current policy trade-offs due to implied policy in the next period. First indirect loss—reflected by the second term in the GEE—comes from the marginal effect of having extra inflation in the next period, which tightens the Phillips curve, (2.17), today and pushes government revenues down making revenue collection required to service existing amount of debt more distortionary. Seconds indirect loss—reflected by the third term in the GEE—comes from the marginal effects on the next period consumption, inflation, and the price of government bonds, which make the Euler equation, (2.15), tighter today and so that it becomes more expensive to borrow today in order to roll-over the existing amount of debt. The indirect effect on the next period price of government bonds is present only in the case of the bond portfolio with the average maturity longer than one period, $\rho > 0$.

The presence of terms associated with the indirect losses from the intertemporal smoothing in the GEE introduces a transitory component and affects the ultimate persistence of the dynamic behavior of the economy. In particular, it follows directly from the GEE that detnamics of inflation, $\hat{\pi}_t$, is described by an autoregressive process with a time-varying coefficient. Using equations (2.21)–(2.23) that are discussed in more details below, it is then straightforward to see that private consumption, $\hat{C}_t$, government spending, $\hat{G}_t$, and output, $\hat{Y}_t$ inherit persistence of the inflation rate, $\hat{\pi}_t$. In general, the GEE implicitly determines persistence of the underlying endogenous state of the economy described by the outstanding amount of government debt. The stronger are the indirect losses from the intertemporal smoothing the faster has to be the debt adjustment required to bring economy back to the steady state. Importantly, the weights of the terms corresponding to the indirect losses in the GEE depend on the level of government debt in the steady state through its market value, $\bar{\Gamma}$, and parameter ρ governing average maturity of the portfolio of government bonds. Therefore economies that differ along these two characteristics are going to have different paths and speed of the stead-state convergence.

The larger is the parameter ρ the lower is the weight attached to the GEE term capturing effect of anticipated future policy on the current roll-over price. In other words, for a given anticipated marginal effect of intertemporal smoothing of fiscal financing on the roll-over price, $\hat{q}_t$, the corresponding indirect loss associated with the gradual debt adjustment decreases with the average maturity of government bonds. This is a direct consequence of the reduction in sensitivity of government budget to variations of the roll-over price, $\hat{q}_t$, at longer average maturities captured by condition (2.20). The lower is the sensitivity of the current budget with respect to the roll-over price, $\hat{q}_t$, the lower is the urge to deviate from the intertemporal smoothing in order to strategically manipulate future policy so as to improve current trade-off. Of course, in equilibrium the derivatives that determine anticipated marginal effects from postponing debt adjustment also implicitly depend on the average maturity.

2.4.2. Instruments of Debt Adjustment

Equations (2.21)–(2.23) constitute a set of discretionary "flexible target criteria".[11] These equations describe optimal intratemporal relationship between deviations of allocation of private consumption, $\hat{C}_t$, and government spending, $\hat{G}_t$, output, $\hat{Y}_t$, and inflation, $\hat{\pi}_t$, in the vicinity of the deterministic steady-state. The sign of the government spending gap—robust to alternative plausible parameters values—is the opposite to that of the inflation because $\Omega_g < 0$ in the equation (2.23).[12] This property implies that part of an excessive debt burden is adjusted by reducing government spending until the adjustment is not over. On the other hand, the interrelation between the signs of targets in the remaining two criteria may switch depending on the amount of outstanding government debt in the steady state and the average maturity of government bonds. This dependence can be seen on the two left panels of Figure 2.1 that depict signs of the coefficients Ω_c and Ω_y for various combinations of the steady-state level and the average maturity of government debt whilst other parameters are set equal to their values from the baseline parameterization of the model discussed in the next section.

Consider a standard case of one-period bonds that corresponds to $\rho = 0$.[13] For small steady-state debt-to-GDP ratios deviations of consumption and output are of the opposite sign to deviations of inflation because both Ω_c and Ω_y are negative. Using this interrelation, one can then deduce the role of taxes in debt adjustment. To do so, I rewrite the Phillips curve, (2.17), using flexible targeting criteria (2.21)–(2.22) and equation (2.16) describing optimal intratemporal choice of the representative household between consumption and leisure:

$$\left[\frac{\phi}{\theta-1} - \gamma_c\Omega_c - \gamma_h\Omega_y\right]\hat{\pi}_t = \frac{\bar{\tau}}{1-\bar{\tau}}\hat{\tau}_t + \beta\frac{\phi}{\theta-1}\hat{\pi}_{t+1}. \tag{2.25}$$

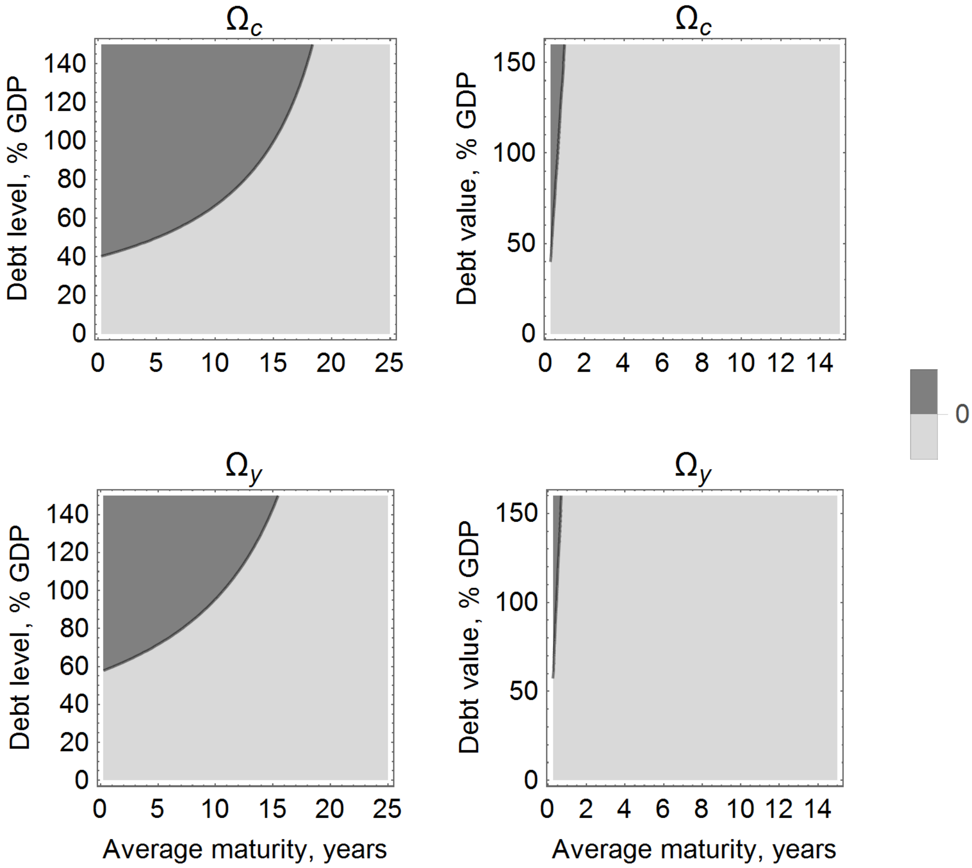
For small steady-state debt-to-GDP ratios the coefficient multiplying current inflation, $\hat{\pi}_t$, on the left-hand-side is positive and larger than the coefficient multiplying next period inflation, $\hat{\pi}_{t+1}$, on the right-hand-side. Therefore, along the monotone debt adjustment path, part of an excessive debt burden is adjusted by increasing tax rates, $\hat{\tau}_t$, until the adjustment is not over. Similarly, one can analyze the role that interest rates play in the adjustment of debt by rewriting the Euler equation, (2.15), using flexible target criterion

[11] See Woodford (2010) for a detailed discussion of the flexible target criteria in the context of optimal monetary policy under commitment.

[12] Burgert and Schmidt (2014) impose an explicit parametric assumption for such an interrelation to hold in their model when analyzing optimal discretionary monetary and fiscal policy with one-period bonds.

[13] Model period is equal to one quarter.

Figure 2.1. – Coefficients in the flexible target criteria



Notes: Dark grey area corresponds to positive values of coefficients. Light greay area corresponds to negative values of coefficients. Left panels differ from right panels in that the former have outstanding debt level on the vertical axis and the latter have market value of the outstanding debt on the vertical axis.

(2.21) and no-arbitrage condition (2.19):

$$\Omega_c \hat{\pi}_t = -\gamma_c^{-1} \hat{r}_t + \Omega_c \hat{\pi}_{t+1}. \tag{2.26}$$

Equation (2.26) implies that for small steady-state debt-to-GDP ratios, when $\Omega_c < 0$, the short-term real interest rate, defined as $\hat{r}_t \equiv \hat{i}_t - \hat{\pi}_{t+1}$, is kept above its steady-state value during the period of (monotone) debt adjustment. Moreover, it is then straightforward to see that the short-term nominal interest rate, $\hat{i}_t$, also stays elevated, compared to its steady-state value, along the (monotone) adjustment path. Such dynamics of the interest rates clearly contributes negatively to the debt adjustment because it keeps the price of borrowing above the long-run level. We therefore conclude that for small steady-state debt-to-GDP ratios it is not optimal to use monetary policy to accommodate fiscal adjustment and the resulting inflation is entirely due to the cost-push effect of fiscal policy—taxes in particular—performing the adjustment. Monetary policy is used conventionally and restrains inflationary pressures resulting from fiscal adjustment.

However, as the steady-state debt-to-GDP ratio increases, and reaches certain thresholds, deviations of consumption and output become of the same sign as the deviations of inflation. Following the previous logic and using equations (2.25) and (2.26), one can show that there exist steady-state debt-to-GDP debt ratios above which the roles of taxes and interest rates in fiscal adjustment are reversed. When the steady-state debt burden is high enough, so that $\Omega_c > 0$, short-term real interest rates start to contribute positively to fiscal adjustment. In other words, monetary policy accommodates fiscal adjustment. The path of the nominal interest rate during fiscal adjustment is however ambiguous: the faster is the adjustment the more likely it is that the nominal interest rate is optimally kept below its steady-state value. Furthermore, there exists another threshold steady-state debt-to-GDP ratio above which the coefficient multiplying current inflation on the left-hand-side of equation (2.25) becomes negative. Before reaching this threshold the role of taxes is ambiguous. Above this threshold it becomes unambiguously optimal to keep taxes below the long-run level in order to restrain inflation despite negative contribution of such policy to fiscal adjustment.

Using baseline parameterization of the model, I find that in the case of one-period bonds monetary policy switches from the conventional regime of inflation restraint to fiscal accomodation starting from the steady-state debt-to-GDP ratio of around 40%.[14] The upper-left panel of the figure (2.1) shows how this threshold varies with the average maturity of the portfolio of government bonds when changing parameter ρ. The threshold of switching monetary policy regime moves to the steady-state debt-to-GDP ratio of 65% for the case of 10 years average maturity of government debt. Increasing average maturity to 15 years moves the threshold further up to the steady-state debt-to-GDP ratio of 100%. This exercise conceals two counteracting effects that variation of parameter ρ has on the optimal policy through the government budget constraint (2.18). On the one hand, given certain level of government debt, $\bar{b}$, higher ρ lengthens average debt maturity, which yields larger payments and therefore raises the associated steady-state

[14] Overall, conclusions for the case of $\rho = 0$ are consistent with those derived in Leith and Wren-Lewis (2013) where the analysis is restricted to one-period bonds.

price, $\bar{q}$, of the portfolio of government bonds. When this effect is not sterilized via adjustment of $\bar{b}$ it leads to an increase of the market value of government debt in the steady-state, $\bar{\Gamma}$.[15] One the other hand, increasing average maturity reduces sensitivity of the government budget to variations of the roll-over price, $\hat{q}_t$.

It is policy relevant to isolate effects of the average maturity of government debt on the policy assignment during debt adjustment periods conditional on sterilizing effects of portfolio composition on the steady-state market value of government debt. The two right panels on Figure 2.1 plot signs of the coefficients Ω_c and Ω_y for various combinations of the steady-state value and the average maturity of government debt. In the case of one-period bonds monetary policy switches from the conventional regime of inflation restraint to fiscal accomodation starting from the steady-state debt-to-GDP market value of slightly above 40%. Increasing average maturity to one year moves the threshold of switching monetary policy regime up to the steady-state debt-to-GDP value of 160%. One can further see that, for a wide range of the steady-state debt-to-GDP market values, empirically plausible deviations of the average maturity from the standard assumption of one-period bonds bring the model into the region where the monetary policy does not accomodate fiscal adjustment. The higher is the average maturity the less effective is the use of contemporaneous monetary policy rate in an attempt to improve the cost of borrowing in the current period because it automatically gets factored into and increases the cost of servicing oustanding debt at a market price.

2.5. Optimal Time-Consistent Policy: Nonlinear Simulations

This section presents numerical solution for the fully nonlinear Markov-perfect equilibrium of the model. I apply numerical methods because the equilibrium can not be solved for in the closed form. First I lay out the parameterization strategy and comment on the solution method. Then I show that analytical insights from using linear-quadratic characterization translate into the nonlinear setting and discuss a number of quantitative results related to the speed of debt adjustment and the underlying policy.

2.5.1. Parameterization and Solution Method

Parameter values are summarized in Table 2.1. Each period in the model represents one quarter of a year. The time discount rate, β, is set equal to 0.99. Curvature parameters of the utility functions, γ_c, γ_g and γ_h, are all set equal to 1. I choose values of the utility weights ν_h and ν_g equal to 20 and 0.25 correspondingly so that in the steady-state households spend one quarter of their unitary time endowment working and government spending amounts to 20 per cent of the value added.

The monopoly power of the firms is described by the elasticity of substitution between intermediate goods, θ, which I set equal to 11 in order to match markup of the price

[15]By design ρ also affects the steady-state tax rate $\bar{\tau}$ and the constant subsidy rate s.

Parameter	Description	Value
β	Discount rate	0.99
γ_c	Interpemoral elasticity for C	1
γ_g	Intertemporal elasticity for G	1
γ_h	Inverse Frisch elasticity	1
ν_g	Utility weight on G	0.25
ν_h	Utility weight on labor	20
φ	Price adjustment cost	116.505
θ	Elasticity of substitution among goods	11

Table 2.1. – Baseline parameterization

over the marginal cost of 10 per cent. The parameter of price adjustment cost, φ, is set to 116.505, which is consistent with a Calvo (1983) price-setting specification where one quarter of the firms reoptimize their prices every period.

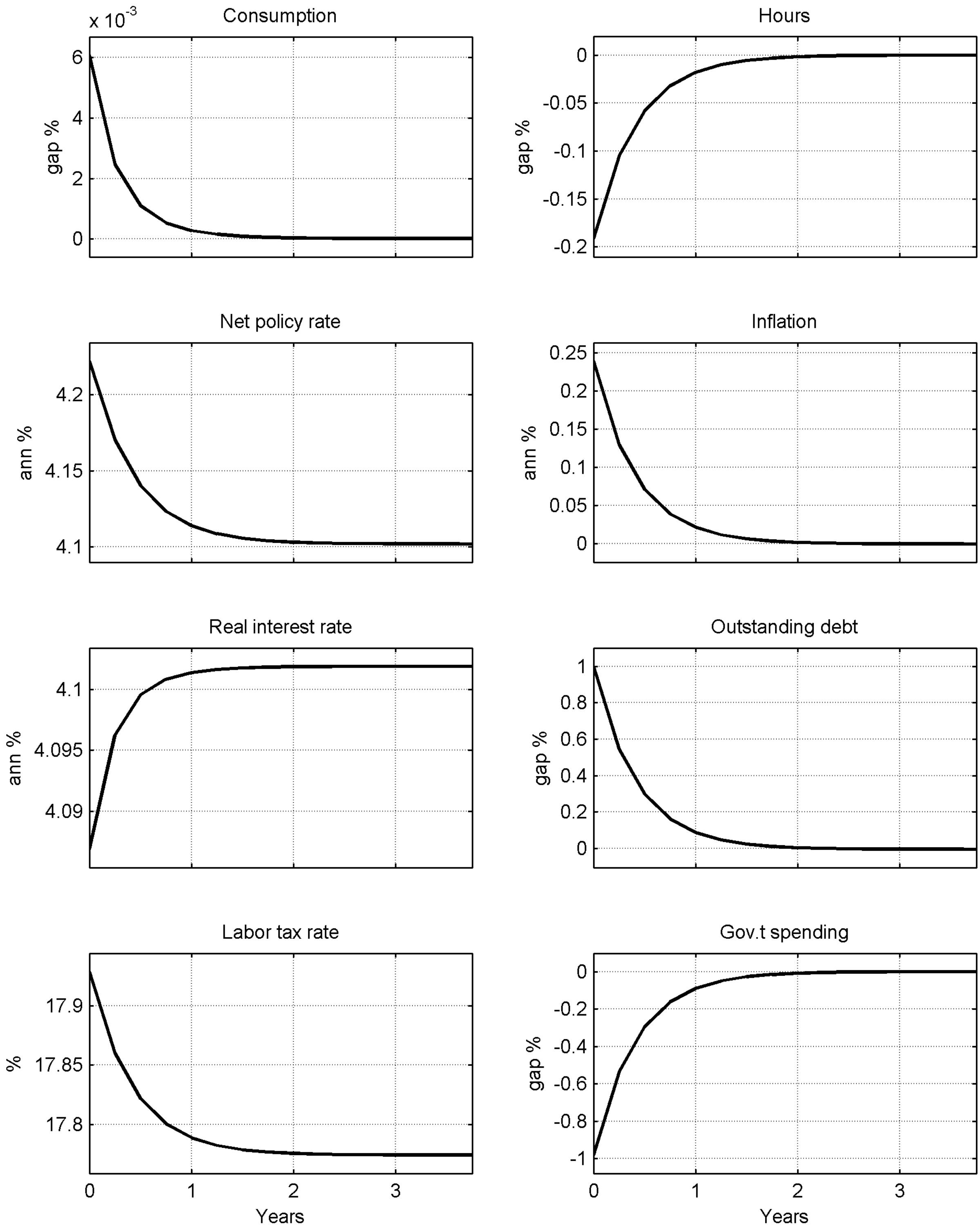
I solve the model numerically by means of a global nonlinear approximation method. Solution method is based on a projection method described in Debortoli and Nunes (2012). I approximate equilibrium functions with cubic splines and solve a system of the first-order conditions of the policy problem looking for a fixed-point of equilibrium functions.

2.5.2. One-Period Bonds

Figure 2.2 shows dynamics of the economy under the optimal discretionary policy for the case of one-period government bonds, that is when $\rho = 0$. I report consumption, government spending, labor supply (hours), and real value of government debt in percentage deviation from the deterministic steady state. Inflation, the (net) nominal and real interest rates are reported in annualized percentage points. The labor tax rate is reported in percentage points. The depicted dynamics start with an initial condition of government debt that is 1 per cent higher than in the steady-state toward which the economy monotonically converges in the long-run.

Along the adjustment path, both fiscal policy instruments contribute to debt reduction positively: government spending is reduced below the long-run level and then follows an increasing profile, whereas tax rates are initially set above the steady-state level and decrease over time. Reduction of labor demand due to fall in government spending meets reduction of labor supply due to high taxes. Therefore, along the adjustment path in equilibrium we observe hours worked below the long-run level despite a boom in private consumption. The latter is due to monetary policy that accomodates fiscal adjustment. Although the short-term nominal interest rates are kept above the long-run level and gradually decline over time, the stance of monetary policy can be characterized as accomodative because the short-term real interest rates, which reflect the real cost of borrowing for the government, appear to be below the long-run level and display and increasing profile over time.

Figure 2.2. – Dynamics with one-period bonds



Notes: The figure plots dynamics of the equilibrium variables over time, for an initial condition of government debt level that is 1% higher than the steady-state value. The steady-state market value of debt is 40% of the annual GDP.

In a given period of time, if the government inherits debt in excess of the steady-state level, it has the incentive to use available policy instruments to reduce the real value of the outstanding liabilities and reduce real interest payments on the newly acquired debt. To reduce the real value of the outstanding debt it suffices to induce contemporaneous inflation. Given expectations of future policies, both high tax rates and loose monetary policy contribute to raising the price index. The former by pushing marginal cost of production up and the latter by pulling private demand up. Loose monetary policy that leads to an increase in private consumption also happens to raise the demand for savings and push up the price at which the newly issued bonds are sold to the household.

Similar discretionary incentives, however, would be faced by the government in the subsequent period if it were to inherit debt in excess of the steady-state level. Increases of future inflation and consumption harm the current bond price. Higher future marginal costs reduce current revenue of the government. Thus, leaving high amount of debt to the successor would worsen the policy trade-offs faced by the current government. Strategic motives to affect its future self in the next period explain why the government deviates from the intertemporal smoothing of debt burden and every period cuts debt in excess of the steady-state level.

One way to assess the speed of debt adjustment is by looking at the half-life of the observed dynamics. Debt adjustment is performed quite fast with the half-life of roughly one quarter, which means that every period the excess of debt is cut by half. Approximating debt process with a simple linear AR(1) process that has equal measure of half-life would deliver persistence coefficient of around 0.5. This means that, even though inflation and interest rates do not exhibit large deviations from the long-run levels, the transitory component of debt dynamics due to discretionary incentives of the government plays an important role in affecting the equilibrium outcome.

2.5.3. Long-Term Bonds

Next, I consider another extreme case of consol bonds, that is when $\rho = 1$. Figure 2.3 (black solid lines) shows dynamics of this economy under the optimal discretionary policy. With consol bonds, government budget is to a large extent not sensitive to changes of the market price of government bonds. Up to the first-order approximation, any gains from a rise in the price of newly issued bonds *ceteris paribus* are offset by capital losses due to the corresponding increase in the value of outstanding liabilities.

The government that finds itself in a given period of time with an outstanding debt in excess of the steady-state level still has an incentive to inflate part of the debt away. The use of the short-term nominal interest rate, however, looses its appeal as a direct way to benefit from influencing the price of newly issued issued bonds. This leads to the first critical difference compared to the economy with one-period bonds. Both short-term nominal and real rates are above the long-run level and gradually decline over time. Although the path of the short-term nominal interest rates has no direct effect on fiscal financing, the path of the real interest rate means that monetary policy does not accomodate fiscal adjustment. Anticipation of high marginal costs and low output due to high taxes along the adjustment path push current consumption gap down and current inflation up.[16] It is optimal to address this trade-off with a strong enough increase of the short-term nominal interest rate that would reduce both wedges. Anti-inflationary stance of monetary policy means that positive inflation observed throughout the adjustment is attributed to high marginal costs due to high tax rates required to bring government debt down over time.

Second critical difference of the economy with consol bonds compared to that with one-period bonds is the speed of debt adjustment. This difference is also a direct consequence of immunity of the government budget to direct effects from variations of the price of bonds. Anticipated effects of future policies on the current choice of the short-term nominal interest rate are irrelevant from the perspective of fiscal financing in the current period. Thus, strategic reasons to affect future policies and obtain better terms of borrowing today by deviating from intertempotal smoothing of debt cease to exist. In every period of time it is still optimal to reduce government debt in excess of the steady-state level because of another strategic motive that is not affected by the change of the maturity structure. It is optimal to reduce debt due to negative effects that anticipated incentive to inflate debt away by the successive government creates on government revenues of the current government. Importantly, monetary policy stance slows down debt adjustment compared to what is implied by the observed fiscal policy alone. Debt adjustment of consol bonds is performed with the half-life of approximately 27 years, which is an increase by factor of 108 compared to one-period bonds. Approximating debt adjustment of consol bonds with a simple linear AR(1) process that has equal measure of half-life would deliver persistence coefficient of around 0.994.

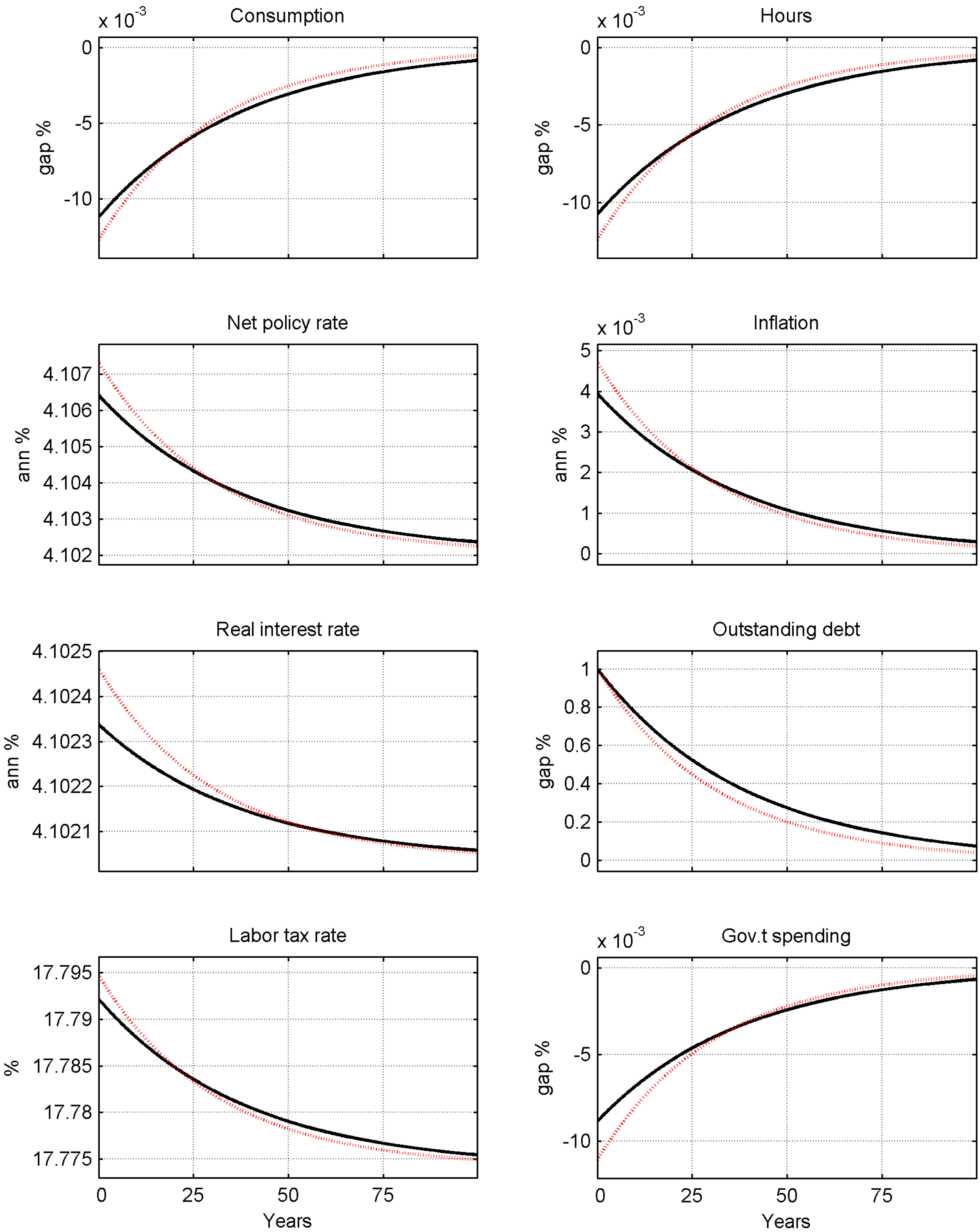
Figure 2.3 (red dotted lines) also demonstrates dynamics of the economy for the case of government bonds with average maturity equal to 4 years. Qualitatively dynamics of this economy is identical to economy with consol bonds. In this case, sensitivity of government budget to changes of the market price of government bonds is still small enough so that monetary policy is not used to accomodate fiscal adjustment. Quantitatively, debt adjustment of government bonds with 4-years average maturity is implemented faster than that of consol bonds. The half-life of debt adjustment is approximately 23 years. Approximating debt adjustment of consol bonds with a simple linear AR(1) process that has equal measure of half-life would deliver persistence coefficient of around 0.992. Faster debt adjustment is a result of the government in every period having strategic reasons to obtain better terms of borrowing, which are absent in the case of consol bonds.

2.6. Conclusion

In the aftermath of the recent global economic downturn, governments of many countries amassed public debt that exceeds historic averages. A pressing policy concern is whether there is a need to reduce the stock of this debt, and if yes, then how fast should the adjustment be performed.

[16] One can think of such anticipation acting isomorphic to two common shocks present in the stochastic New-Keynesian models: positive mark-up shock and positive real rate shock.

Figure 2.3. – Dynamics with long-term bonds



Notes: The figure plots dynamics of the equilibrium variables over time, for an initial condition of government debt level that is 1% higher than the steady-state value. The steady-state market value of debt is 40% of the annual GDP. Solid lines: consol bonds. Dotted red lines: bonds with 4-years average maturity.

In this chapter I focus on the known case where debt reduction is the optimal policy strategy because of the lack of commitment. My analysis shows that the speed of debt adjustment is highly sensitive to the average maturity of government debt. Using a common assumption of government debt composed entirely of one-period bonds, one concludes that it is optimal to adjust government debt at a surprisingly high speed. The speed of adjustment slows down as the average maturity of government debt lengthens. Government with a bond portfolio that contains longer-term bonds finds it less optimal to reduce the stock of debt and prefers to smooth it over time along with the associated distortions. My analysis also shows that, when accounting for a plausible average maturity of government debt, it is not optimal to perform fiscal financing with the help of monetary policy, unlike in the case of one-period bonds.

Analysis of this paper is built under the assumption that the government can always adjust either monetary or fiscal policy instruments to maintain fiscal sustainability. Clearly, there are cases when keeping public debt on a sustainable path requires the government to perform debt adjustment *via* an outright default. Hence, abstracting from the default risk may be not without loss of generality. Next chapter focuses on a risk-sharing role of sovereign default in a monetary union, which is motivated by the recent European debt crisis.

3. Monetary Policy and Sovereign Default in a Monetary Union: a Risk-Sharing Perspective

3.1. Introduction

The problem of debt repayment is not new for emerging economies: Reinhart and Rogoff (2008) discuss that virtually every country has a history of default throughout its stage of being an emerging economy. Recent developments of the European debt crisis demonstrated that the credit risk of debt issued by governments of developed economies may also be not negligible.

The Greek debt restructuring in 2012 has clearly indicated that unsustainable fiscal policies may lead to a possibility of sovereign default by individual countries within a monetary union. A country entering the monetary union abandons independent monetary policy and, therefore, loses the ability to ensure sustainability of its debt by means of monetary instruments. As long as a common central bank stabilizes inflation, any country whose individual fiscal policy does not guarantee its debt sustainability by securing sufficient surpluses is bound to carry out other kind of adjustment of the outstanding debt. A natural candidate for such adjustment mechanism is a sovereign default.[1]

Throughout the European debt crisis, fears that a sovereign default of an insolvent country would inflict losses on private sectors of solvent neighboring countries led their governments to object a default as a viable mechanism to achieve fiscal sustainability. Another point of view—discussed in von Hagen et al. (2010)—emphasizes how essential it is to establish a transparent procedure of an orderly default for sorting out unsustainable debt levels, which would aim at distributing the cost of debt restructuring across the union. This debate calls for a careful consideration of the role of sovereign default as a mechanism for sharing fiscal risks in a monetary union.

In this chapter I set up a model of a monetary union with two countries where government debt of one of the countries, issued in the form of nominal bonds, is held by households of both countries. These government bonds embody the credit risk because tax policy of the corresponding fiscal authority does not ensure intertemporal government budget while facing an exogenous stream of government expenditures. Risky government bonds are assumed to be the only asset available to the households. Such incomplete markets do not provide opportunity to insure against the country-specific variation of government expenditures. I characterize analytically a solution to the model's first-order dynamics and study whether sovereign default can be used to achieve fiscal sustainability

[1] An alternative scenario of a fiscal solvency crisis, discussed in Daniel and Shiamptanis (2010), is for a central bank to give up to a "fiscal dominance" regime.

in the country issuing bonds and help share the fiscal risk across two countries of the monetary union at the same time.

The central bank of the union conducts monetary policy by setting the short-term nominal interest rate according to the Taylor-type feedback rule. Thus, the analysis of this chapter relies on the monetary-fiscal arrangement where both policy branches are "active" in the sense of Leeper (1991). Under such a policy arrangement, default can become necessary to preserve price stability—see a fiscal theory of the sovereign risk by Uribe (2006) for the analysis in a closed economy framework. Current analysis reveals that the monetary union is not different in that nontrivial sovereign default policy is needed to support a stationary equilibrium with stable prices in this economy.

A general formulation of the model allows for an (ex-post) lump-sum subsidy of a constant fraction of aggregate default gains from the insolvent government to its domestic private sector. The case of no subsidy corresponds to a nondiscriminatory default, whereas the positive subsidy captures default discrimination of a certain degree. I characterize the equilibrium allocation of consumption and establish a benchmark of the perfect risk-sharing. The former and the latter do not necessarily coincide. I show that imperfect default discrimination is a necessary condition to attain the perfect risk-sharing in equilibrium. Imperfect default discrimination is needed because it is only under this condition default policy has real effects on the allocation of consumption.

An associated result related to monetary policy emerges under an imperfectly discriminatory default. In the current setup inflation and default are the two alternative channels of achieving fiscal sustainability. Monetary policy that retains control over inflation has an influence over the ultimate amount of debt adjustment carried out through a default. Then, changes in the monetary policy rule may affect the real economic activity during periods of debt adjustment. In particular, the extent of aggressiveness of the central bank in stabilizing inflation affects the aggregate amount of default and the subsequent subsidy received by the domestic private sector. This transmission channel of monetary policy is derived in the absence of nominal rigidities of any kind.

Besides providing a necessary condition for default policy to implement the perfect risk-sharing, I analyze the equilibrium under a simple default rule. I postulate a rule according to which default rates are changed in response to fluctuations of government expenditures. This rule is assessed with the aim of finding further restrictions under which the equilibrium allocation of consumption attains the perfect risk-sharing benchmark. In particular, I derive a set of constraints in the policy parameters space that, for a given inflation feedback coefficient of the monetary policy Taylor-type rule, uniquely pin down two parameters characterizing default policy in this economy – the feedback coefficient to government expenditures in the default rule and the constant subsidy rate that describes the degree of (ex-post) default discrimination.

A large body of the literature on sovereign debt follows Eaton and Gersovitz (1981) and studies the cost of sovereign default under the assumption of strategic behavior of the government. This literature focuses on the inefficiencies and losses created by the sovereign default risk. My analysis, on the other hand, is of the type where government default originates from the need to maintain fiscal sustainability, as in Uribe (2006). My subsequent focus on the potential of the use of sovereign default to improve international risk-sharing is meant to capture and analyze the idea of gains provided by the repudiation of public debt.

In this regard, this chapter is related to the literature that stresses how a sovereign default may prove beneficial in transferring resources between borrowers and lenders to effectively smooth consumption. From this perspective, sovereign default is one of the mechanisms for making government loans issued in the form of non-contingent claims to be implicitly state-contingent. The role of sovereign default as an insurance against fluctuations of domestic income in a small open economy has been emphasized by Grossman and Van Huyck (1988) and more recently by Adam and Grill (2012). These results are not directly applicable to the environment of a monetary union with policy cooperation because they do not take into consideration losses that a sovereign default would inflict on the solvent members of the union. In contrast to these papers, present analysis discusses design of a sovereign default that facilitates international risk-sharing within a two-country monetary union.

A large number of existing models of sovereign default assume that governments can perfectly discriminate between domestic and foreign lenders so as to target and default only on debt held by the foreigners. Clearly, under the current level of integration of financial markets it is nearly impossible for a government to perform perfectly discriminatory default. This observation is put forward by Gennaioli et al. (2012) to rationalize default cost on the domestic economy under a strategic government default. Analysis in this chapter allows for an entire spectrum of ex-post default discrimination. As the main result of the chapter demonstrates, another extreme case of nondiscriminatory default turns out to be detrimental from the perspective of international risk-sharing.

The remainder of the chapter is organized as follows. Section 3.2 introduces the model and defines a competitive equilibrium. Section 3.3 defines and discusses a benchmark of the perfect risk-sharing. Section 3.4 characterizes competitive equilibria and studies equilibrium under a simple default rule. Section 3.5 concludes.

3.2. The Model

I consider a simple endowment two-country open economy model of a monetary union with one traded good and incomplete international asset markets. I refer to the two countries as Home and Foreign for the purpose of distinction. Each country is populated by the same measure of identical households that receive a constant endowment of single good that they can trade and consume. There are no nominal rigidities, hence prices are perfectly flexible. A central bank of the union conducts monetary policy by setting the short-term nominal interest rate. Stochastic nature of the economy is due to a random stream of expenditures faced by the government of the Home country. These expenditures have to be financed either by lump-sum taxes or by borrowing from the households of both countries in the form of nominal non state-contingent one-period discount bonds.

Bonds issued by the government of the Home country are not risk-free. The credit risk is present due to the assumption of the Home country government following tax policy that does not guarantee solvency of its intertemporal budget. In equilibrium, this requires adjustment of the outstanding debt through the combination of inflation and sovereign default. Sovereign default is modeled as a reduction of the face value of redeemed bonds. Default is ex-ante non-discriminatory between households of the two countries, Home and Foreign. It is, however, possible that households of the Home country are treated differently ex-post and receive a lump-sum subsidy in the case of default.

I proceed with a more detailed description of the model.

3.2.1. The Private Sector

Households that reside in the Home country derive utility from consumption. The utility function is identical for all the households and is given by

$$u_t = \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \theta_s U(c_s) \right\}, \tag{3.1}$$

where $\mathbb{E}_t$ denotes the expectation conditional on the information set available at time t, the flow utility function U is increasing and concave, and θ_s is the endogenous intertemporal discount factor specified as in Devereux and Sutherland (2011)

$$\theta_{s+1} = \theta_s \beta(c_{A,s}), \quad s \geqslant t,$$
$$\theta_0 = 1,$$

where $c_{A,s}$ is an aggregate home consumption, $0 < \beta(c_A) < 1$, and $\beta'(c_A) \leqslant 0$. If $\beta(c_A)$ were a constant function then the discount factor would be exogenous. In what follows I assume

$$\beta(c_A) \equiv \omega c_A^{-\eta},$$

where $0 < \eta < 1$ and $0 < \omega \bar{c}_A^{-\eta} < 1$ with $\bar{c}_A$ being the steady state value of domestic aggregate consumption. I use stars to denote Foreign country variables. Households that reside in the Foreign country feature analogous preferences over the sequence of foreign consumption c_t^* discounted with the endogenous discount factor θ_s^*. The endogenous discount factor is merely a technical device that eliminates indeterminacy of the steady state and a non-stationary dynamic behavior, which is a common feature of open economy models with incomplete financial markets approximated up to the first-order.[2]

Households in both countries, Home and Foreign, have access to incomplete international financial markets where they can trade one-period nominal discount bonds issued by the government of the Home country. These bonds are subject to a potential default

[2]The problems of steady states multiplicity and unit root dynamics in a broader class of open economy models along with alternative ways of eliminating them are discussed in Bodenstein (2011). Anagnostopoulos and Tang (2015) demonstrate that the non-stationarity problem is due to the nature of the methods based on local approximation and it disappears when using a global method that accounts for borrowing limits.

when government repays only a fraction $(1-\delta_t)$ of their face value. Besides, within countries households trade a set of complete nominal state-contingent assets that are in zero net supply. The individual nominal flow budget constraint of the Home country household at time t is

$$P_t c_t + R_t^{-1} B_t + \mathbb{E}_t Q_{t,t+1} D_{t+1} = P_t y + (1-\delta_t) B_{t-1} + D_t - P_t \tau_t + P_t z_t, \tag{3.2}$$

where R_t is the contractual nominal interest rate on the domestic government bond holdings B_t purchased at time t with repayment due at time $t+1$, δ_t is the default rate on the government bond holdings B_{t-1} that mature at time t, $Q_{t,t+1}$ is the nominal stochastic discount factor and $\mathbb{E}_t Q_{t,t+1} D_{t+1}$ is the value of investment into state-contingent assets, whereas D_t is the payoff corresponding to a realization of the state at time t, τ_t is the lump-sum taxes collected by the Home country government, z_t is the lump-sum transfers provided by the Home country government, and y is the constant exogenous endowment of the perishable good. Lump-sum transfers are used by the Home country government to compensate part of the domestic households' default losses in equilibrium with an imperfectly discriminatory default. Finally, P_t is the price level in the Home country. The nominal exchange rate in the monetary union is fixed, therefore the same price level prevails in the Foreign country where the individual nominal flow budget constraint of the household reads as

$$P_t c_t^* + R_t^{-1} B_t^* + \mathbb{E}_t Q_{t,t+1}^* D_{t+1}^* = P_t y^* + (1-\delta_t) B_{t-1}^* + D_t^*. \tag{3.3}$$

Within the same country, all the households are assumed to have identical initial wealth levels, which implies that households in the same country face identical budget constraint at every period of time and state of the world. Hence, one can consider a representative households for each country, Home and Foreign. Even though an idiosyncratic risk is pooled among households from the same country, there is imperfect risk-sharing across borders due to incomplete nature of international asset markets.

The maximization problem of the representative household in the Home country consists of maximizing (3.1) with respect to consumption, bond and assets holdings, subject to (3.2) and a no-Ponzi constraint. Along with (3.2) and the no-Ponzi constraint, the resulting first-order conditions are

$$\lambda_t = \frac{U_c(c_t)}{P_t}, \tag{3.4}$$

$$Q_{t,t+1} = \beta(c_t)\frac{\lambda_{t+1}}{\lambda_t}, \tag{3.5}$$

$$\frac{1}{R_t} = \mathbb{E}_t\left[(1-\delta_{t+1})Q_{t,t+1}\right], \tag{3.6}$$

where λ_t is the Lagrange multiplier corresponding to (3.2). Solution of the analogous maximization problem of the representative household in the Foreign country is characterized by the first-order conditions that include

$$\lambda_t^* = \frac{U_c(c_t^*)}{P_t},$$
$$Q_{t,t+1}^* = \beta^*(c_t^*)\frac{\lambda_{t+1}^*}{\lambda_t^*},$$
$$\frac{1}{R_t} = \mathbb{E}_t\left[(1-\delta_{t+1})Q_{t,t+1}^*\right].$$

3.2.2. The Public Sector

The sequence of expenditures of the Home country government, $\{g_t\}_{t=0}^{\infty}$, produced by an exogenous stochastic process is the only source of uncertainty in the model. I set this sequence to be generated by the following logarithmic AR(1) process

$$\log\frac{g_t}{\bar{g}} = \rho\log\frac{g_{t-1}}{\bar{g}} + \varepsilon_t, \quad \varepsilon \sim i.i.d.N(0,\sigma_\varepsilon^2), \tag{3.7}$$

where $\bar{g}$ is the steady-state value of government expenditures.

To finance expenditures government of the Home country can collect lump-sum taxes, τ_t, borrow on the international markets via one-period discount nominal bonds, B_t^g, at the price R_t^{-1} or default on the stock of outstanding debt from the previous period, B_{t-1}^g, by repaying only a fraction $(1-\delta_t)$ of its face value. Ex-ante domestic and foreign households are treated equivalently and are subject to equivalent sovereign default losses. However, the government of the Home country redistributes a fraction λ of its gains from default as a subsidy to domestic households in order to compensate part of the losses incurred by the private sector. Subsidy is distributed via lump-sum transfers and does not depend on the individual amount of government bonds held by a domestic household. The nominal flow budget constraint of the government in the Home country reads as follows

$$R_t^{-1}B_t^g + P_t(\tau_t - g_t) = (1-\delta_t)B_{t-1}^g + \lambda\delta_t B_{t-1}^g, \quad 0 \leqslant \lambda \leqslant 1. \tag{3.8}$$

Introducing subsidy allows me to study how international risk-sharing can be improved through redistributionary effects arising from the ex-post discriminatory treatment of households with different residences in the event of default.[3]

I assume that the government of the Home country adopts "irresponsible" fiscal policy of setting taxes in the sense that it does not by itself guarantees solvency or, in other words, it allows public debt to evolve unsustainably. For the sake of simplicity I focus on the case where the government of the Home country keeps taxes constant, *i.e.* $\tau_t = \tau$. This policy is an example of a broad class of fiscal policies—coined by Woodford (1995) as "non-Ricardian"—that do not guarantee fulfillment of the intertemporal government budget constraint and require an alternative adjustment mechanism for this constraint

[3] Subsidy functioning is different from Corsetti et al. (2013) and Schabert and van Wijnbergen (2011) where redistribution effects of default are neglected and all the gains from default are handed out to all the households affected by default.

to hold in equilibrium.[4] Such policies were used by Leeper (1991), and are otherwise referred to as "active" in the context of his work, to provide rationale for the existence of equilibria with "fiscal dominance" where the monetary authority cedes control over prices to the fiscal authority. For these equilibria to exist, monetary policy has to feature muted feedback from inflation to interest rates or no feedback at all as in the case of interest rate peg, which makes it "passive" in the terminology of Leeper (1991). The mix of "active" fiscal and "passive" monetary policies is also known as a fiscal theory of the price level.

I assume that the union-wide monetary authority is set to conduct its policy by setting the short-term nominal interest rate, R_t, according to the Taylor-type feedback rule of the following form

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\alpha}, \tag{3.9}$$

where $\bar{R}$ is the steady state value of interest rate and $\pi_t = {}^{P_t}/{}_{P_{t-1}}$ is union-wide inflation. I restrict attention to the case of $\alpha > 1$, which corresponds to "active" monetary policy in the terminology of Leeper (1991). In models without sovereign default, having monetary policy that functions in the "active" regime, while also having "active" fiscal policy, creates diverging price paths.[5]

In the closed economy model Uribe (2006) demonstrated that one can restore price stability under the mix of "active" fiscal and "active" monetary policies by allowing the government to repudiate its obligations. I follow Uribe (2006) in allowing for default and modelling it not as an explicit strategic decision of the government but rather as a sequence of government debt adjustments required to support stationary equilibrium and prevent the government from running Ponzi-schemes against households. Default is assumed not to produce neither proportional nor fixed dead-weight losses. Doing so I avoid an explicit bias of making default welfare-inferior adjustment channel relative to inflation because no price rigidities are present in the model.

Finally, I model the fiscal authority of the Foreign country as running a balanced budget.

3.2.3. The Competitive Equilibrium

All households within a given country are identical, hence in equilibrium $D_t = 0$ and $D_t^* = 0$ because state-contingent assets are in zero net supply. Supply of domestic government bonds has to satisfy demand for this asset from both domestic and foreign households. In other words, bonds market clearing requires

$$B_t^g = B_t + B_t^*.$$

[4]Introducing a more sophisticated "non-Ricardian" policy rule with a feedback to endogenous variables that does not guarantee sustainable government debt behavior would not provide any additional insights but would reduce analytical tractability of the model.

[5]Loyo (2000) used this dynamics as underlying for describing episodes of hyperinflation in Brazil.

It then follows that transfers to the private sector of the Home country in equilibrium are

$$z_t = \lambda\delta_t \frac{(b_{t-1} + b^*_{t-1})}{\pi_t} \tag{3.10}$$

where $b_t = B_t/P_t, b^*_t = B^*_t/P_t$ are the real values of bond holdings in the two countries, Home and Foreign correspondingly. Using (3.10), the flow budget constraint of the Home country government, (3.8), in real terms can be written as

$$\frac{(b_t + b^*_t)}{R_t} + (\tau - g_t) = (1 - (1 - \lambda)\delta_t)\frac{(b_{t-1} + b^*_{t-1})}{\pi_t}, \tag{3.11}$$

Using the government budget constraint of the Home country, (3.11), together with the budget constraint of households in the Home country, (3.2), results in the current account identity:

$$\frac{b^*_t}{R_t} - (1 - \delta_t)\frac{b^*_{t-1}}{\pi_t} = (c_t + g_t - y), \tag{3.12}$$

The resource constraint of the economy is given by

$$c_t + c^*_t + g_t = y + y^*. \tag{3.13}$$

I use first-order conditions of the Home country household maximization problem (3.4), (3.5), and (3.6), to derive Euler equation that governs intertemporal profile of domestic consumption and bond holdings:

$$U_c(c_t) = \beta(c_t)R_t\mathbb{E}_t\left[\frac{(1 - \delta_{t+1})}{\pi_{t+1}}U_c(c_{t+1})\right]. \tag{3.14}$$

Analogously I get Euler equation of the foreign country:

$$U_c(c^*_t) = \beta^*(c^*_t)R_t\mathbb{E}_t\left[\frac{(1 - \delta_{t+1})}{\pi_{t+1}}U_c(c^*_{t+1})\right]. \tag{3.15}$$

Transversality conditions are as follows

$$\lim_{T\to\infty} \mathbb{E}_t\left[\Lambda_{t,T}\frac{b_T}{R_T}\right] = 0, \tag{3.16}$$

$$\lim_{T\to\infty} \mathbb{E}_t\left[\Lambda^*_{t,T}\left(\frac{b^*_T}{R_T}\right)\right] = 0, \tag{3.17}$$

where $\Lambda_{t,T} = \theta_T\frac{U_c(c_T)}{U_c(c_t)}$ and $\Lambda^*_{t,T} = \theta^*_T\frac{U_c(c^*_T)}{U_c(c^*_t)}$ are stochastic discount factors.

There are multiple equilibria corresponding to different government policies. I discuss different effects of policies in detail below, and now for a given policy I define a competitive equilibrium as follows:

Definition 3.1 (Competitive Equilibrium)**.** *For a given (stochastic) government default policy* $\{\delta_t\}_{t=0}^{\infty}$ *satisfying government budget constraint (3.11), a competitive rational expectations equilibrium is a sequence of stochastic processes* $\mathcal{X}_t \equiv \{c_t, c_t^*, \pi_t, R_t, b_t, b_t^*\}$ *satisfying equations (3.11) - (3.15), monetary rule (3.9), and transversality conditions (3.16), (3.17), given exogenous process* g_t *defined by (3.7), and initial conditions* b_{-1} *and* b_{-1}^**.*

Results of this chapter are derived analytically in the next section by using solution to the model's dynamics approximated up to the first-order. The solution is derived in the vicinity of a deterministic steady state. I close this section with the derivation of conditions characterizing the steady state by setting government expenditures to their mean value. To proceed with the analysis I assume utility function to be logarithmic. Under this assumption the model is more tractable. However, it is possible to show that all results derived here continue to hold in the more general case of a CRRA utility function.

3.2.4. The Steady State

I derive a system of equations that characterizes the deterministic steady state of the competitive equilibrium for the class of default policies with the steady state default rate of zero. I focus on the steady state with zero inflation. The set of remaining steady state variables consists of the short-term nominal interest rate, $\bar{R}$, consumption levels in Home and Foreign countries, $\bar{c}$ and $\bar{c}^*$, bonds issued by the Home country government and held by households in Home and Foreign countries, $\bar{b}$ and $\bar{b}^*$. Bars over variables denote their steady state values.

Euler equations (3.14) and 3.29 imply that in the steady state inverted subjective discount rates of Home and Foreign countries are equal to the short-term nominal interest rate

$$1/\beta(\bar{c}) = \bar{R}, \tag{3.18}$$

$$1/\beta^*(\bar{c}^*) = \bar{R}. \tag{3.19}$$

Furthermore, the current account identity (3.12), the resource constraint (3.13), and the Home country government budget constraint (3.11) in the steady state are as follows

$$\bar{c} = y - \bar{g} - \frac{\bar{R}-1}{\bar{R}}\bar{b}^*, \tag{3.20}$$

$$\bar{c}^* = y^* + \frac{\bar{R}-1}{\bar{R}}\bar{b}^*, \tag{3.21}$$

$$\bar{b} + \bar{b}^* = \frac{\bar{R}}{\bar{R}-1}(\tau - \bar{g}). \tag{3.22}$$

Equations (3.18)–(3.22) form the system that characterizes the deterministic steady state of the economy. For the remainder of the chapter I assume that parameters of the model are such that the steady state is characterized by a non-zero amount of total debt issued by the Home country government and a non-zero fraction of this debt held by households in the Foreign country.

3.3. The Perfect Risk-Sharing Benchmark

For the subsequent analysis it is useful to establish a benchmark allocation as a solution to the social Planner's problem in a centralized union. The problem of the Planner is to distribute consumption of the fixed aggregate endowment of the union net of government expenditures among the representative households of the two countries. The criterion she uses is a weighted sum of the two countries' expected utilities. Formally, the problem reads as follows

$$\begin{aligned} \max_{\{c_t, c_t^*\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} [\theta_t \log(c_t) + \Phi \theta_t^* \log(c_t^*)] \\ \text{s.t.} \quad & c_t + c_t^* = y + y^* - g_t, \quad t \geqslant 0, \end{aligned} \tag{3.23}$$

where Φ is the Pareto-weight that the Planner assigns to the utility of households in the Foreign country relative to the utility of households in the Home country, θ_t and θ_t^* are the (endogenous) discount rates defined identically to the individual optimization problems in Section 3.2.1.[6]

There is, in general, a continuum of solutions to the Planner's problem that are indexed by different values of the Pareto-weight Φ. In order to establish the benchmark, I pick the Pareto-weight that, in the absence of uncertainty, produces allocation identical to the deterministic steady state of the competitive equilibrium defined above. When solving the Planner's problem with this Pareto-weight (see Appendix C.1), the optimal stochastic allocation of consumption in each country at a given point in time is determined as a constant fraction of the total amount of resources in the union:

$$\frac{c_t^p}{\bar{c}} = \left(\frac{y + y^* - g_t}{y + y^* - \bar{g}}\right), \qquad \frac{c_t^{p*}}{\bar{c}^*} = \left(\frac{y + y^* - g_t}{y + y^* - \bar{g}}\right). \tag{3.24}$$

where $\bar{c}$ and $\bar{c}^*$ correspond to the steady state values of consumption in the competitive equilibrium. The allocation defined above displays an important feature of being independent from the history of shocks to government expenditures. One can see that consumption levels of the two countries in (3.24) are proportional, hence benchmark allocation prescribes the perfect risk-sharing among countries.

[6] In the individual optimization problem it is the average consumption that influences the discount factor, hence individuals do not take into account the effect of their individual decision on the discount rate. I assume that the Planner also does not internalize effects her decisions have on the discount rates. Thus, allocation that solves the Planner's problem is constrained efficient.

For the further analysis I use (3.24) to derive log-linear deviations of consumption allocation from the steady state under perfect risk-sharing

$$\hat{c}_t^p = -\chi_1 \hat{g}_t, \qquad \hat{c}_t^{p*} = -\chi_1 \hat{g}_t, \tag{3.25}$$

where $\chi_1 \equiv \frac{\bar{g}}{\bar{c}+\bar{c}^*}$, and hat variables denote logarithmic deviations from the steady state, *e.g.* $\hat{g}_t = log(g_t) - log(\bar{g})$.

The Fiscal Union A popular topic of discussions during the European debt crisis was related to the need of further integration among the union members and, in particular, a need of working toward setting up a fiscal union. In this section I briefly relate to this topic within the context of the previously developed model. I show that it is possible to attain the benchmark allocation with perfect risk-sharing by establishing an international authority that conducts union-wide fiscal policy.

Available set of instruments for this authority comprises of state-contingent real lump-sum transfers (taxes) to households of Home and Foreign countries, $\mathrm{TR_t}$ and $\mathrm{TR_t^*}$ respectively. These transfers are used to finance government expenditures, g_t, hence the budget constraint of the international fiscal authority is of the following form

$$\mathrm{TR_t} + \mathrm{TR_t^*} = -g_t. \tag{3.26}$$

The allocation resulting from maximizing weighted sum of expected utilities, as in the planner's problem, subject to individual budget constraints and (3.26) is identical to the benchmark allocation described above. The corresponding system of transfers that implements this allocation is as follows

$$\mathrm{TR_t} = -\left(1-\beta\right)\bar{b}^*\left(\frac{y+y^*-g_t}{y+y^*-\bar{g}}\right) - \frac{\bar{g}y^* - \bar{g}g_t}{y+y^*-\bar{g}},$$
$$\mathrm{TR_t^*} = \left(1-\beta\right)\bar{b}^*\left(\frac{y+y^*-g_t}{y+y^*-\bar{g}}\right) + \frac{\bar{g}y^* - (y+y^*)g_t}{y+y^*-\bar{g}}.$$

3.4. Equilibrium Characterization

In this section I characterize how the competitive equilibrium depends on the underlying default policy with the main focus on comparing the equilibrium consumption allocation against the perfect risk-sharing benchmark. Incomplete nature of international markets does not allow to obtain a closed-form solution of the model, therefore to conduct further analysis I focus on the local analysis in the neighborhood of the deterministic steady state defined earlier. I start by log-linearizing equilibrium conditions around the steady state and analyze the resulting system of difference equations.

I make a variable change $d = (1 - \delta)$ so as to avoid taking logarithm of zero steady state default rate. Therefore, a default is associated with negative deviations of d from its steady state value $\bar{d} = 1$. Hat variables denote log deviations from the steady state, e.g. $\hat{d}_t = log(d_t) - log(\bar{d})$. The log-linear version of the monetary policy rule that sets the nominal interest rate as a feedback from contemporaneous inflation rate looks as follows

$$\hat{r}_t = \alpha\hat{\pi}_t, \quad \alpha > 1. \tag{3.27}$$

Other equilibrium conditions in the log-linear form:

$$(1-\eta)\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \left(\hat{r}_t - \mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t\hat{d}_{t+1}\right), \tag{3.28}$$

$$(1-\eta)\hat{c}^*_t = \mathbb{E}_t\hat{c}^*_{t+1} - \left(\hat{r}_t - \mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t\hat{d}_{t+1}\right), \tag{3.29}$$

$$\bar{c}\hat{c}_t + \bar{c}^*\hat{c}^*_t + \bar{g}\hat{g}_t = 0, \tag{3.30}$$

$$\beta\bar{b}^*\left(\hat{b}^*_t - \hat{r}_t\right) = \bar{c}\hat{c}_t + \bar{g}\hat{g}_t + \bar{b}^*\left(\hat{b}^*_{t-1} - \hat{\pi}_t + \hat{d}_t\right), \tag{3.31}$$

$$\beta\left[\bar{b}\hat{b}_t + \bar{b}^*\hat{b}^*_t - (\bar{b}+\bar{b}^*)\hat{r}_t\right] = \bar{g}\hat{g}_t + \bar{b}\hat{b}_{t-1} + \bar{b}^*\hat{b}^*_{t-1} + (\bar{b}+\bar{b}^*)\left((1-\lambda)\,\hat{d}_t - \hat{\pi}_t\right), \tag{3.32}$$

$$\hat{g}_{t+1} = \rho\hat{g}_t + \varepsilon_{t+1}, \tag{3.33}$$

where (3.28) and (3.29) are Home and Foreign country Euler equations, (3.30) is the resource constraint, (3.31) is the current account identity, (3.32) is the flow government budget constraint of the Home country government, and (3.33) is the stochastic process for government expenditures.

I can eliminate two variables—the short-term nominal interest rate, $\hat{r}_t$, and the consumption in the Foreign country , $\hat{c}^*$—and two equations—the monetary policy rule (3.27) and the resource constraint, (3.31). In particular, the government budget constraint of the Home country, (3.32), the current account identity, (3.31), and Euler equations (3.28) and (3.29) with the help of resource constraint (3.30) and monetary policy rule (3.27) can be transformed into the following difference equations

$$(1-\eta)\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - (1-\eta-\rho)\chi_1\hat{g}_t, \tag{3.34}$$

$$\alpha\hat{\pi}_t = \mathbb{E}_t\hat{\pi}_{t+1} + (1-\eta-\rho)\chi_1\hat{g}_t - \mathbb{E}_t\hat{d}_{t+1}, \tag{3.35}$$

$$\beta\bar{b}^*\left(\hat{b}^*_t - \alpha\hat{\pi}_t\right) = \bar{c}\hat{c}_t + \bar{g}\hat{g}_t + \bar{b}^*\left(\hat{b}^*_{t-1} - \hat{\pi}_t + \hat{d}_t\right), \tag{3.36}$$

$$\beta\left[\bar{b}\hat{b}_t + \bar{b}^*\hat{b}^*_t - \alpha(\bar{b}+\bar{b}^*)\hat{\pi}_t\right] = \bar{g}\hat{g}_t + \bar{b}\hat{b}_{t-1} + \bar{b}^*\hat{b}^*_{t-1} + (\bar{b}+\bar{b}^*)\left((1-\lambda)\,\hat{d}_t - \hat{\pi}_t\right), \tag{3.37}$$

Thus, for a given sequence of stochastic processes describing default policy, $\{\hat{d}_t\}_{t=0}^{\infty}$, the (log-linear) competitive equilibrium is characterized by (3.34)–(3.37). Coupled with the process (3.33) driving government expenditures, these equations constitute a system of stochastic difference equations in four endogenous variables, Home country consumption $\hat{c}_t$, union-wide inflation, $\hat{\pi}_t$, and bond holding of the households in Home and Foreign countries, $\hat{b}_t$ and $\hat{b}^*_t$ correspondingly. The former two variables are non-predetermined or jump variables, and the latter two variables are predetermined variables.

3.4.1. Sovereign Default and Monetary Policy Interaction

I solve equation (3.35) forward, taking into account the process of government expenditures (3.33), to characterize union-wide inflation

$$\hat{\pi}_t = \frac{1-\eta-\rho}{\alpha-\rho}\chi_1 \hat{g}_t - \mathbb{E}_t \sum_{j=1}^{\infty} \left(\frac{1}{\alpha}\right)^j \hat{d}_{t+j}. \tag{3.38}$$

First thing to note from (3.38) is that inflation depends on the monetary policy stance described by the value of inflation feedback coefficient, α, in the monetary policy rule (3.27). In the limiting case when $\alpha \to \infty$ monetary authority achieves complete price stability.[7] Second, a default policy characterized by non-zero default expectations influences ability of the monetary authority to control price fluctuations.[8] Higher expectations of future default, *ceteris paribus*, raise current inflation.

Another equilibrium constraint that shapes evolution of inflation in equilibrium and crucially depends on default policy is the government budget constraint. Solving (3.37) forward and taking into account (3.33) and (3.35), I get the following log-linear version of the intertemporal government budget constraint

$$\hat{\pi}_t = \left(\frac{\chi_2}{1-\beta\rho} + \beta\frac{1-\eta-\rho}{1-\beta\rho}\chi_1\right)\hat{g}_t + \frac{\bar{b}}{\bar{b}+\bar{b}^*}\hat{b}_{t-1} + \frac{\bar{b}^*}{\bar{b}+\bar{b}^*}\hat{b}^*_{t-1} + (1-\lambda)\hat{d}_t - \lambda\mathbb{E}_t\sum_{j=1}^{\infty}\beta^j\hat{d}_{t+j}. \tag{3.39}$$

where $\chi_2 \equiv \frac{\bar{g}}{\bar{b}+\bar{b}^*}$.

It can now be demonstrated that a nontrivial default policy is a necessary feature of equilibrium in the studied monetary union economy. The following proposition demonstrates this fact in a formal way.

Proposition 3.1 (Unavoidable default)**.** *There is no stationary equilibrium under the default policy that sets the default rate equal to zero (no default) at all times*

Proof. Set $\hat{d}_t = 0$ for all t, then equations (3.38) and (3.39) become

$$\hat{\pi}_t = \frac{1-\eta-\rho}{\alpha-\rho}\chi_1\hat{g}_t,$$

$$\hat{\pi}_t = \left(\frac{\chi_2}{1-\beta\rho} + \beta\frac{1-\eta-\rho}{1-\beta\rho}\chi_1\right)\hat{g}_t + \frac{\bar{b}}{\bar{b}+\bar{b}^*}\hat{b}_{t-1} + \frac{\bar{b}^*}{\bar{b}+\bar{b}^*}\hat{b}^*_{t-1}.$$

[7] This conclusion holds when restricting attention to default policies characterized by bounded expectations of default rates.

[8] Important for this conclusion is the fact that monetary policy is set in terms of the contractual (risky) interest rate. Bi et al. (2010) and Galí (2013) show that if monetary authority were to be set in terms of the risk-free rate default would not affect stabilization of inflation.

It is impossible to satisfy both equations at the same time, hence the stationary equilibrium with no default does not exist. □

This result is reminiscent to that of the original closed-economy formulation of a fiscal theory of the sovereign risk by Uribe (2006). The reason why nonzero default is needed is to keep inflation rate bounded. Without occasional default, inflation is the only adjustment mechanism available to preserve the intertemporal government budget constraint of the Home country government. "Active" monetary policy then puts inflation on an explosive path—see Loyo (2000) for a discussion.

When both, inflation and default, play roles in stabilizing total government debt, the monetary policy stance has implications not only for the stabilization of prices but also for the ultimate amount of default required to achieve fiscal sustainability. Analytically, this interrelation can be traced by combining equations (3.38) and (3.39) to eliminate inflation so as to get

$$\mathbb{E}_t \sum_{j=0}^{\infty} \left[\lambda\beta^j - \left(\frac{1}{\alpha}\right)^j \right] \hat{d}_{t+j} = \varphi_\delta \hat{g}_t + \frac{\bar{b}}{\bar{b}+\bar{b}^*}\hat{b}_{t-1} + \frac{\bar{b}^*}{\bar{b}+\bar{b}^*}\hat{b}^*_{t-1}, \tag{3.40}$$

where

$$\varphi_\delta \equiv \left[\frac{\chi_2}{1-\beta\rho} + \left(\frac{\beta}{1-\beta\rho} - \frac{1}{\alpha-\rho} \right) (1-\eta-\rho)\,\chi_1 \right]$$

The main take away from this expression is that different degrees of aggressiveness of the central bank reaction to inflation, *ceteris paribus*, require different sequences of sovereign default rates. For instance, given expectations of future default rates, more aggressive monetary policy, characterized by a larger values of α, calls for a higher default rate in the current period. This result is rather intuitive if one thinks of inflation and default as two alternative channels of achieving fiscal sustainability. The monetary policy stance directly influences the extent to which inflation channel can be exploited. Hence, changes of the monetary policy rule indirectly affect the magnitude of default that is needed to bring government debt on a sustainable trajectory.

3.4.2. Risk-Sharing

In what follows I assume that the monetary union has been up and running long enough that initial conditions become unimportant. Analogous results can be derived assuming that the monetary union starts at the steady state.

Iterating equation (3.34) backwards, I express Home country consumption as

$$\hat{c}_t = \sum_{j=0}^{\infty} (1-\eta)^j \Big[(1-\eta-\rho)\chi_1 \hat{g}_{t-1-j} + (\hat{c}_{t-j} - \mathbb{E}_{t-1-j}\hat{c}_{t-j}) \Big]. \tag{3.41}$$

Contrary to the perfect risk-sharing case, the Home country consumption in the equilibrium with incomplete markets is, in general, history-dependent. Equation (3.41) shows that the history of consumption forecast errors influences the current consumption. The history-dependence of the Home country consumption, however, disappears when forecast errors follow a particular pattern. This observation is formally captured in the following lemma.

Lemma 3.1 (History-independent consumption)**.** *If Home country consumption forecast errors in the equilibrium with incomplete markets take the form*

$$\hat{c}_t - \mathbb{E}_{t-1}\hat{c}_t = -\chi_1 \varepsilon_t, \qquad \forall t, \tag{3.42}$$

then Home country consumption deviations from the steady-state are history-independent and take the form identical to the perfect risk-sharing case

$$\hat{c}_t = -\chi_1 \hat{g}_t. \tag{3.43}$$

Proof. Replace forecast errors in expression (3.41) with (3.42) and use the stochastic process of government expenditures (3.33) to express the Home country consumption as

$$\hat{c}_t = \chi_1 \sum_{j=0}^{\infty} (1-\eta)^j \Big[(1-\eta)\hat{g}_{t-1-j} - \hat{g}_{t-j}\Big].$$

Expanding this expression, it is straightforward to see that all the terms but one cancel out and one arrives to expression (3.43). ☐

A straightforward corollary of this Lemma is that the same condition makes deviations of the Foreign country consumption from the steady-state history-independent and identical to the perfect risk-sharing case.

As a next step I focus on consumption forecast errors. Taking into account (3.33)–(3.35), one can solve forward the current account identity, (3.36), and the government budget constraint of the Home country, (3.37). Combining the two and exploiting the fact that bond holdings of the private sector are predetermined variables, I get the following expression characterizing the forecast error of the Home country consumption

$$\hat{c}_t - \mathbb{E}_{t-1}\hat{c}_t = -\frac{\kappa_1}{\bar{c}}\chi_1 \varepsilon_t - \frac{\kappa_2}{\bar{c}}\lambda \left[\sum_{j=0}^{\infty} \beta^j \left(\mathbb{E}_t \hat{d}_{t+j} - \mathbb{E}_{t-1}\hat{d}_{t+j}\right)\right]. \tag{3.44}$$

where coefficients κ_1 and κ_2 do not depend on policy parameters and are defined as follows

$$\begin{aligned} \kappa_1 &\equiv \left[\frac{(1-\eta-\rho)\beta}{1-\beta\rho}\bar{c} + \frac{(1-(1-\eta)\beta)}{1-\beta\rho}\frac{(\bar{c}+\bar{c}^*)}{(\bar{b}+\bar{b}^*)}\bar{b}\right], \\ \kappa_2 &\equiv (1-(1-\eta)\beta)\,\bar{b}^*. \end{aligned} \tag{3.45}$$

The strength of (direct) propagation of innovations to government expenditures, ε_t, onto the forecast error of the Home country consumption is determined by the coefficient attached to ε_t in (3.44). The following lemma characterizes how two terms entering this coefficient are compared to each other depending on the allocation of consumption and debt in the deterministic steady state.

Lemma 3.2. *The coefficient* κ_1 *is greater (less) than the steady state consumption in the Home country,* $\bar{c}$*, whenever*

$$\frac{\bar{b}}{\bar{b}+\bar{b}^*} < (>) \frac{\bar{c}}{\bar{c}+\bar{c}^*} \tag{3.46}$$

Proof. The inequality $\kappa_1 < (>) \; \bar{c}$ can be rewritten, using definition (3.45), as follows

$$(\beta - \eta\beta - \rho\beta)\bar{c} + (1 - \beta + \eta\beta)\left(\frac{\bar{c}+\bar{c}^*}{\bar{b}+\bar{b}^*}\bar{b}\right) < (>) \; (1 - \beta\rho)\bar{c}.$$

Rearranging terms in the last inequality delivers (3.46). □

Using resource constraint (3.30) I derive expression describing the forecast error of the Foreign country consumption

$$\hat{c}_t^* - \mathbb{E}_{t-1}\hat{c}_t^* = -\frac{\kappa_1^*}{\bar{c}^*}\chi_1\varepsilon_t - \frac{\kappa_2}{\bar{c}^*}\lambda\left[\sum_{j=0}^{\infty}\beta^j\left(\mathbb{E}_t\hat{d}_{t+j} - \mathbb{E}_{t-1}\hat{d}_{t+j}\right)\right], \tag{3.47}$$

where $\kappa_1^* \equiv \bar{c} + \bar{c}^* - \kappa_1$.

First important conclusion coming from 3.44 is that with nondiscriminatory default, *i.e.* $\lambda = 0$, default policy does not have real effects on the allocation of consumption. To see that, first note that when $\lambda = 0$, according to (3.44) and (3.47), consumption forecast errors become independent of unexpected changes in default rates. Then, equations (3.41) and (3.30) imply that deviations of consumption from the steady state do not depend on default policy. Under the imperfectly discriminatory default, on the other hand, default policy does have real effects. This conclusion has a notable implication, namely that monetary policy has real effects on the allocation of consumption when default is imperfectly discriminatory. It follows from (3.40) that can be transformed into

$$\sum_{j=0}^{\infty}\left[\lambda\beta^j - \left(\frac{1}{\alpha}\right)^j\right]\left(\mathbb{E}_t\hat{d}_{t+j} - \mathbb{E}_{t-1}\hat{d}_{t+j}\right) = \varphi_\delta\hat{\varepsilon}_t \tag{3.48}$$

because bond holdings are predetermined variables. According to (3.48), for a given λ, different values of monetary rule inflation feedback coefficient α require different adjustments of the path of default rates in response to unforecastable innovations to government expenditures. With imperfect default discrimination, different paths of default rates in turn imply different paths of consumption as determined by (3.44) and (3.41). Note that real effects of changes in the monetary policy rule in this setup arise despite the absence of nominal rigidities of any sort.

Second important conclusion coming from 3.44 is that with nondiscriminatory default, *i.e.* $\lambda = 0$, perfect risk-sharing, in general, is not achieved. To see that, note that when $\lambda = 0$, according to (3.44), forecast errors of the Home country consumption take the form that is different from the one specified in Lemma 3.1, unless $\kappa_1 = \bar{c}$. These conclusions allow to formulate the main result of this chapter.

Proposition 3.2. *In general, imperfect default discrimination, $\lambda > 0$, is a necessary condition for default policy to be able to achieve the perfect risk-sharing.*

To provide further insights it is instructive to consider how the magnitude of forecast errors of the Home country consumption determined by (3.44) in the case of nondiscriminatory default, $\lambda = 0$, compares to the magnitude prescribed by (3.42) to achieve the perfect risk-sharing. Using Lemma 3.2 and equations characterizing deterministic steady state (see Section 3.2.4), one can show that there exists a threshold level of the foreign debt position

$$\bar{b}^*_{\mathrm{tr}} \equiv \frac{y^*}{y + y^* - \tau} \left(\frac{\tau - \bar{g}}{1 - \beta} \right)$$

for which $\kappa_1 = \bar{c}$. Under the nondiscriminatory default, $\lambda = 0$, forecast errors of the Home country consumption satisfy condition (3.42) when $\bar{b}^* = \bar{b}^*_{\mathrm{tr}}$. In this specific case one observes the perfect risk-sharing in the economy despite default policy not having real effects. If government bond holdings of the Foreign country households are in excess of the threshold level, $\bar{b}^* > \bar{b}^*_{\mathrm{tr}}$, then $\kappa_1 > \bar{c}$, which means that forecast errors of the Home country consumption are too low to lead to the perfect risk-sharing. If, on the other hand, government bond holdings of the Foreign country households are below the threshold level, $\bar{b}^* < \bar{b}^*_{\mathrm{tr}}$, then $\kappa_1 < \bar{c}$, which means that forecast errors of the Home country consumption are too high to lead to the perfect risk-sharing.

3.4.3. Equilibrium under a Simple Default Rule

I close this section by studying implications of a particular type of default policy. In this section I show how one can design a simple default rule that is compatible with the existence of stationary competitive equilibrium with the perfect risk-sharing.

Assume that the economy starts at the steady-state and let me specify a simple default rule of the form

$$\hat{d}_t = \phi_g \hat{g}_t \tag{3.49}$$

according to which default rates are set with a feedback to government expenditures. Thus, the model contains three parameters that describe policy. First parameter, α, determines inflation feedback in the rule (3.27) followed by the central bank of the monetary union when setting the short-term nominal interest rates, and the remaining two parameters ϕ_g and λ determine expenditures feedback in the default rule (3.49) and the fraction of default gains transferred to households of the Home country correspondingly. The focus of analysis in this section is on combinations of these three parameters. In particular, I will argue that given α there is in general a unique combination of λ and ϕ_g that leads to equilibrium with the perfect risk-sharing.

First, I derive current consumption in a given period of time when outstanding bonds held by households of both countries are equal to the steady-state levels:

$$\hat{c}_t = -\left(\frac{\kappa_1}{\bar{c}}\chi_1 + \lambda\frac{\kappa_2}{\bar{c}}\frac{\phi_g}{1-\beta\rho}\right)\hat{g}_t.$$

It is straightforward to show that this consumption is identical to the perfect risk-sharing benchmark if the the following condition holds

$$\lambda\phi_g = (1-\beta\rho)(\bar{c}-\kappa_1)\frac{\chi_1}{\kappa_2} \tag{3.50}$$

so that $\hat{c}_t = -\chi_1\hat{g}_t$. Condition (3.50) jointly restricts policy parameters λ and ϕ_g. Recall that, unless the steady state bond holdings of households in the Foreign country, $\bar{b}^*$, are equal to the threshold level $\bar{b}^*_{\mathrm{tr}}$, $\bar{c} \neq \kappa_1$ and therefore both policy parameters, λ and ϕ_g, have to be different from zero to satisfy condition (3.50). Furthermore, under the assumption $\lambda \geqslant 0$, the default rule parameter ϕ_g has to be strictly positive when $\bar{b}^* < \bar{b}^*_{\mathrm{tr}}$ and strictly negative when $\bar{b}^* > \bar{b}^*_{\mathrm{tr}}$.

Second, using (3.36) and (3.37), I derive the following expressions that characterize the amount of newly issued bonds, $\hat{b}_t$ and $\hat{b}^*_t$ in a given period of time when $\hat{b}_{t-1} = 0$ and $\hat{b}^*_{t-1} = 0$:

$$\beta\bar{b}\hat{b}_t = -\bar{c}\hat{c}_t + \bar{b}(\hat{d}_t + (\beta\alpha-1)\hat{\pi}_t) - \lambda(\bar{b}+\bar{b}^*)\hat{d}_t,$$
$$\beta\bar{b}^*\hat{b}^*_t = \bar{c}\hat{c}_t + \bar{b}^*(\hat{d}_t + (\beta\alpha-1)\hat{\pi}_t) + \bar{g}\hat{g}_t.$$

One can then show that if on top of condition (3.50) policy parameters λ and ϕ_g also satisfy condition

$$(\bar{b}+\bar{b}^*)(\alpha-\rho)\lambda\phi_g + \bar{b}\alpha(\beta\rho-1)\phi_g = (\bar{c}(\alpha-\rho) + \bar{b}(\beta\alpha-1)(1-\eta-\rho))\chi_1, \tag{3.51}$$

then, in a given period of time with $\hat{b}_{t-1} = 0$ and $\hat{b}^*_{t-1} = 0$, not only current consumption mimics the perfect risk-sharing benchmark but also the amount of newly issued bonds sold to households in both countries does not deviate from the steady state levels, *i.e.* $\hat{b}_t = 0$ and $\hat{b}^*_t = 0$. Therefore, if policy parameters α, λ, and ϕ_g, satisfy conditions (3.50) and (3.51), and initial conditions of the outstanding liabilities of the Home country government are equal to the steady state levels, *i.e.* $\hat{b}_{-1} = 0$ and $\hat{b}^*_{-1} = 0$, then the monetary union immediately settles on a path with constant bond holdings across the countries and the consumption allocation prescribed by the perfect risk-sharing benchmark. To complete the argument, note that the structure of conditions (3.50) and (3.51) is such that, for a given monetary policy parameter α, they uniquely pin down the remaining default policy parameters λ and ϕ_g.

A couple of remarks are in order. First, I describe the choice of parameters characterizing default policy while taking parameter characterizing monetary policy as given.

This is done with the purpose to focus on whether default policy can achieve the perfect risk-sharing on its own. In general, two conditions (3.50) and (3.51), provide one degree of freedom to choose all the three policy parameters. Hence, one could look for a combination of any two policy parameters provided the value of the third policy parameter is fixed. Second, I search directly for policy parameters that attain the perfect risk-sharing in the competitive equilibrium without specifying an explicit government policy problem where such a policy would be the optimum. One can consider this analysis as if default policy is set cooperatively or as if the Home country government had a goal of the perfect risk-sharing for implicit reasons that are outside of the scope of this model.

Finally, assumption of the steady state being the initial state of the economy is not critical in the following sense. Appendix C.2 shows that under a slightly more general default rule with an additional term that creates feedback from the outstanding amount of debt to default rates the equilibrium in the neighborhood of the steady state is stable and unique. The economy that starts away from the steady state with policy parameters that satisfy conditions (3.50) and (3.51) will experience a transitory period but will approach the steady state at some point in time. The transitory period of this economy features nontrivial debt dynamics and history-dependent allocation of consumption. However, upon reaching the steady state this economy would permanently settle on the path with the constant distribution of debt and the perfect risk-sharing as in the case with restricted initial conditions discussed above.

3.5. Conclusion

In this chapter I discuss design of default policy in a simple two-country model of the monetary union where default is unavoidable. Imperfect default discrimination is a necessary condition to attain the perfect risk-sharing in equilibrium. A simple default rule can be designed to replicate consumption allocation of the perfect risk-sharing benchmark. Moreover, under the imperfect default discrimination changes in the monetary policy rule affect real economic activity and the underlying transmission mechanism does not rely on nominal rigidities.

The model that is used to perform the analysis in this chapter intentionally assumes an ideal world where default does not create direct economic losses. This assumption allows to isolate potential gains from default. Further analysis, however, is needed to asses the relevance of these gains by comparing them to potential losses. Furthermore, introducing distortions produced by various nominal rigidities would be of interest to asses the relative merits of inflation versus default as channels of achieving fiscal sustainability.

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A. Appendix to Chapter 1

A.1. The Planner's Problem

Lagrangian corresponding to the Planner's problem is

$$L \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=-1}^{t-1} \xi_s \right) \left[u(C_t) + g(G_t) - v(Y_t) + \gamma_t \left(Y_t - C_t - G_t \right) \right].$$

First-order conditions are as follows

$$\begin{aligned} u'(C_t) &= \gamma_t, \\ g'(G_t) &= \gamma_t, \\ v'(Y_t) &= \gamma_t. \end{aligned}$$

Eliminating Lagrange multiplier γ_t leaves the system with two equations

$$\begin{aligned} u'(C_t) &= v'(Y_t), \\ g'(G_t) &= v'(Y_t), \end{aligned}$$

that together with resource constraint $Y_t = C_t + G_t$ characterize first-best allocation as a solution of the Planner's problem.

A.2. Characterization of the Ramsey Equilibrium

Lagrangian corresponding to the Ramsey problem is given by

$$
\begin{aligned}
\mathcal{L} \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=-1}^{t-1} \xi_s \right) \Bigg[& u(C_t) + g(G_t) - v(Y_t) \\
& - \lambda_{1,t} \left(u_{c,t} - R_t \xi_t \beta \mathbb{E}_t \left\{ \frac{u_{c,t+1}}{\pi_{t+1}} \right\} \right) \\
& - \frac{\lambda_{1,t-1}}{\xi_{t-1}\beta} \left(u_{c,t-1} - R_{t-1}\xi_{t-1}\beta \frac{u_{c,t}}{\pi_t} \right) \\
& - \lambda_{2,t} \left(\begin{array}{c} \left[\theta(1-s)w_t - (\theta-1) - \varphi(\pi_t - 1)\pi_t\right] Y_t u_{c,t} \\ +\varphi \xi_t \beta \mathbb{E}_t \left\{ (\pi_{t+1}-1)\pi_{t+1} Y_{t+1} u_{c,t+1} \right\} \end{array} \right) \\
& - \frac{\lambda_{2,t-1}}{\xi_{t-1}\beta} \left(\begin{array}{c} \left[\theta(1-s)w_{t-1} - (\theta-1) - \varphi(\pi_{t-1} - 1)\pi_{t-1}\right] Y_{t-1} u_{c,t-1} \\ +\varphi \xi_{t-1} \beta (\pi_t - 1)\pi_t Y_t u_{c,t} \end{array} \right) \\
& - \lambda_{3,t} \left(w_t(1-\tau_t)u_{c,t} - v_{y,t} \right) \\
& - \lambda_{4,t} \left(\left(1 - \frac{\varphi}{2}(\pi - 1)^2 \right) Y_t - C_t - G_t \right) \\
& - \lambda_{5,t} \left(R_t^{-1} b_t - \frac{b_{t-1}}{\pi_t} - (G_t + \varsigma_t - \tau_t w_t Y_t) \right) \\
& - \mathbb{E}_t \left\{ \lambda_{5,t+1}\xi_{t+1}\beta \left(R_{t+1}^{-1} b_{t+1} - \frac{b_t}{\pi_{t+1}} - (G_{t+1} - \tau_{t+1} w_{t+1} Y_{t+1}) \right) \right\} \\
& + \nu_t (R_t - 1) + \eta_t b_t \Bigg],
\end{aligned}
$$

where $\lambda_{1,-1} = \lambda_{2,-1} = 0$. First-order conditions with respect to the decision variables C_t, Y_t, π_t, w_t, τ_t, G_t, R_t, b_t are

$$
\begin{aligned}
u_{c,t} &= \lambda_{1,t} u_{cc,t} + \lambda_{2,t} \left[\theta(1-s)w_t - (\theta-1) - \varphi(\pi_t - 1)\pi_t\right] Y_t u_{cc,t} && \text{(A.1)} \\
&\quad + \lambda_{3,t} w_t (1-\tau_t) u_{cc,t} - \lambda_{4,t} \\
&\quad - \lambda_{1,t-1} R_{t-1} \frac{u_{cc,t}}{\pi_t} - \lambda_{2,t-1} \varphi(\pi_t - 1)\pi_t Y_t u_{cc,t} \\
v_{y,t} &= -\lambda_{2,t} \left[\theta(1-s)w_t - (\theta-1) - \varphi(\pi_t-1)\pi_t\right] u_{c,t} && \text{(A.2)} \\
&\quad + \lambda_{3,t} v_{yy,t} - \lambda_{4,t} \left(1 - \frac{\varphi}{2}(\pi-1)^2\right) - \lambda_{5,t} \left((\tau_t - s) w_t\right) \\
&\quad - \lambda_{2,t-1} \varphi(\pi_t - 1)\pi_t u_{c,t} \\
0 &= \lambda_{2,t} \varphi(2\pi_t - 1) Y_t u_{c,t} + \lambda_{4,t} \varphi(\pi_t - 1) Y_t - \lambda_{5,t} \frac{b_{t-1}}{\pi_t^2} && \text{(A.3)} \\
&\quad - \lambda_{1,t-1} R_{t-1} \frac{u_{c,t}}{\pi_t^2} - \lambda_{2,t-1} \varphi(2\pi_t - 1) Y_t u_{c,t} \\
0 &= \lambda_{2,t} \theta(1-s) Y_t u_{c,t} + \lambda_{3,t} (1-\tau_t) u_{c,t} + \lambda_{5,t} \left((\tau_t - s) Y_t\right) \\
0 &= \lambda_{3,t} u_{c,t} - \lambda_{5,t} Y_t && \text{(A.4)} \\
g_{G,t} &= -\lambda_{4,t} - \lambda_{5,t} && \text{(A.5)} \\
0 &= \lambda_{1,t} \xi_t \beta \mathbb{E}_t \left\{ \frac{u_{c,t+1}}{\pi_{t+1}} \right\} + \lambda_{5,t} \frac{b_t}{R_t^2} + \nu_t && \text{(A.6)} \\
0 &= \beta \mathbb{E}_t \left\{ \xi_{t+1} \frac{\lambda_{5,t+1}}{\pi_{t+1}} \right\} - \frac{\lambda_{5,t}}{R_t} + \eta_t && \text{(A.7)}
\end{aligned}
$$

Multipliers $\nu_t \geqslant 0$, $\eta_t \geqslant 0$ and have to satisfy Kuhn-Tucker complementary slackness conditions

$$
\begin{aligned}
0 &= \nu_t \left(R_t - 1\right), \\
0 &= \eta_t b_t.
\end{aligned}
$$

A.3. Characterization of the Markov-Perfect Equilibrium

Lagrangian for the Markov problem of the government is as follows

$$
\begin{aligned}
\mathcal{L} \equiv & \, u(C) + g(G) - v(Y) + \beta\xi\mathbb{E}\left\{\mathcal{V}\left(b;\xi'\right)\right\} \\
& - \lambda_1\left[u_c - R\beta\xi\mathbb{E}\left\{S\left(b;\xi'\right)\right\}\right] \\
& - \lambda_2\left[\left[\theta(1-s)w - (\theta-1) - \varphi(\pi-1)\pi\right]Yu_c + \varphi\beta\xi\mathbb{E}\left\{Z\left(b;\xi'\right)\right\}\right] \\
& - \lambda_3\left[w(1-\tau)u_c - v_y\right] \\
& - \lambda_4\left[\left(1 - \frac{\varphi}{2}(\pi-1)^2\right)Y - C - G\right] \\
& - \lambda_5\left[R^{-1}b - \frac{b_{-1}}{\pi} - (G + \varsigma - \tau wY)\right] \\
& + \nu\left[R-1\right] + \eta b,
\end{aligned}
$$

where

$$
\begin{aligned}
S\left(b;\xi'\right) \equiv & \frac{u_c\left(\mathcal{C}\left(b;\xi'\right)\right)}{\Pi\left(b;\xi'\right)}, \\
Z\left(b;\xi'\right) \equiv & \mathcal{Y}\left(b;\xi'\right) \cdot \left(\Pi\left(b;\xi'\right) - 1\right) \cdot \Pi\left(b;\xi'\right) \cdot u_c\left(\mathcal{C}\left(b;\xi'\right)\right).
\end{aligned}
$$

Multipliers $\nu \geqslant 0$, $\eta \geqslant 0$ and have to satisfy Kuhn-Tucker conditions

$$
\begin{aligned}
0 &= \nu\left(R-1\right), \\
0 &= \eta b.
\end{aligned}
$$

Ignoring functions' arguments, the corresponding first-order conditions, apart from the competitive equilibrium constraints, include

$$
[C]: \quad u_c = \left(\lambda_1 + \lambda_2\left[\theta(1-s)w - (\theta-1) - \varphi(\pi-1)\pi\right]Y + \lambda_3 w(1-\tau)\right)u_{cc} - \lambda_4 \tag{A.8}
$$

$$
[Y]: \quad v_y = -\lambda_2\left[\theta(1-s)w - (\theta-1) - \varphi(\pi-1)\pi\right]u_c + \lambda_3 v_{yy} - \lambda_4\left(1 - \frac{\varphi}{2}(\pi-1)^2\right) - \lambda_5(\tau-s)w \tag{A.9}
$$

$$
[\pi]: \quad 0 = \varphi\left(\lambda_2(2\pi-1)u_c + \lambda_4(\pi-1)\right)Y - \lambda_5\frac{b_{-1}}{\pi^2} \tag{A.10}
$$

$$
[w]: \quad 0 = \left(\lambda_2\theta(1-s)Y + \lambda_3(1-\tau)\right)u_c + \lambda_5(\tau-s)Y \tag{A.11}
$$

$$
[\tau]: \quad 0 = \lambda_3 u_c - \lambda_5 Y \tag{A.12}
$$

$$
[G]: \quad g_G = -\lambda_4 - \lambda_5 \tag{A.13}
$$

$$
[R]: \quad 0 = \lambda_1\beta\xi\mathbb{E}\left\{S'\right\} + \lambda_5 bR^{-2} + \nu \tag{A.14}
$$

$$
[b]: \quad 0 = \beta\xi\mathbb{E}\left\{\mathcal{V}_b'\right\} + \beta\xi\lambda_1 R\mathbb{E}\left\{S_b'\right\} - \lambda_2\beta\xi\varphi\mathbb{E}\left\{Z_b'\right\} - \lambda_5 R^{-1} + \eta \tag{A.15}
$$

where subscripts denote partial derivatives. Envelope theorem implies

$$
\mathcal{V}_b' = \frac{\lambda_5'}{\Pi'} \tag{A.16}
$$

Simplifying the system

One can work around with the system of first order conditions and simplify it. Using resource constraint (1.8) I solve for government expenditures

$$G = \hat{G}(Y, C, \pi) \equiv \left(1 - \frac{\varphi}{2}(\pi - 1)^2\right) Y - C.$$

Analogously I use Phillips curve (1.7) and consumption leisure trade-off (1.4) to express wage and labor tax

$$w = \hat{w}\left(Y, C, \pi, b\right) \equiv \frac{(\theta - 1)}{\theta(1 - s)} + \frac{\varphi}{\theta(1 - s)}\left((\pi - 1)\pi - \beta\frac{\mathbb{E}\left\{Z'\right\}}{Y u_c}\right),$$

$$\tau = \hat{\tau}\left(Y, C, \pi, b\right) \equiv 1 - \frac{1}{\hat{w}}\frac{v_y}{u_c}.$$

Private Euler equation (1.5) along with zero-lower bound condition (1.9) deliver

$$R = \hat{R}\left(C, b\right) \equiv \max\left\{1, \frac{u_c}{\beta\xi\mathbb{E}\left\{S'\right\}}\right\},$$

I proceed by explicitly solving for Lagrange multipliers $\lambda_2, \lambda_3, \lambda_4$ and λ_5 from equations (A.10)-(A.13).

$$\lambda_2 = \hat{\lambda}_2\left(Y, C, \pi, b\right) \equiv \Omega\frac{g_G}{u_c}\left[\frac{\hat{\tau}}{\theta} + \frac{Y}{(1 - \hat{\tau})u_c}\right], \tag{A.17}$$

$$\lambda_3 = \hat{\lambda}_3\left(Y, C, \pi, b\right) \equiv -\Omega\frac{g_G}{u_c}Y, \tag{A.18}$$

$$\lambda_4 = \hat{\lambda}_4\left(Y, C, \pi, b\right) \equiv \left(\Omega - 1\right) g_G, \tag{A.19}$$

$$\lambda_5 = \hat{\lambda}_5\left(Y, C, \pi, b\right) \equiv -\Omega g_G, \tag{A.20}$$

where $\Omega < 1$ is defined as follows

$$\Omega\left(Y, C, \pi, b\right) \equiv \frac{\varphi\pi^2(\pi - 1)Y}{\varphi\pi^2(\pi - 1)Y + \varphi\pi^2(2\pi - 1)Y\left(\frac{\hat{\tau}}{\theta} + \frac{Y}{(1-\hat{\tau})u_c}\right) + b_{-1}}.$$

Non-binding ZLB

If zero-lower bound is not binding, then from (A.14)

$$\lambda_1 = \hat{\lambda}_1\left(Y, C, \pi, b\right) \equiv \Omega\frac{g_G}{\left(u_c\right)^2}b\beta\xi\mathbb{E}\left\{S'\right\}, \quad \text{if } \eta = 0, \tag{A.21}$$

and I am left with 4 unknowns (Y, C, π, b) and the same number of equations

$$0 = u_c - \left(\hat{\lambda}_1 + \hat{\lambda}_2 \left[\theta\hat{w} - (\theta-1) - \varphi(\pi-1)\pi\right] Y + \hat{\lambda}_3 \hat{w}(1-\hat{\tau})\right) u_{cc} + \hat{\lambda}_4, \tag{A.22}$$

$$0 = v_y + \hat{\lambda}_2 \left[\theta(1-s)\hat{w} - (\theta-1) - \varphi(\pi-1)\pi\right] u_c - \hat{\lambda}_3 v_{yy} + \hat{\lambda}_4 \left(1 - \frac{\varphi}{2}(\pi-1)^2\right) + \hat{\lambda}_5 \hat{\tau}\hat{w}, \tag{A.23}$$

$$0 = \hat{R}^{-1} b - \frac{b_{-1}}{\pi} - \left(\hat{G} + \hat{\varsigma} - \hat{\tau}\hat{w}Y\right), \tag{A.24}$$

$$0 = \beta\xi\mathbb{E}\left\{\frac{\hat{\lambda}_5'}{\Pi'}\right\} + \beta\xi\hat{\lambda}_1\hat{R}\mathbb{E}\left\{S_b'\right\} - \hat{\lambda}_2\beta\xi\varphi\mathbb{E}\left\{Z_b'\right\} - \hat{\lambda}_5\hat{R}^{-1}, \tag{A.25}$$

where first two correspond to first-order conditions (A.8) and (A.9), equation (A.24) is the government budget constraint, and (A.25) is the so-called generalized Euler equation (GEE) characterizing optimal bond purchases choice. The GEE equates the discounted expected utility loss resulting from a tighter budget constraint in the future with the current direct and indirect gains resulting from a marginal relaxation of the budget constraint today and the other competitive equilibrium constraints. There is also ad-hoc lending constraints (1.11), which means that GEE has to hold only for the interior solution $b > 0$.

Binding ZLB

For the case of binding zero-lower bound I can not use (A.14) to express λ_1. Instead, the very fact of binding zero-lower bound delivers extra equation of the form

$$\xi\beta\frac{\mathbb{E}\left\{S\left(b;\xi'\right)\right\}}{u_c} = 1. \tag{A.26}$$

One can then use equation (A.8) to solve for λ_1 as follows

$$\lambda_1 = \hat{\lambda}_1\left(Y, C, \pi, b\right) \equiv \frac{u_c - g_g}{u_{cc}} + \Omega Y \frac{g_G}{u_c}\left(\frac{1}{Y}\frac{u_c}{u_{cc}} + \hat{w}(1-\hat{\tau}) - \left[\frac{\hat{\tau}}{\theta} + \frac{Y}{(1-\hat{\tau})u_c}\right]\left[\theta(1-s)\hat{w} - (\theta-1) - \varphi(\pi-1)\pi\right]\right)$$

and use system of equations (A.23)-(A.25) along with (A.26) to solve for unknown (Y, C, π, b).

Solution

Described system of equations has to be satisfied for each $(b_{-1}; \xi)$, hence it is a system of functional equations with solution described by a tuple of functions $\{\mathcal{Y}, \mathcal{C}, \mathit{\Pi}, \mathcal{B}\}$. As it is common, the system characterizing Markov-perfect equilibria appears to contain yet unknown policy functions and their derivatives.

A.4. Analysis of the Steady State

To analyze time-invariant deterministic long run steady states of both Ramsey and Markov-perfect equilibria set $\sigma = 0$, $\xi_t = 1$ for all $t \geqslant 0$. I restrict attention by considering interior steady states where inequlity constraints on nominal interest rate R and government debt holdings b do not bind, i.e. $\nu = 0$ and $\eta = 0$. Until further notice, consider the case when there is no employment subsidy, $s = 0$.

Start with the Ramsey policy regime. Standard to this class of models, there is in fact a continuum of Ramsey steady states indexed with initial level of government debt. To put it the other way, a continuous range of debt levels can be supported in equilibrium, hence the model exhibits indeterminacy of degree one. I characterize this continuum of steady states with the following proposition.

Proposition A.1 (Ramsey Steady State)**.** *Level of debt in the Ramsey steady state is indeterminate. For a given debt level b, Ramsey steady state is characterized by*

$$\pi = 1 \text{ and } R = \beta^{-1},$$

and marginal utility conditions

$$\left(\frac{\theta - 1}{\theta} - \frac{G + (1-\beta)b}{Y}\right) u'(C) = v'(Y), \tag{A.27}$$

$$g'(G) \geqslant v'(Y). \tag{A.28}$$

Proof. Euler equation at the steady state implies

$$R\beta = \pi.$$

The same condition is implied by the tirst-order condition w.r.t. b_t, (A.7), at the steady state. For this reason, the system of equations determining equilibrium at the steady state exhibits indeterminacy of dimension one. There is a continuum of steady states that can be indexed with the level of debt. One can freely choose steady state level of debt knowing there exist initial condition that would support it. First-order considiton w.r.t. nominal interest rate R_t, (A.6), at the steady state becomes $\lambda_5 b R^{-2} = -\lambda_1 \beta \pi^{-1} u_c$. Combining the two and using it together with first-order condition w.r.t. inflation π_t, (A.3), one can derive the following steady state condition

$$\lambda_4 \varphi(\pi - 1) = 0.$$

Largange multiplier λ_4 corresponds to resource constraint and using envelope theorem argument is equal to the marginal value of relaxing resource constraint. Clearly $\lambda_4 \neq 0$ and it then follows that at the steady state

$$\pi = 1 \text{ and } R = \beta^{-1}.$$

I take into account previous result in all the following derivations. Government budget constraint, Phillips curve, and consumption-leisure optimality condition of the private sector at the steady state can be correspondingly rewritten as

$$\tau w = \frac{G + (1-\beta)b}{Y},$$
$$w = \frac{(\theta - 1)}{\theta},$$
$$\frac{v_y}{u_c} = w(1-\tau).$$

First two equations can be used to eliminate real wage w and tax rate τ from the third one in order to get

$$\underbrace{\left(\frac{\theta-1}{\theta} - \frac{G+(1-\beta)b}{Y}\right)}_{\in(0,1)} u_c = v_y,$$

and the implied inequality

$$u_c > v_y \tag{A.29}$$

First-order conditions w.r.t government spending G_t, (A.5), at the steady state is

$$g_G = -\lambda_4 - \lambda_5. \tag{A.30}$$

Combining it with first-order condition w.r.t. labor supply Y_t, (A.2), to get the following steady state condition $g_G - v_y = -\lambda_3 \left(v_{yy} + (1-\tau w)\right)$. By assumption $v_{yy} \leqslant 0$. For any plausible calibrations $(1-\tau w) \geqslant 0$. Therefore necessary and sufficient condition for the last part of the proposition is

$$g_G \geqslant v_y \text{ } \textit{iff} \text{ } \lambda_3 \leqslant 0, \tag{A.31}$$

where λ_3 is the Lagrange multiplier attached to consumption-leisure optimality condition of the private sector.

The remaining part is proved by contradiction. Using first-order conditions w.r.t. tax rate τ_t, (A.4), and inflation π_t, (A.3), another first-order condition w.r.t. consumption C_t, (A.1), can be rearranged into the following steady state condition

$$\underbrace{u_c}_{\geqslant 0} = \lambda_3 \underbrace{\left(\frac{(1-\beta)b}{Y} + \frac{v_y}{u_c}(1-\tau)\right)}_{\geqslant 0} \underbrace{u_{cc}}_{\leqslant 0} - \lambda_4.$$

Assume $\lambda_3 > 0$, then from the last condition it follows that

$$-\lambda_4 \geqslant u_c \geqslant 0.$$

Recall that $u_c > v_y$ by (A.29). Moreover, if $\lambda_3 > 0$ then $v_y > g_G$ by (A.31). The chain of inequalities then becomes

$$-\lambda_4 \geqslant u_c > v_y > g_G \geqslant 0. \tag{A.32}$$

Recall that Lagrange multiplier λ_4 corresponds to the resource constraint. Resource constraint can be relaxed in a number of ways adjusting labor supply, public and private consumption. It therefore has to be the case that the marginal gain from relaxing budget constraint has to be larger or equal than the marginal cost of reducing public consumption, i.e. $\lambda_4 \geqslant g_G$,which contradicts (A.32). Therefore $\lambda_3 \leqslant 0$ and $g_G \geqslant v_y$. □

This result is similar to Adam (2011) and Motta and Rossi (2013). All Ramsey steady states share the same zero inflation condition and identical positive nominal interest rate.

Compare marginal utility conditions for Ramsey steady state with analogous conditions for the first-best allocation. Conditions (A.27) and (A.28) show that in general there are wedges between marginal utilities of private and public consumption and marginal disutility from labor. Wedge in equation (A.27) implies $u'(C) \geqslant v'(Y)$ so that private consumption in Ramsey steady state falls below first-best optimum. This wedge appears due to firms charging monopolistic mark-up and distortionary nature of taxes required to finance steady state government spending and debt interest payments (for positive levels of debt). The fact that government spending requires collection of distortionary tax provides incentive for the planner to reduce government spending below first-best optimum as described by condition (A.28). Clearly the output also appears to be below efficient level.

If no-lending constraint (1.18) was not in place, one could show that there exists a unique negative level of government debt for which Ramsey steady state is in fact efficient. In this steady state government uses interest receipts from holding assets in order to finance government spending and offset monopolistic distortion without resorting to distortionary labor tax. The magnitude of such efficient asset evel is generally a large fraction of the GDP.[1]

Now turning to the Markov-perfect equilibrium. Steady-state of the Markov-perfect equilibrium in general is hard to characterize, because first-order conditions of the government problem contain derivatives of unknown functions. One can still devise a limited analytical insight into properties of the Markov steady state allocation. To do so, I use Ramsey steady state as a reference and see what happens if policymaker were to achieve price stability $\pi = 1$ in Markov steady state. Under price stability, a subset of first-order conditions imply the reaction function

$$u'(C) = v'(Y),$$
$$g'(G) = v'(Y),$$

[1]See for instance Aiyagari et al. (2002) for a neoclassical model without monopolistic competition and Gnocchi and Lambertini (2014) for a New Keynesian model.

under which the marginal utilities of private and public consumption are equated to marginal disutility of labor just like in the first-best allocation. Such behavior is counterproductive in the presence of economic distortions and cannot be sustained in equilibrium, which leads to deviation of Markov steady state from Ramsey steady state. In particular the following results regarding inflation holds.

Proposition A.2 (Markov steady state)**.** *Given discount factor β close enough to 1, Markov-perfect steady state exhibits positive trend inflation*

$$\pi > 1.$$

Proof. Like in the Ramsey steady state, set of the competitive equilibrium constraints imply

$$R\beta = \pi, \tag{A.33}$$

$$u_c > v_y. \tag{A.34}$$

Condition (A.33) together with zero lower bound on interest rate $R \geqslant 1$ imply $\pi \geqslant \beta$. Moreover, $\pi = 1$ is not a equilibrium. To see this first note that equations (A.17)-(A.20) and (A.21) for $\pi = 1$ would imply $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_5 = 0$ and $\lambda_4 = -g_G$. This fact together with equations (A.8) and (A.9) would imply $u_c = v_y$, which contradicts (A.34). Therefore for β close to one it must be that $\pi > 1$. □

It is then straightforward to show that government spending in Markov steady state is in general different from its counterpart in Ramsey steady state. This result shows that lack of credibility has consequences for the long run properties of optimal policies. Discretionary policymaker in attempt to boost inefficiently low output in the long run leads to biases in inflation and government spending. The former is a reminiscence of the debated outcome due to credibility problem from Kydland and Prescott (1977), and the latter is an accompaniment that arises due to distortionary financing of endogenous government spending akin Adam and Billi (2014) where compared to my setup there is no coordination between monetary and fiscal government authorities.[2]

A.4.1. Efficient Subsidy Rate

Here I relax the assumption of zero subsidy and provide proof of the proposition 1.1.

Proposition. *For a given debt level b, there exists a unique employment subsidy rate $s^e(b)$ such that*

- *corresponding Ramsey steady state is efficient,*
- *there is a Markov steady state with debt level b that is also efficient.*

[2] The idea that discretionary conduct of the monetary policy will lead to a (higher) average inflation bias was criticized for instance in Blinder (1997).

Proof. When $s \neq 0$, implimentability constraints of the competitive equilibrium at the steady state imply

$$\left(\frac{\theta-1}{\theta}\frac{1}{(1-s)}-\frac{G+(1-\beta)b}{Y}\right)u_c = v_y.$$

Given efficiency conditions derived from the Planner's problem in Appendix A.1, necessary condition for competitive equilibrium to be efficient is

$$\left(\frac{\theta-1}{\theta}\frac{1}{(1-s)}-\frac{G+(1-\beta)b}{Y}\right) = 1.$$

Subsidy rate $s^e(b)$ is a solution to this equation. One can then verify that with $s^e(b)$ remaining first-order conditions in Ramsey and Markov equilibria coincide and remaining efficiency conditions are satisfied. $\square$

A.5. Solution Method

I use value function iteration procedure to search for a fixed point of value function and policy functions. Starting with a guess of the next period value function and future policy functions, on a discretized state space, I solve Markov optimization problem of the discretionary government. New solution is used to update guesses of the value and policy functions, and repeat the procedure until convergence when value and policy functions from two consecutive iterations become arbitrarily close.

Off the grid points I use cubic splines to interpolate value and policy functions. Expectations are computed with Gauss-Hermite quadrature. As for the grid, I start with equidistant points and then augment them with a set of adaptive points. Borrowing idea from Brumm and Grill (2014), I choose adaptive part of the grid to better capture the kink due to occasionally binding zero lower bound.

I implement solution method in Matlab using open source nonlinear optimization solver IPOPT. Interface of IPOPT solver for Matlab environment is implemented in a freeware third-party OPTI toolbox presented in Currie and Wilson (2012). To improve computation speed, solution of the Markov optimization problem over the grid is done via parallel computing.

B. Appendix to Chapter 2

B.1. Coefficients in the Flexible Target Criteria

The optimal flexible target criteria take the form

$$\hat{C}_t = \Omega_c \hat{\pi}_t, \quad \hat{Y}_t = \Omega_y \hat{\pi}_t, \quad \hat{G}_t = \Omega_g \hat{\pi}_t$$

where the coefficients depend on the stuctural parameters of the model and are defined as follows

$$\Omega_c \equiv \frac{\phi\bar{Y}\left(\bar{G}\left(-\beta\theta(\rho-1)\bar{b}\gamma_c\gamma_h-\bar{Y}(\beta\rho-1)(\theta\gamma_h(-\gamma_c+\gamma_g+1)+\gamma_g)\right)+\bar{Y}\gamma_g\left(\bar{Y}(\beta\rho-1)(\theta(\gamma_c+\gamma_h)+1)-\beta\theta(\rho-1)\bar{b}\gamma_c\right)+\theta\bar{G}^2(\beta\rho-1)\gamma_h\right)}{\left(\bar{G}-\bar{Y}\right)\left(\phi\bar{Y}(\beta\rho-1)-\theta\bar{b}\right)\left(\bar{G}\gamma_h(\gamma_c-\gamma_g)+\bar{Y}\gamma_g(\gamma_c+\gamma_h)\right)},$$

$$\Omega_y \equiv \frac{\phi\bar{Y}\left(\gamma_g\left(\bar{Y}(\beta\rho-1)(\theta(\gamma_c+\gamma_h)+1)-\beta\theta(\rho-1)\bar{b}\gamma_c\right)+\bar{G}(\beta\rho-1)(\gamma_c(\theta\gamma_h+\theta+1)-\gamma_g(\theta\gamma_h+1))\right)}{\left(\theta\bar{b}+\phi\bar{Y}(1-\beta\rho)\right)\left(\bar{G}\gamma_h(\gamma_c-\gamma_g)+\bar{Y}\gamma_g(\gamma_c+\gamma_h)\right)},$$

$$\Omega_g \equiv \frac{\phi\bar{Y}\left(\theta\gamma_h\left(\beta(\rho-1)\bar{b}\gamma_c+\bar{G}(1-\beta\rho)\right)+\bar{Y}(\beta\rho-1)((\theta+1)\gamma_c+\theta\gamma_h)\right)}{\left(\theta\bar{b}+\phi\bar{Y}(1-\beta\rho)\right)\left(\bar{G}\gamma_h(\gamma_c-\gamma_g)+\bar{Y}\gamma_g(\gamma_c+\gamma_h)\right)}.$$

C. Appendix to Chapter 3

C.1. The Planner's Problem

The first-order conditions of the planner's problem are

$$
\begin{aligned}
[c_t]: \quad & \frac{\theta_t}{c_t^p} = \mu_t, \\
[c_t^*]: \quad & \frac{\theta_t^*}{c_t^{p*}} = \frac{1}{\Phi}\mu_t.
\end{aligned}
$$

where μ_t is the Lagrange multiplier of the union resource constraint (3.23). Dividing these first-order conditions one by another I get the rule that describes interrelation between consumption levels in Foreign and Home countries

$$
c_t^{p*} = \left(\Phi \frac{\theta_t^*}{\theta_t}\right) c_t^p \tag{C.1}
$$

One can recursively obtain optimal sequences of consumption in the two countries by starting at $t = 0$, when $\theta_0^*/\theta_0 = 1$, and using (C.1) together with (3.23) to determine c_0^p and c_0^{p*}, which in turn delivers $\theta_1^*/\theta_1 = 1$ and we can proceed with the same procedure for $t = 1$ and so on. The procedure above is general and works for any Pareto-weight Φ. Moreover, following this procedure shows that condition (C.1) simplifies to

$$
c_t^{p*} = \Phi c_t^p. \tag{C.2}
$$

This condition says that optimal allocation is characterized by consumption in two the countries being proportional over time, i.e. perfect risk-sharing is in place. Using resource constraint, (3.23), one can show that optimal consumption in both countries is expressed as constant fractions of the aggregate union endowment net of government expenditures

$$
\begin{aligned}
c_t^p &= \frac{1}{1+\Phi}(y + y^* - g_t), \\
c_t^{p*} &= \frac{\Phi}{1+\Phi}(y + y^* - g_t).
\end{aligned}
$$

Given solution of the planner's problem for a generic Pareto-weight, next thing is to determine the value of the Pareto-weight that produces allocation that in the absence of uncertainty is identical to the deterministic steady state of the competitive equilibrium.

Such Pareto-weight is equal to the ratio of values of consumption in Foreign and Home countries in the deterministic steady state of the competitive equilibrium

$$\Phi = \frac{\bar{c}^*}{\bar{c}}. \tag{C.3}$$

Using equations (3.20) and (3.21) characterizing $\bar{c}$ and $\bar{c}^*$, it is straightforward to show that $c_t^p = \bar{c}$ and $c_t^{p*} = \bar{c}^*$ when $g_t = \bar{g}$ and

$$\frac{c_t^p}{\bar{c}} = \left(\frac{y + y^* - g_t}{y + y^* - \bar{g}}\right), \qquad \frac{c_t^{p*}}{\bar{c}^*} = \left(\frac{y + y^* - g_t}{y + y^* - \bar{g}}\right).$$

otherwise.

C.2. Stability and Uniqueness under a Simple Default Rule

Set of competitive equilibrium equations (3.34)–(3.37) and (3.33) can be rewritten as

$$\begin{aligned}
(1-\eta)\hat{c}_t &= \mathbb{E}_t\hat{c}_{t+1} - (1-\eta-\rho)\chi_1\hat{g}_t,\\
\alpha\hat{\pi}_t &= \mathbb{E}_t\hat{\pi}_{t+1} + (1-\eta-\rho)\chi_1\hat{g}_t - \mathbb{E}_t\hat{d}_{t+1},\\
\hat{b}^*_{t-1} &= \beta\hat{b}^*_t - \frac{\bar{c}}{\bar{b}^*}\hat{c}_t - (\beta\alpha - 1)\hat{\pi}_t - \hat{d}_t - \frac{\bar{g}}{\bar{b}^*}\hat{g}_t,\\
\hat{b}_{t-1} &= \beta\hat{b}_t + \frac{\bar{c}}{\bar{b}}\hat{c}_t - (\beta\alpha - 1)\hat{\pi}_t - \left(1 - \lambda\frac{\bar{b} + \bar{b}^*}{\bar{b}}\right)\hat{d}_t,\\
\hat{g}_{t+1} &= \rho\hat{g}_t + \varepsilon_{t+1}.
\end{aligned}$$

Assume default policy is governed by a simple rule

$$\hat{d}_t = \phi_g\hat{g}_t + \phi_b\left(\hat{b}^*_{t-1} + \hat{b}_{t-1}\right). \tag{C.4}$$

I partition endogenous variables into $\hat{\mathrm{y}}_t^1 = \begin{bmatrix}\hat{c}_t & \hat{\pi}_t\end{bmatrix}^{\mathrm{T}}$ the vector of free (jump) endogenous state variables, and $\hat{\mathrm{y}}_{t-1}^2 = \begin{bmatrix}\hat{b}^*_{t-1} & \hat{b}_{t-1}\end{bmatrix}$ the vector of predetermined (sluggish) endogenous state variables. Then, using (C.4), I write the set of equilibrium equations in the state-space form

$$\mathrm{A}\begin{bmatrix}\hat{\mathrm{y}}_t^1\\ \hat{\mathrm{y}}_{t-1}^2\\ \hat{g}_t\end{bmatrix} = \mathbb{E}_t\mathrm{B}\begin{bmatrix}\hat{\mathrm{y}}_{t+1}^1\\ \hat{\mathrm{y}}_t^2\\ \hat{g}_{t+1}\end{bmatrix},$$

where

$$
\mathrm{A} = \begin{bmatrix} 1-\eta & 0 & 0 & 0 & (1-\eta-\rho)\chi_1 \\ 0 & \alpha & 0 & 0 & -(1-\eta-\rho)\chi_1 \\ \frac{\bar{c}}{b^*} & \beta\alpha - 1 & 1+\phi_b & \phi_b & \frac{\bar{g}}{b^*} + \phi_g \\ -\frac{\bar{c}}{\bar{b}} & \beta\alpha - 1 & \phi_b & 1+\phi_b & \left(1-\lambda\frac{\bar{b}+\bar{b}^*}{\bar{b}}\right)\phi_g \\ 0 & 0 & 0 & 0 & \rho \end{bmatrix},
$$

and

$$
\mathrm{B} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\phi_b & -\phi_b & -\phi_g \\ 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},
$$

Depending on the eigenvalues of the matrix $\mathrm{D} \equiv \mathrm{B}^{-1}\mathrm{A}$, the model features multiple, unique or no stationary equilibrium. In order to have unique stationary equilibrium there have to be two forward stable eigenvalues with values outside of the unit circle. Having less forward stable eigenvalues leads to multiple equilibria, whereas more results in equilibrium being explosive.[1]

Eigenvalues of the matrix D are

$$
\left\{\frac{1}{\beta}, \frac{1}{\beta}, \alpha(1+2\phi_b), 1-\eta, \rho\right\}.
$$

Note that default rule coefficient ϕ_g does not affect stability and uniqueness properties of an equilibrium. A sign of the third eigenvalue is ambiguous and depends on the value of default rule coefficient ϕ_b. Therefore, a necessary and sufficient condition for there to be a unique stationary equilibrium is

$$
\phi_b < \frac{1-\alpha}{2\alpha},
$$

where the right-hand is unambiguously negative because of the assumption $\alpha > 1$.

[1]See, *e.g.*, Blanchard and Kahn (1980).