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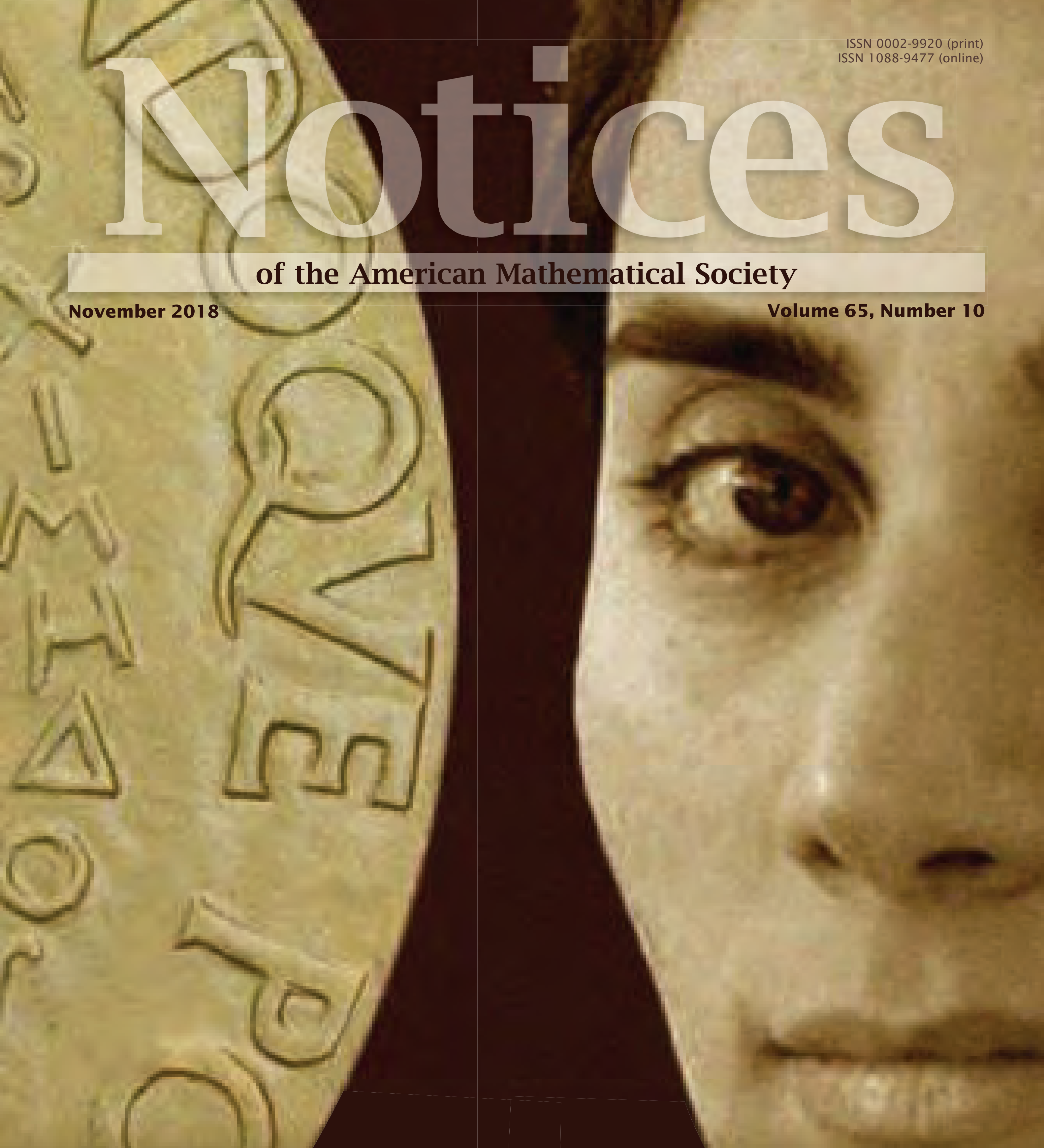
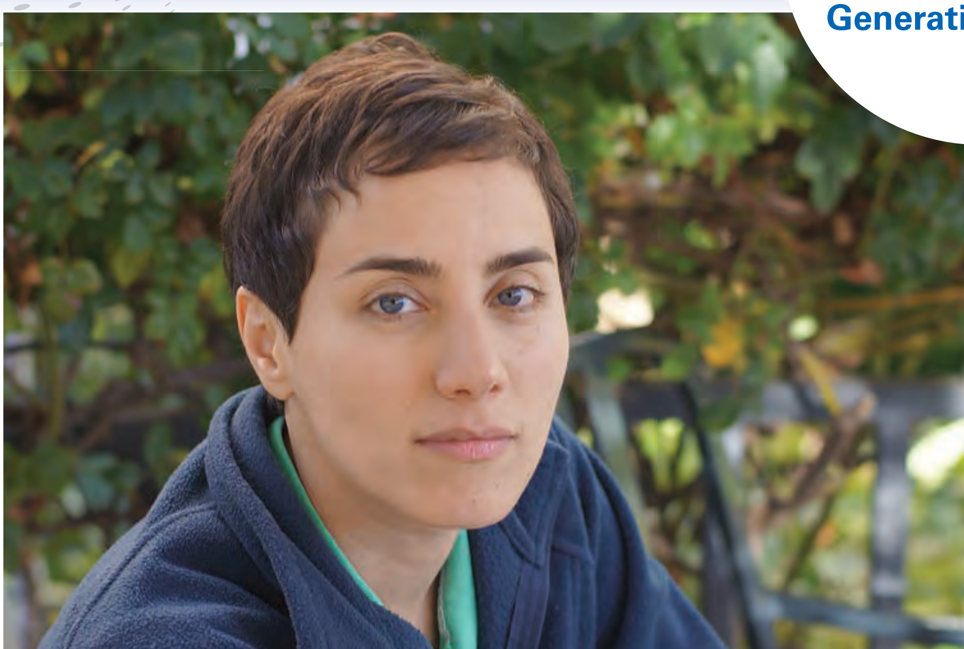


AMERICAN MATHEMATICAL SOCIETY

A Tribute to Maryam Mirzakhani, p. 1221



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I like crossing the imaginary boundaries people set up between different fields…
—*Maryam Mirzakhani*

FEATURED

In this month of the American Thanksgiving, it seems appropriate to give thanks and honor to Maryam Mirzakhani, who in her short life contributed so greatly to mathematics, our community, and our future. In this issue her colleagues and students kindly share with us her mathematics and her life.

—***Frank Morgan, Editor-in-Chief***

ALSO IN THIS ISSUE

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FROM THE AMS SECRETARY:

JMM 2019:

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IN EVERY ISSUE

Cover: Original image from *IFP News* (*Iran Front Page*) ifpnews.com/exclusive/jewel-name-maryam-mirzakhani.

AMERICAN MATHEMATICAL SOCIETY
Advancing research. Creating connections.

A Note from the President and the Executive Director

Joint Mathematics Meetings—2022 and beyond

The Joint Mathematics Meetings (JMM) is a long-standing and popular event for the mathematics community. Since 1998, this important conference has been co-managed by the American Mathematical Society (AMS) and the Mathematical Association of America (MAA). The AMS and MAA have decided that, starting in 2022, the AMS will become solely responsible for the management and hosting of this successful conference.

The JMM welcomes the diverse mathematical community from around the world to present research, share insights on the teaching and learning of mathematics, see the wide range of exhibitors, and meet colleagues, old and new. Several mathematical organizations already participate in the JMM, including the Association for Symbolic Logic (ASL), Association for Women in Mathematics (AWM), National Association of Mathematicians (NAM), Society for Industrial and Applied Mathematics (SIAM), the mathematics institutes, as well as many exhibitors and the National Science Foundation (NSF).

Events at the JMM support mathematicians at every stage of their careers and create important connections. The JMM engages attendees in sessions related to research, teaching, science and education policy, and professional development, and we expect this to continue in 2022 and beyond. The AMS expects to continue offering travel grants, the Department Chairs Workshop, the Employment Center, short courses, special lectures, social events, and poster, contributed, and special sessions. The mathematical community's shared interests in advancing mathematical research, engaging creatively with teaching, and supporting mathematicians throughout their careers, along with our commitment to diversity, inclusion, education, and policy issues affecting our discipline, will continue to be reflected in the JMM.

Many existing joint activities involving the MAA will continue at JMM, such as the Gerald and Judith Porter Public Lecture; the ceremony to honor recipients of the Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student; and the presentation of the Joint Policy Board for Mathematics Communications Award.

A new model for the JMM will be launched more than three years from now. There is ample time to engage in thoughtful discussions and careful planning with the help and input of our community. In the years leading up to this 2022 change, the AMS will work to ensure that any new vision for the JMM is responsive to the interests of the entire mathematical community.

We look forward to seeing everyone at the next JMM: January 16–19, 2019, in Baltimore, Maryland!

Thank you.

Kenneth A. Ribet
AMS President

Catherine A. Roberts
AMS Executive Director

September 2018

Notices
of the American Mathematical Society

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Maryam Mirzakhani: 1977–2017

Communicated by Hélène Barcelo and Stephen Kennedy

Maryam Mirzakhani's Harvard PhD dissertation under Curt McMullen was widely acclaimed and contained already the seeds of what would become her first three major papers. All three of these results—a new proof of Witten's conjecture, a computation of the volume of the moduli space of curves, and an asymptotic count of the number of simple closed geodesics on a hyperbolic surface—were deep, beautiful, and unexpected. Subsequent papers revealed the geometry and dynamics of the moduli space of hyperbolic surfaces, culminating in the rigidity result achieved jointly with Alex Eskin. Her work exhibits great depth of insight, wide-ranging technical mastery, and creativity and imagination of a very high order. It earned her a Fields Medal in 2014.

Mirzakhani was born on May 12, 1977, in Tehran and grew up in postrevolutionary Iran during the Iran–Iraq War. One of four children, her father was an engineer, her mother a homemaker. All three of her siblings became engineers. It was in middle school that she discovered her passion and talent for mathematics. In high school she participated in the International Mathematical Olympiad, winning gold medals in 1994 and 1995. She earned a bachelor's degree in mathematics from Sharif University in Tehran and headed to Cambridge, Massachusetts, for graduate study at Harvard.

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Mirzakhani spoke at the 2015 Clay Research Conference at Oxford after receiving the Fields Medal in 2014.

When Mirzakhani was awarded the Fields Medal in 2014 she had already been diagnosed with the cancer that would eventually claim her life. Always a private and humble person, she did not welcome the attention and acclaim that accompanied the prize. Together with her husband, Jan Vondrák, she wanted to raise their daughter, live her life, and pursue mathematics.

Notices asked friends, collaborators, and students of Mirzakhani to write brief reminiscences of her life. We also sought expository work on her mathematical accomplishments from Ursula Hamenstädt, Scott Wolpert, and, writing jointly, Alex Wright and Anton Zorich. The portrait that emerges from these pieces is of a warm, generous, and modest friend, teacher, and colleague—a mathematician of great talent and imagination coupled with grit and persistence to a very unusual degree.

Maryam Mirzakhani died, aged forty, on July 14, 2017.

The following videos provide more information and testimonials on Maryam Mirzakhani's life and work. The Harvard and Stanford memorial services contain personal reflections and reminiscences from friends, colleagues, and family members who have asked that their contribution to this article be via this medium.

McMullen's Fields laudation: `www.youtube.com watch?v=VC7MZv1JH8w&t=3s`.

The Stanford memorial service: `www.youtube.com/watch?v=IUfB2HadIBw`.

The Harvard memorial service: `www.youtube.com/watch?v=HUBnzTTQ5jk`.

Image Credits

Opening photo courtesy of Alex Wright and Anton Zorich.

Photo of Mirzakhani lecturing courtesy of Roya Beheshti.

Roya Beheshti

Maryam Mirzakhani in Iran

When we were teenagers, Maryam and I would often talk about where in life we wanted to be at forty. In our minds, forty was the peak of everything in life. What I could not imagine was that one of us would be writing in memory of the other at forty.

I met Maryam in 1988 when we were both eleven and had just started middle school. We became close friends almost instantly, and for the next seven years at school we sat next to each other at a shared desk. My early memories of her are that she was well read, had a passion for writing, could easily get into heated debates over social or political issues, and would get very bitter about any kind of prejudice against women. Our friendship did not have its genesis in math. In fact, math was the only subject at which Maryam was not among the top students in the sixth grade. A memory that I distinctly recall is when our math teacher was returning our tests to us toward the end of the academic year. Maryam had received a score of sixteen out of twenty, and although the test had been difficult, there were students who had done better than that. Maryam was so frustrated by her score that before putting the test in her bag, she tore it apart, and announced that that was it for her and she was not going to even try to do better at math! That did not last long. After the summer break, Maryam came back with her confidence regained and started to do very well. Soon after that math became a shared passion between us, and we started to spend a lot of time thinking and talking about math.

The middle/high school that we went to was a special school for gifted female students in Tehran. Each year around one hundred students were admitted to that school through a competitive entrance exam. The principal of the school, Ms. Haerizadeh, was a strong woman with a vision who would always do anything to make sure that the students in our school had the same opportunities as the students in the equivalent boys' school. We had a strong team of teachers, and the overall environment was very encouraging for students who were interested in math or science.

When we entered middle school the Iran–Iraq War had been over for a month, and the country was becoming more stable.[1] Around that time a few programs for high school students were initiated that I think played a big role in developing interest in mathematics for several Iranian mathematicians of my generation. One of them was the Math Olympiad competitions. Another was a summer workshop run by Sharif University for high school

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[1] *After the Islamic Revolution in 1979, all the universities in Iran were closed for almost three years as part of the Cultural Revolution, the Islamization of education. When the colleges resumed classes in 1983, the country was in the middle of a costly war that made it difficult for the educational system to move forward.*

As eleventh-grade students in 1994 Mirzakhani and Behesti became the first female students on the Iranian Mathematical Olympiad team. Maryam Mirzakhani (gold), Roya Beheshti (silver), Ali Norumohammadi (bronze), Omid Naghshineh (bronze), Maziyar Raminrad (gold), and Reza Sadeghi (silver).

students to introduce them to college-level mathematics. We participated in that workshop the summer after ninth grade. That workshop had a major impact on Maryam's growing interest in math and resulted in her first publication (joint with Professor E. Mahmoodian). We participated in the Math Olympiad competitions in the eleventh grade and made it to the national team as the first female students on the team (see above).

From 1995 to 1999 we attended Sharif University, the leading technical university in Iran, where we got a lot of support from our professors and the chair of the mathematics department, Y. Tabesh. By then Maryam had become very ambitious about her future in math, and her life was not imaginable without it. Although the universities, unlike the high schools, are coeducational in Iran, many forms of gender segregation were imposed in the 1990s. The atmosphere within the math department, however, was relaxed and friendly, and there were various activities from workshops to math competitions to reading seminars that kept us motivated and stimulated an even stronger passion for mathematics. There were six of us in our class who decided to apply to graduate schools in the US during our junior year. To increase the chances of all of us getting admitted to top-tier schools, we decided to each choose a school to which the others would not apply. Harvard and MIT were natural choices for Maryam and me because we wanted to stay close to each other. We all moved to universities in the northeastern US in 1999.

Maryam and I continued to see each other very often until 2003, when I left Boston. She was very generous with her time when it came to discussing math and was always happy to meet and talk about any problem I was thinking about. I remember in fall 1999 I took a course that Maryam was auditing. The final was a take-home exam, which I had planned to complete overnight. Maryam came to my office for moral support in case I had to stay up late to finish the exam. The last problem looked difficult, so I discussed it with Maryam. She got interested in it, and we thought on it for a while without success. After a few hours I gave up, as I was too sleepy and decided that I had done enough for the exam. But, as would always be her pattern, Maryam persisted. She stayed up all night working on it, and when I woke up early in the morning, she had figured out how to do the problem and was checking the last steps of her solution.

Maryam's work was always driven by a certain pure joy that did not change over the years by experience or wisdom. She drew pleasure from doing advanced mathematics in the same way as she did from solving high school-level Math Olympiad problems. She was driven by this joy and certainly not by a desire for fame or influence, and she continued to be driven by it even when fame and influence came within her reach. She avoided public attention religiously and that helped her stay focused on her research in the midst of the celebrity she attained winning the Fields Medal while also dealing with cancer.

I was at a workshop at MSRI in May 2013 when I received a message from Maryam informing me of her diagnosis. She had recently come back from a trip to Europe and Iran, and I had planned to meet with her after the workshop. When I saw her a couple of days later at Stanford, she was worried about the effect of the illness on her daughter and her career but was determined to stay hopeful and

Mirzakhani (right) and Beheshti (left) on Cape Cod in 2000.

upbeat. In the four years that followed, she stubbornly focused on the positive and refrained from complaining about the problems she was dealing with in the same way that she always avoided talking about her achievements. Even after she was diagnosed with recurrent cancer and the odds were not in her favor, she maintained hope and handled the situation with grit and grace.

Maryam Mirzakhani's parents heard the news of her receiving the Fields Medal through public media. She explained later that she did not think it was such a big deal! This accords with what I remember from my interactions with her when she was a student at Harvard. While explaining her beautiful work in math, Maryam made it sound as if what she was doing was simple.

Her humanity was very pronounced and she was kindhearted. I recall her helping disabled students at Harvard in getting around. After receiving the Fields Medal, the Iranian students in the Boston area were hoping to celebrate her accomplishments by organizing a big party for her. Maryam refused this, instead opting for attending, unannounced, their small monthly lunch gathering.

Maryam beautifully exemplified that the pursuit of knowledge is a timeless, borderless, and, yes, genderless adventure. Her untimely passing has deprived the world of math of a brilliant, yet humble talent at her peak. Her legacy will live forever.

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Mirzakhani with her daughter.

In her final month, the last time I met Maryam at her home, I told her that I had just read how difficult and painful one of her treatments was and expressed surprise that she had never complained about it. She dismissed that as something not worth focusing on and changed the topic. We then talked about various audiobooks that she planned to listen to. She was, as always, warm and engaging and got radiant and happy when her daughter interrupted us, jumping around Maryam and telling her that she loved her new bike. This is how I am going to always remember Maryam.

Photo Credits

Photo of Mirzakhani with daughter courtesy of Jan Vondrák.

All other section photos courtesy of Roya Beheshti.

Izzet Coskun, Laura DeMarco, and David Dumas

Maryam Mirzakhani in Graduate School

We met in David's office to talk about our memories of Maryam in graduate school and to write some words that might convey what she was like. But how do we even begin to write about Maryam, one of our closest friends in those years and an integral part of our development as mathematicians, when the pain of her absence is so fresh? We started with the present, when we first learned about her illness, when each of us saw her last, the recent interactions we had, whether talking with her about math or talking about our children, and little by little the memories and stories from graduate school started to flow.

Some themes quickly emerged: Maryam's determination to figure things out, her intense and insightful questioning, her excitement about new ideas, and the gleam in those brilliant blue eyes after making some absolutely hilarious comment. We recalled her seminar lectures and her struggles and also her jokes and her cooking, and we recalled how easy it was to spend time with her—and the countless hours we all spent together—though we never truly knew her most personal or private thoughts and feelings.

We were part of a tight-knit group of students at Harvard, and we were all actively involved in Curt McMullen's informal geometry and dynamics seminar. Every semester, each of us had to give a lecture in this seminar. In our first couple of years, we gave expository talks about research articles we were reading, and one of Maryam's first talks was on McShane's identity for the punctured torus. All three of us were there. Before the talk, she was anxious about her presentation, which was partly her apprehension about speaking in English, though, in fact, she was perfectly fluent.

Maryam's lecture style that day was unforgettable. She would race ahead of herself at times, unable to contain her excitement, and the barrage of words was accompanied by large circular hand gestures. At the time, we were all struggling with new mathematical ideas; we often felt

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we were swimming through a mathematical soup as we tried to find our thesis problems. Even then, however, Maryam seemed to have a direction and purpose, and the mathematical content of her lecture was potent and clear, even if the presentation itself was amusing.

I met Maryam in 1993. I had an appointment to meet Ebad Mahmoodian at IPM, the Institute for Research in Fundamental Sciences in Tehran. When I got there, Mahmoodian told me that he wanted to check some math that a high school student had handed to him and he asked if I'd be willing to help him. So, I spent the morning with Mahmoodian going over the arguments with him. Mahmoodian was teaching a summer course on graph theory for gifted school children at Sharif University. One of the topics he had talked about was decomposing graphs into disjoint unions of cycles, including the rather curious example of decomposing a tripartite graph into a union of 5-cycles. This was considered the first difficult case of the general problem. Mahmoodian had asked the students to find examples of tripartite graphs that were decomposable as unions of 5-cycles, offering one dollar for each new example. By the time of the next lecture, Maryam had found an infinite family of examples. She had also found a number of necessary and sufficient conditions for the decomposability. These were the results that Mahmoodian and I checked that day. Checking everything carefully took the whole morning. At the time we joked that Maryam was obviously smart, but not that smart; otherwise she could have milked Mahmoodian for all he was worth by revealing one example a day.

Those of us who knew Maryam in person probably have a hard time thinking of her as "the genius" that she has been portrayed as in the media. She didn't have any of the pretensions of the stereotypical genius of children's books. She had the same qualms and worries as the rest of us: she too had wondered at some point whom to work with, whether she would finish her thesis, whether she would find a decent academic job, whether she could balance the demands of motherhood with being a professor, whether she'd be tenured at some point in this century. And she was a lovely person. We loved her for who she was, and we would have loved her just the same even without her honors and awards.

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In later years, her lectures became much more polished, and she gave talks on enumerating simple closed geodesics on surfaces and on the earthquake flow on Teichmüller space. We still laugh about our own talks in the McMullen seminar sometimes being all show and no substance in contrast with hers that were all substance and no show. She really understood the material deeply. Looking back, it is remarkable that each of her talks in that informal setting was later developed into a substantial contribution to mathematics.

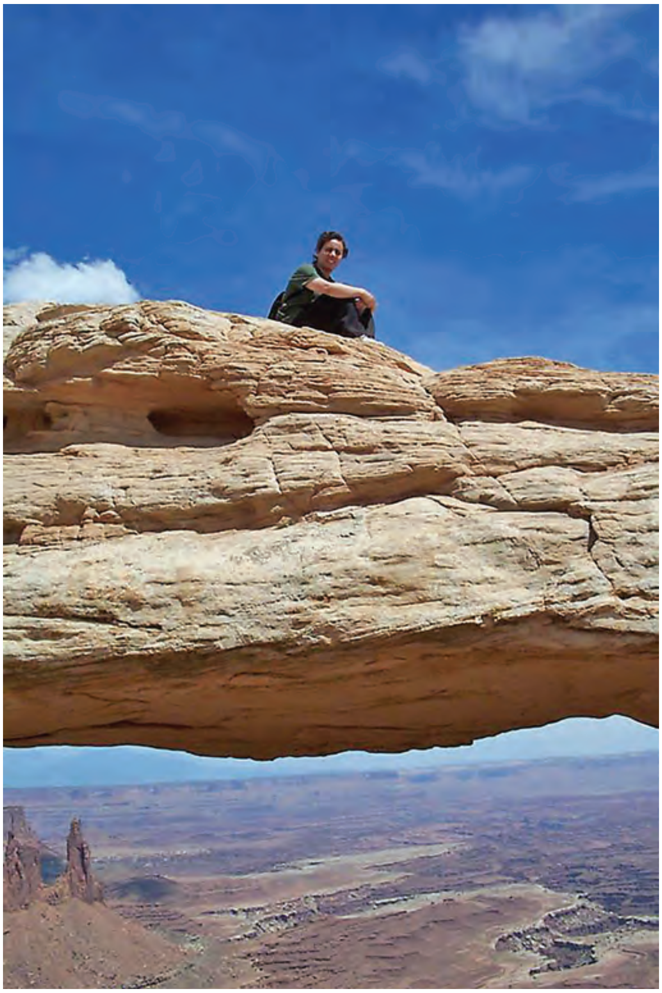
Maryam's perseverance stands out in our minds. In the early spring of her last year in graduate school, she rushed into one of our offices close to tears. She was holding what must have been the twentieth draft of her thesis, covered in McMullen's famous red ink. "I am not sure I will ever graduate," she moaned, and she joked that the last ten pages did not have as much red only because McMullen's pen must have run out of ink. After commiserating for half an hour, Maryam steeled herself and with her characteristic determination marched back to her office for yet another round of editing.

In some ways, Maryam was like all the other students, struggling with writing and expressing herself clearly. All of McMullen's students went through the ritual of countless rewrites. But Maryam had a rare dogged tenacity always to do better. She revised and revised tirelessly. The red on the page subsided, and she completed her thesis. Later she would adapt this thesis into three beautiful papers tying together Witten's conjecture, the volume of the moduli space of curves, and the counting of simple closed geodesics on hyperbolic surfaces.

Maryam had a whimsical side that was not often seen. We had many dinner parties together, hosted in our homes. Once when she was cooking us an Iranian meal, in the middle of preparing a rice dish she put on some music and started to dance. Soon we were all dancing around her kitchen, bumping hips and laughing as she stirred the pot. Another time, we were helping Maryam move into a new apartment, and we had to travel twenty-five miles to return the moving truck. While the rest of us were cranky and exhausted, Maryam spontaneously broke into song. She taught us some Iranian children's songs, and we sang the rest of the way, like school kids on a field trip.

Occasionally Maryam's lighter side would come out in mathematical conversations, too. We all have memories of standing in the hallways of the department with her, laughing raucously about something mathematical, probably disturbing all those seemingly serious professors in their offices. If she was really excited about an idea, she would start explaining it with energetic gestures. She would laughingly say "it's sooo complicated," though in truth the complexity never stopped her.

Maryam had an incredibly warm and infectious smile, but she eschewed the spotlight. While her warmth was apparent among friends, she was often quite reserved in her interactions with others. In 2002 David and Maryam attended a two-week summer program in complex hyperbolic geometry at the University of Utah. A group of students went on a weekend hiking trip together; Maryam came along and seemed to enjoy these quiet hikes in the mountains. When the group stopped to rest, she would often find a place to sit and reflect on the scenery in silence (see next page). Maryam was intensely private. Even after knowing her for so many years, we do not know her deepest religious, social, or political beliefs.



On a 2002 hike in Utah, Mirzakhani liked to reflect on the scenery in silence.

Maryam was extremely generous, both with her mathematical ideas and as a true friend. In her fourth year in graduate school, David had a serious fall and had to recover at home from several broken bones. Maryam organized a group of students to visit him and cheer him up during his recuperation. Maryam would not settle for half measures. When Laura married, Maryam teamed up with Izzet and another fellow graduate student, Alina Marian, to find something that would really please her. Laura had a cutlery set in her registry, but Maryam was not satisfied with ordering a few knives and forks—she insisted that only the entire set would do.

Maryam was equally gracious with her mathematical ideas. When David was working on a paper about Thurston's skinning maps, Maryam realized that part of his work had implications for some counting problems related to the mapping class group action on Teichmüller space that she and her collaborators had recently explored; she was happy to have David include this application in his paper.

Maryam was a brilliant mathematician, a Fields Medalist, an inspiration to many young mathematicians. To us, however, she was first a friend. We are lucky to have studied math with her, benefiting from her deep insights and her penetrating questions. We are equally lucky to have danced and cooked together, laughed together and struggled together through those hard but precious years of graduate school. It seems so wrong that she has died so young. We love and miss her dearly.

Photo Credit

Photo of Mirzakhani in Utah courtesy of David Dumas.

Elon Lindenstrauss

Maryam Mirzakhani at Princeton I

Maryam and I both came to Princeton in 2004. She came just after completing a stellar PhD thesis and had a very visible presence in the department—energetic, sharp, and interested in everything. Getting to know Maryam was to me an unexpected joy. Maryam and I were both going to a course given by Peter Sarnak that first year, and we probably started talking to each other then.

I was intrigued by one aspect of her remarkable thesis that was closest to me: Maryam's asymptotics for the number of simple closed geodesics of length at most T on a hyperbolic surface. Counting the number of all closed geodesics of length at most T is classical in dynamics—it is well known that for surfaces of constant curvature -1 the number of such geodesics grows like e^T/T. The simple closed geodesics—those that as curves on the surface do not cross themselves—are governed by a very different law, which is influenced by the topology of the surface, by the genus g, and the number of cusps n: their number is proportional to $T^{6g-6+2n}$. As sometimes happens when counting objects, the asymptotic formula for the number of simple closed geodesics is intimately related to an equidistribution statement, and as an ergodic theorist I was excited by the fact that the heart of the matter was a beautiful application of ergodicity but in a context I was not familiar with, namely, the action of the mapping class group on the space of measured laminations supported on the surface.

When I met her, Maryam was already dreaming about classifying the invariant measures and orbit closures for the action of $\mathrm{SL}(2,\mathbb{R})$ on the moduli space of quadratic differentials. This quest was a somewhat speculative one at the time, motivated by the fantastic results of Marina Ratner (who died unexpectedly only a week before Maryam's untimely death) on invariant measures and orbit closures for the action of Ad-unipotent subgroups on quotients of Lie groups. Alex Eskin was also obsessed with this problem, and some years later they would join forces (in part aided also by Amir Mohammadi) to fulfill this mathematical dream in a pair of long and exceedingly difficult papers, using what seems like almost every mathematical technology known to mankind.

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We started discussing mathematics, and in particular Maryam patiently explained to me some bits of her thesis. On the third floor of Fine Hall, the tower housing the Princeton mathematics department, there are some niches with blackboards, and they were the favorite places for us to sit and discuss. We decided to work together on an intermediate problem, motivated by the quest for an analogue of Ratner's theorems in the moduli space setting but quite closely connected to Maryam's thesis: the classification of locally finite measures on the space of measured laminations invariant under the action of the mapping class group.

This problem can be viewed as the analogue of the problem of classifying measures invariant under horospheric groups in the Lie group setting (horospheric groups are a special case of Ad-unipotent groups that are "big" in an appropriate sense), which in the Lie group setting is easier than the general case of Ratner's results and where rigidity was established earlier by S. G. Dani and H. Furstenberg. Working with Maryam on this project and learning from her all the multiple points of view on this problem was a pleasure. While working we also talked about life, politics,…. In some sense Iran and Israel are similar, with a tension between tradition/religion and modernity, and we both hoped a time would come when the conflicts between our nations would subside. Sadly, this does not seem to be the direction the world is heading.

When Maryam came to Princeton, she had to meet very high expectations due to her remarkable PhD thesis. Eventually she exceeded these expectations, but because she was not trying to reap low-hanging fruits, this took time, and I am sure it was probably not easy initially. When we worked together she was modest but sharp and persistent.

After Maryam moved to Stanford I had less frequent contact with her, but we had some email exchanges from time to time. In June 2013 she told me she would not be able to come to the Fields Symposium in Toronto due to "an unexpected medical issue that I will have to deal with," and in the ensuing email exchange she told me she had been diagnosed with breast cancer. In 2014–15 I was with my family on sabbatical in Berkeley. Just after we arrived at Berkeley I heard about her Fields Medal. I knew she was in the running, and she certainly very much deserved it, but since I had not heard anything I had assumed this did not work out. I was very pleased to be wrong, and we exchanged some emails about our experiences with the publicity of the medal (though of course her prize generated much greater public interest than mine).

During this year, and particularly in the second semester, when both of us participated in a special semester at MSRI, it was a pleasure to see her back with her full energy and insight, often asking prescient questions from the balcony in the MSRI main lecture auditorium. My family and I hosted Maryam and her family in the house we were renting, and her little girl enjoyed the trampoline. Everything seemed very idyllic. Unfortunately, this full recovery proved to be a mirage, and the cancer returned not long afterwards.

We all miss her terribly.

Peter Sarnak

Maryam Mirzakhani at Princeton II

The first I heard of Maryam was her spectacular thesis, written with Curt McMullen as her adviser. In it she resolved a long-standing question on the asymptotic count of the number of simple closed geodesics on a hyperbolic surface, as well as giving an ingenious new proof of Witten's conjecture relating certain intersection numbers on moduli spaces of surfaces of genus g with n punctures and a KdV hierarchy. These results won her immediate international acclaim, and until her untimely passing, she continued to produce one striking breakthrough after another, reshaping and molding the theory of hyperbolic surfaces and especially the dynamics connected with their moduli spaces. Her contributions will be celebrated and studied for years to come.

Maryam's first faculty position was at Princeton University, and we cherish the period that she spent with us. I met with Maryam many times over the years to learn about her latest work and other developments and to discuss problems of mutual interest. While she was very modest in both private and public exchanges, her deep mathematical intuition and brilliance shone right through in any discussion; one always felt elevated in her presence. On many occasions I would find myself standing at the back of the lecture hall together with Maryam, as we both preferred to stand while listening to a lecture. The bonus for me was that I would get her private clarifications and insights into what the speaker was saying.

Maryam's most recent visit to Princeton was with her husband, Jan, and daughter, Anahita. They spent the fall of 2015 at the Institute for Advanced Study as part of a core activity centered around her far-reaching works with Alex Eskin. This was a good time; Maryam was in remission and doing very well in all respects. She delivered to a packed audience the Minerva Lectures at Princeton University.[2] Her and Jan's research was moving forward handsomely, and there were many exciting opportunities for them going forward. Sadly, a few months after they got back to Stanford I got an email from Maryam saying that she had had a big setback healthwise. She confronted her cancer with the same courage and boldness that she attacked all the obstacles that her very rich and too short life presented. She is missed by so many of us—those whose lives were enriched by knowing her as well as those who are inspired by her story and her mathematics.

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[2] `https://www.math.princeton.edu/events/seminars/minerva-lectures/archive/13`.

Maryam Mirzakhani, Roya Beheshti, and others at the Math Olympiad preparation camp in summer 1994.

Maryam was a member of the Iranian Math Olympiad teams in 1994 and 1995, with whom she went on to receive gold medals at IMO with a perfect score on the later one. I was director of the Math Olympiad in Iran during that period, and it was exciting to see with what ease she could solve difficult problems. Later she joined the training camp of Math Olympiad to train the next generation of Olympians. It was so amazing for me to work with her to design new challenging problems! She also developed resources and wrote a number theory book with her lifelong friend, Roya Beheshti, as a reference for Olympiad camps. The book still is in use after a twelfth printing!

Maryam joined the math department of Sharif University for her undergraduate studies in 1995. I was the chairman of the math department during that period, and with a few colleagues we did our best to generate a center of excellence. We attracted many talented students, but Maryam was truly exceptional in all her courses.

Saeed Akbari, a young faculty member at that time, brought a twenty-four-year-old problem by Paul Erdős to the attention of his algorithmic graph theory students and offered a ten-dollar prize. The problem was to prove the existence of a planar 3-colorable graph that is not 4-choosable. Maryam solved it and received the prize. Her solution appeared in the BICA journal in 1996.

Then she went to Harvard and continued to be among the greatest mathematicians in her generation, but she always remembered the good old days of her undergraduate studies at Sharif. Her heart was always open to concerns about how we can help young and talented teenagers to have the opportunity to flourish and develop their own mindset and talent.

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Photo Credit

Photo courtesy of Roya Beheshti.

Alex Eskin

Maryam Mirzakhani as Collaborator

Maryam and I first met in person at Princeton. She was a postdoc, and I was a visitor at the Institute for Advanced Study. We started collaborating right away. For me it was an amazing experience; she was obviously brilliant, but she was also the nicest and most positive person I have ever met. We spent a lot of time at the IAS talking about math and other things, and every week we would walk to the university to sit in on Elon Lindenstrauss's course about homogeneous dynamics, which included a description of his "low-entropy method."

After we both left Princeton, we continued working together. Maryam's philosophy was to never go after the "low-hanging fruit," and our projects got increasingly ambitious. But the biggest open problem in our subfield, since Curt McMullen's pioneering work ten years before, was trying to find analogues of Marina Ratner's celebrated theorems in moduli space by proving that the $SL(2,\mathbb{R})$ orbit closures were always manifolds. For a few years we never talked about it, but we both could sense that the other person was working on it. Eventually we decided to join forces. For me it was an easy decision, since working with her was so incredibly fun.

At first we just had some vague dreams and no concrete plan. At one point we read a breakthrough paper by two French mathematicians, Yves Benoist and Jean-François Quint. The paper had already been available for a few years, and reading it was on our to-do list, but it was written in French, which neither of us knew. At some point I was visiting IHES, and there was a group of people there who wanted to read the paper, and that provided the needed push. It was ostensibly a result in a different area of mathematics, but we could tell immediately that it was relevant to our problem. At this point we both dropped all of our other projects. After a few months of struggling, we plotted a potential route to the solution based on a variant of their method.

In my senior year at Stanford I had an independent study with Professor Mirzakhani in topology. I learned from her the art of thinking through problems—her diligence and calm methodology of thinking through problems that we would formulate was like watching an artist construct a masterpiece. She would brush on the blackboard the basic fundamentals that she would ensure I had a strong grasp of and then slowly layer on various colors of detailing that would give a more vivid illustration of the problem we were tackling and, more importantly, a profound excitement for our work

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and potential as mathematicians. She would teach in a way to instill in her students a deep curiosity coupled with a heavy thirst for a greater mastery.

During a few of our meetings, I told her about my interests in utilizing structures for grasping insight into various biological phenomena, and we discussed a few examples I had been thinking about. She asked me about my career goals post graduation, and I told her that I was unsure, but that I loved what we were doing. She discussed with me her motivations that fuel her work. She propelled me to apply to PhD programs, which I ended up doing later that year.

Her impact on my life is unforgettable, and I would not be where I am now if not for her support and encouragement.

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We worked on this for two years while getting more and more excited. But at some point, seemingly near the end, we realized that there was a major problem and that our path was blocked. Our method needed a fact called the semisimplicity of the algebraic hull, which we could not prove. We could understand the $SL(2,\mathbb{R})$-invariant measures, but, because of the nonamenability of $SL(2,\mathbb{R})$, in order to understand the $SL(2,\mathbb{R})$ orbit closures we needed to understand the P-invariant measures, where P is the upper-triangular subgroup of $SL(2,\mathbb{R})$. We looked for ways to fix the argument but could not find any. We were still talking once a week on Skype, but we were making no progress. During this period, Maryam's daughter was born, but she continued working; she had this amazing inner strength.

After more than a year of being stuck, we realized that there was a different approach to the problem that was based on a distant relative of the "low-entropy method" that we had learned from Lindenstrauss. That eventually worked, but not all the way. However, it was enough to show that P-invariant measures are $SL(2,\mathbb{R})$-invariant, and for a long time we had an argument (based on the work of Giovanni Forni) that $SL(2,\mathbb{R})$-invariance implies the semisimplicity of the algebraic hull. Then we could fall back on our original argument based on the Benoist-Quint method to complete the proof. It took us another two years to work out all the details. During this time Maryam was first diagnosed with cancer.

Later, the cancer was in remission, and I remember a lot of happy times at MSRI, at IAS, and at Stanford. But eventually the cancer came back, and even Maryam could not beat it. I miss her both personally and professionally.

Mirzakhani working at home.

Photo Credits

Photos courtesy of Jan Vondrák.

Eugenia Sapir

Maryam Mirzakhani as Thesis Advisor

I think back often to my time in grad school as a student of Maryam Mirzakhani. The memories of her are still very much alive in my head. These memories are so full of life that sometimes I feel they will pop out of my mind and materialize in front of me. I will attempt to draw a picture here of the vibrant person who lives on in my mind.

Maryam was full of boundless energy and enthusiasm for her work. This was evident when watching her do math or give a talk or when working with her on a problem. This excitement toward her subject was part of her driving force. When giving talks, she would bounce slightly while laying out all the disparate ideas that came together into the picture. When she got to the part when all the ideas came together, she'd talk faster, excited about getting to the punchline.

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Maryam would paint beautiful, detailed landscapes in her lectures. If she were giving a talk about concepts A, B, and C, she would not just explain that A implies B implies C. Rather, she would paint a mathematical landscape where A, B, and C lived together and interacted with one another in various complicated ways. More than that, she made it seem like the rules of the universe were working harmoniously together to make A, B, and C come about. I was often amazed by what I imagined her inner world to be like. In my imagination it contained difficult concepts from disparate fields of mathematics all living together and influencing one another. Watching them interact, Maryam would learn the essential truths of her mathematical universe.

Maryam's vision occasionally made for cryptic conversations. When, for example, we would talk about my thesis problem, there was a set of concepts that Maryam would repeatedly bring up, rather like a recurrent cast of characters in a television show. At first, it was unclear to me why these characters were relevant. Only after working on my problem for several months could I look back and see that the hints that Maryam dropped formed the backbone of the best, most illuminating way of understanding what was going on.

I met Maryam when she was a graduate student at Harvard. At that time I already saw her passion for mathematics. A couple of years later she did her fabulous dissertation. Subsequently, I saw her at conferences and the couple of visits she made to Chicago to work with Alex Eskin. Her work had a profound influence on me and many others. It was also always a joy to see her. She loved talking mathematics with everybody, and she was also just a wonderful person to be around. I got to know her a little better personally in the spring of 2015 when she and I were both at the Mathematical Sciences Research Institute. She had just won the Fields Medal. I remember how gracious and generous she was with everyone. I saw her for the last time at a seminar at Stanford in November 2016. I knew at that time that her health was deteriorating, so I was both a little surprised but also very happy to see her. She listened to the talk and then participated in a discussion for almost an hour afterwards. In spite of all she was going through, she exhibited the same enthusiasm and passion for mathematics. I think it was important to Maryam not to be identified with her illness. I miss her as a mathematician and as a friend.

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On the other hand, Maryam was completely down to earth. I've heard many of my peers say that they also found her completely approachable even though she didn't know them very well. It was easy for me to talk to her about my ideas. I would come into her office each week while I was in graduate school. I'd tell her what I'd been thinking about, and we'd just talk. We'd throw ideas back and forth and get excited when things worked. We would make math jokes and laugh. Sometimes I'd bring her an annoying problem I couldn't get around. Then her favorite phrase was, "How can that be?" She would repeat it several times, until she found the fissure we could use to crack the problem open. Our meetings would skip along happily until we came to a problem where we didn't see an immediate solution. We'd become silent and pensive, each of us retreating to our paper and pencil until we came up with something new to try. Maryam became engrossed in her drawing pad, sketching small drawings related to what we were discussing. When the silence dragged on for long enough and it had become clear that it would take too long for the next idea to come, the meeting was over. I knew that we'd discussed everything possible that week and that it was time to table the discussion for next time. I'd leave feeling energized and excited for the week to come.

I saw Maryam as my math coconspirator and also as a bottomless well of inspiration. She was an unpretentious person who held great depths. She was the embodiment of someone who studied math for the joy of the pursuit of knowledge. She is someone we have lost much too soon and who has left much undone.

Mirzakhani with her husband, Jan Vondrák, and daughter, Anahita.

Photo Credits

Photo courtesy of Jan Vondrák.

Hélène Barcelo and David Eisenbud

Maryam Mirzakhani and MSRI

Everyone in the mathematical community knows that Maryam Mirzakhani was an outstanding mathematician, a winner of the Fields Medal in 2014. Many have seen a video[3] with her on the floor drawing figures that reflect her deep mathematical imagination and suggest some of the charm of her fresh and vibrant personality. Still others knew Maryam from her other distinctions—for example, as one of *Nature Magazine*'s "10 People Who Matter" in 2014. But only a few know how deeply Maryam was engaged with the Mathematical Sciences Research Institute (MSRI) located in Berkeley, California.

Maryam was a key member of the MSRI Scientific Advisory Committee from 2012 to 2016, the committee charged with selecting the scientific programs and their members. During part of that time she was very ill with what turned out to be her terminal cancer, but she never missed a meeting—even when she couldn't travel from Stanford to Berkeley she attended by video between harsh medical treatments—and she faithfully did the substantial committee homework. It is fair to say that she was an inspiration to the entire committee.

She played other important roles at MSRI too. She was one of the main organizers of the semester-long program on Teichmüller theory and Kleinian groups in 2007 and a research professor for the spring of 2015. She was even a member of the complementary program when her husband, Jan Vondrák, came to MSRI for a computational program in the spring of 2005.

Finally, in the spring before her death, Maryam had agreed to be nominated as a trustee of MSRI, although she warned David Eisenbud, MSRI's director, in a long and deeply sad meeting that she might well not live to take office.

We believe that Maryam's engagement with MSRI came from a close alignment of priorities: first, for mathematics of the highest quality, done in a playful collaborative style; second, for the cultivation of talent also in young women, and the support of women with children; and, finally, for welcoming mathematicians from abroad, understanding that they enormously enrich the US mathematical scene.

It was an honor to know Maryam: brilliant, charming, and courageous to the end. We miss her deeply.

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[3] https://www.simonsfoundation.org/2017/07/20/fields-medalist-maryam-mirzakhani-dies-at-40/.

Ursula Hamenstädt

Zooming In and Out: The Work of Maryam Mirzakhani through the Eyes of a Geometer

Prelude

Let us imagine that we intend to embark on an attempt to understand the geometry of the universe. How would we proceed?

A perhaps naive idea is as follows. Identify some small region in the universe we are reasonably familiar with. What this means can be argued about, but the amount of information used to gain the level of understanding we feel comfortable with should be controlled. Then we send a spacecraft from our region in some arbitrary direction to collect information on the change of shape with respect to the fixed reference region. As the journey of the spacecraft continues and we collect more information, we may have to acquire more knowledge about the reference region to detect hidden similarities with the pictures taken along the journey. The vague hope is that the geometric interaction of nearby regions on the small scale reveals the laws of nature which control the large-scale shape of the universe as recorded in the random sampling.

To develop an understanding of the shape of the universe along such lines requires an intimate understanding of what "shapes" are supposed to represent, how we can describe their interaction, how simultaneously zooming in and out can be done in such a way that pattern formation can be correctly observed.

In my mind, Maryam Mirzakhani successfully implemented this program for the universe formed by Riemann surfaces and their canonical bundles, i.e., for the moduli space of Riemann surfaces and the moduli space of abelian differentials. These moduli spaces are well-studied mathematical objects which naturally emerge in almost any mathematical context. Building on the work of many distinguished researchers, Mirzakhani developed a new geometric way to study these spaces that puts earlier, seemingly unrelated, results into complete harmony and, at the same time, creates many new research directions which will be explored by future generations of mathematicians.

In the sequel I will discuss three of Mirzakhani's theorems which were obtained using the principle of zooming in and zooming out. Geometry should be understood in the broad sense of measurement, which includes counting of shapes sorted by sizes, etc.

Riemann Surfaces

An oriented *surface* is a two-dimensional oriented manifold and as such a quite simple object, in particular, if it is closed (i.e., compact, without boundary). Viewed as topological or smooth manifolds, surfaces can easily be classified. The most basic closed surface is the two-sphere, and any other closed surface arises from the two-sphere

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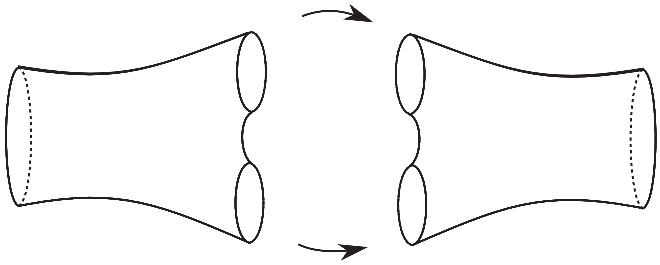

Figure 1. Mirzakhani (Theorem 1 here) proved estimates on the number of pair of pants decompositions.

by attaching a nonnegative number g of so-called *handles*. The number of handles attached is the *genus* of the surface.

Once this classification is established, seemingly there is not much more to say, with the exception of understanding the different geometric shapes a fixed closed surface S admits. The perhaps best known classical result that restricts the geometric structures on S is the *Gauss-Bonnet theorem*. It says that if we denote by K the Gauss curvature of a smooth Riemannian metric on S, then the integral $\frac{1}{2\pi}\int_S K d\text{vol}$ equals the *Euler characteristic* $\chi(S) = 2 - 2g$ of S, where vol is the volume form of the metric. Furthermore, for $g \geq 2$ there exists a *hyperbolic metric* on S, i.e., a metric of constant Gauss curvature -1.

The space of all isometry classes of such metrics can be described as follows. First, the set of all *marked* hyperbolic structures on S is defined to be the set of equivalence classes of hyperbolic metrics where two such metrics are identified if they can be transformed into each other by a diffeomorphism of S isotopic to the identity. This space, *Teichmüller space* $\mathcal{T}(S)$, admits a natural topology that identifies it with an open cell homeomorphic to $\mathbb{R}^{6g-6}$. The group of isotopy classes of diffeomorphisms of S coincides with the quotient of the automorphism group of $\pi_1(S)$ by the subgroup of inner automorphisms. This *mapping class group* $\text{Mod}(S)$ of S acts from the left on $\mathcal{T}(S)$ by precomposition. The space of isometry classes of hyperbolic surfaces is the quotient $\text{Mod}(S)\backslash\mathcal{T}(S)$, called the *moduli space* of S.

Zooming In: Local from Global

For a fixed hyperbolic metric on a closed surface S of genus g, there exists a decomposition of S into geometric pieces, *hyperbolic pairs of pants* as in Figure 1. Topologically, such a hyperbolic pair of pants is a two-sphere from which the interiors of three disjoint compact disks have been removed. This decomposition is done by cutting S open along a collection of $3g - 3$ pairwise disjoint *simple* geodesics. Here a geodesic is simple if it does not have self-intersections.

Clearly, the geometric description of the hyperbolic surface obtained from such a decomposition into hyperbolic pairs of pants depends on the choice of the boundary geodesics. A natural question is how many different such decompositions exist for a fixed hyperbolic metric? The easy answer is: This number is infinite, as there are simple closed geodesics on S of arbitrary length, and each simple closed geodesic belongs to such a decomposition. But what if we restrict the geometry by bounding the length of these geodesics?

A result going back to Huber and Selberg states that the number $c(L)$ of singly-covered closed geodesics of length at most L on a hyperbolic surface S is asymptotic to e^L/L as $L \to \infty$. In particular, the growth rate of the number of such geodesics is not only independent of the specific hyperbolic metric on S but also independent of the genus of S.

However, most of these geodesics have self-intersections, and if we want to understand the geomety of a hyperbolic surface S by decomposing it into pairs of pants, we need to understand simple closed geodesics on S. And, as we vary the hyperbolic metric over moduli space, we may expect that these simple geodesics and their growth rate give information on the geometry of the surface.

In her thesis, Mirzakhani gave an affirmative answer to quite a few questions of this type. Denoting by $\mathcal{M}_g$ the moduli space of closed surfaces of genus g, she showed:

Theorem 1 (Mirzakhani [7, Thm. 1.1])**.** *For a closed hyperbolic surface X of genus $g \geq 2$ and $L > 0$ denote by $s_X(L)$ the number of distinct geodesic pair of pants decompositions of X of total length at most L. Then*

$$\lim_{L\to\infty} \frac{s_X(L)}{L^{6g-6+2n}} = n(X),$$

where $n : \mathcal{M}_g \to \mathbb{R}_+$ is a continuous proper function.

Thus as moduli space is noncompact, the growth rate of the number of pants decompositions of bounded length depends not only on the genus of the surface but also on the surface itself!

Mirakhani showed this result by first zooming out and then zooming in. The first step is to prove that for fixed L, the locally bounded function $X \to s_X(L)$ can be integrated over moduli space, where this moduli space is equipped with the so-called Weil-Petersson volume form, the volume form of an (incomplete) Kähler metric. She discovers that this integral, denoted here by $p(L)$, is a polynomial in L of degree $6g - 6$.

Some pointwise estimates on the growth rate $s_X(L)$ were known earlier. But it requires far-reaching insight and courage to believe that one can basically effectively compute this integral by finding a global relation between these integrals for all $L > 0$. Mirzakhani [7, p. 99] explains this for a fixed pants decomposition γ of S, "the crux of the matter is to understand the density of $\text{Mod}_g \cdot \gamma$ in $\mathcal{ML}_g$." Here $\mathcal{ML}_g$ is the space of so-called *measured geodesic laminations* which is homeomorphic to a euclidean space of dimension $6g - 6$ with the point 0 removed. It contains *weighted simple multicurves*, i.e., collections of disjoint closed geodesics equipped with some positive weights as a dense subset, and it admits an action of the multiplicative group $(0, \infty)$ by scaling.

Each *marked* hyperbolic metric on S defines a continuous length function on $\mathcal{ML}_g$. This function associates to a multicurve the length of its geodesic representative for this metric.

The mapping class group $\mathrm{Mod}(S)$ acts transitively on the multicurves which define a pants decomposition of a fixed combinatorial type. If one equips each component of such a multicurve with the weight $T > 0$, then these weighted point masses define a $\mathrm{Mod}(S)$-invariant Radon measure μ_T on $\mathcal{ML}_g$. Mirzakhani observes that dividing by T^{6g-6}, the total mass of the set of laminations of length at most one for a fixed marked hyperbolic metric determines the value of $s_X(T)$. Furthermore, the measures μ_T converge as $T \to \infty$ to a $\mathrm{Mod}(S)$-invariant Radon measure λ on $\mathcal{ML}_g$ which was earlier constructed by Thurston. Thus zooming in again to the unit length ball defined by a fixed hyperbolic metric, she obtains the asymptotics of the function $s_X(L)$ as $L \to \infty$.

Mirzakhani's result indicates that the action of the mapping class group on $\mathcal{ML}_g$ has properties which resemble a more familiar setting in homogeneous dynamics: The group $\mathrm{SL}(2,\mathbb{Z})$ acts by linear transformations on the puncture plane $\mathbb{R}^2 - \{0\}$. This action preserves the Radon measure, which is just the sum of the Dirac masses on the standard integral grid $\mathbb{Z}^2$ or on any dilation of this grid, and it also preserves the Lebesgue measure of $\mathbb{R}^2$. These measures are all ergodic under the action of $\mathrm{SL}(2,\mathbb{R})$, which means that there are no invariant Borel subsets B of $\mathbb{R}^2$ of positive measure so that the measure of $\mathbb{R}^2 - B$ is positive as well. In fact, they are the only ergodic $\mathrm{SL}(2,\mathbb{Z})$-invariant Radon measures up to scale, with λ being the only nonwandering measure.

It turns out that the dynamics of the action of $\mathrm{Mod}(S)$ on $\mathcal{ML}_g$ has very similar properties: In joint work [6] with Lindenstrauss, Mirzakhani proved that the Thurston measure λ is the only nonwandering invariant Radon measure on $\mathcal{ML}_g$ up to scale.

Hyperbolicity

Mirzakhani's thesis was built on the interplay between geometric information on individual hyperbolic surfaces and the geometry of moduli space, and it culminated in a counting result for simple closed multicurves on hyperbolic surfaces. But in which way does moduli space resemble the Riemann surfaces it is made of?

It is classical that moduli space admits a natural complete geometric structure in its own right, the so-called *Teichmüller metric*. Although this metric is not Riemannian, nor is it nonpositively curved in any reasonable sense, it shares many global features of a hyperbolic metric on a surface. In particular, any two points in Teichmüller space can be connected by a unique geodesic for the pullback of this metric, and periodic geodesics in moduli space are in bijection with conjugacy classes of so-called pseudo-Anosov mapping classes. These periodic geodesics can be sorted and counted according to their length.

In joint work with Alex Eskin [2] and with Eskin and Rafi [5], Mirzakhani uses random sampling and local information on surfaces to obtain precise counting results for periodic geodesics in moduli space. The starting point for this endeavor is the (classical) fact that the cotangent space of moduli space at a surface X can naturally be identified with the vector space of holomorphic quadratic differentials for X. More precisely, the hyperbolic metric of X determines a complex structure, and a holomorphic quadratic differential for this structure is a holomorphic section of the bundle $(K^*)^2$, where K^* is the cotangent bundle.

The geometric beauty of this observation lies in the fact that each nontrivial quadratic differential defines a new geometric structure on the surface S in the same conformal class as the hyperbolic structure. This geometric structure is a flat (Euclidean) metric with finitely many cone points of cone angle an integral multiple of π. As one varies over the fibre of the cotangent bundle at a given point, these flat metrics vary as well, and the space of all such flat metrics reflects the geometric shape of the hyperbolic metric.

Even more is true: Each such flat metric comes equipped with a pair of preferred orthogonal directions, the so-called horizontal and vertical directions, which are defined on the complement of the cone points Σ of the differentials and which are just the line bundles on $S - \Sigma$ on which the evaluation of the differential is positive or negative, respectively. These directions define orthogonal foliations of the surface away from the cone points. Furthermore, there is a natural one-parameter group Φ^t of transformations of such flat surfaces which consists of scaling the horizontal direction with the scaling factor $e^{t/2}$ and the vertical direction with the scaling factor $e^{-t/2}$. These transformations clearly preserve the surface area of the flat metric. Two such flat surfaces define the same cotangent vector in moduli space if they can be transformed into each other with finitely many cut-and-paste operations.

The flow thus defined by Φ^t on the unit area subbundle of the cotangent bundle of moduli space can be analyzed by zooming out: The local change of shape along a flow line can easily be described. A main result is the following:

Theorem 2 (Eskin and Mirzakhani [2])**.** *As $R \to \infty$, the number of periodic trajectories of the flow Φ^t of length at most R is asymptotic to $e^{(6g-6)R}/(6g-6)R$.*

This result was later extended in joint work [5] with Eskin and Rafi with a much more sophisticated argument to *strata* in the moduli space of abelian and quadratic differentials.

The idea of the proof consists of zooming out in random directions and averaging the random samples. It was noted earlier that the flow Φ^t has properties resembling the properties of a hyperbolic flow as long as the trajectories remain in a fixed compact set. What this means precisely was clarified by Mirzakhani in joint work [1] with Athreya, Bufetov, and Eskin. But the moduli space is very notably noncompact, and the metric near the cusp resembles a sup metric which can be thought of as a metric of infinite *positive* curvature. There are many recent results which make this idea precise.

Now moduli space is the quotient of Teichmüller space $\mathcal{T}(S)$ under the action of the mapping class group. Moving in Teichmüller space in a random direction can be thought of as looking at a randomly chosen finite set of points in some large ball in $\mathcal{T}(S)$ and trying to understand the change of shape of a hyperbolic or flat surface sampled on these finitely many points. A careful analysis of this sampling process reveals a bias of such random samples to return to the so-called thick part of Teichmüller space. This is defined to be the preimage of a fixed compact subset of moduli space. Understanding the hyperbolic behavior of the flow Φ^t on the cotangent bundle of the thick part allows Eskin and Mirzakhani to show that the growth rate of periodic orbits that are disjoint from some fixed compact subset of moduli space is strictly smaller than the growth rate of all orbits. That this is sufficient for the above counting result can be established by an adaptation of a classical strategy of Margulis.

Hidden Symmetries and Homogeneous Dynamics

The singular Euclidean metric on the surface S defined by a quadratic differential is given by a collection of charts on the complement of the finite number of singular points with values in the complex plane. Chart transitions are translations or compositions of translations with the reflection $z \to -z$. The differential is the square of a holomorphic one-form or, equivalently, a holomorphic section of the cotangent bundle K^* of S if all chart transitions can be chosen to be translations. The differential is, in this case, called *abelian*. Postcomposition of charts then defines a right action of the group $\mathrm{SL}(2,\mathbb{R})$ on the moduli space of abelian or quadratic differentials. The action of the diagonal group is just the Teichmüller flow Φ^t.

The $\mathrm{SL}(2,\mathbb{R})$-action preserves the *strata* which consists of differentials with the same number and order of singular points. Strata are complex suborbifolds in the moduli space of holomorphic one-forms on Riemann surfaces.

Let P be the set of singular points of an abelian differential ω. Then ω determines by integration over a smooth path a class in $H^1(S,P;\mathbb{C})$. In fact, this class determines ω locally uniquely in the following sense. Integration over a fixed basis of $H_1(S,P;\mathbb{Z})$ yields so-called period coordinates for the stratum. These period coordinates equip the stratum with an $\mathrm{SL}(2,\mathbb{R})$-invariant affine structure, i.e., local charts with affine chart transitions. These chart transitions are volume-preserving and, hence, define a Lebesgue measure on the stratum. This *Masur–Veech measure* is finite and invariant under the action of $\mathrm{SL}(2,\mathbb{R})$.

Orbits of the $\mathrm{SL}(2,\mathbb{R})$-action on the moduli space of area-one abelian differentials can be identified with the unit tangent bundle of *Teichmüller disks* which are totally geodesic immersed hyperbolic disks in moduli space. The analog of periodic Teichmüller geodesics are *closed* $\mathrm{SL}(2,\mathbb{R})$-orbits. Such orbits project to finite-volume immersed hyperbolic surfaces in moduli space. It is known that such closed orbits are dense in any stratum. But how can one globally understand the dynamics of the $\mathrm{SL}(2,\mathbb{R})$-action?

In homogenous dynamics, the celebrated solution of a conjecture of Raghunathan by Marina Ratner (who sadly passed away one week before the death of Maryam Mirzakhani) states for example the following.

Consider a simple Lie group G of noncompact type and a lattice Γ in G. Let $H < G$ be a closed subgroup that is generated by unipotent elements. The group H acts by right translation on $\Gamma\backslash G$. Then for every H-invariant Borel probability measure μ on $\Gamma\backslash G$ there exists a closed subgroup $L < G$ with the following properties:

(1) $\Gamma \cap L$ is a lattice in L.
(2) μ is the projection of the Haar measure on L to $\Gamma \cap L\backslash L$.

Furthermore, the closure of each H-orbit on $\Gamma\backslash G$ equals the closed orbit of a Lie subgroup $L < G$ which intersects Γ in a lattice.

Thus Ratner's theorem gives a complete classification of orbit closures on quotients $\Gamma\backslash G$ as well as of invariant measures.

Can one formulate a conjecture which translates Ratner's theorem into the framework of the action of $\mathrm{SL}(2,\mathbb{R})$ on a stratum of area-one abelian differentials? Remembering that an orbit closure in $\Gamma\backslash G$ is algebraic, an ad hoc guess could be that such an orbit closure is an affine suborbifold, i.e., cut out by polynomial equations in affine coordinate charts. But the affine structure on strata of moduli space is obtained by taking periods of holomorphic one-forms over relative homology classes, with an integral structure that arises from integral homology, and a priori this is not in harmony with algebraic structures obtained from global algebraic geometric information.

The amazing rigidity result of Eskin and Mirzakhani [3] states as a special case:

Theorem 3 (Eskin and Mirzakhani [3])**.** *Every $SL(2,\mathbb{R})$-invariant Borel probability measure on the moduli space of area-one abelian differentials is an affine measure of an affine invariant manifold.*

Together with Mohammadi, they also show that every orbit closure is an affine invariant manifold. By a 2016 result of Filip, this result can be translated into an algebraic-geometric statement for moduli space: The quotient of an affine invariant manifold by the natural circle group of rotations which multiplies an abelian differential by a complex number of absolute value one is a projective subvariety of the projective bundle over moduli space whose fibre at a point X equals the projectivization of the vector space of holomorphic one-forms on X.

But how can one prove this result? By an amazing construction which simultaneously zooms in and out. Random sampling is used to reveal the global geometry. One of the main tools is random walks, and vast and sophisticated knowledge about such random walks which provides information on deviation, stationary measures, drifts, and structural insights related to the idea of proximality enters into the argument.

It requires the amazing insight, the unsurpassed optimism, and the enormous technical strength of Maryam Mirzakhani to carry out this program, using along the way tools from areas of mathematics as diverse as geometry, ergodic theory, homogeneous dynamics, algebraic groups, random walks, algebraic geometry, and mathematical physics. Her vision is the door to a wonderland where very concrete but seemingly unrelated mathematical structures combine to a composition of extreme complexity and breathtaking beauty, yet which is made from simple tunes and is orchestrated in perfect harmony. There is no coda, but there is an unspoken invitation to everyone to extract his or her favorite line and explore its variations and ramifications.

I met Maryam for the first time on the occasion of a workshop in Chicago (probably in 2002). Curtis McMullen introduced her to me during lunch, and, as mathematicians do, the small lunch group chatted about math. She fully participated in the discussion, making interesting comments, and answered a question that came up. Right after giving the answer, she realized that what she had said was not quite right, and she corrected herself with a laugh, mockingly embarrassed about her inaccuracy.

During our encounters in later years, I found many times again this laughter, reflecting her professional ambition, passion, and optimism which enabled her to adjust and fine-tune her fantastic vision until the next step had emerged in complete clarity and in an aesthetically pleasing way. Looking backwards, I am very grateful that I had the privilege to know her in person, early on.

With her creativity, persistence, and courage, Maryam Mirzakhani made a contribution to mathematics which I believe will be remembered and further developed by many future generations of mathematicians. Her memory should in particular serve as an inspiration and role model for all young women interested in pursuing a career in mathematics.

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Image Credit

Figure 1 courtesy of Ursula Hamenstädt.

Scott A. Wolpert

Maryam Mirzakhani and Hyperbolic Geometry

Reflection

In the spring of 2004, I set about reading a manuscript from one of Curt McMullen's graduating students. The manuscript included many of my favorite topics and concluded with a proof, beginning with hyperbolic geometry, of Witten's conjecture from $2d$-quantum gravity. In 1992 Maxim Kontsevich first proved the conjecture using a matrix Airy integral model and ribbon graphs. The conjecture posits that the intersection numbers on the moduli space $\mathcal{M}$ of Riemann surfaces of two tautological line bundles satisfy the recursion of the Korteweg–de Vries (KdV) hierarchy. I soon realized that the manuscript was closely interrelated with two other manuscripts, one on a recursive scheme in genus for integration over the moduli space $\mathcal{M}$ and one on the asymptotic count by length of simple geodesics on a hyperbolic surface. The constants in the asymptotic count formulas were given by the tautological intersection numbers and Thurston volumes. The graduate student was Maryam Mirzakhani, and so began my journey of studying and contemplating her beautiful results. Mirzakhani's number of publications is not so large, but the breadth and achievements are very special. Often her results are given by simple answers—broad statements without special hypotheses.

In my experience, studying Mirzakhani's works is somewhat akin to signing on for a hike with a young, freshly trained guide. You embark on familiar terrain but soon arrive at surprising new vistas. Also the pace of the hike can be quite varied—conceptual steps can be large, and detailed calculations with formulas go by quickly. I've thought that the latter might be the product of her Math Olympiad training. I always personally found Maryam to be full of energy, most gracious, and intellectually ambitious—always eager to unravel the next piece of the puzzle. For me, Maryam always embodied the mathematician persona that we aspire to fulfill.

Background

We describe Mirzakhani's thesis results from *Simple geodesics and Weil-Petersson volumes of moduli spaces of bordered Riemann surfaces, Weil-Petersson volumes and intersection theory on the moduli space of curves*, and *Growth of the number of simple closed geodesics on hyperbolic surfaces*, as well as the main results from the joint work with Elon Lindenstrauss. An exposition of the results and necessary background material are provided in our CBMS lectures *Families of Riemann surfaces and Weil–Petersson geometry* and our PCMI lectures *Mirzakhani's volume recursion and approach for the Witten–Kontsevich theorem on moduli tautological intersection numbers.*

Mirzakhani considers Riemann surfaces R of finite topological type with hyperbolic metrics, possibly with

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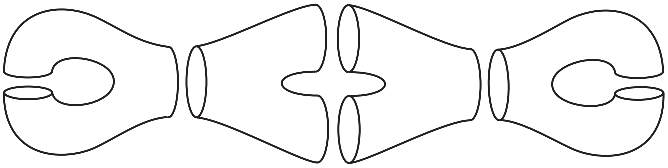

Figure 1. Assembling a genus 3 surface from pairs of pants.

punctures and geodesic boundaries. A surface is marked by choosing a homotopy class of a homeomorphism from a reference surface F. Teichmüller space $\mathcal{T}$ is the space of equivalence classes of marked Riemann surfaces. If the Riemann surfaces are compact or compact with punctures, then the Teichmüller space is a complex manifold. The mapping class group MCG consists of homotopy classes of orientation-preserving homeomorphisms of F and the group acts by precomposition on marking homeomorphisms. The quotient $\mathcal{T}/MCG$ is the classical moduli space $\mathcal{M}$ of Riemann surfaces. For compact genus one surfaces, Teichmüller space is the upper half-plane $\mathbb{H}$ and $\mathrm{PSL}(2 : \mathbb{Z})$ is the MCG. A point $\tau \in \mathbb{H}$ describes the lattice L_τ with generators 1 and τ and the Riemann surface $\mathbb{C}/L_\tau$. Surfaces of negative Euler characteristic are uniformized by the upper half-plane with hyperbolic metric. We consider Teichmüller spaces for negative Euler characteristic surfaces—compact surfaces, surfaces with a finite number of punctures, and surfaces with a finite number of geodesic boundaries. Denote the genus of a surface R by g and the number of punctures or boundaries by n.

A nontrivial, nonpuncture homotopic, free homotopy class α on the reference surface has a unique geodesic representative on a marked surface R—the geodesic length ℓ_α provides a natural function on $\mathcal{T}$. Collections of geodesic-length functions provide local coordinates and global immersions to Euclidean space for $\mathcal{T}$. A hyperbolic surface can be cut open on a simple geodesic. Since a neighborhood of a simple geodesic is an annulus with an S^1 symmetry, the cut boundaries can be reassembled with a relative rotation to form a new hyperbolic structure. For unit-speed hyperbolic relative displacement of reference points, the infinitesimal deformation is the Fenchel–Nielsen (FN) twist vector field t_α on $\mathcal{T}$.

Hyperbolic surfaces can be assembled from pairs of pants—from genus zero surfaces with three geodesic boundaries, as in Figure 1. For pants, boundary reference points are provided by the unique orthogonal geodesics between boundaries. At an assembly seam, the common boundary length ℓ and Fenchel–Nielsen twist displacement parameter τ are unrestricted. The length varies in $\mathbb{R}_{>0}$, and the twist varies in $\mathbb{R}$.

Theorem 1 (FN coordinates)**.** *The parameters* $\prod_{j=1}^{3g-3+n}(\ell_j,\tau_j)$ *provide a real analytic equivalence of* $\mathcal{T}$ *to* $(\mathbb{R}_{>0}\times\mathbb{R})^{3g-3+n}$.

The symplectic geometry of $\mathcal{T}$ begins with the MCG-invariant Weil–Petersson Kähler metric $g_{WP\,Kähler}$ and its symplectic form $\omega = \omega_{WP\,Kähler}$. Symmetry reasoning shows that ℓ and τ provide action-angle coordinates.

Theorem 2 ([W])**.** *The WP symplectic form satisfies twist-length duality*

$$\omega(\ ,t_\alpha) = d\ell_\alpha$$

and has the expansion

$$\omega = \sum_{j=1}^{3g-3+n} d\ell_j \wedge d\tau_j.$$

In particular geodesic-length functions are Hamiltonian, and the symplectic structure is independent of pants decomposition. Also the quotient $\mathcal{T}/MCG$ is at least a real analytic manifold.

The Bers fiber space $\mathcal{B}$ is the complex disc holomorphic bundle over $\mathcal{T}$ with fiber over a marked surface R being the universal cover $\widetilde{R}$. A point on a fiber can be declared to be a puncture, and so $\mathcal{B}_g \approx \mathcal{T}_{g,1}$ and $\mathcal{B}_{g,n} \approx \mathcal{T}_{g,n+1}$. An extension $MCG_{\mathcal{B}}$ of MCG by the fundamental group of the fiber $\pi_1(F)$ acts properly discontinuously and holomorphically on $\mathcal{B}$. The resulting map $\pi : \mathcal{B}/MCG_{\mathcal{B}} \to \mathcal{T}/MCG$ describes an orbifold bundle, the *universal curve* over $\mathcal{M}$; the fibers are Riemann surfaces modulo their automorphism groups. A fiber of the vertical line bundle $(\mathrm{Ker}\, d\pi)$ is a tangent to a Riemann surface; the hyperbolic metrics of individual surfaces provide metrics for the fibers. The Chern form $\mathbf{c}_1(\mathrm{Ker}\, d\pi)$ on $\mathcal{B}/MCG_{\mathcal{B}}$ can be used to define forms/cohomology classes on $\mathcal{M}$ by integrating over fibers $\kappa_k = \int_{\pi^{-1}(R)} \mathbf{c}_1^{k+1}$.

Theorem 3 ([W])**.** *For the hyperbolic metrics on fibers,* $2\pi^2\kappa_1 = \omega$, *pointwise on* $\mathcal{M}$ *and in cohomology on the Deligne–Mumford compactification* $\overline{\mathcal{M}}$.

There is a unique way to fill in a puncture for a conformal structure. Accordingly a labeled puncture determines a section s of $\widetilde{\mathcal{B}}/MCG_{\mathcal{B}} \to \mathcal{M}$ (punctures on fibers are now filled in). Along a puncture section s we consider the family of tangent lines $(\mathrm{Ker}\, d\pi)|_s$ or the dual family of cotangent lines $(\mathrm{Ker}\, d\pi)^*|_s$—the pullback to $\mathcal{M}$ by the puncture section s of the dual family of cotangent lines is the tautological canonical class ψ.

Mirzakhani–McShane and the Volume Recursion

On a compact manifold, the number of closed geodesics of length less than a given bound is finite. In 2003 Mirzakhani wrote, "My work has been motivated by the problem of estimating $s_R(L)$, the number of primitive simple closed geodesics of hyperbolic length less than L on R." A natural first step is to average the count over all Riemann surfaces—this requires an approach to computing integrals over the moduli space of Riemann surfaces. Fundamental domains in Teichmüller space cannot provide a workable approach. With an especially ingenious and useful insight, Mirzakhani considered Greg McShane's remarkable identity to provide a *partition of unity* relative to the mapping class group. The Mirzakhani–McShane identity is based on studying the geodesics emanating orthogonally from a geodesic boundary of a Riemann surface. The behavior of emanating geodesics

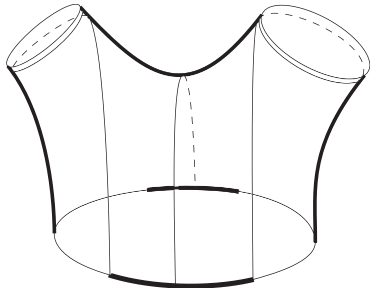

Figure 2. On the waist boundary: an ortho-boundary geodesic, a pair of distinguished intervals, and a pair of spirals.

prescribes a measure zero Cantor subset of the boundary. The remarkable identity is the formula that the sum of lengths of complementary intervals equals the boundary length.

A geodesic boundary β has an infinite number of *ortho-boundary geodesics*—simple geodesic segments orthogonal to the boundary at segment endpoints. Given an ortho-boundary geodesic γ, an ϵ-neighborhood of $\beta \cup \gamma$ is a topological pair of pants—the pants cuff boundaries are homotopic to simple geodesics α and λ. Along the boundary β there is a pair of distinguished intervals; each interval contains an endpoint of γ and has as endpoints spirals (one clockwise and one counterclockwise) about α and λ. A spiral is a simple infinite geodesic emanating orthogonally from β. The pair of pants bounded by β, α, and λ contains only the four spirals emanating orthogonally from β. The (equal) length of the two distinguished intervals is an elementary hyperbolic trigonometric expression in ℓ_β, ℓ_α, and ℓ_λ. The association of distinguished intervals to ortho-boundary geodesics can also be reversed. Given a pair of pants bounded by β, α', and λ', there is a corresponding ortho-boundary geodesic γ' and distinguished intervals bounded by spirals about α' and λ'. Distinguished intervals for distinct ortho-boundary geodesics are disjoint, and the complement of all distinguished intervals on a given boundary is a null Cantor set.

To state the resulting identity, let

$$\mathcal{D}(x, y, z) = 2 \log \frac{e^{\frac{x}{2}} + e^{\frac{y+z}{2}}}{e^{\frac{-x}{2}} + e^{\frac{y+z}{2}}}.$$

Theorem 4 (The Mirzakhani–McShane identity)**.** *For a hyperbolic surface R with a single boundary β of length L,*

$$L = \sum_{\alpha,\lambda} \mathcal{D}(L, \ell_\alpha, \ell_\lambda),$$

where the sum is over unordered pairs of simple geodesics with β, α, and λ bounding an embedded pair of pants.

Mirzakhani applied the length identity to reduce the action of the mapping class group to actions of mapping class groups of subsurfaces. Computing the volume of the moduli space is reduced to an integral of products of lower-dimensional volumes—the desired recursion.

The integration approach is illustrated by computing the genus one, one boundary, volume. For such tori, the selection of an ortho-boundary geodesic is equivalent to selecting a simple closed geodesic. The length identity is

$$L = \sum_{\alpha\ simple} \mathcal{D}(L, \ell_\alpha, \ell_\alpha).$$

Introduce FN coordinates relative to a particular geodesic α and introduce $Stab(\alpha) \subset MCG$, the stabilizer for MCG acting on free homotopy classes. The stabilizer is generated by the Dehn twist about α.

The Dehn twist acts in FN coordinates by $(\ell, \tau) \to (\ell, \tau + \ell)$. The sector $\{0 \le \tau < \ell\}$ is a fundamental domain for the $Stab(\alpha)$ action. A mapping class $h \in MCG$ acts on a geodesic-length function by $\ell_\alpha \circ h^{-1} = \ell_{h(\alpha)}$.

Now we apply an unfolding argument and write the length identity as

$$L = \sum_{\alpha} \mathcal{D}(L, \ell_\alpha, \ell_\alpha) = \sum_{h \in MCG/Stab(\alpha)} \mathcal{D}(L, \ell_{h(\alpha)}, \ell_{h(\alpha)}),$$

use the MCG action on geodesic-length functions to find

$$LV(L) = \int_{\mathcal{T}(L)/MCG} \sum_{MCG/Stab(\alpha)} \mathcal{D}(L, \ell_\alpha \circ h^{-1}, \ell_\alpha \circ h^{-1})\, \omega,$$

change variables on $\mathcal{T}$ by $p = h(q)$ to find

$$= \sum_{h \in MCG/Stab(\alpha)} \int_{h(\mathcal{T}(L)/MCG)} \mathcal{D}(L, \ell_\alpha, \ell_\alpha)\, d\tau d\ell$$

$$= \int_{\mathcal{T}(L)/Stab(\alpha)} \mathcal{D}(L, \ell_\alpha, \ell_\alpha)\, d\tau d\ell,$$

and use the $Stab(\alpha)$ fundamental domain to obtain the integral

$$\int_0^\infty \int_0^\ell \mathcal{D}(L, \ell, \ell)\, d\tau d\ell.$$

The integral in τ gives a factor of ℓ. The derivative $\partial \mathcal{D}(x, y, z)/\partial x$ is a simpler function. Apply this observation and differentiate in L to obtain a formula for the derivative of $LV(L)$:

$$\frac{\partial}{\partial L} LV(L) = \int_0^\infty \frac{1}{1 + e^{\ell + \frac{L}{2}}} + \frac{1}{1 + e^{\ell - \frac{L}{2}}}\, \ell d\ell = \frac{\pi^2}{6} + \frac{L^2}{8}.$$

The genus one volume formula $V(L) = \frac{\pi^2}{6} + \frac{L^2}{24}$ results.

The general volume recursion is a sum over topological configurations for the complement of a pair of pants containing a surface boundary; see Figure 3 (see p. 1238). The possible configuration types for the complement are: connected with two internal cuffs, disconnected with two internal cuffs and one internal cuff. For the tuple of boundary lengths $L = (L_1, \ldots, L_n)$ and $V_g(L)$ the volume of $\mathcal{T}_g(L)/MCG$, the recursion is

$$(1)\quad \frac{\partial}{\partial L_1} L_1 V_g(L) = \mathcal{A}_g^{connect}(L) + \mathcal{A}_g^{disconnect}(L) + \mathcal{B}_g(L),$$

where $\mathcal{A}_g^*(L)$ and $\mathcal{B}_g(L)$ are integral transforms of products of smaller topological-type volume functions. By direct calculation the volume functions are polynomials of lengths-squared with coefficients positive rational multiples of powers of π—products of factorials and Riemann zeta at nonnegative even integers. Mirzakhani very astutely recognized that symplectic geometry provides

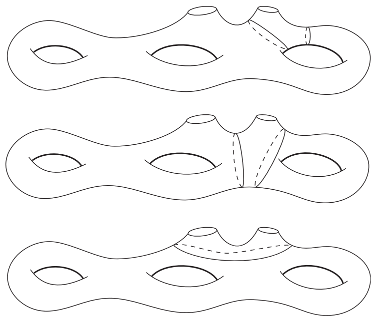

Figure 3. Pants complement configurations.

the setting for the volume formulas to be intersection numbers and for the recursion to give a solution for the Witten–Kontsevich conjecture. The relations between volumes of moduli spaces are the relations of $2d$-quantum gravity.

Symplectic Reduction

Mirzakhani considered equivalences of symplectic spaces. She introduced the space $\widehat{\mathcal{T}}_{g,n}$ of decorated Riemann surfaces with geodesic boundaries of varying lengths and a reference point on each boundary. The $\mathbb{R}$ dimension of $\widehat{\mathcal{T}}_{g,n}$ is $2n$ greater than that of $\mathcal{T}_g(L)$. To a Riemann surface with reference points on n boundaries is also associated a tailored surface with $2n$ cusps and no boundaries. For a boundary of length ℓ, an almost tight pair of pants with boundary lengths ℓ, 0, and 0 (zero length prescribes a puncture) can be attached to cap off the boundary. Almost tight pants are attached by aligning reference points. The space $\widehat{\mathcal{T}}_{g,n}$ is equivalent to $\mathcal{T}_{g,2n}$. The seams and variable points can be included in a pants decomposition of the resulting tailored surface. Theorems 1 and 2 can be applied to prescribe a symplectic form $\omega_{\widehat{\mathcal{T}}_{g,n}}$ on $\widehat{\mathcal{T}}_{g,n}$. Consider the resulting symplectic geometry. For a Riemann surface with boundaries $\beta_1, \dots, \beta_n$, let L_j be the length of β_j, τ_j describe the location of the reference point, and t_j be the infinitesimal FN twist of the jth almost tight pants. Introduce the moment map

$$\widehat{\mathcal{T}}_{g,n} \xrightarrow{\mu} \hat{L} = (L_1^2/2, \dots, L_n^2/2) \in \mathbb{R}^n_{\geq 0}.$$

The function $L_j^2/2$ is the Hamiltonian potential for the rescaled twist $\omega_{\widehat{\mathcal{T}}_{g,n}}(-L_j t_j, \ ) = d(\frac{1}{2}L_j^2)$, and $\omega_{\widehat{\mathcal{T}}_{g,n}}$ is twist flow invariant. In particular the vector field $-L_j t_j$ is the infinitesimal generator for an S^1 action on the jth reference point. The group $(S^1)^n$ acts on the level sets of $\mu : \widehat{\mathcal{T}}_{g,n} \to \mathbb{R}^n$ and the quotient of a level set of the moment map is naturally $\mathcal{T}_g(L)$. Theorem 2 leads to symplectic reduction: for small L, the level set quotient $\mu^{-1}(\hat{L})/(S^1)^n$ is diffeomorphic to $\mathcal{T}_g(L)$ with

$$\omega_{\widehat{\mathcal{T}}_{g,n}}\big|_{\mu^{-1}(\hat{L})/(S^1)^n} \approx \omega_{\mathcal{T}_g(L)},$$

and the level set quotients are mutually diffeomorphic. The volumes $V_g(L)$ are the volumes of $(S^1)^n$ quotients of μ level sets in $\widehat{\mathcal{T}}_{g,n}$.

Mirzakhani noted that the reference points on circle boundaries describe S^1 bundles. $\widehat{\mathcal{T}}_{g,n}$ is the total space of an $(S^1)^n$ bundle over $\mathcal{T}_g(L)$. Circle bundles can be endowed with S^1 principal connections with the associated curvature 2-forms giving the first Chern classes of the bundles in the cohomology of the compactification of $\mathcal{T}_g(L)/MCG$. What are the cohomology classes? Instead of capping off with almost tight pants, a boundary β can be capped off by a Euclidean disc of radius L with the boundary reference point prescribing a radius vector at the origin. The orientation for the twist parameter prescribes the orientation for rotating the radius vector. The fiber is equivalent to the cotangent plane at the origin—the description of the tautological class ψ.

An application of the Duistermaat–Heckman theorem now gives the desired cohomology relationship

$$\omega_{\mathcal{T}_g(L)} \equiv \omega_{\mathcal{T}_g(0)} + \sum_{j=1}^{n} \frac{L_j^2}{2} c_1(\hat{\beta}_j),$$

for $\hat{\beta}_j$ the S^1 principal bundle for β_j. The bundles $\hat{\beta}_j$ and ψ_j are smoothly equivalent. Combining with Theorem 3's formula $2\pi^2\kappa_1 = \omega$ gives Mirzakhani's beautiful relationship of volumes to intersection numbers.

Theorem 5 (Volume and intersections)**.** *For* $d = \dim_{\mathbb{C}} \mathcal{T}_{g,n}$,

$$V_g(L) = \frac{1}{d!} \int_{\mathcal{T}_{g,n}/MCG} \left(2\pi^2\kappa_1 + \sum_{j=1}^{n} \frac{L_j^2}{2}\psi_j\right)^d.$$

A very pleasing result: the coefficients of the volume polynomials are the κ_1-ψ tautological intersection numbers. The tautological classes are rational and nonnegative. The observations explain the positive coefficients and the pattern of powers of π^2 and L^2 in the volume polynomials.

Witten–Kontsevich and Counting Geodesics by Length

Witten posited that a generating function for the ψ intersection numbers should satisfy the KdV relations. To encode intersections, for any set of nonnegative integers $\{d_1, \dots, d_m\}$, write for top intersections

$$\langle \tau_{d_1}, \dots, \tau_{d_m} \rangle_g = \int_{\overline{\mathcal{M}}_{g,n}} \prod_{i=1}^{m} \psi_i^{d_i},$$

for the integral over the compactified moduli space. A product is defined for $\sum_{i=1}^m d_i = 3g-3+n$ and is otherwise set equal to zero. Introduce formal variables $t_i, i \geq 0$, and define F_g, the generating function for genus g, by

$$F_g(t_0, t_1, \dots) = \sum_{\{d_i\}} \langle \prod \tau_{d_i} \rangle_g \prod_{r>0} t_n^{n_r}/n_r!,$$

where the sum is over all finite sequences of nonnegative integers and $n_r = \#\{i \mid d_i = r\}$. The generating function

$$\mathbf{F} = \sum_{g=0}^{\infty} \lambda^{2g-2} F_g$$

is a partition function (probability of states) for a $2d$-quantum gravity model.

Known relations for the intersection numbers include the string equation for adding a puncture to a surface without a ψ factor and the dilaton equation for adding a puncture with a single ψ factor. For $n > 0$ and $\sum_i \alpha_i = 3g - 2 + n > 0$, the relations are

$$\text{string equation } \langle \tau_0 \tau_{\alpha_1} \cdots \tau_{\alpha_n} \rangle_g = \sum_{\alpha_i \neq 0} \langle \tau_{\alpha_1} \cdots \tau_{\alpha_i - 1} \cdots \tau_{\alpha_n} \rangle_g,$$

and for $n \geq 0$ and $\sum_i \alpha_i = 3g - 3 + n \geq 0$, the dilaton equation

$$\langle \tau_1 \tau_{\alpha_1} \cdots \tau_{\alpha_n} \rangle_g = (2g - 2 + n) \langle \tau_{\alpha_1} \cdots \tau_{\alpha_n} \rangle_g.$$

Witten's posited relations are expressed in terms of the Virasoro Lie algebra generated by differential operators $\mathcal{L}_n = -z^{n+1} \frac{\partial}{\partial z}, n \geq -1$, with commutators $[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m}$. The leading terms of the volume polynomials $V_g(L)$ are given by top ψ intersections. Mirzakhani was able to decipher the mystery—the leading terms of the volume recursion (1) give the desired relations $\mathcal{L}_*(e^{\mathrm{F}}) = 0$.

Theorem 6 (The Witten-Kontsevich conjecture)**.** *The moduli volume recursion gives the Virasoro constraints.*

Mirzakhani's prime simple geodesic theorem is based on studying the action of MCG on the space $\mathcal{MGL}$ of compactly supported measured geodesic laminations. A geodesic lamination is a foliation of a closed subset of a surface by geodesics. A transverse measure for a geodesic lamination $\mathcal{G}$ is an assignment for each smooth transverse arc τ with endpoints in the complement, a nonnegative measure which supports the intersection $\mathcal{G} \cap \tau$. Assigned measures coincide for smooth transversals that are homotopic through smooth arcs with endpoints in $\mathcal{G}^c$. Assigning masses to transverse arcs is a functional, and the assignment provides for a weak* topology—$\mathcal{MGL}$ is the space of compactly supported geodesic laminations. The space $\mathcal{MGL}$ is the closure of the set of positively weighted simple closed geodesics; $\mathbb{R}_+$ acts on $\mathcal{MGL}$ by multiplying measures. Typically a convergent sequence of simple closed geodesics limits to a disjoint union of infinite simple complete geodesics. The cross section of a measured geodesic lamination is the union of a finite set and a Cantor set. A multicurve is a finite positive weighted sum $\gamma = \sum_{j=1}^m a_j \gamma_j$ of disjoint, distinct simple closed geodesics with length $\ell_\gamma = \sum_{j=1}^m a_j \ell_{\gamma_j}$. For a general element ν of $\mathcal{MGL}$, the product of the transverse measure and hyperbolic length along leaves gives a measure whose total mass is ℓ_ν the length of ν. Multicurves with integer weights $\mathcal{MGL}(\mathbb{Z})$ form an MCG-invariant lattice-like subset. The space $\mathcal{MGL}$ has a natural volume element, the Thurston measure μ_{Th}. For a convex open set $U \subset \mathcal{MGL}$,

$$\mu_{Th}(U) = \lim_{T \to \infty} \frac{\#\{T \cdot U \cap \mathcal{MGL}(\mathbb{Z})\}}{T^{6g-6+2n}};$$

equivalently μ_{Th} can be computed from the masses of leaves intersecting a system of transverse arcs; μ_{Th} is the top exterior power of Thurston's train-track symplectic form. The volumes of geodesic-length balls $B(R) = \mu_{Th}(\{\nu \in \mathcal{MGL} \mid \ell_\nu(R) \leq 1\})$ and total volume $b(R) = \int_{\mathcal{M}_{g,n}} B(R) dV$ provide $\mathcal{MGL}(\mathbb{Z})$ normalizing factors in the following formulas.

Margulis's general counting result is that for a compact negatively curved closed manifold, the count of closed geodesics is asymptotic in length L to e^{hL}/hL, for h the topological entropy of the geodesic flow. The homeomorphisms of a surface act on the free homotopy classes of curves. Mirzakhani's focus is the length counting function for an MCG orbit

$$s_R(L, \gamma) = \#\{\alpha \in MCG\gamma \mid \ell_\alpha(R) \leq L\}$$

for a multicurve on a compact surface with possible cusps.

Theorem 7 (Prime simple geodesic theorem)**.** *For a rational multicurve γ, there is a positive constant $c(\gamma)$ such that*

$$\lim_{L \to \infty} \frac{s_R(L, \gamma)}{L^{6g-6+2n}} = \frac{c(\gamma) B(R)}{b(R)},$$

where the constant is computed from the weights for γ, the symmetries of γ, and the volume polynomial $V_{R(\gamma)}(L)$ for the cut open surface $R - \gamma$.

For a multicurve γ, introduce FN coordinates for $\mathcal{T}$, including the component curves of γ. Similar to the above unfolding argument for

$$\int_{\mathcal{T}/MCG} \sum_\alpha \mathcal{D}(L, \ell_\alpha, \ell_\alpha) dV = \int_0^\infty \mathcal{D}(L, \ell, \ell) \ell \, d\ell,$$

we have

$$\int_{\mathcal{T}/MCG} s_R(L, \gamma) dV = (\#Sym(\gamma))^{-1} \times \int_{\{0 < \sum a_j x_j \leq L, x_j > 0\}} V(R(\gamma); \mathbf{x}) \, \mathbf{x} \cdot d\mathbf{x}$$

for the symmetry group of γ and $V(R(\gamma); \mathbf{x})$ the volume of the moduli space of the cut open surface $R(\gamma)$ with boundary lengths $\mathbf{x}$. The right-hand integral is calculated by the volume recursion.

Howard Masur showed that the MCG-invariant measure μ_{Th} is ergodic—an MCG-invariant subset of $\mathcal{MGL}$ either has full or zero μ_{Th} measure. Now if k_0 is a common denominator for the coefficients of γ, then the MCG orbit satisfies $MCG\gamma \subset 1/k_0 \mathcal{MGL}(\mathbb{Z})$ and counts for $s_R(L, \gamma)$ are bounded by counts for $1/k_0 \mathcal{MGL}(\mathbb{Z})$. It follows for a convex open set U that a limit

$$\mu_\gamma(U) = \lim_{T_j \to \infty} \frac{\#\{T \cdot U \cap MCG\gamma\}}{T_j^{6g-6+2n}}$$

is bounded by $k_0^{-6g+6-2n} \mu_{Th}(U)$; the measure μ_γ is absolutely continuous with respect to μ_{Th}. By Masur ergodicity, we have that $\mu_\gamma = c' \mu_{Th}$. Now evaluate the measures on the unit balls $\{\nu \in \mathcal{MGL} \mid \ell_\nu \leq 1\}$ and integrate over the moduli space. The left-hand side is calculated above, and the right-hand side gives the normalizing factors $B(R)$ and $b(R)$, completing the argument.

Understanding all the invariant measures and all the possible closures of orbits for a group action is often very challenging. In the joint paper with Lindenstrauss, *Ergodic theory of the space of measured geodesic laminations,* the authors provide an understanding for the mapping class group MCG acting on $\mathcal{MGL}$. The description is in terms of *complete pairs* (S, γ), a multicurve γ, and a subsurface

S that is a union of components of the cut open $R(\gamma)$. For the natural embedding $\mathcal{I}_S$ of $\mathcal{MGL}(S)$ into $\mathcal{MGL}(R)$, define the subset

$$\mathcal{G}^{(S,\gamma)} = \{\gamma + \nu \mid \nu \in \mathcal{I}_S(\mathcal{MGL}(S))\},$$

sums of pairs: a multicurve and a measured geodesic lamination contained in the complement. By using either the lattice-like subset $\mathcal{MGL}(S,\mathbb{Z})$ or Thurston's train-track symplectic form for S, there is a natural measure $\mu_{Th}^{(S,\gamma)}$ on $\mathcal{G}^{(S,\gamma)}$. The measure can be distributed to a locally finite measure μ_{Th}^{S} on the orbit $MCG \cdot \mathcal{G}^{(S,\gamma)}$ in $\mathcal{MGL}(R)$. Mirzakhani and Lindenstrauss give an especially straightforward answer to the MCG orbit questions.

Theorem 8. *Let μ be a locally finite MCG-invariant measure on $\mathcal{MGL}$. Then μ is a constant multiple of a μ_{Th}^{S} for a complete pair (S,γ). Given $\nu \in \mathcal{MGL}$, there exists a complete pair (S,γ) such that $\overline{MCG \cdot \nu} = MCG \cdot \mathcal{G}^{(S,\gamma)}$.*

Image Credits

Section figures 1–3 courtesy of Scott A. Wolpert.

Maryam Mirzakhani with Greg McShane on the terrace at Luminy.

Alex Wright and Anton Zorich

The Magic Wand Theorem

From Problems of Physics to Billiards in Polygons

In a sense, dynamical systems concern anything which moves. The things that move might be the planets in the solar system, or a system of particles in a chamber, or a billiard ball on an unusually shaped table, or currents in the ocean, or electrons in a metal, etc. One can observe certain common phenomena in large classes of dynamical

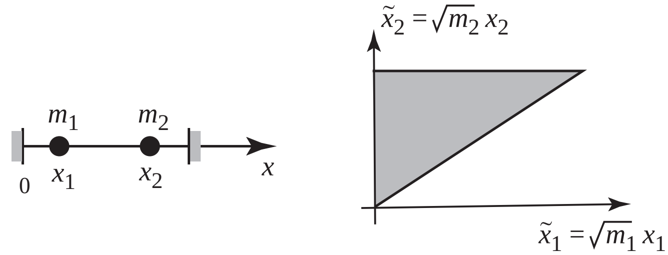

Figure 1. The configuration space of two molecules in a one-dimensional chamber is a triangle.

systems; in particular, ideal billiards might be interpreted as toy models of a gas in a chamber. One can argue that one cannot model a gas with only one molecule. But if you let your molecule move and take a picture of it every second, then superposing millions of pictures you will get a simplified model of the gas, where we pretend that molecules do not interact between themselves.

A very simple model of a gas where molecules do strongly interact also leads to a billiard. In this model there are only two molecules and the chamber is one-dimensional. Consider a system of two elastic beads confined to a rod placed between two walls,[1] as in Figure 1. (To the best of our knowledge this construction originates in lectures of Ya. G. Sinai.)

The beads have different masses m_1 and m_2; they collide between themselves and also with the walls. Assuming that the size of the beads is negligible, we can describe the configuration space of our system using coordinates $0 \le x_1 \le x_2 \le a$ of the beads, where a is the distance between the walls. Rescaling the coordinates as

$$\begin{cases} \tilde{x}_1 &= \sqrt{m_1}x_1, \\ \tilde{x}_2 &= \sqrt{m_2}x_2, \end{cases}$$

we see that the configuration space in the new coordinates is given by a right triangle Δ; see Figure 1. Consider now a trajectory of our dynamical system. It is not difficult to verify that in coordinates $(\tilde{x}_1, \tilde{x}_2)$ trajectories of the system of two beads on a rod correspond to billiard trajectories in the triangle Δ.

Dynamics of a triangular billiard is anything but trivial. For example, it is not known whether every acute triangle has at least one closed billiard trajectory different from the *Fagnano trajectory* presented in Figure 2. For a general obtuse triangle we do not know whether it has any closed billiard trajectories at all! (See papers of Richard Schwartz for recent results in this direction.)

From Billiards in Polygons to Flat Surfaces

Let us discuss now how billiards in rational polygons (polygons having angles rational multiples of π) give rise to surfaces and how billiard trajectories unfold to straight lines as in Figure 3 (see p. 1242).

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[1]*This paper uses some extracts from publications of the second author in* Gazette des Mathèmaticiens ***142** and **154**.*

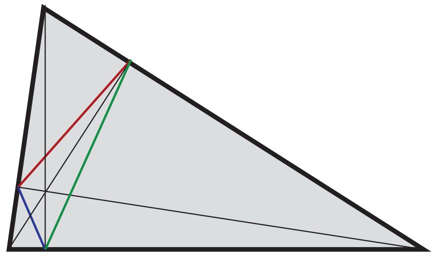

Figure 2. Every acute triangle has a closed Fagnano trajectory, joining the bases of the three heights. It is not known whether it always has another.

Consider a rectangular billiard and launch a billiard ball. When the ball hits the wall we can reflect the billiard table instead of letting the ball bounce from the wall. The trajectory unfolds to a straight line. Folding back the copies of the billiard table we project this line to the original trajectory. Note that at any moment the ball moves in one of the four directions defining the four types of copies of the billiard table. Copies of the same type are related by a parallel translation respecting the labeling of the corners.

Identifying the equivalent patterns by a parallel translation we obtain a torus; the billiard trajectory unfolds to a straight line on the corresponding torus, as in Figure 4 (see p. 1242).

One can apply a similar unfolding construction to any polygon with angles which are rational multiples of π to pass from a billiard to a flat surface.

Consider, for example, the triangle with angles $3\pi/8, 3\pi/8, \pi/4$, as in Figure 5 (see p. 1242). It is easy to check that a generic trajectory of such billiard moves at any time in one of 8 directions (compared to 4 for a rectangle). We can unfold the triangle to a regular octagon glued from 8 copies of the triangle. Identifying opposite sides of the octagon we get a flat surface. All straight lines on this surface project to the initial billiard trajectories.

The geometry of flat surfaces provides important information on billiards. For example, it was proved by H. Masur that rational billiards have plenty of closed trajectories. Closed trajectories appear in bands of parallel trajectories of the same length as in Figure 6 (see p. 1242). To quantify the number of closed billiards trajectories in a rational polygon one counts the number of such bands. It is natural to take into account how thick the band is by giving the band a weight equal to the ratio of the area of the band over the area of the billiard table. When some regions of the billiard table are covered by the band several times, we count their contribution to the area of the band with the corresponding multiplicity. In other words, the area of the band is measured on the flat surface obtained after unfolding the rational billiard as in Figure 10 (see p. 1245).

The results of Masur imply that the number $N(\Pi, L)$ of closed trajectories of length at most L in a rational polygon Π admits lower and upper bounds

$$\liminf_{L\to+\infty} \frac{N(\Pi, L)}{L^2} = c(\Pi) > 0, \limsup_{L\to+\infty} \frac{N(\Pi, L)}{L^2} = C(\Pi).$$

It is not known, however, whether the number of closed trajectories in every rational polygon Π admits exact quadratic asymptotics, i.e., with $c(\Pi) = C(\Pi)$. We are not even aware of an algorithm which produces realistic estimates of the constants $c(\Pi)$ and $C(\Pi)$ for a general rational polygon Π.

Translation Surfaces

We have seen that unfolding a billiard in a rectangle gives us a flat torus. Unfolding more complicated rational polygons gives us more complicated translation surfaces. We present now a construction of a general translation surface, not necessarily related to a billiard.

Consider a collection of vectors $\vec{v}_1, \dots, \vec{v}_n$ in $\mathbb{R}^2$ and arrange these vectors into a broken line as in Figure 7 (see p. 1242). Construct another broken line starting at the same point as the first one arranging the same vectors in the order $\vec{v}_{\pi(1)}, \dots, \vec{v}_{\pi(n)}$, where π is some permutation of n elements. By construction the two broken lines share the same endpoints; suppose that they bound a polygon as in Figure 7. Identifying the pairs of sides corresponding to the same vectors $\vec{v}_j$, $j = 1, \dots, n$, by parallel translations we obtain a closed topological surface.

By construction, the surface is endowed with a flat metric. When $n = 2$ and $\pi = (2, 1)$ we get a usual flat torus glued from a parallelogram. For a larger number of elements we might get a surface of higher genus, where the genus is determined by the permutation π. It is convenient to impose from now on some simple restrictions on the permutation π which guarantee nondegeneracy of the surface; see the original *Annals of Mathematics* papers of H. Masur and W. Veech for details.

For example, a regular octagon gives rise to a surface of genus two as in Figure 8 (p. 1243). Indeed, identifying pairs of horizontal and vertical sides of a regular octagon we get a usual torus with a hole in the form of a square. We slightly cheat in the next frame, where we turn this hole by 45° and only then glue the next pair of sides. As a result we get a torus with two isolated holes as on the third frame. Identifying the remaining pair of sides (which represent the holes) we get a torus *with a handle* or, in other words, a surface of genus two.

Similar to the torus case, the surface glued from the regular octagon or from an octagon as in Figure 7 also inherits from the polygon a flat metric, but now the resulting flat metric has a singularity at the point obtained from identified vertices of the octagon.

Note that the flat metric thus constructed is very special: since we identify the sides of the polygon only by translations, the parallel transport of any tangent vector along a closed cycle (avoiding conical singularities) on the resulting surface brings the vector back to itself. In other words, our flat metric has *trivial holonomy*. In particular, since a parallel transport along a small loop around any conical singularity brings the vector to itself, the cone

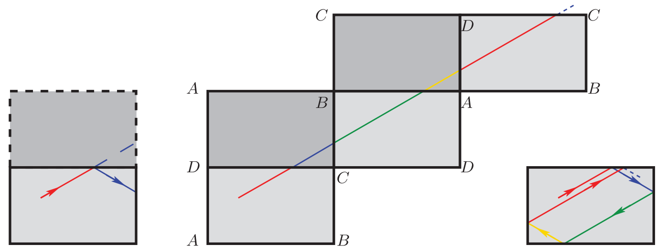

Figure 3. A billiard trajectory (right) can be unfolded to a straight line.

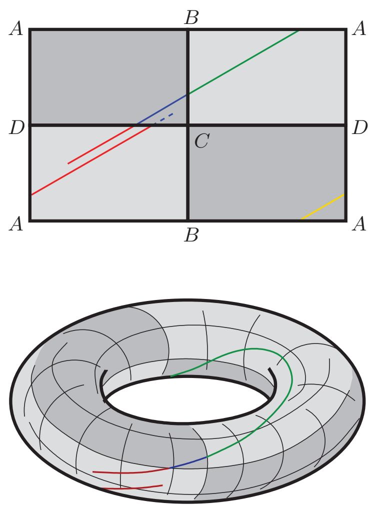

Figure 4. An unfolded billiard trajectory is a straight line on the flat torus.

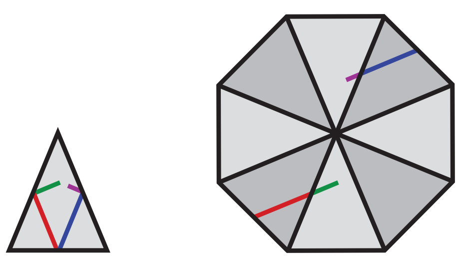

Figure 5. Billiard trajectories on the descriptor $\frac{3\pi}{8}$-$\frac{3\pi}{8}$-$\frac{\pi}{4}$ triangle unfold to straight lines on the octagon with opposite sides identified.

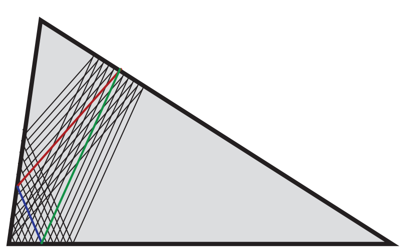

Figure 6. Closed trajectories on the triangle appear in parallel bands.

angle at any singularity is an integer multiple of 2π. In the most general situation the flat surface of genus g would have several conical singularities with cone angles $2\pi(d_1+1), \ldots, 2\pi(d_m+1)$, where $d_1 + \cdots + d_m = 2g-2$.

It is convenient to consider the vertical direction as part of the structure. A surface endowed with a flat metric with trivial holonomy and with a choice of a vertical direction is called a *translation surface*. Two polygons in the plane obtained one from another by a parallel translation give rise to the same translation surface, while polygons obtained one from another by a nontrivial rotation (usually) give rise to distinct translation surfaces.

We can assume that the polygon defining our translation surface is embedded into the complex plane $\mathbb{C} \simeq \mathbb{R}^2$ with coordinate z. The translation surface obtained by identifying the corresponding sides of the polygon inherits the complex structure. Moreover, since the gluing rule for the sides can be expressed in local coordinates as $z = \tilde{z} + \text{const}$, the closed 1-form $dz = d(\tilde{z} + \text{const})$ is well-defined not only in the polygon but on the surface. An exercise in complex analysis shows that the complex structure extends to the points coming from the vertices of the polygon and that the 1-form $\omega = dz$ extends to the holomorphic 1-form on the resulting Riemann surface. This 1-form ω has zeroes of degrees $d_1, \ldots, d_m$ exactly at the points where the flat metric has conical singularities of angles $2\pi(d_1+1), \ldots, 2\pi(d_m+1)$.

Conversely, given a holomorphic 1-form ω on a Riemann surface one can always find a local coordinate z (in a simply-connected domain not containing zeroes of ω) such that $\omega = dz$. This coordinate is defined up to an

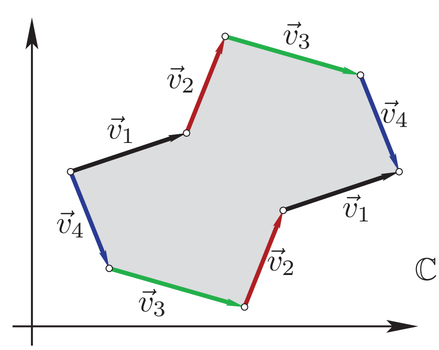

Figure 7. Identifying corresponding pairs of sides of this polygon we obtain a flat surface of genus two with a single conical singularity.

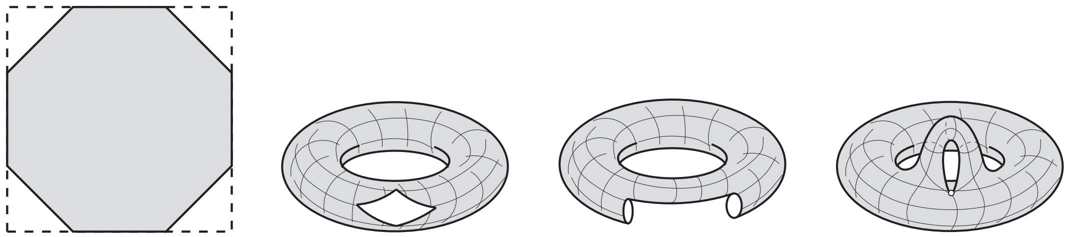

Figure 8. Identifying opposite sides of a regular octagon gives rise to a surface of genus two.

additive constant. It defines the translation structure on the surface. Cutting up the surface along an appropriate collection of straight segments joining conical singularities, we can unwrap the Riemann surface into a polygon as above. Polygons that differ by cut and paste correspond to the same ω; we will elaborate on this later.

This construction shows that the two structures are completely equivalent: the flat metric with trivial holonomy plus a choice of distinguished direction or a Riemann surface endowed with a nonzero holomorphic 1-form.

Families of Translation Surfaces and Dynamics in the Moduli Space

The polygon in our construction depends continuously on the vectors $\vec{v}_i$. This means that the topology of the resulting translation surface (its genus g, the number and the types of the resulting conical singularities) does not change under small deformations of the vectors $\vec{v}_i$. For every collection of cone angles $2\pi(d_1+1), \dots, 2\pi(d_m+1)$ satisfying $d_1 + \dots + d_m = 2g-2$ with integer d_i for $i = 1, \dots, n$, we get a *space* $\mathcal{H}(d_1, \dots, d_m)$ *of translation surfaces*. Vectors $\vec{v}_1, \dots, \vec{v}_n$ can be viewed as local complex coordinates on this space, called *period coordinates*. These coordinates define a structure of a *complex orbifold* (manifold with moderate singularities) on each space $\mathcal{H}(d_1, \dots, d_m)$. The geometry and topology of spaces of translation surfaces is not yet sufficiently explored.

Readers preferring algebro-geometric language may view the space of translation surfaces with fixed conical singularities $2\pi(d_1+1), \dots, 2\pi(d_m+1)$ as the stratum $\mathcal{H}(d_1, \dots, d_m)$ in the moduli space $\mathcal{H}_g$ of pairs (Riemann surface C; holomorphic 1-form ω on C), where the stratum is specified by the degrees $d_1, \dots, d_m$ of zeroes of ω, where $d_1 + \dots + d_m = 2g-2$. Note that while the moduli space $\mathcal{H}_g$ is a holomorphic $\mathbb{C}^g$-bundle over the moduli space $\mathcal{M}_g$ of Riemann surfaces, individual strata are not. For example, the minimal stratum $\mathcal{H}(2g-2)$ has complex dimension $2g$, while the moduli space $\mathcal{M}_g$ has complex dimension $3g-3$. The very existence of a holomorphic form with a single zero of degree $2g-2$ on a Riemann surface C is a strong condition on C.

To complete the description of the space of translation surfaces we need to present one more very important structure: the action of the group $\mathrm{GL}(2,\mathbb{R})$ on $\mathcal{H}_g$ minus the zero section. This action preserves strata. The description of this action is particularly simple in terms of our polygonal model Π of a translation surface S. A linear transformation $g \in \mathrm{GL}(2,\mathbb{R})$ of the plane maps the polygon Π to a polygon $g\Pi$. The new polygon again has all sides arranged into pairs, where the two sides in each pair are parallel and have equal length. We can glue a new translation surface and call it $g \cdot S$. It is easy to see that unwrapping the initial surface into different polygons would not affect the construction. Note also that we explicitly use the choice of the vertical direction: any polygon is endowed with an embedding into $\mathbb{R}^2$ defined up to a parallel translation.

The subgroup $\mathrm{SL}(2,\mathbb{R}) \subset \mathrm{GL}(2,\mathbb{R})$ preserves the flat area. This implies that the action of $\mathrm{SL}(2,\mathbb{R})$ preserves the real hypersurface $\mathcal{H}_1(d_1, \dots, d_m)$ of translation surfaces of area one in any stratum $\mathcal{H}(d_1, \dots, d_m)$. The codimension-one subspace $\mathcal{H}_1(d_1, \dots, d_m)$ can be compared to the unit hyperboloid in $\mathbb{R}^{2n}$.

Recall that under appropriate assumptions on the permutation π, the n vectors

$$\vec{v}_1 = \begin{pmatrix} v_{1,x} \\ v_{1,y} \end{pmatrix}, \dots, \vec{v}_n = \begin{pmatrix} v_{n,x} \\ v_{n,y} \end{pmatrix}$$

as in Figure 7 define local coordinates in the space $\mathcal{H}(d_1, \dots, d_m)$ of translation surfaces. Let $dv := dv_{1x}dv_{1y} \dots dv_{nx}dv_{ny}$ be the associated volume element in the corresponding coordinate chart $U \subset \mathbb{R}^{2n}$. It is easy to verify that dv does not depend on the choice of coordinates, $\vec{v}_1, \dots, \vec{v}_n$, so it is well-defined on $\mathcal{H}(d_1, \dots, d_m)$. Similarly to the case of Euclidean volume element, we get a natural induced volume element dv_1 on the unit hyperboloid $\mathcal{H}_1(d_1, \dots, d_m)$. It is easy to check that the action of the group $\mathrm{SL}(2,\mathbb{R})$ preserves the volume element dv_1.

The following theorem proved independently and simultaneously by Masur and Veech is the keystone of this area. (The two papers were published in the same volume of *Annals of Mathematics* in 1982).

Theorem (H. Masur, W. A. Veech)**.** *The total volume of every stratum* $\mathcal{H}_1(d_1, \dots, d_m)$ *is finite.*

The group $\mathrm{SL}(2,\mathbb{R})$ *and its diagonal subgroup act ergodically on every connected component of every stratum* $\mathcal{H}_1(d_1, \dots, d_m)$.

Here "ergodically" means that every measurable subset invariant under the action of the group has necessarily measure zero or full measure. The Birkhoff Ergodic Theorem then implies that the orbit of almost every point homogeneously fills the ambient connected component. In plain terms, a consequence of the ergodicity of the action

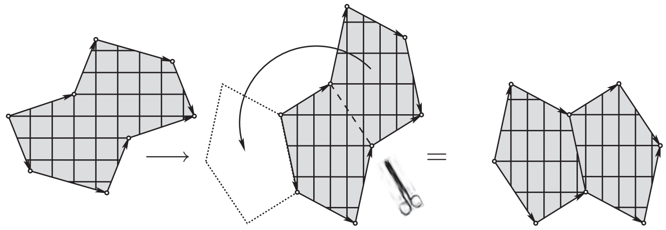

Figure 9. Expansion-contraction, combined with cut-and-paste transformations, approximates any other prescribed octagon.

of the diagonal subgroup can be interpreted as follows. Having almost any polygon as above, we can choose an appropriate sequence of times t_i such that contracting the polygon horizontally with a factor e^{t_i} and expanding it vertically with the same factor e^{t_i} and modifying the resulting polygonal pattern of the resulting translation surface by an appropriate sequence of cut-and-paste transformations (see Figure 9) we can get arbitrarily close to, say, a regular octagon rotated by any angle chosen in advance. Note that expansion-contraction (action of the diagonal group) changes the translation surface, while cut-and-paste transformations change only the polygonal pattern and do not change the flat surface.

Now everything is prepared to present the first marvel of this story. Suppose that we want to find out some fine properties of the vertical flow on an individual translation surface. Applying a homothety we can assume that the translation surface has area one. Masur and Veech suggest the following approach. Consider our translation surface (endowed with a vertical direction) as a point S in the ambient stratum $\mathcal{H}_1(d_1, \dots, d_m)$. Consider the orbit $\mathrm{SL}(2,\mathbb{R}) \cdot S$ (or the orbit of S under the action of the diagonal subgroup $\begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}$ depending on the initial problem). Numerous important properties of the initial vertical flow are encoded in the geometrical properties of the closure of the corresponding orbit. This approach places the problem of finding the orbit closures under the action of $\mathrm{SL}(2,\mathbb{R})$ and studying their geometry at the center of the work in this area for the last three decades.

At first glance, we have just reduced the study of a rather simple dynamical system, namely, the vertical flow on a translation surface, to a really complicated one, the study of the action of the group $\mathrm{GL}(2,\mathbb{R})$ on the space $\mathcal{H}(d_1, \dots, d_m)$. Nevertheless, this approach proved to be extremely fruitful, despite the fact that geometry and topology of spaces of translation surfaces is still under exploration. It is the Magic Wand Theorem of A. Eskin, M. Mirzakhani, and A. Mohammadi which made this approach particularly powerful.

"Almost All" versus "All"

From the dynamical point of view, the moduli space of holomorphic differentials can be viewed as a "homogeneous space with difficulties." We are citing Eskin, who knows both facets very well: how the dynamics on the moduli space might mimic the homogeneous dynamics in some situations and how deep the difficulties might be.

The rigidity theorems including and generalizing the theorems proved by Marina Ratner at the beginning of the 1990s show why homogeneous dynamics is so special. General dynamical systems usually have some very peculiar trajectories living in very peculiar fractal subsets. Such trajectories are rare, but there are still plenty of them. In particular, the question of identification of all (versus almost all) orbit closures or of all invariant measures makes no sense for most dynamical systems: the jungle of exotic trajectories is too large. In certain situations this diversity creates a major difficulty: even when you know plenty of fine properties of the trajectory launched from almost every starting point, you have no algorithm to check whether the particular initial condition you are interested in is generic or not. Ergodic theory answers statistical questions but says nothing about specific initial data.

Consider, for example, the geodesic flow on a compact Riemannian surface of constant negative curvature. We get a very nice dynamical system on the three-dimensional total space of the unit tangent bundle. It was observed long ago by H. Furstenberg and B. Weiss that the closures of individual geodesics might have almost arbitrary Hausdorff dimension from 1 (closed trajectories) to 3 (typical trajectories).

The situation in homogeneous dynamics is radically different. In certain favorable cases (such as certain unipotent flows) one manages to prove that *every* orbit closure is a nice homogeneous space, *every* invariant measure is the corresponding Haar measure, etc. This kind of rigidity gives rise to fantastic applications to number theory.

For several decades it was not clear to what extent the dynamics of the $\mathrm{SL}(2,\mathbb{R})$ action on the moduli space of Abelian and quadratic differentials resembles homogeneous dynamics. For Eskin, who came to the dynamics in the moduli space from homogeneous dynamics, it was, probably, the main challenge for fifteen years. Mirzakhani joined him in working on this problem about 2006. She was challenged by the result of her scientific advisor, C. McMullen, who had solved the problem in the particular case of genus two. After several years of collaboration, the first major part of the theorem, namely, the measure classification for $\mathrm{SL}(2,\mathbb{R})$-invariant measures, was proved in 2010.

To illustrate the importance of this theorem we can cite what Artur Avila has said about this result of Eskin and Mirzakhani to S. Roberts for the *New Yorker* article in memory of Mirzakhani:

> Upon hearing about this result, and knowing her earlier work, I was certain that she would be a front-runner for the Fields Medals to be given in 2014, so much so that I did not expect to have much of a chance.

We do not think that Maryam thought much about the Fields Medal at this time. Several years later she took the email message from I. Daubechies announcing that Maryam got the Fields Medal for a joke and just ignored it. But she certainly knew how important the theorem was. We'd say that most recent papers in our domain have used this Magic Wand Theorem in one way or another.

It took Eskin and Mirzakhani another several years of extremely hard work to extend their result from $\mathrm{SL}(2,\mathbb{R})$ to its subgroup of upper-triangular 2×2 matrices. The difference might seem insignificant, but exactly this difference is needed for the most powerful version of the Magic Wand Theorem.

Magic Wand Theorem

Now comes the Magic Wand Theorem of Eskin, Mirzakhani [EMi], and Mohammadi [EMiMh]. (A. Mohammadi joined the collaboration for the part of the theorem concerning orbit closures.)

As we have already mentioned, the moduli space is not a homogeneous space. Nevertheless, the orbit closures of $\mathrm{GL}(2,\mathbb{R})$ in the space of translation surfaces are as nice as one can only hope: they are complex manifolds, possibly with very moderate singularities (so-called "orbifolds"). In this sense the action of $\mathrm{GL}(2,\mathbb{R})$ and $\mathrm{SL}(2,\mathbb{R})$ on the space of translation surfaces mimics certain properties of the dynamical systems in homogeneous spaces.

Magic Wand Theorem (A. Eskin, M. Mirzakhani, and A. Mohammadi [EMiMh])**.** *The closure of any* $\mathrm{GL}(2,\mathbb{R})$*-orbit is a complex suborbifold (possibly with self-intersections). In period coordinates* $\vec{v}_1,\dots,\vec{v}_n$ *in the corresponding space* $\mathcal{H}(d_1,\dots,d_m)$ *of translation surfaces it is locally represented by a linear subspace.*

Every ergodic $\mathrm{SL}(2,\mathbb{R})$*-invariant measure is supported on a suborbifold. In coordinates* $\vec{v}_1,\dots,\vec{v}_n$ *this suborbifold is represented by the intersection of the hypersurface* $\mathcal{H}_1(d_1,\dots,d_m)$ *of translation surfaces of area one with a linear subspace, and the invariant measure is induced from the usual Lebesgue measure on this subspace.*

As a vague conjecture (or optimistic dream) this property was discussed long ago, and there was not the slightest hint of a general proof. The only exception is the case of surfaces of genus two, for which ten years ago C. McMullen proved a very precise statement, classifying all possible orbit closures. He used a special case of Ratner's results which are applicable here. And he, very ingeniously, used the special properties of surfaces of genus two.

The proof of the Magic Wand Theorem is a titanic work, which absorbed numerous fundamental recent developments in dynamical systems. Most of these developments do not have any direct relation to moduli spaces. It incorporates certain ideas of the low entropy method of M. Einsiedler, A. Katok, and E. Lindenstrauss; results of G. Forni and of M. Kontsevich on Lyapunov exponents of the Teichmüller geodesic flow; the ideas from the works of Y. Benoit and J.-F. Quint on stationary measures; iterative improvement of the properties of the invariant measure inspired by the approach of G. Margulis and G. Tomanov to the actions of unipotent flows on homogeneous spaces; some fine ergodic results due to Y. Guivarch and A. Raugi; and many others.

For many experts in the area it remains a miracle that Eskin and Mirzakhani managed to accomplish this project. Very serious technical difficulties appeared at every stage of the project. Not to mention that in the four years between 2010 and 2014 Maryam gave birth to a daughter and overcame the first attack of cancer. Since then we believed that Maryam could do anything.

The theorem itself really works like a Magic Wand, which allows one to touch a given billiard and obtain precious information describing its geometry and dynamics.

This Magic Wand works best if the corresponding orbit closure in the moduli space of translation surfaces can be computed. Unfortunately, until recently, the orbit closures associated to most billiards remained impossible to calculate. As the result of recent progress stemming from the Magic Wand Theorem, this situation is undergoing rapid improvement, some of which is witnessed by one of the last results of Mirzakhani [MiW]:

Theorem (M. Mirzakhani and A. Wright[MiW])**.** *There are infinitely many triangular billiards that unfold to translation surfaces whose* $\mathrm{GL}(2,\mathbb{R})$*-orbit is dense in the ambient stratum of flat surfaces.*

For example, the triangle Δ with angles $\left(\frac{2\pi}{11},\frac{4\pi}{11},\frac{5\pi}{11}\right)$ unfolds to a surface whose orbit is dense in $\mathcal{H}(1,3,4)$. A simple consequence for billiards is the existence of bands of parallel periodic trajectories which cover the

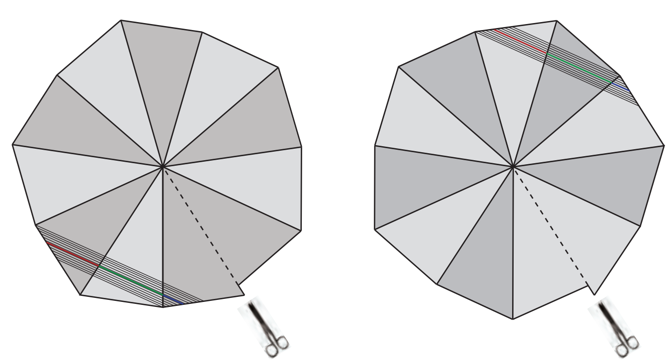

Figure 10. There are bands of closed trajectories which cover an arbitrarily large part of the 22-fold unfolding of the $\frac{2\pi}{11}$– $\frac{4\pi}{11}$– $\frac{5\pi}{11}$ triangle (obtained by sewing the above images along the dotted cut).

table almost 22 times. In other words, one can find bands of parallel closed geodesics as in Figure 10 covering an arbitrarily large part of the flat surface obtained by unfolding the triangle Δ. This follows directly from the density of the orbit, because there is an open set of translation surfaces in the ambient stratum with a band of parallel geodesics covering an arbitrarily large part of the surface.

This also gives a much more precise counting result for the number $N(\Delta, L)$ of bands of parallel periodic billiard trajectories of length less than L, weighted by the width of the band. Under some additional averaging one has

$$N(\Delta, L) \sim \frac{3350523}{760400\,\pi} \cdot \frac{L^2}{\text{area of the billiard table}}.$$

Here the constant in front of L^2 (the *Siegel–Veech constant*) is a function of the geometry of the orbit closure, so a billiard giving a different orbit closure would have a different constant.

There are only countably many $\mathrm{GL}(2,\mathbb{R})$-orbit closures. Each parameterizes surfaces with exceptional flat geometry, and by work of Filip, each is an algebraic variety that can be defined in terms of special properties of the Jacobian. Despite important progress, in general it remains an open problem to classify $\mathrm{GL}(2,\mathbb{R})$-orbit closures.

At first, the proof of the Magic Wand Theorem seemed to use the special properties of the $\mathrm{GL}(2,\mathbb{R})$ action on strata of translation surfaces. However, the techniques have already proved very useful far outside their original setting. For example, they have been applied to random dynamics on surfaces by Brown and Rodriguez Hertz and to homogeneous spaces by Eskin and Lindenstrauss.

In fact, we use not only the results but also the ideas of Mirzakhani all the time. Rereading her papers often echoes something which you were thinking about for a long time and all of a sudden it reveals a simple and unexpected solution. Mirzakhani foresaw many beautiful further results but did not have time to work on them. We often feel that we are following the paths and trails which she imagined and outlined.

Epilogue

We would feel insincere limiting ourselves to mathematical results of Maryam. We feel happy to have known Maryam as a person. Alex spent the last three years in Stanford working with Maryam, speaking with Maryam. We reproduce his recollection presented at the Stanford memorial for Maryam.

I remember vividly the first time I met Maryam in person. It was in her office at Stanford, and I was a graduate student, visiting at her invitation. I sat on the couch opposite the blackboard, which was layered with equations and mathematical doodles. Whenever she needed to write something, she would erase only the smallest portion of the board possible, and squeeze her writing into the small space. She spoke excitedly, sharing her ideas without reservation. Sometimes she repeated herself, sometimes she abandoned a sentence midway through, and her thoughts came out somewhat in a jumble. As her former student Jenya Sapir puts it, she would present different ideas like a cast of characters, all talking to each other. But as I started to grasp the depth of her understanding and vision, I felt awe.

Following the Prize Sessions at the 2009 Joint Mathematics Meetings, Mirzakhani stands with a group of Iranian-American mathematicians, including Ali Enayat, Mojtaba Moniri, Iraj Kalantari, Vali Siadat, and Bahman Kalantari.

Two years later, I was back in that same office, having just moved to Stanford to work with Maryam. She was so eager to get to work that she arranged for me to stay in a hotel for the week before my lease began, so that we could get started that much earlier.

We would discuss mathematics for hours at a time, standing at her blackboard, sometimes pacing in excitement or concentration. She was always optimistic. As her student Ben Dozier recalls, she would find some aspect of any question, even if naive, to be fascinating. Her enthusiasm was infectious.

I would leave our meetings determined, even if the mathematical challenges we faced were daunting. Maryam was not intimidated, and with her working with me, any fear of failure evaporated. I was energized to work on problems I previously would have judged to be too difficult to even attempt.

We had lunch together frequently, always at the business school cafeteria, right next to this auditorium. There our mathematical discussions would pause. Maryam spoke of her daughter, Anahita, and of the joys and challenges of being a parent. She spoke of teaching and complained that she was unable to stick with just one textbook, so that her house would be covered with different textbooks whenever she taught.

She rarely gave advice and spoke in mildly worded suggestions rather than instructions. On career issues, like the best journal to submit a paper to or how to get the best job, she was often silent. Perhaps, as a result of her own meteoric career, she had different firsthand experience with such struggles. But more than that, she radiated a sense of *perspective and focus, as if the adversities she had faced had taught her that which journal accepts a paper is not the most important thing and as if her intellectual curiosity dwarfed academic politics. She once told me: "Know what you want, and don't get distracted."*

She seemed to follow that advice herself. After she was awarded the Fields Medal, she wanted to get right back to thinking deeply. She tried to ignore reporters and the press coverage and seemed unchanged by her newfound recognition and celebrity.

But she did want to help fellow mathematicians, especially young mathematicians. One of the graduate students at Stanford, Pedram Safee, contacted her from Iran as a high school student. They had never met, but she still took the time to reply to him and give him advice on studying mathematics. Recently, Maryam became an editor of one of the most prestigious journals in mathematics. Most papers submitted to that journal get rejected, and the review process is very long. She told me repeatedly, "I just want to accept a paper, I just want to press the accept button."

When Maryam's cancer recurred, she wanted to keep working. Instead of her office, we met at the Coupa Cafe in front of Green Library, closer to her home. Between the meetings, now much less frequent, I was sad and worried, knowing that Maryam's medical situation was dire. But when I saw her in person, even when she spoke of her struggle, I felt almost elated. Maryam was still herself, still curious, brilliant, perseverant, and brave, and her optimism would make me feel, if only briefly, as if she would surely be ok.

Now, as we mourn her, I want to remember the beauty and power of her mathematics and the force of her personality, which could convince me, even as she was in pain, to smile.

For more on the the Magic Wand Theorem, see [W]. If you want to learn more about Maryam as a person, read the article by Erica Klarreich [K] in *Quanta*.

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Earlier this year, the American Mathematical Society governance developed and approved two ways by which to commemorate Maryam Mirzakhani and her mathematical legacy.

The Maryam Mirzakhani Lecture will be an annual AMS Invited Address at the Joint Mathematics Meetings (JMM). The lecture will allow leading scholars to present their research, while commemorating the exceptional accomplishments of Maryam Mirzakhani. *The first lecture will be at the 2020 JMM in Denver.*

The Maryam Mirzakhani Fund for The Next Generation is an endowment that exclusively supports programs for early career mathematicians; i.e., doctoral and postdoctoral scholars. It is part of a special initiative and aims to assist rising scholars each year at modest but impactful levels. A donation to the Maryam Mirzakhani Fund honors her memory by supporting emerging mathematicians now and in the future.

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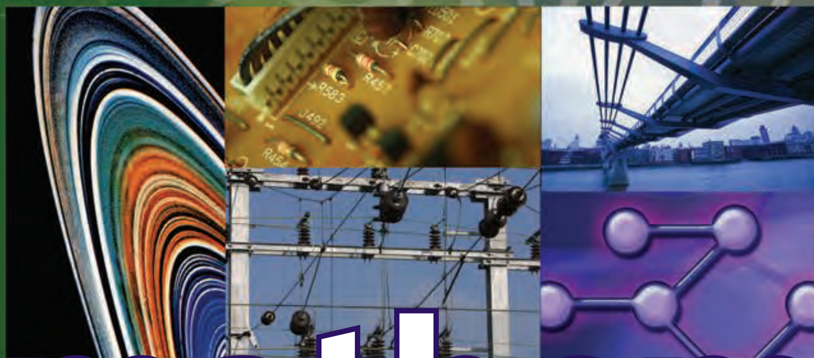

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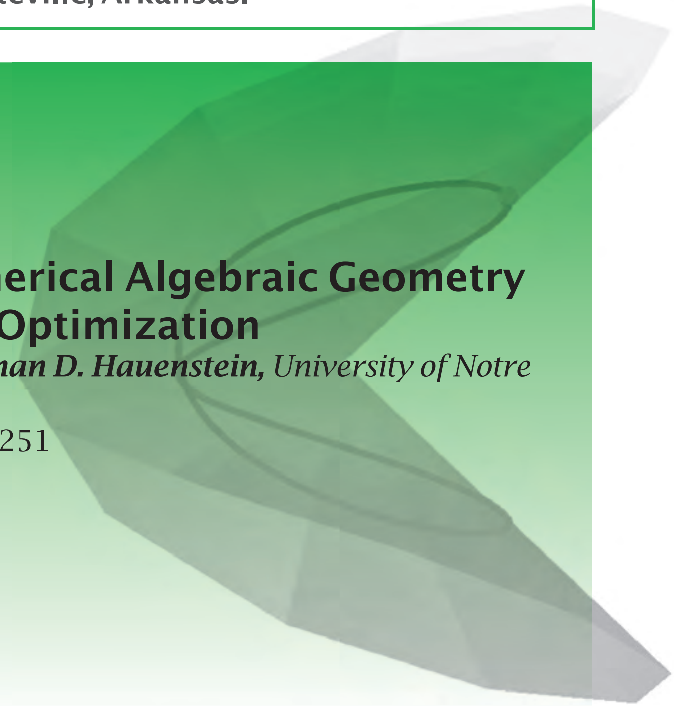
SOUTHEASTERN SECTIONAL SAMPLER

In this sampler, the speakers below have kindly provided introductions to their Invited Addresses for the AMS Fall Southeastern Sectional taking place November 3–4 at University of Arkansas, Fayetteville, Arkansas.



Numerical Algebraic Geometry and Optimization

Jonathan D. Hauenstein, *University of Notre Dame*

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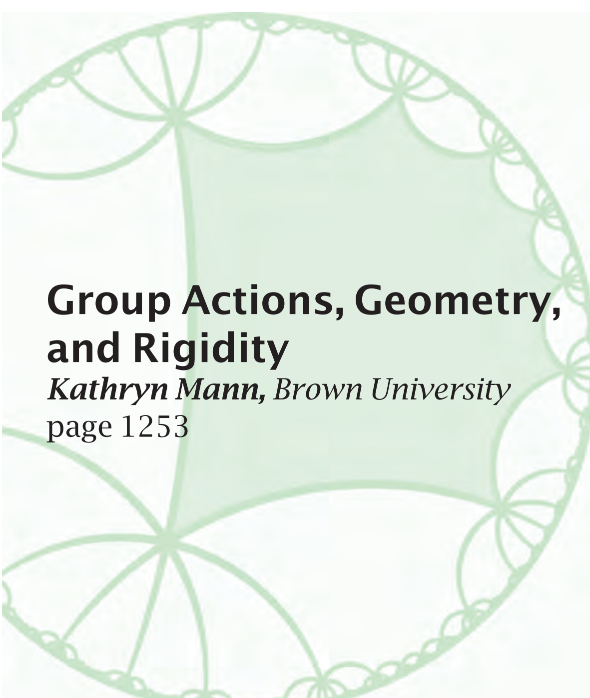

Group Actions, Geometry, and Rigidity

Kathryn Mann, *Brown University*

page 1253

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Jonathan D. Hauenstein

Numerical Algebraic Geometry and Optimization

Convex programming aims to minimize a convex objective function over a convex set, called the feasible set. For example, linear programming minimizes a linear function over a polytope (intersection of finitely many linear half-spaces as in Figure 1(a)) while semidefinite programming minimizes a linear function over a spectrahedron (intersection of the cone of positive semidefinite matrices with a linear space as in Figure 1(b)).

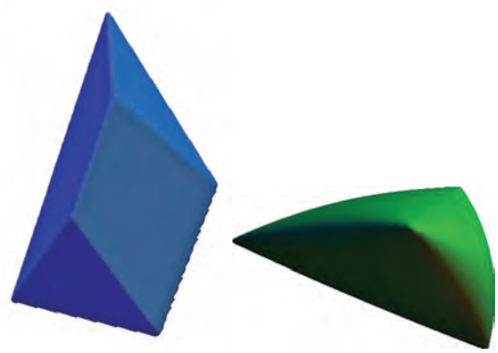

Figure 1. Example of (a) a polytope and (b) a spectrahedron.

When the feasible set has a nonempty interior, a standard approach for solving convex programs are interior point methods. Conversely, when the feasible set is empty, the program is said to be infeasible and the traditional Farkas' lemma is a standard approach for verifying infeasibility. For example, every infeasible linear program can be verified using the traditional Farkas' lemma. However, there are so-called weakly infeasible semidefinite programs where this is not the case. To illustrate, consider the following semidefinite program:

$$\text{(1)} \qquad \begin{array}{ll} \text{minimize} & x_{11} \\ \text{subject to} & \left[\begin{array}{cc} x_{11} & 1 \\ 1 & 0 \end{array}\right] \succeq 0 \end{array}$$

where $A \succeq 0$ means that A is a positive semidefinite matrix. Since the determinant of the matrix in (1) is -1, the program (1) is clearly infeasible. Moreover, (1) is weakly infeasible since the corresponding alternative via the traditional Farkas' lemma is also infeasible, i.e., there does not exist $y \in \mathbb{R}^2$ such that

$$\begin{array}{rcl} \left[\begin{array}{cc} 0 & y_1 \\ y_1 & y_2 \end{array}\right] & \succeq & 0 \\ 2 \cdot y_1 + 0 \cdot y_2 & = & -1. \end{array}$$

One numerical challenge in identifying weakly infeasible semidefinite programs is that perturbations can be strongly infeasible or strictly feasible. For example,

$$\begin{array}{ll} \text{minimize} & x_{11} \\ \text{subject to} & \left[\begin{array}{cc} x_{11} & 1 \\ 1 & \epsilon \end{array}\right] \succeq 0 \end{array}$$

is strongly infeasible for $\epsilon < 0$ and strictly feasible for $\epsilon > 0$. Liu and Pataki [3] showed that many commonly-used software packages in semidefinite programming have difficulty identifying weakly infeasible semidefinite programs when the reason for infeasibility is not trivially obvious. Such *messy* instances were obtained by obscuring their structure via row operations and rotations. Thus, a change of perspective was needed for identifying weakly infeasible semidefinite programs.

Using the lens of numerical algebraic geometry [1,4], the mathematical foundation of traditional interior point methods is to numerically track a solution path of a homotopy from a point in the interior of the feasible set to an optimizer. With this viewpoint, weakly infeasible semidefinite programs can be identified [2] using the following three techniques from numerical algebraic geometry: projective space for compactifying infinite length solution paths, adaptive precision path tracking for navigating through ill-conditioned areas, and endgames for accurately computing singular endpoints.

To illustrate, we consider the following convex program modified from (1):

$$\text{(2)} \qquad \begin{array}{ll} \text{minimize} & \lambda \\ \text{subject to} & \left[\begin{array}{cc} x_{11}+\lambda & 1 \\ 1 & \lambda \end{array}\right] \succeq 0. \end{array}$$

The corresponding optimal value is easily observed to be $\lambda^* = 0$, but this is actually an infimum that is not attained as a minimum, a condition that is equivalent to (1) being weakly infeasible. Therefore, optimizers to (2) are "at infinity" meaning that a solution path defined by traditional interior point methods will have infinite length and approach an asymptote as represented in Figure 2(a). Compactification using projective space yields a finite length path that can be efficiently tracked as represented in Figure 2(b).

Complex analysis enters the scene to accurately compute the endpoint. The winding number (also called the cycle number) of the endpoint for the path displayed in Figure 2(b) is 2, meaning that the path over the complex numbers locally behaves like the complex square root function. Hence, the Cauchy integral theorem can be used to compute the endpoint of this path by integrating along

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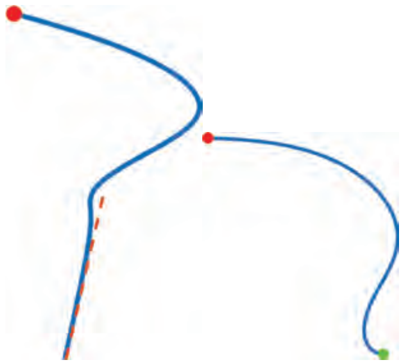

(a) (b)

Figure 2. (a) A plot of paths at a given (red) point may have infinite length with limiting asymptote corresponding with $\lambda^* = 0$.
(b) Compactification using projective space yields a finite-length path that can be efficiently tracked.

(a) (b)

Figure 3. To compute the endpoint of a path as in Figure 2(b), one uses the Cauchy integral theorem and integrates along a closed loop like the one with winding number 2 with real (a) and imaginary (b) parts pictured here.

a closed loop as shown in Figure 3. Due to periodicity, numerical integration by the trapezoid rule is exponentially convergent [5]. Such a procedure for computing the endpoint is called the Cauchy endgame. Since any endpoint with winding number larger than 1 is necessarily singular, ill-conditioning that necessarily arises near the endpoint can be controlled using adaptive precision path tracking methods.

This viewpoint for identifying weakly infeasible semidefinite programs using numerical algebraic geometry and the software package `Bertini` [1] along with several other interactions of numerical algebraic geometry and optimization will be discussed in Arkansas.

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Jonathan D. Hauenstein

ABOUT THE AUTHOR

Jonathan D. Hauenstein has been honored with a DARPA Young Faculty Award, Sloan Research Fellowship, Army Young Investigator Award, and Office of Naval Research Young Investigator Award. Outside of mathematics, he enjoys spending time with his wife and four daughters.

Kathryn Mann

Group Actions, Geometry, and Rigidity

Classical representation theory is concerned with representations of discrete groups into Lie groups. In topological or smooth dynamics we are concerned with representations of discrete groups into the group of *homeomorphisms* or *diffeomorphisms* of a manifold M, and the behavior of these representations under perturbation or deformation. Our recent result says that rigidity can arise only in certain geometric ways.

Rigidity

A representation ρ of a discrete group Γ into a topological group G is *rigid*, loosely speaking, if it has no non-obvious deformations. "Obvious deformations" arise by conjugacy: if $\rho : \Gamma \to G$ is a representation and g_t a path based at the identity in G, then $\gamma \mapsto g_t \rho(\gamma) g_t^{-1}$ gives a continuous path of representations starting at ρ. Thus, one way to formalize the notion of rigidity is to define ρ to be rigid if it is an *isolated point* in the space of representations up to conjugacy, $\mathrm{Hom}(\Gamma, G)/G$.

What is remarkable is that such examples exist at all. Perhaps the most famous rigidity result—and the first theorem that I remember being truly astounded by as a graduate student—is *Mostow rigidity.* In geometric language, it says that a compact manifold of dimension at least 3 admits at most one hyperbolic structure. In representation-theoretic language, it states that the inclusion $\Gamma \to SO(n,1)$ of a co-compact lattice Γ into the Lie group $SO(n,1)$, for $n \geq 3$, is *rigid* in the sense above.

Mostow rigidity completely fails in dimension 2; in fact a genus g compact surface has a much studied $(6g-6)$-dimensional moduli space of hyperbolic structures. My talk is about how to recover rigidity by passing to the *nonlinear*, dynamical setting of groups of homeomorphisms.

Geometry and Group Actions

The story begins with hyperbolic structures on surfaces. If Σ_g is a surface of genus $g \geq 2$, equipped with a hyperbolic structure, then the universal cover $\tilde{\Sigma}_g$ can be identified with the hyperbolic plane and $\pi_1(\Sigma_g)$ with a subgroup of the isometry group $SO(2,1) \cong PSL(2,\mathbb{R})$. The hyperbolic plane has a natural compactification—in the Poincaré disk model depicted in Figure 1, the compactification adds the circle at the boundary of the open disk—and the action of $PSL(2,\mathbb{R})$ by hyperbolic isometries of the disc extends to an action on $S^1 = \mathbb{R} \cup \{\infty\}$ by Möbius transformations. This is an example of what we call a "geometric" action

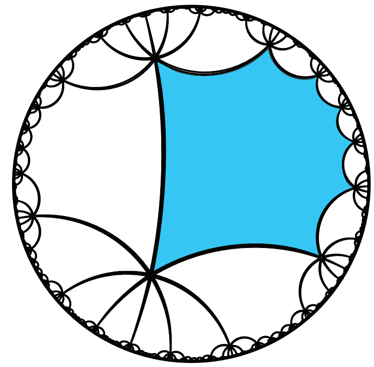

Figure 1. The Poincaré disc model of $\mathbb{H}^2$, tiled by fundamental domains for a genus 2 surface. Hyperbolic isometries extend to the boundary and provide an example of a geometric action.

of $\pi_1(\Sigma_g)$ on the circle. More generally, we say an action of a discrete group Γ on a manifold M is *geometric* if the action $\Gamma \to \mathrm{Homeo}(M)$ factors through an embedding $\Gamma \to G \to \mathrm{Homeo}(M)$, where G is a connected Lie group acting transitively on M, and $\Gamma \subset G$ a co-compact lattice.

It is not difficult to classify all geometric actions of groups on the circle; they are virtually all surface groups, embedded into copies of $PSL(2,\mathbb{R})$ and its central extensions by finite cyclic groups. In earlier work, I showed that these geometric examples were all rigid—they are isolated points in the moduli space of representations of a surface group into $\mathrm{Homeo}(S^1)$. Alternate, independent proofs have since been proposed by S. Matsumoto and J. Bowden.

Hidden Lie Groups

Recently, Maxime Wolff and I proved the remarkable converse: *if $\rho : \pi_1(\Sigma_g) \to \mathrm{Homeo}(S^1)$ is rigid, then ρ is geometric.*[1] In other words, an underlying geometric structure is the *only* source of dynamical rigidity for surface groups acting by homeomorphisms on the circle.

This result is much more difficult than the original "geometric implies rigid" direction. In that first direction, one is given a geometric representation ρ—which can be written down completely explicitly—and one just needs to show that it is stable under perturbation. For the converse, one starts with a completely mysterious representation, save for the knowledge that whatever it is, it can't be deformed. From there, the goal is to conjure up an ambient Lie group.

The proof uses classical dynamical tools such as the *rotation number* of Poincaré, and various refinements of our own invention, but also mapping class groups and surface topology, a combination theorem for actions admitting Markov-partition-like structures due to Matsumoto, and perspectives borrowed from Calegari, Ghys, and others.

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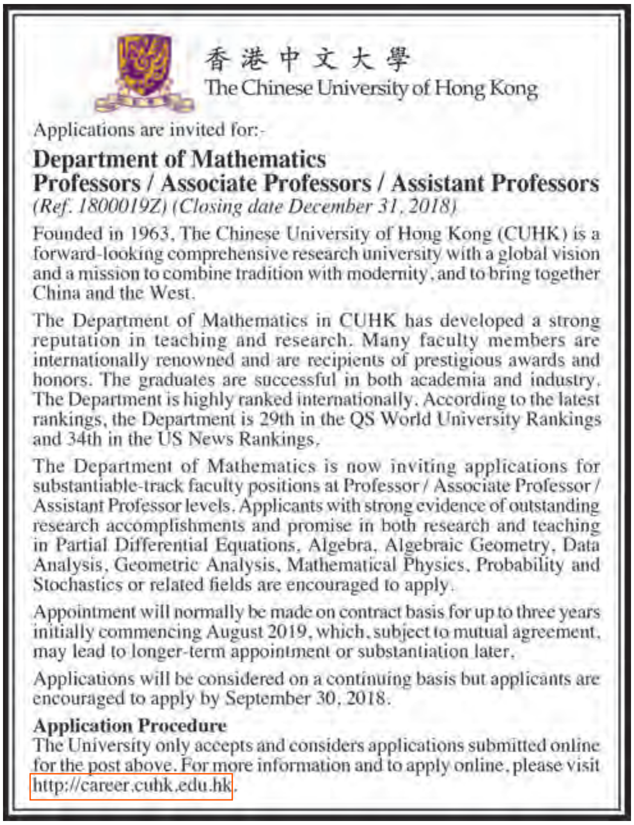
[1] *Technically, this holds after passing to a Hausdorff quotient of the representation space.*



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While the end result has turned into a cohesive narrative, the process felt like a three-year ordeal of "hit it with everything we've got." Fortunately, the philosophy of the proof—constructing geometry from rigidity—can be communicated quite easily in a simplified setting that avoids all the technical nightmare. That's the version you'll see in my talk.

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Kathryn Mann

ABOUT THE AUTHOR

Kathryn Mann works in geometric topology and low-dimensional dynamics, studying moduli spaces of group actions on manifolds. She is the 2019 recipient of the AWM-Birman Research Prize in Topology and Geometry.

Linear Hydrodynamic Stability

Y. Charles Li

Communicated by Christina Sormani

Introduction

Hydrodynamic instability and turbulence are ubiquitous in nature, such as the air flow around an airplane of Figure 1. Turbulence was also the subject of paintings by Leonardo da Vinci and Vincent van Gogh (Figure 2). Turbulence is the last unsolved problem of classical physics. Mathematically, turbulence is governed by the Navier–Stokes equations, for which even the basic question of well-posedness is not completely solved. Indeed, the global well-posedness of the 3D Navier–Stokes equations is one of the seven Clay millennium prize problems [1]. Hydrodynamic stability goes back to such luminaries as Helmholtz, Kelvin, Rayleigh, O. Reynolds, A. Sommerfeld, Heisenberg, and G. I. Taylor. Many treatises have been written on the subject [5].

Linear hydrodynamic stability theory applies the simple idea of linear approximation of a function through its tangent to function spaces. Hydrodynamicists have been trying to lay down a rigorous mathematical foundation for linear hydrodynamic stability theory ever since its beginning [5]. Recent results on the solution operators of Euler and Navier–Stokes equations [2] [3] [4] imply that the simple idea of tangent line approximation does not apply because the derivative does not exist in the inviscid case, thus the linear Euler equations fail to provide a linear approximation to inviscid hydrodynamic stability. Even for the linearized Navier–Stokes equations, instabilities are often dominated by faster than exponential growth, especially when the viscosity is small.

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Figure 1. Hydrodynamic instability and turbulence are ubiquitous in nature, such as the turbulent vortex air flow around the tip of an airplane wing.

Linear Approximation in the Simplest Setting

Let us start with a real-valued function of one variable

$$y = f(x).$$

If the function is differentiable, then

$$\Delta y = f'(x)\Delta x + o(|\Delta x|).$$

Thus in a small neighborhood of $\Delta x = 0$, the tangent line

$$dy = f'(x)dx$$

offers a linear approximation. Sometimes there is a nice change of variables

$$\Delta \eta = \Delta y + g(\Delta y),$$

Figure 2. Vincent van Gogh's painting "Starry Night" exhibits a turbulent fluid pattern.

where $g(0) = g'(0) = 0$ such that

$$\Delta\eta = f'(x)\Delta x$$

in a small neighborhood of $\Delta x = 0$. This is called linearization. The same idea can be applied to differential equations. Let us consider a simple first order differential equation,

$$\frac{dx}{dt} = f(x).$$

Denote by $S^t(x_0)$ the solution operator

$$x(t) = S^t(x_0),$$

where $x(0) = x_0$. The solution operator can be viewed as a family of maps parametrized by t. If $S^t(x_0)$ is differentiable in x_0, then

$$\Delta x(t) = \frac{d}{dx_0}S^t(x_0)\Delta x_0 + o(|\Delta x_0|).$$

In a small neighborhood of $\Delta x_0 = 0$, the tangent line

$$dx(t) = \frac{d}{dx_0}S^t(x_0)dx_0$$

offers a linear approximation, and $dx(t)$ satisfies the linear equation

$$\frac{d}{dt}(dx) = f'(x)dx,$$

whereas Δx satisfies

$$\frac{d}{dt}(\Delta x) = f'(x)\Delta x + o(|\Delta x|).$$

Sometimes there is a nice change of variables

$$\Delta\eta = \Delta x + g(\Delta x)$$

where $g(0) = g'(0) = 0$ such that

$$\frac{d}{dt}(\Delta\eta) = f'(x)\Delta\eta$$

in a small neighborhood of $\Delta\eta = 0$. This is called linearization of the differential equation.

Mathematical Foundation of Inviscid Linear Hydrodynamic Stability Theory

We start with the basic incompressible, inviscid (no viscosity) Euler equations for the velocity $u(t,x)$ of a fluid:

$$\partial_t u + u \cdot \nabla u = -\nabla p, \ \nabla \cdot u = 0$$

under various boundary conditions, where the spatial dimension is either 2 or 3. Here p is the pressure. In order to establish linear hydrodynamic stability theory, first of all the (nonlinear) Euler equations must be well posed. The usual well-posedness requires that there is a unique solution which depends on the initial condition continuously. Linear hydrodynamic stability theory requires a stronger well-posedness: the solution needs to depend on the initial condition *differentiably*. The well-posedness of the Euler equations depends critically on the function space in which the Euler equations are posed. First of all, the function space consists of functions that satisfy the boundary condition. Second, a norm is defined on the function space so that every function with a finite norm belongs to the function space. The most natural function spaces for the well-posedness of Euler equations are the Sobolev spaces H^n. A fluid velocity function $u(t,x_1,x_2)$ belonging to H^n must have a finite Sobolev norm which is the square root of the integral over the spatial domain of the sum of squares of n^{th} order partial derivatives of velocity components u_j:

(0.1)

$$\|u\|_n^2 = \int \sum_{\ell+m\le n,\ j=1,2} \left[\left(\frac{\partial}{\partial x_1}\right)^\ell \left(\frac{\partial}{\partial x_2}\right)^m u_j\right]^2 dx_1 dx_2,$$

and similarly for spatial dimension 3. Under the boundary conditions of either decaying at infinity or spatial periodicity, the Euler equations are locally well posed (for a short time) in the Sobolev spaces H^n when $n > 1 + d/2$, where d is the spatial dimension. In two spatial dimensions, the local well-posedness can be extended to global (all time) well-posedness. There are also well-posedness results for other boundary conditions, and boundary conditions usually do not pose any substantial issue for well-posedness.

Once we know well-posedness, we can define a solution map S^t (a flow, see Figure 3) in the function space:

$$S^t(u(0)) = u(t), \ \text{ for any } u(0) \in H^n,$$

where $u(t)$ is the solution starting at the initial condition $u(0)$. The well-posedness of the Euler equations in H^n implies that $S^t(u(0))$ is continuous in $u(0)$. To study the stability of any solution $u(t)$, one needs to introduce a perturbation in its initial condition

$$u(0) + \Delta u(0),$$

which generates a new solution to the Euler equations

$$\hat{u}(t) = S^t(u(0) + \Delta u(0)).$$

The stability of the solution $u(t)$ is studied via investigating the growth or decay of the difference

$$\begin{aligned} \Delta u(t) &= \hat{u}(t) - u(t) \\ &= S^t(u(0) + \Delta u(0)) - S^t(u(0)). \end{aligned}$$

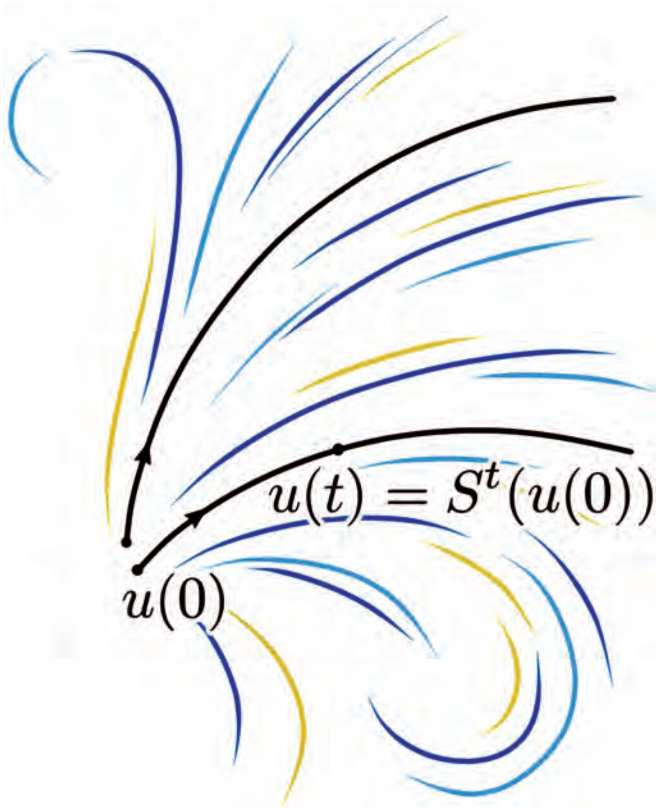


Figure 3. A solution of the Euler equation may be viewed as an orbit in the function space, and the solution map S^t is the flow map.

The linear hydrodynamic stability theory must be based on the linear approximation to $\Delta u(t)$ as a function of $\Delta u(0)$, which requires that $S^t(u(0))$ is differentiable in $u(0)$. If $S^t(u(0))$ were differentiable in $u(0)$, then we would have

$$\Delta u(t) = [\nabla_{u(0)} S^t(u(0))]\,\Delta u(0) + o\left(\|\Delta u(0)\|_n\right), \tag{0.2}$$

where $\nabla_{u(0)}$ represents the derivative in $u(0)$ and $\| \; \|_n$ is the H^n norm (0.1). In a small neighborhood of $\Delta u(0) = 0$, we would have the linear approximation

$$du(t) = [\nabla_{u(0)} S^t(u(0))]\, du(0). \tag{0.3}$$

Indeed, $du(t)$ would satisfy the linearized Euler equations

$$\partial_t du(t) + du \cdot \nabla u + u \cdot \nabla du = -\nabla dp, \;\; \nabla \cdot du = 0. \tag{0.4}$$

Unfortunately, H. Inci ([2], 2015) proved that $S^t(u(0))$ is nowhere differentiable in either 2 or 3 spatial dimensions under decaying boundary condition at infinity. That is, at any initial condition in H^n, the solution operator $S^t(u(0))$ is not differentiable in $u(0)$. Recently, Inci and the author proved that differentiability fails for other boundary conditions as well. The derivative (0.2) never exists, the differential (0.3) never exists, and the linearized Euler equations (0.4) always fail to provide a linear approximation for the inviscid hydrodynamic stability. In conclusion, *the inviscid linear hydrodynamic stability theory always fails to have a rigorous mathematical foundation!*

There are different ways for the derivative (0.2) to fail to exist. The most common way is that the norm of the derivative $\nabla_{u(0)} S^t(u(0))$ is infinite. In such a case, the amplification from $\Delta u(0)$ to $\Delta u(t)$ can be expected to be much faster than the exponential growth associated with any unstable eigenvalue of the linearized Euler equations (0.4). This is the phenomenon that we call "rough dependence upon initial data" [4]. Such superfast growth can reach substantial magnitude in very short time during which the exponential growth is very small as in Figure 6 (see p. 1258). Thus the classical inviscid instability due to unstable eigenvalues does not capture the true inviscid instability of superfast growth.

Rayleigh Equation as an Equation for a Directional Differential

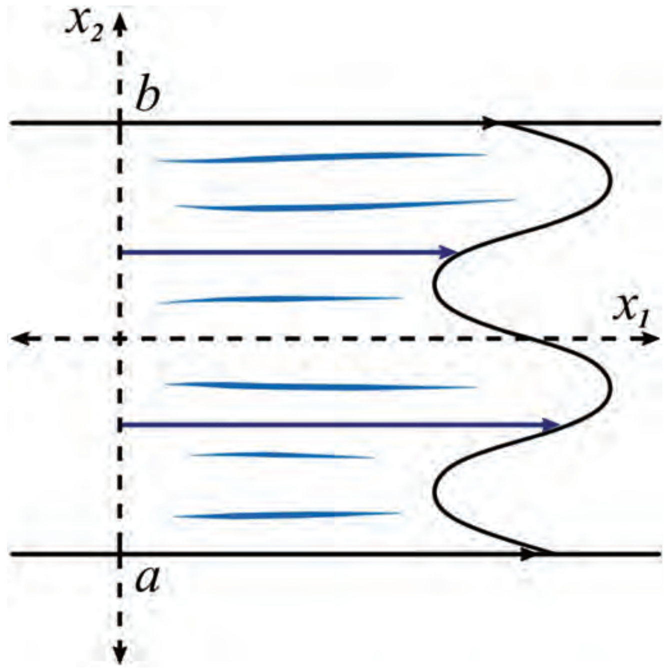


Figure 4. Simple channel flows for which the approximating Rayleigh equation cannot capture the dominant instability of superfast growth.

Now we focus on a 2D channel flow as in Figure 4, where x_1 is the streamwise direction and x_2 is the transverse direction with the boundaries at $x_2 = a, b$. The classical interest is to study the linear stability of the steady state

$$u_1 = U(x_2), \;\; u_2 = 0,$$

which satisfies the boundary condition

$$u_2 = 0 \;\text{ at } x_2 = a, b.$$

The solution operator is not differentiable at the steady state, so the linearized Euler equations at the steady state cannot provide a linear approximation. Nevertheless, the classical inviscid linear hydrodynamic stability theory assumed that the solution operator was differentiable. Formally starting from the linearized Euler equations, Rayleigh derived the so-called Rayleigh equation as follows. Let

(0.5) $$du = (\partial_{x_2}\psi, -\partial_{x_1}\psi),\ \psi = \phi(x_2)e^{ik(x_1-\sigma t)}.$$

Then $\phi(x_2)$ satisfies the Rayleigh equation

(0.6) $$(U-\sigma)(\phi'' - k^2\phi) - U''\phi = 0,$$

with the boundary condition

$$\phi(a) = \phi(b) = 0.$$

Setting $t = 0$ in (0.5), we have

(0.7) $$du(0) = (\partial_{x_2}\psi(0), -\partial_{x_1}\psi(0)),\ \psi(0) = \phi(x_2)e^{ikx_1}.$$

One can view the $du(0)$ in (0.7) as a single Fourier mode in x_1 out of the whole Fourier series in x_1. Indeed, (0.5) represents a directional differential with the direction of $du(0)$ specified by k and $\phi(x_2)$. Even though the full differential $du(t)$ does not exist, the directional differential (0.5) can still exist once the Rayleigh equation (0.6) produces an eigenfunction. Thus the directional differential (0.5) generated from the Rayleigh equation (0.6) cannot capture the nature of the full differential (0.3), which does not exist. The most common way for the nonexistence of (0.3) is that the norm of the derivative $\nabla_{u(0)}S^t(u(0))$ is infinite. This will result in superfast growth of certain perturbations $\Delta u(0)$, which is much faster than any exponential growth predicted by Rayleigh equation. Such superfast growth can reach substantial magnitude in very short time. The superfast growing perturbations are not the directional perturbation (0.5) of Rayleigh. Instability is usually dominated by the fastest growing perturbation. Thus the Rayleigh equation cannot capture the dominant inviscid instability of superfast growth, which can reach substantial magnitude in very short time—short term unpredictability.

High Reynolds Number Linear Hydrodynamic Stability Theory

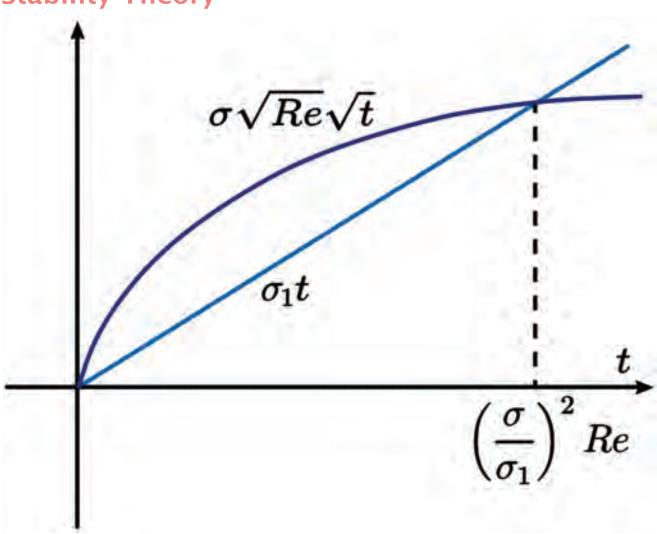
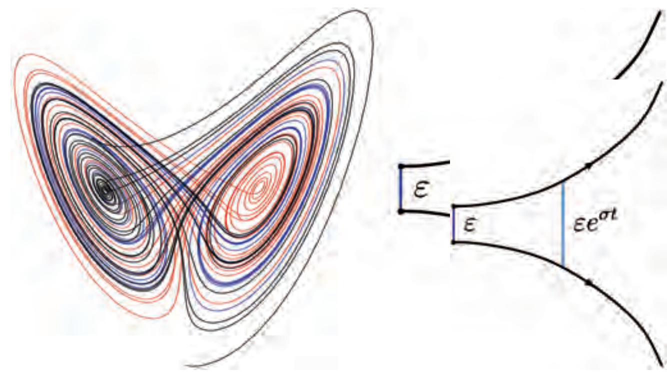


Figure 5. The upper-bound bound (0.11) on the gradient of the flow has two terms. For small t, the one involving the Reynolds number Re is larger.

(a) Ocean Turbulence



(b) Lorenz Attractor

Figure 6. (a). Low viscosity yields rough dependence on initial conditions, as in ocean turbulence. (b). Higher viscosity yields smoother but still sensitive dependence on initial conditions, as in the famous Lorenz-Attractor chaos.

We now consider the incompressible Navier–Stokes equations (with viscosity)

$$\partial_t u + u\cdot\nabla u = -\nabla p + \frac{1}{Re}\nabla^2 u,\ \nabla\cdot u = 0$$

under various boundary conditions, where the spatial dimension d is either 2 or 3. Here Re is the Reynolds number, large when the viscosity is small. Under the boundary conditions of either decaying at infinity or spatial periodicity, like Euler equations Navier–Stokes equations are locally well posed in the Sobolev space H^n when $n > 1 + d/2$. In two spatial dimensions, the local well-posedness can be extended to global well-posedness. As for the Euler equations, given an initial condition $u(0)$ in H^n, we consider the Naiver-Stokes solution $u(t) = S^t(u(0))$. Unlike for the Euler equations, the solution operator $S^t(u(0))$ for the Navier–Stokes equations is not only continuous but also everywhere differentiable in $u(0)$. Then we have

(0.8) $$\Delta u(t) = \left[\nabla_{u(0)}S^t(u(0))\right]\Delta u(0) + o\left(\|\Delta u(0)\|_n\right).$$

In a small neighborhood of $\Delta u(0) = 0$, we have the linear approximation

(0.9) $$du(t) = \left[\nabla_{u(0)}S^t(u(0))\right]du(0).$$

As a result, $du(t)$ satisfies the linearized Navier–Stokes equations

(0.10)
$$\partial_t du(t) + du\cdot\nabla u + u\cdot\nabla du = -\nabla dp + \frac{1}{Re}\nabla^2 du,\ \nabla\cdot du = 0.$$

The norm of the derivative $\nabla_{u(0)}S^t(u(0))$ as a map which maps $du(0)$ to $du(t)$, is defined as

$$\|\nabla_{u(0)}S^t(u(0))\| = \sup_{du(0)} \frac{\|du(t)\|_n}{\|du(0)\|_n}.$$

Under either decaying at infinity or periodic boundary condition in both two and three spatial dimensions, the norm of the derivative $\nabla_{u(0)}S^t(u(0))$ is bounded as follows [3]

$$\|\nabla_{u(0)}S^t(u(0))\| \leq e^{\sigma\sqrt{Re}\sqrt{t}+\sigma_1 t},\ t\in[0,T], \tag{0.11}$$

where

$$\sigma_1 = \frac{\sqrt{2e}}{2}\sigma,\ \sigma = \frac{8}{\sqrt{2e}}\sup_{\tau\in[0,T]}\|u(\tau)\|_n,$$

and $[0,T]$ is a time interval on which well-posedness holds. The exponent of the bound has two parts: the first part depends on the square root of the Reynolds number Re and t, while the second part is independent of Re as in Figure 5. As Re approaches infinity, the bound also approaches infinity in agreement with the intuition that the norm of the derivative approaches its inviscid counterpart, which is infinite. The peculiar feature of the first part is the square root in both t and Re. At $t = 0$, the time derivative of the first part is infinite. During the time interval $t \in (0, \frac{2}{e}Re)$, the first part remains greater than the second part. When the Reynolds number Re is large (and the viscosity is small), this time interval is very large. Thus, when the Reynolds number Re is large, the first part corresponds to superfast growth. We also call such superfast growth as "rough dependence upon initial data" or "short term unpredictability" as in the turbulent waves of Figure 6(a). The second part corresponds to the exponential growth with unstable eigenvalues, the sensitive dependence of chaos as in Figure 6(b). Examples show that as the Reynolds number approaches infinity, the unstable eigenvalues approach the corresponding inviscid unstable eigenvalues.

Numerical simulations on the linearized Navier–Stokes equations (0.10) under periodic boundary condition have verified the $e^{c\sqrt{t}}$ nature in (0.11) for the amplification of abundant perturbations (0.9) along abundant base solutions. Our conclusion is that high Reynolds number turbulence appears from the outset as superfast amplifications of perturbations.

Conclusion

We discover that the linearized Euler equations with their exponential growth of perturbations due to unstable eigenvalues fail to provide a good approximation, because they miss the actual super-exponential growth. Even though the linearized Navier–Stokes equations can provide a decent approximation for high viscosity, they still miss the initially dominant super-exponential growth of perturbations.

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ABOUT THE AUTHOR

Y. Charles Li's research focuses on chaos in partial differential equations and theoretical aspects of fluid turbulence. He also has interest in nanotechnology, mathematical biology, and complex systems.

Y. Charles Li

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Ryan Hynd Interview

Conducted by Alexander Diaz-Lopez

Ryan Hynd is associate professor of mathematics at the University of Pennsylvania (Penn). He works on partial differential equations and likes problems involving optimization, geometry, and probability. Ryan is an NSF CAREER grant recipient and is co-founder of the Bridge to PhD program at Penn.

Diaz-Lopez: *When did you know you wanted to be a mathematician?*

Hynd: I started my college education as a basketball player at my local community college in Florida. At some point I got serious about school and thought about transferring to a university to major in computer engineering. My best friend's older brother was a computer technician, and I thought maybe I could do that, too.

One of the math prerequisites I needed for computer engineering was differential equations; I took this course over the summer at Florida Atlantic University from Tomas Schonbek. He was the first mathematician I had ever met. I really liked his lecturing style and the way he told stories in class. During office hours, I prodded him for details in one of his stories and he lent me a copy of E. T. Bell's *Men of Mathematics*. I made up my mind that I would be a mathematician after reading a few chapters.

Diaz-Lopez: *Who else encouraged or inspired you?*

Hynd: The summer after I took Tomas Schonbek's class, I did an REU with John McCuan at Georgia Tech. This was my first experience doing research. We didn't have much success initially, but we kept chipping away at a few problems during the school year. I think I eventually impressed him with my commitment to research. John was the first person who told me that I had the ability to become a research mathematician and who encouraged me to go to graduate school in mathematics. I'll never forget him for that.

Not long after I arrived at Berkeley to begin graduate school, I started attending Craig Evans' partial differential equations (PDE) seminar. It was eye-opening to learn about the variety of topics people were doing research on related to PDE. I was really inspired by how much Craig cared about the theory of PDE. He seemed to be conversant in everything going on in PDE theory, and he had a great sense of when the audience needed perspective during a lecture. He would often interrupt the speakers to give commentary for the students in the audience. This is how I got my research foundation.

Ted Hill also became my mentor when I was in graduate school. I met him when I was an undergraduate at Georgia Tech, but I really got to know him during his many visits to Berkeley to meet with his advisor Lester Dubins. We bonded as we both have nontraditional backgrounds for mathematicians (he started college at West Point). Ted told me a lot of stories about his career and really taught me

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about how great life could be as an academic. Ted and I are still very close to this day.

Diaz-Lopez: *How would you describe your work to a graduate student?*

Hynd: In 1757, the first partial differential equations were written down by Euler in his seminal work on fluid mechanics. Since then, these types of equations have found their way into seemingly countless mathematical models. PDE theorists such as myself use real and functional analysis to understand properties of solutions of equations that we usually have no chance of solving explicitly.

We ask questions such as: Are there solutions? Are solutions unique? What do they look like globally and locally? Do they have distinctive properties? Are they stable in some sense? Ironically, the answers to most of these basic questions for Euler's equations are still unknown.

Diaz-Lopez: *What theorem are you most proud of?*

Hynd: Probably the main result I obtained in my PhD dissertation. It involved the solution of a nonlinear eigenvalue PDE problem that had an application in financial modeling with transaction costs. I had come up with a problem related to the eigenvalue problem that I could solve for any given approximation parameter, but I was unable to justify sending that parameter to zero to get a solution of my original problem. I needed certain estimates to get control of these approximate solutions.

Fortunately, I ran into Scott Armstrong at a cafe in Berkeley one afternoon and explained one of the inequalities that I suspected to be true for the approximate solutions but couldn't prove. He saw how to verify the inequality with a clever trick: he put an epsilon somewhere I hadn't thought to try. It turned out that my discussion with Scott opened up a new way of looking at the approximate solutions, and a few weeks later I essentially had all the estimates I needed to solve the original eigenvalue problem.

I'm most proud of this result, as I was about halfway through my last year in grad school with no results to show. Moreover, I had failed pretty miserably at analyzing two other PDEs and was starting to wonder if I was cut out for this line of work. I'm glad that I didn't give up, and I'm lucky my persistence paid off.

Diaz-Lopez: *What advice do you have for graduate students?*

Hynd: Keep an open mind and be opportunistic.

Diaz-Lopez: All mathematicians feel discouraged occasionally. How do you deal with discouragement?

Hynd: With the amount of failure that is typical in research, it is hard not to get discouraged. I try to remember this harsh reality and see failure as an opportunity to be more inventive. Unfortunately, neither option solves the problem of being discouraged from time to time.

Diaz-Lopez: *As part of your CAREER grant, you created the Bridge to PhD program in the math department at the University of Pennsylvania. What are your objectives and goals for the program?*

Hynd: I was on the graduate admissions committee one of my first few years at Penn. The chair instructed us to be on the lookout for applicants from underrepresented groups. When we met to discuss applications, I presented a few very promising minority applicants. However, I encountered hesitancy because their GRE math subject test scores were a bit too low and because they were coming from institutions not known for having strong math departments.

So I spoke with Shaun Harper, who was in our education school at the time, and he suggested the idea of creating a bridge program. We could accept students at the master's level and groom them for our PhD program. It also turned out that a program like this already existed between Fisk and Vanderbilt Universities in physics, biology, and chemistry. I learned a lot from speaking with Keivan Stassun, who started that program, and from using the "toolkit" material they have posted online.

To answer your question, our goal is the same as it was before—more diversity at the PhD level, especially from students from underrepresented groups. We are hoping that the Bridge to PhD program will make a difference.

Diaz-Lopez: *If you could recommend one book to graduate students, what would it be?*

Hynd: Tough question! There are so many good books to choose from. One little book that really inspired me, that perhaps isn't so well known, is Robert Osserman's *Poetry of the Universe: A Mathematical Exploration of the Cosmos*. I think it's special in the way it emphasizes how powerful mathematical thinking can be.

Diaz-Lopez: *Any final comment or advice?*

Hynd: Enjoy your time in graduate school! Make some friends outside of the mathematics department, pick up a hobby, travel during the summers, learn another language. I believe that activities such as these, done with appropriate balance, will only serve to enhance your experience.

Photo Credit

Photo of Ryan Hynd by Felice Macera.

Alexander Diaz-Lopez

ABOUT THE INTERVIEWER

Alexander Diaz-Lopez, having earned his PhD at the University of Notre Dame, is now assistant professor at Villanova University. Diaz-Lopez was the first graduate student member of the *Notices* Editorial Board.

WHAT IS...

a Borel Reduction?

Matthew Foreman

Communicated by Cesar E. Silva

ABSTRACT. Borel reductions provide a method of proving that certain problems are impossible using countably infinitary techniques based on countable information and provide a hierarchy of difficulty for classification problems. This is illustrated with examples, including a recent result that a classification problem in dynamical systems proposed by von Neumann in 1932 is impossible to solve with inherently countable tools.

Mathematics is uniquely capable of producing *impossibility results*. The most famous examples include the impossibility of

- proving the parallel postulate
- squaring the circle
- solving a general quintic polynomial
- solving the word problem for finitely presented groups.

What do these results have in common? They have rules that determine what methods are considered legal for a solution. For example, the quintic is *unsolvable by radicals*. Explicitly there is no algebraic formula for solving the general quintic that uses expressions of the form $a^{1/n}$ ($a \in \mathbb{Q}$). Quintics are trivially solvable if you allow expressions that stand for solutions to arbitrary equations. Similarly it is impossible to square the circle *using ruler and compass*; it is impossible to prove the parallel postulate *using the other Euclidean axioms*, and so forth.

The notion of *unsolvability* has various alternate meanings, including the related notion of *independence*. In the context of the *word problem*, being solvable would mean the existence of a *recursive algorithm* for deciding whether two words in the generators represent the same element of the group. Heuristically, this would mean that there is a protocol using inherently finite information that converges in finite time with a yes-no answer to the question.

In contrast, here we describe a method for proving an emerging form of impossibility result that says

Doing **X** is impossible using
inherently countable techniques.

Note that being unsolvable using *inherently countable* techniques is a much stronger result than unsolvability using inherently finite techniques. Moreover the objects we describe here give a "hierarchy of difficulty" for many types of problems in mathematics.

What precisely does the phrase *inherently countable technique* mean? The context is Polish Spaces—those spaces whose topology can be induced by a complete separable metric. The collection of Borel sets is the smallest σ-algebra that contains the open sets. The Borel sets can be viewed as the broadest class of sets for which membership can be modeled as passing a countable—possibly transfinite—protocol of yes/no questions asked of an arbitrary countable collection of basic open sets. Thus the statement that "A is *not* Borel" says that there is **no** inherently countable method of determining membership in A. The natural setting for considering the Borel/non-Borel distinction is that of *analytic sets*, where a subset A of a Polish space X is *analytic* if it is the continuous image of a Borel subset B of a Polish space Y. Similarly, C is coanalytic if $Y \setminus C$ is analytic.

An example of an impossibility result of this sort is due independently to Kaufman and Solovay, who in 1983-84 showed that the collection of closed sets of uniqueness for trigonometric series is not a Borel set. (A set $E \subseteq [0,1]$

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is a *set of uniqueness* if whenever $\sum c_n e^{2\pi i n x} = 0$ on $[0,1] \setminus E$ the series $\sum c_n e^{2\pi i n x}$ is identically 0.) Hence the classical problem of deciding whether the complement of a given closed set determines the values of a trigonometric series is simply not possible using anything resembling even a countable transfinite computation. Following these results there have been a plethora of similar results in many areas, including one by Beleznay and the author in 1995 showing that the classically studied collection of so-called *distal* dynamical system is not a Borel set.

B. Weiss and the author[1] hope to publish soon a proof that the program initiated by von Neumann in 1932 ([3]) to classify the statistical behavior of Lebesgue measure-preserving diffeomorphisms of the 2-torus is impossible to carry out using inherently countable techniques. It is currently unknown if isomorphism for diffeomorphisms is strictly above graph isomorphism.

Borel reduction is the main tool for proving certain procedures are impossible. It originated in the late 1980s in the work of Friedman and Stanley (1989) and independently Harrington, Kechris, and Louveau. The idea starts with the cliche that:

> To solve A you reduce it to a problem B which you already know how to solve.

Turning this on its head:

> To show that solving B is impossible, you start with a *known* impossible problem A and reduce it to B.

Formally:

Definition. Let A and B be subsets of Polish spaces X and Y. Then A is *Borel reducible* to B if and only if there is a Borel function $f : X \to Y$ such that for $x \in X$:

$$x \in A \text{ if and only if } f(x) \in B.$$

Thus if A is not Borel, B cannot be either, since the inverse image of a Borel set by a Borel function is Borel. The function f is a *Borel reduction*.

Define $A \leq_{\mathcal{B}} B$ if A is Borel reducible to B. Then $\leq_{\mathcal{B}}$ is transitive since one can compose Borel reductions. Defining the equivalence relation $A \sim_{\mathcal{B}} B$ if $A \leq_{\mathcal{B}} B$ and $B \leq_{\mathcal{B}} A$ we see that $\leq_{\mathcal{B}}$ induces a partial ordering of the $\sim_{\mathcal{B}}$ equivalence classes.

The heuristic above interprets $A \leq_{\mathcal{B}} B$ as saying that B is at least as complicated as A (with respect to countably feasible computations) and $A \sim_{\mathcal{B}} B$ as saying that they have the same complexity. Among analytic sets, there is a $\leq_{\mathcal{B}}$-maximal equivalence class, called the *complete* analytic sets.

For Borel reductions to be useful we must have an example of a non-Borel set A to start with. There are many choices. One canonical example can be found by taking X to be the space of connected acyclic countable graphs (allowing infinite valence) and $A \subseteq X$ to be the set of graphs with a nontrivial end (an end is an infinite path through the graph). Equivalently we can take X to be the space of rooted connected countable trees and A to be the collection of *ill-founded trees*—those trees that have an infinite branch. (Figure 1 represents a tree with an infinite branch.) In each example, the set A is complete analytic and not Borel. Thus if there is a Borel reduction of A to any set B then B is not Borel (and by transitivity B is also complete).

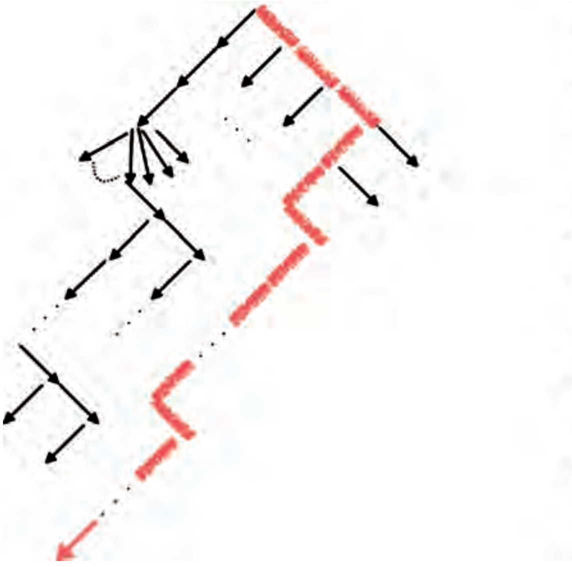

Figure 1. The set of trees with an infinite path, like the tree pictured here, is an example of a complete analytic subset of the space of trees that is not Borel—i.e., cannot be determined by a countable process based on countable information.

An extension of the ordering $\leq_{\mathcal{B}}$ from subsets to relations is its two-dimensional version, which we write $\leq^2_{\mathcal{B}}$. For $E \subseteq X \times X$ and $F \subseteq Y \times Y$, we let $E \leq^2_{\mathcal{B}} F$ if and only if there is a Borel $f : X \to Y$ such that for $(x_1, x_2) \in X$:

$$x_1 E x_2 \text{ if and only if } f(x_1) F f(x_2).$$

The function f is again called a *Borel reduction.*

Classification problems are the most common objects of study here, because they are naturally given by equivalence relations, such as those coming from attaching invariants to collections of objects being studied. Saying that one classification problem E is Borel reducible to another classification problem F is a precise way of saying that determining whether $y_1 F y_2$ is at least as hard as determining whether $x_1 E x_2$. This subject has been studied extensively over the last thirty years by many mathematicians (see [2]).

Analytic equivalence relations fall into five basic intersecting categories (see Figure 2): countable equivalence relations, S^∞-actions, Polish group actions, Borel, and non-Borel. The first three are qualitative:

$$\{\text{countable equivalence relations}\}$$
$$\cap$$
$$\{S^\infty\text{-actions}\}$$
$$\cap$$
$$\{\text{Polish group actions}\}$$

To these we add the Borel/non-Borel distinction. The countable equivalence relations are all Borel, hence this

[1] *"Measure Preserving Diffeomorphisms of the Torus are Unclassifiable,"* https://arxiv.org/abs/1705.04414

distinction only applies to S^∞-actions, Polish group actions, and those equivalence relations that are neither.

We now define these classes, give examples of each type, and describe which are more complex than others. Many more examples are completely understood; we only scratch the surface of the subject.

Countable equivalence relations

A Borel equivalence relation with countable classes is called a *countable* equivalence relation. It is a theorem of Feldman and Moore (1975) that every such equivalence relation is the orbit relation of a countable group of Borel isomorphisms.

Group actions

The most ubiquitous examples of equivalence relations come from group actions. If G is a Polish group acting on a Polish space X in a Borel manner, then we get the *orbit equivalence relation*, namely $x \sim y$ if and only if there is a $g \in G, gx = y$. Especially important classes of Polish group actions include those of S^∞, the group of permutations of the natural numbers, the group of unitary operators on a separable Hilbert space, the group MPT of measure-preserving transformations of $[0,1]$, and groups of homeomorphisms of compact separable metric spaces.

S^∞-actions

We let S^∞ be the group of permutations of the natural numbers. We illustrate the importance of S^∞-actions with an example. We can identify a countable group $G = \langle g_n : n \in \mathbb{N}\rangle$ with its multiplication table $\{(l,m,n) : g_l \cdot g_m = g_n\}$. Defining $\chi_G : \mathbb{N}\times\mathbb{N}\times\mathbb{N} \to \{0,1\}$ by setting $\chi_G(l,m,n) = 1$ if and only if $g_l \cdot g_m = g_n$, we get an element of $\{0,1\}^{\mathbb{N}\times\mathbb{N}\times\mathbb{N}}$ which, endowed with the product topology, is a compact space homeomorphic to the Cantor set. Let S^∞ act on

$$CG = \{\chi_G : G \text{ is a countable group}\}$$

by setting $(\phi\chi)(l,m,n) = \chi(\phi^{-1}l, \phi^{-1}m, \phi^{-1}n)$. Let $G = \langle g_n\rangle_n$ and $H = \langle h_n\rangle_n$ be isomorphic. Then there is a $\phi \in S^\infty$ such that this isomorphism takes g_n to $h_{\phi(n)}$. Thus $\phi\chi_G = \chi_H$. For two countable groups G and H we've shown:

G is isomorphic to H if and only if χ_G and χ_H are in the same S^∞ orbit.

We conclude that the isomorphism relation for countable groups is naturally encoded as the orbit equivalence relation of an S^∞-action.

Clearly there is nothing special here about groups: *for any class of countable algebraic structures the isomorphism relation is coded by an S^∞-action.* Being Borel reducible to an S^∞-action is thus equivalent to being able to assign countable algebraic structures as invariants. Showing that a given classification problem is *not* reducible to an S^∞-action is an impossibility result interpreted as saying there are no complete algebraic invariants for the equivalence relation. In the mid 1990s, Hjorth gave a general method for doing this—the method of *turbulence*.

Polish group actions

More generally, many classification problems are given as orbit equivalences of Polish group actions. Commonly the group action is some form of *conjugacy*. We now place some benchmarks into the setting being described (Figure 2).

At the bottom of $\leq^2_{\mathcal{B}}$

Since the Cantor set can be injected into every perfect Polish space, the identity equivalence relation on $\{0,1\}^{\mathbb{N}}$ (the diagonal relation), denoted $\mathbf{Id}_{2^{\mathbb{N}}}$, is at the bottom of the $\leq^2_{\mathcal{B}}$ ordering. A given relation E being reducible to $\mathbf{Id}_{2^{\mathbb{N}}}$ is equivalent to being able to attach complete numerical invariants to the equivalence classes of E in a Borel way.

Another important benchmark is E_0: the equivalence relation of eventual agreement of sequence of 0's and 1's. A fundamental result is due to Harrington, Kechris, and Louveau, who proved for a Borel equivalence relation F that either E_0 is reducible to F or F has complete numerical invariants (i.e., is reducible to $Id_{2^{\mathbb{N}}}$).

Maximal relations in a class

Several of the classes have a *maximal* equivalence relation—in the sense that every equivalence relation in that class is reducible to it. We describe these as follows:

For countable equivalence relations

Let F_2 be the free group on 2 generators. Then we can identify the power set of F_2 with the product space $\{0,1\}^{F_2}$ and let F_2 act by left translation on the exponent. (This is the *Bernoulli Shift* for F_2.) The resulting equivalence relation is denoted E_∞. It has countable classes, and every countable Borel equivalence relation is reducible to E_∞.

Another natural example of a maximal Borel equivalence relation among those with countable classes was identified by Hjorth and Kechris (2000): the relation of conformal equivalence among (noncompact) Riemann surfaces.

A third example is isomorphism for finitely generated groups.

For S^∞-actions

A graph whose vertices are natural numbers can be identified with an element of $\{0,1\}^{\mathbb{N}\times\mathbb{N}}$ by setting $\mathrm{X}_G(n,m) = 1$ if and only if n and m are connected by an edge. By letting S^∞ act on the exponent, we code the equivalence relation of *isomorphism of countable graphs.* Every S^∞-action is reducible to isomorphism of countable graphs.

For Polish group actions

Becker and Kechris (1996) proved that for every Polish group there is a $\leq^2_{\mathcal{B}}$-maximal orbit equivalence relation. It then follows from a result of Uspenskiy, showing there

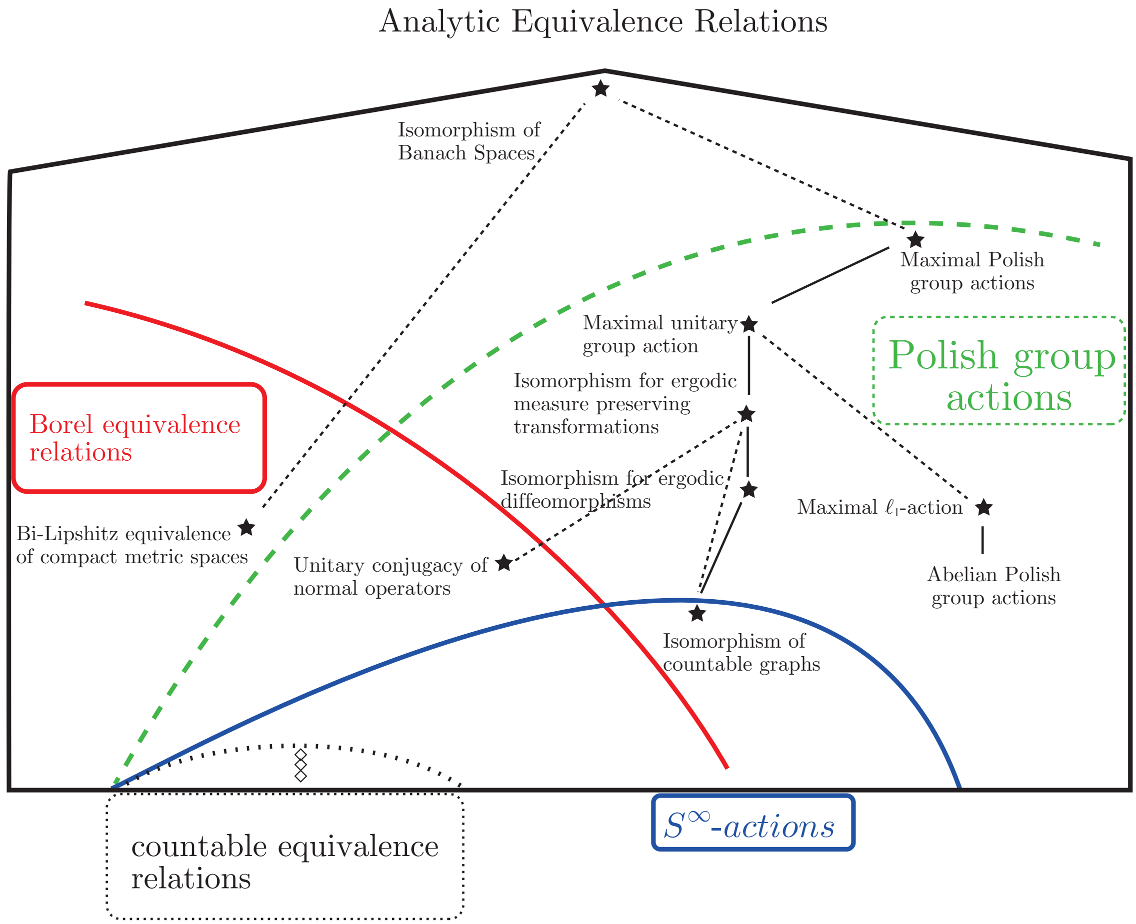

Figure 2. Five basic types of equivalence relations: analytic (the whole box), Borel (in red), those induced by Polish group actions (in green), those induced by S^∞-actions (in blue) and those that have countable classes (at the bottom). In the diagram a line indicates the lower equivalence relation is Borel reducible to the upper equivalence relation and a dotted line indicates the upper equivalence relation is *not* reducible to the lower relation. Other relationships remain open.

is a universal Polish group, that there is a maximal equivalence relation among all Polish group actions.

Borel equivalence relations

Friedman and Stanley (1998) showed that there is no maximal Borel equivalence relation.

For Borel Polish group actions

Hjorth, Kechris, and Louveau (1998) showed there were no maximal Borel Polish group orbit equivalence relations.

Analytic equivalence relations

Harrington proved the existence of a maximal analytic equivalence relation, but it wasn't until the remarkable work of Ferenczi, Louveau, and Rosendal (2009) that a natural example was given. It is *isomorphism for Banach Spaces.*

Placing mathematical examples in the ordering

Many well-known classification results have been placed into the Borel Reducibility ordering. We now give only a tiny sample of the known examples, ending with a recent solution of von Neumann's classification problem for measure-preserving diffeomorphisms.

At the bottom are the countable equivalence relations—those that have countable classes. These are always induced by Borel actions of countable groups. Among many possibilities we take as typical examples questions from the classification of finite-rank torsion-free abelian groups. Thomas showed that they form a collection of problems of strictly increasing complexity as the rank increases. Define the following equivalence relations.

- $\cong_{fg}$ the isomorphism relation on finitely generated groups
- $\cong_n$ the isomorphism of torsion-free abelian groups of rank n
- $\cong_n^p$ the isomorphism relation on p-local abelian groups of rank n.

The relationships between these equivalence relations are given in Figure 3:

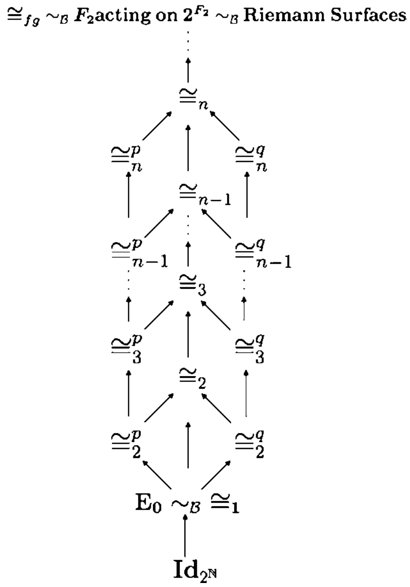

Figure 3. The Borel reducibility ($\leq_{\mathcal{B}}^2$ ordering) of some countable equivalence relations. The central spine consists of isomorphism for torsion free abelian groups of rank n. For primes p the relation $\cong_n^p$ is strictly reducible to $\cong_n$ and if $p \neq q$ are prime the relations $\cong_n^p$ and $\cong_n^q$ are $\leq_{\mathcal{B}}^2$ incomparable.

We now consider the following examples of equivalence relations with uncountable classes (Figure 2):

Unitary conjugacy for normal operators

Here the classical spectral theorem shows the equivalence relation is Borel and it is trivially reducible to the maximal unitary group action. This relation is strictly below isomorphism for measure-preserving transformations.

Bi-Lipschitz equivalence of metric spaces

Rosendal (2005) showed that the relation on pairs of metric spaces given by having a Lipschitz homeomorphism with a Lipschitz inverse is a Borel equivalence relation that is not reducible to a Polish group action.

ℓ_1-actions

Every action of an abelian Polish group can be reduced to an action of the abelian group $\ell_1(\mathbb{N})$ with pointwise addition, hence to the maximal ℓ_1-action. This in turn can be reduced to the maximal unitary group action by results of Gao and Pestov.

Isomorphism for MPTs

This is the equivalence relation of isomorphism (the conjugacy action of MPT) of ergodic measure-preserving transformations of $[0, 1]$. Classifying this equivalence relation was proposed by Halmos in 1956. In 2008 Rudolph, Weiss, and the author [1] showed that this equivalence relation is not Borel. The author observed that the graph isomorphism problem can be reduced to isomorphism of ergodic measure-preserving transformations. Furthermore, with Weiss (2003), the author showed the equivalence relation is *turbulent*, hence strictly above every equivalence relation induced by an S^∞-action.

Open problems

We now note some open problems. We give two questions related to geometry and end with a problem internal to the subject.

Classification up to homeomorphism:

Von Neumann was concerned with classifying the statistical behavior of diffeomorphisms. Hence the relevant equivalence relation was *isomorphism by measure-preserving transformations*. In 1967, Smale suggested classifying diffeomorphisms of surfaces up to *conjugation by homeomorphisms*. This spawned a large and successful literature that solved the problem for structurally stable diffeomorphisms, but not in general.

Let M be a compact surface. Where does the equivalence relation *conjugacy by homeomorphism* of pairs of diffeomorphisms of M sit in Figure 2? In particular *is it Borel*?

Classifying smooth $\mathbb{R}^4$ structures:

Taubes proved in 1987 that there are a continuum of smooth structures on $\mathbb{R}^4$ up to equivalence by diffeomorphisms. What is the complexity of this equivalence relation on smooth structures?

What happens at the top?

Many problems, such as isomorphism of ergodic diffeomorphisms of the 2-torus, are reducible to the maximal Polish group action, but it is not known if the reductions are strict. While it seems unlikely to practitioners, it could be that the problems shown are all $\leq_{\mathcal{B}}^2$-equivalent.

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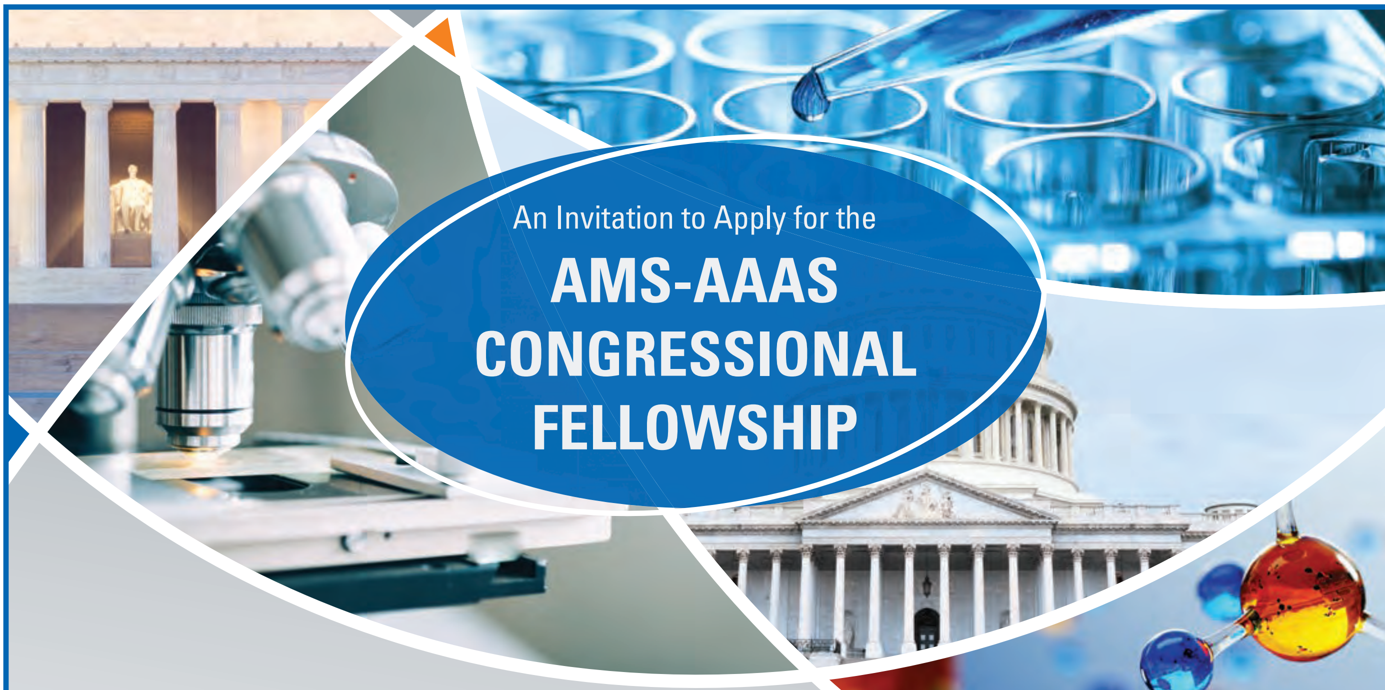
Matthew Foreman

ABOUT THE AUTHOR

Matthew Foreman sailed his C&C 44 across the Atlantic, and around Europe and the Mediterranean. He also circumnavigated Newfoundland. In a different sailboat, he rounded Cape Horn.



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OPINION

Ethics Lessons Learned While Editing the *Monthly*: Modern Publishing Is Raising New Issues

Scott T. Chapman

Note: The opinions expressed here are not necessarily those of Notices.

ABSTRACT. A former *Monthly* editor discusses self-plagiarism, forged documents, and the ethics of double-blind reviewing.

As a pragmatist who instinctively trusts people, my five-year term as editor of the *American Mathematical Monthly* ended much differently than it began. After four years of editing with little or no ethical issues, two significant ones emerged during year five. The first involved an author who republished in the *Monthly* a work previously published elsewhere; it became the first retraction in the 121-year history of the *Monthly*. The second involved an author who forged acceptance letters for papers that he never had submitted. While these are extreme cases, they made me think deeply about ethical issues in mathematical publication and how they have changed so drastically since I began publishing papers thirty years ago.

I doubt that many of the professional mathematicians who are reading this have ever taken a course in academic ethics or even listened to a seminar on the subject. Most of us (and I include myself) learned mathematical ethics secondhand as we worked our way through graduate school or our first tenure-track jobs. We basically know what not to do, but with the ever-changing landscape of academic publishing, the lines that seemed so clear thirty years ago are sometimes much less fine. For instance, does posting a paper to the arXiv constitute a publication? Or, if I use a photograph or table taken from the internet public domain, do I need to credit where it came from? The internet has opened an entire new can of ethical worms that we are just now beginning to sort out.

So what are the rules? I found when I started dealing with the self-plagiarism case that there is no Bible for mathematical ethics. There are many different sources that one can use. I was quite surprised to learn that the Mathematical Association of America has no written ethics guidelines with regards to publishing. The three best sources that I could find were ethical guidelines adopted by the American Mathematical Society [2], the European Mathematical Society [4], and the London Mathematical Society [5]. The Procedures Manual of the AMS Committee on Professional Ethics (COPE) [1] has an appendix with many hypothetical ethics violations and the following disclaimer: "These are guidelines, not a collection of rigid rules." Even in the most clear-cut of cases, one must be ready to deal with gray area.

Obviously, forging acceptance letters is not a gray area. The rules involving plagiarism of another author's work are pretty clear-cut, but those involving self-plagiarism are not. Retractions are liable to become more commonplace

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as the number of online journals continues to increase. If you've written something previously, when and how can you use it again? Here is an excerpt from the Australian Code for the Responsible Conduct of Research [3, Section 4.7]:

> It is not acceptable to include the same research findings in several publications, except in particular and clearly explained circumstances, such as review articles, anthologies, collections, or translations into another language. An author who submits substantially similar work to more than one publisher, or who submits work similar to work already published, must disclose this at the time of submission.

Moreover, authors need to keep in mind that copyright laws apply. Yes, you wrote it, but if a publisher holds a copyright on an article, then technically you have given up ownership. For academic publication, the doctrine of "fair use" comes into play, and reproduction of short passages can normally be satisfied by the appropriate referencing methods. Using more than snippets from a copyrighted article requires written permission from the copyright holder, and this in theory could involve monetary considerations. When the *Monthly* wanted to republish a Pi Day crossword puzzle from the *New York Times*, I was dismayed to learn that this would cost thousands of dollars.

To be on the safe side, if you plan on using significant portions of your prior work in a new article, be completely up-front about this. Include in an introductory paragraph a statement to that effect ("This paper includes portions of previous work [XX] …"). Carefully reference theorems and other results. If you are taking more than a few lines verbatim, obtain a copyright release and footnote this on the first page. Most academic publishers will do this with no charge. It is best here to err on the side of caution—self-plagiarism can be dealt with as severely as the more traditional plagiarism of another's works.

The remainder of this note will tread on even grayer area. In August of 2012, the MAA instituted a policy of double-blind reviewing for all papers submitted to the *Monthly* and its other two journals (*Mathematics Magazine* and the *College Math Journal*). Double-blind reviewing was already in use at the *College Math Journal*, but its implementation at the *Monthly* broke new ground. The theory behind double-blind reviewing is that most of the bias in publishing can be eliminated if the authors and referees are unknown to each other.

Question 1. Whose responsibility is it that the manuscript does not contain information that identifies the author(s)?

Question 2. If authors submit a double-blind manuscript, can they still make copies of it available (either online or otherwise) that identify the authors? More specifically, can the authors post a copy on the arXiv?

Question 3. If a potential reviewer of a double-blind manuscript determines the identity of the author(s), can he or she still act as referee?

As I began navigating the world of double-blind, I developed strong opinions on all three of these questions. As an editor who handled nearly 900 manuscripts a year, my answers are a mix of practical methods and the philosophy behind double-blind.

As to Question 1, the initial answer seems simple: it is up to the author to remain anonymous. Yet I saw many instances where a highly regarded author would have benefited if the referees knew who he or she was. How much can the editor be expected to do? The job of editing is hard enough without adding the additional burden of screening every manuscript line by line for subtle clues on who the author is. Moreover, in today's Google-like atmosphere, if referees want to know who wrote an article, then they can likely find out. The system cannot be completely foolproof. The best we can do is to give the author the responsibility to keep the submission anonymous. I state this as the first in a short series of double-blind editorial axioms.

Double-Blind Axiom 1. The onus of anonymity in the double-blind process lies mainly on the author.

Question 2 involves perhaps the most frequently asked question I got while editing the *Monthly*. Can I post a paper I have submitted to my webpage or to the arXiv? The practice of posting preprints is widespread and universally accepted. When we used traditional reviewing, the answer was clearly "yes." Logic might seem to indicate that in the double-blind situation that answer is "no." Unlike the previous situation, in this case I think the onus is on the journal and the editor, rather than the author.

I think it would be completely within a double-blind journal's right to prohibit authors from openly posting submitted work. During my double-blind years at the *Monthly*, I was hesitant to do this. I was concerned that a complete prohibition on using the arXiv might have a significant impact on our submission rate. Thus, a double-blind journal needs a clear policy on this, as I state in Double-Blind Axiom 2.

Double-Blind Axiom 2. A journal that uses double-blind reviewing should have a clear and easily found policy concerning the posting of work that is under consideration.

I think Question 3 boils down to an issue that we are all used to dealing with in the traditional refereeing system: Can I objectively referee this work? There seems to be no concrete definition of "conflict of interest." If you cannot read the paper and make a decision that is universal regardless of the identity of the author, then you should not referee the paper. A good editor should police this and not send papers out to co-authors or others who have clearly worked closely with the author. If you believe you know who the author is and think this could play a role in your decision, then bring it to the editor's attention. It is then his or her job to the make the call.

Double-Blind Axiom 3. Issues involving conflict of interest for potential referees should be brought to the attention of the editor, who is the final arbiter in such matters.

Shortly after my term as editor of the *Monthly* ended in 2016, the MAA convened a task force (chaired by former *Monthly* Editor Dan Velleman) to review the situation at its journals with respect to double-blind refereeing. Its report discusses, among other issues, the three questions above and offers recommendations on how the MAA should proceed with developing editorial policy.

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EDITOR'S NOTE. For a review of how the AMS handled ethics see the April 2010 *Notices* article on "Awareness of Ethical Pitfalls: A Requirement for Professional Protection" by Catherine Roberts, https://www.ams.org/notices/201004/rtx100400485p.pdf.

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OPINION

Why Do We Need Minorities Among Our Faculty?

William Yslas Vélez

Note: The opinions expressed here are not necessarily those of Notices.

Many years ago, when I was just beginning in our profession, the southwestern university that I was visiting had no Chicano[1] faculty in the mathematics department, a common state of affairs even now. I asked the head of the mathematics department at the time why they had no Chicano faculty, and his answer was, "Why do we need one?"

Have departments asked themselves this question: Why do we need minority faculty? In my forty years as a faculty member I never witnessed substantive faculty deliberations on this topic. Of course, there are obvious answers to this question. The lack of minority faculty speaks to the failure of our educational system to educate the US population. Why is it that there are so few minority students in our graduate programs in mathematics? This represents a failure of our departments to attract minority students to the study of mathematics. Is this intentional or is it simply not a concern that comes up in departmental discussions [2]?

When I was a child growing up in Tucson, our Catholic church was staffed with priests from Spain and Ireland. Their mission was to convert or re-make the Mexicans and Indians of the Southwest in their own image. Could mathematics departments be viewed as such missionaries? Their faculty commonly do not look like the native populations they are supposed to be educating.

I do not think that mathematicians view their role as one of motivating their students to join them.[2] Then, what is the role of these faculty? Gatekeepers? Dedicated Guardians of Mathematical Knowledge?

"Why do we need one?" If mathematics departments were research institutes, with only one goal, the production of mathematical knowledge, then I could see a monk-like existence for mathematics departments. We sit and meditate, think deep thoughts, separated from the local community.

But we are not research institutes. Mathematics departments are part of a university, a teaching university, with students! The goal of a university is multi-faceted. Not only does it produce knowledge, it now patents it. It is linked to the world. It has goals. And a mathematics department's activities and hiring practices should be linked to those goals: effective teaching, the use of technology in teaching and diverse teaching methods, outreach to the community, the promotion of career opportunities for its students, scholarly endeavors, and research with students. This complexity of activities necessitates a more diverse and caring workforce. Yes, I said caring.

Professional meetings are an important part of our profession. We cannot minimize the importance of face-to-face conversations among our colleagues. As scholars we address many issues. As we participate in conferences, conversations spark thought and launch us into new directions, provide insights that we were searching for. So it was at a recent meeting of Transforming Post-Secondary Education (TPSE) held at Worcester Polytechnic Institute. Conversations there gave me a better answer than the one I presented in [4] to why we need diverse faculty.

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[1] *At that time, Chicano meant Mexican-American, though with an activist connotation.*

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[2] *See [1] for examples of how departments can have productive discussions on motivating their students.*

During a session on equity, Maria Mercedes Franco of Queensborough Community College, originally from Colombia, mentioned that in Hispanic interactions there are laughter and smiles, handshakes and hugs. I understood Maria's comments as indicating that there is a cultural divide between students and faculty at our universities. We all know this though we don't talk about it. My own belief is that one of the unfortunate current hidden functions of a mathematics department is to change the culture of the mathematics majors, to make mathematics majors become like the faculty. I decided to earn a PhD in mathematics in 1965, in my second year of undergraduate studies at The University of Arizona. This was not a friendly place for Mexican-Americans. It has been my experience that mathematics departments ignore the culture of the local community. One attends mathematical functions, and if there is music it is classical. Physically, what does a mathematics department look like? Is there any part of the local community that is reflected in the department? There are 21 Native American reservations in the state of Arizona. Our universities are built on the lands of these Native peoples. When one walks into a mathematics department, is there any indication of the rich culture of the local communities?

Maria's comments point to the interactions between individuals in our mathematics departments, and this ties into a comment that Uri Treisman made during the conference. I have written about my efforts to recruit students into the mathematics major by inviting them to add the mathematics major [4]. Uri commented that this was a very wise tactic. Though it was kind of him to say that, it is not correct. It is not wise. It is cultural.

When I was a child, when someone walked into our home, my mother would say, "Saluda, mi hijito" (Say hello, my dear son). As a young professor, perhaps I did not heed these words. I had been acculturated in the way a mathematician behaves. However, no matter what I did, I was often viewed as a minority mathematician. At a math party that was held at my home, one of the graduate students had a bit too much to drink and told me that the only reason that I was hired was because I was a minority. With time, I began to see my value to the department as a Chicano mathematician. With time my upbringing won out over this attempt to make me fit the mold.

Now, when a student comes into my office, I listen to those words, "Saluda, mi hijito." I extend my hand and a welcome. I want to share my good fortune with others. I decided to study mathematics and it greatly enriched my life. Because of that upbringing, I invite students to study mathematics. This is not a wisdom. It is culture.

Even though I was brought up in the Mexican culture, I am not promoting the view that Hispanics are better suited to educate the next generation of students. The characteristic, the ability to care and to motivate others to join us in mathematical studies, is not restricted to one culture. I have used the word "minority" because I think that those of us in the minority community who have managed to be successful in this profession bring not only cultural elements, but also a recognition of the struggles that US students have gone through to get to the university. Our experiences, plus our culture, make us a valuable asset to the profession. We bring to the workplace a different dynamic, a different interaction with students [1]. Given the importance of mathematical training for our current workforce and the dramatic changes in the make-up of the student populations, minority faculty are needed now more than ever.

In 1974, as a fourth year graduate student at The University of Arizona, I had the opportunity to be an intern at Bell Labs. What an amazing mathematical experience that was! At the end of the summer a recruiter came to see me and asked if I would be interested in applying for a position upon completing my PhD the following year. My response was visceral: "I would not work here if it was the last place on earth. This is the coldest bunch of people I have ever been with."

That was an incredibly sad response. I denied myself an absolutely amazing mathematical experience. Yet, that was my reaction. I grew up in the warm cultural embrace of the Mexican-American community in Tucson, and the cultural divide that I felt was not one that I thought I could live with.

I believe that this is all too common. How many students from all walks of life, men and women, have turned away from the study of mathematics because of the mathematical culture that is so impersonal? It does not have to be this way. Just because we are serious about what we are doing does not mean we should be somber.

That is why we need minority and female faculty. It is not just the production of mathematics that goes on in a department. There is also an acculturation process and it is this that needs diversification. It is time for our graduate programs to place more emphasis on recruiting and retaining minority students. It is especially important for the top 20 graduate programs in mathematics to take the lead in this effort.

How are these graduate programs going to succeed in recruiting a diverse student body while affirmative action is again under attack? How do we defend bringing in a diverse student body while at the same time saying that we have recruited the best?

The Problem is the Word "Best."

1. Do we mean best in terms of GRE scores and GPAs? Anyone who has run a graduate program knows that these are poor indicators of student success. As mathematicians, we value creativity; yet in determining whom to admit we evaluate only knowledge.
2. Does best mean what is best for the faculty? The more prepared a student is the less that we have to teach them.
3. Could best be taken to mean what is best for the nation? Wouldn't educating a diverse workforce produce a faculty better able to connect with the US population? Is it really best for our nation that half of our PhDs in mathematics are international?
4. It may come as a surprise to graduate programs but our graduates do not all become mathematical researchers. What are the best characteristics in an applicant to link to the many job opportunities that are now available?

The above points are in line with a recent publication [3], where the summary states: "Over the course of their education, graduate students become involved in advancing the frontiers of discovery, as well as in making significant contributions to the growth of the US economy, its national security, and the health and well-being of its people."

As a start, I would like to pass on to all of my colleagues the wisdom that my mother passed on to me. "Say hello, my dear child." "Saluda, mi hijito." "Saluda, mi hijita."

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Author photo courtesy of William Yslas Vélez.

ABOUT THE AUTHOR

After 50 years at The University of Arizona (UA), nine as a student and 41 as member of the faculty, Bill stepped down. Bill has high hopes that his interest in promoting the study of mathematics among UA students, especially minority students, will continue.

William (Bill) Yslas Vélez and his wife Bernice enjoy his retirement party from UA.

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Thomas Scott Fiske, PhD founder of the American Mathematical Society, November 24, 1888

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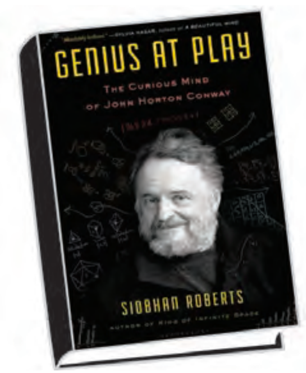
Genius at Play: The Curious Mind of John Horton Conway



A review by Elizabeth T. Milićević

Communicated by Thomas Garrity

Genius at Play: The Curious Mind of John Horton Conway
By Siobhan Roberts
Bloomsbury USA, 2015
480 pages
ISBN-13: 978-1620405932

In her biography of John H. Conway titled *Genius at Play*, Siobhan Roberts explains that the only medium through which Conway is capable of reaching out to other humans is through "a giant prosthetic carapace of mathematical knowledge and mathematical appetite" (p. 296). If there were a reliable way to document such mathematical encounters with Conway, and to apply a reasonable metric for the distance between encounters, I would wager that a significant number of us in the mathematical community would achieve a somewhat lower "Conway number" than we have "Erdős number." On their first trip to the United States from Belgrade, Serbia even my non-mathematician in-laws encountered Conway in his preferred natural habitat: the common room in the Princeton mathematics department. He somehow manages to both be everywhere and to have something to say to everyone; one of his USA/Canada Math Camp students deemed Conway "the closest thing the world has to a polymath" (p. 346).

Even so, I find the title of Roberts' book to be an imperfect reflection of the inspiring story she reveals. Perhaps the general public, having a rich fascination with the notion of a crazed mathematician as portrayed in a wide variety of media, will cling to the promise on the book cover of exploring a mathematical "genius," and thus be drawn to read Roberts' honest and detailed account of this intriguing character. There is sufficient support provided throughout this biography that Conway is indeed a genius, often in the form of direct quotations from other prominent research mathematicians who are in a position to evaluate his work. However, Conway's world view of mathematics, and his own position therein, is considerably more inclusive than the jacket cover of this biography reveals. As a working professional in the field, I find that the title does not work its mainstream "mathematical genius" magic on me. Once I started to read, however, I found a surprising and compelling resonance between my own methods for engaging with

The biography's title plays into public fascination by mathematical genius, reflecting only a small fraction of the story inside.

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my work, my profession, and myself and those of this curious Princeton professor.

The dominant first-person narrative of direct quotations gives the biography its true backbone. My own experience of the book and subsequent reaction to it are thus molded in large part by my direct response to the protagonist himself, and this review consequently weaves back and forth between a discussion of the writing itself and the impression the author's portrayal of Conway left upon me. I myself have never met Conway, and my impressions of him are exclusively a result of my experience reading Roberts' biography. It is intended as a great compliment to Roberts as a biographer that my opinion of the book is centered primarily around the subject himself, rather than its author.

There is absolutely no denying Conway's supernatural gift of being able to see patterns and connections that no one else would be able to see, and the biography explores Conway's primary mathematical contributions in great detail. Initially, when arriving to a point in which Roberts was poised to explain some actual mathematics, I held my breath and prepared for secondhand embarrassment to overcome me, as the non-mathematician's explanation of a theorem often over simplifies the mathematics or paints too broad a landscape in a language of sweeping exaggerations. To my great relief, the mathematics student and professor alike can read these passages in Roberts' account without blushing, since these sections are portrayed in the same raw, truthful, and intimate manner as the personal accounts. Indeed, I was delighted to actually learn some very intriguing mathematics by reading a book not penned by a fellow mathematician! For a beautiful summary of the main theorems of Conway which are highlighted in the book, I refer the reader to Joseph O'Rourke's meticulous mathematical review in the *College Math Journal* [1].

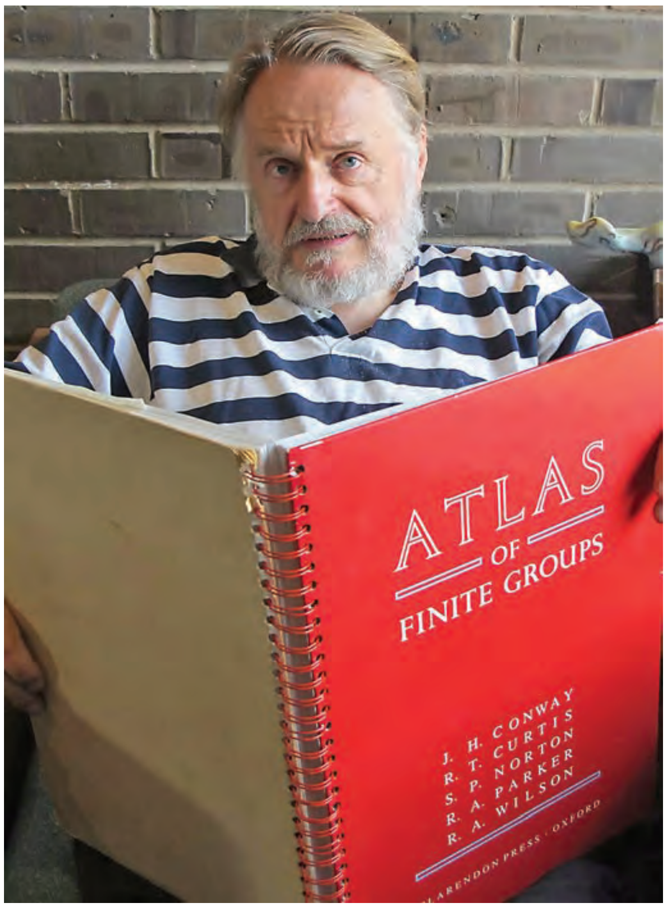

Figure 1: Conway's contribution to the classification of finite simple groups represented a turning point in his battle with the "imposter syndrome."

In sharp contrast to the implications of the biography's title, highlighting an eccentric personality and an eclectic body of work, Roberts' presentation humanizes Conway to an unexpected degree. We are provided a firsthand account of the plight of a working mathematician, as we are taken behind the scenes to witness the imperfect struggle of one mathematician's obsessive quest to reveal truth and its simplest possible explanation. We are introduced early on to Conway as a graduate student at Cambridge, where he spent the majority of his time not working on any of his first several thesis problems, but then frantically conjuring up a means to either cover up this fact or distract his advisor with some other interesting mathematical musings. Roberts directly addresses the realities of trying to do mathematics while working through depression, and simultaneously steering clear of the manic state, which lures one to the other side. Throughout his career, Conway also struggled greatly with the "imposter syndrome," working alongside mathematicians who Conway was sure viewed his professional accomplishments and mathematical fascinations as beneath them. Conway claimed momentary victory over the imposter syndrome when he discovered what we now call the "Conway group," an indisputably important contribution to the classification of the sporadic simple groups, now chronicled in the *Atlas of Finite Groups* proudly displayed in Figure 1. At that point, Conway vowed to stop feeling guilty about pursuing ideas that some mathematicians might consider recreational—he fully released himself to *floccinaucinihilipilificate*, a word he created that means "to engage in mathematics which others would deem worthless" (p. 98). He will think about absolutely anything, but when he thinks about anything, he thinks deeply. Although Conway claims to possess mathematical taste, he also chooses not to exercise it (p. 282), frittering away his time on studying what some might perceive as childish frivolities, but then occasionally beating the "grown-ups" at their own games (p. 196).

In crafting a mathematical argument, Conway's goal is to find the simplest explanation possible. These pursuits were typically the result of countless hours spent toiling over an inconceivable number of examples until the patterns emerged. Conway has an uncanny ability to sort through and process an incredible number of bifurcations on a single problem, elevating this messy case splitting to an art form (p. 332). As Conway explains, "It's a mistake to assume that what mathematicians do is esoteric, deep, and difficult. All the great discoveries are very simple" (p. 338). Conway does mathematics because he likes knowing things. He writes because he has sought to understand something and has finally distilled that understanding into the simplest possible terms. Professionally, the impetus to write is typically that we *know* something is true, and that we have found one explanation for *why* it is true. To Conway, however, a proof is not a synonym for understanding, and he always seeks the latter.

Figure 2: Conway seeks opportunities to talk mathematics to nearly anyone, but especially students like these participants in the "Modern Mathematics" International Summer School in Bremen, Germany.

Conway calls himself a "professional nonunderstander" (p. 264), claiming to be "confused at all times" (p.150). He even carries this permanent state of confusion into all of his classes and invited talks, categorically refusing to prepare beforehand. Roberts explains the rationale behind Conway's "more honest way to lecture, showing the false starts and 'stuckedness' that are crucial to the mathematical process" (p. 74). Ironically, somehow a man who dives headfirst into his lectures with minimal planning has provided me with a deep source of pedagogical self-reflection, as I acknowledge the value of uncovering the tracks we work so hard to hide in our own articles and textbooks, and thus seek ways to make the behind-the-scenes more transparent to our students and our research colleagues. Such contradictions were often the richest source for professional inspiration as I read this biography, their controversial nature whispering in my ear to turn one more page. For example, Conway is a firm believer that almost everyone is capable of doing mathematics (p. 369) and notoriously spends hours discussing mathematics with students, as seen in Figure 2. On the other hand, Conway simultaneously instructs a group of high school students at the conclusion of one of his USA/Canada Math Camp mini-courses to "take it as axiomatic that you are stupid" (p. 348), a divisive quote which nevertheless fully reflects his own mathematical self-image.

Roberts' biography of Conway is not a new release, having been published in 2015, and consequently there is an extensive sequence of beautiful and thorough reviews you can consult if you are trying to determine for yourself whether or not you should read this book. I am not an expert on finite groups, combinatorial game theory, or other fields on which Conway has made a lasting impression, so the impetus for me to continue turning the pages did not arise from a fascination with the specific mathematical results presented in the book. Rather, I found myself mesmerized by Conway's personality and tenacity and their compelling and often inspirational portrayal. I was stimulated by the self-examination inspired by reflecting upon Conway's many internal contradictions. I was haunted by meta questions that arose in Conway's excerpted monologues. Is mathematical knowledge created or discovered? (p. 52) Does the cathartic effect of math permit someone like Conway to completely forget the outside world? (p. 355)

Conway would be delighted that, up to this point, I have not mentioned what is arguably his most famous mathematical attribution. But what Roberts depicts as Conway's most despised creation permits the most salient closing analogy. Conway's "Game of Life" continues to tantalize and thrill those who are in search of specific constructions illustrating the theorem that the Game of Life is a Turing machine, or—as Conway prefers to explain—Life contains all the digits of π. Delightfully, Roberts' biography, much like the protagonist's Game of Life, permits the possibility that *everyone*—the casual mathematical spectator up through the professional mathematician—will find themselves repeatedly embedded and reflected somehow in Conway's story.

Reference

[1] Joseph O'Rourke, Review of *Genius at Play: The Curious Mind of John Horton Conway*, *College Math. J.* **46** (September 2015), 309–314.

Image Credits

Figure 1 courtesy of Siobhan Roberts.

Figure 2 courtesy of Dierk Schleicher and the 'Modern Mathematics' summer school 2015.

Author photo courtesy of Patrick Montero/Haverford College.

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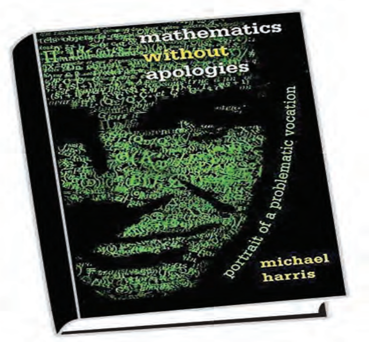
Elizabeth Milićević earned her PhD from the University of Chicago in 2009. Her research program studies flag varieties using the methods of algebraic combinatorics, representation theory, and geometric group theory. She has been supported by grants from the Association for Women in Mathematics, the Simons Foundation, the National Science Foundation, and the Australian Research Council.

BOOK REVIEW



Mathematics without Apologies

A Review by John McCleary

Communicated by Thomas Garrity

Mathematics without Apologies: Portrait of a Problematic Vocation
By Michael Harris
Princeton University Press, 2015
464 pages

"Is any wonder that, in popular culture's more serious precincts, the mathematician has become the romantic figure of choice?"

—*From* Mathematics without Apologies

In *A Mathematician's Apology*, G. H. Hardy asks: "Why is it really worth while to make a serious study of mathematics? What is the justification of a mathematician's life?" Michael Harris takes up these questions 75 years later in a world where Hardy would have felt alien. Nowadays mathematics, even pure mathematics, has made significant changes to the lives of everyone. Information is available instantly and securely almost everywhere. Economic forces influence individual lives more directly. Hardy would certainly recognize that mathematics is not well understood by most. Harris's unapologetic answers to Hardy's questions are placed in our "age when everything has to prove immediate profitability."

Harris bases his account on his own career, not as autobiography, but as raw material for his journalistic impulse. He also draws on his insatiable thirst for ideas—in philosophy, sociology, film, music, religion, and more. Frankly, to review this book in detail is beyond my pay grade. The torrent of associations Harris makes produces a "deliberately elusive style." But a book on mathematics that can leave many readers with many different impressions is welcome. Let me share mine.

How do we understand mathematical activity? Especially pure mathematics? I recall a spirited discussion of "What is mathematics?" in an institute's coffee room. The positions were *a philosophy*, for how could the most trivial computation require knowledge of the homotopy groups of spheres; *a chess game*, for we need not know what we are moving, only how to move it correctly, and cleverly; and *a garden*, for we plant ideas and coax them to grow and propagate. Harris's answer is in his subtitle: "Portrait of a Problematic Vocation." Problems are the stuff that determines what being a mathematician is about.

In the practice of mathematics, some activity (philosophical insight, chess move, or weeding) takes place—at a blackboard, in conversation, at a desk—after which mathematicians produce a proof, a notion that has evolved considerably since its inception in ancient Greece. A written proof, however, cannot be identified with what it means to do mathematics. Atiyah has written "I may think that I understand, but the proof is the check that I have understood, that's all. It is the last stage in the operation—an ultimate check—but it isn't the primary thing at all." Harris goes on to say that "answers are less important than how they change the way we look at the questions."

So what questions are the right ones? How does a mathematician find problems? The first chapter of the book opens with a quote from Hilbert's Paris 1900 ICM address—"to lift the veil behind which the future lies." Did the Hilbert problems, as he suggested, lead to "an advancement of science"? Here we have a chicken-and-egg problem. In the framework that Harris outlines in the book, Hilbert was a charismatic leader, whose followers found value in his suggestions. By working on these prob-

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lems individuals could increase their own stature, their *charisma*, in the mathematical community. Today there are many research programs that generate problems for groups of specialists. Harris is part of a number theory tribe, for which he has contributed significantly to the Langlands program. Generalizing from his own example, he describes mathematical socialization through the lens of Max Weber's sociological analysis of authority. Legal or traditional authority does not motivate mathematicians to prove theorems; charisma does. Harris introduces *routinized charisma*. For Weber this is how the charisma of a leader turns into a new bureaucracy. For mathematics, those with charisma shape mathematical activity—by the problems leaders work on or dispense and by the public value placed on such work. Success in mathematics begins when ability is identified, usually by a teacher or parent, training begins, and the milestones of an academic career are passed—PhD, post-doc, tenure, prizes, better positions—each step with enhanced charisma.

In the wider world charisma is rewarded, usually with wealth. Mathematicians and scientists generally appear to be outside this kind of valuation. For example, Gauss wrote that there is "a pure, disinterested joy in study." Harris focuses on a trilogy of motivations signaled by the words *good, truth, and beauty*. The good, called the "Golden Goose" by Stephen Shapin, is the promise that research in pure mathematics may one day bring unanticipated applications that benefit humankind. Hardy dismisses the "Golden Goose" by suggesting that it is the least interesting parts of mathematics that find their way into applications. The higher parts of mathematics are unsullied by applications. Hardy writes, just five years before Hiroshima, "No one has yet discovered any warlike purpose to be served by the theory of numbers or relativity, and it seems very unlikely that anyone will do so for many years."

Truth, the certitude of mathematical results, has been a philosophical touchstone since Descartes. Harris has a lot to say about the rise of interest in machine verification of mathematics and the danger of proofs that are so complex they defy human verification. There is great satisfaction in discovering a truth, old or new. But providing the bedrock of a philosophical program does not motivate most mathematicians.

That leaves beauty, the deepest source for Hardy, and the most challenging motivation for mathematical activity. Challenging because it calls into question how external support of mathematics is justified. Hardy tells us that "a mathematician, like a painter or poet, is a maker of patterns." Throwing mathematicians in with the painters and poets leads to justification of mathematics as an art. Harris situates his discussion of beauty in the notion of play. Play needs to take place in a "relaxed field," a notion introduced by ethologist Gordon Burghardt to describe animal behavior when not preoccupied with food, mating, territory, or predators—"the opposite of stress." Cantor's *Paradies* and Hausdorff's *Spielraum* are the relaxed fields of thought where mathematics arises. And when it does, it is a source of pleasure. Harris makes a good case for the mathematician's sense of beauty to be located in the rush of pleasure that accompanies mathematical activity. Today neuroscientists have identified the places activated in our brains by pleasure. In fact, in a small study employing fMRI imaging, the field A1 of the mOFC (medial orbito-frontal cortex) lights up when presented with beautiful mathematics. The mOFC is the same part of the brain that responds to other experiences of beauty.

In his *A Mathematician's Apology*, Hardy asks: "Why is it really worth while to make a serious study of mathematics?"

If mathematicians are motivated by play, how can the vocation be justified to those who support it? The golden eggs of pure mathematics are plentiful. Chapter 4 presents an introduction to mathematical finance as an example of the modeling of reality, and as a Faustian bargain. Students are attracted by high-paying positions as quants, and academic mathematicians train such students to think carefully, analytically, and critically about mathematics that will "make money for the firm." The pressure on the mathematical community to feed the needs of industry appears in several places in the book. Harris, from his charismatic position, participated in workshops about the future of mathematics hosted by the *decideurs*, who acknowledge the difficulty of measuring pure mathematical research as a resource—but measure they will, posing a threat to the relaxed field enjoyed by mathematicians.

In Hardy's time, mathematicians were immersed in the beginnings of World War II. The ethical challenges were clear, and of course many contributed to the war effort. For Hardy, however, "real mathematics," that is, pure mathematics, "has no effects on war." He was justifying not only his place in mathematics, but his response to the war. By contrast, 35 years later, in the midst of the Cold War, Stanislaw Ulam published his *Adventures of a Mathematician*, another account of what is it like to be a mathematician, in this case, an applied mathematician. His apology is brief:

"…I failed to realize fully the immense importance of nuclear armament and the influence it would have on the course of world events.… I would describe myself as having taken a middle course between completely naive idealism and extreme jingoism. I followed my instincts (or perhaps lack of instincts) and was mainly interested in the scientific aspects of the work."

Ulam's book is entertaining with its great cast of prominent mathematicians and physicists who led the A-bomb and H-bomb efforts—a relaxed field of a different sort.

Michael Harris takes up Hardy's question about the purpose of mathematics 75 years later in a world where Hardy would have felt alien.

For Harris, it is the "profoundly contingent" nature of mathematics that worries him, as he told an interviewer at CIRM. Without taking foundational questions seriously, mathematicians will be faced with machines that will produce a substitute for mathematics. The humanity of mathematics is indispensable, and it needs to be better understood and encouraged.

The other chapters of the book take up the humanity of mathematics with gusto. There is a bonus chapter in which Harris offers an insightful critical reading of Thomas Pynchon's novels organized around the conic sections. Chapter 6 focuses on the film *Rites of Love and Math* by Edward Frenkel and Riene Graves, in which a mathematical formula of love leads to the death of its discoverer, but not before he has tattooed it on his lover. Harris takes the film as an occasion to consider the perception of the (mad/martyr) mathematician by the film-going public, giving *Rites* a countercultural quality.

Chapter 7 goes to the place left empty by Atiyah—if a proof is the last stage of the operation, what is the first step? Much is made of the use of "scare quotes," a form of punctuation that indicates where an analogy is straining to communicate an unproved relation or an undefined object. This is the "what if?" that is often part of a mathematical conversation. The discussion covers a dizzying selection of topics, including a description of the Langlands program, higher category theory, and *avatars*—the reflections of one theory in another. Chapter 8 plays with the meaning of the word "trick" inside and outside mathematics. The playful nature of this discussion supports Harris's image of the relaxed field and beauty as pleasure. A quote of Steven Pinker is a good summary. Pinker describes music as "cheesecake," "unlike anything in the natural world because it is a brew of megadoses of agreeable stimuli which we concocted for the express purpose of pressing our pleasure buttons." Harris counts pure mathematics a cheesecake.

Woven through the book are the Greek letter chapters entitled "How to explain number theory at a dinner party." The Number Theorist N.T. explains a large swath of number theory at a dinner party to a Performing Artist P.A., who challenges Number Theorist in dialogue to higher levels of clarity. Performing Artist also provides a foil for Number Theorist's analogies of mathematical objects. From primes and the irrationality of their square roots through algebraic numbers, Galois theory, transcendental numbers, congruences, and elliptic curves, the theme of numbers as answers to questions about numbers is explored. The monologues and dialogues depict how Number Theorist thinks, a sort of sampler of the rest of the book.

Chapter 9 is my favorite. The excitement of being on the trail of an idea, an idea that came in a dream, is engagingly written. As in Villani's *Birth of a Theorem*, the reader is not expected to keep up with the mathematics, but the thrill is evident. I can add another dream result to the discussion—Ulam's obituaries in various newspapers report that his insight that shock waves would play a role in the design of the H-bomb came in a dream. The unconscious as a source of ideas is not new, but another piece of the puzzle of the sources of mathematics.

Chapter 10, "No Apologies," contains a recapitulation of the main points of the book. The narrative ends with an afterword in which Felix Hausdorff (aka Paul Mongré) appears. A poet, dramatist, philosopher, and mathematician, Hausdorff is a model for the kind of free thinker that Harris admires and whose example mathematicians can follow.

There are more than 70 pages of endnotes. I dutifully consulted them as I read, and I found many references that I will return to.

The goal of Harris's *Mathematics without Apologies* is not "to justify mathematics; no one needs another *Mathematician's Apology*." Nor is it to explain mathematics to "ordinary people" (as the British philosopher Crispin Wright would put it), but to tell "what it is like to be a mathematician, freely choosing a tradition to which to adapt, not to serve the Powerful Beings of market rationality …" The image I leave with is his final one, that each veil we turn aside has behind it another veil: mathematics is about problems and the problems that are born from their answers. The book is about "how hard it is to write a book about mathematics," and failing to tell us what mathematics is, Harris has "failed better," to paraphrase Samuel Beckett, and given us a rich, playful, and personal portrait that will raise significant questions for every reader, and give each of us even more problems to try to solve.

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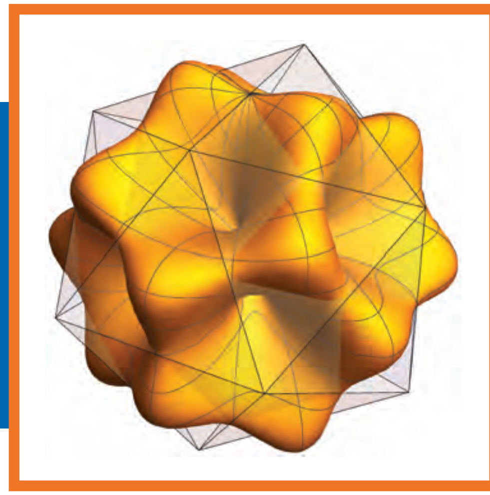
Photo of Hardy courtesy of the Master and Fellows of Trinity College Cambridge.
Photo of Harris By Béatrice Fernandez.
Author photo by Carlie Graves.


John McCleary

ABOUT THE AUTHOR

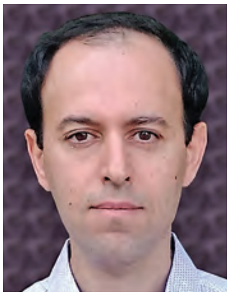
John McCleary teaches and writes mathematics. He also sings, and works at singing better.



2018 Fields Medals

The Fields Medal is awarded to recognize outstanding mathematical achievement for existing work and for the promise of future achievement. On August 1, 2018, the 2018 Fields Medals were awarded at the opening ceremony of the International Congress of Mathematicians (ICM) in Rio de Janeiro, Brazil. Following are the official prize citations issued by the International Mathematical Union. A future issue of the *Notices* will highlight the recipients of other awards given at ICM.

The Work of Caucher Birkar



Caucher Birkar

Caucher Birkar has made fundamental contributions to birational geometry in two particular areas: the *minimal model program* (MMP) and the boundedness of Fano varieties. The original MMP involves two kinds of projective varieties Y with so-called terminal singularities whose canonical divisors K have opposite properties: for a *minimal model* K is non-negative on curves on Y; while for a Fano fibering Y has a surjective morphism onto a lower dimensional projective variety with $-K$ relatively ample. The MMP attempts to construct for each smooth projective variety a birational map to either a minimal model or a Fano fibering.

Although the MMP is not always known to work, Birkar, jointly with Cascini, Hacon, and McKernan, made a stunning contribution: a special version of the MMP works for complex varieties of arbitrary dimension whose canonical divisor is either big or not pseudo effective, a situation which covers many important cases. They actually established the MMP for a wider class of singularities, which was essential for the induction on dimension in the proof, and it implies many important consequences, such as the finite generation of canonical rings of arbitrary smooth projective varieties. The MMP is now a fundamental tool which is used extensively.

It was Birkar who further proved that complex Fano varieties (i.e., Fano fiberings over a point) of arbitrary fixed dimension with terminal singularities are parametrized by a (possibly reducible) algebraic variety. Since these Fano varieties constitute one of the main outputs of MMP as applied to smooth projective varieties, their boundedness, previously considered unreachable, is fundamentally important. Birkar has settled the more general Borisov-Alexeev-Borisov conjecture building upon results by Hacon, McKernan, Xu, and others. Birkar's boundedness will be crucial as a paradigm for the full MMP.

Biographical Sketch

Caucher Birkar's dedication to the winding and multidimensional world of algebraic geometry, with its ellipses, lemniscates, Cassini ovals, among so many other forms defined by equations, granted him the Philip Leverhulme Prize in 2010 for exceptional scholars whose greatest achievement is yet to come. Eight years later, the Cambridge University researcher joins the select group of Fields Medal winners at the age of forty. Birkar, who was recognized earlier this year with the Whitehead Prize of the London Mathematical Society, was born in 1978 in Marivan, a Kurdish province in Iran bordering Iraq with about 200,000 inhabitants. His curiosity was awakened by algebraic geometry, an area that had attracted the attention of Omar Khayyam (1048–1131) and Sharaf al-Din al-Tusi (1135–1213) in previous centuries.

After graduating in mathematics from Tehran University, Birkar sought refugee status in the United Kingdom, where he became a British citizen. In 2004, he completed his PhD at the University of Nottingham with the thesis "Topics in Modern Algebraic Geometry." Throughout his trajectory, birational geometry has stood out as his main area of interest. He has devoted himself to the fundamental aspects of key problems in modern mathematics—such as minimal models, Fano varieties, and singularities. His theories have solved long-standing conjectures. In 2010 Birkar wrote (alongside Paolo Cascini, Imperial College London, Christopher Hacon, University of Utah, and James McKernan, University of California, San Diego) an article called "Existence of Minimal Models for Varieties of General Log Type" that revolutionized the field. The article earned the quartet the AMS Moore Prize in 2016.

COMMUNICATION

The Work of Alessio Figalli

Alessio Figalli

ALESSIO FIGALLI has made multiple fundamental advances in the theory of optimal transport, while also applying this theory in novel ways to other areas of mathematics. Only a few of his numerous results in these areas are described here.

Figalli's joint work with De Philippis on regularity for the Monge-Ampère equation is a groundbreaking result filling the gap between gradient estimates discovered by Caffarelli and full Sobolev regularity of the second derivatives of the convex solution of the Monge-Ampère equation with merely bounded right-hand side. The result is almost optimal in view of existing counterexamples. It has direct implications on regularity of the optimal transport maps and on regularity to semigeostrophic equations.

Figalli initiated the study of the singular set of optimal transport maps and obtained the first definite results in this direction: he showed that it has null Lebesgue measure in full generality. He has also given significant contributions to the theory of obstacles problems, introducing new methods to analyze the structure of the free boundary.

Figalli and his coauthors have also applied optimal transport methods in a striking fashion to obtain sharp quantitative stability results for several fundamental geometric inequalities, such as the isoperimetric and Brunn-Minkowski inequalities, without any additional assumptions of regularity on the objects to which these inequalities are applied; the methods are also not reliant on Euclidean symmetries, extending in particular to the Wulff inequality to yield a quantitative description of the low-energy states of crystals.

Biographical Sketch

Born in Naples, Italy, on April 2, 1984, Alessio Figalli discovered an interest in science later than some. Until high school, he was more interested in playing football. Preparation for the International Mathematical Olympiad (IMO) awakened his interest in the subject, and he chose to study math when he joined the Scuola Normale Superiore di Pisa. Figalli completed his PhD in 2007 at the École Normale Supérieure de Lyon in France, with the guidance of Fields Medal laureate Cédric Villani. He has worked at the French National Center for Scientific Research, École Polytechnique, the University of Texas, and ETH Zürich. A specialist in calculating variations and partial differential equations, he was invited to speak at the 2014 ICM in Seoul. He has won several awards, including the Peccot-Vimont Prize (2011) and Cours Peccot (2012) of the College of France, the European Mathematical Society Prize (2012), the Stampacchia Gold Medal of the Italian Mathematical Union (2015), and the Feltrinelli Prize (2017).

The Work of Peter Scholze

Peter Scholze

PETER SCHOLZE has transformed arithmetic algebraic geometry over p-adic fields.

Scholze's theory of perfectoid spaces has profoundly altered the subject of p-adic geometry by relating it to geometry in characteristic p. Making use of this theory, Peter Scholze proved Deligne's weight-monodromy conjecture for complete intersections. As a further application, he constructed Galois representations that are attached to torsion cohomology classes of locally symmetric spaces, resolving a long-standing conjecture.

Scholze's version of p-adic Hodge theory extends to general p-adic rigid spaces. Together with Bhatt and Morrow, Scholze developed an integral version of p-adic Hodge theory that establishes a relation between the torsion in Betti and crystalline cohomologies.

On the way to the revolution that he launched in arithmetic geometry, Scholze took up a variety of topics that he reshaped, such as algebraic topology and topological Hochschild homology.

Scholze developed new cohomological methods. Beyond p-adic fields, Scholze's vision of a cohomology theory over the integers has become a guideline that fascinates the entire mathematical community.

Biographical Sketch

Peter Scholze was born in Dresden, Germany, on December 11, 1987. At thirty years old, he is already considered by the scientific community as one of the most influential mathematicians in the world. In 2012, at age twenty-four, he became a full professor at the University of Bonn. Scholze impresses his colleagues with the intellectual ability he has shown since was a teenager, when he won four medals—three gold and one silver—at the International Mathematical Olympiad (IMO). The German mathematician completed his university graduate and master's in record time—five semesters—and gained notoriety at the age of twenty-two, when he simplified a complex mathematical proof of number theory from 288 to 37 pages. A specialist in arithmetic algebraic geometry, he stands out for his ability to understand the nature of mathematical phenomena and to simplify them during presentations.

At age sixteen, still a student at the Heinrich-Hertz-Gymnasium—a school with a strong scientific focus—Scholze decided to study Andrew Wiles's solution to Fermat's Last Theorem. Faced with the complexity of the result, he realized that he was on the right track in choosing mathematics as a profession. He was a guest speaker at ICM 2014 in Seoul, South Korea, and was a plenary member this year at ICM 2018. Scholze is well decorated for his contributions to arithmetic algebraic geometry, and he has collected several major mathematics awards, such as the Prix and Cours Peccot of the College de France (2012), the SASTRA Ramanujan Prize (2013), a Clay Research Award

(2014), the Frank Nelson Cole Prize of the AMS (2015), the Fermat Prize (2015), the Ostrowski Prize (2015), the European Mathematical Prize (2016), and the Leibniz Prize (2016). In 2018 he was appointed a director of the Max Planck Institute for Mathematics in Bonn.

The Work of Akshay Venkatesh

Akshay Venkatesh

Akshay Venkatesh has made profound contributions to an exceptionally broad range of subjects in mathematics, including number theory, homogeneous dynamics, representation theory, and arithmetic geometry. He solved many long-standing problems by combining methods from seemingly unrelated areas, presented novel viewpoints on classical problems, and produced strikingly far-reaching conjectures.

What follows is a small sample of his major achievements.

Venkatesh introduced a general and unifying technique based on representation theory and homogeneous dynamics in the subconvexity problem for L-functions and (partly in collaboration with Michel) used these ideas to give a complete treatment of all cases of subconvexity for $GL(2)$ over number fields.

He made major progress on the local-global principle for the representations of one quadratic lattice by another, in joint work with Ellenberg.

In joint work with Einsiedler, Lindenstrauss, and Michel, Venkatesh proved equidistribution of the periodic torus orbits in $SL(3, Z)\backslash SL(3, R)$ that are attached to the ideal classes of totally real cubic number fields as the discriminant tends to infinity.

Venkatesh established effective equidistribution of periodic orbits of many semisimple groups both in the local and adelic settings, in joint work with Einsiedler, Margulis, and in part with Mohammadi.

With Ellenberg and Westerland, Venkatesh established significant special cases of the Cohen–Lenstra conjectures concerning class groups in the function field setting.

Biographical Sketch

Conquering the greatest honor among the world's mathematicians before the age of forty is a notable accomplishment, but the life of Akshay Venkatesh was already marked with precocious feats. Born in New Delhi in 1981 and raised in Australia, Venkatesh became a medalist at the International Mathematical Olympiad when he was twelve years old. This propelled him into the world of mathematics and kickstarted his illustrious career. He began his bachelor's degree in mathematics and physics at the University of Western Australia at the age of thirteen, and seven years later, age twenty, had completed his PhD at Princeton University. Venkatesh was appointed [as a C.L.E. Moore instructor] at the Massachusetts Institute of Technology (MIT), a prestigious position offered to recent graduates in the area of pure mathematics, previously occupied by prominent figures such as John Nash. Upon leaving MIT in 2004, he became a Clay Research Fellow and was appointed associate professor at the Courant Institute of Mathematical Sciences at New York University. He became a professor at Stanford University at the age of twenty-seven and, as of this year, is a faculty member at the Institute for Advanced Study (IAS).

Venkatesh researches number theory—an area that deals with abstract issues and had no known application until the arrival of cryptography in the late 1970s—and he roves with ease through related topics, such as theory of representation, ergodic theory, and automorphic forms. Armed with a meticulous, investigative, and creative approach to research, detecting impressive connections between diverse areas, Venkatesh's contributions are fundamental to several fields of research in mathematics. He has won several accolades for his work, including the Salem Prize (2007), the SASTRA Ramanujan Prize (2008), the Infosys Prize (2016), and the Ostrowski Prize (2017). Previously a guest speaker at the 2010 ICM, Venkatesh was invited to speak at ICM 2018.

Photo Credits

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Photo of Akshay Venkatesh by Rod Searcey.

Notes from my First Press Conference, at ICM 2018

Frank Morgan

From his trip to the 2018 International Congress of Mathematicians (ICM) in Rio de Janeiro, Brazil, *Notices* Editor-in-Chief Frank Morgan offers a firsthand account of the ICM prizewinners' press conference featuring Fields Medal winners Akshay Venkatesh, Alessio Figalli, and Peter Scholze, along with Gauss prize winner David Donoho, Chern medal winner Masaki Kashiwara, and Nevanlinna Prize winner Constantinos Daskalakis.

As Editor-in-Chief of *Notices*, I got to attend my first press conference at the 2018 International Congress of Mathematicians. Fields Medalist Alessio Figalli was asked how his advisor Cedric Villani reacted when he told him the news of his win. Villani is himself a Fields Medalist and now also an elected member of the French Parliament. Figalli responded something like this:

“We are not allowed to share the news, but there are some exceptions. Now that Villani is a member of Parliament, it sometimes takes him days to respond to email messages. I sent him an email message requesting a phone number. He responded within an hour. He must have known that something was up. When I called, he was very happy to hear the news.”

When I talked to Villani later, he verified the story and added:

“When Alessio called, I was sitting on the floor of Parliament and had to excuse myself to take the call.”

I put forward a question to all the prize winners about the moment when they realized they had solved a problem. Figalli said he got a key idea one night at 1 am after going out for beers with friends.

Another reporter asked for a comparison of the joy of winning a prize with the joy of doing math. Figalli said the prize is a “peculiar joy, great, but something you cannot share.” Constantinos Daskalakis, winner of the Nevanlinna Prize, said that recognition is a nice honor, but that finding truth after great struggle is a unique joy. Fields Medalist Akshay Venkatesh agreed that “we're not in this for the prize”and wished he could share it. Gauss prize winner David Donoho said he had a different perspective having received an award for his whole career: “Solving a problem is a short-lived joy, not equal to the satisfaction of seeing your ideas grow up and have a large impact.”

I asked Daskalakis afterwards whether they really had inscribed his name on the edge of his medal, and he showed us that they had done so *in Greek*, at his request.

Chair Marcelo Viana features awardees at the ICM 2018 closing ceremony.

Photo Credit

ICM photo is in the public domain.

Backlog of Mathematics Research Journals

Journal (Print and Electronic)	Number issues per Year	Approximate Number Pages per Year	2017 Median Time (in Months) from: Submission to Final Acceptance	Acceptance to Print	Acceptance to Electronic Posting	Current Estimate of Waiting Time between Submission and Publication (in Months): Print	Electronic
Acta Math.	4	800	NA	6	4	NA	NA
Adv. in Appl. Math.	10	1608	9	2	0.9	11	11
Adv. Math.	18	18052	16	1	0.8	21	19
Algebr. Geom. Topol.	7	3700	9	7	5	16	14
Algebra Number Theory	10	2500	9	7	5	16	14
Amer. J. Math.	6	1728	NA	NA	NA	16–18	15–17
Anal. PDE	8	2100	10	5	3	14	12
Ann. Appl. Probab.	6	3900	11	10	10.5	19.5	19.5
Ann. Inst. H. Poincare Anal. Non Lineaire	7	1948	10	12	0.7	22	14
Ann. K-Theory	4	800	9	11	9	18	16
Ann. of Math. (2)	6	2100	14	5	3	5	4
Ann. Polon. Math	6	590	7	1.2	2.1	8	6
Ann. Probab.	6	4800	13	15.5	15.5	27	27
Ann. Pure Appl. Logic	12	2200	16	4	1	22	20
Ann. Statist.	6	2760	10	12.5	12.5	27	27
Appl. Anal.	16	2944	3.5	12	0.7	18.1	5.6
Appl. Comput. Harmon. Anal.	6	1168	11	16	0.4	11	11
Appl. Math. Comput.	24	7520	9	1	0.9	10	10
Ark. Mat.	2	400	9	6	4	15	13
Automatica J. IFAC	12	4368	13	2	2.2	16	16
Balkan J. Geom. Appl.	2	240	5	5	3	8	6
Bernoulli	4	3950	9	15.5	15.5	24	24
Bull. Aust. Math. Soc.	6	1056	0.6	7.5	3	9	4
Bull. Lond. Math. Soc.	6	1152	7.7	4	1.3	11.7	9

The Backlog of Research Journals is reported each year in the November issue of the *Notices*. The journals covered in this report are representative of mathematics and are from publishers who have agreed to participate and who continue to provide backlog information. Publishers whose journals are not currently included can request that their journals be added. Such requests should be made in e-mail to Marcia Almeida, backlogreport@ams.org. To be considered for inclusion in the backlog report, a journal must be on the list of journals receiving cover-to-cover treatment in *Mathematical Reviews* (www.ams.org/msnhtml/serials.pdf).

Once a publisher's journals are accepted for inclusion, the publisher must designate a contact person or persons to supply data about the journals to the AMS. While the AMS makes every effort to obtain the data from the designated contacts, if data about a journal is not supplied, then that journal will not appear in the backlog report.

Journal (Print and Electronic)	Number issues per Year	Approximate Number Pages per Year	2017 Median Time (in Months) from: Submission to Final Acceptance	Acceptance to Print	Acceptance to Electronic Posting	Current Estimate of Waiting Time between Submission and Publication (in Months): Print	Electronic
Bull. Sci. Math.	8	936	9	4	1.8	16	10
Canad. J. Math.	6	1440	6	9	4	18	13
Canad. Math. Bull.	4	896	5	10	2	14	7
Combinatorica	6	1500	8	27	14	22	10
Commun. Appl. Math. Comput. Sci.	2	250	9	6	4	14	12
Commun. Pure Appl. Anal.	6	2400	4	3	1	7	5
Complex Var. Elliptic Equ.	12	1800	5.5	6	1	15.1	5.5
Compos. Math.	12	2760	10.4	6.1	4.3	15.7	14.7
Comput. Aided Geom. Design	9	828	6	2	0.5	5	5
Comput. Geom.	9	588	11	5	2.4	13	12
Comput. Math. Appl.	24	6208	6	3	0.9	10	7
Constr. Approx.	6	1200	10	10	2	16	9
Differential Geom. Appl.	6	1588	7	2	0.7	11	9
Discrete Appl. Math	18	4408	13	4	1	20	14
Discrete Comput. Geom.	8	2000	8.9	2.9	1	12	9.5
Discrete Contin. Dyn. Syst.	12	6500	5	2	1	7	6
Discrete Contin. Dyn. Syst. Ser. B	10	4100	5	2	1	7	6
Discrete Math.	12	3328	10	2	1.1	13	12
Discrete Optim.	4	704	12	9	0.9	21	17
Duke Math. J.	18	3600	14	8	5	22	19
Dyn. Syst.	4	736	7.4	9.3	1	16.5	7.8
Eur. J. Math.	4	1200	5.5	4	1	10	8
European J. Combin.	8	1960	10	1	1	11	11
Expo. Math.	4	512	8	8	0.2	22	20
Finite Fields Appl.	6	1904	9	1	0.5	8	8
Found. Comput. Math.	6	1500	10	12	2	19	12
Geom. Dedicata	6	1282–1400	NA	6	0.3	11	NA
Geom. Topol.	7	3700	12	10	8	22	20
Graphs Combin.	6	1399	6.7	1.5	0.6	6	5
Historia Math.	4	472	7	4	0.8	17	14
Homology Homotopy Appl.	2	800	6.5	12.2	9	17	14
Houston J. Math.	4	1400	3	18	17	21	20
Illinois J. Math.	4	600	6	6	4	6	4
Indag. Math. (N.S.)	6	1304	8	4	1.5	14	11
Indiana Univ. Math. J.	6	2100	23	18	18	24	5
Infor. Process. Lett.	12	720	12	1	0.3	13	13
Inform. and Comput.	6	2204	10	10	5.2	12	11
Involve	5	900	7	7	5	15	13
Israel J. Math.	6	4500	8	13	11	17	14
J. Algebra	24	12688	10	1	0.3	13	10
J. Amer. Math. Soc.	4	1200	17.4	10.2	1.3	26.6	21.5
J. Anal. Math.	3	2400	6	30	30	27	24
J. Approx. Theory	12	1240	10	1	0.4	14	13
J. Aust. Math. Soc.	6	888	5	14	5	22	13
J. Combin. Theory Ser. A	8	2564	14	1	0.7	16	16
J. Combin. Theory Ser. B	6	2008	24	3	0.6	33	30
J. Complexity	6	720	7	4	0.4	9	6

Journal (Print and Electronic)	Number issues per Year	Approximate Number Pages per Year	2017 Median Time (in Months) from: Submission to Final Acceptance	Acceptance to Print	Acceptance to Electronic Posting	Current Estimate of Waiting Time between Submission and Publication (in Months) Print	Electronic
J. Comput. Appl. Math.	18	7348	8	3	0.5	12	10
J. Comput. System Sci.	8	1980	14	4	1.5	12	8
J. Convex Anal.	4	1400	8	9	2	13	7
J. Difference Equ. Appl.	12	2112	3.5	6.8	0.8	8.6	5.4
J. Differential Equations	24	15240	7	2	0.5	10	9
J. Differential Geom.	9	1630	10	15	15	18	18
J. Discrete Algorithms	6	368	15	2	0.3	15	15
J. Eur. Math. Soc. (JEMS)	12	3000	7	24	18	24	18
J. Funct. Anal.	24	9496	9	2	0.3	11	8
J. Geom. Phys.	12	3784	9	3	0.5	8	8
J. Ind. Manag. Optim.	4	2150	6	6	1	12	7
J. Lie Theory	4	1200	6	9	1	15	7
J. Log. Algebr. Program.	8	1420	12	3	0.7	12	11
J. Lond. Math. Soc. (2)	6	1600	8	3.4	1.2	10.4	9.2
J. Math. Anal. Appl.	24	18480	6	1	0.3	8	7
J. Math. Ecom.	6	944	10	2	0.7	12	11
J. Math. Phys.	12	9000	5.5	1	0.8	7	6
J. Math. Pures Appl. (9)	12	1860	4	8	2.3	12	9
J. Math. Soc. Japan	4	1320	9	16	16	18	9
J. Mod. Dyn.	1*	600	7	6	1	13	8
J. Multivariate Anal.	10	2204	10	1	0.4	10	10
J. Nonlinear Math. Phys.	4	650	2.4	6	3	7.8	7.1
J. Number Theory	12	5480	7	1	0.4	9	7
J. Pure Appl. Algebra	12	3208	9	4	0.4	13	10
J. Statist. Plann. Inference	12	1160	8	2	0.5	15	11
J. Symbolic Comput.	6	2304	8	7	2.6	7	10
J. Symbolic Logic	4	1576	11	11	8	16	14
J. Théor. Nombres Bordeaux	3	1000	7	13	10	NR	NR
J. Topol.	4	1120	11.4	3.4	1.9	14.8	13.3
Kodai Math. J.	3	700	5	7	7	12	12
Kyoto J. Math.	4	900	6	14	13	28	23
Linear Algebra Appl.	24	8552	7	1	0.2	10	8
Linear Multilinear Algebra	12	2592	4.6	7.3	0.7	15.8	5.1
Math. Comp.	6	3000	11	15.8	8.1	25.6	18.1
Math. Control Signals Systems	4	500	9	3.5	0.8	9.5	6
Math. Mech. Complex Syst.	4	400	5	5	3	9	7
Math. Oper. Res.	4	1400	15.8	10.8	5.9	20.5	17
Math. Res. Lett.	6	1300	7	9	9	9	9
Math. Scand.	4	640	8	21	20	26	25
Math. Social Sci.	6	658	11	4	0.6	13	11
Mathematika	3	825	6.8	6.1	2.9	11.2	9.7
Mem. Amer. Math. Soc.	6	4600	15.1	29	NA	41	NA
Methods Appl. Anal.	4	400	5	4	4	7	7
Michigan Math. J.	4	896	8	10	9	12	11
Multiscale Model. Simul.	4	1900	9.5	5.7	3.7	15.2	13.2
Nagoya Math. J.	4	820	9	14	2.1	22	11
Nonlinear Anal.	18	4736	4	2	1	6	6

Journal (Print and Electronic)	Number issues per Year	Approximate Number Pages per Year	2017 Median Time (in Months) from: Submission to Final Acceptance	Acceptance to Print	Acceptance to Electronic Posting	Current Estimate of Waiting Time between Submission and Publication (in Months): Print	Electronic
Nonlinear Anal. Hybrid Syst.	4	1200	10	2	1.2	13	13
Nonlinear Anal. Real World Appl.	6	2648	8	1	1	9	9
Notre Dame J. Form. Log.	4	632	9	31	29	17	14
Osaka J. Math.	4	850	7	16	16	22	22
Pacific J. Math.	12	3000	9	5	3	15	13
Proc. Amer. Math. Soc.	12	5240	5.1	9.2	4.7	14.4	10.1
Proc. Lond. Math. Soc. (3)	12	3200	9.8	4.2	2	13	11.8
Quantum Topol.	4	800	9	14	12	14	10
Quart. Appl. Math.	4	800	1.6	7.7	1.2	10.8	4.9
Real Anal. Exchange	2	500	4	9	7	13	5
Semigroup Forum	6	1300	9	8	1	16	10
SIAM J. Appl. Math.	6	2325	7.4	5.3	3.3	12.7	10.7
SIAM J. Comput.	6	1950	15.4	5.7	3.7	21.1	19.1
SIAM J. Control Optim.	6	4250	13.3	5.3	3.3	18.6	16.6
SIAM J. Discrete Math.	4	2800	12.8	5.8	3.8	18.6	16.6
SIAM J. Math. Anal.	6	5250	9.6	4.5	3.5	14.1	13.1
SIAM J. Matrix Anal. Appl.	4	1625	8.9	5.2	3.2	14.1	12.1
SIAM J. Numer. Anal.	6	3225	10	4.5	3.5	14.5	13.5
SIAM J. Optim.	4	2675	10.6	5.6	3.6	16.2	14.2
SIAM J. Sci. Comput.	6	5800	10	4.4	3.4	14.4	13.4
SIAM Rev.	4	900	11.3	10	9	21.3	20.3
Stochastic Process. Appl.	12	4232	10	7	0.6	21	14
Theoret. Comput. Sci.	48	5692	10	4	0.5	12	11
Topol. Methods Nonlinear Anal.	4	1600	7	12	10	12	10
Topology Appl.	18	5040	8	4	3.4	9	9
Trans. Amer. Math. Soc.	12	8880	9.2	21.4	14.3	26.7	21.7

Journal (Electronic)	Number of Articles Posted in 2017	2017 Median Time (in days) from: Submission to Final Acceptance	Acceptance to Posting	Format(s)
Abstr. Appl. Anal. www.hindawi.com/journals/aaa/	32	79	34	html, pdf, ps, dvi, tex, ePUB
Adv. Math. Commun. aimsciences.org/journal/1930-5346	62	90	20	html, pdf
Appl. Math. E-Notes www.math.nthu.edu.tw/~amen/	30	168	233	pdf
Conform. Geom. Dyn. www.ams.org/publications/journals/journalsframework/ecgd	14	210	48	pdf
C. R. Math. Acad. Sci. Paris www.journals.elsevier.com/comptes-rendus-mathematique	174	149	47.6	html, pdf
Differ. Geom. Dyn. Syst. www.mathem.pub.ro/dgds/	13	100	180	pdf
Differ. Uravn. Protsessy Upr. www.math.spbu.ru/diffjournal/EN/about.html	35	35	15	html, pdf, tex, doc
Discrete Math. Theor. Comput. Sci. dmtcs.episciences.org/	56	284	17	pdf
Electron. J. Combin. www.combinatorics.org/	224	269	15	pdf
Electron. J. Differential Equations ejde.math.txstate.edu	312	67	8	pdf, tex
Electron. J. Qual. Theory Differ. Equ. www.math.u-szeged.hu/ejqtde/	100	153	27	pdf
Electron. Res. Announc. Math. Sci. aimsciences.org/journal/1935-9179	13	30	14	html, pdf
Electron. Trans. Numer. Anal. etna.ricam.oeaw.ac.at/submissions/	20	301.5	53	pdf
ESAIM Control Optim. Calc. Var. www.esaim-cocv.org/	53	219	261	html, pdf
ESAIM Math. Model. Numer. Anal. www.esaim-m2an.org/	69	227	242	html, pdf
ESAIM Probab. Stat. www.esaim-ps.org/	17	218	128	html, pdf
Int. J. Math. Math. Sci. www.hindawi.com/journals/ijmms/	20	67	36	html, pdf, ps, dvi, tex, ePUB
Int. J. Stoch. Anal. www.hindawi.com/journals/ijsa/	4	61	22	html, pdf, ps, dvi, tex, ePUB
Integers www.integers-ejcnt.org	69	388	19	pdf
Inverse Probl. Imaging aimsciences.org/journal/1930-8337	60	110	14	html, pdf
J. Appl. Math. www.hindawi.com/journals/jam/	38	81	36	html, pdf, ps, dvi, tex, ePUB
J. Integer Seq. cs.uwaterloo.ca/journals/JIS/	80	139	8	html, pdf, ps, dvi, tex
Math. Biosci. Eng. aimsciences.org/journal/1551-0018	90	120	20	html, pdf
Netw. Heterog. Media aimsciences.org/journal/1556-1801	28	80	14	html, pdf
Open Math. openmathematics.com/	140	169	60	html, pdf
Proc. Amer. Math. Soc. Ser. B www.ams.org/publications/journals/journalsframework/bproc	6	207	240	pdf

Journal (Electronic)	Number of Articles Posted in 2017	2017 Median Time (in days) from: Submission to Final Acceptance	Acceptance to Posting	Format(s)
Reliab. Comput. interval.louisiana.edu/reliable-computing-journal/	18	250	0	pdf
Represent. Theory www.ams.org/publications/journals/journalsframework/ert	20	252	42	pdf
Sém. Lothar. Combin. www.mat.univie.ac.at/~slc/	93	95	136	pdf, ps, dvi, tex
SIAM J. Appl. Algebra Geom. epubs.siam.org/journal/siaga	32	216	91	pdf
SIAM J. Appl. Dyn. Syst. epubs.siam.org/journal/siads/	74	207	106	pdf
SIAM J. Financial Math. epubs.siam.org/journal/sifin/	31	396	102	pdf
SIAM J. Imaging Sci. epubs.siam.org/journal/siims	76	186	112	pdf
SIAM/ASA J. Uncertain. Quantif. epubs.siam.org/journal/juq	48	315	99	pdf
Theory Appl. Categ. www.tac.mta.ca/tac/	46	333	5	pdf
Theory Comput. theoryofcomputing.org	20	403	221	html**, pdf, ps, tex
Trans. Amer. Math. Soc. Ser. B www.ams.org/publications/journals/journalsframework/btran	5	127	146	pdf

NR means no response received. NA means not available or not applicable.
*One volume in any year, no issues.
**Applies to abstract, bibliography, author info, but not to full article.

AMS DEPARTMENT CHAIRS WORKSHOP

Tuesday, January 15, 2019
8:00 am – 6:30 pm

Marriott Inner Harbor Hotel
Baltimore, MD

What makes a chair different than any other engaged faculty member in the department? This workshop examines the chair's role in leading a department. The day will be structured to include and encourage networking and sharing of ideas amongst participants and will include four sessions:

1. Reassessing the relationship between pure and applied mathematics.
2. Math in the data movement.
3. Professional development and evaluating faculty.
4. The next step: moving to higher administration.

Join mathematical sciences department chairs and leaders for this annual one-day workshop.

Registration fee: $200
Includes lunch and post-workshop reception.

If you are interested in attending, please register online by December 19th at http://bit.ly/2MHngPu.

Workshop leaders will be:

Malcolm Adams, *University of Georgia*
Gloria Marí-Beffa, *University of Wisconsin-Madison*
Douglas Mupasiri, *University of Northern Iowa*
Jennifer Zhao, *University of Michigan-Dearborn*

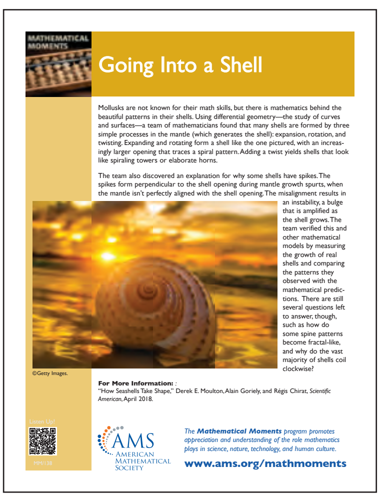
AMS AMERICAN MATHEMATICAL SOCIETY
Advancing research. Creating connections.

Inside the AMS

2019 Class of Fellows of the AMS Selected

A list of those who have been selected for the 2019 Class of Fellows of the AMS will appear on the AMS website prior to November 1, 2018. The list will be located at: www.ams.org/profession/new-fellows.

From the AMS Public Awareness Office



MATHEMATICAL MOMENTS

Going Into a Shell

Mollusks are not known for their math skills, but there is mathematics behind the beautiful patterns in their shells. Using differential geometry—the study of curves and surfaces—a team of mathematicians found that many shells are formed by three simple processes in the mantle (which generates the shell): expansion, rotation, and twisting. Expanding and rotating form a shell like the one pictured, with an increasingly larger opening that traces a spiral pattern. Adding a twist yields shells that look like spiraling towers or elaborate horns.

The team also discovered an explanation for why some shells have spikes. The spikes form perpendicular to the shell opening during mantle growth spurts, when the mantle isn't perfectly aligned with the shell opening. The misalignment results in an instability, a bulge that is amplified as the shell grows. The team verified this and other mathematical models by measuring the growth of real shells and comparing the patterns they observed with the mathematical predictions. There are still several questions left to answer, though, such as how do some spine patterns become fractal-like, and why do the vast majority of shells coil clockwise?

©Getty Images.

For More Information: :
"How Seashells Take Shape," Derek E. Moulton, Alain Goriely, and Régis Chirat, *Scientific American*, April 2018.

Listen Up!

MM/138

AMS American Mathematical Society

*The **Mathematical Moments** program promotes appreciation and understanding of the role mathematics plays in science, nature, technology, and human culture.*

www.ams.org/mathmoments

Mathematical Moments

Recent topics include the mechanical forces behind the shapes of shells, the Netflix Challenge, *Hidden Figures*, and bus bunching. You can hear a podcast interview with Derek Moulton, University of Oxford, on the shapes of shells, as well as many other interviews, and download the free PDFs at www.ams.org/mathmoments.

AMS on Social Media

We invite AMS members to follow AMS on Facebook (facebook.com/amermathsoc), Twitter (twitter.com/amermathsoc), YouTube (youtube.com/user/amermathsoc), LinkedIn (linkedin.com/company/American-mathematical-society/) or Instagram (Instagram.com/amermathsoc/) to:

- See our posts as one way to keep abreast of mathematics, people, events, and AMS news
- Like, comment, share, and retweet posts to be an "ambassador" for the AMS and mathematics.

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
paoffice@ams.org

AMS Congressional Fellowship

The AMS, in conjunction with the American Association for the Advancement of Science (AAAS), will sponsor a Congressional Fellow from September 2019 through August 2020. The Fellow will spend the year working on the staff of a Member of Congress or a congressional committee, as a special legislative assistant in legislative and policy areas requiring scientific and technical input.

The Fellowship is designed to provide a unique public policy learning experience, to demonstrate the value of science-government interaction, and to bring a technical background and external perspective to the decision-making process in the Congress.

An AMS Fellowship Committee will select the AMS Congressional Fellow. The Fellowship stipend is US$81,548 for the Fellowship period, with allowances for relocation and professional travel and a contribution toward health insurance. Applicants must have a PhD or an equivalent doctoral-level degree in mathematics by the application deadline.

For information and to apply, go to bit.ly/AMSCongressionalFellowship. The deadline for applications is **February 15, 2019**.

—AMS Office of Government Relations

Jump into Science Communication

With placements at national print media such as *National Geographic, Wired, Scientific American,* the *Washington Post* and *LA Times,* national radio and TV stations such as NPR and CNN, TV programs such as *NOVA,* and local newspapers and radio and TV stations, the AAAS Mass Media Fellowship is the premier experience for mathematicians who want to try out science communication.

During the ten-week Fellowship, scientists, mathematicians, and engineers are placed in newsrooms around the country to report on today's headlines in the fast-paced world of science journalism.

Fellows observe and participate in the process by which events and ideas become news, improve their communication skills by describing complex technical subjects in a manner understandable to the lay public, and increase their understanding of editorial decision making and how information is effectively disseminated. They go through a short boot camp of journalism training at the beginning of summer, keep in touch with each other throughout the summer, and meet again at the end to wrap up the fellowship.

After the Fellowship, alumni participate in science communication in a variety of career fields, including science journalism, advocacy, education, and community outreach. In the forty-four-year program history, the more than 700 fellows have become renowned leaders in the scientific community, award-winning TV and documentary producers, authors, producers, and leaders in cutting-edge science communication.

The AMS sponsors one Fellowship placement each year. This year's Fellow, Yen Duong, finished her PhD in geometric group theory at University of Illinois at Chicago in fall 2017. Over her summer at the *Raleigh News and Observer,* Yen published more than twenty articles explaining science to the lay public. From a computer science paper (which included sigma algebras) to psychology papers that used statistical distributions and models to animal science stories, her topics varied in terms of mathematical content but held one thing in common: a respect for and foundation in evidence and logical thinking.

Read about the rest of the 2018 Fellowship class at `www.aaas.org/page/2018-mass-media-fellows`. See more information about the AMS Mass Media fellows at `www.ams.org/programs/ams-fellowships/media-fellow/massmediafellow`.

Applications may be filed at either URL for the AMS-AAAS 2019 Mass Media Fellowship during the application period, **October 16, 2018–January 15, 2019**.

—Rebekah Corlew, PhD
Project Director, Public Engagement, AAAS

Deaths of AMS Members

Theodore W. Anderson, of Stanford, California, died on September 17, 2016. Born on June 5, 1918, he was a member of the Society for 76 years.

I. Edward Block, of Elkins Park, Pennsylvania, died on February 18, 2015. Born on August 8, 1924, he was a member of the Society for 66 years.

Alan Cobham, of East Granby, Connecticut, died on June 28, 2011. Born on November 4, 1927, he was a member of the Society for 57 years.

Gordon McCrea Fisher, of Ridgefield, Connecticut, died on February 27, 2016. Born on October 5, 1925, he was a member of the Society for 61 years.

Michael B. Freeman, of Ridgefield, Washington, died on November 1, 2016. Born on July 5, 1936, he was a member of the Society for 55 years.

Wolfgang Gaschutz, of Kiel, Germany, died on November 7, 2016. Born on June 11, 1920, he was a member of the Society for 61 years.

John Gregory, of Ithaca, New York, died on April 27, 2012. Born on October 7, 1940, he was a member of the Society for 41 years.

Harold B. Hanes Jr., of Richmond, Indiana, died on April 19, 2008. Born on April 10, 1931, he was a member of the Society for 49 years.

Roger D. Johnson Jr., professor, Georgia Institute of Technology, died on April 22, 2016. Born on March 27, 1930, he was a member of the Society for 64 years.

Bjarni Jonsson, of Cincinnati, Ohio, died on September 30, 2016. Born on February 15, 1920, he was a member of the Society for 70 years.

Joseph B. Keller, professor, Stanford University, died on September 7, 2016. Born on July 31, 1923, he was a member of the Society for 56 years.

Paul Levy, of Brooklyn, New York, died on March 28, 2016. Born on May 25, 1941, he was a member of the Society for 49 years.

Sibe Mardesic, of Croatia, died in 2016. Born on June 20, 1927, he was a member of the Society for 58 years.

John D. Nelligan, of Mt. Prospect, Illinois, died on September 12, 2016. Born on September 10, 1936, he was a member of the Society for 51 years.

Nancy C. Ogden, of Austin, Texas, died on August 23, 2012. Born on July 30, 1927, she was a member of the Society for 42 years.

Robert E. Reed, of Voorheesville, New York, died on July 1, 2002. Born on April 27, 1929, he was a member of the Society for 41 years.

Cris T. Roosenraad, of Northfield, Minnesota, died on October 24, 2016. Born on July 28, 1941, he was a member of the Society for 50 years.

Joseph J. Rotman, professor, University of Illinois, died on October 10, 2016. Born on May 26, 1934, he was a member of the Society for 59 years.

Frank S. Scalora, of Bronxville, New York, died on October 25, 2013. Born on June 16, 1927, he was a member of the Society for 63 years.

Paul C. Shields, of Cotati, California, died on September 15, 2016. Born on November 10, 1933, he was a member of the Society for 58 years.

M. Stephanie Sloyan, professor, Georgian Court University, died on December 21, 2007. Born on April 18, 1918, she was a member of the Society for 44 years.

Alexander Vasil'ev, professor, University of Bergen, died on October 20, 2016. Born on April 1, 1962, he was a member of the Society for 24 years.

William A. Veech, professor, Rice University, died on August 30, 2016. Born on December 24, 1938, he was a member of the Society for 55 years.

Marcellus E. Waddill, of Winston-Salem, North Carolina, died on August 24, 2016. Born on April 28, 1930, he was a member of the Society for 45 years.

Jerome H. Weiner, professor, Brown University, died on September 19, 2016. Born on April 5, 1923, he was a member of the Society for 67 years.

Robert J. Wisner, of Las Cruces, New Mexico, died on October 29, 2016. Born on November 18, 1925, he was a member of the Society for 28 years.

AMS Reciprocity Agreements

The American Mathematical Society (AMS) has reciprocity agreements with a number of mathematical organizations around the world. A current list of the reciprocating societies appears here; for full details of the agreements, see www.ams.org/membership/individual/mem-reciprocity.

Allahabad Mathematical Society
Argentina Mathematical Society
Australian Mathematical Society
Austrian Mathematical Society
Azerbaijan Mathematical Society
Balkan Society of Geometers
Bangladesh Mathematical Society
Belgian Mathematical Society
Berliner Mathematische Gessellschaft
Bharata Ganita Parisad
Brazilian Mathematical Society
Brazilian Society of Computational and Applied Mathematics
Calcutta Mathematical Society
Canadian Mathematical Society
Catalan Society of Mathematicians
Chilian Mathematical Society
Colombian Mathematical Society
Croatian Mathematical Society
Cyprus Mathematical Society
Danish Mathematical Society
Dutch Mathematical Society
Edinburgh Mathematical Society
Egyptian Mathematical Society
European Mathematical Society
Finnish Mathematical Society
German Mathematical Society
German Society for Applied Maths & Mechanics
Glasgow Mathematical Association
Hellenic Mathematical Society
Icelandic Mathematical Society
Indian Mathematical Society
Indonesian Mathematical Society
Iranian Mathematical Society
Irish Mathematical Society
Israel Mathematical Union
Italian Mathematical Union
János Bolyai Mathematical Society
Korean Mathematical Society
London Mathematical Society
Luxembourg Mathematical Society
Macedonian Society Association Mathematics/Computer Science
Malaysian Mathematical Society
Mathematical Society of France
Mathematical Society of Japan
Mathematical Society of the Philippines
Mathematical Society of the Republic of China
Mathematical Society of Serbia
Mexican Mathematical Society
Mongolian Mathematical Society
Nepal Mathematical Society
New Zealand Mathematical Society
Nigerian Mathematical Society
Norwegian Mathematical Society
Palestine Society for Mathematical Sciences
Parana's Mathematical Society
Polish Mathematical Society
Portuguese Mathematical Society
Punjab Mathematical Society
Ramanujan Mathematical Society
Romanian Mathematical Society
Romanian Society of Mathematicians
Royal Spanish Mathematical Society
Saudi Association for Mathematical Sciences
Singapore Mathematical Society
Sociedad Matemática de la Republica Dominicana
Sociedad Uruguaya de Matemática y Estadística
Société Mathématiques Appliquées et Industrielles
Society of Mathematicians, Physicists, and Astronomers of Slovenia
South African Mathematical Society
Southeast Asian Mathematical Society
Spanish Mathematical Society
Swedish Mathematical Society
Swiss Mathematical Society
Tunisian Mathematical Society
Turkish Mathematical Society
Ukrainian Mathematical Society
Union of Bulgarian Mathematicians
Union of Czech Mathematicians and Physicists
Union of Slovak Mathematicians and Physicists
Vietnam Mathematical Society
Vijnana Parishad of India

Call For Suggestions

YOUR VOTE COUNTS! AMS

YOUR SUGGESTIONS ARE WANTED BY:

the Nominating Committee, for the following contested seats in the 2019 AMS elections:
vice president, trustee,
and five members at large of the Council.

Deadline for suggestions: November 1, 2018

the President, for the following contested seats in the 2019 AMS elections:
three members of the Nominating Committee and
two members of the Editorial Boards Committee.

Deadline for suggestions: January 31, 2019

the Editorial Boards Committee, for appointments to various editorial boards of AMS publications.

Deadline for suggestions: Can be submitted any time

Send your suggestions for any of the above to:
Carla D. Savage, Secretary
American Mathematical Society
Department of Computer Science
North Carolina State University
Raleigh, NC 27695-8206 USA
secretary@ams.org
or submit them online at **www.ams.org/committee-nominate**

AMS AMERICAN MATHEMATICAL SOCIETY

Famous Maryam Mirzakhani quotes:

"[Doing] mathematics for me is like being on a long hike with no trail and no end in sight."

"If we knew things would be so complicated, I think we would have given up," she said. Then she paused. "I don't know; actually, I don't know," she said. "I don't give up easily."

Pi = 3.141...

Submitted by Adam Coffman.

"There is a concept that is the corrupter and destroyer of all others. I speak not of Evil, whose limited empire is that of ethics; I speak of the infinite."

—Jorge Luis Borges

"Nature laughs at the difficulties of integration."

—Pierre-Simon de Laplace

"Go down deep enough into anything and you will find mathematics."

—attributed to Charles Sumner Slichter, noted professor of applied mathematics and dean of the graduate school at the University of Wisconsin.

Mathematics People

Munshi Awarded ICTP-IMU Ramanujan Prize

Ritabrata Munshi

Ritabrata Munshi of the Indian Statistical Institute and the Tata Institute of Fundamental Research has been awarded the 2018 Ramanujan Prize for Young Mathematicians from Developing Countries for his outstanding work in number theory. The prize is awarded by the Abdus Salam International Centre for Theoretical Physics (ICTP), the International Mathematical Union (IMU), and the Department of Science and Technology of the Government of India.

The prize citation reads: "Ritabrata Munshi has made profound contributions to analytic number theory, in particular to the study of analytic properties of L-functions and automorphic forms. L-functions were defined in great generality by Robert Langlands, and while much is known about them from the representation theoretic and arithmetic geometry points of view, their deeper analytic properties are largely unknown.

"In recent years, the work of Henryk Iwaniec and his collaborators has started to shed light on growth properties of these L-functions in the case of the group $GL(2)$, proving what are now called subconvexity theorems. These theorems, which are actually estimates for the L-function on the 'critical' line, represent progress towards the proof of the Lindelof hypothesis, which is one of the big open problems in analytic number theory, perhaps second only to the Riemann hypothesis.

"Munshi takes these techniques to new levels by proving subconvexity theorems for some L-functions that come from $GL(3)$. In a series of remarkable papers he has extended the reach of the classical Hardy-Littlewood-Ramanujan 'circle method' to obtain sharp subconvexity estimates for L-functions arising from cusp forms on higher rank groups.

"The progress from $GL(2)$ to $GL(3)$ is very hard won and involves a lot of technical prowess as well as ingenuity. While many authors have established some special cases, Ritabrata's results are perhaps the most far-reaching and most general. In addition, he has made striking contributions to other areas in number theory like Diophantine equations, quadratic forms and elliptic curves. His work also makes clear that he is far from done, and that we should expect to see many more interesting results from him in the future."

Munshi received his PhD from Princeton University in 2006 under the direction of Andrew Wiles. His honors include the 2015 Shanti Swarup Bhatnagar Prize for Science and Technology in mathematical sciences and the 2017 Infosys Prize in Mathematical Sciences.

The Ramanujan Prize is awarded annually to a young researcher from a developing country. The prize carries a cash award of US$15,000, and the recipient is invited to deliver a lecture at ICTP.

—From an ICTP announcement

Sisto Receives 2018 Duszenko Award

Alessandro Sisto

Alessandro Sisto of ETH Zürich has been named the recipient of the 2018 Duszenko Award for his significant contributions to the study of generalizations of hyperbolic groups. According to the prize citation, "he proved deep and interesting results, addressing wide range of questions and using wide range of techniques: random walks, bounded cohomology, embedding obstructions." He received his PhD from the University of Oxford in 2013 under the direction of Cornelia Drutu. He was a postdoctoral fellow at ETH Zürich before becoming an assistant professor. In 2018 he co-organized the Young Geometric Group Theory conference in Les Diablerets, Switzerland. He has written papers on various topics in geometric group theory, as well as other fields. When not doing mathematics, he climbs.

The Duszenko Award is given by the Wrocław Mathematicians Foundation (WMF) for outstanding work or research that has significantly contributed to the deepening of knowledge and further progress in the field of mathematics. It was founded in honor of Kamil Duszenko, a young mathematician who died of acute lymphoblastic leukemia at the age of twenty-eight. It will be given at least every two years in the fields of mathematics and hematology.

—From a WMF announcement

2018 Dirac Medals Awarded

The Dirac Medals for 2018 have been awarded by the International Centre for Theoretical Physics (ICTP) to Subir Sachdev of Harvard University, Dam Thanh Son of the University of Chicago, and Xiao-Gang Wen of the Massachusetts Institute of Technology "for their independent contributions towards understanding novel phases in strongly interacting many-body systems, introducing original transdisciplinary techniques."

According to the prize citations, Sachdev "has made pioneering contributions to many areas of theoretical condensed matter physics. Of particular importance were the development of the theory of quantum critical phenomena in insulators, superconductors and metals; the theory of spin-liquid states of quantum antiferromagnets and the theory of fractionalized phases of matter; the study of novel deconfinement phase transitions; the theory of quantum matter without quasiparticles; and the application of many of these ideas to a priori unrelated problems in black hole physics, including a concrete model of non-Fermi liquids."

Dam Thanh Son

Son "was the first to understand that gauge/gravity duality could be used to address basic questions in strongly interacting many-body problems from cold trapped atoms to the quark-gluon plasma. He was able to show that one could compute transport coefficients, such as viscosity and conductivity, analytically in these systems, and that strong coupling typically gives rise to a bound on these coefficients. More recently, he has argued for the emergence of a Dirac fermion at the half-filled Landau level, work which has stimulated rapid developments in our understanding of three-dimensional gauge theories."

Xiao-Gang Wen

Wen "has pioneered the concept of topological order as a new principle to understand gapped quantum systems. He found that states with topological order contain non-trivial boundary excitations, and he developed chiral Luttinger theory for the boundary states of quantum Hall systems. He realized that quantum Hall states fall outside of the usual Landau paradigm for characterizing phases of matter, and he showed how to classify them. He unveiled deep connections between topological order and entanglement. More recently, he has developed the concept of symmetry protected topological phases. These ideas have close connections to anomalies in quantum field theory."

The medals are awarded to scientists who have made significant contributions to theoretical physics and carry a cash award of US$5,000.

—From an ICTP announcement

2018 Poincaré Prizes Awarded

The International Association of Mathematical Physics (IAMP) has awarded the 2018 Henri Poincaré Prizes for mathematical physics to Michael Aizenman of Princeton University, Percy Deift of New York University, and Giovanni Gallavotti of Universitá di Roma La Sapienza.

Michael Aizenman

Percy Deift

Giovanni Gallavotti

Aizenman was honored "for his seminal contributions to quantum field theory, statistical mechanics, and disordered systems in which he pioneered innovative techniques that demonstrate the beautiful and effective interplay between physical ideas, mathematical analysis, geometric concepts, and probability theory." Deift was recognized "for his seminal contributions to Schrödinger operators, inverse scattering theory, nonlinear waves, asymptotic analysis of Fredholm and Toeplitz determinants, universality in random matrix theory, and his deep analysis of integrable models." Gallavotti was honored "for his outstanding contributions to equilibrium and non-equilibrium statistical mechanics, quantum field theory, classical mechanics, and chaotic systems, including, in particular, the renormalization theory for interacting fermionic systems and the fluctuation relation for the large deviation functional of entropy production."

Semyon Dyatlov of the University of California Berkeley and the Massachusetts Institute of Technology was selected the recipient of the 2018 Early Career Award of the IAMP "for the introduction and the proof of the fractal uncertainty principle, which has important applications to quantum chaos and to observability and control of quantum systems."

The Poincaré Prizes, sponsored by the Daniel Iagolnitzer Foundation, recognize outstanding contributions that lay the groundwork for novel developments in mathematical physics. The prizes recognize and support young people of exceptional promise who have already

Semyon Dyatlov

made outstanding contributions to the field. The prize is awarded every three years at the International Congress on Mathematical Physics. The Early Career Award, sponsored by Springer Publishing Company, is given in recognition of a single achievement in mathematical physics and is reserved for scientists under the age of thirty-five.

—From IAMP announcements

2018 Computer-Aided Verification Award

The Computer-Aided Verification (CAV) Award is given for fundamental contributions to the field of computer-aided verification. Six researchers were selected in 2018 for their outstanding contributions to the enhancement and scalability of model checking by introducing bounded model checking based on Boolean satisfiability (SAT) for hardware (BMC) and software (CBMC). They are:

- Armin Biere, Johannes Kepler University
- Alessandro Cimatti, Fondazione Bruno Kessler
- Edmund M. Clarke, Carnegie Mellon University
- Daniel Kroening, University of Oxford
- Flavio Lerda, Carnegie Mellon University
- Yunshan Zhu, Synopsys

The CAV award carries a cash prize of US$10,000, shared equally among recipients.

—From a CAV announcement

Perfekt Awarded Inaugural Zemánek Prize

Karl-Mikael Perfekt of the University of Reading has been named the first recipient of the Jaroslav and Barbara Zemánek Prize, given in functional analysis with an emphasis on operator theory. Perfekt was honored for his "essential input in a variety of topics in operator theory," especially his "breakthrough work on spectral theory of singular integral operators, in particular on the essential spectrum of the double layer operators, and his penetrating study of 'multiplicative' Hankel operators."

The prize was founded by the Institute of Mathematics of the Polish Academy of Sciences (IMPAN) to encourage research in functional analysis, operator theory, and related topics. The prize recognizes the work of mathematicians under thirty-five years of age who have made important contributions to the field. The monetary amount of the prize is 12,000 PLN (approximately US$3,200). More information about the prize is available at `www.impan.pl/en/events/awards/b-and-j-zemanek-prize`.

—Nikolai Nikolski, University of Bordeaux

MAA Awards Presented

The Mathematical Association of America (MAA) presented several awards for writing and education at its 2018 summer MathFest.

The Carl B. Allendoerfer Award for excellent mathematical writing published in *Mathematics Magazine* was presented to Fumiko Futamura and Robert Lehr of Southwestern University for their joint paper, "A New Perspective on Finding the Viewpoint."

The Trevor Evans Award for excellent writing for an undergraduate audience published in *Math Horizons* was presented to James Propp of the University of Massachusetts, Lowell, for his article "The Paintball Party."

The Paul R. Halmos–Lester R. Ford Awards for exceptional writing published in *The American Mathematical Monthly* were presented to the following: Paul E. Becker and Jennifer Ulrich of Pennsylvania State University Behrend, Martin Derka of Car Media 2.0, and Sheridan Houghten of Brock University for their article "Build a Sporadic Group in Your Basement"; to Maria Deijfen of Stockholm University, Alexander E. Holroyd of the University of Cambridge, and James B. Martin of St. Hugh's College, University of Oxford, for their article "Friendly Frogs, Stable Marriage, and the Magic of Invariance"; to Francis E. Su of Harvey Mudd College for his article "Mathematics for Human Flourishing"; and to Michael Barnsley of Australian National University and Andrew Vince of the University of Florida for their article "Self-Similar Polygonal Tiling."

The George Pólya Awards for exceptional papers published in the *College Mathematics Journal* were presented to Ben Blum-Smith of TED and Samuel Coskey of Boise State University for their article "Fundamental Theorem on Symmetric Polynomials: History's First Whiff of Galois Theory"; and to Stephen Kaczkowski of the South Carolina Governor's School for Science and Mathematics for his article "Mathematical Models for Global Mean Sea Level Rise."

The Daniel Solow Author's Award for authors of undergraduate teaching materials was presented to the following for their coauthored textbook, *Introduction to Statistical Investigations*: Nathan Tintle of Dordt College; Beth Chance, Allan Rossman, and Soma Roy, all of California Polytechnic State University San Luis Obispo; George Cobb of Mt. Holyoke College; and Todd Swanson and Jill VanderStoep of Hope College.

The Henry L. Alder Awards honor beginning college or university faculty members whose teaching has been highly effective and successful in undergraduate mathematics. Chad Awtrey of Elon University has mentored a total of thirty-seven undergraduates and seven high school students on thirty-three different research projects; these students have given sixty-five presentations at national and regional meetings and have coauthored seventeen research papers with Awtrey. David Clark of Grand Valley State University has mentored twenty-two students, many of whom have received awards and grants for their work; is coauthoring a book on mathematical enrichment activities; and participates in MathPath, a summer math program for students ages eleven to fourteen. Mohamed Omar of Harvey Mudd College has created YouTube videos to help students study for the GREs, has published several articles on the teaching and learning of mathematics, and has mentored more than twenty-five undergraduates on more than twenty research projects, several of which have been published in top combinatorics and algebra research journals.

The Mary P. Dolciani Award was presented to Al Cuoco, Distinguished Scholar at Education Development Center, for his contributions to mathematics education, especially the highly original and highly mathematical nature of these contributions to mathematics education and the national stature of his programs.

The Annie and John Selden Prize for Research in Undergraduate Mathematics Education, given for a significant record of published research in undergraduate mathematics education, was presented to Elise Lockwood of Oregon State University for her publication record and her conclusions about the role of example-based reasoning that focuses on sets of outcomes in providing insights to combinatorics tasks.

—From MAA announcements

NSF Postdoctoral Research Fellowships Awarded

The Mathematical Sciences Postdoctoral Research Fellowship Program of the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) awards fellowships each year for postdoctoral research in pure mathematics, applied mathematics and operations research, and statistics. Following are the names of the fellowship recipients for 2018, together with their PhD institutions (in parentheses) and the institutions at which they will use their fellowships.

- Carolyn Abbott (University of Wisconsin-Madison), University of California Berkeley
- Hannah Alpert (Massachusetts Institute of Technology), The Ohio State University
- Paul Apisa (University of Chicago), Yale University
- Dori Bejleri (Brown University), Massachusetts Institute of Technology
- John Berman (University of Virginia), University of Texas at Austin
- Daniel Bernstein (North Carolina State University), Massachusetts Institute of Technology
- Harold Blum (University of Michigan), University of Utah
- Sarah Cannon (Georgia Institute of Technology), University of California Berkeley
- Charlotte Chan (University of Michigan), Princeton University
- Ian Charlesworth (University of California Los Angeles), University of California Berkeley
- Anastasia Chavez (University of California Berkeley), University of California Davis
- William Chen (Pennsylvania State University), McGill University
- Yi Chen (Rutgers University), Princeton University
- Michelle Chu (University of Texas at Austin), University of California Santa Barbara
- Laure Flapan (University of California Los Angeles), Northeastern University
- Robert Fraser (University of British Columbia), University of Edinburgh
- Chris Gerig (University of California Berkeley), Harvard University
- Julian Gold (University of California Los Angeles), Northwestern University
- Jeremy Hahn (University of Wisconsin-Madison), Massachusetts Institute of Technology
- Kyle Hayden (Boston College), Columbia University
- Samuel Hopkins (Massachusetts Institute of Technology), University of Minnesota
- Sameer Iyer (Brown University), Princeton University
- Ian Jauslin (University of Rome Sapienza), Princeton University
- William Johnson (University of California Berkeley), Fudan University
- Daniel Kriz (Princeton University),Massachusetts Institute of Technology
- Eric Larson (Massachusetts Institute of Technology), Stanford University
- Xue Liu (Massachusetts Institute of Technology), Max Planck Institute
- Nicole Looper (Northwestern University), University of Cambridge
- Benjamin Lund (University of Georgia), Princeton University
- Krithika Manohar (University of Washington), California Institute of Technology
- Matthew Mills (University of Nebraska-Lincoln), Michigan State University
- Courtney Paquette (University of Washington), University of Waterloo
- Samuel Punshon-Smith (University of Maryland), Brown University
- David Rolnick (Massachusetts Institute of Technology), University of Pennsylvania
- Anna Romanov (University of Utah), University of Sydney
- Jonathan Rubin (University of Chicago), University of California Los Angeles
- Sophie Spirkl (Princeton University), Rutgers University
- Jonathan Wang (University of Chicago), Massachusetts Institute of Technology
- Jonathan Zhu (Harvard University), Princeton University
- Andrew Zucker (Carnegie Mellon University), Université Paris Diderot

—NSF announcement

NEWS

2018 Pi Mu Epsilon Student Presentation Awards

Pi Mu Epsilon, the student mathematics honor society, awards outstanding student presentations given at the conference held in conjunction with the Mathematical Association of America's (MAA) annual MathFest. In 2018 the MathFest was held in Denver, Colorado.

The AMS, the American Statistical Association, and the Budapest Semesters in Mathematics for Excellence in Student Exposition or Research funded the Pi Mu Epsilon Speaker Awards. The awardees, along with their institutions and the titles of their presentations, are:

- Preston Biro, Texas A&M University, "A Statistical Approach to the Effect of Suspensions in the NFL"
- Keller Blackwell, University of South Florida-Tampa, "Structural Properties of Twisted Hermitian Codes and Applications to Cryptography"
- Kathleen Buch, Xavier University, "Optimizing Congressional Voting Districts Using a Genetic Algorithm"
- William Craig, Virginia Institute of Technology, "Quiver Hall-Littlewood Functions and Kostka-Shoji Polynomials"
- Brian Darrow Jr., Southern Connecticut State University, "On Developing an Early Warning System"
- Samuel Delatore, Youngstown State University, "A Not-So-Fair Guessing Game and the Math Behind It"
- Anthony Dickson, Youngstown State University, "The Prime Number Theorem: A Historical Look at How Mathematicians Proved It"
- Caroline Howell, Troy University, "Mapping Sound Waves in Octave"
- Jacob Kirsch, Saint John's University, "Image Recognition Using Polynomial Regression and Artificial Neural Networks"
- Robert Lehr, Southwestern University, "Perspective Drawing: How to Find the Immersion Point"
- Katherine Mantych, Elmhurst College, "The Rational-Float Data Type"
- Bridget Mueller-Brennan, University of Illinois at Urbana-Champaign, "New Songs in the Deep: A Passive Acoustic Analysis of the Temporal and Spatial Distribution of Omura's Whales (*Balaenoptera omurai*)"
- Daniel Plummer, Howard University, "Bitcoin, Blockchain Technology and the Future of Commerce"
- Henry Potts-Rubin, College of Wooster, "A Convention for Drawing Knots and Links on the Real Projective Plane"
- Victoria Robinson, University of Mississippi, "On a Generalization of the Fibonacci Sequence"
- Bao Van, St. Norbert College, "Building Low Rank Matroids"

The Council for Undergraduate Research Award for Outstanding Student Research was awarded to Vladimir Sworski, Cleveland State University, for "Problem 21: An Exploration of Dial Rings." The Janet L. Andersen Award for Outstanding Student Exposition or Research in Mathematical or Computational Biology was awarded to Allison Gerk, St. Norbert College, "Columnaris Disease and the Population Dynamics of Infected Fish."

—From a Pi Mu Epsilon announcement

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Mathematics Opportunities

Listings for upcoming math opportunities to appear in Notices *may be submitted to* notices@ams.org *.*

Call for Proposals for the 2020 AMS Short Courses

The AMS Short Course Subcommittee invites submissions of preliminary proposals for Short Courses to be offered on January 13–14, 2020, in coordination with the 2020 Joint Mathematics Meetings in Denver, Colorado. Members of the mathematical community are also welcome to suggest names of colleagues as potential organizers.

Preliminary proposals may be as short as one page, and suggestions and questions are welcome. Proposals should be sent via email to the Associate Executive Director (aed-mps@ams.org) with a cc to Robin Hagan Aguiar (rha@ams.org).

A short course typically incorporates a sequence of survey lectures and other activities focused on a single theme of applied mathematics. The Subcommittee is also interested in proposals that go beyond the traditional course in methodology and subject matter. Proposers might be interested in a webinar format or other mechanisms for reaching an audience that extends beyond those at the JMM site, or they may want to appeal to mathematicians who are considering careers in business, industry, government, and nonprofit sectors that utilize mathematical training and experience.

For full consideration, 2020 Short Course proposals should be submitted by **December 18, 2018**. More detailed guidance on proposals is available at www.ams.org/meetings/short-courses/2019call.

American Mathematical Society Centennial Fellowship

Invitation for Applications for Awards for 2019–2020
***Deadline* December 1, 2018**

Description: The AMS Centennial Research Fellowship Program makes awards annually to outstanding mathematicians to help further their careers in research. The number of fellowships to be awarded is small and depends on the amount of money contributed to the program. The Society supplements contributions as needed. At least one fellowship will be awarded for the 2019–2020 academic year. A list of previous fellowship winners can be found at www.ams.org/profession/prizes-awards/ams-awards/centennial-fellow.

Eligibility: The eligibility rules are as follows: The primary selection criterion for the Centennial Fellowship is the excellence of the candidate's research. Preference will be given to candidates who have not had extensive fellowship support in the past. Recipients may not hold the Centennial Fellowship concurrently with another research fellowship such as a Sloan or NSF Postdoctoral fellowship. Under normal circumstances, the fellowship cannot be deferred. A recipient of the fellowship shall have held his or her doctoral degree for at least three years and not more than twelve years at the inception of the award (that is, received between September 1, 2007, and September 1, 2016). Applications will be accepted from those currently holding a tenured, tenure track, postdoctoral, or comparable (at the discretion of the selection committee) position at an institution in North America. Applications should include a cogent plan indicating how the fellowship will be used. The plan should include travel to at least one other institution and should demonstrate that the fellowship will be used for more than reduction of teaching at the candidate's home institution. The selection committee will consider the plan, in addition to the quality of the candidate's research, and will try to award the fellowship to those for whom the award would make a real difference in the development of their research careers. Work in all areas of mathematics, including interdisciplinary work, is eligible.

Deadline: The deadline for receipt of applications is **December 1, 2018**. The award recipient will be announced in February 2019 or earlier, if possible.

Application information: Find Centennial application information at www.ams.org/ams-fellowships. For questions, contact the Professional Programs Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294; prof-serv@ams.org; 401-455-4096.

—AMS Professional Programs Department

AMS Epsilon Fund

The AMS Epsilon Fund awards grants to summer mathematics programs that support and nurture mathematically talented high school students in the United States. The deadline to apply for funding for summer 2019 programs is **December 15, 2018**. Applications are now being accepted online at MathPrograms.org (www.mathprograms.org).

For more information, go to www.ams.org/programs/edu-support/epsilon/emp-epsilon or contact the AMS Professional Programs Department by email at prof-serv@ams.org or by telephone at 800-321-4267, ext. 4060.

—AMS Professional Programs Department

*NSF Graduate Research Fellowships

The National Science Foundation's Graduate Research Fellowship Program supports outstanding graduate students in NSF-supported science, technology, engineering, and mathematics disciplines who are pursuing research-based master's and doctoral degrees at US institutions. The NSF welcomes applications from all qualified students and strongly encourages underrepresented populations, including women, underrepresented racial and ethnic minorities, and persons with disabilities, to apply. Fellows receive a three-year annual stipend of US$34,000 and opportunities for international research and professional development. Fellowships may be used to support graduate research at any accredited US institution. The deadline for the mathematical sciences is **October 26, 2018**. For further information, visit www.nsf.gov/funding/pgm_summ.jsp?pims_id=6201.

—From NSF announcements

*NSF Mathematical Sciences Research Institutes

The NSF Division of Mathematical Sciences invites proposals for Mathematical Sciences Research Institutes that will advance research in the mathematical sciences, increase the impact of the mathematical sciences in other disciplines, and expand the talent base engaged in mathematical research in the United States. This competition welcomes new projects from US sites, as well as renewal proposals from any of the US-based institutes that have had previous funding from NSF. Required letters of intent are due **December 14, 2018**. Full proposals are due **March 14, 2019**. For further information, visit www.nsf.gov/funding/pgm_summ.jsp?pims_id=5302.

—NSF announcement

Call for Nominations for the Alan T. Waterman Award

The National Science Foundation (NSF) is soliciting nominations for the 2019 Alan T. Waterman Award. The award recognizes an outstanding young researcher in any field of science or engineering supported by the NSF. The award consists of a US$1,000,000 grant over a five-year period for research at the institution of the recipient's choice. The deadline for nominations is **October 21, 2018**. Nominations must be submitted at www.fastlane.nsf.gov/honawards. For more details, see www.nsf.gov/od/waterman/waterman.jsp.

—From an NSF announcement

Jefferson Science Fellows Program

The Jefferson Science Fellows (JSF) program at the US Department of State involves the American academic science, technology, and engineering communities in the formulation and implementation of US foreign policy. Each fellow spends one year at the US Department of State or the US Agency for International Development (USAID) for an on-site assignment in Washington, DC, that may also involve extended stays at US foreign embassies and/or missions. The deadline for applications is **October 31, 2018**. For further information, email jsf@nas.edu or see sites.nationalacademies.org/PGA/Jefferson/index.htm.

—From a National Academies announcement

AAUW Educational Foundation Fellowships and Grants

The American Association of University Women (AAUW) has programs supporting women students and scholars at various stages of their careers. American Fellowships support women in full-time study to complete dissertations, conduct postdoctoral research, or prepare research for publication. Selected Professions Fellowships support women students in areas in which women's participation has traditionally been low, including computer/information sciences and mathematics/statistics. For further information about the fellowships, application procedures,

The most up-to-date listing of NSF funding opportunities from the Division of Mathematical Sciences can be found online at: www.nsf.gov/dms and for the Directorate of Education and Human Resources at www.nsf.gov/dir/index.jsp?org=ehr. To receive periodic updates, subscribe to the DMSNEWS listserv by following the directions at www.nsf.gov/mps/dms/about.jsp.

and deadlines, visit the website www.aauw.org/what-we-do/educational-funding-and-awards/.

—From AAUW website

AWM Essay Contest

The Association for Women in Mathematics (AWM) and Math for America cosponsor an annual essay contest for biographies of contemporary women mathematicians and statisticians in academic, industrial, and government careers. This contest is open to students in Grades 6-8, Grades 9-12, and College Undergraduates. The deadline for the 2019 AWM Essay Contest is **January 31, 2019**. AWM is also currently seeking women mathematicians to volunteer as the subjects of these essays. See sites.google.com/site/awmmath/programs/essay-contest for complete information.

—AWM announcement

News from MSRI

Workshops

The Mathematical Sciences Research Institute (MSRI) will hold the following workshops during the spring of 2019. Established researchers, postdoctoral fellows, and graduate students are invited to apply for funding. It is the policy of MSRI to actively seek to achieve diversity in its workshops. Thus, a strong effort is made to remove barriers that hinder equal opportunity, particularly for those groups that have been historically underrepresented in the mathematical sciences.

MSRI has a resource to assist visitors with finding child care in Berkeley. For more information, please contact Sanjani Varkey at sanjani@msri.org.

The workshops are as follows:

January 28–30, 2019: Connections for Women: Derived Algebraic Geometry, Birational Geometry and Moduli Spaces. www.msri.org/workshops/861

January 31–February 8, 2019: Introductory Workshop: Derived Algebraic Geometry and Birational Geometry and Moduli Spaces. www.msri.org/workshops/862

March 6–8, 2019: Critical Issues in Mathematics Education 2019: Mathematical Modeling in K–16: Community and Cultural Context. www.msri.org/workshops/919

March 25–29, 2019: Derived Algebraic Geometry and Its Applications. www.msri.org/workshops/873

April 8–12, 2019: Hot Topics: Recent Progress in Langlands Program. www.msri.org/workshops/855

May 6–10, 2019: Recent Progress in Moduli Theory. www.msri.org/workshops/869

MSRI is supported by the National Science Foundation, the National Security Agency, over 100 academic sponsor departments, a range of private foundations, and individual contributions.

Summer Research for Women in Mathematics

MSRI invites applications for their Summer Research for Women in Mathematics program. The purpose of the program is to provide space and funds to groups of women mathematicians to work on a research project at MSRI. Research projects can arise from work initiated at a women's conference or can be freestanding activities.

Groups of two to six women with partial results on an established project may submit an application to the program through MathPrograms.Org. Applications require a Program Description and a bio-sketch for each group member. The visits must take place between June 10, 2019 and August 2, 2019. Each group may apply to be in residence at MSRI for a minimum of two weeks, though longer visits are preferred.

Lodging, meals and reimbursement of travel expenses will be provided. For participants with children, MSRI will provide funding that makes it possible for the member to fully take part in the program.

Applications are due by **December 1, 2018**. Decisions will be announced by January 15, 2019. For more information, see msri.org/srw2019.

—MSRI announcements

INSTITUTE FOR PURE AND APPLIED MATHEMATICS

MACHINE LEARNING FOR PHYSICS AND THE PHYSICS OF LEARNING

September 4 - December 8, 2019 | Los Angeles

Organizers: Steve Brunton (University of Washington), Cecilia Clementi (Rice University, Chemistry), Yann LeCun (New York University and Facebook), Marina Meila (University of Washington, Statistics), Frank Noe (Freie Universität Berlin), and Francesco Paesani (University of California, San Diego).

SCIENTIFIC OVERVIEW

Machine Learning (ML) is quickly providing new powerful tools for physicists and chemists to extract essential information from large amounts of data, either from experiments or simulations. Significant steps forward in every branch of the physical sciences could be made by embracing, developing and applying the methods of machine learning to interrogate high-dimensional complex data in a way that has not been possible before.

As yet, most applications of machine learning to physical sciences have been limited to the "low-hanging fruits," as they have mostly been focused on fitting pre-existing physical models to data and on discovering strong signals. We believe that machine learning also provides an exciting opportunity to learn the models themselves–that is, to learn the physical principles and structures underlying the data–and that with more realistic constraints, machine learning will also be able to generate and design complex and novel physical structures and objects. Finally, physicists would not just like to fit their data, but rather obtain models that are physically understandable; e.g., by maintaining relations of the predictions to the microscopic physical quantities used as an input, and by respecting physically meaningful constraints, such as conservation laws or symmetry relations.

The exchange between fields can go in both directions. Since its beginning, machine learning has been inspired by methods from statistical physics. Many modern machine learning tools, such as variational inference and maximum entropy, are refinements of techniques invented by physicists. Physics, information theory and statistics are intimately related in their goal to extract valid information from noisy data, and we want to push the cross-pollination further in the specific context of discovering physical principles from data.

WORKSHOP SCHEDULE

- Opening Day : September 4, 2019.
- Tutorials Workshop: September 5-10, 2019.
- Workshop I: From Passive to Active: Generative and Reinforcement Learning with Physics : September 23-27, 2019.
- Workshop II: Interpretable Learning in Physical Sciences : October 14-18, 2019.
- Workshop III: Validation and Guarantees in Learning Physical Models: October 28 - November 1, 2019.
- Workshop IV: Using Physical Insights for Machine Learning : November 18-22, 2019.
- Culminating Workshop at Lake Arrowhead Conference Center : December 3-8, 2019.

PARTICIPATION

This long program will involve senior and junior researchers from several communities relevant to this program. You may apply for financial support to participate in the entire fourteen-week program, or a portion of it. We prefer participants who stay for the entire program. Applications will be accepted through **June 4, 2019**, but offers may be made up to one year before the start date. We urge you to apply early. Mathematicians and scientists at all levels who are interested in this area of research are encouraged to apply for funding. Supporting the careers of women and minority researchers is an important component of IPAM's mission, and we welcome their applications.

www.ipam.ucla.edu/mlp2019

NSF UCLA ipam

Classified Advertisements

Positions available, items for sale, services available, and more

KANSAS

University of Kansas
Department of Mathematics

The Department of Mathematics, University of Kansas invites applications for a non-tenure-track, Visiting Assistant Professor (Teaching Postdoctoral Fellow) expected to begin as early as January 1, 2019. Requirements for the position include a PhD in mathematics or mathematical sciences. For a complete announcement and to apply online, go to: `https://employment.ku.edu/academic/12760BR`. A complete online application includes: a cover letter, CV, teaching statement, and the names and contact information for three references. In addition, at least three recommendation letters (teaching ability must be addressed in at least one letter) should be submitted electronically to `https://www.mathjobs.org/jobs/jobs/12398`. Only complete applications will be considered. Initial review of applications will begin October 4, 2018 and will continue as long as needed to identify a qualified pool. KU is an EO/AAE. All qualified applicants will receive consideration for employment without regard to race, color, religion, sex (including pregnancy), age, national origin, disability, genetic information or protected Veteran status.

037

MASSACHUSETTS

Massachusetts Institute of Technology
Cambridge, MA

The Mathematics Department at MIT is seeking to fill positions in Pure and Applied Mathematics at the level of Assistant Professor or higher beginning July 2019 (for the 2019–2020 academic year, or as soon thereafter as possible). Appointments are based primarily on exceptional research qualifications. Appointees will be required to fulfill teaching duties and pursue their own research program. PhD in Mathematics or related field required by employment start date.

For more information and to apply, please visit `www.mathjobs.org`. To receive full consideration, submit applications by December 1, 2018. MIT is an Equal Opportunity, Affirmative Action Employer.

018

Massachusetts Institute of Technology
Cambridge, MA

The Mathematics Department at MIT is seeking to fill positions in Pure and Applied Mathematics, and Statistics at the level of Instructor beginning July 2019 (for the 2019–2020 academic year). Appointments are based primarily on exceptional research quali-

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services. The publisher reserves the right to reject any advertising not in keeping with the publication's standards. Acceptance shall not be construed as approval of the accuracy or the legality of any advertising.

The 2018 rate is $3.50 per word with a minimum two-line headline. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: November 2018—August 29, 2018; December 2018—September 21, 2018; January 2019—October 17, 2018; February 2019—November 15, 2018; March 2019—December 17, 2018; April 2019—January 17, 2019; May 2019—February 18, 2019.

US laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the US cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to US laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the US and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02904; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

fications. Appointees will be expected to fulfill teaching duties and pursue their own research program. PhD in Mathematics or related field required by employment start date.

For more information and to apply, please visit www.mathjobs.org. To receive full consideration, submit applications by December 1, 2018. MIT is an Equal Opportunity, Affirmative Action Employer.

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Northeastern University
Department of Mathematics
Tenured/Tenure-Track Position, Open Level

The Department of Mathematics at Northeastern University invites applications for a tenured/tenure-track position at the Assistant, Associate, or Full Professor level in Mathematics to start as early as Fall of 2019.

Appointments will be based on exceptional research contributions in Mathematics combined with a strong commitment and demonstrated success in teaching.

The Department seeks to build excellence in the core, interrelated areas of Algebraic Geometry, Representation Theory, and Topology. Outstanding candidates in these or synergetic areas are encouraged to apply and will be seriously considered. The Department currently has a strong research program in Algebraic Geometry and Representation Theory supported by an NSF RTG grant.

Candidates must have a PhD in Mathematics or a related field by the start date, strong record of research, and demonstrated evidence of excellent teaching ability. Qualified candidates should be committed to fostering diverse and inclusive environments as well as to promoting experiential learning, which are central to a Northeastern University education.

Responsibilities will include teaching undergraduate and graduate courses, mentoring students and conducting an independent research program.

Review of applications will begin immediately. Complete applications received by November 1, 2018 will be guaranteed full consideration. Additional applications will be considered until the position is filled.

To apply, please submit the documentation requested on the mathjobs.org web site. Applicants invited to interviews will be asked to complete a Northeastern University application on the appropriate website.

Northeastern University is an Equal Opportunity, Affirmative Action Educational Institution and Employer, Title IX University. Northeastern University particularly welcomes applications from minorities, women and persons with disabilities. Northeastern University is an E-Verify Employer.

034

The Williams College
Department of Mathematics and Statistics

The Williams College Department of Mathematics and Statistics invites applications for two two-year visiting positions in mathematics, to begin fall 2019. Candidates should have earned a PhD in mathematics, applied mathematics, or a related field by summer, 2019. We will consider candidates with any area of mathematical expertise.

Visiting Assistant Professors are asked to teach four courses per year on our 12-week semester schedule, advise several undergraduate student colloquia (our capstone experience for seniors), and make small contributions to service activities in the department. This set of professional duties provides a window into the experience of being a mathematician in a liberal arts setting.

Our department offers a vibrant undergraduate program with majors in mathematics (including an applied mathematics emphasis) and in statistics. For more information, see https://math.williams.edu. The multidisciplinary environment is a rich and collegial setting for student education and faculty research. Williams College provides: the opportunity to apply for student research assistant support; a standard, annual allocation of funds to support travel and research; and a shared computer cluster for parallel computation. Visiting Assistant Professors are also eligible to participate in the college's comprehensive First Three professional development program (https://faculty-networks.williams.edu/networking-opportunities).

Approximately one hour from the Albany, NY airport, Williams College is located in Williamstown, a thriving destination proximate to: three major art museums; theater, music, and dance festivals; community supported agriculture farms; a highly-rated public school system; and many other resources.

The Williams undergraduate student body has 40% US minority enrollment and nearly 10% international enrollment. Reflecting the institution's values, our department is diverse and inclusive, with 50% of our faculty being women, people of color, and/or members of the LGBTQ+ community. We encourage applications from members of underrepresented groups with respect to gender, race and ethnicity, religion, sexual orientation, disability status, socioeconomic background, and other axes of diversity.

Applications should be submitted via www.mathjobs.org. Your application should include the following components.

1) Please provide a cover letter. This letter might describe your interest in Williams and in the liberal arts, and provide a brief summary of your professional experience and future goals. We ask you to address how your teaching, scholarship, mentorship and/or community service might support Williams's commitment to diversity and inclusion.
2) Please provide a current curriculum vitae.
3) Please provide a teaching statement. Ideally, this statement should be 2–3 pages long, and it might address your teaching philosophy, teaching experience, and any other reflections or relevant information you would like to share.
4) Please provide a brief research statement. Ideally, it should help our faculty, who come from a wide range of mathematical disciplines, understand the nature of your work and think about how to support you during your post-PhD years.
5) Please have at least three recommenders submit letters of recommendation. If possible, at least one of these letters should comment on your experience as a teaching assistant or on any other instructional capacities in which you have served.

We also ask applicants to fill out this brief EEOC demographic survey: https://goo.gl/forms/xqT52JBGKXSonPUn1. While completing this form is voluntary, we hope you will fill it out. Responses will be accessible only by administrators and EEO officers.

If you have questions about this position, contact search committee chair Chad Topaz (cmt6@williams.edu). Review of applications will begin on or after November 1 and will continue until the positions are filled. All offers of employment are contingent upon completion of a background check. Further information is available at https://faculty.williams.edu/prospective-faculty/background-check-policy.

Williams College is a coeducational liberal arts institution located in the Berkshire Hills of western Massachusetts. The college has built its reputation on outstanding teaching and scholarship and on the academic excellence of its approximately 2,000 students. Please visit the Williams College website (www.williams.edu). Beyond meeting fully its legal obligations for non-discrimination, Williams College is committed to building a diverse and inclusive community where members from all backgrounds can live, learn, and thrive.

030

NEW HAMPSHIRE

Dartmouth College
Department of Mathematics

John Wesley Young Research Instructorship, 2–3 years, new or recent PhD graduates whose research overlaps a department member's. Teach 3 ten-week courses spread over 3 terms. Appointment for 26 months, with possible 12 month renewal. Salary will begin at a monthly rate of $5,229. The assumption is that the Instructor will be in residence during all but one of the summers spanned by their contract (three out of the four from 2019 to 2022 under normal circumstances), and that residence is defined to be two of the three summer months. Those Instructors who choose not to satisfy the summer residence requirement will have their salary adjusted accordingly. To initiate an application go to www.mathjobs.org – Position ID: JWY #12260. You can also access the application through a link at www.math.dartmouth.edu/activities/recruiting/. Applicants received by February 1, 2019 will receive first consideration. General inquiries can be directed to Tracy Moloney, Administrator, Department of Mathematics, tfmoloney@math.dartmouth.edu.

Dartmouth College is an equal opportunity/affirmative action employer with a strong commitment to diversity and inclusion. We prohibit discrimination on the basis of race, color, religion, sex, age, national origin, sexual orientation, gender identity or expression, disability, veteran status, marital status, or any other legally protected status. Applications by members of all underrepresented groups are encouraged.

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Dartmouth College
Department of Mathematics

The Department of Mathematics announces a tenure-track opening for the 2019–2020 academic year. There is a preference for a junior appointment, but we welcome applicants suitable for a high initial rank. The department has an interest in hiring in number theory/algebra, broadly defined, but also welcomes applications from excellent candidates in other areas of pure mathematics, especially those that strengthen or build connections among our relevant research areas, which include (but are not limited to) combinatorics, analysis, geometry, and topology. Applicants should apply online at www.mathjobs.org Position ID: TTNT#12261. Applicants received by December 15, 2018 will receive first consideration. For more information about this position, please visit our website: https://www.math.dartmouth.edu/activities/recruiting/.

Dartmouth College is an equal opportunity/affirmative action employer with a strong commitment to diversity and inclusion. We prohibit discrimination on the basis of race, color, religion, sex, age, national origin, sexual orientation, gender identity or expression, disability, veteran status, marital status, or any other legally protected status. Applications by members of all underrepresented groups are encouraged.

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NEW JERSEY

Rutgers University
New Brunswick
Mathematics Department

The Mathematics Department of Rutgers University—New Brunswick invites applications for the following positions which may be available September 2019.

TENURE-TRACK ASSISTANT/TENURED ASSOCIATE/ TENURED FULL PROFESSORSHIP:

Subject to availability of funding, the Department expects at least two openings at the level of Tenure-Track Assistant Professor. In exceptional cases, there may be the possibility of appointment at a higher level. Candidates must have a PhD and have a strong record of research accomplishments in pure or applied mathematics as well as effective teaching. The Department has hiring priorities in Probability and Applied Mathematics, but will consider outstanding candidates in any field of pure or applied mathematics. The normal teaching load for research-active faculty is 2–1. Review of applications begins November 1, 2018.

HILL AND OTHER ASSISTANT PROFESSORSHIPS:

These are three-year nontenure-track, nonrenewable, Post-Doctoral appointments. Subject to availability of funding, the department expects five positions of this nature. These positions carry a reduced teaching load of 2–1 for research; candidates should have received a PhD and show outstanding promise of research ability in pure or applied mathematics as well as a capacity for effective teaching. Review of applications begins December 1, 2018.

Applicants for the above positions should submit a curriculum vitae (including a publication list) and arrange for four

letters of reference to be submitted, one of which evaluates teaching. Applicants should first go to the website https://www.mathjobs.org/jobs and fill out the AMS Cover Sheet electronically. It is essential to fill out the cover sheet completely, including naming the positions being applied for (TT and PD, respectively).

Please read carefully for special instructions in our posting on mathjobs. Be aware that Rutgers conducts background checks for all new hires.

Rutgers, the State University of New Jersey, is an Equal Opportunity / Affirmative Action Employer. Qualified applicants will be considered for employment without regard to race, creed, color, religion, sex, sexual orientation, gender identity or expression, national origin, disability status, genetic information, protected

veteran status, military service or any other category protected by law. As an institution, we value diversity of background and opinion, and prohibit discrimination or harassment on the basis of any legally protected class in the areas of hiring, recruitment, promotion, transfer, demotion, training, compensation, pay, fringe benefits, layoff, termination or any other terms and conditions of employment.

036

UTAH

University of Utah
Department of Mathematics

The Department of Mathematics at the University of Utah invites applications for the following positions:

- Full-time tenure-track or tenured appointments at the level of Assistant, Associate, or Full Professor in all areas of mathematics.
- Full-time tenure-track or tenured appointments at the level of Assistant, Associate, or Full Professor in all areas of statistics. These positions are part of a University-wide cluster hiring effort in statistics, with particular emphasis in mathematics,

computer science, and bioengineering. Successful candidates will have strong interdisciplinary interests.

- Three-year Burgess, Kollár, Tucker, and Wylie Assistant Professor Lecturer positions.

Please see our website at www.math.utah.edu/positions for information regarding available positions and application requirements. Applications must be completed through www.mathjobs.org/jobs/Utah. Review of complete applications for tenure-track positions will begin on October 5, 2018, and will continue until the positions are filled. Completed applications for postdoctoral positions received before January 1, 2019, will receive full consideration.

The University of Utah is an Equal Opportunity/Affirmative Action employer and educator. Minorities, women, veterans, and those with disabilities are strongly encouraged to apply. Veterans' preference is extended to qualified veterans. Reasonable disability accommodations will be provided with adequate notice. For additional information about the University's commitment to equal opportunity and access see: www.utah.edu/nondiscrimination/.

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CHINA

Southern University of Science and Technology (SUSTech)
Faculty Positions of Mathematics
The Department of Mathematics

The Department of Mathematics at Southern University of Science and Technology (SUSTech) is founded in 2015 with a dual mission of creating a first-class research and education organization for mathematics and providing service courses in support of other academic departments at SUSTech. We currently have 36 full-time faculty members, including 6 Chair Professors & 7 Full Professors, 3 Associate Professors, 12 Assistant Professors, and 8 teaching faculty members. Research interests of the faculty members cover a broad array of Mathematics including Pure Mathematics, Computational and Applied Mathematics, Probability and Statistics, and Financial Mathematics.

Call for Applications

We invite applications for full-time faculty positions at all ranks and in all areas of Mathematics, including Financial Mathematics and Statistics. SUSTech has a tenure system. Qualified candidates may apply for appointments with tenure.

Candidates should have demonstrated excellence in research and a strong commitment to teaching. A doctoral degree is required at the time of appointment. A candidate for a senior position must have an established record of research and teaching, and a track-record in securing external funding.

To apply, please visit www.mathjobs.org and look up our job ad for instructions. For an informal discussion about applying to one of our positions, please contact Ms. Xianghui Yu, the Secretary of Department of Mathematics, by phone +86-755-88018703 or email: yuxh@sustc.edu.cn.

SUSTech offers competitive salaries, fringe benefits including medical insurance, retirement and housing subsidy, which are among the best in China. Salary and rank will be commensurate with qualifications and experiences of an appointee.

About the University

Established in 2012, SUSTech is a public institution funded by Shenzhen, a city with a designated special economic zone status in Southern China bordering Hong Kong. As one of China's key gateways to the world, Shenzhen is the country's fastest-growing city in the past three decades. From a small fishing village 30 years ago to a modern city with a population of over 10 million, the city has become the high-tech and manufacturing hub of southern China. It is home to the world's third-busiest container port and the fourth-busiest airport on the Chinese mainland. Being a picturesque coastal city, Shenzhen is also a popular tourist destination.

SUSTech is a pioneer in higher education reform in China. Its mission is to become a globally recognized institution that excels in research and promotes innovation, creativity and entrepreneurship. Ninety percent of SUSTech faculty members have overseas work experiences, and sixty percent studied or worked in top 100 universities in the world. The languages of instruction are English and Chinese. Sitting on five hundred acres of subtropical woodland with hills, rivers and a natural lake in Nanshan District of Shenzhen, the SUSTech campus is a beautiful place for learning and research.

The prosperity of Shenzhen is built on innovations and entrepreneurship of its citizens. The city has some of China's most successful high-tech companies such as Huawei and Tencent. SUSTech strongly supports innovations and entrepreneurship, and provides funding for promising initiatives. The university encourages candidates with intention and experience on entrepreneurship to apply.

010

Tianjin University, China
Tenured/Tenure-Track/Postdoctoral Positions at the Center for Applied Mathematics

Dozens of positions at all levels are available at the recently founded Center for Applied Mathematics, Tianjin University, China. We welcome applicants with backgrounds in pure mathematics, applied mathematics, statistics, computer science, bioinformatics, and other related fields. We also welcome applicants who are interested in practical projects with industries. Despite its name attached with an accent of applied mathematics, we also aim to create a strong presence of pure mathematics. Chinese citizenship is not required.

Light or no teaching load, adequate facilities, spacious office environment and strong research support. We are prepared to make quick and competitive offers to self-motivated hard workers, and to potential stars, rising stars, as well as shining stars.

The Center for Applied Mathematics, also known as the Tianjin Center for Applied Mathematics (TCAM), located by a lake in the central campus in a building protected as historical architecture, is jointly sponsored by the Tianjin municipal government and the university. The initiative to establish this center was taken by Professor S. S. Chern. Professor Molin Ge is the Honorary Director, Professor Zhiming Ma is the Director of the Advisory Board. Professor William Y. C. Chen serves as the Director.

TCAM plans to fill in fifty or more permanent faculty positions in the next few years. In addition, there are a number of temporary and visiting positions. We look forward to receiving your application or inquiry at any time. There are no deadlines.

Please send your resume to mathjobs@tju.edu.cn.

For more information, please visit cam.tju.edu.cn or contact Ms. Erica Liu at mathjobs@tju.edu.cn, telephone: 86-22-2740-6039.

001

MISCELLANEOUS

A New Solution to the 3x + 1 Problem

I am seeking a mathematician with two very rare qualities in combination: (1) the courage to give a fair, open-minded reading to a paper not written by an academic mathematician (my degree is in computer science, and for most of my career I have been a researcher in the computer industry), and (2) the ability to understand new, counter-intuitive ideas.

I need such a person to help me prepare the paper containing the solution for submission to a journal. The new solution (a proof of the 3x + 1 Conjecture) is the best of the three proofs of the Conjecture I have discovered. It is in Appendix H in the paper, "A Solution to the 3x + 1 Problem", on occampress.com. The proof, including supporting material, can be read in less than 20 min. (instructions for doing so are on the first page of the paper).

Considerable prestige awaits the mathematician who helps prepare a paper setting forth a solution to the Problem. I will be glad to pay any reasonable consulting fee. I will also give generous credit in the Acknowledgements (but only with the mathematician's prior written approval).

—*Peter Schorer*, peteschorer@gmail.com

033

FOR SALE

Math and Science
Neckties for Sale

Need a Gift for the Integral Person in your Life?
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035

ADVERTISEMENT

Ingrid Daubechies receives William Benter Prize in Applied Mathematics


Ingrid Daubechies

City University of Hong Kong (CityU) has awarded the William Benter Prize in Applied Mathematics 2018 to Ingrid Daubechies, James B. Duke Professor of Mathematics and Electrical and Computer Engineering at Duke University, for her exceptional contributions and pioneering work in a wide spectrum of scientific and mathematical subjects.

Daubechies is the first female recipient of the William Benter prize. Her work in functional analysis, particularly related to wavelets in image-compression technology, has had a profound impact in mathematics, science and engineering. The results of her work are evident in many aspects of our daily life, including digital communication systems, medical image compression, audio and videos coders, and even tools for art history and art authentication. The impact of her work is symbolic of our era.

The William Benter Prize in Applied Mathematics was set up in 2010 by the Liu Bie Ju Centre for Mathematical Sciences at CityU in honour of Mr William Benter, the donor of the prize, for his dedication and generous support for the enhancement of the University's strength in mathematics. The Prize recognises outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, finance and engineering applications. It includes a cash prize of US$100,000 and is given once every two years.

The Prize was presented to Ingrid Daubechies at the opening ceremony of the International Conference on Applied Mathematics, organised by the Liu Bie Ju Centre for Mathematical Sciences at CityU, on 4 June 2018.

Biographical Sketch

Ingrid Daubechies was born in Houthalen, Belgium. She obtained her Bachelor's degree in physics in 1975 and her PhD in 1980 from Vrije Universteit Brussel. After teaching for 12 years at her alma mater, she joined AT&T Bell Laboratories in 1987. She was a Professor of Mathematics at Rutgers University from 1991 to 1993 and moved to Princeton University in 1994. Daubechies was the first-ever female professor of mathematics at Princeton and was the William R. Kenan Jr. Professor of Mathematics from 2004 to 2010. She joined Duke University in January 2011 and is currently the James B. Duke Professor of Mathematics and Electrical and Computer Engineering.

Daubechies has received many awards and honours for her achievements and contributions over the years. She is a member of the US National Academy of Sciences, a member of the US National Academy of Engineering, and a foreign member of the French Academy of Sciences. Daubechies received two AMS Steel Prizes: one for Exposition (1994) and one for her seminal contribution to research (2011). Her monograph *Ten Lectures on Wavelets* has been cited more than 20,000 times. She was a plenary speaker at the International Congress of Mathematics in 1994 and gave the SIAM John von Neuman lecture in 2011. She received the National Academy of Sciences Award in Mathematics in 2000 and the Nemmers Prize in 2012. Daubechies was also the first female president of the International Mathematical Union from 2011 to 2014.

Citation

Over the past 20 years, digital signal processing has exploded in significance. The mobile smartphone revolution has completely changed the face of commerce, education and ultimately human culture. At the core of this revolution is the transformation of digital data from one format to another for transmission in compact forms... Daubechies' work on wavelet transforms figures prominently in the literature of compression and noise removal. Her work is truly symbolic of the technology that has enabled the massive digital media content revolution.

Daubechies' work spans an amazing breadth of scientific disciplines, with deep impacts in signal and image processing, numerical computation and data analysis.

She has made numerous other contributions to scientific and mathematical problems in a wide spectrum of subjects, ranging from computer graphics, analysis of internet traffic, machine learning and randomized algorithms to mathematical biology and functional MRI, and even mathematical tools for art history and art authentication. She is unique in her ability to penetrate a completely new subject and contribute to it in a novel and fundamental way.

– City University of Hong Kong

New Publications Offered by the AMS

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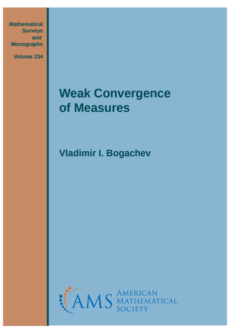
Analysis



Weak Convergence of Measures

Vladimir I. Bogachev, *Lomonosov Moscow State University, Russia, and National Research University Higher School of Economics, Moscow, Russia*

This book provides a thorough exposition of the main concepts and results related to various types of convergence of measures arising in measure theory, probability theory, functional analysis, partial differential equations, mathematical physics, and other theoretical and applied fields. Particular attention is given to weak convergence of measures. The principal material is oriented toward a broad circle of readers dealing with convergence in distribution of random variables and weak convergence of measures.

The book contains the necessary background from measure theory and functional analysis. Large complementary sections aimed at researchers present the most important recent achievements. More than 100 exercises (ranging from easy introductory exercises to rather difficult problems for experienced readers) are given with hints, solutions, or references. Historic and bibliographic comments are included.

The target readership includes mathematicians and physicists whose research is related to probability theory, mathematical statistics, functional analysis, and mathematical physics.

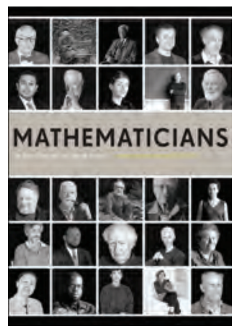
This item will also be of interest to those working in probability and statistics.

Contents: Weak convergence of measures on $\mathbb{R}^d$; Convergence of measures on metric spaces; Metrics on spaces of measures; Convergence of measures on topological spaces; Spaces of measures with the weak topology; Comments; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 234

November 2018, 286 pages, Hardcover, ISBN: 978-1-4704-4738-0, 2010 *Mathematics Subject Classification:* 60B10, 28C15, 46G12, 60B05, 60B11, 60B12, 60B15, 60E05, 60F05, 54A20, **AMS members US$97.60**, List US$122, Order code SURV/234

General Interest



Mathematicians

An Outer View of the Inner World

Mariana Cook

Mathematicians is a remarkable collection of ninety-two photographic portraits, featuring a selection of the most impressive mathematicians of our time. Acclaimed photographer Mariana Cook captures the exuberance and passion of these brilliant thinkers. The superb images are accompanied by autobiographical texts written by each mathematician. Together, the photographs and words illuminate a diverse group of men and women dedicated to the absorbing pursuit of mathematics.

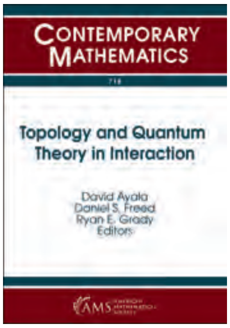
The compelling black-and-white portraits introduce readers to mathematicians who are both young and old and from notably diverse backgrounds. They include Fields Medal winners, those at the beginning of major careers, and those who are long-established celebrities in the discipline. Their candid personal essays reveal unique and wide-ranging thoughts, opinions, and humor. The mathematicians discuss how they became interested in mathematics, why they love the subject, how they remain motivated in the face of mathematical challenges, and how their greatest contributions have paved new directions for future generations. Mathematicians in the book include Jean-Pierre Serre, Henri Cartan, Karen Uhlenbeck, David Blackwell, Eli Stein, John Conway, Timothy Gowers, Frances Kirwan, Peter Lax, William Massey, John Milnor, Cathleen Morawetz, John Nash, Pierre Deligne, and James Simons.

This book conveys the beauty and joy of mathematics to readers outside the field as well as those in it. These pictures and their texts are an inspiration, and a perfect gift for those who love mathematics as well as for those who think they can't do it!

Contents: Preface by Mariana Cook; Introduction by Robert Clifford Gunning; Mathematicians: Portraits; Afterword by Brandon Fradd; List of Mathematicians.

October 2018, 199 pages, Softcover, ISBN: 978-1-4704-4838-7, LC 2018026735, 2010 *Mathematics Subject Classification:* 01A65, **AMS members US$28,** List US$35, Order code MBK/116

Mathematical Physics



Topology and Quantum Theory in Interaction

David Ayala, *Montana State University, Bozeman, MT*, **Daniel S. Freed**, *University of Texas at Austin, TX*, and **Ryan E. Grady**, *Montana State University, Bozeman, MT*, Editors

This volume contains the proceedings of the NSF-CBMS Regional Conference on Topological and Geometric Methods in QFT, held from July 31–August 4, 2017, at Montana State University in Bozeman, Montana.

In recent decades, there has been a movement to axiomatize quantum field theory into a mathematical structure. In a different direction, one can ask to test these axiom systems against physics. Can they be used to rederive known facts about quantum theories or, better yet, be the framework in which to solve open problems? Recently, Freed and Hopkins have provided a solution to a classification problem in condensed matter theory, which is ultimately based on the field theory axioms of Graeme Segal.

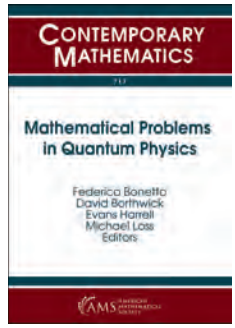
Papers contained in this volume amplify various aspects of the Freed–Hopkins program, develop some category theory, which lies behind the cobordism hypothesis, the major structure theorem for topological field theories, and relate to Costello's approach to perturbative quantum field theory. Two papers on the latter use this framework to recover fundamental results about some physical theories: two-dimensional sigma-models and the bosonic string. Perhaps it is surprising that such sparse axiom systems encode enough structure to prove important results in physics. These successes can be taken as encouragement that the axiom systems are at least on the right track toward articulating what a quantum field theory is.

This item will also be of interest to those working in geometry and topology.

Contents: D. R. Morrison, Geometry and physics: An overview; **I. Saberi**, An introduction to spin systems for mathematicians; **A. Debray** and **S. Gunningham**, The Arf-Brown TQFT of pin$^-$ surfaces; **A. Beaudry** and **J. A. Campbell**, A guide for computing stable homotopy groups; **D. Ayala** and **J. Francis**, Flagged higher categories; **O. Gwilliam** and **T. Johnson-Freyd**, How to derive Feynman diagrams for finite-dimensional integrals directly from the BV formalism; **R. Grady** and **B. Williams**, Homotopy RG flow and the non-linear σ-model; **O. Gwilliam** and **B. Williams**, The holomorphic bosonic string.

Contemporary Mathematics, Volume 718

November 2018, approximately 262 pages, Softcover, ISBN: 978-1-4704-4243-9, 2010 *Mathematics Subject Classification:* 18A05, 53C44, 55Q10, 55T15, 81S40, 81Txx, **AMS members US$93.60**, List US$117, Order code CONM/718



Mathematical Problems in Quantum Physics

Federico Bonetto, *Georgia Institute of Technology, Atlanta, GA*, **David Borthwick**, *Emory University, Atlanta, GA*, **Evans Harrell**, *Georgia Institute of Technology, Atlanta, GA*, and **Michael Loss**, *Georgia Institute of Technology, Atlanta, GA*, Editors

This volume contains the proceedings of the QMATH13: Mathematical Results in Quantum Physics conference, held from October 8–11, 2016, at the Georgia Institute of Technology, Atlanta, Georgia.

In recent years, a number of new frontiers have opened in mathematical physics, such as many-body localization and Schrödinger operators on graphs. There has been progress in developing mathematical techniques as well, notably in renormalization group methods and the use of Lieb–Robinson bounds in various quantum models.

The aim of this volume is to provide an overview of some of these developments. Topics include random Schrödinger operators, many-body fermionic systems, atomic systems, effective equations, and applications to quantum field theory. A number of articles are devoted to the very active area of Schrödinger operators on graphs and general spectral theory of Schrödinger operators. Some of the articles are expository and can be read by an advanced graduate student.

Contents: R. L. Frank, **P. T. Nam**, and **H. Van Den Bosch**, A short proof of the ionization conjecture in Müller theory; **M. Porta**, Mean field dynamics of interacting fermionic systems; **H. Abdul-Rahman**, **R. Sims**, and **G. Stolz**, Correlations in disordered quantum harmonic oscillator systems: The effects of excitations and quantum quenches; **J. Z. Imbrie**, The lattice Anderson model with discrete disorder; **V. Mastropietro**, Interacting fermions with quasi-random disorder; **M. Correggi**, **D. Lundholm**, and **N. Rougerie**, Local density approximation for almost-bosonic anyons; **B. Nachtergaele**, **R. Sims**, and **A. Young**, Lieb-Robinson bounds, the spectral flow, and stability of the spectral gap for lattice fermion systems; **J. Bolte** and **G. Garforth**, Solvable models of interacting n-particle systems on quantum graphs; **R. Band** and **A. J. Krueger**, Nonlinear Sturm oscillation: From the interval to a star; **C. Cacciapuoti**, Existence of the ground state for the NLS with potential on graphs; **V. Rabinovich**, Fredholm theory of differential operators on periodic graphs; **P. Exner** and **V. Lotoreichik**, Optimization of the lowest eigenvalue for leaky star graphs; **V. Bruneau** and **G. Raikov**, Local eigenvalue asymptotics of the perturbed Krein Laplacian; **P. von Soosten** and **S. Warzel**, Singular spectrum and recent results on hierarchical operators; **D. Auckly** and **P. Kuchment**, On Parseval frames of exponentially decaying composite Wannier functions; **M. Ballesteros**, **N. Crawford**, **M. Fraas**, **J. Fröhlich**, and **B. Schubnel**, Non-demolition measurements of observables with general spectra; **P. Naaijkens**, Subfactors and quantum information theory; **R. Dick**, Dressing up for length gauge: Mathematical aspects of a debate in quantum optics; **A. Joye** and **M. Merkli**, Random phase infinite coherent states: Construction and dynamics; **M. Merkli**, Effective evolution of open dimers.

Contemporary Mathematics, Volume 717

November 2018, approximately 336 pages, Softcover, ISBN: 978-1-4704-3681-0, 2010 *Mathematics Subject Classification:* 81V45, 81V70, 81V10, 82B28, 82B44, 82B20, 82B23, 82D77, **AMS members US$93.60**, List US$117, Order code CONM/717

New AMS-Distributed Publications

Algebra and Algebraic Geometry

Notes on Jacquet-Langlands' Theory

Roger Godement

The Jacquet-Langlands correspondence is an important case of the functorial principle in the Langlands program. This book is written by the founder of the eminent French school of automorphic representations and gives an accessible introduction to the Jacquet-Langlands correspondence. It starts from the basic results of automorphic representations and ends with the converse theorem for L-functions.

This book is suitable for everyone who is interested in the Langlands program, in particular automorphic representations, L-functions and the Jacquet-Langlands correspondence.

This item will also be of interest to those working in number theory.

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Classical Topics in Mathematics, Volume 8

August 2018, 134 pages, Hardcover, ISBN: 978-7-04-050303-6, 2010 *Mathematics Subject Classification:* 11F41, 11F66, 22E50, **AMS members US$47.20**, List US$59, Order code CTM/8

Kuga Varieties: Fiber Varieties Over a Symmetric Space Whose Fibers are Abelian Varieties

Michio Kuga

Kuga varieties are fiber varieties over symmetric spaces whose fibers are Abelian varieties and have played an important role in the theory of Shimura varieties and number theory. This book is the first systematic exposition of these varieties and was written by their creators.

This book also contains one of Weil's letters and a paper by Satake which are relevant to the topic of the book.

This item will also be of interest to those working in number theory.

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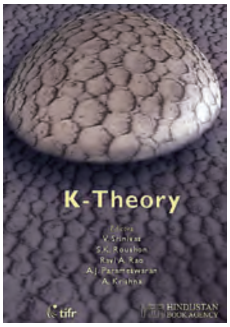
Classical Topics in Mathematics, Volume 9

August 2018, 172 pages, Hardcover, ISBN: 978-7-04-050304-3, 2010 *Mathematics Subject Classification:* 14G35, 11G18, **AMS members US$47.20**, List US$59, Order code CTM/9



K-Theory

V. Srinivas, **S. K. Roushon**, **Ravi A. Rao**, **A. J. Parameswaran**, and **A. Krishna**, *Tata Institute of Fundamental Research, Mumbai, India*, Editors

This volume contains the proceedings of the international colloquium organized by the Tata Institute of Fundamental Research in January 2016, one of a series of colloquia going back to 1956.

The talks at the colloquium covered a wide spectrum of mathematics, ranging over algebraic geometry, topology, algebraic K-theory and number theory. Algebraic theory, $\mathbb{A}^1$-homotopy theory, and topological K-theory formed important sub-streams in this colloquium.

Several branches of K-theory, like algebraic cycles, triangulated categories of motives, motivic cohomology, motivic homotopy theory, Chow groups of varieties, Euler class theory, equivariant K-theory as well as classical K-theory have developed considerably in recent years, giving rise to newer directions to the subject as well as proving results of "classical" interest. The colloquium brought together experts in these various branches and their talks covered this wide spectrum, highlighting the interconnections and giving a better perspective of the whole subject area.

This volume contains refereed articles by leading experts in these fields and includes original results as well as expository materials in these areas.

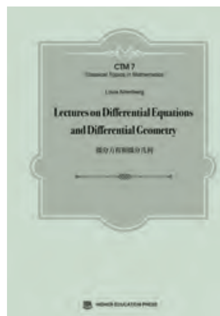
This item will also be of interest to those working in geometry and topology and number theory.

A publication of the Tata Institute of Fundamental Research. Distributed worldwide except in India, Bangladesh, Bhutan, Maldavis, Nepal, Pakistan, and Sri Lanka.

Tata Institute of Fundamental Research

September 2018, 400 pages, Hardcover, ISBN: 978-9-386279-74-3, 2010 *Mathematics Subject Classification:* 14F05, 19D45, 14C15, 19E15, 14C40, 14F42, 14F20, 19E20, 14C25; 13D09, 18G10, 55P42, 19C40, 14L30, 55N15, 20G15, 13D15, 19D10, 19D35, 19D55, 19E08, 19M99, 14C30, 14C35, 14E08, 14F43, **AMS members US$160**, List US$200, Order code TIFR/19

Differential Equations



Lectures on Differential Equations and Differential Geometry

Louis Nirenberg, *Courant Institute of Mathematical Sciences, New York University, NY*

This book is superbly written by a world-leading expert on partial differential equations and differential geometry. It consists of two parts. Part I covers the existence and uniqueness of solutions of elliptic differential equations. It is direct, to the point, moves smoothly and quickly, and there are no unnecessary discussions or digressions. Many topics discussed in Part II will be new and surprising to many students, even to some experts in differential geometry.

This item will also be of interest to those working in geometry and topology.

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Classical Topics in Mathematics, Volume 7

August 2018, 134 pages, Hardcover, ISBN: 978-7-04-050302-9, 2010 *Mathematics Subject Classification:* 35-XX, 53-XX, **AMS members US$47.20**, List US$59, Order code CTM/7

Attend the 2019 AMS "Until Next Time" Social

at the Joint Mathematics Meetings in Baltimore, Maryland

Guests will enjoy the evening at this Baltimore destination with **access to exhibits** and **hands-on displays.** Join us for **live music,** various **food stations, interactive activities** and the opportunity to connect with enthusiastic members of the mathematics community. It is a great chance to wish your colleagues well **"until next time!"**

Saturday, January 19

7:00 pm–9:30 pm

Maryland Science Center,
601 Light Street,
Baltimore, MD

0.6 miles from the Convention Center

A limited number of tickets will be available at a special price for students!

Complimentary Shuttle Service

from: the Hilton & Convention Center
to: the Venue,
every 15 Minutes.
first shuttle 6:45PM.

Accessible shuttle service will be available.

Photos courtesy of Maryland Science Center.

AMS AMERICAN MATHEMATICAL SOCIETY
Advancing research. Creating connections.

Purchase your tickets when registering for the 2019 Joint Mathematics Meetings at: jointmathematicsmeetings.org.

MEETINGS & CONFERENCES OF THE AMS

NOVEMBER TABLE OF CONTENTS

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at: **www.ams.org/meetings**.

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to page 88 in the January 2018 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit **www.ams.org/cgi-bin/abstracts/abstract.pl**. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

MEETINGS IN THIS ISSUE

2018

2019

2020

2021

2022

2023

See `www.ams.org/meetings` *for the most up-to-date information on the meetings and conferences that we offer.*

ASSOCIATE SECRETARIES OF THE AMS

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; email: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; email: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, email: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; email: lapidus@math.ucr.edu; telephone: 951-827-5910.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See www.ams.org/meetings/.

Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL .

Ann Arbor, Michigan

University of Michigan, Ann Arbor

October 20–21, 2018

Saturday – Sunday

Meeting #1143

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: July 2018
Program first available on AMS website: August 30, 2018
Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Elena Fuchs, University of California, Davis, *Primes and local to global in circle packings*.

Andrew Putman, University of Notre Dame, *The mapping class group of a surface*.

Charles K Smart, The University of Chicago, *The Abelian sandpile and circle packings*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Commutative Algebra, **Jack Jeffries**, University of Michigan, **Linquan Ma**, Purdue University, and **Karl Schwede**, University of Utah.

Advances on Analytical and Geometric Aspects of Differential Equations, **Alessandro Arsie**, **Chunhua Shan**, and **Ekaterina Shemyakova**, University of Toledo.

Analytical and Numerical Aspects of Turbulent Transport, **Michele Coti Zelati**, Imperial College London, and **Ian Tobasco** and **Karen Zaya**, University of Michigan.

Aspects of Geometric Mechanics and Dynamics, **Anthony M Bloch** and **Marta Farre Puiggali**, University of Michigan.

Bio-inspired Mechanics and Propulsion, **Silas Alben**, University of Michigan, and **Longhua Zhao**, Case Western Reserve University.

Canonical Operators in Several Complex Variables and Related Topics, **David Barrett** and **Luke Edholm**, University of Michigan, and **Yunus Zeytuncu**, University of Michigan, Dearborn.

Cell Motility: Models and Applications, **Magdalena Stolarska**, University of St. Thomas, and **Nicoleta Tarfulea**, Purdue University Northwest.

Cluster Algebra, Poisson Geometry, and Related Topics, **Eric Bucher**, Michigan State University, and **Maitreyee Kulkarni** and **Bach Nguyen**, Louisiana State University.

Combinatorics in Algebra and Algebraic Geometry, **Zachary Hamaker**, **Steven Karp**, and **Oliver Pechenik**, University of Michigan.

Commutative Algebra and Complexity, **Harm Derksen**, **Francesca Gandini**, and **Visu Makam**, University of Michigan.

Commutative Ring Theory, **Joe Stickles**, Millikin University, and **Darrin Weber**, University of Evansville.

Ergodic and Topological Quantum Systems, **Matthew Cha**, **Ilya Kachkovskiy**, and **Shiwen Zhang**, Michigan State University.

Extensions-Interpolation-Shape Matching in R^d, Symmetry-Invariance, Algorithms and Related Topics, **Steven Damelin**, American Mathematical Society, and **Nir Sharon**, Princeton University.

From Hyperelliptic to Superelliptic Curves, **Tony Shaska**, Oakland University, **Nicola Tarasca**, Rutgers University, and **Yuri Zarhin**, Pennsylvania State University.

Geometry of Submanifolds, in Honor of Bang-Yen Chens 75th Birthday, **Alfonso Carriazo**, University of Sevilla, **Ivko Dimitric**, Penn State Fayette, **Yun Myung Oh**, Andrews University, **Bogdan D. Suceava**, California State University, Fullerton, **Joeri Van der Veken**, University of Leuven, and **Luc Vrancken**, Universite de Valenciennes.

Interactions between Algebra, Machine Learning and Data Privacy, **Jonathan Gryak**, University of Michigan, **Kelsey Horan**, CUNY Graduate Center, **Delaram Kahrobaei**, CUNY Graduate Center and New York University, **Kayvan Najarian** and **Reza Soroushmehr**, University of Michigan, and **Alexander Wood**, CUNY Graduate Center.

Large Cardinals and Combinatorial Set Theory, **Andres E. Caicedo**, Mathematical Reviews, and **Paul B. Larson**, Miami University.

Mathematics of the Genome, **Anthony Bloch**, **Daniel Burns**, and **Indika Rajapakse**, University of Michigan.

Modern Trends in Integrable Systems, **Deniz Bilman**, **Peter Miller**, **Amber Music**, and **Guilherme Silva**, University of Michigan.

Multiplicities and Volumes: An Interplay Among Algebra, Combinatorics, and Geometry, **Federico Castillo**, University of Kansas, and **Jonathan Montaño**, New Mexico State University.

New Trends in Numerical Methods for Partial Differential Equations: Theory and Applications, **Fatih Celiker**, Wayne State University.

Nonlocality in Models for Kinetic, Chemical, and Population Dynamics, **Christopher Henderson**, University of Chicago, **Stanley Snelson**, Florida Institute of Technology, and **Andrei Tarfulea**, University of Chicago.

Probabilistic Methods in Combinatorics, **Patrick Bennett** and **Andrzej Dudek**, Western Michigan University, and **David Galvin**, University of Notre Dame.

Random Matrix Theory Beyond Wigner and Wishart, **Elizabeth Meckes** and **Mark Meckes**, Case Western Reserve University, and **Mark Rudelson**, University of Michigan.

Recent Advances in Nonlinear PDE, **Jessica Lin**, McGill University, and **Russell Schwab**, Michigan State University.

Recent Developments in Discontinuous Galerkin Methods for Differential Equations, **Mahboub Baccouch**, University of Nebraska at Omaha.

Recent Developments in Mathematical Analysis of Some Nonlinear Partial Differential Equations, **Mimi Dai**, University of Illinois at Chicago.

Recent Developments in the Mathematics of Tomography and Scattering, **Shixu Meng**, University of Michigan, and **Yang Yang**, Michigan State University.

Recent Trends on Local, Nonlocal and Fractional Partial Differential Equations, **Pablo Raúl Stinga**, Iowa State University, **Peiyong Wang**, Wayne State University, and **Jiuyi Zhu**, Louisiana State University.

Representations of Reductive Groups over Local Fields and Related Topics, **Anne-Marie Aubert**, Institut Mathématiques de Jussieu, Paris Rive Gauche, **Jessica Fintzen**, IAS, University of Michigan, University of Cambridge, and **Camelia Karimianpour**, University of Michigan.

Self-similarity and Long-range Dependence in Stochastic Processes, **Takashi Owada**, Purdue University, **Yi Shen**, University of Waterloo, and **Yizao Wang**, University of Cincinnati.

Structured Homotopy Theory, **Thomas Fiore**, University of Michigan, Dearborn, **Po Hu** and **Dan Isaksen**, Wayne State University, and **Igor Kriz**, University of Michigan.

The Mathematics of Decisions, Elections, and Games, **Michael A. Jones**, Mathematical Reviews, and **David McCune**, William Jewell College.

Topics in Graph Theory, Hypergraphs and Set Systems, **John Engbers**, Marquette University, and **Cliff Smyth**, University of North Carolina, Greensboro.

San Francisco, California

San Francisco State University

October 27–28, 2018
Saturday – Sunday

Meeting #1144

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: July 2018
Program first available on AMS website: September 6, 2018
Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Srikanth B Iyengar, University of Utah, *Finite free complexes over polynomial rings.*

Sarah Witherspoon, Texas A&M University, *Derivatives, derivations, and Hochschild Cohomology.*

Abdul-Aziz Yakubu, Howard University, *Population cycles in discrete-time infectious disease models.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Operator Theory, Operator Algebras, and Operator Semigroups, **Asuman G. Aksoy**, Claremont McKenna College, **Michael Hartglass**, Santa Clara University, **Zair Ibragimov**, California State University, Fullerton, and **Marat Markin**, California State University, Fresno.

Algebraic Geometry, **Emily Clader** and **Dustin Ross**, San Francisco State University, and **Mark Shoemaker**, Colorado State University.

Analysis and Geometry of Fractals, **Kyle Hambrook**, University of Rochester, **Chun-Kit Lai**, San Francisco State University, and **Sze-Man Ngai**, Georgia Southern University.

Applied Harmonic Analysis: Frame Theory and Applications, **Chun-Kit Lai** and **Shidong Li**, San Francisco State University.

Big Data and Statistical Analytics, **Tao He**, **Mohammad Kafai**, and **Alexandra Piryatinska**, San Francisco State University.

Combinatorial and Categorical Aspects of Representation Theory, **Nicholas Davidson** and **Jonathan Kujawa**, University of Oklahoma, and **Robert Muth**, Tarleton State University.

Coupling in Probability and Related Fields, **Sayan Banerjee**, University of North Carolina, Chapel Hill, and **Terry Soo**, University of Kansas.

Geometric Analysis, **Ovidiu Munteanu**, University of Connecticut, and **David Bao**, San Francisco State University.

Geometric Methods in Hypercomplex Analysis, **Paula Cerejeiras**, Universidade de Aveiro, **Matvei Libine**, Indiana University, Bloomington, and **Mihaela B. Vajiac**, Chapman University.

Geometric and Analytic Inequalities and their Applications, **Nicholas Brubaker**, California State University, Fullerton, **Isabel M. Serrano**, University of California, Berkeley, and **Bogdan D. Suceavă**, California State University, Fullerton.

Homological Aspects in Commutative Algebra and Representation Theory, **Srikanth B. Iyengar**, University of Utah, and **Julia Pevtsova**, University of Washington.

Homological Aspects of Noncommutative Algebra and Geometry, **Dan Rogalski**, University of California San Diego, **Sarah Witherspoon**, Texas A&M University, and **James Zhang**, University of Washington, Seattle.

Markov Processes, Gaussian Processes and Applications, **Alan Krinik** and **Randall J. Swift**, California State Polytechnic University.

Mathematical Biology with a focus on Modeling, Analysis, and Simulation, **Jim Cushing**, The University of Arizona, **Saber Elaydi**, Trinity University, **Suzanne Sindi**, University of California, Merced, and **Abdul-Aziz Yakubu**, Howard University.

Mathematical Methods for the study of the Three Dimensional Structure of Biopolymers, **Javier Arsuaga** and **Mariel Vazquez**, University of California Davis, Davis, and **Robin Wilson**, Cal Poly Pomona.

Noncommutative Geometry and Fundamental Applications, **Konrad Aguilar**, Arizona State University, and **Frederic Latremoliere**, University of Denver.

Nonlocal PDEs via Harmonic Analysis, **Tadele Mengesha**, University of Tennessee, Knoxville, and **Armin Schikorra**, University of Pittsburgh.

Probabilistic and Statistical Problems in Stochastic Dynamics, **Tao He**, **Mohammad Kafai**, and **Alexandra Piryatinska**, San Francisco State University.

Research in Mathematics by Early Career Graduate Students, **Michael Bishop**, **Marat Markin**, **Jenna Tague**, and **Khang Tran**, California State University, Fresno.

Social Change In and Through Mathematics and Education, **Federico Ardila** and **Matthias Beck**, San Francisco State University, **Jamylle Carter**, Diablo Valley Community College, and **Kimberly Seashore**, San Francisco State University.

Statistical and Geometrical Properties of Dynamical Systems, **Joanna Furno** and **Matthew Nicol**, University of Houston, and **Mariusz Urbanski**, University of North Texas.

Fayetteville, Arkansas

University of Arkansas

November 3–4, 2018
Saturday – Sunday

Meeting #1142
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: July 2018
Program first available on AMS website: August 16, 2018
Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Mihalis Dafermos, Princeton University, *On falling into black holes.*

Jonathan D. Hauenstein, University of Notre Dame, *Numerical algebraic geometry and optimization.*

Kathryn Mann, Brown University, *Group actions, geometry and rigidity.*

Special Sessions

Advances in Birational Geometry, **Roi Docampo**, University of Oklahoma, and **Lance Edward Miller** and **Wenbo Niu**, University of Arkansas.

Commutative Algebra, **Alessandro De Stefani**, University of Nebraska-Lincoln, **Paolo Mantero**, University of Arkansas, and **Thomas Polstra**, University of Utah.

Groups in Low-dimensional Topology and Dynamics, **Matt Clay**, University of Arkansas, and **Kathryn Mann**, Brown University.

Harmonic Analysis and Partial Differential Equations, **Ariel Barton**, University of Arkansas, and **Simon Bortz**, University of Minnesota.

Interactions Between Combinatorics and Commutative Algebra, **Ashwini Bhat**, **Chris Francisco**, and **Jeffrey Mermin**, Oklahoma State University.

Interactions Between Contact and Symplectic Geometry and Low-dimensional Topology, **Jeremy Van Horn-Morris**, University of Arkansas, and **David Shea Vela-Vick**, Louisiana State University.

Non-associative Algebraic Structures and their (Co)homology Theories, **Michael Kinyon**, University of Denver, **Jozef H Przytycki**, The George Washington University, and **Petr Vojtechovsky** and **Seung Yeop Yang**, University of Denver.

Numerical Methods for Nonlinear Systems, **Jonathan Hauenstein** and **Tingting Tang**, University of Notre Dame.

Operator Theory and Function Spaces of Analytic Functions, **Daniel Luecking** and **Maria Tjani**, University of Arkansas.

Partial Differential Equations in Several Complex Variables, **Phillip Harrington** and **Andrew Raich**, University of Arkansas.

Recent Advances in Mathematical Fluid Mechanics, **Zachary Bradshaw**, University of Arkansas.

Recent Developments on Fluid Turbulence, **Eleftherios Gkioulekas**, University of Texas Rio Grande Valley.

The Geometry of Curves and Applications, **Jason Cantarella** and **Philipp Reiter**, University of Georgia.

Validation and Verification Strategies in Multiphysics Problems, **Tulin Kaman**, University of Arkansas.

Baltimore, Maryland

Baltimore Convention Center,
Hilton Baltimore, and
Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019

Wednesday – Saturday

Meeting #1145

Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2018
Program first available on AMS website: November 1, 2018
Issue of *Abstracts*: Volume 40, Issue 1

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/national.html`.

Joint Invited Addresses

Sarah Koch, University of Michigan, *What is the shape of a rational map?* (AMS-MAA Invited Address).

Bryna Kra, Northwestern University, *Dynamics of systems with low complexity* (AWM-AMS Noether Lecture).

Cathy O'Neil, ORCAA, *Big data, inequality, and democracy* (MAA-AMS-SIAM Gerald and Judith Porter Public Lecture).

Daniel Spielman, Yale University, *Miracles of Algebraic Graph Theory* (AMS-MAA Invited Address).

AMS Invited Addresses

Jesus A. De Loera, University of California, Davis, *Algebraic, Geometric, and Topological Methods in Optimization.*

Benedict H. Gross, University of California San Diego, *Complex multiplication: past, present, future* (AMS Colloquium Lectures: Lecture I).

Benedict H. Gross, University of California San Diego, *Complex multiplication: past, present, future* (AMS Colloquium Lectures: Lecture II).

Benedict H. Gross, University of California San Diego, *Complex multiplication: past, present, future* (AMS Colloquium Lectures: Lecture III).

Peter Oszvath, Princeton University, *Title to be announced.*

Lior Pachter, University of California Berkeley, *Title to be announced.*

Karen Hunger Parshall, University of Virginia, *The Roaring Twenties in American Mathematics.*

Alan Perelson, Los Alamos National Laboratory, *Title to be announced* (AMS Josiah Willard Gibbs Lecture).

Lillian Pierce, Duke University, *On Torsion subgroups in class groups of number fields.*

AMS Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `jointmathematicsmeetings.org/meetings/abstracts/abstract.pl?type=jmm`.

Some sessions are cosponsored with other organizations. These are noted within the parenthesis at the end of each listing, where applicable.

25 years of Conferences for African-American Researchers in the Mathematical Sciences (CAARMS times 25), **William A. Massey**, Princeton University.

A Showcase of Number Theory at Undergraduate Institutions, **Adriana Salerno**, Bates College, and **Lola Thompson**, Oberlin College.

Advances and Applications in Integral and Differential Equations, **Jeffrey T. Neugebauer**, Eastern Kentucky University, and **Min Wang**, Kennesaw State University.

Advances by Early Career Women in Discrete Mathematics, **Jessalyn Bolkema**, State University of New York at Oswego, and **Jessica De Silva**, California State University, Stanislaus.

Advances in Operator Theory, Operator Algebras, and Operator Semigroups, **Joseph Ball**, Virginia Tech, **Marat Markin**, California State University, Fresno, **Igor Nikolaev**, St. John's University, and **Ilya Spitkovsky**, New York University, Abu Dhabi.

Advances in Quantum Walks, Quantum Simulations, and Related Quantum Theory, **Radhakrishnan Balu**, US Army Research Lab, **Chaobin Liu**, Bowie State University, and **Takuya Machida**, Nihon University, Japan.

Agent-based Modeling in Biological and Social Systems (a Mathematics Research Communities Session), **Maryann Hohn**, University of California Santa Barbara, **Angelika Manhart**, Imperial College, London, and **Christopher Miles**, Courant Institute, New York University.

Algebraic Structures Motivated by Knot Theory, **Mikhail Khovanov**, Columbia University, and **Jozef H. Przytycki** and **Alexander Shumakovitch**, George Washington University.

Algebraic and Geometric Methods in Discrete Optimization, **Amitabh Basu**, Johns Hopkins University, and **Jesus De Loera**, University of California, Davis.

Algebraic, Discrete, Topological and Stochastic Approaches to Modeling in Mathematical Biology, **Olcay Akman**, Illinois State University, **Timothy D. Comar**, Benedictine University, **Daniel Hrozencik**, Chicago State University, and **Raina Robeva**, Sweet Briar College.

Algorithmic Dimensions and Fractal Geometry, **Jack H. Lutz**, Iowa State University, and **Elvira Mayordomo**, University of Zaragoza, Spain (AMS-ASL).

Analysis and Geometry of Nonlinear Evolution Equations, **Marius Beceanu**, University at Albany, State University of New York, and **Dan-Andrei Geba**, University of Rochester.

Analysis of Fractional, Stochastic, and Hybrid Dynamic Systems with Applications, **John R. Graef**, University of Tennessee at Chattanooga, **G. S. Ladde**, University of South Florida, and **A. S. Vatsala**, University of Louisiana at Lafayette.

Analytic Number Theory, **Thomas A. Hulse**, Boston College, **Angel V. Kumchev** and **Nathan McNew**, Towson University, and **John Miller**, The Johns Hopkins University.

Arithmetic Statistics, **Michael Chou** and **Robert Lemke Oliver**, Tufts University, and **Ari Shnidman**, Center for Communications Research-Princeton.

Bifurcations of Difference Equations and Discrete Dynamical Systems with Applications, **Arzu Bilgin**, Recep Tayyip Erdogan University, Turkey, and **Toufik Khyat**, Trinity College.

Commutative Ring Theory: Research for Undergraduate and Early Graduate Students, **Nicholas Baeth**, Franklin and Marshall College, and **Branden Stone**, Hamilton College.

Continued Fractions, **Geremías Polanco Encarnación**, Hampshire College, **James McLaughlin**, West Chester University, **Barry Smith**, Lebanon Valley College, and **Nancy J. Wyshinski**, Trinity College.

Counting Methods in Number Theory, **Lillian Pierce**, Duke University, **Arindam Roy**, Rice University, and **Jiuya Wang**, University of Wisconsin.

Definability and Decidability Problems in Number Theory, **Kirsten Eisenträger**, Pennsylvania State University, **Deidre Haskell**, McMaster University, Ontario, Canada, **Jennifer Park**, University of Michigan, and **Alexandra Shlapentokh**, East Carolina University (AMS-ASL).

Differential Equations on Fractals, **Patricia Alonso-Ruiz**, University of Connecticut, **Joe Chen**, Colgate University, **Luke Rogers**, University of Connecticut, **Robert Strichartz**, Cornell University, and **Alexander Teplyaev**, University of Connecticut.

Enumerative Combinatorics, **Miklos Bona**, University of Florida, and **Cheyne Homberger**, University of Maryland, Baltimore County.

Financial Mathematics, **Maxim Bichuch**, Johns Hopkins University, **Anja Richter**, Baruch College, City University of New York, and **Stephan Sturm**, Worcester Polytechnic Institute.

Geometric and Topological Combinatorics, **Anastasia Chavez** and **Jamie Haddock**, University of California, Davis, and **Annie Raymond**, University of Massachusetts, Amherst.

Geometric and Topological Generalization of Groups, **Amrita Acharyya**, University of Toledo, and **Bikash C. Das**, University of North Georgia.

Geometry Labs United: Research, Visualization, and Outreach, **Marianne Korten**, Kansas State University, and **Sean Lawton** and **Anton Lukyanenko**, George Mason University.

Geometry and Dynamics of Continued Fractions, **Anton Lukyanenko**, George Mason University, and **Joseph Vandehey**, Ohio State University.

Geometry of Representation Spaces, **Sean Lawton**, George Mason University, **Chris Manon**, University of Kentucky, and **Daniel Ramras**, Indiana University-Purdue University Indianapolis.

Group Representation Theory and Character Theory, **Mohammad Reza Darafsheh**, University of Tehran, Iran, and **Manouchehr Misaghian**, Prairie View A&M University.

Harmonic Analysis, Partial Differential Equations, and Applications, **Russell Brown**, University of Kentucky, and **Irina Mitrea**, Temple University.

Harmonic Analysis: Recent Developments on Oscillatory Integrals (a Mathematics Research Communities Session), **Xiumin Du**, University of Maryland, **Taryn C. Flock**, University of Massachusetts Amherst, and **Yakun Xi**, University of Rochester.

History of Mathematics, **Sloan Despeaux**, Western Carolina University, **Jemma Lorenat**, Pitzer College, **Daniel E. Otero**, Xavier University, and **Adrian Rice**, Randolph-Macon College (AMS-MAA-ICHM).

Hopf Algebras and Tensor Categories, **Siu-Hung Ng**, Louisiana State University, **Julia Plavnik**, Texas A&M

University, and **Henry Tucker**, University of California, San Diego.

How to Guard an Art Gallery and Other Discrete Mathematical Adventures (In Memory of T. S. Michael, 1960 to 2016), **Joseph Bonin**, The George Washington University, **Carolyn Chun**, US Naval Academy, and **Nancy Neudauer**, Pacific University.

If You Build It They Will Come: Presentations by Scholars in the National Alliance for Doctoral Studies in the Mathematical Sciences, **David Goldberg**, Purdue University, and **Phil Kutzko**, University of Iowa.

Latinx in Math, **Alexander Diaz-Lopez**, Villanova University, **Laura Escobar**, University of Illinois, and **Juanita Pinzón-Caicedo**, North Carolina State University.

Lattice Path Combinatorics and Applications, **Christian Krattenthaler**, University of Vienna, Austria, and **Alan Krinik** and **Randall J. Swift**, California State Polytechnic University.

Localization and Delocalization for Disordered Quantum Systems, **Peter D. Hislop**, University of Kentucky, **Christoph A. Marx**, Oberlin College, and **Jeffery Schenker**, Michigan State University.

Low Complexity Models in Data Analysis and Machine Learning, **Emily J. King**, University of Bremen, Germany, **Nate Strawn**, Georgetown University, and **Soledad Villar**, New York University.

Mappings on Metric and Banach Spaces with Applications to Fixed Point Theory, **Torrey M. Gallagher**, Bucknell University, and **Christopher J. Lennard**, University of Pittsburgh.

Mathematical Analysis in Fluid Dynamics, **Yanqiu Guo**, Florida International University, **Jinkai Li**, South China Normal University, **Jing Tian**, Towson University, and **Yuncheng You**, University of South Florida.

Mathematical Investigations of Spatial Ecology and Epidemiology, **Leah Shaw** and **Junping Shi**, College of William and Mary, and **Zhisheng Shuai**, University of Central Florida.

Mathematical Models in Ecology, Epidemiology, and Medicine, **Richard Schugart**, Western Kentucky University, and **Najat Ziyadi**, Morgan State University.

Mathematicians at Sea (in the Sky, or on Land): Defense Applications of Mathematics, **Tegan Emerson**, **Timothy Doster**, and **George Stantchev**, Naval Research Laboratory.

Mathematics in the Realm of Cyber Research, **Daniel Bennett**, Army Cyber Institute, **Paul Goethals**, United States Military Academy, and **Natalie Scala**, Towson University.

Mathematics of Coding Theory and Applications, **Hiram Lopez-Valdez** and **Felice Manganiello**, Clemson University, and **Gretchen L. Matthews**, Virginia Tech.

Multiscale Problems in the Calculus of Variations, **Elisa Davoli**, University of Vienna, Austria, and **Rita Ferreira**, King Abdullah University of Science and Technology, Saudi Arabia.

Natural Resources Modeling, **Julie Blackwood**, Williams College, and **Shandelle M. Henson**, Andrews University.

Network Science, **David Burstein**, Swarthmore College, **Franklin Kenter**, United States Naval Academy, and **Feng 'Bill' Shi**, University of North Carolina.

New Directions in the Theory of Complex Multiplication, **Henri Darmon**, McGill University, **Samit Dasgupta**, University of California, Santa Cruz, and **Benedict Gross**, Harvard University.

Nonlinear Evolution Equations and Their Applications, **Mingchao Cai**, Morgan State University, **Gisele Mophou Loudjom**, University of French West Indies, Guadeloupe, France, and **Gaston N'Guerekata**, **Alexander Pankov**, **Xuming Xie**, and **Guoping Zhang**, Morgan State University.

Not KNerds: A Community for Knot Theory, **Moshe Cohen**, Vassar College, **Elizabeth Denne**, Washington and Lee University, and **Adam Lowrance**, Vassar College.

Number Theoretic Methods in Hyperbolic Geometry (a Mathematics Research Communities Session), **Samantha Fairchild**, University of Washington, **Junxian Li**, Universität Göttingen, and **Richard Vradenburgh**, University of Virginia.

Number Theory, Arithmetic Geometry, and Computation, **Brendan Hassett**, Brown University, **Drew Sutherland**, Massachusetts Institute of Technology, and **John Voight**, Dartmouth College.

Numerical Methods for PDEs and Applications, **Wenrui Hao**, **Qingguo Hong**, and **Jinchao Xu**, Pennsylvania State University.

Optimal Methods in Applicable Analysis: Variational Inequalities, Low Rank Matrix Approximations, Systems Engineering, Cyber Security, **Aritra Dutta**, King Abdullah University of Science and Technology, Saudi Arabia, **Ram Mohapatra**, University of Central Florida, **Gayatri Pany**, Singapore University of Technology and Design, Singapore, and **Nabin Kumar Sahu**, Dhirubhai Ambani Institute of Information and Communication Technology, India.

Orthogonal Polynomials, Quantum Probability, Harmonic and Stochastic Analysis, **Nobuhiro Asai**, Aichi University of Education, Kariya, Japan, **Rodica Costin**, The Ohio State University, **Aurel I. Stan**, The Ohio State University at Marion, and **Hiroaki Yoshida**, Ochanomizu University, Tokyo, Japan.

Partition Theory and Related Topics, **Dennis Eichhorn**, University of California, Irvine, **Tim Huber**, University of Texas, Rio Grande Valley, and **Amita Malik**, Rutgers University.

Problems in Partial Differential Equations, **Alex Himonas**, University of Notre Dame, and **Curtis Holliman**, The Catholic University of America.

Quantum Symmetries: Subfactors and Fusion Categories (a Mathematics Research Communities Session), **Cain Edie-Michell** and **Lauren Ruth**, Vanderbilt University, and **Yilong Wang**, Louisiana State University.

Quaternions, **Terrence Blackman**, Medgar Evers College, City University of New York, and **Johannes Familton** and **Chris McCarthy**, Borough of Manhattan Community College, City University of New York.

Recent Advancements in Mathematical Modeling of Cancer, **Kamila Larripa**, Humboldt State University, and **Hwayeon Ryu**, University of Hartford.

Recent Advances and Trends in Computable Structure Theory (in honor of J. Remmel), **Jennifer Chubb**, University of San Francisco, and **Tim McNicholl**, Iowa State University.

Recent Advances in Biological Modeling and Related Dynamical Analysis, **Joshi Raj Hem**, Xavier University, and **Yanyu Xiao**, University of Cincinnati.

Recent Advances in Homological and Commutative Algebra, **Neil Epstein**, George Mason University, **Claudiu Raicu**, Notre Dame University, and **Alexandra Seceleanu**, University of Nebraska.

Recent Advances in Inverse Problems and Imaging, **Kui Ren**, University of Texas at Austin, and **Yang Yang**, Michigan State University.

Recent Advances in Regularity Lemmas, **Gabriel Conant**, University of Notre Dame, **Rehana Patel**, and **Julia Wolf**, University of Bristol, UK.

Recent Progress in Multivariable Operator Theory, **Dmitry Kaliuzhnyi-Verbovetsky** and **Hugo Woerdeman**, Drexel University.

Research in Mathematics by Early Career Graduate Students, **Marat Markin**, **Morgan Rodgers**, **Khang Tran**, and **Oscar Vega**, California State University, Fresno.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs, **Darren A. Narayan**, Rochester Institute of Technology, **Khang Tran**, California State University, Fresno, **Mark David Ward**, Purdue University, and **John Wierman**, The Johns Hopkins University (AMS-MAA-SIAM).

Riordan Arrays, **Alexander Burstein** and **Dennis Davenport**, Howard University, **Asamoah Nkwanta**, Morgan State University, **Lou Shapiro**, Howard University, and **Leon Woodson**, Morgan State University.

Statistical, Variational, and Learning Techniques in Image Analysis and their Applications to Biomedical, Hyperspectral, and Other Imaging, **Justin Marks**, Gonzaga University, **Laramie Paxton**, Washington State University, and **Viktoria Taroudaki**, Eastern Washington University.

Stochastic Analysis and Applications in Finance, Actuarial Science and Related Fields, **Julius N. Esunge**, University of Mary Washington, **See Keong Lee**, University of the Sciences, Malaysia, and **Aurel I. Stan**, The Ohio State University at Marion.

Stochastic Differential Equations and Applications, **Carey Caginalp**, University of Pittsburgh.

Symbolic Dynamics, **Van Cyr**, Bucknell University, and **Bryna Kra**, Northwestern University.

The Mathematics of Gravity and Light (a Mathematics Research Communities Session), **Sougata Dhar**, University of Maine, **Chad R. Mangum**, Niagara University, and **Nadine Stritzelberger**, University of Waterloo.

The Mathematics of Historically Black Colleges and Universities (HBCUs) in the Mid-Atlantic, **Edray Goins**, Purdue University, **Janis Oldham**, North Carolina A&T, **Talithia Washington**, Howard University, and **Scott Williams**, University at Buffalo, State University of New York.

Topological Data Analysis: Theory and Applications, **Justin Curry**, University at Albany, State University of New York, **Mikael Vejdemo-Johansson**, College of Staten Island, City University of New York, and **Sara Kalisnik Verovsek**, Wesleyan University.

Topology, Structure and Symmetry in Graph Theory, **Lowell Abrams**, George Washington University, and **Mark Ellingham**, Vanderbilt University.

Using Modeling to Motivate the Study of Differential Equations, **Robert Kennedy**, Centennial High School, Ellicott City MD, **Audrey Malagon**, Virginia Wesleyan University, **Brian Winkel**, SIMIODE, Cornwall NY, and **Dina Yagodich**, Frederick Community College.

Women in Topology, **Jocelyn Bell**, Hobart and William Smith Colleges, **Rosemary Guzman**, University of Chicago, **Candice Price**, University of San Diego, and **Arunima Ray**, Max Planck Institute for Mathematics, Germany.

Auburn, Alabama

Auburn University

March 15–17, 2019
Friday – Sunday

Meeting #1146
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: January 2019
Program first available on AMS website: January 31, 2019
Issue of *Abstracts*: Volume 40, Issue 2

Deadlines
For organizers: Expired
For abstracts: January 29, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses
Grigoriy Blekherman, Georgia Institute of Technology, *To be announced*.

Carina Curto, Pennsylvania State University, *To be announced*.

Ming Liao, Auburn University, *To be announced*.

Special Sessions
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic and Discrete Methods in Mathematical Biology (Code: SS 21A), **Carina Curto**, The Pennsylvania State University, **Katherine Morrison**, University of Northern Colorado, and **Nora Youngs**, Colby College.

Applications of Algebraic Geometry (Code: SS 25A), **Greg Blekherman**, Georgia Institute of Technology, **Michael Burr**, Clemson University, and **Tianran Chen**, Auburn University at Montgomery.

Clustering Methods and Applications (Code: SS 23A), **Benjamin McLaughlin**, Naval Surface Warfare Center Panama City Division (NSWCPCD), and **Sung Ha Kang**, Georgia Institute of Technology.

Combinatorial Matrix Theory (Code: SS 2A), **Zhongshan Li**, Georgia State University, and **Xavier Martínez-Rivera**, Auburn University.

Commutative and Combinatorial Algebra (Code: SS 3A), **Selvi Kara Beyarslan**, University of South Alabama, and **Alessandra Costantini**, Purdue University.

Developments in Commutative Algebra (Code: SS 1A), **Eloísa Grifo**, University of Michigan, and **Patricia Klein**, University of Kentucky.

Differential Equations in Mathematical Biology (Code: SS 7A), **Guihong Fan**, Columbus State University, **Zhongwei Shen**, University of Alberta, and **Xiaoxia Xie**, Idaho State University.

Discrete and Convex Geometry (Code: SS 17A), **Andras Bezdek**, Auburn University, **Ferenc Fodor**, University of Szeged, and **Wlodzimierz Kuperberg**, Auburn University.

Evolution Equations and Applications (Code: SS 9A), **Dmitry Glotov**, **Wenxian Shen**, and **Paul G. Schmidt**, Auburn University.

Experimental Mathematics in Number Theory, Analysis, and Combinatorics (Code: SS 6A), **Amita Malik**, Rutgers University, and **Armin Straub**, University of South Alabama.

Geometric Flows and Minimal Surfaces (Code: SS 20A), **Theodora Bourni**, University of Tennessee, and **Giuseppe Tinaglia**, King's College London and University of Tennessee.

Geometric Methods in Representation Theory (Code: SS 15A), **Jiuzu Hong** and **Shrawan Kumar**, University of North Carolina, Chapel Hill, and **Yiqiang Li**, University at Buffalo, the State University of New York.

Geometric and Combinatorial Aspects of Representation Theory (Code: SS 19A), **Mark Colarusso**, University of South, and **Jonas Hartwig**, Iowa State University.

Geometry and Topology of Low Dimensional Manifolds, and Their Invariants (Code: SS 13A), **John Etnyre**, Georgia Institute of Technology, **Bulent Tosun**, University of Alabama, and **Shea Vela-Vick**, Louisiana State University.

Graph Theory in Honor of Robert E. Jamison's 70th Birthday (Code: SS 4A), **Robert A Beeler**, East Tennessee State University, **Gretchen Matthews**, Virginia Tech, and **Beth Novick**, Clemson University.

Hopf Algebras and Their Applications (Code: SS 10A), **Robert Underwood**, Auburn University at Montgomery, and **Alan Koch**, Agnes Scott College.

Mapping Class Groups (Code: SS 27A), **Joan Birman**, Columbia University, and **Kevin Kordek** and **Dan Margalit**, Georgia Institute of Technology.

Mathematical Analysis and Control Theory of Coupled Partial Differential Equation Models (Code: SS 11A), **George Avalos** and **Pelin Gu˝ven Geredeli**, University of Nebraska-Lincoln, and **László Kindrat**, University of New Hampshire.

Nonlinear Reaction-Diffusion Equations and Their Applications (Code: SS 18A), **Jerome Goddard,II**, Auburn University at Montgomery, **Nsoki Mavinga**, Swarthmore College, **Quinn Morris**, Appalachian State University, and **R. Shivaji**, University of North Carolina at Greensboro.

Probability and Stochastic Processes (Code: SS 5A), **Ming Liao**, **Erkan Nane**, and **Jerzy Szulga**, Auburn University.

Random Discrete Structures (Code: SS 24A), **Lutz P Warnke**, Georgia Institute of Technology, and **Xavier Pérez-Giménez**, University of Nebraska-Lincoln.

Recent Advances in Coarse Geometry (Code: SS 12A), **Jerzy Dydak**, University of Tennessee.

Recent Advances in Numerical Methods for PDEs and PDE-constrained Optimization (Code: SS 26A), **Yanzhao Cao**, **Thi-Thao-Phuong Hoang**, and **Junshan Lin**, Auburn University.

Recent Developments in Graph Theory (Code: SS 16A), **Xiaofeng Gu**, **Jeong-Hyun Kang**, **David Leach**, and **Rui Xu**, University of West Georgia.

Representations of Lie Algebras, Algebraic Groups, and Quantum Groups (Code: SS 8A), **Joerg Feldvoss**, University of South Alabama, **Lauren Grimley**, Spring Hill College, and **Cornelius Pillen**, University of South Alabama.

The Modeling and Analysis of Spatially Extended Structures (Code: SS 22A), **Shibin Dai**, University of Alabama, **Keith Promislow**, Michigan State University, and **Qiliang Wu**, Ohio University.

Topological Data Analysis, Statistics and Applications (Code: SS 14A), **Yu-Min Chung**, University of North Carolina at Greensboro, and **Vasileios Maroulas**, University of Tennessee.

Honolulu, Hawaii

University of Hawaii at Manoa

March 22–24, 2019

Friday – Sunday

Meeting #1147

Central Section
Associate secretaries: Georgia Benkart and Michel L. Lapidus
Announcement issue of *Notices*: January 2019
Program first available on AMS website: February 7, 2019
Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: Expired
For abstracts: January 29, 2019

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Barry Mazur, Harvard University, *On the arithmetic of curves* (Einstein Public Lecture in Mathematics).

Aaron Naber, Northwestern University, *Analysis of geometric nonlinear partial differential equations.*

Deanna Needell, University of California, Los Angeles, *Simple approaches to complicated data analysis*.

Katherine Stange, University of Colorado, Boulder, *Title to be announced*.

Andrew Suk, University of California, San Diego, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances in Iwasawa Theory (Code: SS 12A), **Frauke Bleher**, University of Iowa, **Ted Chinburg**, University of Pennsylvania, and **Robert Harron**, University of Hawaii at Manoa.

Advances in Mathematical Fluid Mechanics (Code: SS 32A), **Kazuo Yamazaki**, University of Rochester, and **Adam Larios**, University of Nebraska - Lincoln.

Algebraic Groups, Galois Cohomology, and Local-Global Principles (Code: SS 3A), **Raman Parimala**, Emory University, **Andrei Rapinchuk**, University of Virginia, and **Igor Rapinchuk**, Michigan State University.

Algebraic Number Theory and Diophantine Equations (Code: SS 20A), **Claude Levesque**, University of Laval.

Algebraic Points (Code: SS 36A), **Barry Mazur** and **Hector Pasten**, Harvard University.

Algebraic and Combinatorial Structures in Knot Theory (Code: SS 9A), **Sam Nelson**, Claremont McKenna College, **Natsumi Oyamaguchi**, Shumei University, and **Kanako Oshiro**, Sophia University.

Algebraic and Geometric Combinatorics (Code: SS 17A), **Andrew Berget**, Western Washington University, and **Steven Klee**, Seattle University.

Analysis of Nonlinear Geometric Equations (Code: SS 23A), **Aaron Naber**, Northwestern University, and **Richard Bamler**, University of California Berkeley.

Analytic and Probabilistic Methods in Convex Geometry (Code: SS 27A), **Alexander Koldobsky**, University of Missouri, **Alexander Litvak**, University of Alberta, **Dmitry Ryabogin**, Kent State University, **Vladyslav Yaskin**, University of Alberta, and **Artem Zvavitch**, Kent State University.

Applications of Ultrafilters and Nonstandard Methods (Code: SS 33A), **Isaac Goldbring**, University of California, Irvine, and **Steven Leth**, University of Northern Colorado.

Arithmetic Dynamics (Code: SS 29A), **Andrew Bridy**, Texas A&M University, **Michelle Manes**, University of Hawaiʻi at Manoa, and **Bianca Thompson**, Harvey Mudd College.

Arithmetic Geometry and Its Connections (Code: SS 51A), **Laura Capuano**, University of Oxford, and **Amos Turchet**, University of Washington.

Arithmetic and Transcendence of Special Functions and Special Values (Code: SS 56A), **Matthew A. Papanikolas**, Texas A&M University, and **Federico Pellarin**, Université Jean Monnet, St. Étienne.

Coarse Geometry, Index Theory, and Operator Algebras: Around the Mathematics of John Roe (Code: SS 53A), **Erik Guentner**, University of Hawaiʻi at Manoa, **Nigel Higson**, Penn State University, and **Rufus Willett**, University of Hawaiʻi at Manoa.

Coding Theory and Information Theory (Code: SS 39A), **Manabu Hagiwara**, Chiba University, and **James B. Nation**, University of Hawaii.

Combinatorial and Experimental Methods in Mathematical Phylogeny (Code: SS 47A), **Sean Cleary**, City College of New York and the CUNY Graduate Center, and **Katherine St. John**, Hunter College and the American Museum of Natural History.

Commutative Algebra and its Environs (Code: SS 4A), **Olgur Celikbas** and **Ela Celikbas**, West Virginia University, and **Ryo Takahashi**, Nagoya University.

Computability, Complexity, and Learning (Code: SS 45A), **Achilles A. Beros** and **Bj\orn Kjos-Hanssen**, University of Hawaiʻi at Manoa.

Computational and Data-Enabled Sciences (Code: SS 54A), **Roummel Marcia**, **Boaz Ilan**, and **Suzanne Sindi**, University of California, Merced.

Constructive Aspects of Complex Analysis (Code: SS 7A), **Ilia Binder** and **Michael Yampolsky**, University of Toronto, and **Malik Younsi**, University of Hawaii at Manoa.

Differential Geometry (Code: SS 10A), **Vincent B. Bonini**, Cal Poly San Luis Obispo, **Jie Qing**, University of California, Santa Cruz, and **Bogdan D. Suceava**, California State University, Fullerton.

Dynamical Systems and Algebraic Combinatorics (Code: SS 57A), **Maxim Arnold**, University of Texas at Dallas, **Jessica Striker**, North Dakota State University, and **Nathan Williams**, University of Texas at Dallas.

Emerging Connections with Number Theory (Code: SS 43A), **Katherine Stange**, University of Colorado, Boulder, and **Renate Scheidler**, University of Calgary.

Equivariant Homotopy Theory and Trace Methods (Code: SS 58A), **Andrew Blumberg**, University of Texas, **Teena Gerhardt**, Michigan State University, **Michael Hill**, UCLA, and **Michael Mandell**, Indiana University.

Factorization and Arithmetic Properties of Integral Domains and Monoids (Code: SS 31A), **Scott Chapman**, Sam Houston State University, **Jim Coykendall**, Clemson University, and **Christopher O'Neill**, University California, Davis.

Generalizations of Symmetric Spaces (Code: SS 22A), **Aloysius Helminck**, University of Hawaii at Manoa, **Vicky Klima**, Appalachian State University, **Jennifer Schaefer**, Dickinson College, and **Carmen Wright**, Jackson State University.

Geometric Approaches to Mechanics and Control (Code: SS 55A), **Monique Chyba**, University of Hawaii at Manoa, **Tomoki Ohsawa**, The University of Texas at Dallas, and **Vakhtang Putkaradze**, University of Alberta.

Geometry, Analysis, Dynamics and Mathematical Physics on Fractal Spaces (Code: SS 8A), **Joe P. Chen**, Colgate University, **Lũ (Tim) Hùng**, Hawaiʻi Pacific University,

Machiel van Frankenhuijsen, Utah Valley University, and **Robert G. Niemeyer**, University of the Incarnate Word.

Homotopy Theory (Code: SS 48A), **Kyle Ormsby** and **Angélica Osorno**, Reed College.

Interactions between Geometric Measure Theory, PDE, and Harmonic Analysis (Code: SS 41A), **Mark Allen**, Brigham Young University, **Spencer Becker-Kahn**, University of Washington, **Max Engelstein**, Massachusetts Institute of Technology, and **Mariana Smit Vega Garcia**, University of Washington.

Interactions between Noncommutative Algebra and Noncommutative Algebraic Geometry (Code: SS 24A), **Garrett Johnson**, North Carolina Central University, **Bach Nguyen** and **Xingting Wang**, Temple University, and **Daniel Yee**, Bradley University.

Lie Theory in the Representations of Groups and Related Structures (Code: SS 14A), **Christopher Drupieski**, DePaul University, and **Kay Magaard**, University of Arizona.

Mapping Class Groups (Code: SS 35A), **Asaf Hadari**, University of Hawaii.

Mathematical Analysis of Nonlinear Phenomena (Code: SS 16A), **Mimi Dai**, University of Illinois at Chicago.

Mathematical Methods and Models in Medicine (Code: SS 19A), **Monique Chyba**, University of Hawaii, and **Jakob Kotas**, University of Hawaii and University of Portland.

New Trends in Geometric Measure Theory (Code: SS 37A), **Antonio De Rosa**, Courant Institute of Mathematical Sciences, New York University, and **Luca Spolaor**, Massachusetts Institute of Technology.

New Trends on Variational Calculus and Non-Linear Partial Differential Equations (Code: SS 44A), **Craig Cowan**, University of Manitoba, **Michinori Ishiwata**, Osaka University, **Abbas Moameni**, Carleton University, and **Futoshi Takahashi**, Osaka City University.

Nonlinear Wave Equations and Applications (Code: SS 42A), **Boaz Ilan**, University of California, Merced, and **Barbara Prinari**, University of Colorado, Colorado Springs.

Numerical Methods for Partial Differential Equations (Code: SS 50A), **Evan Gawlik**, **Michael Holst**, and **Martin Licht**, University of California, San Diego.

Real and Complex Singularities (Code: SS 34A), **Leslie Charles Wilson**, University of Hawaii, Manoa, **Goo Ishikawa**, Hokkaido University, and **David Trotman**, Université de Provence.

Recent Advances and Applications of Modular Forms (Code: SS 1A), **Amanda Folsom**, Amherst College, **Pavel Guerzhoy**, University of Hawaii at Manoa, **Masanobu Kaneko**, Kyushu University, and **Ken Ono**, Emory University.

Recent Advances in Lie and Related Algebras and their Representations (Code: SS 28A), **Brian D. Boe**, University of Georgia, and **Jonathan Kujawa**, University of Oklahoma.

Recent Advances in Numerical Methods for PDEs (Code: 2249A), **Hengguang Li**, Wayne State University, and **Sara Pollock**, University of Florida.

Recent Advances in Numerical Methods for PDEs (Code: SS 49A), **Hengguang Li**, Wayne State University, and **Sara Pollock**, University of Florida.

Recent Developments in Automorphic Forms (Code: SS 2A), **Solomon Friedberg**, Boston College, and **Jayce Getz**, Duke University.

Recent Trends in Algebraic Graph Theory (Code: SS 26A), **Sebastian Cioaba**, University of Delaware, and **Shaun Fallat**, University of Regina.

SYZ Mirror Symmetry and Enumerative Geometry (Code: SS 11A), **Siu Cheong Lau**, Boston University, **Naichung Leung**, The Chinese University of Hong Kong, and **Hsian-Hua Tseng**, Ohio State University.

Several Complex Variables (Code: SS 5A), **Peter Ebenfelt**, University of California, San Diego, **John Erik Fornaess**, University of Michigan and Norwegian University of Science and Technology, **Ming Xiao**, University of California, San Diego, and **Yuan Yuan**, Syracuse University.

Spaces of Holomorphic Functions and Their Operators (Code: SS 21A), **Mirjana Jovovic** and **Wayne Smith**, University of Hawaii.

Sparsity, Randomness, and Optimization (Code: SS 15A), **Deanna Needell** and **Jamie Haddock**, University of California, Los Angeles.

Spectral Geometry: The Length and Laplace Spectra of Riemannian Manifolds (Code: SS 25A), **Benjamin Linowitz**, Oberlin College, and **Jeffrey S. Meyer**, California State University at San Bernardino.

Stability and Singularity in Fluid Dynamics (Code: SS 40A), **Tristan Buckmaster**, Princeton University, **Steve Shkoller**, University of California, Davis, and **Vlad Vicol**, Princeton University.

Structural Graph Theory (Code: SS 30A), **Zixia Song**, University of Central Florida, **Martin Rolek**, College of William and Mary, and **Yue Zhao**, University of Central Florida.

The Mathematics of Cryptography (Code: SS 18A), **Shahed Sharif**, California State University, San Marcos, and **Alice Silverberg**, University of California, Irvine.

Three-dimensional Floer Theory, Contact Geometry, and Foliations (Code: SS 6A), **Joan Licata**, Australian National University, and **Robert Lipshitz**, University of Oregon.

Topics at the Interface of Analysis and Geometry (Code: SS 38A), **Alex Austin** and **Sylvester Eriksson-Bique**, University of California, Los Angeles.

Valuations on Algebraic Function Fields and Their Subrings (Code: SS 46A), **Ron Brown**, University of Hawaii, **Steven Dale Cutkosky**, University of Missouri, and **Franz-Viktor Kuhlmann**, University of Szczecin.

What is Happening in Mathematical Epidemiology? Current Theory, New Methods, and Open Questions (Code: SS 52A), **Olivia Prosper**, University of Kentucky.

Hartford, Connecticut

University of Connecticut Hartford (Hartford Regional Campus)

April 13–14, 2019

Saturday – Sunday

Meeting #1148

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: February 2019
Program first available on AMS website: February 21, 2019
Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: Expired
For abstracts: February 5, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Olivier Bernardi, Brandeis University, *Title to be announced.*

Brian Hall, Notre Dame University, *Title to be announced.*

Christina Sormani, Lehman College and CUNY Graduate Center, *Compactness Theorems for Sequences of Riemannian Manifolds.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Number Theory (Code: SS 22A), **Harris Daniels**, Amherst College, and **Alvaro Lozano-Robledo** and **Erik Wallace**, University of Connecticut.

Analysis, Geometry, and PDEs in Non-smooth Metric Spaces (Code: SS 1A), **Vyron Vellis**, University of Connecticut, **Xiaodan Zhou**, Worcester Polytechnic Institute, and **Scott Zimmerman**, University of Connecticut.

Banach Space Theory and Metric Embeddings (Code: SS 11A), **Mikhail Ostrovskii**, St. John's University, and **Beata Randrianantoanina**, Miami University.

Chip-firing and Divisor Theory (Code: SS 19A), **Caroline Klivans**, Brown University, and **David Perkinson**, Reed College.

Cluster Algebras and Related Topics (Code: SS 12A), **Emily Gunawan** and **Ralf Schiffler**, University of Connecticut.

Combinatorial Commutative Algebra and Polyhedral Geometry (Code: SS 13A), **Elie Alhajjar**, US Military Academy, and **McCabe Olsen**, Ohio State University.

Computability Theory (Code: SS 2A), **Damir Dzhafarov** and **Reed Solomon**, University of Connecticut, and **Linda Brown Westrick**, Pennsylvania State University.

Convergence of Riemannian Manifolds (Code: SS 17A), **Lan-Hsuan Huang** and **Maree Jaramillo**, University of Connecticut, and **Christina Sormani**, City University of New York Graduate Center and Lehman College.

Discrete Dynamical Systems and Applications (Code: SS 20A), **Elliott J. Betrand**, Sacred Heart University, and **David McArdle**, University of Connecticut.

Invariants of Knots, Links, and Low-dimensional Manifolds (Code: SS 15A), **Patricia Cahn**, Smith College, and **Moshe Cohen** and **Adam Lowrance**, Vassar College.

Knot Theory, the Colored Jones Polynomial, and Khovanov Homology (Code: SS 18A), **Adam Giambrone**, Elmira College, and **Katherine Hall**, University of Connecticut.

Mathematical Cryptology (Code: SS 8A), **Lubjana Beshaj**, United States Military Academy, and **Jaime Gutierrez**, University of Cantabria, Santander, Spain.

Mathematical Finance (Code: SS 14A), **Oleksii Mostovyi**, University of Connecticut, **Gu Wang**, Worcester Polytechnic Institute, and **Bin Zhou**, University of Connecticut.

Modeling and Qualitative Study of PDEs from Materials Science and Geometry. (Code: SS 6A), **Yung-Sze Choi**, **Changfeng Gui**, and **Xiaodong Yan**, University of Connecticut.

Recent Advances in Structured Matrices and Their Applications (Code: SS 16A), **Maxim Derevyagin**, University of Connecticut, **Olga Holz**, University of California, Berkeley, and **Vadim Olshevsky**, University of Connecticut.

Recent Development of Geometric Analysis and Nonlinear PDEs (Code: SS 3A), **Ovidiu Munteanu**, **Lihan Wang**, and **Ling Xiao**, University of Connecticut.

Representation Theory of Quantum Algebras and Related Topics (Code: SS 10A), **Drew Jaramillo**, University of Connecticut, **Garrett Johnson**, North Carolina Central University, and **Margaret Rahmoeller**, Roanoke College.

Special Session on Regularity Theory of PDEs and Calculus of Variations on Domains with Rough Boundaries (Code: SS 5A), **Murat Akman**, University of Connecticut, and **Zihui Zhao**, University of Washington.

Special Values of L-functions and Arithmetic Invariants in Families (Code: SS 21A), **Ellen Eischen**, University of Oregon, **Yifeng Liu**, Yale University, **Liang Xiao**, University of Connecticut, and **Wei Zhang**, Massachusetts Institute of Technology.

Stochastic Analysis and Related Fields (Code: SS 7A), **Fabrice Baudoin**, University of Connecticut, and **Cheng Ouyang**, University of Illinois at Chicago.

Stochastic Processes, Random Walks, and Heat Kernels (Code: SS 4A), **Patricia Alonso Ruiz**, University of Connecticut, and **Phanuel Mariano**, Purdue University.

Sub-Riemannian and CR Geometric Analysis (Code: SS 9A), **Fabrice Baudoin**, University of Connecticut, and **Luca Capogna**, Worcester Polytechnic Institute.

Quy Nhon City, Vietnam

Quy Nhon University

June 10–13, 2019

Monday – Thursday

Meeting #1149

Associate secretary: Brian D. Boe
Announcement issue of *Notices*: April 2019
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: November 30, 2018
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses

Henry Cohn, Microsoft Research, *To be announced*.

Robert Guralnick, University of Southern California, *To be announced*.

Le Tuan Hoa, Hanoi Institute of Mathematics, *To be announced*.

Nguyen Dong Yen, Hanoi Institute of Mathematics, *To be announced*.

Zhiwei Yun, Massachusetts Institute of Technology, *To be announced*.

Nguyen Tien Zung, Toulouse Mathematics Institute, *To be announced*.

Madison, Wisconsin

University of Wisconsin-Madison

September 14–15, 2019

Saturday – Sunday

Meeting #1150

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: June 2019
Program first available on AMS website: July 23, 2019
Issue of *Abstracts*: Volume 40, Issue 3

Deadlines

For organizers: February 14, 2019
For abstracts: July 16, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Nathan Dunfield, University of Illinois, Urbana-Champaign, *Title to be announced*.

Teena Gerhardt, Michigan State University, *Title to be announced*.

Lauren Williams, University of California, Berkeley, *Title to be announced* (Erdős Memorial Lecture).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Computability Theory in honor of Steffen Lempp's 60th birthday (Code: SS 6A), **Joseph S. Miller**, **Noah D. Schweber**, and **Mariya I. Soskova**, University of Wisconsin–Madison.

Homological and Characteristic $p > 0$ Methods in Commutative Algebra (Code: SS 1A), **Michael Brown**, University of Wisconsin-Madison, and **Eric Canton**, University of Michigan.

Model Theory (Code: SS 5A), **Uri Andrews** and **Omer Mermelstein**, University of Wisconsin-Madison.

Recent Developments in Harmonic Analysis (Code: SS 3A), **Theresa Anderson**, Purdue University, and **Joris Roos**, University of Wisconsin-Madison.

Recent Work in the Philosophy of Mathematics (Code: SS 4A), **Thomas Drucker**, University of Wisconsin-Whitewater, and **Dan Sloughter**, Furman University.

Special Functions and Orthogonal Polynomials (Code: SS 2A), **Sarah Post**, University of Hawai'i at Manoa, and **Paul Terwilliger**, University of Wisconsin-Madison.

Binghamton, New York

Binghamton University

October 12–13, 2019

Saturday – Sunday

Meeting #1151

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: August 2019
Program first available on AMS website: August 29, 2019
Issue of *Abstracts*: Volume 40, Issue 3

Deadlines

For organizers: March 12, 2019
For abstracts: August 20, 2019

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Richard Kenyon, Brown University, *Title to be announced*.

Tony Pantev, University of Pennsylvania, *Title to be announced*.

Lai-Sang Young, New York University, *Title to be announced*.

Gainesville, Florida

University of Florida

November 2–3, 2019

Saturday – Sunday

Meeting #1152

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: September 2019
Program first available on AMS website: September 19, 2019
Issue of *Abstracts*: Volume 40, Issue 4

Deadlines

For organizers: April 2, 2019
For abstracts: September 10, 2019

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Jonathan Mattingly, Duke University, *To be announced*.

Isabella Novik, University of Washington, *To be announced*.

Eduardo Teixeira, University of Central Florida, *To be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Geometric and Topological Combinatorics (Code: SS 1A), **Bruno Benedetti**, University of Miami, **Steve Klee**, Seattle University, and **Isabella Novik**, University of Washington.

Riverside, California

University of California, Riverside

November 9–10, 2019

Saturday – Sunday

Meeting #1153

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: September 2019
Program first available on AMS website: September 12, 2019
Issue of *Abstracts*: Volume 40, Issue 4

Deadlines

For organizers: April 9, 2019
For abstracts: September 3, 2019

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Robert Boltje, University of California, Santa Cruz, *Title to be announced*.

Jonathan Novak, University of California, San Diego, *Title to be announced*.

Anna Skripka, University of New Mexico, Albuquerque, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Inverse Problems (Code: SS 3A), **Hanna Makaruk**, Los Alamos National Laboratory, and **Robert Owczarek**, University of New Mexico, Albuquerque and University of New Mexico, Los Alamos.

Random Matrices and Related Structures (Code: SS 2A), **Jonathan Novak**, University of California, San Diego, and **Karl Liechty**, De Paul University.

Topics in Operator Theory (Code: SS 1A), **Anna Skripka** and **Maxim Zinchenko**, University of New Mexico.

Denver, Colorado

Colorado Convention Center

January 15–18, 2020
Wednesday – Saturday

Meeting #1154
Joint Mathematics Meetings, including the 126th Annual Meeting of the AMS, 103rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM)
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2019
Program first available on AMS website: November 1, 2019
Issue of *Abstracts*: To be announced

Deadlines
For organizers: April 1, 2019
For abstracts: To be announced

Charlottesville, Virginia

University of Virginia

March 13–15, 2020
Friday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Medford, Massachusetts

Tufts University

March 21–22, 2020
Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Fresno, California

California State University, Fresno

May 2–3, 2020
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

El Paso, Texas

University of Texas at El Paso

September 12–13, 2020
Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Washington, District of Columbia

Walter E. Washington Convention Center

January 6–9, 2021
Wednesday – Saturday
Joint Mathematics Meetings, including the 127th Annual Meeting of the AMS, 104th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: October 2020
Program first available on AMS website: November 1, 2020
Issue of *Abstracts*: To be announced

Deadlines
For organizers: April 1, 2020
For abstracts: To be announced

Grenoble, France

Université Grenoble Alpes

July 5–9, 2021
Monday – Friday
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Buenos Aires, Argentina

The University of Buenos Aires

July 19–23, 2021
Monday – Friday
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Omaha, Nebraska

Creighton University

October 9–10, 2021
Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 5–8, 2022
Wednesday – Saturday
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2021
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2023

Wednesday – Saturday
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2022
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Photos by: Kate Awtrey, Atlanta Convention Photography

Will you be attending the Joint Mathematics Meetings in Baltimore, MD?

Visit the AMS Membership Booth to learn more about the benefits of membership: In addition to receiving a discount on books purchased through the online bookstore and at meetings, members are also entitled to receive free shipping on their purchases, free and discounted subscriptions to journals, and access to colleagues via the Member Directory. Join or renew your membership at JMM and receive a complimentary gift!

Availability:
THURSDAY,
JANUARY 17TH,
9:30AM–4:25PM
FRIDAY,
JANUARY 18TH,
9:30AM–4:25PM

AMS Members, have your professional portrait taken at the AMS Membership Booth!

Back by popular demand! The AMS Membership Department has arranged for a photographer to take your professional portrait and have it emailed to you in just a few minutes. Consider uploading this photo to your MathSciNet® Author Profile page, using it on your university website, submitting it as the professional photograph for your book publication, or using it as your profile picture in email and on social platforms.

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amermathsoc.simplybook.me

2019 JOINT MATHEMATICS MEETINGS

January 16-19, 2019 • Baltimore, Maryland

Visit Baltimore

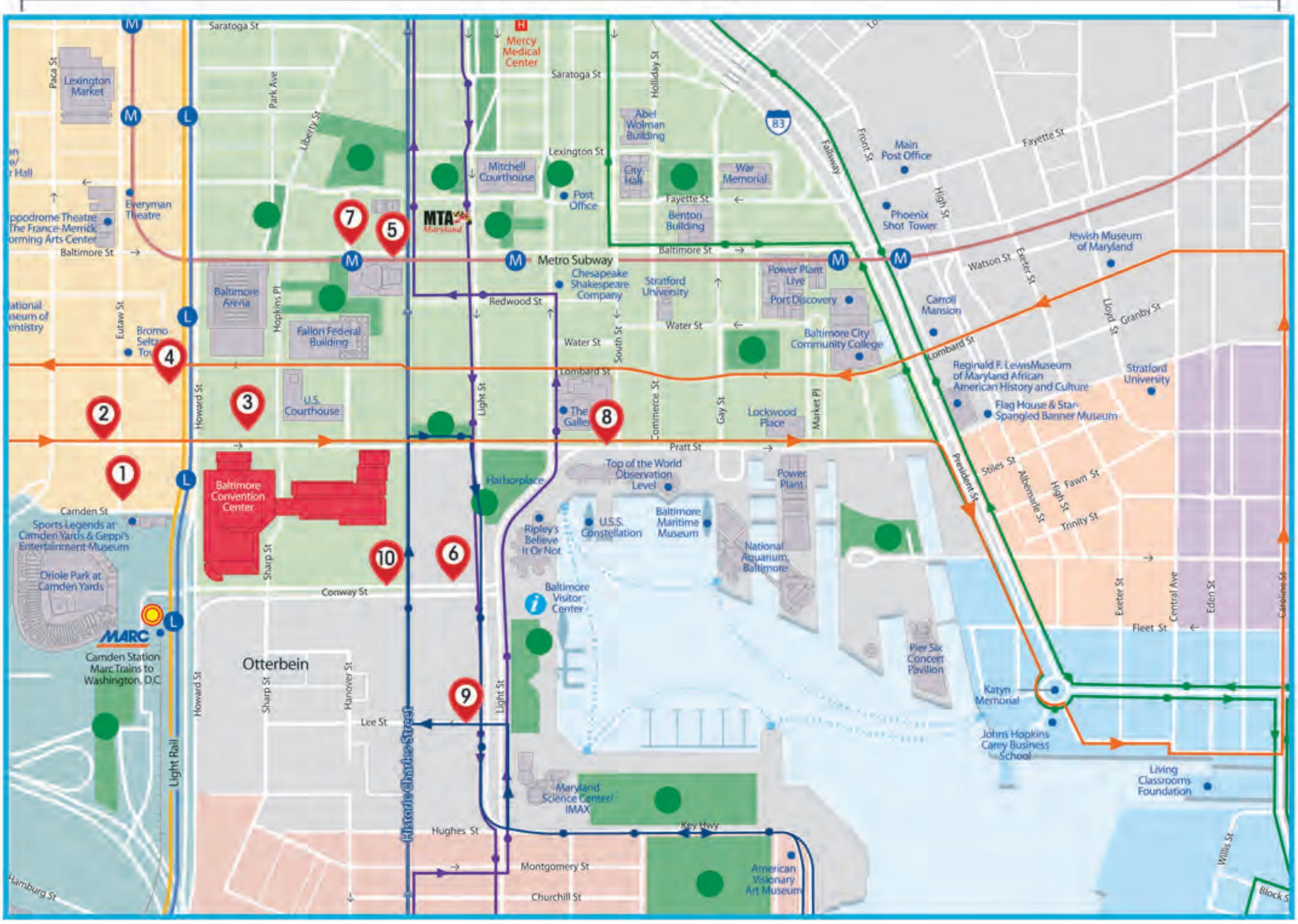
HOTELS

1. Hilton Baltimore (co-HQ)
   401 W Pratt St. • Baltimore, MD 21201
2. Baltimore Marriott Inner Harbor at Camden Yards (co-HQ)
   110 S Eutaw St. • Baltimore, MD 21202
3. Days Inn Inner Harbor
   100 Hopkins Plaza • Baltimore, MD 21201
4. Holiday Inn Inner Harbor
   301 W Lombard St. • Baltimore, MD 21202
5. Hotel Monaco
   2 N Charles St. • Baltimore, MD 21201
6. Hyatt Regency Baltimore
   300 Light St. • Baltimore, MD 21202
7. Lord Baltimore Hotel
   20 W Baltimore St. • Baltimore, MD 21201
8. Renaissance Baltimore Harborplace
   202 E Pratt St. • Baltimore, MD 21202
9. Royal Sonesta Harbor Court
   550 Light St. • Baltimore, MD 21202
10. Sheraton Inner Harbor Hotel
    300 S Charles St. • Baltimore, MD 21201

TRANSPORTATION

- Metro Line
- Light Rail
- MARC Train
- Water Taxies
- Visitors Center
- Charm City Circulator - Green Route
- Charm City Circulator - Orange Route
- Charm City Circulator - Purple Route
- Charm City Circulator - Banner Route
- Parks and green spaces

MAP IS APPROXIMATELY 1.5 MILES ACROSS



2019 Joint Mathematics Meetings Advance Registration/Housing Form

JOINT MATHEMATICS MEETINGS
BALTIMORE • JAN 16-19, 2019
AMERICAN MATHEMATICAL SOCIETY
MATHEMATICAL ASSOCIATION OF AMERICA

Name ____
(please print your name as you would like it to appear on your badge)

Mailing Address ____

____

Telephone ____________________ Fax: ____________________

In case you have an emergency at the meeting: Day #: ____________ Evening #: ____________

Email Address ____________________ Additional email address for receipt ____________________

Acknowledgment of this registration and any hotel reservations will be sent to the email address(es) given here. **Check this box to receive a copy in U.S. Mail:** ☐

Affiliation for badge ____________________ (company/university) Nonmathematician guest badge name: ____________________ (Note fee of US$ 22)

PLEASE NOTE THAT BADGES WILL NOT BE MAILED IN ADVANCE FOR THIS MEETING. YOU MAY OPT TO HAVE YOUR PROGRAM MAILED ON DEC. 12 (SEE BELOW)

Registration Fees

Membership please ✓ all that apply. First row is eligible to register as a member. For undergraduate students, membership in PME and KME also applies.

☐ AMS & MAA ☐ AMS but not MAA ☐ MAA but not AMS ☐ ASL ☐ CMS ☐ SIAM

Undergraduate Students Only: ☐ PME ☐ KME

Other Societies: ☐ AWM ☐ NAM ☐ YMN ☐ AMATYC

Joint Meetings	**by Dec 27**	**at mtg**	**Subtotal**
☐ Member AMS, MAA, ASL, CMS, or SIAM	US$ 345	US$ 455	
☐ Nonmember	US$ 548	US$ 699	
☐ Graduate Student Member (AMS, MAA ASL, CMS, or SIAM)	US$ 78	US$ 90	
☐ Graduate Student (Nonmember)	US$ 124	US$ 137	
☐ Undergraduate Student (Member AMS, ASL, CMS, MAA, PME, KME, or SIAM)	US$ 78	US$ 90	
☐ Undergraduate Student (Nonmember)	US$ 124	US$ 137	
☐ High School Student	US$ 7	US$ 15	
☐ Unemployed	US$ 78	US$ 90	
☐ Temporarily Employed	US$ 281	US$ 322	
☐ Developing Countries Special Rate	US$ 78	US$ 90	
☐ Emeritus Member of AMS or MAA	US$ 78	US$ 90	
☐ High School Teacher	US$ 78	US$ 90	
☐ Librarian	US$ 78	US$ 90	
☐ Press	US$ 0	US$ 0	
☐ Exhibitor (Commercial)	US$ 0	US$ 0	
☐ Artist Exhibitor (work in JMM Art Exhibit)	US$ 0	US$ 0	
☐ Nonmathematician Guest of registered mathematician	US$ 22	US$ 22	
			$ ______

AMS Short Course: *Sum of Squares (1/16-1/19)*

☐ Member of AMS	US$ 124	US$ 158	
☐ Nonmember	US$ 190	US$ 225	
☐ Student, Unemployed, Emeritus	US$ 72	US$ 93	
			$ ______

MAA Minicourses (see listing in text)

I would like to attend: ☐ One Minicourse ☐ Two Minicourses

Please enroll me in MAA Minicourse(s) #______ and #______

Price: US$ 100 for each minicourse.

(For more than 2 minicourses, call or email the MMSB.) $ ______

Graduate School Fair Table

☐ Graduate Program Table US$ 125 US$ 125
(includes table, posterboard & electricity)

Dept. or Program to be represented (write below or email)

______________________________ $ ______

Receptions & Banquets

☐ Graduate Student/First-Time Attendee Reception (1/16) (no charge)

☐ NAM Banquet (1/17)

#___Chicken #___Salmon #___Vegetarian US$ 65

#___Kosher (*Additional fees apply for Kosher Meals.*) US$ 204

Total for NAM Banquet $ ______

☐ AMS Social (1/19) Regular Price #___US$ 75
Student Price #___US$ 35

Total for AMS Social $ ______

Printed Meeting Program (PLEASE CHOOSE)

☐ Meeting Program (pick up at mtg only) US$ 5

☐ Meeting Program mailed (U.S. residents only) US$ 10

Registration must be received by Nov. 20 to be eligible for shipping.

☐ I do not want a printed program.

Total for Meeting Program/Shipping $ ______

Total for Registrations and Events $ ______

Payment

Registration & Event Total (total from column on left)	$ ______
Hotel Deposit (only if paying by check) **If you send a hotel deposit check, the deadline for this form is December 1.**	$ ______
Total Amount To Be Paid	**$** ______

Method of Payment

☐ **Check.** Make checks payable to the AMS. For all check payments, please keep a copy of this form for your records.

☐ **Credit Card.** All major credit cards accepted. For your security, we do not accept credit card numbers by email, fax, or postal mail. If the MMSB receives your registration form by any of these methods, it will contact you at the phone number provided on this form.

Signature: ______________________________

☐ **Purchase Order #** ____________ (please enclose copy)

Other Information

Mathematical Reviews primary field of interest # ______

☐ I am willing to serve as a judge for the MAA Undergraduate Student Poster Session.

☐ If you are an undergraduate, are you interested in participating in the Radical Dash, a multi-day scavenger hunt sponsored by the MAA?

☐ For planning purposes for the MAA Two-year College Reception, please check if you are a faculty member at a two-year college.

☐ Please ✓ this box if you have a disability requiring special services. ♿

To respect your privacy and to better serve you, please indicate your preferences for the following:

☐ Please include my name and affiliation on the JMM Participant List.

☐ Please include my name and postal address on promotional mailing lists.

Registration for the Joint Meetings is not required for the short course but it is required for the minicourses and the Employment Center. To register for the Employment Center, go to www.ams.org/profession/employment-services. For questions, email emp-info@ams.org.

Registration Deadlines

To be eligible for the complimentary hotel room lottery:	**Oct. 30, 2018**
In time to receive programs in the mail:	**Nov. 20, 2018**
Hotel reservations with check deposit:	**Dec. 1, 2018**
Hotel reservations, changes/cancellations through the JMM website:	**Dec. 13, 2018**
Advance registration for the Joint Meetings, short course, minicourses, and dinner tickets:	**Dec. 27, 2018**
Cancel in time to receive 50% refund on advance registration, banquets, minicourses, and short course	**Jan. 8, 2019***

***no refunds issued after this date.**

Mailing Address/Contact:

Mathematics Meetings Service Bureau (MMSB)
P. O. Box 6887
Providence, RI 02940-6887 **Fax**: 401-455-4004; **Email:** mmsb@ams.org
Telephone: 401-455-4144 or 1-800-321-4267 x4144 or x4137

2019 Joint Mathematics Meetings Hotel Reservations – Baltimore, MD

Please see the hotel information in the announcement or on the web for detailed information on each hotel. To ensure accurate assignments, please rank hotels in order of preference by writing 1, 2, 3, etc. in the column on the left and by circling the requested bed configuration. If your requested hotel and room type is no longer available, you will be assigned a room at the next available comparable rate. Please call the MMSB for details on suite configurations, sizes, availability, etc. All reservations, including suite reservations, must be made through the MMSB to receive the JMM rates. Reservations made directly with the hotels before **December 14, 2018** may be changed to a higher rate. All rates are subject to applicable local and state taxes in effect at the time of check-in; currently 15.5% state tax. **Guarantee requirements: First night deposit by check (add to payment on reverse of form) or a credit card guarantee. Please note that reservations with check deposits must be received by the MMSB by December 1, 2018.** People interested in suites should contact the MMSB directly at mmsb@ams.org or by calling 800-321-4267, ext. 4137; (401-455-4137).

☐ **Deposit enclosed (see front of form)**

☐ **Hold with my credit card. For your security, we do not accept credit card numbers by email, postal mail or fax.** If the MMSB receives your registration form by any of these methods, it will contact you at the phone number provided on the reverse of this form.

Date and Time of Arrival ______________ **Date and Time of Departure** ______________ **Number of adult guests in room** ________ **Number of children** ________

Name of Other Adult Room Occupant (s) ______________________ **Arrival:** ______________ **Departure:** ______________

Housing Requests: (example: rollaway cot, crib, nonsmoking room, low floor) ______________________

☐ I have disabilities as defined by the ADA that require a sleeping room that is accessible to the physically challenged. My needs are: ______________________

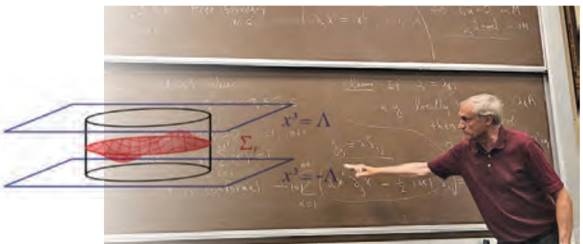
☐ I am a member of a hotel frequent-travel club and would like to receive appropriate credit. The hotel chain and card number are: ______________________

☐ I am not reserving a room. I am sharing with ______________________, who is making the reservation.

Order of choice	Hotel	Single	Double 1 bed-2 people	Double 2 beds- 2 people	Triple 3 adults-2 beds	Quad 4 adults-2 beds	Rollaway/Cot Information (add to special requests if reserving online)
	Hilton Baltimore (co-headquarters)	US$ 179	US$ 179	US$ 179	US$ 199	US $219	No charge for rollaway cots, available in king-bedded rooms only
	Student Rate	US$ 149	US$ 149	US$ 149	US $169	US $189	
	Baltimore Marriott Inner Harbor (co-headquarters)	US$ 169	US$ 169	US$ 169	US $189	US$ 209	No charge for rollaway cots, available in king-bedded rooms only
	Student Rate	US$ 138	US$ 138	US$ 138	US $158	US$ 178	
	Sheraton Inner Harbor	US$ 159	US$ 159	US$ 159	US$ 179	US$ 199	No charge for rollaway cots, available in king-bedded rooms only
	Student Rate	US$ 145	US$ 145	US$ 145	US$ 165	US$ 185	
	Renaissance Harborplace Hotel	US$ 155	US$ 155	US$ 155	US$ 175	US$ 195	Rollaway cots available for US$ 25 per stay, in king-bedded rooms only
	Student Rate	US$ 138	US$ 138	US$ 138	US$ 158	US$ 178	
	Hyatt Regency Baltimore	US$ 150	US$ 150	US$ 150	US$ 175	US$ 200	Rollaway cots available for US$ 25 per stay in king-bedded rooms only
	Student Rate	US$ 140	US$ 140	US$ 140	US$ 165	US$ 190	
	Royal Sonesta	US$ 149	US$ 149	US$ 149	US$ 169	US$ 189	Rollaway cots available for US$ 30 per night in king-bedded rooms only.
	Student Rate	US$ 134	US$ 134	US$ 134	US$ 154	US$ 174	
	Lord Baltimore Hotel	US$ 139	US$ 139	US$ 139	US$ 159	US$ 179	Rollaway cots available for US$ 20 per night in king-bedded rooms only
	Student Rate	US$ 129	US$ 129	US$ 129	US$ 149	US$ 169	
	Hotel Monaco	US$ 135	US$ 135	US$ 135	US$ 155	US$ 175	Rollaway cots available for US$ 25 per stay in king-bedded rooms only.
	Student Rate	US$ 125	US$ 125	US$ 125	US$ 145	US$ 165	
	Days Inn Baltimore Inner Harbor	US$ 139	US$ 139	US$ 139	N/A	N/A	Rollaway cots available for US$ 10 per night based on availability
	Student Rate	US$ 129	US$ 129	US$ 129	N/A	N/A	
	Holiday Inn Inner Harbor	US$ 139	US$ 139	US$ 139	US$ 159	US$ 179	No charge for rollaway cots, available in king-bedded rooms only
	Student Rate	US$ 129	US$ 129	US$ 129	US$ 149	US$ 169	

IN THE NEXT ISSUE OF NOTICES

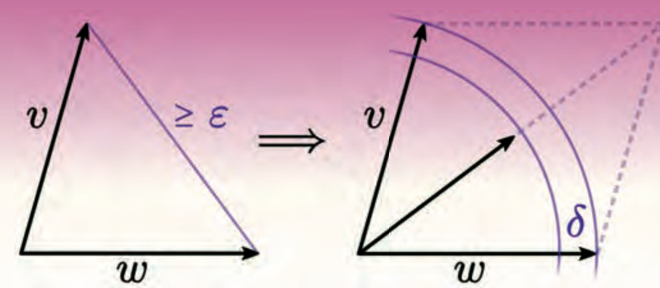
DECEMBER 2018



The Mathematics of Richard Schoen

Ad Honorem Yves Meyer

Felix Browder (1927–2016)



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◆ = Textbook

How Euler Did Even More

C. Edward Sandifer

The sum total of years of work and study of the most prolific mathematician of history, this volume will leave you marveling at Euler's clever inventiveness and Sandifer's wonderful ability to explicate and put it all in context.

Spectrum, Volume 77; 2015; 236 pages; Softcover; ISBN: 978-0-88385-584-3; List US$36; AMS members US$27; MAA members US$27; Order code SPEC/77

In the Dark on the Sunny Side

A Memoir of an Out-of-Sight Mathematician

Larry W. Baggett

Besides describing the various successes and failures that visually impaired mathematician Larry Baggett experienced living in the dark on the sunny side, he displays in this volume his love of math and music by interspersing short musings on both topics.

Spectrum, Volume 71; 2012; 206 pages; Hardcover; ISBN: 978-0-88385-581-2; List US$40; AMS members US$30; MAA members US$30; Order code SPEC/71

Ordinary Differential Equations ◆

From Calculus to Dynamical Systems

Virginia W. Noonburg

Aimed at students with a good calculus background that want to learn more about how calculus is used to solve real problems in today's world, this book presents a modern treatment of material traditionally covered in the sophomore-level course in ordinary differential equations.

AMS/MAA Textbooks, Volume 25; 2014; 315 pages; Hardcover; ISBN: 978-1-93951-204-8; List US$65; AMS members US$48.75; MAA members US$48.75; Order code TEXT/25

Beyond the Quadratic Formula

Ron Irving

The study of formulas for solutions of cubic and quartic equations forms the heart of this book as part of the broader theme that a polynomial's coefficients can be used to obtain detailed information on its roots.

Classroom Resource Materials, Volume 43; 2013; 228 pages; Hardcover; ISBN: 978-0-88385-783-0; List US$55; AMS members US$41.25; MAA members US$41.25; Order code CLRM/43

A Mathematical Space Odyssey

Solid Geometry in the 21st Century

Claudi Alsina and **Roger B. Nelsen**

This book presents techniques for proving a variety of geometric results in three dimensions.

Dolciani Mathematical Expositions, Volume 50; 2015; 271 pages; Hardcover; ISBN: 978-0-88385-358-0; List US$55; AMS members US$41.25; MAA members US$41.25; Order code DOL/50

Varieties of Integration ◆

C. Ray Rosentrater

This volume explores the critical contributions by Riemann, Darboux, Lebesgue, Henstock, Kurzweil, and Stieltjes to the theory of integration and provides a glimpse of more recent variations.

Dolciani Mathematical Expositions, Volume 51; 2015; 325 pages; Hardcover; ISBN: 978-0-88385-359-7; List US$58; AMS members US$43.50; MAA members US$43.50; Order code DOL/51

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