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Beyond Value-at-Risk: GlueVaR Distortion Risk Measures

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Abstract

We propose a new family of risk measures, called GlueVaR, within the class of distortion risk measures. Analytical closed-form expressions are shown for the most frequently used distribution functions in financial and insurance applications. The relationship between GlueVaR, Value-at-Risk (VaR) and Tail Value-at-Risk (TVaR) is explained. Tail-subadditivity is investigated and it is shown that some GlueVaR risk measures satisfy this property. An interpretation in terms of risk attitudes is provided and a discussion is given on the applicability in non-financial problems such as health, safety, environmental or catastrophic risk management.

1 Introduction

Financial and insurance risk management practitioners typically have to deal with two opposing demands: on the one hand, they want business units to achieve or outperform the objectives fixed by the firm's executive committee, yet, on the other, they are responsible for controlling their economic risks. Finding a trade-off between these two demands is the challenging task that risk managers face on a daily basis. At the same time, they need to decide how risk should be quantified.

Financial and insurance firms are subject to the capital requirements established by regulators' guidelines and directives. These requirements are typically equal to, or proportional to, a risk measure value that determines a minimum cushion of economic liquidity. The selection of such risk measures and tolerance levels is crucial therefore from the regulators' point of view.

Our aim is to propose a new family of risk measures, which we name GlueVar, and which have analytical closed-form expressions for many statistical distributions that are frequently used in financial and insurance applications. This new family combines the most popular risk measures and considers more than just one parameter to capture managerial and regulatory attitudes towards risk.

Financial institutions and insurance companies prefer to minimize the level of capital reserves required by solvency regulations, because they must contend with many restrictions on how this capital can be invested and, as such, the return on their capital reserves is usually lower than that provided by other opportunities. For this reason, companies typically favor regulations that impose risk measures and tolerance levels that are not overly conservative. Managers also prefer simple, straightforward risk measures rather than more complicated alternatives, since they claim that the former are more easily communicated.

From the regulators' perspective, controlling the risk of financial institutions and insurance companies is fundamental in order to protect consumers and investors, which may have conflicting objectives. Strict solvency capital requirements may limit the capacity of firms, but they also reassure consumers and guarantee the position of the financial industry in the economy. Thus, the debate as to what constitutes a suitable risk measure and what represents a suitable tolerance level is interminable, without their apparently having been much investigation as to what might represent an appropriate compromise.

We contend that the GlueVaR family could be useful in helping regulators and practitioners reach a consensus. As we discuss below, the GlueVaR family should enhance the way in which regulatory capital requirements are calculated, as GlueVaR can incorporate more information about agents' attitudes to risk. It is our belief that the incorporation of qualitative information in decision making tools is essential for risk managers and, as such, the GlueVaR risk measures can play a key role in achieving this goal.

2 Background and motivation

Value-at-Risk (VaR) has been adopted as a standard tool to assess the risk and to calculate capital requirements in the financial industry. Value-at-Risk at level α is the α-quantile of a random variable X (which we often call loss), i.e. $\mathrm{VaR}_\alpha(X) = \inf\{x \mid F_X(x) \geq \alpha\} = F_X^{-1}(\alpha)$, where F_X is the cumulative distribution function (cdf) of X and α is the confidence or the tolerance level $0 \leq \alpha \leq 1$. However, VaR is known to present a number of pitfalls when applied in practice. A disadvantage when using VaR in the financial context is that the capital requirements for catastrophic losses based on the measure can be underestimated, i.e. the necessary reserves in adverse scenarios may well be less than they should be. The underestimation of capital requirements may be aggravated when fat-tailed losses are incorrectly modeled by mild-tailed distributions, such as the Normal distribution. There are attempts to overcome this kind of model risk when using VaR or, at least, to quantify the risk related to the modelling, as shown in Alexander and Sarabia[(2)]. A second drawback is that the VaR may fail the subadditivity property. A risk measure is subadditive when the aggregated risk is less than or equal to the sum of individual risks. Subadditvity is an appealing property when aggregating risks in order to preserve the benefits of diversification. VaR is subadditive for elliptically distributed losses (see, for example, McNeil et al.[(42)]). However, the subadditivity of VaR is not granted as it has been shown, for instance, in Artzner et al.[(3)] and Acerbi and Tasche[(1)].

Tail Value-at-Risk (TVaR) may be interpreted as the mathematical expectation beyond VaR, and is defined as $\mathrm{TVaR}_\alpha(X) = \dfrac{1}{1-\alpha}\displaystyle\int_\alpha^1 VaR_\lambda(X)\,d\lambda$. The TVaR risk measure does not suffer the two drawbacks discussed above for VaR and, as such, would appear to be a more powerful measure for assessing the actual risks faced by companies and financial institutions. However, TVaR has not been widely accepted by practitioners in the financial and insurance industry. VaR is currently the risk measure contemplated in the European solvency regulation for the insurance sector (Solvency II), and this is also the case of solvency regulation for the banking sector (Basel accords). The TVaR measures average losses in the most adverse cases rather than just the minimum loss, as the

VaR does. Therefore, capital reserves based on the TVaR have to be considerably higher than those based on VaR and significant differences in the size of capital reserves can be obtained depending on which risk measure is adopted.

This paper is motivated, therefore, by an attempt to respond to the following question. Can a risk measure be devised that would provide a risk assessment that lies somewhere between that offered by the VaR and the TVaR? To this end, we propose a new family of risk measures (GlueVaR) which forms part of a wider class referred to as distortion risk measures. We analyze the subadditivity properties of these GlueVaR risk measures and show that a subfamily of GlueVaR risk measures satisfies tail-subadditivity.

GlueVaR risk measures are defined by means of a four-parameter function. By calibrating the parameters, GlueVaR risk measures can be matched to a wide variety of contexts. Specifically, once a confidence level has been fixed, the new family contains risk measures that lie between those of VaR and TVaR and which may adequately reflect the risk of mild-tailed distributed losses without having to resort to VaR. In certain situations, however, more conservative risk measures even than TVaR may be preferred. We show that these highly conservative risk measures can also be defined by means of the GlueVaR family. We derive analytical closed-form expressions of GlueVaR for commonly used statistical distributions in the financial context. These closed-form expressions should enable practitioners to undertake an effortless transition from the use of VaR and TVaR to GlueVaR.

3 Distortion risk measures

Consider a probability space and the set of all random variables defined on this space. Any risk measure[48] ρ is a mapping from the set of random variables to the real line $\mathbb{R}$, $X \mapsto \rho(X) \in \mathbb{R}$. Distortion risk measures were introduced by Wang[51,52] and are closely related to the distortion expectation theory. For instance, Tsanakas and Desli[49] provide a review on how risk measures can be interpreted from several perspectives, and include a clarifying explanation of the relationship between distortion risk measures and distortion expectation theory. There are two key elements to define a distortion risk measure: first, the associated distortion function; and, second, the concept of the Choquet[14] Integral. A detailed literature review of distortion risk measures is available in Denuit et al.[20] and Balbás et al.[6]. The distortion function, Choquet Integral and the distortion risk measure can be defined as follows:

- **Distortion function.** Let $g : [0,1] \to [0,1]$ be a function such that $g(0) = 0$, $g(1) = 1$ and g is non-decreasing. Then g is called a distortion function.

- **Choquet Integral** The (asymmetric) Choquet Integral with respect to a set function μ of a μ-measurable function $X : \Omega \to \overline{\mathbb{R}}$ is denoted as $\int X d\mu$ and is equal to $\int X d\mu = \int_{-\infty}^{0} [S_{\mu,X}(x) - \mu(\Omega)]\, dx + \int_{0}^{+\infty} S_{\mu,X}(x) dx$, if $\mu(\Omega) < \infty$, where $S_{\mu,X}(x) = \mu(\{X > x\})$ denotes the *survival function* of X with respect to μ. Note that Ω denotes a set, which in financial and insurance applications is the sample space of a probability space. A set function μ in this context is a function defined from 2^{Ω} (the set of all subsets of Ω) to $\overline{\mathbb{R}}$. A μ-measurable

Table 1: VaR and TVaR distortion functions

Risk measure	Distortion function
VaR	$\psi_\alpha(u) = \begin{cases} 0 & \text{if} \quad 0 \leq u < 1-\alpha \\ 1 & \text{if} \quad 1-\alpha \leq u \leq 1 \end{cases}$
TVaR	$\gamma_\alpha(u) = \begin{cases} \dfrac{u}{1-\alpha} & \text{if} \quad 0 \leq u < 1-\alpha \\ 1 & \text{if} \quad 1-\alpha \leq u \leq 1 \end{cases}$

For a confidence level $\alpha \in (0,1)$.

function X is, widely speaking, a function defined on Ω so that expressions like $\mu(\{X > x\})$ or $\mu(\{X \leq x\})$ make sense. See Denneberg[19] for more details.

- **Distortion risk measure.** Let g be a distortion function. Consider a random variable X and its survival function $S_X(x) = P(X > x)$. Function ρ_g defined by $\rho_g(X) = \int_{-\infty}^{0} [g(S_X(x)) - 1]\,dx + \int_{0}^{+\infty} g(S_X(x))\,dx$ is called a distortion risk measure.

From the previous definitions, it is straightforward to see that for any random variable X, $\rho_g(X)$ is the Choquet Integral of X with respect to the set function $\mu = g \circ P$, where P is the probability function associated with the probability space in which X is defined.

The mathematical expectation is a distortion risk measure whose distortion function is the identity function, $\rho_{id}(X) = \mathbb{E}(X)$ (see, for instance, Denuit et al.[20]). Therefore, a straightforward way to interpret a distortion risk measure is as follows: first, the survival function of the random variable is distorted ($g \circ S_X$); second, the mathematical expectation of the distorted random variable is computed. From a theoretical point of view, note that this interpretation fits the discussion by Aven[5], who considers that risk may be defined as an expected value in many situations.

VaR and TVaR measures are in fact distortion risk measures. The associated distortion functions of these risk measures are shown in Table 1.

Based on the distortion functions shown in Table 1, once α is fixed it can be proved that $\mathrm{VaR}_\alpha(X) \leq \mathrm{TVaR}_\alpha(X)$ for any random variable X.

Remark 3.1 *Let g and g^* be two distortion functions and let ρ_g and ρ_{g^*} be their respective distortion risk measures. Suppose that $g(u) \leq g^*(u)$ for all $u \in [0,1]$. Then $\rho_g(X) \leq \rho_{g^*}(X)$ for any random variable X.*
This result follows immediately from the definition of distortion risk measures, because

$$\int_{-\infty}^{0} [g(S_X(x)) - 1]dx + \int_{0}^{+\infty} g(S_X(x))\,dx \leq \int_{-\infty}^{0} [g^*(S_X(x)) - 1]dx + \int_{0}^{+\infty} g^*(S_X(x))\,dx.$$

4 A new family of risk measures: GlueVaR

We define a new family of risk measures, named GlueVaR. Any GlueVaR risk measure can be described by means of its distortion function. Given a confidence level α, the distortion function for GlueVar is:

$$\kappa_{\beta,\alpha}^{h_1,h_2}(u) = \begin{cases} \dfrac{h_1}{1-\beta} \cdot u & \text{if} \quad 0 \le u < 1-\beta \\ h_1 + \dfrac{h_2 - h_1}{\beta - \alpha} \cdot [u - (1-\beta)] & \text{if} \quad 1-\beta \le u < 1-\alpha \\ 1 & \text{if} \quad 1-\alpha \le u \le 1 \end{cases} \tag{1}$$

where $\alpha, \beta \in [0,1]$ so that $\alpha \le \beta$, $h_1 \in [0,1]$ and $h_2 \in [h_1, 1]$. Parameter β is the additional confidence level besides α. The shape of the GlueVaR distortion function is determined by the distorted survival probabilities h_1 and h_2 at levels $1-\beta$ and $1-\alpha$, respectively. We call parameters h_1 and h_2 the heights of the distortion function.

A wide range of risk measures may be defined under this framework. Note that VaR_α and TVaR_α are particular cases of this new family of risk measures. Namely, for a random variable X, $\text{VaR}_\alpha(X)$ and $\text{TVaR}_\alpha(X)$ correspond to distortion functions $\kappa_{\alpha,\alpha}^{0,0}(u)$ and $\kappa_{\alpha,\alpha}^{1,1}(u)$, respectively. By establishing suitable conditions on the heights h_1 and h_2, the GlueVaR family is very flexible. For example, risk managers might like to select α, β, h_1 and h_2 so that $\text{VaR}_\alpha(X) \le \text{GlueVaR}_{\beta,\alpha}^{h_1,h_2}(X) \le \text{TVaR}_\alpha(X)$: this can be achieved by selecting a set of parameters for their associated distortion functions to ensure that $\psi_\alpha(u) \le \kappa_{\beta,\alpha}^{h_1,h_2}(u) \le \gamma_\alpha(u)$ for any $u \in [0,1]$, following remark 3.1, i.e. by forcing condition $h_1 \le \dfrac{1-\beta}{1-\alpha}$. An example of such a case is shown in Figure 1 (left-hand side).

The GlueVaR family also allows us to define a highly conservative risk measure $\text{GlueVaR}_{\beta,\alpha}^{h_1,h_2}$, so that $\text{TVaR}_\alpha(X) \le \text{GlueVaR}_{\beta,\alpha}^{h_1,h_2}(X) \le \text{TVaR}_\beta(X)$ for any X and that the associated distortion function $\kappa_{\beta,\alpha}^{h_1,h_2}(u)$ is concave in $[0,1]$. In this case, $\dfrac{1-\beta}{1-\alpha} \le h_1$ and $h_2 = 1$ must be fulfilled, as occurs in the example shown in Figure 1 (right-hand side).

5 Linear combination of risk measures

Given a random variable X and for fixed tolerance levels α and β so that $\alpha < \beta$, $\text{GlueVaR}_{\beta,\alpha}^{h_1,h_2}(X)$ can be expressed as a linear combination[1] of $\text{TVaR}_\beta(X)$, $\text{TVaR}_\alpha(X)$ and $\text{VaR}_\alpha(X)$. This result allows us to translate the initial graphical-based construction of GlueVaR risk measures into an algebraic construction based on standard risk measures.

If the following notation is used,

$$\begin{cases} \omega_1 = & h_1 - \dfrac{(h_2 - h_1)\cdot(1-\beta)}{\beta - \alpha} \\ \omega_2 = & \dfrac{h_2 - h_1}{\beta - \alpha} \cdot (1-\alpha) \\ \omega_3 = & 1 - \omega_1 - \omega_2 \quad = 1 - h_2, \end{cases} \tag{2}$$

[1] An interpretation of GlueVaR risk measures as aggregation operators can be undertaken. An aggregation operator is a function that combines inputs into a single value, where inputs may be degrees of preference, membership or likelihood, or support of a hypothesis. Therefore, a linear combination of risk measures may be understood as an aggregation operator. A complete state of the art on aggregation operators can be found in Grabisch et al.(34,35). Additionally, VaR and TVaR may be understood as aggregation operators for discrete distributed random variables, as it has been shown in Belles-Sampera et al.(7). Dhaene et al.(22) discuss the relationships between quantiles and distortion risk measures.

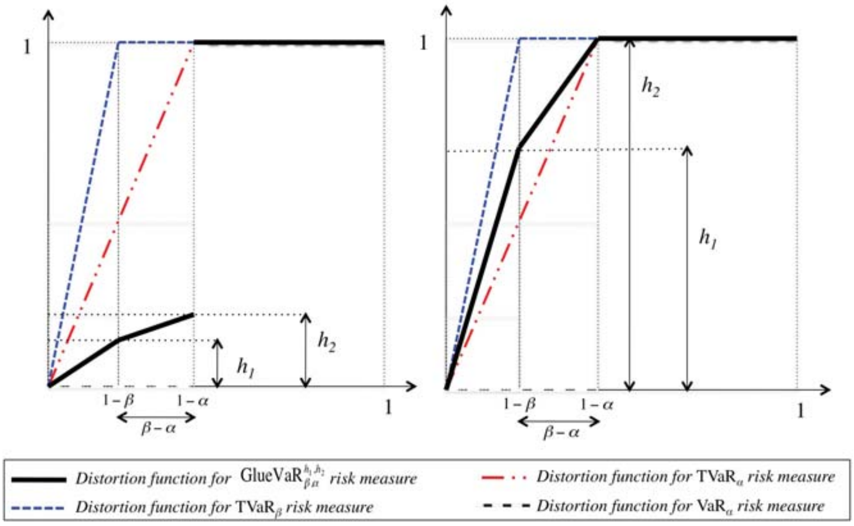

Figure 1: Examples of GlueVaR distortion functions.
Left. Distortion function is concave in $[0, 1-\alpha)$ and $\text{VaR}_\alpha(X) \leq \text{GlueVaR}_{\beta,\alpha}^{h_1,h_2}(X) \leq \text{TVaR}_\alpha(X)$ for a random variable X;
Right. Distortion function is concave in the whole range $[0, 1]$ and $\text{TVaR}_\alpha(X) \leq \text{GlueVaR}_{\beta,\alpha}^{h_1,h_2}(X) \leq \text{TVaR}_\beta(X)$ for a random variable X.

then the distortion function $\kappa_{\beta,\alpha}^{h_1,h_2}(u)$ in (1) may be rewritten as (details can be found in appendix A):

$$\kappa_{\beta,\alpha}^{h_1,h_2}(u) = \omega_1 \cdot \gamma_\beta(u) + \omega_2 \cdot \gamma_\alpha(u) + \omega_3 \cdot \psi_\alpha(u) \tag{3}$$

where γ_β, γ_α, ψ_α are the distortion functions of TVaR at confidence levels β and α and of VaR at confidence level α, respectively (see Table 1). Therefore GlueVaR is a risk measure that can be expressed as a linear combination of three risk measures: TVaR at confidence levels β and α and VaR at confidence level α,

$$GlueVaR_{\beta,\alpha}^{h_1,h_2}(X) = \omega_1 \cdot TVaR_\beta(X) + \omega_2 \cdot TVaR_\alpha(X) + \omega_3 \cdot VaR_\alpha(X). \tag{4}$$

Given this relationship, some abuse of notation may be employed for $\text{GlueVaR}_{\beta,\alpha}^{h_1,h_2}(X)$ and its related distortion function. The notation $\text{GlueVaR}_{\beta,\alpha}^{\omega_1,\omega_2}(X)$ or $\kappa_{\beta,\alpha}^{\omega_1,\omega_2}(u)$ may, on occasions, be preferred to that based on heights h_1 and h_2. The bijective relationship between pairs (h_1, h_2) and (ω_1, ω_2) is shown in appendix B.

5.1 Analytical closed-form expressions of GlueVaR

A useful consequence of (4) is that when analytical closed-form expressions of $\text{VaR}_\alpha(X)$ and $\text{TVaR}_\alpha(X)$ are known for a random variable X, we can automatically derive the closed-form expression of $\text{GlueVaR}_{\beta,\alpha}^{h_1,h_2}(X)$ without further complications. Otherwise, using the definition of GlueVaR as a distortion risk measure, the Choquet Integral of X with respect to the set function $\kappa_{\beta,\alpha}^{h_1,h_2} \circ P$ should be calculated.

5.1.1 Illustration: GlueVaR expression for Student t distribution

If X is a random variable such that $\tilde{X} = \dfrac{X-\mu}{\sigma}$ is distributed as a Student t random variable with ν degrees of freedom (df)[2], then

$$VaR_\alpha(X) = \mu + \sigma \cdot t_\alpha$$
$$TVaR_\alpha(X) = \mu + \sigma \cdot \frac{\tau(t_\alpha)}{1-\alpha} \cdot \left(\frac{\nu + t_\alpha^2}{\nu - 1}\right),$$

where t_α is the α-quantile of a Student t distribution with ν df and τ is its density function.

Using (4) the GlueVaR of X random variable is

$$GlueVaR_{\beta,\alpha}^{h_1,h_2}(X) = \omega_1 \cdot \left[\mu + \sigma \cdot \frac{\tau(t_\beta)}{1-\beta} \cdot \left(\frac{\nu + t_\beta^2}{\nu - 1}\right)\right] + \omega_2 \cdot \left[\mu + \sigma \cdot \frac{\tau(t_\alpha)}{1-\alpha} \cdot \left(\frac{\nu + t_\alpha^2}{\nu - 1}\right)\right]$$
$$+ (1 - \omega_1 - \omega_2) \cdot (\mu + \sigma \cdot t_\alpha) =$$
$$\mu + \sigma \cdot \left[\left(\frac{h_1}{1-\beta} - \frac{h_2 - h_1}{\beta - \alpha}\right) \cdot \tau(t_\beta) \cdot \left(\frac{\nu + t_\beta^2}{\nu - 1}\right) + \frac{h_2 - h_1}{\beta - \alpha} \cdot \tau(t_\alpha) \cdot \left(\frac{\nu + t_\alpha^2}{\nu - 1}\right) + (1 - h_2) \cdot t_\alpha\right]$$

[2] In a case such as this, X has μ mean and a standard deviation equal to $\sqrt{\dfrac{\nu \cdot \sigma^2}{\nu - 2}}$

5.1.2 Analytical expressions for other frequently used distributions

Normal ($\mathcal{N}$), Log-normal ($\mathcal{LN}$) and Generalized Pareto ($\mathcal{GP}$) distributions[3] have simple closed-form expressions of GlueVaR. Notation conventions are used. Namely, ϕ and Φ stand for the standard Normal pdf and cdf, respectively. The standard Normal distribution α and β quantiles are denoted as $q_\alpha = \Phi^{-1}(\alpha)$ and $q_\beta = \Phi^{-1}(\beta)$. For the $\mathcal{GP}$ distribution, the definition provided by Hosking and Wallis[(39)] is considered, where the scale parameter is denoted by σ and k is the shape parameter. The $\mathcal{GP}$ distribution contains the Uniform ($k = 1$), the Exponential ($k = 0$), the Pareto ($k < 0$) and the type II Pareto ($k > 0$) distributions as special cases. Closed-form expressions of GlueVaR for several distributions are presented in Table 2.

[3]There are some exceptions to this rule. When X follows a Pareto distribution with $k \leq 1$ and for any confidence level α, $\mathrm{TVaR}_\alpha(X) = +\infty$. But when $h_1 = 0$ $\mathrm{GlueVaR}^{h_1,h_2}_{\beta,\alpha}(X)$ is finite. There is a compensation effect between $\mathrm{TVaR}_\alpha(X)$ and $\mathrm{TVaR}_\beta(X)$. This is taken into account in Table 2.

Table 2: Closed-form expressions of GlueVaR for some selected distributions

Distribution	$\mathbf{GlueVaR}_{\beta,\alpha}^{h_1,h_2}$ **expression**
Normal: $\mathcal{N}(\mu,\sigma^2)$	$\mu+\sigma\cdot q_\alpha\cdot(1-h_2)+\sigma\cdot\frac{h_2-h_1}{\beta-\alpha}\cdot[\phi(q_\alpha)-\phi(q_\beta)]+\sigma\cdot\frac{h_1}{1-\beta}\cdot\phi(q_\beta)$
Log-normal: $\mathcal{LN}(\mu,\sigma^2)$	$\exp(\mu+\sigma\cdot q_\alpha)\cdot(1-h_2)+\exp\left(\mu+\frac{\sigma^2}{2}\right)\cdot\frac{h_2-h_1}{\beta-\alpha}\cdot[\Phi(\sigma-q_\alpha)-\Phi(\sigma-q_\beta)]+ +\exp\left(\mu+\frac{\sigma^2}{2}\right)\cdot\frac{h_1}{1-\beta}\cdot\Phi(\sigma-q_\beta)$
Exponential: $\mathcal{GP}(k,\sigma)$, with $k=0$	$\sigma\cdot[h_2-\ln(1-\alpha)]+\sigma\cdot(1-\beta)\cdot\ln\left(\frac{1-\beta}{1-\alpha}\right)\cdot\left[\frac{h_2-h_1}{\beta-\alpha}-\frac{h_1}{1-\beta}\right]$
Pareto: $\mathcal{GP}(k,\sigma)$, with $k<0$	$\begin{cases} +\infty & if\quad k\le-1,\quad h_1\neq0 \\ \frac{\sigma}{k}\cdot\left[1-(1-\alpha)^k\right]+\frac{h_2-h_1}{\beta-\alpha}\cdot(1-\beta)\cdot\frac{\sigma}{k}\cdot\left[(1-\beta)^k-(1-\alpha)^k\right]+ +\frac{h_2-h_1}{\beta-\alpha}\cdot\frac{\sigma}{k+1}\cdot\left[(1-\alpha)^{k+1}-(1-\beta)^{k+1}\right] & if\quad k<-1,\quad h_1=0 \\ \sigma\cdot\left[\frac{1}{1-\alpha}-1\right]-\frac{h_2-h_1}{\beta-\alpha}\cdot(1-\beta)\cdot\sigma\cdot\left[\frac{1}{1-\beta}-\frac{1}{1-\alpha}\right]+ +\frac{h_2-h_1}{\beta-\alpha}\cdot\sigma\cdot\ln\left(\frac{1-\alpha}{1-\beta}\right) & if\quad k=-1,\quad h_1=0 \\ \frac{\sigma}{k}\cdot\left[1-(1-\alpha)^k\right]+\frac{\sigma}{k}\cdot\left(\frac{h_2-h_1}{\beta-\alpha}-\frac{h_1}{1-\beta}\right)\cdot\left[(1-\alpha)^k\cdot(1-\beta)\right]+ +\frac{h_2-h_1}{\beta-\alpha}\cdot\frac{\sigma}{k}\cdot\left[\frac{k\cdot(1-\alpha)^{k+1}}{k+1}\right]+\left(\frac{h_2-h_1}{\beta-\alpha}-\frac{h_1}{1-\beta}\right)\cdot\frac{\sigma}{k}\cdot\left[\frac{(1-\beta)^{k+1}}{k+1}\right] & if\quad k\in(-1,0) \end{cases}$
Type II Pareto: $\mathcal{GP}(k,\sigma)$, with $k>0$	$\frac{\sigma}{k}\cdot\left[1-(1-\alpha)^k\right]+\frac{\sigma}{k}\cdot\left(\frac{h_2-h_1}{\beta-\alpha}-\frac{h_1}{1-\beta}\right)\cdot\left[(1-\alpha)^k\cdot(1-\beta)\right]+ +\frac{h_2-h_1}{\beta-\alpha}\cdot\frac{\sigma}{k}\cdot\left[\frac{k\cdot(1-\alpha)^{k+1}}{k+1}\right]+\left(\frac{h_2-h_1}{\beta-\alpha}-\frac{h_1}{1-\beta}\right)\cdot\frac{\sigma}{k}\cdot\left[\frac{(1-\beta)^{k+1}}{k+1}\right]$

6 Subadditivity in the tail

This section is devoted to an analysis of the properties of the GlueVaR family of risk measures, with special attention to subadditiviy. Our reason for defining these GlueVaR risk measures is a response to the concerns expressed by risk managers regarding the choice of risk measures in the case of regulatory capital requirements. However, an axiomatic approach to define or represent risk measures is more frequent in the literature(3,28,29,21,47,12,24,32,36).

Artzner et al.(3) established the following set of axioms that a risk measure should satisfy: positive homogeneity, translation invariance, monotonicity and subadditivity. They referred to such risk measures as "coherent risk measures". Distortion risk measures always satisfy the first three properties, but subadditivity is only guaranteed when the distortion function is concave(19,54,56). Therefore, VaR, unlike TVaR, is not coherent. In some situations, coherence of risk measures is a requirement (see, for instance, Cox(15)) but, nonetheless, some criticisms can be found, for example, in Dhaene et al.(23). Additional properties for distortion risk measures are provided by Jiang(41) and Balbás et al.(6).

As shown in the previous section, GlueVaR risk measures may be interpreted as a linear combination of VaR and TVaR risk measures. Therefore, a GlueVaR risk measure is coherent when the weight assigned to VaR is zero and the weights of TVaR are non-negative. In terms of the parameters of the distortion function, GlueVaR is subadditive (and thus coherent) if $h_2 = 1$ and $\frac{1-\beta}{1-\alpha} \leq h_1$. More generally, any property satisfied by TVaR but not by VaR will be inherited by GlueVaR if $\omega_1 \geq 0$ and $\omega_3 = 0$ in expression (2).

Subaddtitivity in the whole domain is a strong condition. When dealing with fat tail losses (i.e. low-frequency and large-loss events), risk managers are especially interested in the tail region. Fat right-tails have been extensively studied in insurance and finance(53,25,26,17,44,13). To the best of our knowledge, however, previous studies of the subadditivity of risk measures in the tail region are scarce(16,40). The milder condition of subadditivity in the tail region is investigated here.

We introduce the concept of subadditivity in the right tail for a pair of risks. Note that if interested in the left -as opposed to the right- tail, a simple change of sign in the random variable suffices. Subadditivity in the right tail is defined in this discussion for distortion risk measures. Consider a probability space with sample space Ω. Let $s_\alpha(Z)$ the α-quantile of random variable Z, $s_\alpha(Z) = \inf\{z \mid S_Z(z) \leq 1-\alpha\}$. Let $\mathcal{Q}_{\alpha,Z}$ be defined by $\mathcal{Q}_{\alpha,Z} := \{\omega \mid Z(\omega) > s_\alpha(Z)\} \subseteq \Omega$, so $\mathcal{Q}_{\alpha,Z}$ means here the tail region of random variable Z given a confidence level α. Let X, Y be two risks defined on the same probability space. When aggregating two risks, the common tail for both risks must be taken into account. This common tail region is defined here as follows: $\mathcal{Q}_{\alpha,X,Y} := \mathcal{Q}_{\alpha,X} \cap \mathcal{Q}_{\alpha,Y} \cap \mathcal{Q}_{\alpha,X+Y}$.

Definition 6.1 *Given a confidence level $\alpha \in [0,1]$, a distortion risk measure ρ_g is subadditive in the tail for the pair X, Y if $\mathcal{Q}_{\alpha,X,Y} \neq \emptyset$ and*

$$\int_{\mathcal{Q}_{\alpha,X,Y}} (X+Y)\, d(g \circ P) \leq \int_{\mathcal{Q}_{\alpha,X,Y}} X d(g \circ P) + \int_{\mathcal{Q}_{\alpha,X,Y}} Y d(g \circ P),$$

where the integral symbol stands for Choquet Integrals with respect to the set function $g \circ P$.

When there is no ambiguity as to which confidence level α and random variables X, Y are taken into account, *tail-subadditivity* is used to refer to this property. If notation $m_\alpha = \sup\{s_\alpha(X), s_\alpha(Y), s_\alpha(X+Y)\}$

is introduced, the integral condition used in the definition can be rewritten, in terms of survival functions, as

$$\int_{\inf\{0,m_\alpha\}}^{0} \left[g\left(S_{X+Y}(z)\right)-1\right]dz+\int_{\sup\{0,m_\alpha\}}^{+\infty} g\left(S_{X+Y}(z)\right)dz \leq \int_{\inf\{0,m_\alpha\}}^{0} \left[g\left(S_X(x)\right)-1\right]dx+$$
$$+\int_{\sup\{0,m_\alpha\}}^{+\infty} g\left(S_X(x)\right)dx+\int_{\inf\{0,m_\alpha\}}^{0} \left[g\left(S_Y(y)\right)-1\right]dy+\int_{\sup\{0,m_\alpha\}}^{+\infty} g\left(S_Y(y)\right)dy.$$

Theorem 6.1 *Given a confidence level α and a pair of risks X and Y so that $\mathcal{Q}_{\alpha,X,Y} \neq \emptyset$, a GlueVaR risk measure is tail-subadditive if its associated distortion function $\kappa_{\beta,\alpha}^{h_1,h_2}(u)$ is concave in $[0, 1-\alpha)$.*

The proof is contained in appendix C.

Tail-subadditivity is a desirable property, because it implies that the benefits of diversification may not be valid in every situation but, at least, they hold in extreme cases.

Note that, in terms of parameters h_1 and h_2, a GlueVaR risk measure may be tail-subadditive if, and only if, $h_2 \leq h_1 \cdot \dfrac{1-\alpha}{1-\beta}$, as a corollary of Theorem 6.1.

7 Risk attitudes in GlueVaR

An interesting interpretation in the context of decision making and risk management is that GlueVaR risk measures arise as a linear combination of three possible scenarios. So, two levels of severity can be fixed, namely α and β, with $\alpha < \beta$. Then, the risk can be measured in the highly conservative scenario with TVaR at level β; in the conservative scenario with TVaR at level α; and in the less conservative scenario with VaR at level α.

Each combination of these risk scenarios reflects a specific risk aversion attitude. Therefore, we can say that the combination of these risk attitudes in this context is something that is directly identified by an explicit GlueVaR risk measure. To some extent, these risk attitudes could be related to risk appetite as shown in Aven(4).

From the practitioner's point of view, four parameters must be fixed in order to define the GlueVar risk measure. The α and β values correspond to the confidence levels used for bad and very bad scenarios, respectively. We could select, for instance, $\alpha = 95\%$ and $\beta = 99.5\%$, which is equivalent to one bad event every twenty years or one bad event every two hundred, respectively. The other two parameters are directly related to the weights given to these scenarios. For instance, we could say that the three components of GlueVaR in (4) are equally important. This would imply $\omega_1 = \omega_2 = \omega_3 = 1/3$, so we could find the corresponding h_1 and h_2 parameters. When $\omega_1 = \omega_2 = \omega_3 = 1/3$ and $\alpha = 95\%$, $\beta = 99.5\%$, these parameters are $h_1 = 11/30$ and $h_2 = 2/3$.

7.1 Geometrical discussion on risk attitudes

Given α and β, the shaded areas in Figure 2 delimit feasible weights (ω_1, ω_2) for $\text{GlueVaR}_{\beta,\alpha}^{\omega_1,\omega_2}$. The point $(1/3, 1/3)$ corresponds to a balanced risk attitude on the part of risk managers when faced by the three components shown in (4). The corresponding distortion function $\kappa_{\beta,\alpha}^{\omega_1,\omega_2}$ is concave on $[0, 1-\alpha)$ in the lightly shaded area and, thus, the associated GlueVaR risk measure can be tail-subadditive. Yet, the distortion function is not concave on $[0, 1-\alpha)$ in the darkly shaded area and, thus, the associated GlueVaR risk measure cannot be tail-subadditive. The distortion function is concave in $[0, 1]$ in the boldest continuous segment and, thus, the associated GlueVaR risk measure is subadditive.

If $\omega_1 < 0$, risk managers are optimistic regarding the impossibility of the occurrence of the worst case scenario, and so attach a negative weight to it.

Note that any pair of weights (ω_1, ω_2) on the boldest line in Figure 2 leads to $\omega_3 = 0$. This means that a zero weight is allocated to the less conservative scenario, i.e. the one associated with the $\mathrm{VaR}_\alpha(X)$. This is indicative of the decision makers' conservative approach. Nonetheless, differences in just how restrictive this conservative attitude is can be found among the weights lying on this line: the nearer to $(\omega_1, \omega_2) = \left(\frac{\beta - 1}{\beta - \alpha}, \frac{1 - \alpha}{\beta - \alpha}\right)$, the less restrictive it is, while the nearer to $(\omega_1, \omega_2) = (1, 0)$, the more conservative it is.

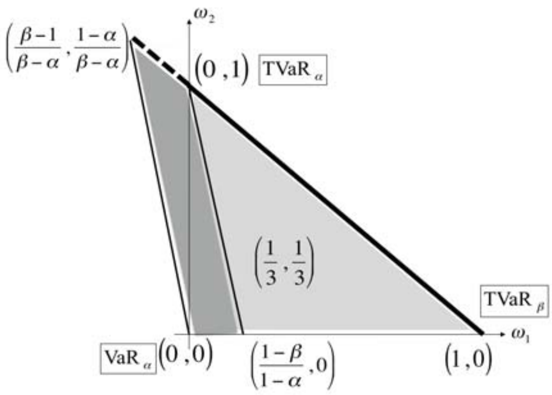

Figure 2: Given α and β, the shaded areas delimits feasible weights (ω_1, ω_2) for $\mathrm{GlueVaR}_{\beta,\alpha}^{\omega_1,\omega_2}$.

7.2 Illustration

Data for the cost of claims for property damage from a major Spanish motor insurer are used to illustrate the applicability of these results. These data contain $n = 519$ observations of the cost of individual claims in thousands of euros, and were analyzed in Bolancé et al.[(8)] and Guillén et al.[(37)]. The risk measures for these data are displayed in Table 3. In the first row, our results obtained using the empirical distribution are presented. In subsequent rows Normal, Log-normal, Student t with 4 df and Generalized Pareto distributions are fitted and their respective risk values are shown.

The values in Table 3 indicate that the cost of individual claims is fat right-tailed: differences between $\mathrm{TVaR}_{95\%}(X)$ and $\mathrm{VaR}_{95\%}(X)$, and also between $\mathrm{TVaR}_{99.5\%}(X)$ and $\mathrm{TVaR}_{95\%}(X)$ are huge for the empirical distribution (and also for the rest of the selected distributions). In this case,

Table 3: Risk measures for the data on claims' cost

	$\text{VaR}_{95\%}(X)$	$\text{TVaR}_{95\%}(X)$	$\text{TVaR}_{99.5\%}(X)$	$\text{GlueVaR}^{11/30,2/3}_{99.5\%,95\%}(X)$
Empirical	38.8	112.5	440.0	197.1
Normal	78.9	96.1	130.4	101.8
Log-normal	42.5	106.3	364.0	170.9
Student t (4 df)	99.0	143.2	272.1	171.4
Pareto	38.3	82.4	264.5	128.4

X stands for "cost of individual claims in thousands of euros".
Notation $\text{GlueVaR}^{h_1,h_2}_{\beta,\alpha}(X)$ is used in this table.
For $\alpha = 95\%$, $\beta = 99.5\%$, $\omega_1 = 1/3$, $\omega_2 = 1/3$ and $\omega_3 = 1/3$.

it seems clear that GlueVaR is more conservative than TVaR at the 95% level but less so than TVaR at the 99.5% level, independently of the selected distribution. On the other hand, GlueVaR is not, unlike TVaR, subadditive, but it is a candidate to be tail-subbadditive in many situations, because its associated distortion function is concave in $[0, 0.05)$.

Calculations have been made in R. R programmes are available from the authors.

8 Other non-financial applications

New risk measures based on distortion functions can be valuable outside the scope of finance and insurance. There is a natural bridge from financial applications to any discipline where the choice of a risk measure plays a role for decision making. GlueVaR risk measures can be applied to health, safety, environmental, adversarial risks or catastrophic risks including terrorism. Health or safety regulations report quantile risks and could be enhanced with GlueVaR, as the latter allows to combine risk measures and risk levels.

A crucial feature is that in GlueVaR risk measures, the decision maker sets up two tolerance levels: one for the "bad cases" and another one for the "very bad cases". Then, weights are set up according to a decision maker's or a regulator's risk aversion. If a decision maker is very risk averse, then he should give all weight to the "very bad case" outcome, whereas a less risk averse decision maker would assign all weight to a lesser "very bad case". An intermediate decision maker could set up a reasonable position, where he could balance a position between Value-at-Risk versus Tail Value-at-Risk, and a trade-off between a lower tolerance level versus a higher tolerance level.

One good example of difficulties in agreeing upon a suitable risk measure is found in a recent article by Mohtadi and Agiwal[(43)] on the optimal security investments and extreme risks with an application to terrorism risks. These authors focus on both, amount and timing of security investment, but they fundamentally model risk based on the principle of expected net benefit of investment in security. An analogous principle is also used by Hausken[(38)] to investigate security in information systems and was earlier proposed by Gordon and Loeb[(33)] for the same purpose. The expected net benefit of investment in security is a simple analytical model that maximizes the gain when investing in security procedures. Optimization is straightforward once a level of loss severity and a probability of occurrence are assumed (see section 2 in Mohtadi and Agiwal[(43)]). When addressing terrorism risk in form of intentional attacks on the food sector using chemical, biological and radio nuclear tools, Mohtadi and Agiwal[43] establish several scenarios for casualties resulting from a terrorist attack, which have a very low probability. The choice of those scenarios is done subjectively, even if there is a technical part that relies on extreme value theory to approximate the tail probabilities. At least two scenarios are necessary to carry out the optimization procedure and their corresponding anti-cumulative distributions. A GlueVaR risk measure could be used directly in the optimization procedure. Moreover, GlueVaRs could help to compare different contexts, such as terrorist attacks on the food sector versus other sectors, where the number of casualties could be much lower. GlueVaRs could provide a single value for every phenomenon and there would be no need to define interval scenarios, whose choice could potentially be controversial.

Extensions can be found in many other applications. Let us imagine an employer who has to manage worker compensation reserves. In order to assess the risk of being short in reserves in a one-year horizon, the classical procedure is to estimate Value-at-Risk, which would equal the minimum reserve amount that would be likely to cover the compensations to be paid. If the manager has a tolerance of one in one hundred years, that would correspond to a confidence level equal to 99% and Value-at-Risk at that level should provide the estimate of sufficient reserve. In the GlueVaR framework, the manager could be slightly more cautious. He would recalculate the risk with a higher confidence and raise reserves accordingly, in case he might face a bad scenario. He could fix a confidence level of 99.5% or tolerate shortfall of reserves once in two hundred years. The fund manager could select transitional position and assign weights so that a GlueVaR reflects his risk attitude. He could give equal weight to the two scenarios and be equally positioned with regard to the classical risk measure. Not only risk managers, but regulators could leave freedom to fund managers to choose their preferred GlueVaR risk measure to set up their reserves and thus, regulators could inform about the risk measure that is being used by the workers compensation fund managers.

Generalizations and extensions to disaster management could be implemented. A public safety agency could use GlueVaR risk measures to help better plan how many resources to set aside to meet the needs of next year's hurricane season. Let us assume that resources are established proportional to a Value-at-Risk at certain level α. Similarly, as in the example concerning workers compensation, agencies could allow for a risk attitude, so that there would be two tolerance levels, α and β and then weights would determine, whether the agency would rather have a position in between those two levels, as it is natural to do with GlueVaRs.

Managers of scarce resources with uncertain supply and demand (e.g., strategic petroleum reserve, antibiotic stockpiles, blood bank, etc.) could use the GlueVaR measures instead of classical measures to improve decisions. Quantile-based risk measures are applied to solve a variety of optimization issues in presence of uncertainty in supply and/or demand, such as strategic planning for hospital care services (Dehlendorff et al.[18]), operational planning of chemical and petrochemical plants (Verderame and Floudas[50], Pongsakdi et al.[46]), level of capacity in auto industry facilities (Eppen et al.[27]), water resources management (García-González et al.[30], Webby et al.[55]) or hydrocarbon supply-chain designs (Gebreslassie et al.[31], Carneiro et al.[11]). Many decisions are taken based on an optimization procedure where an expected loss is minimized subject to the risk being lower that an upper bound. Risk could be defined by a GlueVaR rather than by a traditional risk measure. In that respect, GlueVaR would allow for defining the bad and very bad scenarios and would weight them according to risk aversion.

A related area where the application of GlueVaR measures might be useful is the reservoir management decision making. A set of percentiles are commonly computed to subdivide hydrocarbon reserve estimates into categories that describe the probability of extracting a certain volume, such as proven (P1), probable (P2) and possible (P3) reserves (Bret-Rouzart and Favennec[9], Owen et al.[45], Campbell and Laherrère[10]). Management decisions are taken under these alternative scenarios. For instance, they might represent the bad, average and good cases to evaluate the impact of additional drillings. A unified evaluation may be performed by means of the GlueVaR measures. These three scenarios may be jointly considered by the selection of tolerance levels and weights for the Value-at-Risk and Tail Value-at-Risk measures in accordance with the management decision maker's risk profile.

The choice of a risk measure for regulatory purposes in a matter of strong debate in the financial and the insurance sector, as solvency requirements limit the potential benefits of a firm; however the discussion is not unique to that sector.

9 Conclusions

We have shown that GlueVaR measures can be expressed as linear combinations of standard risk measures and that, similarly, they can be defined based on a straightforward distortion function. Attractive properties of a GlueVaR risk measure are, therefore, readily derived from the definition of its associated distortion function. This is the case of the tail-subadditivity property defined in this paper. Basically, concavity of the distortion function on the subrange $[0, 1-\alpha)$ assures tail-subadditivity. This milder condition in the distortion function than concavity over the whole range might be a sufficient requisite for risk measures when fat right-tail risks are assessed: the benefits of diversification are attained in adverse scenarios but capital requirements are not excessively high.

The results provided in this article are directly applicable in financial industry. Closed-form expressions of GlueVaR risk measures are shown for commonly used distributions in finance and insurance. We encourage regulators and financial and insurance risk managers to seek an equilibrium between their different demands. The two levels of qualitative information that GlueVaR risk measures incorporate (one related to the confidence levels of bad and worst-case scenarios; the other related to the plausibility of those scenarios) can help achieve this goal. We believe that GlueVaR risk measures can play a leading role in helping to reach a satisfactory consensus.

There is potential for extending the application of GlueVaR measures to non-financial disciplines where the choice of a risk measure matters.

A Equivalent expression for the GlueVaR distortion function

We provide details as to how to define the GlueVaR distortion function $\kappa_{\beta,\alpha}^{h_1,h_2}(u)$ as a linear combination of the distortion functions of TVaR at confidence levels β and α, and VaR at confidence level α, i.e. details as to how to obtain expression (3). Expression (1) of the distortion function $\kappa_{\beta,\alpha}^{h_1,h_2}(u)$ can be rewritten as,

$$\kappa_{\beta,\alpha}^{h_1,h_2}(u) = \begin{aligned} & h_1 \cdot \gamma_\beta(u) \cdot \mathbb{1}_{[0,1-\beta)}(u) + \\ & + \left(h_1 + \frac{h_2-h_1}{\beta-\alpha} \cdot (1-\alpha) \cdot \gamma_\alpha(u) - \frac{h_2-h_1}{\beta-\alpha} \cdot (1-\beta)\right) \cdot \mathbb{1}_{[1-\beta,1-\alpha)}(u) + \\ & + \psi_\alpha(u), \end{aligned} \tag{5}$$

where $\mathbb{1}_{[x_1,x_2)}(u)$ is an indicator function so that it takes a value of 1 if $u \in [x_1, x_2)$ and 0 otherwise.

Note that

$$\gamma_\beta(u) \cdot \mathbb{1}_{[0,1-\beta)}(u) = \gamma_\beta(u) - \psi_\beta(u), \tag{6}$$

$$\mathbb{1}_{[1-\beta,1-\alpha)}(u) = \psi_\beta(u) - \psi_\alpha(u), \tag{7}$$

$$\gamma_\alpha(u) \cdot \mathbb{1}_{[1-\beta,1-\alpha)}(u) = \gamma_\alpha(u) - \psi_\alpha(u) - \left(\frac{1-\beta}{1-\alpha}\right) \cdot [\gamma_\beta(u) - \psi_\beta(u)]. \tag{8}$$

Taking into account expressions (6), (7) and (8), expression (5) may be rewritten as,

$$\begin{aligned} \kappa_{\beta,\alpha}^{h_1,h_2}(u) = & \left[h_1 - \frac{(h_2-h_1)\cdot(1-\beta)}{\beta-\alpha}\right] \cdot \gamma_\beta(u) + \\ & + \left[-h_1 + h_1 - \frac{(h_2-h_1)\cdot(1-\beta)}{\beta-\alpha} + \frac{(h_2-h_1)\cdot(1-\beta)}{\beta-\alpha}\right] \cdot \psi_\beta(u) + \\ & + \frac{h_2-h_1}{\beta-\alpha} \cdot (1-\alpha) \cdot \gamma_\alpha(u) + \left[1 - h_1 + \frac{(+h_2-h_1)\cdot(1-\beta)}{\beta-\alpha} - \frac{h_2-h_1}{\beta-\alpha} \cdot (1-\alpha)\right] \cdot \psi_\alpha(u). \end{aligned} \tag{9}$$

Given that $\omega_1 = h_1 - \frac{(h_2-h_1)\cdot(1-\beta)}{\beta-\alpha}$, $\omega_2 = \frac{h_2-h_1}{\beta-\alpha} \cdot (1-\alpha)$ and $\omega_3 = 1 - h_2$, expression (3) follows directly from (9).

B Bijective relationship between heights and weights

Pairs of GlueVaR heights (h_1, h_2) and weights (ω_1, ω_2) are linearly related to each other. The parameter relationships are $(h_1, h_2)' = H \cdot (\omega_1, \omega_2)'$ and, inversely, $(\omega_1, \omega_2)' = H^{-1} \cdot (h_1, h_2)'$, where H and H^{-1} matrices are $H = \begin{pmatrix} 1 & \frac{1-\beta}{1-\alpha} \\ 1 & 1 \end{pmatrix}$ and $H^{-1} = \begin{pmatrix} \frac{1-\alpha}{\beta-\alpha} & \frac{\beta-1}{\beta-\alpha} \\ \frac{\alpha-1}{\beta-\alpha} & \frac{1-\alpha}{\beta-\alpha} \end{pmatrix}$, respectively.

C Tail-subadditivity for GlueVaR risk measures

This appendix is devoted to the proof of Theorem 6.1. Given a confidence level α and a pair of random variables X and Y so that $\mathcal{Q}_{\alpha,X,Y} \neq \emptyset$, a GlueVaR risk measure is tail-subadditive if its associated distortion function $\kappa_{\beta,\alpha}^{h_1,h_2}$ is concave in $[0, 1-\alpha)$.

Following Denneberg[19], the subadditivity theorem and the integration on subsets of Ω are defined as:

- **Subadditivity theorem.** Let $\mu : 2^{\Omega} \to \overline{\mathbb{R}}_+$ be a monotone, submodular set function. Then for functions $X, Y : \Omega \to \overline{\mathbb{R}}$ being μ-essentially$> -\infty$

$$\int (X+Y)\, d\mu \leq \int X d\mu + \int Y d\mu.$$

If μ is continuous from below the assumption on X, Y being μ-essentially $> -\infty$ can be dropped.

- **Integration on subsets.** Let μ be a monotone set function on a set system $\mathcal{S} \subset 2^{\Omega}$ with $\Omega \in \mathcal{S}$ and closed under intersection. For $A \in \mathcal{S}$ define $\mu_A(B) := \mu(B \cap A)$, $B \in \mathcal{S}$. Then μ_A is a monotone set function on $\mathcal{S}$ and we define $\int_A X d\mu := \int X d\mu_A$.

A set system is, generally speaking, a collection of sets. Definitions of monotone, modular or submodular set functions, as well as the definition of continuity from below, are given in next paragraphs[4].

According to definition 6.1, given a confidence level α and taking into account that $\mathcal{Q}_{\alpha,X,Y} \neq \emptyset$ for the fixed pair of random variables, i.e. $X, Y : \Omega \to \overline{\mathbb{R}}$, the tail-subadditivity property is satisfied by a distortion risk measure ρ_g if the subadditivity theorem can be applied to the set function $(g \circ P)_{\mathcal{Q}_{\alpha,X,Y}}$, i.e. the set function so that for any $B \in 2^{\Omega}$, $(g \circ P)_{\mathcal{Q}_{\alpha,X,Y}}(B) = g(P(B \cap \mathcal{Q}_{\alpha,X,Y}))$.

Therefore, subadditivity in the tail for a pair of risks is proven if $(g \circ P)_{\mathcal{Q}_{\alpha,X,Y}}$ is submodular and continuous from below.

If ρ_g is a distortion risk measure so that its associated distortion function g is concave in $[0, 1-\alpha)$, then it is shown that $(g \circ P)_{\mathcal{Q}_{\alpha,X,Y}}$ is submodular[5]. Consider the set function ν defined by $\nu(B) := P(B \cap \mathcal{Q}_{\alpha,X,Y})$, for any $B \in 2^{\Omega}$. Note that $\nu(B) \in [0, 1-\alpha)$ because $P(\mathcal{Q}_{\alpha,X,Y}) < 1-\alpha$ and P is a monotone set function. The set function ν is modular because P is modular, i.e. $\nu(A \cup B) + \nu(A \cap B) = \nu(A) + \nu(B)$ for any $A, B \in 2^{\Omega}$. Given $A, B \in 2^{\Omega}$ suppose, without loss of generality, that $A \subseteq B$. Let us rename $a := \nu(A)$, $b := \nu(B)$, $i := \nu(A \cap B)$ and $u := \nu(A \cup B)$. Because ν is monotone then it holds that $i \leq a \leq b \leq u$ due to $A \cap B \subseteq A \subseteq B \subseteq A \cup B$. The modularity of ν implies that $i + u = a + b$, i.e. $[i, u]$ and $[a, b]$ have common centers, $\frac{i+u}{2} = \frac{a+b}{2}$. Then, because g is concave in $[i, u]$ we can conclude that $g(u) + g(i) \leq g(a) + g(b)$ or, equivalently, that $g \circ \nu = (g \circ P)_{\mathcal{Q}_{\alpha,X,Y}}$ is submodular.

The property of continuity from below of $g \circ \nu = (g \circ P)_{\mathcal{Q}_{\alpha,X,Y}}$ must also be satisfied to use the subadditivity theorem. An arbitrary set function μ is continuous from below if for any increasing collection of subsets in the set system ($A_n \in \mathcal{S}$, $A_n \subseteq A_{n+1}$ for $n \in \mathbb{N}$) so that $A := \cup_{n=1}^{\infty} A_n \in \mathcal{S}$ then equality $\lim_{n\to\infty} \mu(A_n) = \mu(A)$ holds. So $\mu = (g \circ P)_{\mathcal{Q}_{\alpha,X,Y}}$ is continuous from below because $(g \circ P)_{\mathcal{Q}_{\alpha,X,Y}} : 2^{\Omega} \longrightarrow g([0, 1-\alpha))$ and g is continuous on $[0, 1-\alpha)$.

Given that $(g \circ P)_{\mathcal{Q}_{\alpha,X,Y}}$ is submodular and continuous from below, applying the subadditivity theorem and using integration on subsets, it is true that, given X and Y:

$$\int_{\mathcal{Q}_{\alpha,X,Y}} (X+Y)\, d(g \circ P) \leq \int_{\mathcal{Q}_{\alpha,X,Y}} X d(g \circ P) + \int_{\mathcal{Q}_{\alpha,X,Y}} Y d(g \circ P),$$

[4] A proper definition of a function X μ-essentially $> -\infty$ is not needed in the proof and, thus, not provided. Interested readers can find this definition in Denneberg[(19)].

[5] A set function μ is modular if $\mu(A \cup B) + \mu(A \cap B) = \mu(A) + \mu(B)$, and it is submodular if $\mu(A \cup B) + \mu(A \cap B) \leq \mu(A) + \mu(B)$. A set function μ is monotone if $\mu(A) \leq \mu(B)$ for any $A \subseteq B$ in 2^{Ω}.

which prove that the associated risk measure ρ_g is tail-subadditive.

Consider a GlueVaR risk measure so that weights (ω_1, ω_2) belong to the lightly shaded area in Figure 2. This is a sufficient condition to guarantee concavity of the distortion function on $[0, 1-\alpha)$. Therefore, these GlueVaR risk measures are candidates to satisfy the tail-subadditivity property.

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