

MARKETING COOPERATION FOR DIFFERENTIATED PRODUCTS*

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ABSTRACT

In a three-stage duopoly game with product design at stage 1, advertising & marketing at stage 2, and price competition at stage 3, advertising & marketing enable customers to distinguish the goods from each other thus relaxing price competition. The subgame perfect equilibria of the three stage Hotelling-type model are characterized under two institutional arrangements: independant decision making at stage 2 or a joint marketing organization. The two firms will gain from marketing cooperation implying more expenditure on advertising & marketing due to the specification of the model that both firms benefit from advertising & marketing of either firm.

JEL-Classification: L13, M37, D43

KEY WORDS: Informative Advertising, Spatial Competition, Product Differentiation, Marketing Cooperation.

1 Introduction

Advertising and marketing belong to the firms' strategic instruments to create market power. In modern times, where there is a huge number of branded consumer goods available, advertising and marketing are crucial for firms to attract customers (for an overview see Schmalensee, 1986). Goods, which I have in mind, are durable goods such as radios, TV sets, microwaves and bicycles where customers usually do not have an experience with the goods for sale. In the management literature it is widely recognized that differentiating a firm's products from its competitors' products is essential in order to setting prices above unit costs and attracting a positive share of customers (see for instance Porter, 1980 and 1985). It is not sufficient to differentiate a product by its physical properties (which I call product design) but these distinguishing properties must be relevant to at least some customers¹ and must be realized by them before they buy. At this point marketing and advertising instruments come into play. In the paper I consider a market of fixed size for two goods which are substitutes. Here advertising and marketing are used to make the goods distinguishable from each other. The aim of the paper is to single out the equilibrium strategies of the firms in such a framework. The paper focuses on one particular view on advertising and marketing which is called *informative advertising* providing information of the existence, the price and/or the physical specification of a good. It is frequently argued that advertising which increases the objective knowledge about goods helps "perfect" competition. In this paper where informative advertising reveals

¹Porter (1985) writes that these properties must be valuable to customers or at least some of them. However, from models of vertical product differentiation it can be learnt that in a duopoly also the low-quality producer has an incentive to move away from the high-quality producer thus producing a less valuable product. In this framework Porter's statement is right again if one assumes that there are more firms in the market and that there is Bertrand competition for a particular generic good. Then indeed prices above marginal costs can only occur in price equilibrium if a firm is able to differentiate a product by making it more valuable for at least some customers.

information on the specification of the goods the converse holds. Advertising and marketing are in the interest of the firms because doing so enables the customers to distinguish the one good from the competitor's good thus relaxing price competition. One can distinguish two different cases. If a firm increases its expenditures on advertising and marketing this leads to higher or lower profits of the competitor. In the second case there is rivalry suggesting that there will be too much expenditure compared to the situation where firms jointly decide upon advertising and expenditure. In the first case for which I will present a model firms benefit from the advertising and marketing expenditure of their competitor. I consider the extreme specification where firms can only provide information on their good in comparison to the one of their competitor. Hence they provide information on the two goods at the same time. The opposite specification has been chosen by Eaton and Grossman (1986), see below.

One example which fits well into this specification is the training of the retail staff such that they are able to tell customers the differences between the consumer goods which they are supposed to sell. It does not matter which of the firms supplying the goods does the training.

The spatial modelling approach, which was initiated by Hotelling (1929), seems well-suited to analyze this aspect of advertising and marketing. Grossman and Shapiro (1984) also study informative advertising in a spatial market, see also Tirole (1988, pp. 292). They analyze a one-stage game with two-dimensional product differentiation where advertising adds the second dimension. In their model advertising informs customers about the existence of a good whereas in my model advertising provides information on the physical specification, customers always know of the existence of the goods. These different roles of advertising lead to different conclusions about price competition: in their model advertising fosters competition whereas in my model price competition is relaxed.

Closest in spirit is Eaton and Grossman (1986) where firms decide whether to inform customers about the product characteristics of their own product.

This is modelled as a 0,1-decision which is costless. Consequently, in their model two firms choose maximal differentiation and customers are perfectly informed. Furthermore, there is no externality in the firms' decision making. The spatial approach has been applied to the marketing literature before (see for instance Hauser, 1988, and Choi, Desarbo, and Harker, 1990 and Ansari, Economides, and Ghosh, 1994). There the locational choice is called product positioning. These models can be seen as variants of Hotelling-type models. Also Anderson, de Palma, and Thisse (1992) remark the applicability of models of product differentiation to marketing problems (using the logit approach).

Contrary to the just mentioned literature, I explicitly model that advertising and marketing influence customers in their degree of information on the specification of the goods and thus is an additional strategic variable besides product design and price. A virtue of the model is its simplicity because I am able to provide unique analytic solutions.

An important element of the paper is to compare two different institutional arrangements concerning decisions on advertising and marketing. Since in the model both firms benefit from advertising and marketing of either firm, joint advertising and marketing decisions are beneficial to both firms. This result is driven by the fact that advertising and marketing are a public good from the viewpoint of the firms.

The paper is organized as follows. Section 2 describes the relationship between advertising and customers' utilities. Customers are indirectly affected by advertising via an information function. Section 3 presents the three-stage model. Then, in Section 4, the perfect equilibrium is determined depending on the shape of the information function. Section 5 concludes.

2 Advertising and Customers' Utility

In this section I describe customers and their perception of advertising and marketing. For convenience, I will only refer to advertising and no longer to

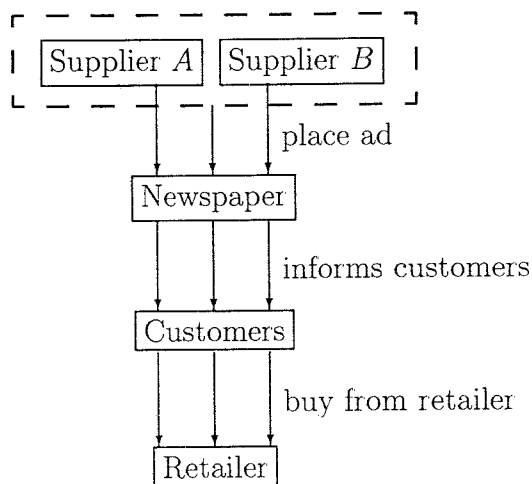


Figure 1: Informative Advertising - an Illustration.

marketing. I present a very simple specification of informative advertising. Figure 1 illustrates how advertising works in the model. Firms A and B place their ads for example in a newspaper describing their good in comparison to their competitor's good. The newspaper is read by the customers providing them with some degree of information on the true specification of the goods. Given that information, each customer buys one unit of the good which maximizes his utility at the prevailing prices, where the prices at the retailer's shop are set by the suppliers, firms A and B . The role of the retailer is reduced to a joint market place of the suppliers.

Firms design their goods on an interval. This means that customers perceive product differentiation to be one-dimensional. However, customers do not observe the differences between the goods and it is advertising which gives them information on the physical specification of the goods. The parameter measuring how informed customers are is denoted by $\theta \in [0, 1]$. All customers are always informed equally well. However, the value this information has

for the customers depends on their location in the product space. For $\theta = 1$ all customers are fully informed.

I take θ as the value of a function of advertising expenditure. This *information function* is defined as a function $f : \mathbb{R}_+ \cup \{\infty\} \rightarrow [0, 1]$ which satisfies

(I-1) $f(0) = 0$,

(I-2) there exists an \bar{e} such that $f(e) = 1$ for all $e \geq \bar{e}$,²

(I-3) $f(e + \Delta) \geq f(e)$ for $e \in (0, \bar{e})$, $\Delta > 0$.

Property (I-1) says that without advertising customers know nothing about the product specification and implies that goods are considered as homogeneous by the customers when buying them in the case of zero advertising. Property (I-2) implies that a firm always chooses from a compact strategy set when deciding about advertising expenditure. Property (I-3) states that advertising expenditure $e \geq 0$ affects the degree of being informed positively. In the proofs one does not need it but there seems to be no economic argument to give the information function a shape which violates (I-3).

Advertising has been taken as a one-dimensional variable. This is not to argue that the choice of advertising instruments is of less importance for the success of the firm but in this simple model this is not the issue.

With v I denote the indirect utility of a customer whose ideal good is $\omega \in [0, 1]$ and who buys one unit of the good of specification $l \in [0, 1]$. k is a constant and p the price of the good.

$$v(p; l, \theta) = \theta^2 - (\theta(\omega - l))^2 + k - p$$

Hence, for $\theta = 0$, goods are perfect substitutes. For $\theta > 0$ customers have a utility function which is similar to the one used in the Hotelling model with quadratic transportation costs.³

At a first glance at the utility function it might seem cumbersome that the

²For \bar{e} being infinity this property says that $\lim_{e \rightarrow \infty} f(e) = 1$.

³In order to solve for the equilibrium in Section 4 it is important that the transportation cost function is quadratic. The maximal differentiation result does not hold for all power transportation cost functions, see Economides (1986)

utility function depends on the advertising expenditure suggesting a violation of consumer sovereignty. This is not the case if one understands v as an *ex ante* utility function. This means that v defines the utility which customers expect to derive from one unit of a particular good before actually consuming the good. The higher the advertising expenditure the more the customers learn about the *true* specification.⁴

Once a customer ω consumes a particular good he will learn about the ‘true’ specification of the good and derives for example the *ex post* utility $\hat{v}(p, l) = k + 1 - (\omega - l)^2 - p$. According to this specification advertising does not have any intrinsic value to the customer. It only reduces the lack of information on the goods at the point where customers enter the shop. The customer will be informed in any case once he has bought the good. Hence advertising only allows customers to make better informed choices. An alternative point of view would be to see advertising as a mean to improve the usefulness of a good. Then also *ex post* utility would depend on θ .

Informative advertising points out the true difference between goods. It will be argued that advertising is in the interest of the firms because it relaxes price competition.

Note that $\partial v / \partial \theta > 0$ for $|\omega - l| < 1$. For given prices customers are in favor of informative advertising because this helps them to know better about the goods they can buy. The closer a good is positioned to the ideal good of a customer ($|\omega - l|$ small) the more the utility increases with increased advertising expenditure. Being better informed is always better except for the worst case of a customer and a good situated at opposite end points. Advertising, which contains information on how to use a good, is such a case where customers expect to benefit even from information on a good which they do not like that much. Alternatively, one can work with a utility function where utility decreases in advertising expenditure for goods far away

⁴This is why I call this kind of advertising ‘informative advertising’. I do not analyze situations where firms can misstate the specification of their good. This contrasts to the signaling literature on advertising, an issue not explored in this paper.

from the ideal good without affecting the analysis (e.g. one can replace the additive θ^2 term by $(1/4)\theta^2$).

Customers behave in a very simplistic way: they receive information on the specification of the goods by advertising, they evaluate one unit of the goods according to their indirect utility function and buy one unit of the good which gives the highest value.⁵

Individual demand functions then are

$$\xi_A^\omega(p_A, p_B; l_A, l_B, \theta) \in \begin{cases} \{1\} & \text{if } v(p_A; l_A, \theta) > v(p_B; l_B, \theta) \\ \{0, 1\} & \text{if } v(p_A; l_A, \theta) = v(p_B; l_B, \theta) \\ \{0\} & \text{else} \end{cases}$$

and $\xi_B = 1 - \xi_A$.

The heterogeneity of customers is expressed by a distribution over ω . It is assumed that ω is uniformly distributed over $[0, 1]$. The population is of mass M . Functional form of the utility function and the other characteristics of a customer are assumed to be identical over the population.

3 The Three Stage Model

In this section I will describe the full model. The process of product design, advertising and price competition will be analyzed as a multi-stage game. One interpretation for such a model is that of a delay supergame. In a delay supergame decisions at all stages are made simultaneously but the time which passes until a managerial decision becomes effective decreases with each stage. In the case of the model with joint advertising one can establish equivalence results for subgame perfect equilibria of the multi-stage game and any finite delay supergame (for details see Selten (1994)'s Nobel Prize Lecture).⁶ Consider any delay supergame of length $T < \infty$ in which firms

⁵Prices are assumed to be such that v is always greater or equal to 0, which holds for $p \leq k$. Hence there is no need to introduce an outside option

⁶According to his Theorem 1 a subgame perfect equilibrium of the multi-stage game G with strategy combination b induces a subgame perfect equilibrium of a delay supergame

have three variables to choose every period: product design, joint advertising, and price. A decision becomes effective after a delay of some periods smaller than T depending on the particular variable. The equivalence of the subgame perfect equilibria in such a delay supergame and the three-stage model with joint advertising holds when the delay of the decision on the product design is longer than the delay of the decision on advertising which is longer than the delay of the price decision. This seems to be the natural delay structure in most industries.

The game structure is the following:

Stage 1: Product Design,

Stage 2: Advertising,

Stage 3: Price Competition.

At stage 1, firms A and B simultaneously locate their goods A and B at l_A and l_B which are element of the bounded characteristics space $[0, 1]$. Hence I only consider inside location games.⁷ It is implicitly assumed that the market only sustains two one-product firms. This can be made endogenous by assuming adequate entry costs.

G^* of G with the strategy combination which is generated by b . To apply his Theorem 2 the set of all subgame perfect equilibria needs to be a subgame perfect equilibrium set. This holds under joint advertising. (If one imposes symmetry as an additional refinement the multi-stage model under separate advertising also has this property.) In order to rewrite the model as a delay supergame one also needs to be careful about specifying the impact of advertising on customers' utilities over time. The probably simplest specification is to consider expected demand generated by a sample which is drawn independently each period out of a continuum of potential customers. By this one avoids reputation effects. The relevant demand for firms is expected demand. In addition, to exclude long-run effect of advertising one assumes that only actual customers are exposed to advertising. This seems reasonable because customers are much more likely to pay attention to informative advertising to a particular category of goods if they are in the process of buying from it.

⁷Inside location games are compared to outside location games for instance in Gabszewicz and Thisse (1992).

At stage 2, firms decide upon the advertising expenditure. Two institutional arrangements are considered. In the first, each firm has its own advertising department. Firm A chooses e_A and firm B chooses e_B . In the second, they have a joint advertising organization. Advertising enables customers to distinguish the two goods from each other. It is assumed that it does not matter which firm does the advertising, only the over-all advertising expenditure is relevant in order to affect the perception of the customers. This variable is $e = e_A + e_B$. Firm A prefers firm B rather than itself spending on advertising. As it will be shown, firms can increase their equilibrium profits when replacing the institutional arrangement of separate advertising departments by a joint advertising organization. Because of the symmetry of the model no conflict will arise within this joint organization if the cost sharing rule is the equal split between the two firms.

At stage 3, firms compete in prices. They produce the goods at zero marginal cost of production. Cooperation at this stage is excluded from the analysis: firms set prices non-cooperatively.

The solution concept is subgame perfect Nash equilibrium. Firms only choose pure strategies.

4 Results

In this section I analyze the three-stage game. Beginning with the derivation of market demand I determine equilibrium prices at stage 3. Then I show that whenever there is positive advertising expenditure firms will choose maximum differentiation in the characteristics space. These results follow from the analysis of d'Aspremont, Gabszewicz and Thisse (1979). Then at stage 2 the equilibrium expenditure on advertising is determined. In isolation a firm increases its advertising as long as the marginal gain from relaxed product differentiation outweighs the marginal cost of advertising.

Since transportation costs $t_\theta(|\omega - l|) = (\theta(\omega - l))^2$ are strictly convex for any $\theta > 0$ there is at most one *marginal customer* $m(p_A, p_B; l_A, l_B, \theta)$ who is

indifferent between buying from firm A or B .

$$m(p_A, p_B; l_A, l_B, \theta) = \frac{\frac{1}{2\theta^2}(p_B - p_A) + \frac{1}{2}(l_B^2 - l_A^2)}{l_B - l_A}$$

Firms are labelled according to their position: the firm which is denoted by A is always to the left of firm B , i.e. $l_A \leq l_B$. All customers to the left-hand side of m buy from firm A and all customers to the right-hand side of m buy from firm B . Market demand for firm A is

$$X_A(p_A, p_B; l_A, l_B, \theta) = \begin{cases} 0 & \text{if } p_A + t_\theta(l_A) > p_B + t_\theta(l_B) \\ M m(p_A, p_B; l_A, l_B, \theta) & \text{if } m \in [0, 1] \\ M & \text{if } p_A + t_\theta(1 - l_A) < p_B + t_\theta(1 - l_B) \end{cases}$$

Market demand for firm B is $X_B = M - X_A$.

At stage 3 firms set prices in order to maximize profits $\pi_A = p_A X_A$ and $\pi_B = p_B X_B$ where all costs which are fixed at this stage are ignored. The following lemma states a well-known result on the equilibrium prices of stage 3.

Lemma 1.

For any admissible parameters l_A, l_B, θ at stage 3, the unique price equilibrium is determined by the following equilibrium prices

$$\begin{aligned} p_A^* &= (l_B - l_A) \frac{2\theta^2}{3} + (l_B^2 - l_A^2) \frac{\theta^2}{3}, \\ p_B^* &= (l_B - l_A) \frac{4\theta^2}{3} - (l_B^2 - l_A^2) \frac{\theta^2}{3}. \end{aligned}$$

Proof. For $l_A = l_B$ or $\theta = 0$ the model is identical to a Bertrand model with homogeneous goods. Hence the unique equilibrium is the one where price equals marginal costs. In all other cases the price equations follow from simple computations, see d'Aspremont, Gabszewicz and Thisse (1979). \square

One still can show the existence of a unique price equilibrium for particular distributions of customers' ideal goods which are not uniform. This

has been done by Neven (1986) and Caplin and Nalebuff (1991). However, it is also quite clear that maximum differentiation in the location-then-price game no longer necessarily holds, see also Goeree and Ramer (1994) and, for a similar model, Economides (1994). In order to retain a simple algebraic structure I will only analyze the model with a uniform distribution of customers' ideal goods.

In the next lemma advertising is taken as given. It will be convenient to solve first for stage 1 for any possible situation at stage 2 and taking into account the equilibrium at stage 3. The reason for solving the game in this unconventional manner is that by doing so one only needs to consider the case $l_A = 0$ and $l_B = 1$ when determining the advertising decisions of the firms. The following maximal differentiation result is due to d'Aspremont, Gabszewicz, and Thisse (1979), see also Neven (1985).

Lemma 2.

For any $\theta > 0$ firms choose maximal differentiation in the unique subgame perfect equilibrium, i.e. $l_A^* = 0$ and $l_B^* = 1$.

For $\theta = 0$ product differentiation does not matter and there exists a continuum of subgame perfect equilibria with $l_A^*, l_B^* \in [0, 1]$, $l_A \leq l_B$.

Proof. Computations show that $\partial \pi_A(p_A^*, p_B^*, l_A, l_B, e_A, e_B) / \partial l_A < 0$ and $\partial \pi_B(p_A^*, p_B^*, l_A, l_B, e_A, e_B) / \partial l_B > 0$ for $l_A, l_B \in [0, 1]$, $l_A \leq l_B$ and $\theta > 0$. Since for any l_B firm A locates at $l_A = 0$ and for any l_A firm B locates at $l_B = 1$, firms choose maximal differentiation in the unique subgame perfect equilibrium. For $\theta = 0$ the two goods are homogeneous and the Bertrand result applies irrespective of the product design (l_A, l_B) . \square

Note that the continuum of equilibria in the case $\theta = 0$ does not pose difficulties since equilibrium profits are always equal to zero. In the eyes of the customers goods are indistinguishable. Hence the product design is irrelevant.

Note that an ϵ -amount of advertising makes goods distinguishable from each other. Now consider the situation of maximal differentiation and an ϵ -amount of advertising. Firms will compete fiercely on stage 3 resulting in small mark-ups. Since the model is symmetric the same customers will buy from firm A as in the case where customers are perfectly informed. From the viewpoint of the customers a little bit of advertising is in their interest but more than that leaves their decisions in equilibrium unaffected leading only to higher prices. This result is due to the symmetry of the model.

What matters from a welfare point of view are only the associated costs of the firms because demand is inelastic and, in equilibrium, the decision from which firm to buy is unaffected by strictly positive advertising. As advertising has no intrinsic value and serves only as a “matching device” between firms and customers, advertising does not affect customers’ utilities as long as they do not switch from one good to the other. Hence if θ turns out to be “large” in equilibrium there is excessive advertising because the only welfare effect is the real cost of advertising. According to the model, an institutional arrangement which leads to more excessive advertising than an alternative arrangement should be prohibited by competition law because it is welfare reducing. This unambiguous result is due to the symmetry because in the model for any $\theta > 0$ there is the right match between customers and firms.⁸ This is all I want to say on welfare effects.

At stage 2 I solve for the equilibrium value of advertising e given that firms have chosen maximum differentiation at stage 1. Two institutional arrangements are considered. Firms either advertise separately or they jointly set up an organization which does the advertising in their interest. In this case the cost-sharing rule is assumed to be that each firm pays half of the total advertising expenditure. Since the model is symmetric this rule can be shown

⁸Let me come back to the newspaper story, where advertising is an important source of revenue for the newspaper and the printing of the ad often represents a negligible cost. In this case advertising supports the sales of a different good (the newspaper) and a welfare analysis of this situation would have to go well beyond the scope of the model.

to be optimal for the two firms. If there is demand uncertainty at stage 2 (but not at stage 3) both firms will agree on this rule.

I first analyze the case of the class of linear information functions. The following piecewise linear functional form will be chosen:

$$f_L(e) = \begin{cases} e/\bar{e} & \text{for } e \leq \bar{e} \\ 1 & \text{else} \end{cases}$$

This class of functions is only defined for $\bar{e} \in \mathfrak{R}_+$. When a non-linear information function has a finite full information advertising expenditure, it is said to correspond to the particular linear one. An information function f is said to be *dominated by the corresponding linear information function* f_L if $f_L(e) \geq f(e)$ for all $e \in [0, \bar{e}]$.

In the linear case firms' profits are

$$\begin{aligned} \pi_A(p_A^*, p_B^*, l_A = 0, l_B = 1, e_A, e_B) &= \frac{M}{2} \left(\frac{e_A + e_B}{\bar{e}} \right)^2 - e_A \\ \pi_B(p_A^*, p_B^*, l_A = 0, l_B = 1, e_A, e_B) &= \frac{M}{2} \left(\frac{e_A + e_B}{\bar{e}} \right)^2 - e_B \end{aligned}$$

Since profits are convex in advertising, in equilibrium either customers will be perfectly informed which means that $\theta = 1$ or customers will not perceive any difference between the goods.

The non-negativity conditions of profits for the case $\bar{e} = e_A + e_B$ become $M/2 \geq e_A$ and $M/2 \geq e_B$.

Let me first look for symmetric equilibria in which $e_A = e_B$. Under separate decision making firm A will spend $e_A = \bar{e}/2$ if

$$\frac{M}{2} - \frac{\bar{e}}{2} \geq \frac{M}{8}.$$

To economize on the number of cases it is assumed that in case of equality firms choose strictly positive advertising. On the left-hand side of the above inequality one has the equilibrium profit in the case that firm A spends $e_A = \bar{e}/2$ and firm B spends the same amount. The right hand side is equal to the price equilibrium with maximal differentiation when $e_A = 0$ and

$e_B = \bar{e}/2$. Hence the symmetric equilibrium under separate decision making is the following:

Lemma 3.

Assume an information function with full information expenditure \bar{e} which is dominated by the corresponding linear information function and assume maximal differentiation. Under separate decision making the two firms will do the following advertising in the unique symmetric subgame perfect equilibrium of stages 2 and 3

$$e_A^* = e_B^* = \begin{cases} 0 & \text{if } \frac{3}{4}M < \bar{e} \\ \bar{e}/2 & \text{else.} \end{cases}$$

Proof. See the Appendix. \square

Now if advertising will occur in equilibrium a firm will want the other firm to do the larger share of advertising. This is stated in the following lemma. Note that the larger M/\bar{e} the larger becomes the set of asymmetric equilibria.

Lemma 4.

Assume an information function with full information expenditure \bar{e} which is dominated by the corresponding linear information function and assume maximal differentiation. Under separate decision making there exists a continuum of subgame perfect equilibria of stages 2 and 3 if and only if $(3/4)M > \bar{e}$. The associated normal form game is one of conflicting interest. Otherwise, there exists a unique subgame perfect equilibrium.

Proof. See the Appendix. \square

According to the alternative institutional arrangement advertising is jointly determined and the costs are split equally between the two firms.

Lemma 5.

Assume an information function with full information expenditure \bar{e} which is dominated by the corresponding linear information function and maximal differentiation. Under joint decision making the two firms will do the following advertising in the unique subgame perfect equilibrium of stages 2 and 3

$$e_A^* = e_B^* = \begin{cases} 0 & \text{if } M < \bar{e} \\ \bar{e}/2 & \text{else.} \end{cases}$$

Proof. Joint profits at stage 2 are $\pi = M(e/\bar{e})^2 - e$ for $e \in [0, \bar{e}]$. Hence $e = \bar{e}$ for $M > \bar{e}$ and $e = 0$ else. Since $e = e_A + e_B$ and $e_A = e_B$, the result follows. \square

Since both firms benefit from the advertising of either firm it is in their interest to establish a joint advertising organization. Note that the binding agreement that one firm does all the advertising but that the other firms pays half of the cost, leads to the same conclusion. All I want to say at this point is that marketing cooperation is worthwhile. To formally state this result, I introduce an additional stage.

Stage 0: Institutional Design.

At stage 0, firms decide whether to establish a joint advertising organization or whether to advertise separately. The agreement is binding. The information structure is as follows: the market size M is a random variable. Its distribution with measure μ on its support $\text{supp } \mu \subseteq \mathbb{R}_+$ is known by the firms at stage 0. The expected market size is finite, i.e. $\int_{\text{supp } \mu} M \mu(dM) < \infty$. After stage 0 the realization of the random variable is observed by both firms. Now I am in the position to summarize all the preceding results. The intuition from above holds if the distribution of M is of positive measure on $(\bar{e}, (4/3)\bar{e})$.

Theorem 1.

Assume an information function with full information expenditure \bar{e} which is dominated by the corresponding linear information function and $\mu((\bar{e}, (4/3)\bar{e})) > 0$. Any symmetric subgame perfect equilibrium is characterized by:

- (0) At stage 0, firm set up a joint advertising organization.
- (1) At stage 1, they choose maximal differentiation if $M \geq \bar{e}$. Otherwise, product differentiation will be irrelevant.
- (2) At stage 2, they pay $e_A = e_B = \bar{e}/2$ if $M \geq \bar{e}$. Otherwise, $e_A = e_B = 0$.
- (3) At stage 3, they set prices $p_A = p_B = 1$ if $M \geq \bar{e}$ and $p_A = p_B = 0$ else.

Proof. See the Appendix. \square

Note that I only characterized *symmetric* subgame perfect equilibria. There may be asymmetric subgame perfect equilibria in which firms choose separate decision making at stage 0. These can only be ruled out if M is sufficiently small.

Corollary.

Under the assumptions of the Theorem, if $\text{supp } \mu \subseteq [0, (4/3)\bar{e})$ any subgame perfect equilibrium is characterized by (0) to (3).

Proof. It follows from Lemma 4 that in any subgame perfect equilibrium at stage 2 under separate decision making and maximal differentiation firms choose zero advertising and equilibrium profits are zero at stage 1 for any M in the support of μ . Then the proof of Theorem 1 applies. \square

In general, marketing cooperation results if the expected profit under co-operation is greater than the upper bound on expected profits under separate decision making, i.e. the word ‘symmetric’ can be eliminated in the Theorem

if

$$\begin{aligned} & \int_{[\bar{e}, \infty) \cap \text{supp } \mu} M - \frac{\bar{e}}{2} \mu(dM) \\ & > \int_{[(4/3)\bar{e}, 2\bar{e}] \cap \text{supp } \mu} M - \left(2\frac{\bar{e}}{M} - 1\right) \bar{e} \mu(dM) + \int_{(2\bar{e}, \infty) \cap \text{supp } \mu} M \mu(dM). \end{aligned}$$

In other cases I cannot say more.⁹

Under the previous class of information functions always boundary solutions for the advertising strategies were derived in equilibrium. This is due to the particular properties of the information functions so far considered. In the remainder I construct a particular example such that there are always interior solutions. The information functions are defined as

$$f_C(e) = \begin{cases} \sqrt[4]{1 - (\bar{e} - e)^2 / \bar{e}^2} & e \leq \bar{e} \\ 1 & \text{else} \end{cases}$$

The important properties to derive only interior solutions for the advertising strategies in equilibrium are $\lim_{e \rightarrow 0} f_C^2(e) = \infty$ and $\lim_{e \rightarrow \bar{e}} f_C^2(e) = 0$.

In the following theorem it is stated that, under both institutional arrangements, firms will choose advertising levels such that $e_A + e_B$ is in the open set between 0 and \bar{e} . At the stage of the institutional design marketing cooperation will result. Contrary to Theorem 1 there is a *unique* symmetric equilibrium because product differentiation is never irrelevant.

⁹In a different model a forward induction argument works against cooperation and in favor of an asymmetric solution: if the rule of the game is that say firm A is forced to choose cooperation at stage 0 when the other firm does so, marketing cooperation will not occur because if firm B has not chosen to cooperate this choice only makes sense if firm B pays a smaller share of the total advertising expenditure. Thus firm A should pay a larger share. By rejecting cooperation firm B induces firm A to play an asymmetric equilibrium. However, this model does not sound interesting enough to be analyzed with more care.

Theorem 2.

Assume the concave information function f_C from above. The unique symmetric subgame perfect equilibrium is characterized by:

- (0) At stage 0, firms set up a joint advertising organization.
- (1) At stage 1, they choose maximal differentiation.
- (2) At stage 2, they spend $e_A = e_B = \frac{1}{2}(1 - \frac{1}{\sqrt{(M/\bar{e})^2 + 1}})\bar{e}$ on advertising. Under separate decision making they would have chosen $e_A = e_B = \frac{1}{2}(1 - \frac{1}{\sqrt{(M/2\bar{e})^2 + 1}})\bar{e}$.
- (3) At stage 3, firms set prices $p_A^* = p_B^* = \sqrt{1 - \frac{1}{(\frac{M}{\bar{e}})^2 + 1}}$.

Proof. See the Appendix. \square

Clearly, this is only one particular example of an information function. I constructed other examples such that there is an interior or a boundary solution depending on the parameter \bar{e} . The whole exercise could also be carried out in the case of transportation cost functions such that the maximal differentiation result does not hold if one looks for loci of zero relocation tendency.

5 Conclusion

In this paper I presented a benchmark model of informative advertising, in which customers are informed about the physical specification of the goods. The first goal was to present a model in which informative advertising relaxes price competition. The second goal was to characterize the subgame perfect equilibria of the multi-stage game with the stages (0) institutional design, (1) product design, (2) advertising, and (3) price competition. The analysis suggests that firms should cooperate in their marketing decisions. However, this result is due to the fact that, in the model, advertising is a public good from the point of view of the firms and the model should be seen as an extreme case. It depends on the particular marketing and advertising instruments used whether this is an adequate assumption.

In reality, when a firm places ads in a newspaper to inform customers about its good it usually does so without referring to the goods of its competitors. Hence it seems to be worthwhile to construct a model with this feature. Doing so one will have to distinguish between the advertising expenditures of each firm and the model will have to look quite different because the symmetry which is very important in my model has to be given up.

Appendix

Proof of Lemma 3: The result is first proved for the class of linear information functions f_L .

Existence. (i) $e_A^* = e_B^* = \bar{e}/2$ for $(3/4)M \geq \bar{e}$: for $e_B = \bar{e}/2$ firm A will spend $e_A = \bar{e}/2$ if

$$\frac{m}{2} - \frac{\bar{e}}{2} \geq \frac{M}{8} = \pi_A(p_A^*, p_B^*, l_A = 0, l_B = 1, e_A = 0, e_B = \frac{\bar{e}}{2}).$$

(ii) $e_A^* = e_B^* = 0$ for $(3/4)M < \bar{e}$: by the convexity of the profit function of firm A in e_A on $[0, \bar{e}]$ it is sufficient to show that

$$\begin{aligned} & \pi_A(p_A^*, p_B^*, l_A = 0, l_B = 1, e_A = 0, e_B = 0) \\ & \geq \pi_A(p_A^*, p_B^*, l_A = 0, l_B = 1, e_A = \bar{e}, e_B = 0) \\ & \quad 0 \geq \frac{M}{2} - \bar{e}. \end{aligned}$$

Analogously for firm B .

Uniqueness. The best response of firm A on e_B is either $\bar{e} - e_B$ or 0. Analogously for firm B . Consequently, only $e_A = e_B = (1/2)\bar{e}$ and $e_A = e_B = 0$ can be symmetric equilibria. By assumption the equilibrium $e_A^* = e_B^* = 0$ for $(3/4)M = \bar{e}$ is ignored. For $(3/4)M > \bar{e}$, $e_A = e_B = 0$ is not an equilibrium and, for $(3/4)M < \bar{e}$, $e_A = e_B = \bar{e}/2$ is not an equilibrium. Except for the equilibrium advertising stated in the existence part of the proof there can be no other symmetric equilibrium.

Since profit functions in the case of information functions which are dominated by the linear ones are also dominated and the profit functions take the same values for every price-product design choice when $e_A + e_B = 0$ or \bar{e} , the extension is straightforward. \square

Proof of Lemma 4:

(i) For $(3/4)M < \bar{e}$, it is shown that $e_A^* = e_B^* = 0$ are the advertising expenditure in the unique subgame perfect equilibrium of stages 2 and 3. This

follows from the proof of Lemma 3 and implies that there is no continuum of equilibria in this case.

(ii) $(3/4)M = \bar{e}$: Firm A will never choose $e_A > \bar{e}/2$ because this will yield negative profits. For $0 < e_A < \bar{e}/2$, firm B will set $e_B = 0$ which cannot be an equilibrium. The case $e_A = e_B = 0$ has been excluded by assumption. It has been shown that $e_A = e_B = \bar{e}/2$ is a subgame perfect equilibrium and it follows that it is unique.

(iii) $(3/4)M > \bar{e}$: for a given e_B , firm A either sets $e_A = \bar{e} - e_B$ or 0. Firm A chooses $e_A = \bar{e} - e_B \equiv (1 - \lambda)\bar{e}$ rather than 0 if and only if

$$\frac{M}{2} - (1 - \lambda)\bar{e} \geq \frac{M}{2}\lambda^2.$$

For the minimal λ which satisfies the above inequality one obtains $\lambda = 2\frac{\bar{e}}{M} - 1$ as the only admissible solution. For any M with $2\bar{e} > M > \frac{4}{3}\bar{e}$, any

$$e_A \in \left[2\frac{\bar{e}^2}{M} - \bar{e}, 2\bar{e} - 2\frac{\bar{e}^2}{M}\right] \text{ and } e_B = \bar{e} - e_A$$

constitutes an equilibrium. For $M \geq 2\bar{e}$, any $e_A \in [0, \bar{e}]$ and $e_B = \bar{e} - e_A$ constitutes an equilibrium.

For $e'_A > e_A$, the associated equilibrium profits for firm A are smaller whereas they are greater for firm B . Hence the associated normal form game is one of conflicting interest.

The extension to information functions which are dominated by the corresponding linear ones is shown as outlined in the proof of Lemma 3. \square

Sketch of the proof of Theorem 1:

(0). In order to establish (0) one has to show that both firms prefer to establish a joint advertising organization. Take advertising according to Lemmata 3 and 5 and maximal differentiation because of Lemma 2 and the corresponding equilibrium prices from Lemma 1. Expected profits in case of cooperation are

$$\int_{[\bar{e}, \infty) \cap \text{supp } \mu} M - \frac{\bar{e}}{2} \mu(dM).$$

Expected profits in case of separate decision making are

$$\int_{[(4/3)\bar{e}, \infty) \cap \text{supp } \mu} (M - \frac{\bar{e}}{2}) \mu(dM).$$

which is strictly smaller under the assumption of the theorem. Both firms unanimously agree on establishing a joint advertising organization.

(1). Maximal differentiation follows from Lemma 2 and part (2) of the characterization. This implies that advertising in case of cooperation is strictly positive if and only if $M \geq \bar{e}$.

(2). Follows from Lemma 5.

(3). Substitute the equilibrium values for advertising and product design into the equilibrium prices of Lemma 1. \square

Sketch of the proof of Theorem 2:

(0). The equilibrium of the last stage is given by Lemma 1. Whenever there is strictly positive advertising maximal differentiation at stage 1 follows from Lemma 2. Under either institutional setting it can be shown that firms choose strictly positive advertising in the unique subgame perfect equilibrium of stages 2 and 3 given maximal differentiation. For any \bar{e} profits under marketing cooperation exceed profits under isolated decision making.

(1) From Lemma 2 it follows that at this stage firms either choose maximal differentiation or that product differentiation does not matter from the firms' point of view. Since the resulting profits in the latter case are dominated by the associated profits under advertising in some positive range, maximal differentiation will always be chosen in any subgame perfect equilibrium.

(2) There exists a unique admissible solution to the first-order conditions for each of the two institutional arrangements. The second-order conditions for a local maximum are satisfied and boundary solutions can be ruled out. Remark that in the case of separate decision making only symmetric equilibria are considered.

(3) Follows from the substitution of the equilibrium values of the variables into the equilibrium prices at stage 3 which are given by Lemma 1. \square

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