

Vault theory in Spain between XVIIIth and XIXth century: Monasterio's unpublished manuscript *Nueva teorica sobre el empuje de bóvedas*

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This paper intends to offer a first introduction to an unpublished manuscript recently found by one of the authors in the Library of the Escuela de Ingenieros de Caminos, Canales y Puertos of the Universidad Politécnica de Madrid and entitled *Nueva teórica sobre el empuje de bóvedas*.

In the opinion of Fernando Sáenz Ridruejo (1998; see also 1990), the author of this important manuscript is, most probably, Joaquín Monasterio, Civil Engineer of the first promotion of the «Old School» of Civil Engineers established in 1802 by Agustín de Betancourt. The information about this obscure, though brilliant, young professor is very scarce. Rumeu (1980) only cites his name and we are indebted to F. Sáenz for the following information: Promotion in 1804; in 1809 figures as professor in a proposal for the Academy of Sciences and in 1810 it was promoted to «Ingeniero de 1^a Clase». After this date there is no other mention in the files. Between 1805 and 1806 the Escuela de Caminos was closed and in Sáenz's opinion he could have written his *Teórica* in this time. This agrees pretty well with the internal evidence in the manuscript: among others, Monasterio cites Coulomb (1773) and Rondelet (first edition 1802) but does not cite Gauthey (first edition 1809), though the matter is not settled yet.

It is certain that the manuscript came to the hands of Eduardo Saavedra, one of the most prominent Spanish engineers of the 19th century. In the first numbered page of the manuscript we find, in fact, the following note by Saavedra: «Este trabajo es de un profesor de la primitiva Escuela de Caminos llamado

Monasterio, y me lo regaló mi maestro D. Francisco de Travesedo».

The manuscript is cited by Saavedra himself in the Dictionary of Clairac (1877) (article «Bóveda»), as part of the Spanish bibliography on the subject. The manuscript belonged then to the private library of Saavedra, who gave it (unknown date) to the Library of the Escuela de Caminos.

THE MANUSCRIPT

The manuscript consists of an Introduction, three chapters devoted to the collapse analysis of non-symmetric and symmetric arches and a fourth chapter on the thickness of the abutments and their way of cracking. In the complex, ninety numbered pages clearly handwritten, with few corrections, and two Plates (Figures 1 and 2, here) with 25 figures.

By simply observing the two plates at the end of the text, it is not difficult to appreciate the general character of Monasterio's theoretical approach. For instance, we find there the illustration of the collapse modes of non-symmetric arches, a topic rarely investigated and probably never tackled before him. Moreover, we recognize the general research of the actual fracture of masonry abutments, an argument requiring new considerations about the stabilising contribution of the arch piers.

As we shall see, not everything of Monasterio's analysis is correct nor his deductive process is always

clearly stated. Anyway, the manuscript contains material enough to guarantee a higher theoretical quality in comparison with preceding and later studies, even the best ones usually celebrated by the official historiography on the subject.

THE INTRODUCCIÓN

The *Introducción* (3-11) to the *Nueva Teórica* is a beautiful manifest of scientific methodology. Monasterio is perfectly aware that the theory of the masonry arch has been treated «hasta el presente con . . . poca exactitud y extension». (3) He writes that some authors «solo se extienden à dar reglas practicas» giving nothing more than «una confusa mezcla de principios de Geometria y Estatica, mal entendidos y aplicados»; other authors have developed «calculos fundados en hipotesis falsas, o formulas inutiles, tanto por la arbitrariedad y restricciones . . . como por que solo sirven para impedir algunos movimientos, y no los que mas comunemente adquieren las bovedas». These last authors «creyeron que para asegurarse de la solidez de un arco, bastaba resolver dos cuestiones que se refieren à impedir separadamente los movimientos de traslacion y rotation».

Monasterio starts from these critical comments in order to establish a general theory of the masonry arch in accordance with the following method of investigation:

Sabemos que un arco es el agregado de varios cuerpos, à quienes se dà el nombre de dovelas, capaces por su figura y colocacion de moverse de diferentes maneras, y al mismo tiempo que para reducir al estado de reposo un sistema de cuerpos ligados entre si y solicitados por varias fuerzas, es necesario tener tantas condiciones que satisfacer. cuantos movimientos diferentes se pueden concebir en èl: asi, pues, si la teorica de bovedas ha de tener por objecto impedir que los arcos se vengán abajo, y que padezean alteraciones susceptibles de perjudicar á su solidez, es claro que no las desempeñaremos devidamente, mientras no se averiguen primero todos los movimientos que pueden adquirir las diferentes partes en que se divide una boveda, para hallar despues con arreglo á este conocimiento las condiciones correspondientes á evitar cada uno de aquellos movimientos en particular: y solo siguiendo el camino trazado por este raciocinio, conseguiremos que nuestras aplicaciones sean utiles á la practica, y esten exentas de los errores cometidos hasta aqui. (3-4)

Now, the movements of the voussoirs —considered as rigid blocks subject to friction and unilateral constraints— may be (absolute and relative) rotations around the joint edges and (absolute and relative) translations along the joint lines. As an arch can collapse «abriendose por uno, dos, tres o mas trozos, y ademas cada uno de estos adquirir el movimiento de rotacion o el de traslacion», it is evident that

segun sean diferentes las circunstancias que acompañen à estos movimientos, asi deberan variar de forma las condiciones que los evitan: de suerte que . . . el exacto conocimiento de todas estas circunstancias ha de depender necesariamente del numero de permutaciones que admiten dos letras que representan la una el movimiento de traslacion, y la otra el de rotación. (4)

By naming the (absolute and relative) translations with t and the (absolute and relative) rotations with r , Monasterio represents the collapse modes of a non-symmetric arch by means of permutations of the letters t and r in accordance with the following rule: for any permutation, the number of letters gives the number of voussoirs in which the arch breaks at collapse, the order (from left to right) gives the type of (absolute and relative) movement taken by the voussoirs.

From a strictly mathematical reasoning, these permutations are infinite and Monasterio writes them in the form $t, r, tt, tr, rt, rr, ttr, trt, rtt, trr, rrt, rrr, ttt, tttr, trrt, rrrt$, etc. From a mechanical point of view, however, some of them are kinematically impossible. For instance, Monasterio observes that the permutations t, r , and rr

no pueden tener cabida . . . por que atendida la figura de cuña de las dovelas, su impenetrabilidad y la convexidad del intrados mirado desde arriba para abajo, ni un trozo puede adquirir aisladamente el movimiento de traslacion, ni menos dos solos el movimiento de rotación. (5)

Moreover, he adds that the permutations with more than three letters can be decomposed, without changing the order, into groups formed by the following eleven permutations $tt, tr, rt, ttr, trt, rtt, trr, rtr, rrt, rrr, ttt$. For instance, he says that the permutation $trrr$ is not relevant as it is formed by the permutations trr e tr which already represent two collapse modes. Finally, he concludes that these

eleven permutations can be further reduced to seven by observing that in the four permutations *ttr*, *rtt*, *rtr*, *ttt* the first two letters already represent a possible collapse mode, so that «todos los diferentes movimientos con que un arco puede venirse abajo» can be given by the seven permutations *tt*, *rrr*, *rrt*, *trr*, *tr*, *rt*, *trt*.

The sequence of these permutations reflects the organisation of chapters 1-3. The first one —*tt*— corresponds to the collapse by translation of two voussoirs (Plate 1, Fig. 1); the second one —*rrr*— corresponds to the collapse by rotation of three voussoirs (Plate 1, Fig. 2); the last five —*rrt*, *trr*, *tr*, *rt*, *trt*— correspond to collapse modes involving both translation and rotation of two or three voussoirs (Plate 1, Figs. 3-7). In particular, the permutations

tr and *rt* imply a composed relative movement of translation and rotation between two voussoirs (Plate 1, Figs. 5-6).

In a few pages it should be impossible to go deeply into the questions discussed by Monasterio in the four dense chapters of the manuscript. We must, then, limit ourselves to present the main features and results of his study. It is of help in these sense the uniform process of reasoning followed throughout the text. Starting with the collapse analysis of non-symmetric arches, Monasterio firstly deduces the general conditions of equilibrium to be fulfilled in order to prevent the seven modes of failure previously mentioned; then he specializes the general conditions to the case of symmetric arches and applies them to vaults and domes with simple geometry, so that he

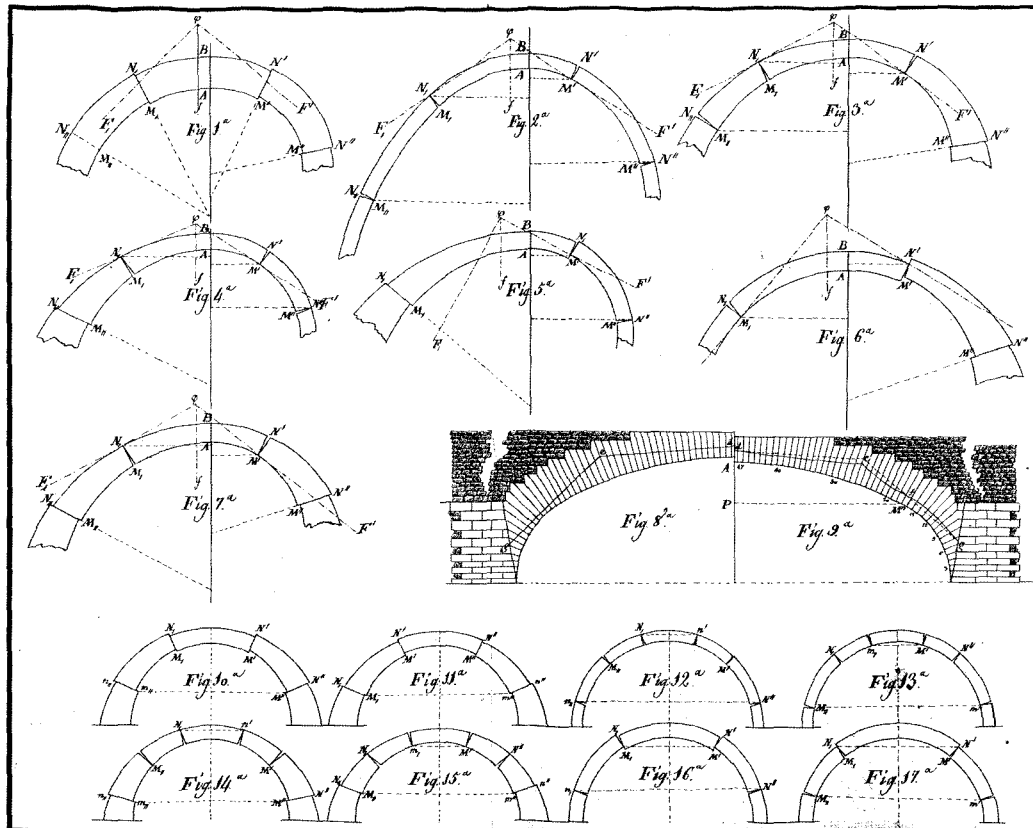


Figure 1
Monasterio's Plate 1, with the collapse analysis of arches

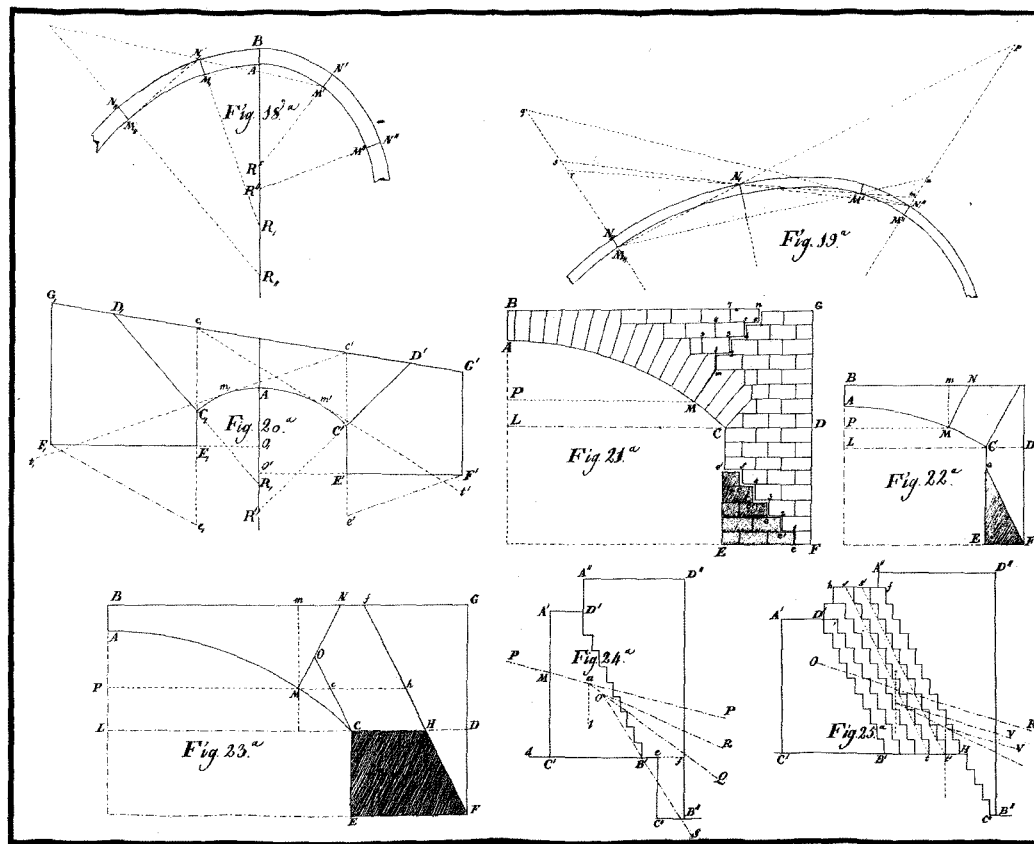


Figure 2
Monasterio's Plate 2, with the collapse analysis of the system «arch'buttress»

can obtain quantitative results. This process is developed with the awareness that,

pudiendose abrir los arcos por tantas partes diferentes, cuantas son sus juntas de hecho, para estar seguros de que aquellas condiciones se verifican en toda la extension de la bóveda, debemos referirnos á aquella poscion de trozos mas poderosa para hacerlos volcar, pues claro está que si el movimiento no se verifica considerados los trozos mas dispuestos a moverse, menos se verificará con cualesquiera otros (8-9)

This means that the search for the collapse condition must be carried out «por medio de la doctrina de maximos y minimos, y no valiendose, como se ha hecho comunemente, de observaciones

practicas». (9) This last remark becomes a rational criterion for judging the previous studies on the arch theory. In this respect, Monasterio acknowledges Coulomb's primacy, even though he observes that his analysis would have been correct

si . . . no hubiera confundido en una sola las dos fuerzas horizontales dimanadas de considerar un trozo, ya como potencia, ya como resistencia: suposiciones que hacen se diferencien entre si aquella fuerzas, constantemente en los movimientos de rotacion, y en los de traslacion cuando se introduce el rozamiento. (9)

The previous quotation reflects a deep understanding of the mechanical questions and probably constitutes an anticipation of Persy's

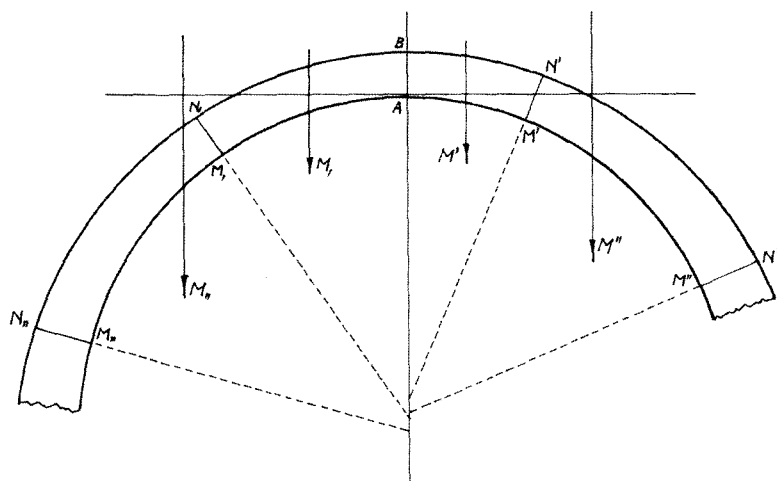


Figure 3
Monasterio's general scheme of a non-symmetric arch

criticism towards Coulomb's application of the method of maxima and minima. With similar rigour Monasterio specifies the hypotheses of his theoretical analysis: he explains that the cohesion can be neglected «por ser insensible o casi nula cuando el mortero se halla blando, esto es, recién contruidos los arcos: tiempo, al cual deben referirse nuestras formulas, por ser la epoca en que aquellos corren mas riesgo de arruinarse» (10); on the contrary, he affirms the necessity of taking into account the role of friction, «por que el objecto de dar à las bóvedas un estado mas firme y permanente que el de l'equilibrio, no puede conseguirse . . . si no se admite una fuerza pasiva, cual el rozamiento, que obrando solo como resistencia sofoque el movimiento que intentan producir las activas»; finally, he observes that, even if the materials have limited compressive strength, «podemos muy bien desentendernos de calcular si las dovelas, pilares y demas partes del arco tienen la suficiente robustez para aguantar sin desmoronarse à la presiones que sufren, seguros de que la tendran, mientras sus dimensiones, por pequeñas que parezcan con relacion à este objecto, guarden las proporciones que exigen las demas consideraciones que llevamos mencionadas». (10-11). As it is easy to understand, this last point focuses a fundamental of the modern limit analysis of the arch as it affirms that global stability, and not local strength, is the main question of masonry structures.

THE FIRST CHAPTER

As anticipated, this first chapter, «De las condiciones necesarias para evitar los movimientos puros de traslación» (12-33), deals with the collapse of the arch by translation of the voussoirs. Monasterio initially studies a non-symmetric arch and analyzes the collapse mode of Plate 1, Fig. 1, representing the permutation *II*. The procedure to establish the stability condition is based on the fact that, at collapse, the direction of the forces at the rupture joints are known.

Before developing Monasterio's analysis, we give here his notations with reference to Figure 3

Coordinates (with respect to the vertical and horizontal axes)

$$\begin{aligned} M_{,,} &= (y_{,,}; x_{,,}) & M_{,,} &= (y_{,,}; x_{,,}) & M' &= (y'; x') & M'' &= (y''; x'') \\ N_{,,} &= (u_{,,}; z_{,,}) & N_{,,} &= (u_{,,}; z_{,,}) & N' &= (u'; z') & N'' &= (u''; z'') \end{aligned}$$

Weights

$M_{,,}$ = weight of the voussoir $AM_{,,}N_{,,}B$
 $M_{,,}$ = weight of the voussoir $AM_{,,}N_{,,}B$
 M'' = weight of the voussoir $AM''N''B$
 M' = weight of the voussoir $AM'N'B$

Angles

$\alpha_{,,}$ = angle between the vertical axis and joint $M_{,,}N_{,,}$,
 α = angle between the vertical axis and joint M,N ,
 α' = angle between the vertical axis and joint $M'N'$
 α'' = angle between the vertical axis and joint $M''N''$

Lever arms

$X_{,,}$ and $Z_{,,}$ = lever arms of the weight $M_{,,}$ with respect to the points $M_{,,}$ and $N_{,,}$,
 X and Z = lever arms of the weight M with respect to the points M and N ,
 X' and Z' = lever arms of the weight M' with respect to the points M' and N'
 X'' and Z'' = lever arms of the weight M'' with respect to the points M'' and N''

Moreover, $k = AB$ is the thickness of the arch at the crown joint and f is the friction angle.

Summarizing Monasterio's reasoning, the following steps can then be developed:

- 1) Decompose the weight $(M+M')$ in two forces F' and F'' which form the angles λ and λ' with the vertical and are equal to the complements of the friction angle f at the joints M,N , and $M'N'$, respectively;
- 2) Impose the equilibrium of the weight $(M''-M')$ and the force F' with respect to the outwards translation along the joint $M''N''$.

By conserving the original numbering of the manuscript, the stability condition is then given by the following disequity

$$'M'' - b'M' - c'M, \geq 0 \quad (A)$$

where the coefficients a' , b' and c' satisfy the relationships

$$\begin{aligned} a' &= \cos(\alpha''-f) \sin(\alpha' + f + \alpha, + f) \\ b' &= \cos(\alpha' + f) \sin(\alpha'' + \alpha,) = a' + c' \\ c' &= \cos(\alpha''+f) \sin(\alpha' - f - a, - f) \end{aligned}$$

Monasterio treats this disequity in order to simplify its discussion and obtain, after some passages, the new form

$$\begin{aligned} &M''\cot(\alpha''-f) - M'\cot(\alpha'+f) \} \sin(\alpha'+f) / \\ &\sin(\alpha''-f - \alpha'+f) + \{ M''\cot(\alpha''-f) - \\ &- M'\cot(\alpha'+f) \} \sin(\alpha, + f) / \sin(\alpha'' + \alpha,) \geq 0 \quad (7) \end{aligned}$$

He observes, then, that the first term depends on α' and α'' and the second term on α , and α'' . Moreover, if one reasonably fixes α'' at the right springing, the two terms only depend on α' and α , respectively. In this case «sus respectivos minimos podran explicitamente hallarse, ó por la diferenciacion, ó por tanteos dando à α' y α , diferentes valores: si la suma de estos minimos es positiva . . . concluiremos de este calculo que el movimiento puro de traslacion no tiene de modo alguno lugar en la boveda». (21)

The disequity (7) is the starting point of various developments. First of all Monasterio studies the case of absence of friction and finds the equilibrium equation

$$M = A \tan \alpha \quad (10)$$

where A is a constant and M is the weight of a generic voussoir subtended by angle α . This equation «se halla bajo diferentes formas en casi todas las obras que tratan del empuje de bovedas, y parece que el objeto principal de sus autores no ha sido otro que de darla a conocer como si en ella solo estibara todo el fundamento de la teorica, siendo asi que es la mas inútil». (11) Anyway, Monasterio uses equation (10) in the case of a vault and a dome of constant infinitesimal thickness. As expected he finds the analytical equation of the «catenaria plana o de cañon seguido» in the first case and the equation of the «catenaria de revolucion o de media naranja» in the second one.

Another use of the disequity (7) is given by Monasterio to treat symmetric arches. In this case he finds that the stability condition with respect to the collapse by translation becomes

$$M'' \cot(\alpha''-f) - M'\cot(\alpha' + f) \geq 0 \quad (B)$$

«donde se vé que la boveda satisfara en todas sus juntas à esta condicion, si el minimo de $M''\cot(\alpha''-f)$ es mayor que el maximo de $M'\cot(\alpha' + f)$ ». (28) This conclusion perfectly agrees with Coulomb's stability condition stated by the method of maxima and minima. The two terms of (B) are nothing else than the values of the horizontal thrust satisfying the equilibrium with respect to the outwards translation

along the joint α'' and the inwards translation along the joint α' . As Monasterio correctly affirms generalizing Coulomb's statement, the disequilibrium (B) can be useful to study the two opposite collapse mode in Plate 1, Figs. 10 and 11, depending on the relative position of the rupture joints.

An application of disequilibrium (B) is given by Monasterio for determining the minimum friction coefficient preventing the collapse of a semi-circular arch of constant thickness. He obtains that the collapse by translation becomes impossible for $\tan f > 0,31$. This value, corresponding to $f = 17^\circ$, is quantitatively correct as we know (Sinopoli et al., 1997) that the collapse of Fig. 10 can occur when $\tan f = 0,305$, with rupture joints for $\alpha' = 29^\circ$ and $\alpha'' = 90^\circ$.

THE SECOND CHAPTER

In this second chapter, «De las condiciones que impiden los movimientos puros de rotación» (33-49), Monasterio deals with the collapse by rotation, starting from the general case of non-symmetric arches. He refers to Fig. 2 of Plate 1, corresponding to the permutation rrr , and observes that at collapse four conditions can be stated:

- 1) the component F' of the weight $(M,+M')$ must go through the intrados edge M' ;
- 2) the component F , of the weight $(M,+M')$ must go through the extrados edge N ;
- 3) the moment of F , with respect to the intrados edge $M_{,,}$ must be smallest than the moment of the weight $(M_{,,} - M_{,,})$ with respect to the same point;
- 4) the moment of F' with respect to the extrados edge N'' must be greater than the moment of the weight $(M'' - M')$ with respect to the same point.

By analytically treating these conditions Monasterio finds the following stability disequilibrium

$$a''M''Z'' - b''M'X' + c''M,Z_{,,} - d''M_{,,}X_{,,} \geq 0 \quad (C)$$

where the coefficients a'' , b'' , c'' and d'' satisfy some relationships among the coordinates of the centers of absolute and relative rotation.

Disequilibrium (C) is specialised by Monasterio for some important cases. The first one concerns vaults

and domes of infinitesimal thickness. Once again he finds the equations deduced in the preceding chapter, «lo que nos prueba que las catenarias plana y de revolucion son las unicas curvas que deben formar las bovedas de cañon seguido y media naranja para que no se vengán abajo en el caso de der su espesor infinitamente pequeño». (40-41)

Monasterio deals then with the case of symmetric arches and specialises the disequilibrium (C) for the two opposite collapse modes of Plate 1, Figs. 12 and 13. In particular, he develops the calculation for the cases in which a rupture joint opens at the crown, with hinge at the crown extrados or at the crown intrados. The corresponding stability conditions are then

$$M''Z''/(z''+k) - M'X'/(x'+k) \geq 0 \quad (F)$$

$$M,Z_{,,}/z_{,,} - M_{,,}X_{,,}/x_{,,} \geq 0 \quad (G)$$

These disequilibria are equivalent to the stability conditions which can be derived by a «proper» application of Coulomb's method of maxima and minima, as the two terms in (F) represent the values of thrust satisfying the rotational equilibrium around the extrados edge and the intrados edge when the thrust is applied at the crown extrados, and the two terms in (G) represent the values of thrust satisfying the rotational equilibrium around the intrados edge and the extrados edge when the thrust is applied at the crown intrados. We underline «proper» application because, as Coulomb's analysis of the rotational collapse mechanisms is fundamentally wrong and an explicit correction of his erroneous conclusions was given only in 1825 by Persy (Foce 2002; see also Foce, Aita in these Proceedings). From this point of view, Monasterio's analysis anticipates the correct application of the method of maxima and minima under the different form given in (F) and (G).

Besides the theoretical analysis, Monasterio applies the disequilibrium (F) for determining the minimum thickness of a semi-circular arch, a case already studied in 1730 by Couplet under the a priori hypothesis that the rupture joints was at 45° from the crown. Couplet had found that the minimum thickness is $k = 0,1061x$, where x is the intrados radius. On the contrary, Monasterio finds by trial and error that the minimum thickness is between $1/8 = 0,125$ and $1/9 = 0,111$ of the intrados radius, and the rupture joint at the haunches is between 54° and 56° from the crown. This result is

quantitatively correct and agrees with the calculation by Petit (1835), who has given a better approximation of the minimum thickness with the value 0,114 of the intrados radius. In 1907 Milankovitch has obtained the rigorous value 0,1136 corresponding to the rupture joint at $54^{\circ}29'$ from the crown.

As concerns the collapse by rotation, Monasterio does not limit the analysis to the mode previously investigated for the semi-circular arch. He observes that also the opposite collapse mode corresponding to the disequality (G) must be considered, in particular for the domes. He applies then the disequality (G) to a semi-circular dome and finds that the minimum thickness is between $1/23 = 0,043$ and $1/24 = 0,041$ of the intrados radius. On the basis of this result, Monasterio rightly criticizes Rondelet and the «formula suya», according to which a semi-circular dome of nil thickness would be equilibrated.

THE THIRD CHAPTER

This third chapter, «De las condiciones necesarias para que no se verifiquen los cinco movimientos mixtos, y reduccion de todas las generales à otras mas sencillas» (50-66), begins with the analysis of the five collapse modes of a non-symmetric arch involving both translation and rotation and corresponding to the permutations *rrt*, *trr*, *tr*, *rt*, *trt*. As in the two preceding chapters, Monasterio determines the stability condition by imposing known requirements about the action of the internal forces at the rupture joints for each mode of collapse.

Without entering into details, we limit ourselves to give the stability conditions whose fulfilment prevents the collapse modes mentioned above. As concerns the «movimiento mixto de primera especie», that is the permutation *rrt* (Plate 1, Fig. 3), Monasterio finds the stability conditions

$$a'''M''Z'' - b'''M'X' + c'''M,Z, - d'''M,,X,, \geq 0 \quad (H)$$

where the coefficients a''' , b''' and c''' are known satisfy known relationships among the coordinates of the centers of rotation and the friction angle.

For the «movimiento mixto de segunda especie», that is the permutation *trr* (Plate 1, Fig. 4), the condition becomes

$$a''M''Z'' - b''M'X' + c''M,Z, - d''M,,X,, \geq 0 \quad (I)$$

where again the coefficients are satisfy known relationships among the coordinates of the centers of rotation and the friction angle.

In the case of a symmetric arch, the two types of collapse are represented in Plate 1, Figs. 14 and 15. In particular, when the collapse occurs with a rupture joint at the crown the stability conditions take the simple form

$$M''\cot(\alpha'' - f) - M'X'/(x' + k) \geq 0 \quad (M)$$

$$M,Z,/z, - M,,\cot(\alpha'' + f) \geq 0 \quad (N)$$

These last disequalities can be easily interpreted in terms of Coulomb's method of maxima and minima.

Coming back to the non-symmetric arch, Monasterio gives also the stability conditions for the others three collapse modes. Without reporting these conditions for the sake of brevity, we only add that an important remark about the collapse modes of fourth and fifth kind, represented in Plate 1, Figs. 5-6 for the non-symmetric arch and Figs. 16-17 for the symmetric arch (these latter are La Hire's mode and its opposite). In fact, the mechanical analysis of these two kinds of collapse modes does not seem to be correct. The mixed movements of fourth and fifth kind show that a composed movement of translation and rotation takes place at the rupture joint $M'N'$. Following Monasterio's reasoning on Fig. 5, for instance, the force F' is contemporaneously required to go through the intrados point M' and to form the friction angle with the normal at the joint. If we interpret this situation in terms of thrust line, we must conclude that the thrust line necessarily goes outside the arch ring under the joint $M'N'$, that is the arch cannot be stable. As any joint can be a rupture joint, the only possibility to remove this contradiction is that the tangent to the intrados curve at the generic point M' coincides with the boundary of the friction cone at the joint $M'N'$. Unfortunately, Monasterio does not realize this contradiction, which holds also for the collapse mode of Fig. 6, so that his discussion of the movements of fourth and fifth kind is substantially wrong.

After Monasterio, this same type of mistake has been made by some post-Coulombian authors, for instance Poncelet (1835). The first author who has clearly described the eight collapse modes of a symmetric arch seems to be Michon in his beautiful

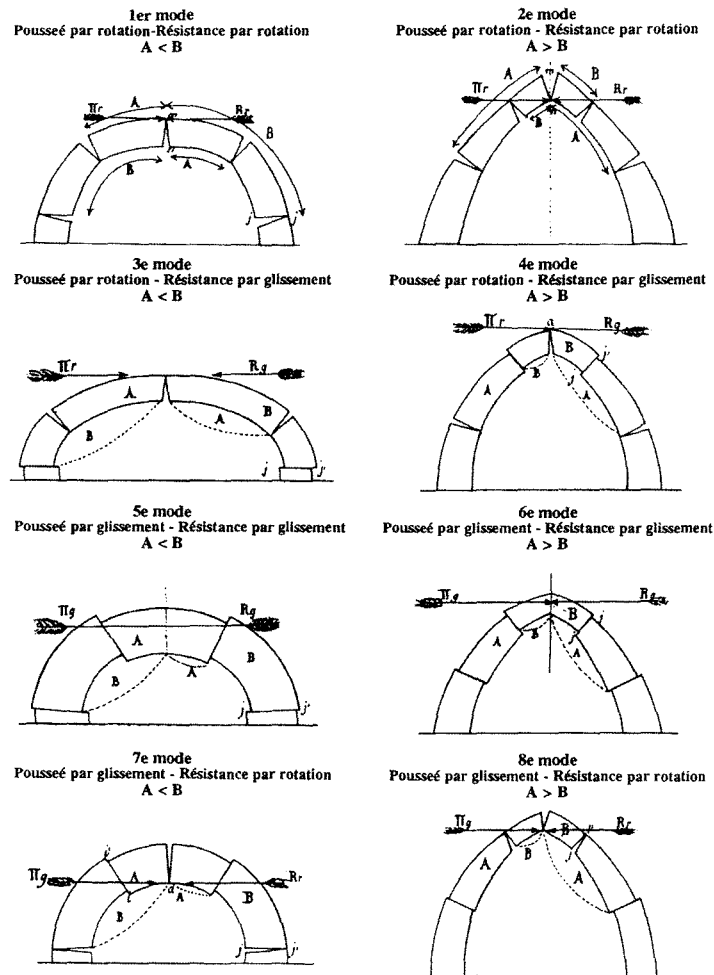


Figure 4
The eight collapse modes of a symmetric arch (redrawn from Michon, 1857)

Instruction of 1857 (Foce 2002). Michon's mechanisms are collected in Figure 4 and can be compared with the eight collapse modes given by Monasterio, provided that for these latter we consider a rupture joint at the crown.

The comparison shows that La Hire's collapse mode and its opposite are excluded from Michon's scheme and substituted by his seventh and fourth mode, respectively. Despite the correctness of Michon's analysis, Saavedra (1860) has repeated Monasterio and Poncelet's mistake in his Spanish

translation of Michon's *Instruction* of 1857. This paradoxical circumstance is an evidence of the difficulty of the matter and may serve as a justification of Monasterio's erroneous discussion.

THE FOURTH CHAPTER

The last fourth chapter, «Aplicacion de la doctrina expuesta en los capitulos anteriores à la determinacion del grueso de machones» (67-90), deals with the

thickness of the abutment in order to avoid the collapse of the system «arch-abutment». Since the beginning, Monasterio restricts the discussion to symmetric systems and limits the collapse analysis to the modes described by the stability conditions (F) and (M).

The point of the problem consists in determining the correct weights of the two parts which enter the stability conditions. In particular, it is necessary to know the fracture lines of arch fill and abutment when the collapse occurs. This question is new. As Monasterio remarks, the authors who have dealt with the matter have always considered «cada pie derecho formado de un solo trozo de piedra, y de consiguiente capaz de oponer al tiempo de volcarse una resistencia mayor de la que tiene realmente». (67) On the contrary, Monasterio reasonably admits that the abutments and the fill must break along fracture lines whose inclination depend on the relative sizes (length and height) of the masonry blocks (see Plate 2, Fig. 21). For instance, if the collapse occurs by rotation and then the stability condition (F) must hold, the weight M' is measured by the area AMNBA and the weight M'' is given by the area of the semi-system, less the triangular surface EeF (see Plate 2, Fig. 22). If the same type of masonry blocks is used for fill and abutment, the angles mMN and EeF are equal and their tangent is half the ratio length/height. In particular, for the sake of safety Monasterio considers square block and finds that the angles mMN and EeF measure $26^{\circ}34'$.

On this theoretical basis, Monasterio develops a very sophisticated analysis which is impossible to report here. We limit ourselves to add that, after Monasterio's unknown contribution, the problem of the fracture of the abutment was independently considered by other authors with less rigour. Dealing with the stability of the system 'arch-abutment', Gauthey remarks that

l'analyse précédente suppose nécessairement que les différentes portions de la voûte forment des masses solides dont toutes les parties sont parfaitement liées entre elles, et ne peuvent éprouver aucun tassement. Elle suppose également que les culées sont établies sur une base entièrement incompressible, et que, dans la chute de la voûte, ces culées tourneraient, sans se disjoindre, autour de leur arête extérieure. Ces suppositions sont, en général bien éloignées de la vérité. La chute d'un pont ne pourrait guère arriver sans qu'il ne se fit quelques

disjonctions dans ses culées, avec quelque soin qu'elles eussent été construites; et quand même il ne s'en ferait aucune, les culées ne pourraient point tourner autour de leur arête extérieure, où les pierres s'écraieraient nécessairement sous l'effort qu'elles auraient à soutenir, et qu'on doit chercher par cette raison à repartir sur une surface suffisamment considérable (Gauthey 1809, 324-325)

In an Appendix «Sur la résistance des murs aux pousées» to Gauthey's treatise, Navier assumes a straight fracture line and tries to demonstrate, on the basis of rather arbitrary hypotheses, that it forms an angle of 45° with the horizontal. He also quotes a result given by Mayniel (1808, 98), according to which

Un mur de 20 pieds de hauteur, dont on avait laissé consolider la maçonnerie, s'est rompu au niveau du sol, en formant une ligne de rupture qui eût pu dans le profil être la diagonale d'un carré qui eût l'épaisseur du mur.

In the *nouvelle* edition of Bélidor's *Science des ingénieurs* of 1813, Navier confirms this idea when affirms that

si le piedroit AE vient à tourner sur l'arête D, il ne sera pas soulevé en entier, à moins que son épaisseur ne soit très-peu considérable et que les pierres ne fassent *parpains*, ou que l'adhésion des mortiers ne suffisamment grande. Ce piedroit tendra à se partager suivant le plan DT, incliné de la moitié d'un angle droit, en sorte que la partie ADT ne sera point soulevée. Donc, on ne doit point en tenir en compte dans l'évaluation de la force du piedroit . . . (186)

The argument is taken again by Navier in his *Leçons* of 1826 and 1839 and in the later editions of Gauthey's *Traité des ponts*. Gauthey's observations have been accepted by Haupt (1851) and Cain (1879).

It is interesting to remark that Navier treats the buttress as a continuum and applies ideas from the soil mechanics. Monasterio, on the contrary, considers the buttress as an assemblage of rigid blocks. Vicat's experiences of 1832, Figure 5, seem to confirm Monasterio's result, even though Navier consider them as a confirmation of his theory. Attention to the influence that the size of the blocks have on the fracture line is given, unexpectedly, also by Walther (1854), Figure 6.

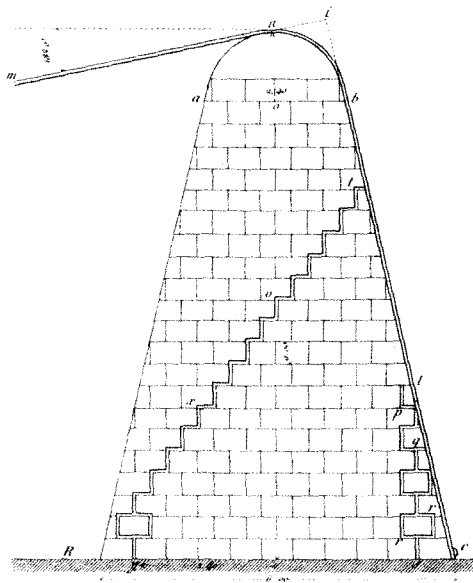


Figure 5

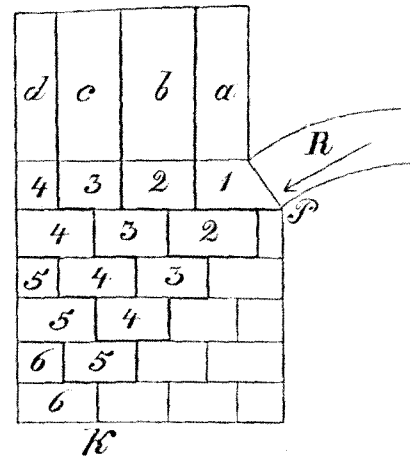


Figure 6

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