THE SOLUTION OF OPTIMAL POST-STRATIFICATION WITH MULTIPLE AUXILIARY INFORMATION: AN STOCHASTIC OPTIMIZATION APPROACH[1]

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ABSTRACT
A proposal for determining post strata using simple information is developed. A stochastic programming with probabilistic constraints models it. Bounds for the approximation error and its expectation are developed. Their behavior is evaluated using Monte Carlo experiments.

KEY WORDS: Chance constrained stochastic optimization, approximation error, bounds, multivariate stratification

MSC 90C15

RESUMEN
Se desarrolla una propuesta para determinar post estratos usando información sencilla.. Se modela mediante un programa estocástico con restricciones probabilísticas. Se desarrollan coatas para el error de aproximación. El comportamiento de ellas es evaluado usando experimentos de Monte Carlo.

1. INTRODUCTION

It is usual that strata should be constructed in statistical applications but they are unknown in advance to the decision maker (DM). Some particular techniques may be used for grouping the units in a population $U=\{u_1 ,...,u_N \}$. Take multivariate clustering for example. The units are identified by a vector, a certain distance is determined and U is partitioned into H mutually disjoint clusters $U_1,..., U_K$. Usually the DM aims to cluster the units in terms of a distance to a certain centre. The DM may determine or not the number of groups and the distance to be used. The implementation of clustering using these criteria may be easily implemented because several specialized software programs are available in commercial packages. The procedures implemented are not necessarily those wanted by the DM, specially if he is a statistician. It is usual that the objective is to determine the strata for developing a survey.

A stratum is considered as a set of population units such that the variable of interest $Y(h)$, evaluated in each unit u_i in U_k , is close to the conditional expectation $\mu_k(h)=E[Y(h) / U_k]$. Ideally, once the strata are determined, the means should be very different. This fact sustains that the estimation based on stratified random sampling (SRS) is very accurate when compared with simple random sampling, see Cochran (1981). The particularities of the classic problem of strata determination is discussed in Section 1.
As pointed out the criteria implemented in commercial software do not satisfy that the determined clusters constitute a set of optimal strata. Then SRS will not satisfy that the sampling error be sufficiently small. Allende-Bouza (1987) studied the problem of determining optimal strata using multivariate information. They assumed that some multivariate information was available and it was used as auxiliary information on

[1] This paper is an extended version of a contribution presented at the VIII International Conference on Approximation, optimisation and Mathematics for Economics in the Caribbean held at Santo Domingo, Dominican Republic in April 2006
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the interest (stratification) variables. The auxiliary information must be obtained for each unit in the population. Unfortunately that information does not necessarily exist. A solution is to select a sample and to evaluate a set of variables considered as important for determining the strata in the sense of having a small intra-stratum variance, $(E(Y(h)-\mu_k(h) / U_k)^2)$, for each variable $Y(h)$ of interest. A source of randomness is present in the new problem. The usual approach is known as post stratification. It assumes $H=1$ and that the strata are known and the sample is used only for classifying the units and estimating within stratum parameters of interest, see Cochran (1981). Section 2 is devoted to posing for this problem the corresponding multivariate optimum stratification counterpart following the approach of Allende-Bouza (1987). This new optimization problem is a stochastic program.

2. STRATA CONSTRUCTION

The usual stratification problems consider that each stratum U_k of size N_k has a population mean $\mu_k(h)$ which is close to the values of the variable of interest in each of its units and that it is very different from the other strata means. Then $\mu_k(h)$ is very different from the population mean $\mu(h)$. Ideally the stratum deviations $\sigma_k(h)$, $k=1,...,K$, and the weighted mean of them should also be very different. Take for each $i \in U$.
A vector $\mathbf{Y}_i=[Y_i(1),\ldots,Y_i(H)]^T$ with H interest variables. The mean of the h-th variable is:
$E[Y(h)]=\sum_{i\in U} Y_i(h)/N=\mu(h)$,

and

$V[Y(h)]=\sum_{i\in U} (Y_i(h)-\mu(h))^2/N=\sigma^2(h)$

Is its variance.

Each variable $Y(h)$ is associated with the stratum parameters:

$E[Y(h)/U_k]=\sum_{i\in Uk} Y_i(h)/N_k=\mu_k(h)$,

and

$V[Y(h)/U_k]=\sum_{i\in Uk} (Y_i(h)-\mu_k(h))^2/N_k=\sigma_k^2(h)$.

The population mean can be expressed by the linear function of the strata means, see Cochran (1981),

$\mu(h)=\sum_{k=1}^{K} W_k\mu_k(h)$,

where $W_k=N_k/N=Prob(u_i \in U_k)$, $\forall k=1,..,K$.

An approach for constructing the strata is to consider that the DM fixes a set of points in $\Re^H$. They represent points that characterize possible centroids of the strata. This hypothesis is sustained by assuming that each centroid is close to the expectation, given the belonging of unit i to a set U_t , of the vector $[Y_i(1),\ldots,Y_i(H)]^T$. Take $\theta_t=[\theta_1(1),\ldots,\theta_t(H)]^T \in \Theta=\{\theta_j, j=1,..,T, T\geq K\}$ as one of the centroids determined by the DM. With these vectors we can define a cost function:

$$C_{it}=\sum_{h=1}^{H} (Y_i(h)-\theta_t(h))^2=\sum_{h=1}^{H} V_{it}(h) \tag{2.1}$$

For each unit. Then we may determine a set of K clusters such that the clustered units are close to the corresponding centroid and the centroids are as far as possible.

The use of (1.1) grant the strata should have a small variance for each variable. This idea was used by Allende –Bouza (1987) for proposing the following program:

$P1{:}\ \min\ \sum^{N}_{i=1} \sum^{T}_{t=1} \sum^{H}_{h=1} P_h(Y_i\,(h)-\theta_t(h)\,)^2 x_{it}=\sum^{N}_{i=1} \sum^{T}_{t=1} c^*_{it} x_{it}$

Subject to :

$\sum^{T}_{t=1} x_{it}=1,\ \forall i\in U$

$\sum^{N}_{i=1} x_{it}\leq N,\ \forall t=1,..,T.$

$\sum^{T}_{t=1} \lambda_t \leq KN\ ,\ K\leq T$

$x_{it}\in\{0,1\},\ \forall i,t$

$\lambda_t\in\{0,1\},\ \forall t1,..,T$

$$x_{it}=\begin{cases}1 & if \quad i \quad is \quad assigned \quad to \quad U_t\\ 0 & if \quad i \quad is \quad not \quad assigned \quad to \quad U_t\end{cases}$$

$$\lambda_t=\begin{cases}1 & if \quad \theta_t \quad is \quad a \quad selected \quad centrori\\ 0 & otherwise\end{cases}$$

The first constraint establishes that each $u_i\ \in U$ belongs only to one stratum. The second one that there will not be a stratum with more than N units. The third constraint fixes that no more than K strata can be determined. The importance of the *h-th* variable is weighted by P_h. The strata determined by solving *P1* may be considered as of ´minimum multivariate variance ´ because if $\theta_t(h)=\mu_t(h)$

$\sum_{i\in Uk} V_{it}(h)/N_k=\ \sigma^2_k(h).$

This equality is not valid for all the variables and strata but we can expect that $\theta_t(h)\approx\mu_t(h)$. Hence the solution of P1 yields an ´approximately minimum multivariate variance ´stratification.

In applications we do not know the set of vectors $\{\ \mathbf{Y}_i\ ,\ i=1,..,N\}$. We can be able to obtain some known auxiliary variables for using them as substitutes in the program. The optimality of the obtained stratification depends on how representative are the auxiliary variables of the unknown ones. Commonly, it is more practical to select a sample and to look for the construction of optimum strata using the information obtained form the sample. .

3. POST-STRATIFICATION

In the univariate case do the post-stratification technique deals with the classification of sample units in previously known strata. That is the case of classifying the interviewed persons in strata determined by an interval of variable age. Once a sample is selected from U it can be divided into subsamples $s_k= s\cap U_k,\ k=1,..,K.$ This technique allows estimating stratum's parameters.

Our objective is to use the information provided by s to establish stratification in *U*. It is basically the idea of clustering: to partition the observed group of unit. The difference with the usual post-stratification is that the strata are not known. The optimal stratification procedures in the literature uses $H=1$ variables but in our case $H>1$. As we are using sample information P1 is not longer adequate because we will have a Stochastic Program...

The modelling of this problem will be based on the proposal approach of Albareda et. al. (2000). They suggested to use a deterministic counterpart.

The new program is:

$P2: \min \sum^{N}_{i=1} \sum^{T}_{t=1} \sum^{H}_{h=1} P_h(Y_i(h)-\theta_t(h))^2 x_{it} = \sum^{N}_{i=1} \sum^{T}_{t=1} c^*_{it} x_{it}$

Subject to :

$\sum^{T}_{t=1} x_{it}=1,\ \forall i \in U$

$Prob\ \{\sum^{N}_{i=1} q_{it}\, x_{it} \leq b_t\} \geq 1-\alpha,\ ,\ \forall t=1,..,T.$

$\sum^{T}_{t=1} \lambda_t \leq PN\ ,\ P \leq T$

$x_{it} \in \{0,1\},\ \forall i,t$

$\lambda_t \in \{0,1\},\ \forall t1,..,T$

$$q_{it} = \begin{cases} 1 & if \quad i \in s \cap U_t \\ 0 & otherwise \end{cases}$$

$$\lambda_t = \begin{cases} 1 & if \quad \theta_t \quad is \quad a \quad selected \quad centroide \\ 0 & otherwise \end{cases}$$

Note that q_{it} is a Bernoulli random variable with expectation $E(q_{it})=Prob\ (u_i \in s \cap U_t)$. For a fixed K $Prob\ (u_i \in s \cap U_t) = W_t = N_t / N$

Which is the weight assigned to the t-th stratum The solution of *P2* determines which strata have high probabilities and a posterior calculus permits to determine the exact value of each stratum (W_t).

The Generalized Assignment Problem can be written as:

$P3: \min \sum^{N}_{i=1} \sum^{T}_{t=1} q_{it} c^*_{it} x_{it}$

Subject to :

$\sum^{T}_{t=1} x_{it} \geq 1,\ \forall i \in U$

$\sum^{N}_{i=1} q_{it}\, x_{it} \leq b_t,\ ,\ \forall t=1,..,T.$

$x_{it} \in \{0,1\},\ \forall i,t$

$$q_{it} = \begin{cases} 1 & if \quad i \in s \cap U_t \\ 0 & otherwise \end{cases}$$

In this model b_t represents the number maximum of units in U_t and qit is the needed resource needed by ´ t for performing the job t´. As we are using this model for optimal strata construction the parameters and variables have another meaning and

- b_t is an upper bound of the number of units that can be assigned to U_t.
- The first constraint becomes equality for establishing which each unit should be classified in only one stratum.

Note that to assign unit u_i to certain stratum do not depends of t. Then we may, $q_{it}=q_{i\bullet},\ \forall i=1,\ldots,N$. As

$\sum^{N}_{i=1} q_{i\bullet} \leq \sum^{T}_{t=1} b_t$

is valid in our case *P3* has at least a feasible solution. Haneveld et. al. (1999) developed a detailed discussion of this problem.

A particular stratum with a set of a distinguished performance of the units may linked with a demand

$\sum^{N}_{i=1} q_{i\bullet}\, x_{it} = q(t)$

Taking this new variable and the effective assignment of the units

$N(t) = \sum^{N}_{i=1} x_{it}\ ,\ \forall t=1,..,T$

We have a frame for studying the post-stratification problem. We are using the information provided by a random sample s for determining post-strata. Due to the particularities of the randomness of our data we will use the modification of *P3*:

$P^*3: \min \sum_{i\in s} \sum^{T}_{t=1} c^*_{it} x_{it}$

Subject to :

$\sum^{T}_{t=1} x_{it} = 1,\ \forall i \in s$

$Prob\ \{q(t) = \sum_{i\in s} q_{i\bullet}\, x_{it} \leq b_t\} \geq 1-\alpha\ ,\ \forall t=1,..,T.$

$x_{it} \in \{0,1\},\ \forall i \in s\ , t=1,\ldots,T$

*P*3* is a stochastic problem with probabilistic constraints. Note that *q(t)* is the sum of independent Bernoulli variables with expectation W_t . Hence it is a Binomial variable with expected value:

$E[q(t)/s] = [\sum_{i\in s} x_{it}] W_t = n(t) N_t / N$

Using these results we obtain that:

$$\Pr ob\left\{\sum_{i\in s} q_{ii}\, x_{it} \geq b_t\right\} = \sum_{h=b_i+1}^{n(t)} \binom{n(t)}{h} W_t^{\,h} \left[1 - W_t\right]^{n(t)-h}$$

The DM is able to fix W_t by determining the proportion of elements that he expect to be in U_t .
Note that we can determine a quantile of order α using the fact that we are dealing with a Binomial distribution. The usual choice is to calculate

$\lambda\{b_t\ , W_t\ , \alpha\} = Max\{h \in Z / Prob\ (b_t\ ) \leq \alpha\}$

The *t-th* constraint is satisfied if:

$n(t) \leq \lambda\{b_t\ , W_t\ , \alpha\}$

Hence the deterministic equivalent program of *P*3* is:

PD1:

$$Min\left\{\sum_{t=1}^{T}\sum_{i\in s} c*_{it}\ x_{it} \middle|\ \sum_{t=1}^{T}x_{it} \geq 1, \forall i \in s, \qquad \sum_{i\in s}x_{it} \leq \lambda(b_t, W_t, \alpha), \forall t = 1,..,T\right\}$$

A relaxation of the integer-ness of $\lambda\{b_t , W_t ,\alpha\}$ allows to use a Transport Model for solving *PD1*.

3. APPROXIMATION ERROR

As quoted previously an approximation to the optimal solution may obtain by using a transport program. As we are using a random sample the vector $(x_{11}{}^{0},\ldots, x_{nT}{}^{0})$ is random and assigns every unit in s a stratum. Then

$$C(s)= \Sigma^{N}{}_{i=1} \Sigma^{T}{}_{t=1} c*_{it}x^{0}{}_{it}$$

Is the cost of assigning the sample units to the strata. It is also a random variable and its properties are conditional to the sample s. If we have an optimum stratification the cost is

$$C= \Sigma^{N}{}_{i=1} \Sigma^{T}{}_{t=1} c*_{it}Y_{it}$$

where

$$Y_{it} = \begin{cases} 1 & ii \quad i \in U_t \cap s \\ 0 & otherwise \end{cases}$$

Note that Y_{it} fixes that i belong to the stratum Ut. It is non random because it was determined once the stratification was fixed. Generally the optimum stratification using sample information will be different to that obtained from complete enumeration. Then the use of the sample generates the error

$$\varepsilon(s)=/C(s)-C/$$

which is the approximation error (AE) of this problem, see Kall-Mayer (2000) and Bouza (1992) for a discussion on the role of it in stochastic optimization. We are interested in evaluating the behaviour of

$$|\Sigma^{N}{}_{i=1} \Sigma^{T}{}_{t=1} c*_{it} [x^{0}{}_{it} - Y_{it}]\ | \tag{3.1}$$

The usual approach is to look for an upper bound of the AE. Note that

$$/\Sigma^{N}{}_{i=1} \Sigma^{T}{}_{t=1} c*_{it} [x^{0}{}_{it} - Y_{it}]/\leq \Sigma^{N}{}_{i=1} \Sigma^{T}{}_{t=1} c*_{it} /[x^{0}{}_{it} - Y_{it}]/$$

Following the procedures discussed and developed by Bouza (1992) and Klamnablath et. al. (2003). They studied the expectation of the AE. The expectation of (3.1) is:

$$E\{|\Sigma^{N}{}_{i=1} \Sigma^{T}{}_{t=1} c*_{it} [x^{0}{}_{it} - Y_{it}]|\}$$

Therefore

$$E\{|\Sigma^{N}{}_{i=1} \Sigma^{T}{}_{t=1} c*_{it} [x^{0}{}_{it} - Y_{it}]|\}\leq \Sigma^{N}{}_{i=1} \Sigma^{T}{}_{t=1} c*_{it} E\{| [x^{0}{}_{it} - Y_{it}]\ |\}$$

We are interested in evaluating the efficiency of a sampled based algorithm in the construction of stratification. It may be measured by comparing the post-stratification, obtained by solving PD1, and that derived by using perfect information, if it is available, and we use *P1*. Hence, we are involved in the study of the matching of the classifications made by *PD*1 and the Optimal Stratification (OS) determined form the use of P1. This interest is characterized by the probabilities *Prob* $\{x^{0}{}_{it} = Y_{it}\}$.

Take

$$I\left(\left|x_{it}^{0}-Y_{it}\right|\right)=\begin{cases}1 & if \quad PD1 \quad and \quad the \quad OS \quad classify \quad i \quad in \quad U_t \\ 0 & otherwise\end{cases}$$

It is a Bernoulli random variable with parameter:

$Prob\ \{x^0_{it} = Y_{it}\} = \sum^{N}_{i=1} \sum^{T}_{t=1} I(|x^0_{it} - Y_{it}|)/N = \pi$

Note that it is the probability of a perfect matching between sample based post-stratification and OS. It is unbiasedly estimated by:

$Q = \sum^{n}_{i=1} \sum^{T}_{t=1} I\{|[x^0_{it} - Y_{it}]|\}/n$

because

$E\{|[x^0_{it} - Y_{it}]|\} = 1 - Prob\ \{x^0_{it} = Y_{it}\} = \pi$

Therefore as

$E[\varepsilon(s)] \leq (1-\pi)\sum^{N}_{i=1}\sum^{T}_{t=1} c^*_{it} = \varepsilon$

An upper bound of the expected AE is then estimated by:

$$\hat{\varepsilon} = (1-Q)\sum_{i=1}^{N}\sum_{t=1}^{T} c^*_{it} \tag{3.2}$$

We can make some statistical analysis of this estimator. As the random variable is Q the variance of (3.2) is

$$\hat{V}(\varepsilon) = V(Q)\left(\sum_{i=1}^{N}\sum_{t=1}^{T} c^*_{it}\right)^2 = \frac{\pi(1-\pi)}{n}\left(\sum_{i=1}^{N}\sum_{t=1}^{T} c^*_{it}\right)^2 = V_1^2$$

We can evaluate the behaviour of the sample based post-stratification calculating a confidence Interval (CI) once we fix the level α where $p=1-\alpha/2$. Let us assume that $Prob\ \{x^0_{it} = Y_{it}\}$ is constant for any i and t. If the sample size is sufficiently large the normal approximation is valid for the distribution of (3.2).

Then the CI estimator is:

$I(1) = [(1-Q) \sum^{N}_{i=1} \sum^{T}_{t=1} c^*_{it} - z_p V_1 , [(1-Q) \sum^{N}_{i=1} \sum^{T}_{t=1} c^*_{it} + z_p V_1]$

In the case in which $Prob\ \{x^0_{it} = Y_{it}\}$ is not constant for any i and t, the solution is to select m independent random samples and determine a post-stratification using *PD1*. $I\{|x^0_{it} - Y_{it}|\}$ is a Bernoulli random variable but its expectation is given by:

$E\{|[x^0_{it} - Y_{it}]|\} = 1 - Prob\ \{x^0_{it} = Y_{it}\} = \pi_{it}$,

An unbiased estimator of these probabilities is $q_{it} = 1 - p_{it}$ where

$p_{it} = \sum_{i \in s} I\{|x^0_{it} - Y_{it}|\}_s/m$

Whenever m>30 the convergence to the normal distribution is valid and the CI for a level α for π_{it} is:

$$\left[q_{it} - z_p \sqrt{\frac{p_{it}(1-p_{it})}{m}}, q_{it} + z_p \sqrt{\frac{p_{it}(1-p_{it})}{m}} \right] = [A_{it}, B_{it}]$$

We have that:

$E\{|\sum^N_{i=1} \sum^T_{t=1} c*_{it} [x^0_{it} - Y_{it}]\,|\} \leq \sum^N_{i=1} \sum^T_{t=1} c*_{it} E\{|\,[x^0_{it} - Y_{it}]\,|\} = \sum^N_{i=1} \sum^T_{t=1} c*_{it}\, \pi_{it}$

An unbiased estimator of this upper bound is

$$\tilde{\varepsilon} = \sum_{i=1}^{N} \sum_{t=1}^{T} c*_{it} \left(1 - q_{it}\right)$$

Then we may use it as CI.

$I(2) = [\sum^N_{i=1} \sum^T_{t=1} c*_{it} A_{it}\,,\ \sum^N_{i=1} \sum^T_{t=1} c*_{it} B_{it}]$

4. A NUMERICAL STUDY

We are going to evaluate the use of PD1 in determining optimal strata. Its implementation was made using a transport algorithm through LINDO. Details on the package can be obtained in (1997).
The data used in the analysis were provided by two investigations developed in Mexico. OS was performed using the information on the interest variables obtained from the populations.
PD1 was used and the post-stratification was determined in each selected sample.

A stratum U_t, determined by using the complete population, was considered equal to the closest post stratum. The closeness was measured by the distance between the corresponding centroids. Then $x^0_{it} = Y_{it} = 1$ when i was in U_t and in the closest post strum to it.

Each population unit was classified in a post-stratum using one of the following rules:

Rule 1. The nearest neighbour

Rule 2. Minimum distance to the centroid defined by the sample means of the units classified in each post-stratum.

$\varepsilon(s)$ was computed and in each sample it was measured if it was included in the CI.

The data bases used in the Monte Carlo experiments were:

- *12 000* children with ages between *8* and *14* years of the State of Guerrero México. The pre-eminence of allergic diseases was studied. Different variables related with their sensibility were measured. We fixed $K=10$ and *20* centroids. Each centroid characterized a basic allergic typology.
- *800* voters of Distrito Electoral of Acapulco. Different characteristics related with their selection of the candidates in the elections of a diputado federal were studied though a questionnaire. The militants of each party, from the *6* involved in the election, were characterized using the means of the variables of interest.

The sizes of the selected samples were equal to the *5%*, *10%* or *20%* of the population. The procedure was evaluated in sets of *30, 50* or *100* samples.

A value of $\alpha = 0, 05$ was used in each Monte Carlo experiment and

H (J) =Number of samples in which $\varepsilon \in I(J)$/*Number of samples* , *J=1,2*

Note in Table 1 that the use of *I (2*) seems to be more efficient than *I (1)*. This fact may be explained taking into account that an incorrect classification depends intrinsically of the stratum and of the individual characteristics. However this is more costly as the post-stratification process has to be repeated m times and the cost is increased. It is remarkable that even if the sampling fraction is as low as *10%* the results can be considered acceptable inclusive for *I (1)*. The characteristics of the data have a decisive influence in the behaviour of the methods. We should expect that the behaviour of the voters depends of the individual political militancy. Therefore is unacceptable that the π_{it}´s are constant. That is the reason of preferring me *I(2)* to *I (1)*. Another suggestion is that the classification based on the second classification rule is the best option

Table 1.Values of *H (J)* in post-strata construction

	Nearest Newburgh		Minimum Distance		Nearest Newburgh	Minimum Distance
	Allergic		Study		Voter of	District X
	H(1)	H(2)	H(1)	H(2)	H(1)	H(2)
f=0,05 *m=30*	*0,46*	*0,59*	*0,51*	*0,70*	*0,55*	*0,68*
m=50	*0,58*	*0,67*	*0,62*	*0,83*	*0,61*	*0,69*
m=100	*0,68*	*0,81*	*0,84*	*0,89*	*0,72*	*0,79*
f=0,10 *m=30*	*0,97*	*0,97*	*0,91*	*0,87*	*0,83*	*0,87*
m=50	*0,90*	*0,99*	*0,99*	*0,90*	*0,92*	*0,93*
m=100	*0,92*	*0,95*	*0,91*	*0,93*	*0,87*	*0,96*
f=0,20 *m=30*	*0,92*	*0,93*	*0,97*	*0,94*	*0,87*	*0,90*
m=50	*0,96*	*0,95*	*0,92*	*0,91*	*0,91*	*0,95*
m=100	*0,94*	*0,93*	*0,94*	*0,97*	*0,92*	*0,96*

Acknowledgements: This paper has been improved by the comments of two anonymous referees. The research developed was benefited by a fellowship of The Academy of Sciences of the Devloping World (TWAS)
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Received September 2006
Revised April 2007

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