https://doi.org/10.34024/prometeica.2022.25.12303

FOURTEEN MODAL THEOREMS CONSISTENT WITH THE THEORY OF MENTAL MODELS

CATORCE TEOREMAS MODALES CONSISTENTES CON LA TEORÍA DE LOS MODELOS MENTALES

QUATORZE TEOREMAS MODAIS CONSISTENTES COM A TEORIA DOS MODELOS MENTAIS

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Recibido: 14/06/2021
Aprobado: 24/10/2021

RESUMEN

Los conceptos de 'posibilidad' y 'necesidad' son muy importantes en la teoría de los modelos mentales. En base a las definiciones que esta teoría psicológica ha entregado para tales conceptos, se ha mostrado que cumple con la exigencia que Fitting y Mendelsonh establecen en general para la lógica modal. Dicha exigencia consiste en ser coherente con la interpretación modal del cuadro de oposición presentado por Aristóteles. Este trabajo trata de avanzar en esta dirección. Analiza catorce teoremas indicados por Carnap bajo el enfoque de la teoría de los modelos mentales. Los resultados revelan que la teoría es también compatible con esos teoremas.

Palabras clave: conjunción. modelo mental. lógica modal. necesidad. posibilidad.

ABSTRACT

Very important concepts in the theory of mental models are those of 'possibility' and 'necessity'. Based on the definitions this psychology theory gives for those concepts, it has been shown that it complies with the requirement Fitting and Mendelsonh provides in general for modal logic. That requirement consists of being coherent with the modal interpretation of the square of opposition presented by Aristotle. This paper tries to move forward in this direction. It analyzes fourteen modal theorems indicated by Carnap under the approach of the theory of mental models. The results reveal that the theory is also compatible with those theorems.

Keywords: conjunction. mental model. modal logic. necessity. possibility.

RESUMO

Os conceitos de 'possibilidade' e 'necessidade' são muito importantes na teoria dos modelos mentais. Com base nas definições que esta teoria psicológica oferece para esses conceitos, mostrou-se que atende à exigência que Fitting e Mendelsohn estabelecem em general para a lógica modal. Esta exigência consiste em ser coerente com a interpretação modal do quadro de oposição apresentado por Aristóteles. Este artigo tenta avançar nesta direção. Analisa quatorze teoremas indicados por Carnap seguindo o enfoque da teoria dos modelos mentais. Os resultados revelam que a teoria é também compatível com esses teoremas.

Palavras-chave: conjunção. modelo mental. lógica modal. necessidade. possibilidade.

Introduction

The theory of mental models (e.g., Khemlani, Byrne, & Johnson-Laird, 2018) is an important cognitive framework nowadays. It offers accounts of many different intellectual phenomena linked to reasoning that are to be found in the literature (see also, e.g., Johnson-Laird & Ragni, 2019). From the philosophical point of view, another relevant point about the theory is that it rejects logic; its proponents try to show that human reasoning works without any relation to logic (see also, e.g., Johnson-Laird, 2010). However, it has been explained that the theory is in accordance with the minimum general criterion given by Fitting and Mendelsonh (1998) to deem a system as a modal logic, that is, with the relations derived from the modal perspective of the 'Aristotelian square of opposition' (López-Astorga, 2020a). Therefore, one might think that perhaps, despite what its adherents claim, it is possible to build a modal logic from the basic theses of the theory of mental models. In fact, there are already works trying to link the former and the latter resorting to systems such as K and a temporal semantics (e.g., López-Astorga, 2020b).

This is not to question the habitual way the theory of mental models is used to account for reasoning. It does not also lead to claim that, if the theory is right, people absolutely and strictly follow logical requirements. This only means that, if the theory fulfills the mentioned modal criterion, maybe it is possible to introduce a logical system from it. That system may not match with the exact steps people take when making inferences. However, it could be somehow coherent with general intellectual processes (of course, assuming that the theory of mental models is correct).

That is the point this paper is intended to keep exploring. To do that, it will review fourteen theorems of modal logic that are in Carnap (1947), which will allow in turn checking the possibility that the theory is coherent with stronger modal systems such as S5. The aim will be to verify whether or not, from the approach of the theory of mental models, what those theorems provide can be admitted. In particular, the theorems will be theorems from 'a' to 'n' in Carnap (1947: 186). They will be presented below, although with symbols different from the original ones.

Thus, the first section will describe what the theory of mental models is from its five 'primary principles': 'representation', 'inference', 'dual systems', 'modulation', and 'verification' (Khemlani et al., 2018, Table 3). Next, this paper will comment on the manner that essential modal concepts such as 'possibility', 'necessity', and 'fact' are understood by the theory (e.g., in Khemlani, Hinterecker, & Johnson-Laird, 2017). Lastly, the theorems will be mentioned and it will be argued that what they imply can be accepted by the theory of mental models.

Five primary principles in the theory of mental models

Five primary principles characterize the theory of mental models. Perhaps, to know them is not absolutely necessary to make to point of this paper. Nevertheless, to offer a brief description of those principles can be useful to understand the general framework of the theory. The first one is the 'principle of representation'. According to it, every sentence including a 'sentential connective' enables to deploy a 'conjunction of possibilities' (see also, e.g., Johnson-Laird & Ragni, 2019). Following Khemlani et al. (2018, Table 3), this principle has received strong support in the literature, for example, in works such as the one of Hinterecker, Knauff, & Johnson-Laird (2016). The cases of two sentential connectives, the conditional and inclusive disjunction, can be illustrative enough to explain what this principle provides.

Regarding the conditional, what the principle of representation means is that a sentence such as (1) leads to a conjunction of possibilities such as (2).

(1) If X then Y.

(2) Possible (X & Y) & Possible (not-X & Y) & Possible (not-X & not-Y).

As far as the inclusive disjunction, that is, a sentence such as (3), is concerned, its conjunction of possibilities should be akin to (4).

(3) Either X or Y, or both of them.

(4) Possible (X & Y) & Possible (X & not-Y) & Possible (not-X & Y).

Conjunctions of possibilities (2) and (4) are expressed in the way usual in the latest version of the theory of mental models (e.g., Johnson-Laird & Ragni, 2019; Khemlani et al., 2018; Khemlani et al., 2017). They seem to show the truth tables of the sentential connectives in classical logic. However, the proponents of the theory insist that they do not. (2) and (4) are only conjunctions of possibilities, and not truth tables (Johnson-Laird & Ragni, 2019). In fact, the theory is different from classical propositional logic in several aspects. Some of them will be indicated below. Of course, the theory of mental models addresses more sentential connectives. Nevertheless, the two ones indicated suffice for the goals of this paper.

The second principle is the 'principle of inference'. It establishes that an inference can be necessary if and only if the conjunction of possibilities of the premises coincides with that of the conclusion. Khemlani et al. (2018, Table 3) indicates that works such as the one of Hinterecker et al. (2016) also confirm this principle (see also, e.g., Orenes & Johnson-Laird, 2012).

An important consequence of the principle of inference is that it reveals that several inferences correct in classical logic cannot be admitted by the theory of mental models. For instance, in classical logic, it is possible to derive a sentence such as (3) from a sentence such as (5) (see, e.g., Khemlani et al., 2018.

(5) Either X or Y, and not both of them.

Indeed, classical propositional calculus allows deducing an inclusive disjunction such as (3) from an exclusive disjunction such as (5). Nonetheless, according to the theory of mental models, that is not what individuals do. The reason is obvious by virtue of the principle of inference. The conjunction of possibilities for (5) is (6).

(6) Possible (X & not-Y) & Possible (not-X & Y).

As it can be checked, (6) is different from (4). The first conjunct in (4) is missing in (6). Hence, by the principle of inference, (3) cannot be inferred from (5).

The next one is the 'principle of dual systems'. The theory of mental models claims that human reasoning works as described by dual-process theories (see also, e.g., Byrne & Johnson-Laird, 2020). Those theories (e.g., Evans, 2009; Johnson-Laird & Wason, 1970; Ragni, Kola, & Johnson-Laird, 2017; Stanovich, 2012) distinguish two levels of reflection: a minimal level and an advanced level. Applied to the theory of mental models, this idea leads to assume that, when the level is minimal, people do not note all the elements belonging to conjunctions of possibilities. Khemlani et al. (2018, Table 3) point out that the literature shows this fact too (in particular, in papers such as that of Khemlani & Johnson-Laird, 2009). But maybe what is most interesting now is how this has an influence on examples such as (1) to (6).

Following Table 1 in Khemlani et al. (2018), when the reflection is not advanced, people tend to consider the conjunction of possibilities corresponding to (1) not to be (2). They only manage to identify the first conjunct in (2). So, what is assigned to the conditional is not actually a conjunction of possibilities, but just one possibility: that in which both clauses, the antecedent and the consequent, happen. In the case of (3), minimal level of reflection means to note, in addition to its first possibility, only the conjuncts that are affirmed, and not negated, in the second and third possibilities in (4). In other words, individuals keep attributing three possibilities to (3), but, while the first one does not change, the second one only includes X and the third one only presents Y. Thereby, not-Y and not-X are ignored, respectively, in these last possibilities of the inclusive disjunction. Finally, in the case of (5), when the level of reflection is not high, what individuals detect is, obviously, just X in the first possibility and Y in the second one. The negative elements, as it occurs with the inclusive disjunction, are not considered for the exclusive disjunction either.

This is a relevant point of the theory of mental models. It enables to account for the reasons why individuals do not offer right answers in reasoning tasks sometimes. They do not identify all of the elements of the possibilities linked to the sentences. From its startup and in previous versions of the theory, this principle was essential (see, e.g., Johnson-Laird, 2012; Oakhill & Garnham, 1996). So, one might think that two, and not only one, logics could be built from the theory of mental models: one of them based on the cases in which reflection is high and the other one based on the cases in which reflection is not that. However, this paper will only consider ideal situations in which people pay attention to all the information regarding the possibilities of each sentence.

The 'principle of modulation' is the following. According to it, what the expressions in sentences mean and the pragmatic circumstances in which those expressions are used can modify the conjuncts of conjunctions of possibilities. Khemlani et al. (2018, Table 3) deem the action of this principle as a fact from the results obtained in works such as Orenes and Johnson-Laird (2012) and Quelhas and Johnson-Laird (2017). Nevertheless, as they indicate as well, much more papers have dealt with this principle (e.g., Johnson-Laird, Khemlani, & Goodwin, 2015; Quelhas, Johnson-Laird, & Juhos, 2010).

The manner the principle of modulation works can be better seen by means of an example. Although it can seem the contrary, this sentence does not have the possibilities of an inclusive disjunction, that is, the three possibilities in (4):

(7) "…Eva read Don Quixote or a novel" (Khemlani et al., 2018: 1899; italics in text).

The second conjunct in (4) cannot be admitted for (7). The reason is evident: if Eva read Don Quixote, then she necessarily read a novel too. This principle is also important in the theory because it seems to make it the only theory that is able to explain certain cognitive phenomena. There is a number of cases in which people do not always apply rules valid in classical logic. One of those cases is the one of the rule that leads from a premise such as Y to a disjunction such as (3). However, according to the theory of mental models, the situations in which individuals use that rule and the situations when they do not are clear. When the conjunction of possibilities associated to the particular disjunction is akin to (4), people tend to reject the rule. That is because the second conjunct in (4) provides that a scenario with X and not-Y is possible. Hence, (4) provides that a scenario inconsistent with premise Y is possible. Nevertheless, in the case of a sentence such as (7), the problem does not exist. If the premise were the second disjunct in it, that is, 'Eva read a novel', (7) could be accepted as a conclusion. As indicated, the possibility incompatible with the premise could not be admitted for (7), since it is internally contradictory. So, these are the circumstances in which the rule indicated is often accepted by individuals (see also, e.g., Orenes & Johnson-Laird, 2012).

As it can be seen below, this principle does not have a direct influence on the arguments of this paper, since the present paper only addresses abstract theorems without thematic content. Nonetheless, perhaps it is important to point out that this principle is one of the most important in the theory. It allows accounting for many circumstances in which people do not reason in accordance with logic (e.g, the principle of modulation can also explain problems related to the conditional, and not only to disjunction; see, e.g., Orenes & Johnson-Laird, 2012).

But the primary principles mentioned by Khemlani et al. (2018) are five. The remaining is the 'principle of verification'. As explained, according to the principle of dual systems, the level of reflection can vary.

Thus, the possibilities recovered for a sentence may not be always the same. This is the main point of the principle of verification. It establishes that what individuals can deem as relevant to verify a sentence is linked to the possibilities they detect for that very sentence. Khemlani et al. (2018, Table 3) consider this principle to be supported by the literature as well. In this way, they cite the work by Goodwin and Johnson-Laird (2018). However, to pay much attention to this last principle is not necessary to achieve the goals of the present paper either. Hence, what has been said on it can be enough.

What is more important for the development of those goals is to take other definitions given by the theory into account. Those are the definitions of 'possibility', 'necessity', and 'fact'. The five previous principles can help to get a general view of the theory of mental models, but these three last definitions can be essential to make the point of this study. The next section addresses the definitions.

Possibility, necessity, and fact in the theory of mental models

Those concepts are explicitly defined by the proponents of the theory. If an element appears in at least one of the alternative conjuncts of a conjunction of possibilities, that element is possible. If an element appears in all of the alternative conjuncts of a conjunction of possibilities, that element is necessary. If actually there is no conjunction of possibilities, but only a possibility that is affirmed, and an element appears in that possibility, that element stands for a fact (Khemlani et al., 2017).

Based on these definitions, both X and Y are possible in (2), (4), and (6). However, in the conjunction of possibilities corresponding to (7), while X, that is, 'Eva read Don Quixote', keeps being possible, Y, that is, 'Eva read a novel', is necessary, since it appears in the two possible conjuncts.

Regarding an example of fact, perhaps it suffices to think about conjunction. The theory of mental models claims that, given a conjunction such as:

(8) X and Y

In principle, only one possibility can be assigned to it:

(9) Possible (X & Y)

Therefore, it is evident that both X and Y are facts in (9) (see, e.g., Johnson-Laird & Ragni, 2019).

From all this machinery, it is not difficult to understand how the theory of mental models works. Maybe an example of inference with the structure of Modus Tollendo Tollens can be illustrative enough in this way. As it is well known, Modus Tollendo Tollens consists of two premises and a conclusion. The premises are a conditional, that is, a sentence such as (1), and the negation of the consequent of that conditional, that is, in the case of (1), not-Y. The conclusion is the negation of the antecedent, that is, following with (1), not-X. The literature shows that this inference is often hard to individuals (see, e.g., Byrne & Johnson-Laird, 2009). Nevertheless, the theory of mental models can give an account of it. The key is in the principle of dual systems. As pointed out, if the level of reflection does not suffice, people only note the first conjunct in (2) for (1). But, to apply Modus Tollendo Tollens, it is necessary to be aware of the three possibilities in (2). That is the only manner individuals can note that the two first conjuncts are incompatible with the second premise, that is, premise not-Y (in those conjuncts, Y is affirmed). Thereby, they can see that, in the only possibility coherent with that very premise, that is, the third one in (2), X is also negated, which leads to conclusion not-X (see, e.g., Byrne & Johnson-Laird, 2009).

Nonetheless, beyond this illustrative description, what is interesting here is related to two points. On the one hand, as indicated, a basic requirement has been provided in general to modal logic: to be consistent with the modal reading of the Aristotelian square of opposition (Fitting & Mendelsohn, 1998; it is said 'in general' because Fitting and Mendelsonh also refers to the existence of some modal logics inconsistent with the square). On the other hand, as also mentioned, the theory of mental models fulfills that condition (López-Astorga, 2020a). Continuing to follow this direction, the rest of the paper is devoted to argue that the theory of mental models, in addition to the fact that it meets that requirement, is coherent with fourteen theorems in modal logic too. Informal arguments in this regard are presented below.

Carnap's modal theorems and the theory of mental models

Carnap (1947) indicates the theorems that will be analyzed here. They are exactly theorems 'a' to 'n' in Carnap (1947: 186). As it is well known, the logical framework offered by Carnap is akin to system S5 (see, e.g., Burguess, 1999; Carnap, 1946). Therefore, it may be necessary to remind the main characteristics of this last system.

Modal logic has a basic rule as far as normal modal logics are considered. That is the rule of necessitation. It establishes that, if a formula is a theorem, that formula has to be always true, that is, the formula has to be necessary. However, S5 adds axioms to this (see, e.g., Lewis & Langford, 1932). One of them is K. K provides that a necessary conditional logically implies that, if its antecedent is necessary, then its consequent is also that. There is a simple system that only includes the rule of necessitation and this axiom. It is system K (the name is given by virtue of Kripke), and the theory of mental models has already been linked to it (see, e.g., López-Astorga, 2020b).

However, this paper is intended to go beyond. It tries to delve into other characteristics a modal system compatible with the theory of mental models could have. The analysis of Carnap's theorems below can offer clues in this sense, since, as it can be noted after reviewing those theorems, which are theorems (10) to (23), the compatibility with them can allow thinking, as pointed out, about at least a system such as S5. This last system enables to speak about, at a minimum, two more axioms: T and 5.

T is the axiom that, together with K, leads to system T. It provides that, when the antecedent of a conditional is necessary, it logically implies that antecedent holds. On the other hand, 5 indicates that, when the antecedent of a conditional is possible, it logically implies that the possibility of that antecedent is necessary.

But, in addition, Carnap's framework also refers to concepts attributed to Lewis such as those of strict implication and strict equivalence. Following Carnap (1947), strict implication can be understood as a conditional relation that is necessary, namely, as a conditional relation for which it is impossible to be false. Likewise, strict equivalence can be understood as a biconditional (in the two directions) relation that is necessary, namely, as a biconditional relation for which it is impossible to be false.

With these clarifications made, the goal of this paper can keep being developed. As mentioned, the paper tries to identify more possible characteristics of the modal logic that could be linked to the theory of mental models. Until now, as also stated, the literature has only shown that logic could be consistent with systems close to K (e.g., López-Astorga, 2020b), that is, to K or extensions of it that are not very strong. Nevertheless, as indicated too, this paper aims to go a step further and argue that the modal logic that can be related to the theses of the theory of mental models can be even as strong as S5. This, as it is claimed in other works relating the theory of mental models to system K (e.g., López-Astorga, 2020b) too, is not intended to be a serious challenge for the theory. Although the theory rejects several logical principles, the possibilities that can be linked to sentences can work in a way compatible with a system of that kind. So, it is absolutely possible to accept the general theses of the theory of mental models and, at the same time, question just one of its aspects: the idea that its framework has nothing to do with logic.

Next, the fourteen theorems are presented. A brief explanation trying to make it evident that they are compatible with the way the theory of mental models interprets possibility and necessity follows each of them. As mentioned, the theorems are (10) to (23).

(10) $NX \Rightarrow X$

Where 'N' is the operator of necessity and '$\Rightarrow$' stands for necessary implication.

Explanation: if X is necessary, it appears in all of the alternative conjuncts of the conjunction of possibilities. So, it can be affirmed that X is the case.

(11) $X \Rightarrow \Diamond X$

Where '$\Diamond$' is the operator of possibility.

Explanation: if X is the case, then it is obvious that it appears in at least one of the alternative conjuncts of the conjunction of possibilities. Therefore, it is possible.

(12) $(X \Rightarrow Y) \Rightarrow (NX \Rightarrow NY)$

Explanation: if it is correct that whenever X is true, Y must also be true, then, if X were necessary, Y would have to be necessary too. In other words, if what is between the first brackets in (12) is right, if X appeared in all of the alternative conjuncts of the conjunction of possibilities, Y would have to appear in all of those alternative conjuncts as well.

(13) $N(X \wedge Y) \Leftrightarrow (NX \wedge NY)$

Where '$\wedge$' represents conjunction and '$\Leftrightarrow$' refers to necessary equivalence.

Explanation: if the conjunction of X and Y is necessary, that means that X and Y appear in all of the alternative conjuncts of the conjunction of possibilities. But, if this is so, two points can be stated. On the one hand, X appears in all of the alternative conjuncts of the conjunction of possibilities, and hence it is necessary. On the other hand, Y appears in all of the alternative conjuncts of the conjunction of possibilities too, and hence it is necessary too. Besides, there is necessary equivalence because the inverse relation holds as well. If X is necessary, it appears in all of the alternative conjuncts of the conjunction of possibilities. Likewise, if Y is necessary, it appears in all of the alternative conjuncts of the conjunction of possibilities. However, if these two circumstances are the case, it is evident that the conjunction of X and Y also appears in all of the alternative conjuncts of the conjunction of possibilities. Accordingly, the conjunction of X and Y is necessary too.

(14) $\Diamond(X \vee Y) \Leftrightarrow \Diamond X \vee \Diamond Y$

Where '$\vee$' denotes disjunction.

Explanation: if the disjunction between X and Y is possible, that denotes that there is at a minimum an alternative conjunct of the conjunction of possibilities in which either X appears or Y appears. So, it can be said that either X is possible or Y is possible, since at least one of them appears in the alternative conjunct of the conjunction of possibilities. In the same way, if either X is possible or Y is possible, then at least one of that pair appears in an alternative conjunct of the conjunction of possibilities. Therefore, it can be said that either X or Y appears in that alternative conjunct. That in turn makes 'either X or Y' possible.

(15) $NNX \Leftrightarrow NX$

Explanation: it is important not to forget that the conjunctions of possibilities linked to sentences in the theory of mental models are conjunctions that people detect. Under this approach, what the first clause in (15) can state is that individuals know that the idea that X is necessary is necessary, that is, appears in all of the alternative conjuncts of the conjunction of possibilities. If this is correct, X is necessary, since it will be, indeed, in all of the alternative conjuncts that individuals imagine and consider for the particular sentence. Thus, the inverse argument can be accepted too. If X appears in all of the alternative conjuncts of the conjunction of possibilities, it is necessary. In addition, the idea that it is necessary (that is, the idea that it is in all of the conjuncts) will appear in all of those possibilities as well.

(16) $N\neg NX \Leftrightarrow \neg NX$

Where '¬' expresses negation.

Explanation: if the idea of the negation of that X is necessary is necessary, that is, appears in all of the alternative conjuncts of the conjunction of possibilities, that means that X is not necessary, that is, it does not appear in all of the alternative conjuncts (remember that the alternative conjuncts are in people's mind). Conversely, if X is not necessary, that idea should appear in all of those conjuncts.

(17) $\Diamond\Diamond X \Leftrightarrow \Diamond X$

Explanation: what the first clause of (17) points out is that the idea of that X is possible is possible, that is, appears in at least one of the alternative conjuncts of the conjunction of possibilities. But, if the idea that X is possible appears in a conjunct, given the way the theory of mental models deems such conjuncts, X really appears in that conjunct and is really possible. Thereby, the same can be claimed for the inverse argument. If X is possible, it appears in at least one of the alternative conjuncts. If that is correct, the idea that it appears in that conjunct also appears in that very conjunct, which revels that the idea is possible too.

(18) $\Diamond NX \Leftrightarrow NX$

Explanation: if the idea that X is necessary appears in at least one of the alternative conjuncts of the conjunction of possibilities, that means that the fact that X is necessary, that is, that it appears in all of the alternative conjuncts, can be thought and is not contradictory. So, X should be actually necessary, as that is coherent with all of the alternative conjuncts. This is true inverted as well. If X is necessary, it is clear that in at least one of the alternative conjuncts (in fact, in any of them) it can be thought that it is necessary. Therefore, that thought, the idea that X is necessary, is possible.

(19) $N\Diamond X \Leftrightarrow \Diamond X$

Explanation: if in all of the alternative conjuncts of the conjunction of possibilities it can be thought that X appears in at least one of those conjuncts, that is, that X is possible, X must be possible. Furthermore, if X appears in at least one of the alternative conjuncts of the conjunction of possibilities, people should be aware of it from all of those alternative conjuncts. This last sentence implies that the idea that X is possible is necessary.

(20) $\forall x\ (NPx) \Leftrightarrow N[\forall x\ (Px)]$

Where '∀' is the universal quantifier and 'Px' stands for the situation in which X has property P.

Explanation: if, for any x, x has property P in all of the alternative conjuncts, it is evident that, in all of the alternative conjuncts, for any x, x has property P. Conversely, if, in all of the alternative conjuncts of the conjunction of possibilities, for any x, x has property P, it is obvious that, for any x, x has property P in all of the alternative conjuncts.

(21) $\exists x\ (NPx) \Rightarrow N[\exists x\ (Px)]$

Where '∃' is the existential quantifier.

Explanation: the relation is just strict implication. If there is a x that, in all of the alternative conjuncts of the conjunction of possibilities, has property P, then, in all of those alternative conjuncts, there is a x having property P.

(22) $\exists x\ (\Diamond Px) \Leftrightarrow \Diamond[\exists x\ (Px)]$

Explanation: the relation is strict equivalence here again. If there is at least a x that, in at least one conjunct, has property P, there is at least an alternative conjunct in which there is at least a x with property P. In the same way, if there is at least an alternative conjunct in the conjunction of possibilities in which there is at least a x with property P, then there is at least a x that, in at least one alternative conjunct, has property P.

(23) $\Diamond[\forall x\ (Px)] \Rightarrow \forall x\ (\Diamond Px)$

Explanation: here is a strict implication one more time. If there is at least one alternative conjunct in the conjunction of possibilities in which, for any x, x has property P, it is clear that, for any x, there is at least a conjunct with x having property P.

Conclusions

Hence, it is possible to offer informal arguments in favor of an interesting idea: theorems (10) to (23) could keep being valid in a logic built from the essential theses of the theory of mental models. As mentioned, this tries to challenge either the theory or some of its most basic elements in no way. The only aim here has been to show that perhaps the theory of mental models is compatible with a logic. However, that logic would have to be consistent with principles such as those described above and in other papers relating the theory to aspects of modal logic (e.g., López-Astorga, 2020a, 2020b).

On the other hand, this is not the first time the theory of mental models is linked to Carnap's (1947) framework (see, e.g., López-Astorga, 2020c). Nevertheless, the relation here has been very indirect. The present paper has resorted to Carnap's work just because it indicates a number of theorems in his framework of modal logic. Those theorems have been the key to make the point of the previous analysis. However, other Carnap's (1947) particular theses have not been even considered. As mentioned, the purpose has been only to continue to argue that the theory of mental models can be coherent with elements of that logic. So, the main conclusion can be that the theory can help in the construction of a logic that somehow is nearer the real way human beings reason (or, at least, the way the theory of mental models deems as the real way they reason).

This is relevant nowadays. In the cognitive science field, it is currently difficult to find works proposing a logic in the human mind. In particular, it is hard even to identify approaches arguing in favor of a syntax of any kind in cognition. Maybe one of the last attempts in this regard was that of the mental logic theory (e.g., Braine & O'Brien, 1998). Nonetheless, in the same sense as other papers in the literature (e.g., López-Astorga, 2020a, 2020b), the previous arguments have not been intended to recover that type of works. The idea has been simply to give syntactic structures to the theory of mental models from the establishment of relations to a part of logic less explored in cognitive science: modal logic. It is true that there have been papers focusing on possible worlds, such as, for example, that of Stalnaker (1968). However, the general tendency in the theories claiming the existence of a mental logic has been to privilege rules or schemata akin to those of standard propositional calculus. Accordingly, it can be thought that the perspective of argumentations such the present one is, at a minimum to some extent, different.

As indicated in the beginning, there is no doubt that the theory of mental models considers classical standard logic not to play any role in reasoning. Given that modal logic is an extension of classical logic, this idea has been the only one raised by the theory of mental models that has been questioned in this paper. Thereby, the present study has tried to start to put the foundations of a modal logic related, to a greater or lesser degree, to reasoning. Nevertheless, it has not been claimed that logic has to reveal the exact manner individuals make inferences. One might think that this is what the theory of mental models already tries to do. So, the attempt to find a logic consistent with the results the experiments in cognitive science offer has been deemed to be enough here. In any case, it is obvious that further work needs to be made in this way. Modal logic has advanced a lot from the time Carnap proposed his philosophy. Accordingly, maybe more recent developments (see, e.g, Carnielli & Pizzi, 2008) could be confronted with the theory of mental models.

Furthermore, there are works in which the proponents of the theory of mental models argue that the theory and modal logic are incompatible. For example, to derive from (3) that one of its disjuncts is possible is a derivation modal logic does not usually allow. Nevertheless, as (4) shows, that is correct in the theory of mental models (see, e.g., Khemlani et al., 2017). So, the differences between the two frameworks are clear. But those differences reveal in even a more obvious way the sense of this paper. To find a modal logic coherent with the theory of mental models (which, given what has been said, could not match the habitual systems in that logic) would have, at least, two advantages. It would enable to note that the theory is not absolutely separated from any kind of modal logic. On the other hand, it would also allow logicians to work with a system that would be consistent with the actual manner people think. The positive consequences of this in fields such as artificial intelligence, cognitive science, or computational linguistics would be evident.

Of course, the theory of mental models is not the only cognitive theory at present. Therefore, it is possible that, although the results shown in the literature seem to indicate that its predictions are correct, this is not the case. Nonetheless, given the arguments above, if it is provisionally assumed that the theory is right, it can be said that, beyond its proponents' view, there are syntactic processes on the basis of modal logic that can be tied to its general thesis.

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